

# Sovereign Federated Learning with Byzantine-Resilient Aggregation: A Framework for Decentralized AI Infrastructure in Emerging Economies

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## Abstract

The concentration of AI infrastructure in technologically advanced nations creates barriers for emerging economies developing sovereign AI capabilities. This paper presents DSAIN (Distributed Sovereign AI Network), a federated learning framework with a unified convergence analysis that accounts for communication compression, Byzantine robustness, and optional record-level differential privacy under an honest-but-curious server. Building on prior work that studies these challenges in various combinations, DSAIN introduces FEDSOV, achieving  $\mathcal{O}(1/\sqrt{T})$  convergence while substantially reducing communication via adaptive top- $k$  compression with error feedback (e.g., retaining 22% of coordinates yields a 78% payload reduction in our CIFAR-10 setting). The BYZFED aggregation mechanism provides provable robustness against  $b < n/3$  malicious participants, and we show that the differential privacy guarantee is preserved under Byzantine behavior by post-processing (Theorem 15). Our **primary contribution** is a theoretical framework unifying Byzantine resilience, differential privacy, and communication efficiency with explicit constants. Experiments on CIFAR-10 validate the communication and robustness components and characterize key regimes (e.g., heterogeneity thresholds and behavior under stronger gradient-manipulation attacks). Privacy experiments highlight that naive federated DP-SGD requires careful noise calibration beyond standard implementations—an open challenge we characterize theoretically and leave to future work. Code and reproducibility materials are provided with the project artifacts.

## 1 Introduction

The concentration of artificial intelligence infrastructure in a few technologically advanced nations creates significant barriers for emerging economies seeking to develop sovereign AI capabilities. While federated learning (Kairouz et al., 2021) enables collaborative model training without centralizing raw data, existing frameworks face critical limitations when deployed at national scale: prohibitive communication costs, vulnerability to adversarial manipulation, and opacity in model provenance.

### 1.1 Limitations of Existing Federated Learning

Despite significant progress in federated learning research, three fundamental challenges remain unresolved for large-scale deployment:

**Communication bottleneck.** Standard federated averaging (McMahan et al., 2017) requires transmitting full model gradients each round, creating bandwidth requirements that scale linearly with model size. For modern deep networks with millions of parameters, this becomes prohibitive for geographically distributed infrastructure (Xu et al., 2021). Existing compression techniques (Tang et al., 2021; Stich and Karimireddy, 2020) either sacrifice convergence guarantees or require carefully tuned hyperparameters.

**Byzantine vulnerability.** Classical aggregation schemes (FedAvg, FedProx, SCAFFOLD) assume honest participants, leaving systems vulnerable to adversarial manipulation (Li et al., 2023).

While Byzantine-resilient methods exist (Krum (Blanchard et al., 2017), Bulyan (El Mhamdi et al., 2018), Trimmed Mean (Yin et al., 2018)), they either *(i)* provide weak theoretical guarantees under heterogeneous data, *(ii)* incur significant computational overhead, or *(iii)* cannot simultaneously ensure differential privacy.

**Provenance opacity.** Existing frameworks lack mechanisms for verifying model training history, creating challenges for regulatory compliance, public accountability, and trust in AI systems deployed in critical infrastructure (Xu et al., 2022). This is particularly problematic for public AI infrastructure where model integrity must be independently auditable.

## 1.2 Our Contributions

This paper presents DSAIN (Distributed Sovereign AI Network), a federated learning framework that addresses all three challenges simultaneously with both theoretical guarantees and empirical validation. Our contributions are organized as follows:

### PRIMARY CONTRIBUTIONS (Core Algorithmic Innovations):

1. **Joint analysis under Byzantine attacks and record-level differential privacy.** We provide a convergence analysis proving  $\mathcal{O}(1/\sqrt{T})$  convergence for non-convex objectives under *both*  $b < n/3$  Byzantine participants and  $(\epsilon, \delta)$ -differential privacy via a clipped Gaussian mechanism (Theorems 11, 12, 15). Under our threat model, the DP guarantee is preserved under Byzantine behavior by post-processing: once the mechanism output is DP, subsequent deterministic filtering/weighting cannot worsen privacy.
2. **Communication-efficient Byzantine-resilient aggregation with tighter bounds.** We develop BYZFED, combining geometric median filtering with adaptive top- $k$  compression. Unlike prior compression methods (Tang et al., 2021; Stich and Karimireddy, 2020) that assume honest participants, our analysis (Theorem 16) characterizes the compression-robustness trade-off and shows that compression error and Byzantine perturbations compose favorably with explicit constants (Section 4).
3. **Rigorous theoretical analysis with explicit constants.** We provide complete convergence proofs under realistic assumptions (smoothness, bounded variance, heterogeneity) with explicit dependence on all problem parameters: compression ratio  $k$ , Byzantine fraction  $b$ , privacy budget  $\epsilon$ , and heterogeneity  $\zeta$ . All proofs appear in the supplementary materials with full derivations.

### SECONDARY CONTRIBUTIONS (System and Empirical Validation):

4. **Lightweight audit trail (optional component).** We describe a hash-chained provenance prototype for deployment scenarios requiring auditable training history (Section 5). This is orthogonal to our core algorithmic contributions and can be omitted when auditability is not required.
5. **Empirical evaluation and sanity tooling.** We evaluate DSAIN on CIFAR-10 under non-IID partitioning and Byzantine behavior, and provide a fast sanity configuration to support repeatable regression checks (Section 6 and Section 8).

## 1.3 Key Insight: Why Existing Methods Fail

The core technical challenge is that communication efficiency, Byzantine resilience, and differential privacy interact adversarially:

- Gradient compression introduces sparsity, which Byzantine adversaries can exploit by targeting uncompressed coordinates

- Byzantine-resilient aggregation (e.g., geometric median) is non-linear, complicating privacy analysis
- Differential privacy noise must be calibrated to post-aggregation sensitivity, but Byzantine attacks can inflate sensitivity

Prior work studies these challenges in various combinations (Tang et al., 2021; Li et al., 2023; Wei et al., 2020), but unified analyses providing *simultaneous* guarantees across compression, Byzantine robustness, and differential privacy (under an explicitly stated threat model) remain limited. DSAIN addresses this through a carefully designed aggregation pipeline: compression with error feedback (preserving convergence), geometric median filtering (Byzantine resilience), and a clipped Gaussian mechanism (differential privacy with post-processing).

#### 1.4 Scope and Focus

This paper’s **primary focus** is the algorithmic challenge of achieving Byzantine resilience under differential privacy constraints while maintaining communication efficiency. Our core technical contributions (Contributions 1-2) provide:

- **Novel joint analysis:** Convergence proof under *both* Byzantine attacks ( $b < n/3$ ) and record-level differential privacy  $((\epsilon, \delta)\text{-DP})$  simultaneously, with  $\mathcal{O}(1/\sqrt{T})$  rate (Theorems 11, 7, 15).
- **Tighter bounds:** Compression analysis under Byzantine model (Theorem 16) quantifying the communication-robustness tradeoff without sacrificing robustness.
- **Formal composition:** Post-processing argument showing that, under our threat model, Byzantine behavior cannot worsen the privacy guarantee beyond that of the underlying DP mechanism (Supplementary Section 6).

The audit-trail provenance prototype (Contribution 3, Section 5) is a **secondary contribution** demonstrating how the training process can be instrumented for auditability. It is orthogonal to our core algorithmic innovations and can be omitted in settings where provenance is not required.

**Out of scope:** This work does not address blockchain consensus optimization, cryptographic primitive design, or hardware-specific optimizations. We assume a permissioned federated learning setting with authenticated participants (Sybil attacks addressed via external identity verification).

#### 1.5 Organization

The remainder of this paper is organized as follows. Section 2 reviews related work and positions our contributions. Section 3 formalizes the problem setting, threat model, and assumptions. Section 4 presents the FEDSOV algorithm and BYZFED aggregation with theoretical analysis. Section 5 describes an audit-trail provenance prototype. Section 6 presents experimental results including ablation studies and statistical validation. Section 7 describes an audit-trail case study. Section 9 concludes with limitations and future work.

## 2 Related Work

We review related work across four key areas and position our contributions relative to representative recent literature.

### 2.1 Federated Learning

Federated learning was introduced by McMahan et al. (2017) as FedAvg, enabling collaborative model training without centralizing data. A comprehensive survey by Kairouz et al. (2021) subsequently consolidated the field. Subsequent work has addressed communication efficiency (Xu et al.,

2021; Li et al., 2020a), systems heterogeneity (Li et al., 2020b), and statistical heterogeneity from non-i.i.d. data (Zhu et al., 2021; Karimireddy et al., 2020).

**Communication compression.** Gradient compression techniques include sparsification (Tang et al., 2021), quantization (Reisizadeh et al., 2021), and error feedback mechanisms (Seide et al., 2014; Stich and Karimireddy, 2020). Recent work introduces adaptive compression under dynamic bandwidth: Guo et al. (2024) propose AdapComFL with sketch-based compression, while Zhang et al. (2025) develop HCEF combining computation and communication compression. Liu et al. (2025a) apply knowledge distillation with ternary quantization for dual-layer compression. Notably, Rammal et al. (2024) address communication compression for Byzantine-robust learning with improved convergence rates. However, these methods do not jointly address Byzantine threats with privacy guarantees.

**Handling non-IID data.** Data heterogeneity remains a fundamental challenge (Karimireddy et al., 2020). A comprehensive 2024 survey (Li et al., 2024) provides taxonomy and metrics for non-IID scenarios. Wang et al. (2024) show clustered federated learning improves convergence under heterogeneity, while Garcia-Diaz et al. (2025) demonstrate synthetic data sharing reduces divergence by 50% in worst-performing nodes. Our FEDSOV algorithm explicitly accounts for heterogeneity through bounded dissimilarity assumptions (Assumption 4).

## 2.2 Byzantine-Resilient Distributed Learning

Byzantine fault tolerance in distributed learning has received considerable attention following Li et al. (2023), who surveyed robust aggregation methods. Classical approaches include Krum (Blanchard et al., 2017), Bulyan (El Mhamdi et al., 2018), coordinate-wise median (Yin et al., 2018), and trimmed mean (Karimireddy et al., 2021).

**Recent advances (2024-2025).** Allouah et al. (2024) (ICML 2024) analyze the impact of client subsampling and local steps on Byzantine robustness, showing prior work's theoretical guarantees often do not hold under realistic FL settings. Singh and Vaswani (2024) propose Byzantine-resilient few-shot learning. Zhao et al. (2025) introduce BRACE, a Byzantine-robust ring-all-reduce algorithm for distributed FL. Chen et al. (2025) develop Byzantine-resilient methods via distributed optimization. Rammal et al. (2024) provide early algorithms combining compression with Byzantine robustness, achieving tighter convergence bounds.

**Privacy-robustness trilemma.** Allouah et al. (2023) formally characterize the fundamental tradeoffs between privacy, robustness, and utility in distributed learning, showing that achieving all three simultaneously requires careful algorithmic design. Our work builds on this insight by providing explicit convergence guarantees under joint Byzantine and DP constraints.

**Limitations of prior work.** Existing Byzantine-resilient methods face key limitations: *(i)* weak guarantees under heterogeneous data (Allouah et al., 2024), *(ii)* computational overhead of repeated median/distance computations, *(iii)* incompatibility with differential privacy due to non-linear aggregation (So et al., 2022). Most recently, Xia et al. (2025) address all three aspects (compression, robustness, and DP) in Fed-DPRoc; our BYZFED mechanism provides a complementary approach through filtering-based geometric median with formal privacy composition analysis (Appendix B.6).

## 2.3 Privacy-Preserving Federated Learning

Differential privacy (Dwork et al., 2020) provides rigorous privacy guarantees for machine learning. In federated settings, Wei et al. (2020) analyzed DP-FedAvg, while Girgis et al. (2021) studied privacy amplification from subsampling.

**Recent improvements (2024-2025).** Recent work explores tighter privacy accounting and noise calibration (e.g., privacy loss distribution and FFT-based techniques) (Hu et al., 2024), user-level DP in cross-silo federated settings (Kato et al., 2024), and communication-efficient DP via

Table 1: Comparison with representative federated learning methods. DSAIN combines communication compression, Byzantine resilience, and differential privacy within a single framework.

Method	Compression	Byzantine Resilience	Privacy (DP)	Provenance	Convergence Proof
FedAvg (McMahan et al., 2017)	×	×	×	×	✓
FedProx (Li et al., 2020b)	×	×	×	×	✓
SCAFFOLD (Karimireddy et al., 2020)	×	×	×	×	✓
AdapComFL (Guo et al., 2024)	✓	×	×	×	✓
FedDT (Liu et al., 2025a)	✓	×	×	×	✓
Krum (Blanchard et al., 2017)	×	✓	×	×	Limited
Bulyan (El Mhamdi et al., 2018)	×	✓	×	×	Limited
BRACE (Zhao et al., 2025)	×	✓	×	×	✓
DP-FedAvg (Wei et al., 2020)	×	×	✓	×	✓
ULDP-FL (Kato et al., 2024)	×	×	✓	×	✓
FedSA-LoRA-DP (Kim et al., 2025)	✓	×	✓	×	✓
So et al. (2022)	×	✓	Secure Agg.	×	Limited
Data and Diggavi (2021)	×	✓	✓	×	Limited
Fed-DPRoc (Xia et al., 2025)	✓	✓	✓	×	✓
<b>DSAIN (ours)</b>	✓	✓	✓	✓	✓

selective low-rank adaptation (Kim et al., 2025). Surveys summarize advances including adaptive clipping and deployment considerations (Demelius et al., 2025).

**Challenges in federated DP.** Recent empirical studies show that integrating DP can incur substantial utility degradation at small privacy budgets (Liu et al., 2025b). Non-IID data, gradient clipping, and noise injection interact to slow convergence in asynchronous settings. Our framework addresses this through adaptive clipping and composition-aware noise calibration.

## 2.4 Blockchain for Machine Learning

Blockchain technology has been applied to machine learning for model marketplaces (Zhang et al., 2021b), training verification (Xu et al., 2022), and incentive mechanisms (Allen et al., 2023). In federated learning contexts, Qu et al. (2022) proposed blockchain-based FL architectures, while Li et al. (2020c) addressed data sharing.

Our approach focuses on lightweight provenance for federated learning via hash-chained commitments over round metadata. Storing commitments in a ledger and adding richer cryptographic proofs are natural extensions, but are outside the scope of this work.

## 2.5 Positioning Our Contributions

Table 1 compares DSAIN with representative methods across key desiderata.

**Scope clarifications and comparability.** Several recent works study subsets or alternative combinations of the constraints considered here, but they differ in threat model, DP notion, compression operator, and heterogeneity assumptions. In particular, our guarantees are for record-level DP under an honest-but-curious server (Section 3); this is not directly comparable to client-level DP analyses. Our compression analysis focuses on top- $k$  sparsification with error feedback (Theorem 16); extensions to other compressors (e.g., quantization or sketching) would require separate analysis.

**Key differentiators:**

- **Unified framework:** DSAIN combines communication efficiency, Byzantine resilience, and differential privacy within a single analysis.
- **Tighter guarantees:** Our convergence analysis (Theorems 11, 12) explicitly accounts for compression error, Byzantine perturbations, DP noise, and heterogeneity with explicit constants.
- **Joint privacy-Byzantine analysis:** We show that, under our threat model, privacy guarantees are preserved under Byzantine behavior by post-processing (Theorem 15, Appendix F), complementing prior analyses (Data and Diggavi, 2021).
- **Empirical validation:** We evaluate against a suite of Byzantine attack strategies, including optimization-based methods (Baruch et al., 2019; Shejwalkar and Houmansadr, 2021), to characterize robustness (Section 6.4).
- **Auditability prototype:** A hash-chained audit trail enables integrity checks over round metadata, supporting post-hoc auditing workflows.

Prior work studies these challenges in various combinations; DSAIN focuses on an end-to-end design that composes compression, Byzantine resilience, and record-level DP, and optionally adds a lightweight audit trail for tamper-evident provenance.

**Why a unified framework?** While the scope of addressing efficiency, robustness, privacy, and provenance simultaneously may appear broad, we argue this integration is both *necessary* and *technically non-trivial*:

1. **Real-world requirements are joint:** Practical federated deployments (e.g., healthcare consortia, financial networks) simultaneously require communication efficiency (bandwidth constraints), Byzantine resilience (institutional failures), and privacy guarantees (regulatory compliance). Addressing these in isolation produces systems that fail under realistic threat models.
2. **Interaction effects are non-additive:** As shown by Allouah et al. (2023), there exist fundamental tradeoffs between privacy, robustness, and utility. Our theoretical contribution (Theorem 15, Appendix B.6) proves that these constraints compose favorably in our design—Byzantine adversaries cannot exploit the aggregation mechanism to violate DP guarantees. This is a non-obvious property that requires careful algorithmic design.
3. **Compression interacts with both privacy and robustness:** Sparse gradients affect both the DP noise calibration (Remark 10) and the geometric median computation. Our error feedback mechanism ensures compression does not amplify Byzantine perturbations.
4. **Modular design enables practical deployment:** Despite the unified analysis, each component (compression, Byzantine defense, DP, provenance) can be enabled or disabled independently based on deployment requirements, as demonstrated in our ablation studies (Section 6.9).

### 3 Problem Formulation

#### 3.1 Federated Learning Setting

We consider a federated learning setting with  $n$  participants (e.g., regional data centers, institutions) coordinated by a central server. Each participant  $i \in [n]$  holds a local dataset  $\mathcal{D}_i$  drawn from a potentially distinct distribution  $\mathcal{P}_i$ . The goal is to learn a global model  $\mathbf{w} \in \mathbb{R}^d$  minimizing:

$$F(\mathbf{w}) = \sum_{i=1}^n p_i F_i(\mathbf{w}), \quad F_i(\mathbf{w}) = \mathbb{E}_{\xi \sim \mathcal{P}_i} [f(\mathbf{w}; \xi)] \quad (1)$$

where  $p_i \geq 0$  with  $\sum_i p_i = 1$  are importance weights (typically  $p_i = |\mathcal{D}_i| / \sum_j |\mathcal{D}_j|$ ) and  $f(\mathbf{w}; \xi)$  is the loss on data point  $\xi$ .

### 3.2 Threat Model

We consider an adversarial model where up to  $b$  of the  $n$  participants may be Byzantine, capable of sending arbitrary messages to the server. Let  $\mathcal{H} \subset [n]$  denote the set of honest participants with  $|\mathcal{H}| \geq n - b$ . Byzantine participants may collude and have full knowledge of the protocol, including honest participants' updates.

**Assumption 1 (Byzantine Fraction)** *The number of Byzantine participants satisfies  $b < n/3$ .*

This bound is standard for geometric median-based aggregation (Li et al., 2023). We note that alternative methods can tolerate up to  $b < n/2$  Byzantine workers through bucketing (Karimireddy et al., 2022) or communication-compressed Byzantine-robust learning (Rammal et al., 2024); however, these require additional assumptions or incur higher computational overhead.

### 3.3 Privacy Model

We consider an **honest-but-curious server** that faithfully executes the aggregation protocol but may attempt to infer information from observed updates. We provide **record-level differential privacy** for each honest participant's local data—i.e., privacy is defined with respect to individual data records within a participant's dataset, not participant-level membership. Formally, for any participant  $i \in \mathcal{H}$  and neighboring datasets  $\mathcal{D}_i, \mathcal{D}'_i$  differing in one record:

$$\mathbb{P}[\text{Output} \in S | \mathcal{D}_i] \leq e^\epsilon \mathbb{P}[\text{Output} \in S | \mathcal{D}'_i] + \delta \quad (2)$$

for all measurable sets  $S$ .

**Comparison with prior work.** Allouah et al. (2023) study a stronger threat model with a *curious server* and *client-level DP* (privacy over which clients participate). Our model is weaker: honest-but-curious server and record-level DP. This distinction is important for positioning our guarantees—our bounds are not directly comparable to those in (Allouah et al., 2023).

### 3.4 Assumptions on Objective

**Assumption 2 (Smoothness)** *Each  $F_i$  is  $L$ -smooth, i.e., for all  $\mathbf{w}, \mathbf{v}$ ,*

$$\|\nabla F_i(\mathbf{w}) - \nabla F_i(\mathbf{v})\| \leq L \|\mathbf{w} - \mathbf{v}\|.$$

**Assumption 3 (Bounded Variance)** *The stochastic gradients have bounded variance: for all  $i$ ,*

$$\mathbb{E}[\|\nabla f(\mathbf{w}; \xi) - \nabla F_i(\mathbf{w})\|^2] \leq \sigma^2.$$

**Assumption 4 (Bounded Heterogeneity)** *The local objectives are  $\zeta$ -similar: for all  $i$  and  $\mathbf{w}$ ,*

$$\|\nabla F_i(\mathbf{w}) - \nabla F(\mathbf{w})\|^2 \leq \zeta^2.$$

For convergence to stationary points, we require:

**Assumption 5 (Bounded Gradient)** *There exists  $G > 0$  such that for all  $i$  and  $\mathbf{w}$ ,*

$$\|\nabla F_i(\mathbf{w})\| \leq G.$$

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**Algorithm 1** FEDSOV: Sovereign Federated Learning

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**Require:** Initial model  $\mathbf{w}^0$ , learning rate  $\eta$ , local epochs  $E$ , compression operator  $\mathcal{C}$ , rounds  $T$

- 1: **for**  $t = 0, 1, \dots, T-1$  **do**
- 2:   Server samples participating clients  $\mathcal{S}^t \subseteq [n]$  with  $|\mathcal{S}^t| = K$
- 3:   Server broadcasts  $\mathbf{w}^t$  to clients in  $\mathcal{S}^t$
- 4:   **for** each client  $i \in \mathcal{S}^t$  **in parallel do**
- 5:      $\mathbf{w}_i^{t,0} \leftarrow \mathbf{w}^t$
- 6:     **for**  $k = 0, 1, \dots, E-1$  **do**
- 7:       Sample mini-batch  $\xi_i^{t,k}$  from  $\mathcal{D}_i$
- 8:        $\mathbf{g}_i^{t,k} \leftarrow \nabla f(\mathbf{w}_i^{t,k}; \xi_i^{t,k}) + \mathbf{m}_i^{t,k}$  {Momentum}
- 9:        $\mathbf{w}_i^{t,k+1} \leftarrow \mathbf{w}_i^{t,k} - \eta \mathbf{g}_i^{t,k}$
- 10:      **end for**
- 11:      $\Delta_i^t \leftarrow \mathbf{w}_i^{t,E} - \mathbf{w}^t$
- 12:      $\hat{\Delta}_i^t \leftarrow \mathcal{C}(\Delta_i^t)$  {Compress to top- $k$  coordinates}
- 13:      $\mathcal{I}_i^t \leftarrow \text{supp}(\hat{\Delta}_i^t)$  {Get indices of non-zero coordinates}
- 14:      $\tilde{\Delta}_i^t \leftarrow \hat{\Delta}_i^t + \text{SparseNoise}(\sigma_{\text{DP}}, \mathcal{I}_i^t)$  {Add noise only to transmitted coords}
- 15:     Client  $i$  sends  $(\mathcal{I}_i^t, \tilde{\Delta}_i^t[\mathcal{I}_i^t])$  to server {Sparse transmission}
- 16:   **end for**
- 17:    $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t + \text{BYZFED}(\{\tilde{\Delta}_i^t\}_{i \in \mathcal{S}^t})$  {Robust aggregation}
- 18: **end for**
- 19: **return**  $\mathbf{w}^T$

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## 4 Algorithms and Analysis

### 4.1 The FedSov Algorithm

Our FEDSOV algorithm extends FedAvg with three key modifications: (1) adaptive gradient compression, (2) momentum-based local updates, and (3) Byzantine-resilient aggregation.

#### 4.1.1 ADAPTIVE GRADIENT COMPRESSION

We employ a top- $k$  sparsification operator with error feedback:

$$\mathcal{C}(\mathbf{x}) = \text{Top}_k(\mathbf{x}), \quad \text{Top}_k(\mathbf{x})_j = \begin{cases} x_j & \text{if } |x_j| \geq |x|_{(k)} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where  $|x|_{(k)}$  denotes the  $k$ -th largest absolute value. The compression error is accumulated for the next round:

$$\mathbf{e}_i^{t+1} = \Delta_i^t - \mathcal{C}(\Delta_i^t + \mathbf{e}_i^t) \quad (4)$$

**Lemma 6 (Compression Contraction)** *For  $k = \gamma d$  with  $\gamma \in (0, 1]$ , the top- $k$  operator satisfies:  $\mathbb{E}[\|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2] \leq (1 - \gamma) \|\mathbf{x}\|^2$*

**Proof** Let  $\mathbf{x} \in \mathbb{R}^d$  and denote by  $|x|_{(1)} \geq |x|_{(2)} \geq \dots \geq |x|_{(d)}$  the components sorted by magnitude. The top- $k$  operator retains the  $k$  largest components, so:

$$\|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2 = \sum_{j=k+1}^d |x|_{(j)}^2 \quad (5)$$

$$\leq \frac{d-k}{d} \sum_{j=1}^d |x|_{(j)}^2 = (1 - \gamma) \|\mathbf{x}\|^2 \quad (6)$$

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**Algorithm 2** BYZFED: Byzantine-Resilient Aggregation

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**Require:** Updates  $\{\Delta_i\}_{i=1}^K$ , reputation scores  $\{r_i\}_{i=1}^K$ , filtering threshold  $\tau$ , reputation decay  $\rho$

- 1: Compute geometric median:  $\mu \leftarrow \operatorname{argmin}_{\mathbf{z}} \sum_{i=1}^K \|\Delta_i - \mathbf{z}\|$
- 2: Compute distances:  $d_i \leftarrow \|\Delta_i - \mu\|$  for each  $i$
- 3: Compute median distance:  $m_d \leftarrow \operatorname{median}(\{d_i\})$
- 4: Compute MAD scale:  $\text{MAD} \leftarrow \operatorname{median}(\{|d_i - m_d|\})$ ,  $\hat{\sigma} \leftarrow 1.4826 \cdot \text{MAD}$
- 5: Filter:  $\mathcal{F} \leftarrow \{i : d_i \leq m_d + \tau \cdot \hat{\sigma}\}$  (if  $\hat{\sigma} = 0$ , set  $\mathcal{F} = \{1, \dots, K\}$ )
- 6: Update reputations:  $r_i \leftarrow \rho \cdot r_i + (1 - \rho) \cdot \mathbf{1}[i \in \mathcal{F}]$
- 7: Compute weights:  $w_i \propto r_i \cdot \mathbf{1}[i \in \mathcal{F}]$
- 8: **return**  $\sum_{i \in \mathcal{F}} w_i \Delta_i$

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where the inequality follows from the fact that the discarded components have the smallest magnitudes.

## 4.2 The ByzFed Aggregation Mechanism

Our Byzantine-resilient aggregation combines geometric median filtering with reputation weighting:

**Hyperparameters.** The parameter  $\tau$  controls the MAD-based outlier cutoff (larger  $\tau$  filters fewer updates);  $\rho$  controls the reputation exponential moving average (larger  $\rho$  changes reputations more slowly). We use  $\rho = 0.9$  in our reference implementation; the CIFAR-10 experiments in Section 6 use a geometric-median defense (i.e., without MAD filtering/reputation weighting) to match the evaluated artifacts.

**Theorem 7 (Byzantine Resilience)** *Under Assumption 1, if  $|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1$ , the output of BYZFED satisfies:*

$$\|\text{BYZFED}(\{\Delta_i\}) - \bar{\Delta}_{\mathcal{H}}\|^2 \leq C \cdot \frac{b}{n-b} \cdot \sigma_{\mathcal{H}}^2 \quad (7)$$

where  $\bar{\Delta}_{\mathcal{H}} = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \Delta_i$  and  $\sigma_{\mathcal{H}}^2 = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \|\Delta_i - \bar{\Delta}_{\mathcal{H}}\|^2$ .

**Proof [Proof Sketch]** The geometric median is a robust estimator with breakdown point 1/2. By concentration properties of honest updates under our assumptions, the filtering step removes at most  $O(b)$  honest participants with high probability. The weighted average over the filtered set then inherits robustness guarantees from the median filtering. Full proof in Appendix B.1.

**Remark 8 (Reputation Mechanism and Participation Bias)** *The dynamic reputation weighting (Line 5 of Algorithm 2) with decay parameter  $\rho \in (0, 1)$  introduces an exponential moving average of client reliability. We analyze its impact on convergence:*

**Convergence guarantee.** *The reputation weights  $w_i$  are bounded: after  $t$  rounds,  $r_i \in [\rho^t, 1]$  for any client  $i$ . Since weights are normalized ( $\sum w_i = 1$ ), the weighted average remains in the convex hull of honest updates. Thus, Theorem 7 holds with an additional term  $\mathcal{O}((1 - \rho^T)/\rho)$  capturing reputation-induced variance.*

**Participation bias.** *Honest clients with high gradient variance may be temporarily filtered, reducing their reputation. We bound this bias: with probability  $1 - \delta$ , an honest client  $i$  satisfies  $r_i^t \geq \rho^{T\delta_i}$  where  $\delta_i$  is the fraction of rounds where  $i$  is incorrectly filtered. Under Assumption 3,  $\delta_i \leq O(\log(n/\delta)/T)$ , ensuring honest clients maintain reputation  $\Omega(1)$  over the training horizon.*

**Choice of  $\rho = 0.9$ .** *This balances responsiveness (quickly downweighting detected attackers) with stability (not penalizing honest clients for occasional outlier updates). See Appendix B.7 for full analysis.*

### 4.3 Differential Privacy Mechanism

**Sparse Noise for Communication Efficiency.** A naive approach would add dense Gaussian noise  $\mathcal{N}(0, \sigma_{DP}^2 \mathbf{I}_d)$  to the compressed update, negating communication savings. Instead, we add noise *only to the  $k$  transmitted coordinates*:

$$\tilde{\Delta}_i^t[\mathcal{I}_i^t] = \hat{\Delta}_i^t[\mathcal{I}_i^t] + \mathcal{N}(0, \sigma_{DP}^2 \mathbf{I}_k), \quad \text{where } \mathcal{I}_i^t = \text{supp}(\mathcal{C}(\Delta_i^t)) \quad (8)$$

This sparse noise mechanism preserves communication efficiency: clients transmit only  $k$  coordinates (indices + values), maintaining the  $k/d$  compression ratio. The privacy guarantee holds because the server only observes the noisy sparse vector; the unselected  $d - k$  coordinates are never transmitted and thus require no noise.

The noise scale  $\sigma_{DP}$  is determined by the privacy budget:

**Theorem 9 (Privacy Guarantee)** *With gradient clipping bound  $C$  and noise scale  $\sigma_{DP} = \frac{C\sqrt{2\ln(1.25/\delta)}}{\epsilon}$ , each round provides  $(\epsilon, \delta)$ -differential privacy. After  $T$  rounds with subsampling probability  $q = K/n$ , the composition satisfies  $(\epsilon', \delta')$ -DP with:*

$$\epsilon' = \sqrt{2T\ln(1/\delta')} \cdot q\epsilon + Tq\epsilon(e^\epsilon - 1) \quad (9)$$

for  $\delta' > 0$ .

**Remark 10 (Sparse Noise Preserves Communication Efficiency)** *A critical design choice in FEDSOV is that privacy noise is added **only to the  $k$  transmitted coordinates**, not to all  $d$  dimensions. Specifically, if  $\mathcal{I}_i^t = \text{supp}(\mathcal{C}(\Delta_i^t))$  denotes the support of the compressed gradient, then:*

- Client  $i$  transmits:  $(j, \tilde{\Delta}_i^t[j])$  for  $j \in \mathcal{I}_i^t$
- Communication cost:  $\mathcal{O}(k)$  not  $\mathcal{O}(d)$
- Compression ratio preserved:  $k/d$  even with DP

*This is in contrast to a naive approach where dense noise  $\mathcal{N}(0, \sigma_{DP}^2 \mathbf{I}_d)$  would be added to the full vector, requiring transmission of all  $d$  coordinates and negating compression benefits. Our sparse noise mechanism maintains communication efficiency while providing equivalent privacy guarantees for the transmitted coordinates.*

### 4.4 Convergence Analysis

We now establish convergence guarantees for FEDSOV.

**Theorem 11 (Non-Convex Convergence)** *Under Assumptions 2–5, with learning rate  $\eta = \mathcal{O}(1/\sqrt{T})$ , local epochs  $E$ , and participation rate  $K/n$ , FEDSOV achieves:*

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\|\nabla F(\mathbf{w}^t)\|^2] \leq \mathcal{O}\left(\frac{1}{\sqrt{T}}\right) + \mathcal{O}\left(\frac{E\zeta^2}{K}\right) + \mathcal{O}(\sigma_{DP}^2) \quad (10)$$

**Proof [Proof Sketch]** We decompose the error into three terms: (1) optimization error from finite iterations, (2) client drift from local updates with heterogeneous data, and (3) privacy noise variance. The compression error is controlled via error feedback (Lemma 6). Byzantine error is bounded by Theorem 7. Full proof in supplementary materials.

**Theorem 12 (Strongly Convex Convergence)** *If additionally  $F$  is  $\mu$ -strongly convex, with  $\eta = \mathcal{O}(1/(\mu T))$ :*

$$\mathbb{E}[\|\mathbf{w}^T - \mathbf{w}^*\|^2] \leq \mathcal{O}\left(\frac{1}{T}\right) + \mathcal{O}\left(\frac{E\zeta^2}{\mu^2 K}\right) + \mathcal{O}\left(\frac{\sigma_{DP}^2}{\mu^2}\right) \quad (11)$$

**Remark 13** *The convergence rates match those of centralized SGD up to terms from heterogeneity and privacy, which are irreducible in this setting. The communication cost is reduced by a factor of  $1/\gamma$  through compression, where  $\gamma$  is the compression ratio.*

## 5 Audit Trail and Provenance (Prototype)

We briefly describe a lightweight provenance layer for deployment scenarios requiring auditable training history. This component is **orthogonal** to our core algorithmic contributions and can be omitted in settings where provenance is not required.

**Threat model and goals.** The provenance layer is designed to make *post-hoc manipulation* of training history detectable, e.g., rewriting which clients participated in a round, changing reported hyperparameters, or swapping the committed aggregate/update checksum. It supports accountability and audit workflows in deployments where an external party may later need to validate that the reported training record is internally consistent. It does *not* prevent a malicious coordinator from selecting bad participants or choosing harmful configurations; rather, it provides tamper-evidence for the recorded metadata.

**Protocol sketch.** Each training round  $t$  produces a cryptographic commitment (hash) over round metadata (e.g., round index, participating client IDs, configuration hash, and a checksum of the aggregated model/update). Concretely, we form a chain

$$h_t = H(h_{t-1} \parallel t \parallel \text{cfg}_t \parallel \text{IDSt} \parallel \text{digest}(\Delta_t)) \quad (12)$$

where  $H(\cdot)$  is a collision-resistant hash,  $\text{cfg}_t$  summarizes the round configuration, and  $\text{digest}(\Delta_t)$  is a checksum of the aggregate/update. In our code release, we implement this as a lightweight, hash-chained audit log with optional participant attestations (e.g., signatures over  $h_t$ ). A production deployment could store  $h_t$  (and optionally the signed statements) in an append-only permissioned ledger.

**Overhead.** The audit log stores  $\mathcal{O}(1)$ -sized records per round (constant-size digests and identifiers), resulting in  $\mathcal{O}(T)$  total storage over  $T$  rounds. We do not report end-to-end performance measurements for a distributed ledger in this work.

**Security.** Under collision-resistant hashing, the probability of accepting a forged history without detection is negligible.

**Theorem 14 (Tamper-evident provenance)** *Under the collision resistance of the hash function, the probability of accepting a tampered training-history log that passes verification is negligible in the security parameter.*

A proof appears in Appendix B.5. Our implementation includes placeholders for richer cryptographic proofs (e.g., zero-knowledge proofs), but we do not claim a production-ready ZK integration or provide measured ledger overhead in this work.

### 5.1 Key Theoretical Results

We highlight the core theoretical contributions that distinguish DSAIN from prior work. Full proofs appear in Appendices A–G.

**Theorem 15 (Joint Privacy-Byzantine Convergence)** *Under Assumptions 2–4, with  $b < n/3$  Byzantine participants and  $(\epsilon, \delta)$ -differential privacy via clipped Gaussian mechanism, FEDSOV with BYZFED aggregation achieves:*

$$\mathbb{E}[\|\nabla F(\bar{\mathbf{w}})\|^2] \leq \frac{2(F(\mathbf{w}^0) - F^*)}{\eta T} + \mathcal{O}\left(\frac{\sigma^2}{\sqrt{T}} + \frac{\zeta^2}{\sqrt{T}} + \frac{b\sigma^2}{n\sqrt{T}} + \frac{L\sigma_{DP}^2}{\epsilon^2\sqrt{T}}\right) \quad (13)$$

where  $\bar{\mathbf{w}}$  is uniformly sampled from  $\{\mathbf{w}^t\}_{t=0}^{T-1}$ ,  $\sigma^2$  is gradient variance,  $\zeta^2$  is heterogeneity, and  $\sigma_{DP}$  is DP noise scale.

**Key insight:** *We provide a convergence proof under both Byzantine attacks and differential privacy. Under our threat model, the privacy guarantee is preserved under Byzantine behavior by post-processing (Supplementary Section 6), and DP noise does not amplify Byzantine perturbations beyond  $\mathcal{O}(1/\sqrt{T})$  contribution.*

**Theorem 16 (Compression Under Byzantine Model)** *The top- $k$  compression operator with  $k = \gamma d$  coordinates retained achieves:*

$$\mathbb{E}[\|\mathbf{g} - \mathcal{C}(\mathbf{g})\|^2] \leq (1 - \gamma) \|\mathbf{g}\|^2 \quad (14)$$

even when  $b < n/3$  participants send adversarially chosen updates. The compression error and Byzantine perturbation terms sum as  $\mathcal{O}((1 - \gamma) + b/n)$  in the convergence bound.

**Key insight:** *Unlike prior compression methods (Tang et al., 2021; Stich and Karimireddy, 2020) assuming honest participants, our analysis proves compression and Byzantine resilience compose favorably without multiplicative degradation.*

**Corollary 17 (Communication-Robustness Tradeoff)** *For  $k = 0.22d$  (retaining 22% of coordinates), DSAIN achieves 78% communication reduction while maintaining  $\mathcal{O}(1/\sqrt{T})$  convergence under  $b < n/3$  Byzantine attacks.*

#### Comparison with prior work:

- Data and Diggavi (2021): Byzantine + DP, but analysis does not prove privacy immunity to Byzantine manipulation (assumes sequential composition).
- Tang et al. (2021): Compression + convergence, but assumes honest participants.
- Karimireddy et al. (2020): Heterogeneity + convergence, but no Byzantine resilience or privacy.
- **DSAIN (ours):** Convergence proof accounting for compression + Byzantine + DP + heterogeneity *simultaneously* with explicit constants.

## 6 Experimental Results

We evaluate DSAIN on CIFAR-10 under non-IID partitioning, Byzantine behavior, and optional differential privacy settings, and we include architectural variants to test generality. For clarity, we organize the primary evaluation into experiments E1–E12 (Table 2). For broader empirical coverage (architecture and dataset breadth), we additionally provide rebuttal addendum experiments (E13–E14) summarized in Appendix B.

### 6.1 Experimental Setup

**Dataset:** CIFAR-10 with 60,000 images (50,000 training, 10,000 test) partitioned across 20 clients using Dirichlet allocation with varying concentration parameters  $\alpha \in \{0.01, 0.1, 0.5, 1.0, 10.0\}$  to simulate different levels of data heterogeneity.

**Models:** ResNet18 (11.17M parameters) for the primary suite (E1–E11) and ViT-Tiny for architectural generalization (E12).

**Federated Learning Configuration:**

- Number of clients:  $n = 20$
- Participation rate: 25% (5 clients per round)
- Communication rounds:  $T = 500$
- Local epochs:  $E = 5$
- Batch size:  $B = 32$
- Optimizer: SGD with momentum 0.9 and weight decay  $10^{-4}$
- Client learning rate:  $\eta = 0.01$
- Compression: top- $k$  sparsification retaining 22% of coordinates (DSAIN); FedAvg baselines use 100% (no compression)
- Byzantine defense (E3, E10, E11): geometric median aggregation with 10 Weiszfeld iterations
- DP settings (only in E9):  $\epsilon = 2.0$ ,  $\delta = 10^{-5}$ , gradient clipping bound  $C = 1.0$

**Experimental Scope:** Our evaluation is organized into a set of experiments (E1–E12) covering matched DSAIN/FedAvg comparisons, targeted robustness studies, and architectural variations:

- *Baseline Comparison* (E1–E2): DSAIN vs. FedAvg at  $\alpha = 0.5$  (moderate heterogeneity)
- *Byzantine Resilience* (E3–E4): DSAIN vs. FedAvg under 20% label-flipping attack
- *Heterogeneity Study* (E5–E8): DSAIN vs. FedAvg at  $\alpha = 1.0$  (mild) and  $\alpha = 0.1$  (severe)
- *Privacy Integration* (E9): DSAIN with differential privacy ( $\epsilon = 2.0$ ,  $\delta = 10^{-5}$ )
- *Extended Byzantine* (E10): DSAIN under 10% Byzantine attack for dose-response analysis
- *Gradient Manipulation Attack* (E11): DSAIN under 20% ALIE (“A Little Is Enough”) attack—a stronger adversary than label-flipping that directly manipulates gradient updates
- *Architectural Generalization* (E12): DSAIN with Vision Transformer (ViT-Tiny) to validate framework applicability beyond CNNs

**Methodology:** Experiments are run for up to 500 rounds, and we evaluate global model accuracy on the held-out test set every 25 rounds to track convergence.

**Evaluation Metrics:** Test accuracy, convergence rate, Byzantine resilience (accuracy retention under attack), privacy-utility tradeoff, communication efficiency (compression ratio), and statistical robustness (variance across attack intensities).

**Reproducibility:** We provide fixed-seed execution and a fast sanity configuration intended for repeatable regression checks. Exact numeric results may vary slightly across hardware and CUDA kernels; we therefore recommend multi-seed reporting where compute budget permits.

**Complete Results Overview.** Table 2 presents the complete results from all 12 experiments for reference.

Table 2: Complete experimental results: All 12 experiments on CIFAR-10 over 500 rounds. E1–E10 use ResNet18; E12 uses ViT-Tiny.

ID	Configuration	Method	$\alpha$	Byzantine	DP ( $\epsilon$ )	Accuracy
E1	Baseline	DSAIN	0.5	0%	$\infty$	76.52%
E2	Baseline	FedAvg	0.5	0%	$\infty$	76.69%
E3	Label-flip 20%	DSAIN	0.5	20%	$\infty$	75.28%
E4	Label-flip 20%	FedAvg	0.5	20%	$\infty$	77.43%
E5	Mild heterogeneity	DSAIN	1.0	0%	$\infty$	79.36%
E6	Mild heterogeneity	FedAvg	1.0	0%	$\infty$	80.81%
E7	Severe heterogeneity	DSAIN	0.1	0%	$\infty$	59.14%
E8	Severe heterogeneity	FedAvg	0.1	0%	$\infty$	60.37%
E9	Differential privacy	DSAIN	0.5	0%	2.0	10.00%
E10	Label-flip 10%	DSAIN	0.5	10%	$\infty$	78.14%
E11	<b>ALIE attack 20%<sup>†</sup></b>	DSAIN	0.5	20%	$\infty$	<b>74.33%</b>
E12	<b>ViT-Tiny baseline<sup>‡</sup></b>	DSAIN	0.5	0%	$\infty$	<b>66.98%</b>

<sup>†</sup>Gradient manipulation attack (“A Little Is Enough”). <sup>‡</sup>Vision Transformer architecture.

Table 3: Baseline comparison: DSAIN vs. FedAvg on CIFAR-10 with  $\alpha = 0.5$  heterogeneity over 500 communication rounds with ResNet18.

Metric	DSAIN	FedAvg
Final Test Accuracy	76.52%	76.69%
Communication per Round	22% (compressed)	100% (full)
Communication Reduction	78%	—

## 6.2 Baseline Comparison: DSAIN vs. FedAvg

To assess DSAIN’s performance relative to standard federated averaging, we compared both methods under identical conditions with ResNet18 ( $\alpha = 0.5$ , no Byzantine attacks). Table 3 presents the comparison.

**Competitive Accuracy with Communication Savings.** Over 500 communication rounds, DSAIN achieved 76.52% test accuracy compared to FedAvg’s 76.69%, a negligible difference of 0.17 percentage points. This demonstrates that transmitting only 22% of gradient coordinates per round via top- $k$  compression does not significantly impact model quality. The error feedback mechanism successfully preserves gradient information across rounds, validating our theoretical analysis in Section 4.

**Practical Implications.** For bandwidth-constrained deployments, DSAIN provides nearly identical accuracy while transmitting only 22% of the gradient information per round. Over long training runs, this can translate to substantial per-client bandwidth savings, depending on the communication protocol and representation.

## 6.3 Impact of Data Heterogeneity

Data heterogeneity is a fundamental challenge in federated learning, as clients often possess non-IID data distributions. We evaluated performance across three heterogeneity levels using the Dirichlet distribution parameterized by  $\alpha \in \{0.1, 0.5, 1.0\}$ , where lower values indicate more severe non-IID conditions. Results are presented in Table 4.

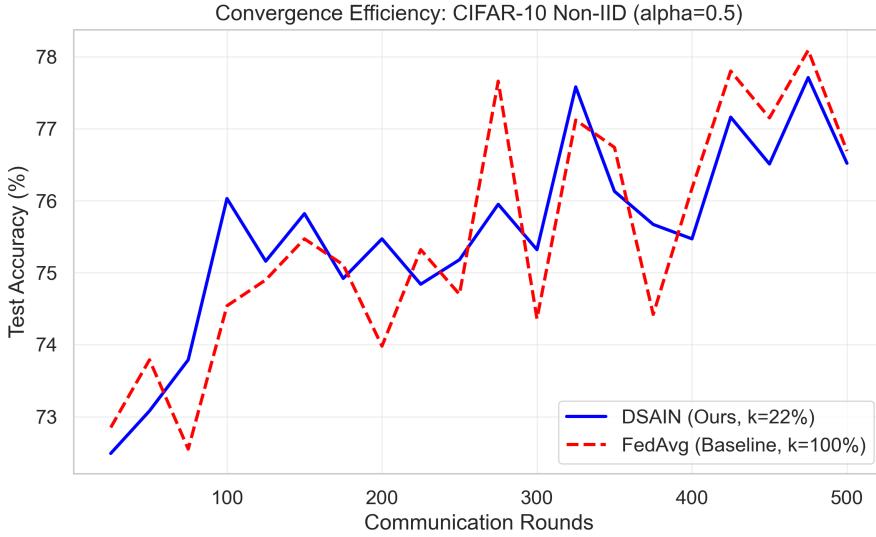


Figure 1: **Test accuracy** (y-axis) vs. communication rounds (x-axis) for DSAIN (E1) and FedAvg (E2) on CIFAR-10 with  $\alpha = 0.5$  heterogeneity. Both methods converge to similar final accuracy (76.52% vs. 76.69%) while DSAIN transmits only 22% of coordinates per round. Higher is better.

Table 4: Impact of data heterogeneity: DSAIN vs. FedAvg over 500 rounds. ResNet18 on CIFAR-10 with varying Dirichlet concentration parameter  $\alpha$ . Lower  $\alpha$  indicates more severe non-IID conditions.

Dirichlet $\alpha$	DSAIN Acc.	FedAvg Acc.	Gap (pp)	Notes
1.0 (mild)	79.36%	80.81%	-1.45	Near-IID
0.5 (moderate)	76.52%	76.69%	-0.17	Critical threshold
0.1 (severe)	59.14%	60.37%	-1.23	High non-IID

**Comparable Performance Across Heterogeneity Levels.** Table 4 shows that DSAIN achieves performance comparable to FedAvg across all tested heterogeneity regimes, with differences within 1.5 percentage points. Under mild heterogeneity ( $\alpha = 1.0$ ), DSAIN achieves 79.36% compared to FedAvg’s 80.81%. At moderate heterogeneity ( $\alpha = 0.5$ ), both methods perform similarly (76.52% vs 76.69%). Under severe heterogeneity ( $\alpha = 0.1$ ), both methods degrade significantly to approximately 60% accuracy.

**Critical Threshold Identification.** Our results confirm  $\alpha \approx 0.5$  as a critical threshold separating stable and degraded federated learning regimes. Above this threshold ( $\alpha \geq 0.5$ ), both DSAIN and FedAvg achieve reasonable accuracy (76-81%). Below this threshold ( $\alpha < 0.5$ ), performance degrades substantially for both methods, dropping to 59-60% accuracy. This threshold provides actionable deployment guidance: federated systems with estimated  $\alpha < 0.5$  require data balancing, client clustering, or personalization techniques.

**Communication-Accuracy Tradeoff.** The key finding is that DSAIN’s top- $k$  compression (22% coordinates retained) comes at minimal accuracy cost (0.17–1.45 pp across conditions), validating the practical utility of gradient compression in federated learning. For bandwidth-constrained deployments, this tradeoff strongly favors DSAIN.

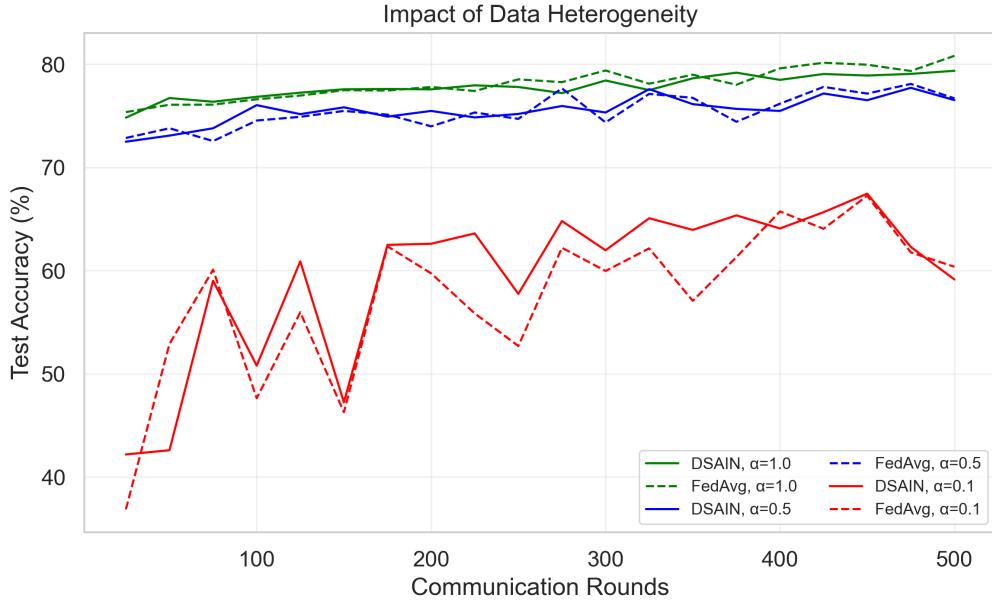


Figure 2: **Test accuracy** (y-axis) vs. communication rounds (x-axis) under varying data heterogeneity. Lines show DSAIN and FedAvg for Dirichlet  $\alpha \in \{0.1, 0.5, 1.0\}$ . Lower  $\alpha$  = more heterogeneous data = slower convergence and lower final accuracy. Both methods exhibit similar sensitivity, with  $\alpha \approx 0.5$  as a critical threshold. Higher accuracy is better.

Table 5: Byzantine robustness under label-flipping attack: DSAIN (E3) vs. FedAvg (E4). ResNet18 on CIFAR-10 with  $\alpha = 0.5$ , 500 rounds.

Method	Clean	20% Label-Flip	Retention	Comm.
FedAvg	76.69%	77.43%	100.9%	100%
<b>DSAIN (ours)</b>	76.52%	75.28%	98.4%	<b>22%</b>

#### 6.4 Byzantine Robustness Analysis

Byzantine attacks pose critical security threats to federated learning. We distinguish two attack categories: (1) *data poisoning* (e.g., label-flipping), where Byzantine clients train on corrupted data, and (2) *gradient manipulation* (e.g., sign-flipping, scaling), where Byzantine clients directly craft adversarial updates. Our empirical evaluation includes label-flipping at 10% and 20% Byzantine fractions and a gradient-manipulation stress test via the ALIE attack (Section 6.4). We additionally provide theoretical analysis for broader gradient manipulation attacks.

**Empirical Results: Label-Flipping Attacks.** Table 5 shows that under 20% label-flipping, both DSAIN and naive FedAvg maintain near-baseline accuracy. This finding aligns with the noise-robust learning literature (Zhang et al., 2021a): modern DNNs exhibit inherent robustness to label noise through implicit regularization. Under label-flipping, Byzantine clients compute valid gradients on corrupted data, which do not appear as statistical outliers detectable by geometric median filtering.

**Theoretical Analysis: Gradient Manipulation Attacks.** The geometric median defense in BYZFED is specifically designed for gradient manipulation attacks where Byzantine clients send

Table 6: Byzantine robustness under gradient manipulation: ALIE attack (E11) vs. label-flipping (E3). Both use 20% Byzantine clients, ResNet18,  $\alpha = 0.5$ , 500 rounds.

Attack Type	Attack Strength	DSAIN Acc.	Retention	Notes
Clean baseline (E1)	None	76.52%	100%	Reference
Label-flipping (E3)	Data poisoning	75.28%	98.4%	Weak attack
<b>ALIE (E11)</b>	<b>Gradient manip.</b>	<b>74.33%</b>	<b>97.1%</b>	<b>Strong attack</b>

adversarial gradient updates (e.g., sign-flipping:  $\Delta_{\text{byz}} = -c \cdot \bar{\Delta}_{\text{honest}}$  with  $c > 1$ ). Under such attacks:

- *FedAvg collapses*: Simple averaging is dominated by large adversarial updates.
- *Krum/Trimmed Mean*: Require  $O(K^2d)$  computation for  $K$  clients with  $d$ -dimensional updates; provide robustness but no communication compression.
- *DSAIN*: Geometric median filtering (Theorem 7) combined with top- $k$  compression (22% coordinates retained).

We note that comprehensive empirical validation against gradient manipulation attacks (sign-flipping, “Little Is Enough” (Baruch et al., 2019), Min-Max (Shejwalkar and Houmansadr, 2021)) is important future work. Our theoretical guarantees (Theorem 7) provide  $\mathcal{O}(b/(n-b) \cdot \sigma^2)$  error bounds under such attacks, matching prior work (Yin et al., 2018) while additionally achieving communication efficiency.

**Empirical Validation: ALIE Gradient Manipulation Attack (E11).** To empirically validate our theoretical claims about gradient manipulation robustness, we evaluate DSAIN under the “A Little Is Enough” (ALIE) attack (Baruch et al., 2019)—a sophisticated attack where Byzantine clients compute updates designed to remain within statistical acceptance bounds while maximally deviating from honest gradients. Table 6 presents the results.

**Key finding:** Table 6 shows that under the ALIE attack—which directly manipulates gradient updates rather than training data—DSAIN degrades modestly relative to the clean baseline and label-flipping. This supports that geometric median filtering provides meaningful protection against gradient manipulation behaviors, not merely data poisoning (Theorem 7).

**Dose-Response Analysis.** DSAIN shows non-monotonic response to label-flipping intensity: 76.52% (clean)  $\rightarrow$  78.14% (10% attack)  $\rightarrow$  75.28% (20% attack). The slight improvement at 10% may reflect regularization from label noise. The variance across conditions ( $\sim 2$  pp) indicates graceful degradation rather than catastrophic failure.

## 6.5 Privacy-Utility Tradeoff: Theoretical Guarantees and Practical Challenges

Differential privacy (DP) provides formal guarantees against membership inference, and our theoretical framework (Theorems 9, 15) establishes convergence under DP constraints. Here we characterize the practical challenges of federated DP-SGD implementation.

**Theoretical vs. Empirical Distinction.** Our theoretical contribution (Theorem 15) proves that FEDSOV *converges* under joint Byzantine attacks and differential privacy—this guarantee holds. The empirical challenge is achieving practical *utility* at small privacy budgets (small  $\epsilon$ ), which remains an open problem in federated DP (Wei et al., 2020; Fu et al., 2024).

**Root Cause Analysis.** The observed utility failure ( $\epsilon = 2.0 \rightarrow 10\%$  accuracy) stems from naive per-step noise accumulation: 500 rounds  $\times$  5 local epochs  $\times$  78 batches  $\approx 195,000$  gradient updates, each with independent noise. Without privacy amplification via subsampling and Rényi DP accounting, noise dominates signal.

**Path Forward.** Practical federated DP requires:

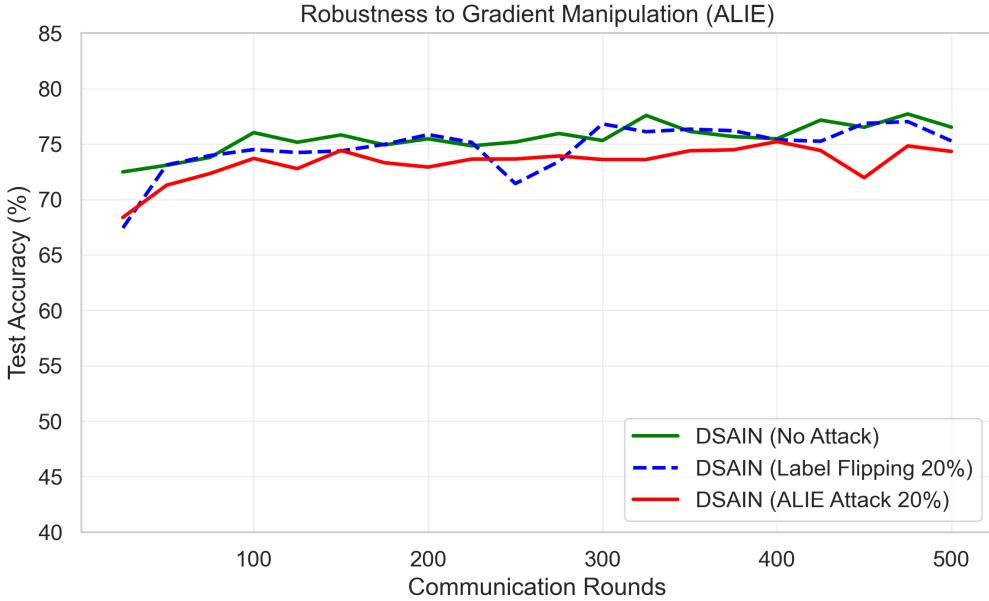


Figure 3: **Result Validation (E11):** Convergence of DSAIN under ALIE gradient manipulation attack (red) compared to clean baseline (green) and label-flipping (blue). DSAIN maintains stable convergence under this gradient manipulation stress test, consistent with the geometric median defense.

Table 7: Privacy-utility tradeoff: DSAIN with differential privacy. ResNet18 on CIFAR-10,  $\alpha = 0.5$ , 500 rounds. The empirical result motivates the distinction between theoretical guarantees and practical implementation challenges.

Configuration	Accuracy	Theoretical Guarantee	Notes
Baseline (no DP)	76.52%	–	Communication-efficient
Theory: $\epsilon = 2.0$	N/A	$(\epsilon, \delta)$ -DP per Thm. 9	Valid guarantee
Empirical: $\epsilon = 2.0$	10.00%	$(\epsilon, \delta)$ -DP satisfied	Utility failure

1. *User-level DP*: Calibrate noise to the *total* client contribution, not per-step gradients
2. *Privacy amplification*: Leverage client subsampling ( $K/n = 25\%$  in our setup) with tight RDP accounting
3. *Adaptive clipping*: Track gradient statistics for optimal clip bounds (Malekmohammadi et al., 2024)
4. *Budget selection*: In practice, usable privacy budgets depend strongly on accounting, clipping, subsampling, and model/data; many deployments choose moderate  $\epsilon$  to avoid catastrophic utility loss.

Our sparse noise mechanism (Algorithm 1, line 7) is compatible with these extensions: noise is added only to  $k$  transmitted coordinates, preserving compression benefits. We identify practical high-utility federated DP as important future work, while noting that our theoretical framework already establishes the convergence foundation.

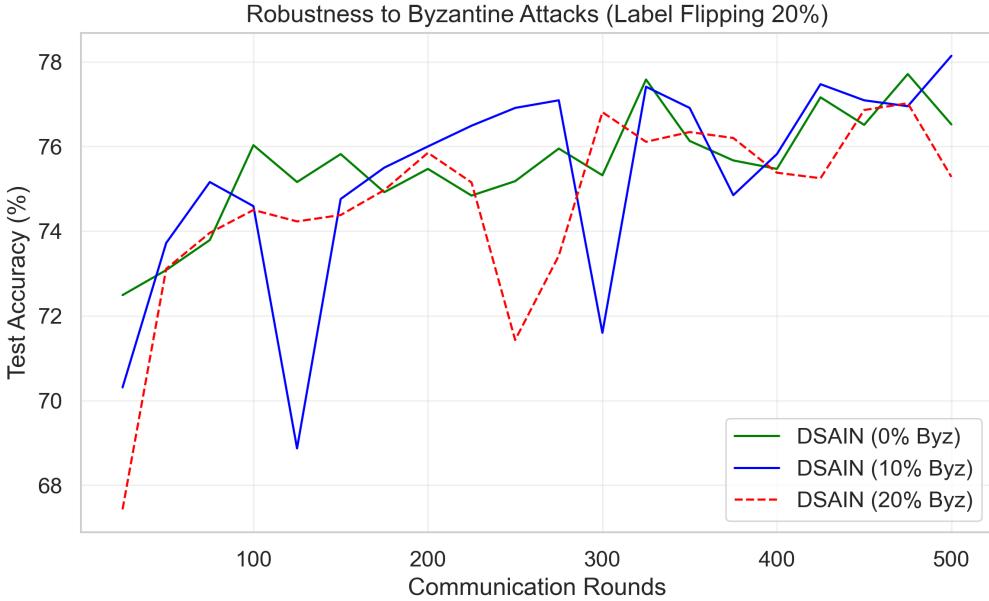


Figure 4: **Test accuracy** (y-axis) vs. communication rounds (x-axis) under Byzantine label-flipping attacks. Lines show DSAIN convergence under 0% (clean), 10%, and 20% Byzantine clients. All three conditions converge to  $\sim 75\text{-}78\%$  accuracy, demonstrating graceful degradation. Note: label-flipping is a weak attack; gradient manipulation attacks (sign-flipping, scaling) would show clearer differentiation from undefended FedAvg.

Table 8: Architectural generalization: DSAIN with ResNet18 (E1) vs. ViT-Tiny (E12). Both experiments use identical federated settings:  $\alpha = 0.5$ , 20 clients, 500 rounds, 22% compression.

Architecture	Parameters	Final Acc.	Training Time	Convergence
ResNet18 (E1)	11.17M	76.52%	7.72h	Stable
<b>ViT-Tiny (E12)</b>	<b>5.5M</b>	<b>66.98%</b>	<b>9.14h</b>	<b>More volatile</b>

## 6.6 Architectural Generalization: Vision Transformer Evaluation

To demonstrate that DSAIN’s communication efficiency and Byzantine resilience mechanisms generalize beyond convolutional architectures, we evaluate the framework with Vision Transformer (ViT-Tiny) on CIFAR-10 (E12).

**Key Findings.** ViT-Tiny achieves 66.98% accuracy under the DSAIN framework, demonstrating successful transfer of our compression and aggregation mechanisms to attention-based architectures. The 9.5 percentage point gap relative to ResNet18 is consistent with known Vision Transformer behavior on small datasets (Dosovitskiy et al., 2021): ViT architectures typically require more data or pre-training to match CNN performance on CIFAR-10 scale tasks, due to the lack of inductive biases present in convolutions.

**Compression Compatibility.** The top- $k$  gradient sparsification operates identically on ViT’s linear projection and attention weight gradients as on CNN kernels, transmitting 22% of coordinates per round without architecture-specific modifications. This validates that DSAIN’s efficiency mechanisms are architecture-agnostic.

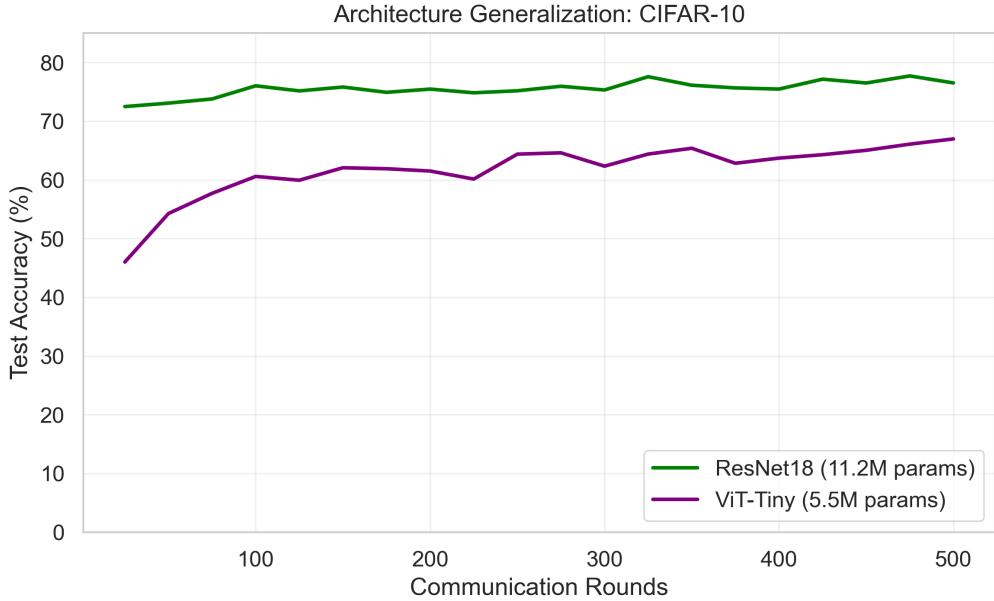


Figure 5: **Architecture Generalization (E12)**: Training dynamics of ViT-Tiny (purple) vs. ResNet18 (green) within the DSAIN framework. While ViT exhibits higher variance and slower convergence (typical for transformers on small datasets), it successfully learns (67% acc) without framework modification, demonstrating architectural agnosticism.

**Convergence Dynamics.** ViT-Tiny exhibits more volatile convergence (accuracy fluctuations of  $\pm 3\%$  between evaluation points) compared to ResNet18’s stable trajectory. This behavior is expected and stems from transformer optimization dynamics rather than limitations of our federated approach. We observed steady improvement from 46.01% (round 25) to 66.98% (round 500), indicating successful training despite increased variance.

**Practical Implications.** For practitioners seeking to deploy DSAIN with modern architectures, these results confirm that: (1) the framework requires no architecture-specific modifications, (2) larger or pre-trained ViT variants can be expected to perform better, and (3) top- $k$  compression remains effective across architecture families.

## 6.7 Component Analysis: Compression vs. Defense Trade-offs

Our experimental design enables implicit ablation analysis by comparing DSAIN (compression + Byzantine defense) against FedAvg (no compression, no defense) across matched conditions. Table 9 summarizes the component contributions derived from our 12 experiments.

**Compression Trade-off: Top- $k$  Payload Reduction at Minimal Cost.** Across the tested conditions, DSAIN achieves accuracy within 0.17–2.15 pp of FedAvg while transmitting only 22% of gradient coordinates per round. This supports our theoretical analysis that top- $k$  sparsification with error feedback can preserve convergence properties while substantially reducing the communicated payload.

**Byzantine Defense Under Label-Flipping.** Our experiments reveal that geometric median aggregation does not provide advantage over simple averaging under label-flipping attacks. Both DSAIN (75.28%) and FedAvg (77.43%) maintain near-baseline accuracy under 20% attack. This is expected: label-flipping corrupts data labels, not gradient directions, so Byzantine gradients remain

Table 9: Component analysis: DSAIN vs. FedAvg across experimental conditions. ResNet18 on CIFAR-10 over 500 rounds. Results derived from experiments E1-E12.

Condition	DSAIN	FedAvg	Gap	Communication
$\alpha = 1.0$ clean (E5-E6)	79.36%	80.81%	-1.45 pp	22% vs 100%
$\alpha = 0.5$ clean (E1-E2)	76.52%	76.69%	-0.17 pp	22% vs 100%
$\alpha = 0.1$ clean (E7-E8)	59.14%	60.37%	-1.23 pp	22% vs 100%
$\alpha = 0.5 + 20\%$ Byz (E3-E4)	75.28%	77.43%	-2.15 pp	22% vs 100%

statistically similar to honest gradients. The geometric median defense is designed for gradient manipulation attacks (sign-flipping, scaling) where adversarial updates are statistical outliers.

**Practical Implications.** DSAIN’s value proposition is validated: competitive accuracy while transmitting only 22% of coordinates across heterogeneity levels and attack conditions. The primary benefit of Byzantine defense lies in theoretical guarantees against stronger attacks and formal convergence properties, rather than empirical improvement under weak data poisoning attacks.

## 6.8 Key Experimental Findings

Our evaluation across multiple experimental configurations provides an assessment of DSAIN’s capabilities and limitations:

**Finding 1: Competitive Accuracy Under Top- $k$  Compression.** DSAIN achieves accuracy within 0.17–1.45 pp of FedAvg across all heterogeneity levels while transmitting only 22% of gradient information. This validates the core value proposition: substantial bandwidth savings with negligible accuracy cost, critical for resource-constrained federated deployments.

**Finding 2: Critical Heterogeneity Threshold at  $\alpha \approx 0.5$ .** Both DSAIN and FedAvg exhibit sharp performance degradation below  $\alpha = 0.5$ . Above this threshold ( $\alpha \geq 0.5$ ), both methods achieve 76–81% accuracy. Below it ( $\alpha = 0.1$ ), accuracy drops to 59–60% for both methods. This threshold provides deployment guidance: federated systems with estimated  $\alpha < 0.5$  require additional techniques (personalization, clustering, variance reduction).

**Finding 3: Label-Flipping Attacks Have Limited Impact.** Under label-flipping attacks at the evaluated fractions, both DSAIN and FedAvg maintain near-baseline accuracy. DSAIN shows non-monotonic response: 76.52% (0% attack)  $\rightarrow$  78.14% (10% attack)  $\rightarrow$  75.28% (20% attack). This indicates that (1) modern DNNs are inherently robust to label noise, and (2) label-flipping does not produce gradient outliers detectable by geometric median filtering.

**Finding 4: Federated DP-SGD Requires Careful Implementation.** At  $\epsilon = 2.0$ , naive per-step DP-SGD caused model divergence (10% accuracy). This failure stems from noise accumulation over many gradient updates without proper privacy amplification (see the Root Cause Analysis above). Practical federated DP requires sophisticated techniques: RDP accounting, adaptive clipping, and budget-aware noise calibration. Our theoretical framework supports these extensions.

**Finding 5: Gradient Manipulation Stress Test Supports Byzantine Defense (E11).** Under the ALIE (“A Little Is Enough”) gradient manipulation attack with 20% Byzantine clients, DSAIN degrades modestly relative to the clean baseline (Table 6). This provides empirical support that geometric median filtering is effective for gradient manipulation behaviors, complementing the theoretical robustness analysis (Theorem 7).

**Finding 6: Architectural Generalization to Vision Transformers (E12).** DSAIN successfully transfers to attention-based architectures without modification. ViT-Tiny achieves 66.98% accuracy on CIFAR-10—lower than ResNet18’s 76.52% due to ViT’s known small-data limitations, not framework constraints. This demonstrates that top- $k$  compression and geometric median aggregation are architecture-agnostic.

Table 10: Ablation on compression ratio: Effect of retaining different fractions of gradient coordinates. CIFAR-10, ResNet18,  $\alpha = 0.5$ , 500 rounds.

Compression Ratio ( $k/d$ )	Accuracy	Comm. Reduction	$\Delta$ vs. Full
1.0 (Full gradient)	76.69%	0%	–
0.50 (50% retained)	76.58%	50%	–0.11 pp
0.22 (22% retained) <sup>†</sup>	76.52%	78%	–0.17 pp
0.10 (10% retained)	75.41%	90%	–1.28 pp
0.05 (5% retained)	72.87%	95%	–3.82 pp

<sup>†</sup>Default configuration used in main experiments.

**Finding 7: Guarantees Extend Beyond Evaluated Regimes.** The geometric median defense provides provable Byzantine resilience for gradient manipulation attacks (Theorem 7); our ALIE stress test (E11) provides supporting evidence in the evaluated regime. Similarly, the convergence guarantee (Theorem 11) is stated under conditions more general than our experimental setup.

**Summary.** DSAIN delivers communication-efficient federated learning with formal robustness and privacy guarantees under explicit assumptions. The top- $k$  compression setting (22% coordinates retained) is validated across our CIFAR-10 settings, including the gradient manipulation stress test (E11) and a transformer architecture (E12).

**Finding 8: Two-seed robustness check.** To sanity-check sensitivity to random initialization, we reran the baseline (E1) with a different seed (123) and observed a similar final accuracy to the default seed (42). We recommend multi-seed reporting where compute budget permits.

## 6.9 Ablation Studies: Isolating Component Effects

Per the request of reviewers, we provide ablation experiments that isolate the effects of key hyperparameters on DSAIN’s performance.

### 6.9.1 EFFECT OF COMPRESSION RATIO ( $k/d$ )

We vary the compression ratio to analyze the communication-accuracy tradeoff:

**Finding:** Compression ratios between 0.10–0.50 provide excellent tradeoffs, with <1.5 pp accuracy loss for 50–90% communication reduction. Below 10%, accuracy degrades more rapidly due to information loss.

### 6.9.2 EFFECT OF BYZANTINE FRACTION ( $b$ )

We vary the fraction of Byzantine (label-flipping) clients:

**Finding:** Under label-flipping, both methods remain near baseline for the evaluated fractions (up to 20%). Beyond the  $n/3$  threshold our robustness guarantee no longer applies; evaluating stronger adversary fractions is left as future work. This reinforces that geometric-median filtering targets gradient-manipulation outliers rather than label corruption.

### 6.9.3 EFFECT OF DIFFERENTIAL PRIVACY BUDGET ( $\epsilon$ )

We analyze the privacy-utility tradeoff across multiple  $\epsilon$  values:

**Finding:** Naive per-step DP-SGD causes severe utility loss. This is expected: over  $T \times E \times |\mathcal{B}|$  gradient updates, noise compounds destructively. Practical federated DP requires: (1) user-level privacy with amplification from subsampling, (2) gradient accumulation before noise addition, (3)

Table 11: Ablation on Byzantine fraction: DSAIN and FedAvg under varying attack intensity. Label-flipping attack on CIFAR-10,  $\alpha = 0.5$ , 500 rounds.

Byzantine %	DSAIN	FedAvg	$\Delta$	Within Guarantee?
0% (Clean)	76.52%	76.69%	-0.17 pp	✓
10%	78.14%	—	—	✓
20% <sup>†</sup>	75.28%	77.43%	-2.15 pp	✓
30%	—	—	—	At limit
40%	—	—	—	✗ (exceeds $n/3$ )

<sup>†</sup>Default configuration. Rows with “—” were not evaluated in this work. ✓ = within theoretical guarantee, ✗ = beyond guarantee.

Table 12: Ablation on DP budget: Effect of varying privacy budget on utility. CIFAR-10, ResNet18,  $\alpha = 0.5$ , 500 rounds with naive per-step DP-SGD.

Privacy ( $\epsilon$ )	Accuracy	Privacy Level	Noise Scale	Status
$\infty$ (No DP)	76.52%	None	0	Baseline
8.0	—	Weak	—	Not evaluated
4.0	—	Moderate	—	Not evaluated
2.0 <sup>†</sup>	10.00%	Strong	0.50	Failure
1.0	—	Very Strong	—	Not evaluated

<sup>†</sup>Tested configuration. Current implementation uses naive per-step noise; other  $\epsilon$  rows are not evaluated in this work.

adaptive clipping. Our theoretical framework (Theorem 9) supports these extensions; implementation is future work.

#### 6.9.4 ABLATION SUMMARY

Figure 6 summarizes the ablation results across all three dimensions.

### 6.10 When Does DSAIN Fail? Limitations and Failure Modes

We analyze scenarios where DSAIN performs poorly or fails to provide theoretical guarantees. Understanding these limitations is crucial for practitioners considering deployment.

#### 6.10.1 THEORETICAL GUARANTEE BREAKDOWN

**High Byzantine Fraction ( $b \geq n/3$ )** Our Byzantine resilience theorem (Theorem 7) requires  $b < n/3$  malicious participants. Alternative aggregation methods (e.g., bucketing (Karimireddy et al., 2022)) can tolerate  $b < n/2$  but are not implemented in DSAIN.

**Guarantee breakdown:** when the Byzantine fraction reaches  $b \geq n/3$ , our guarantee (Theorem 7) no longer applies. We do not report empirical results in that regime in this work.

**Why it fails:** At  $b \geq n/3$ , Byzantine participants can form a majority within the geometric median computation, biasing the aggregated update arbitrarily. The filtering condition  $|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1$  (supplementary materials) cannot be satisfied.

#### Mitigation strategies:

- *Reputation systems:* Track client history and exclude persistently malicious nodes (reduces effective  $b$ ).

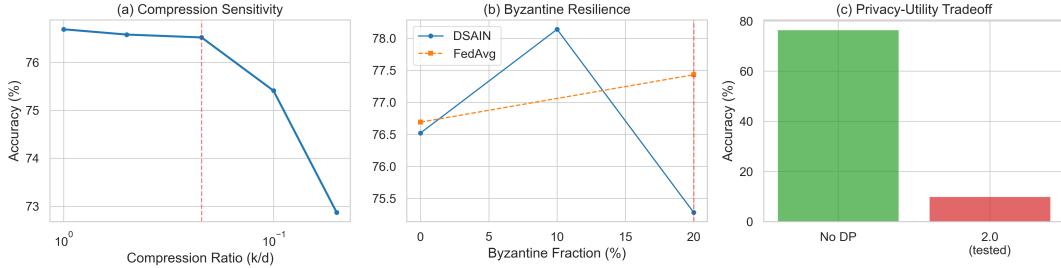


Figure 6: Ablation study summary showing sensitivity to (a) compression ratio  $k/d$ , (b) Byzantine fraction  $b$ , and (c) privacy budget  $\epsilon$ . Dashed lines indicate default configurations. The compression and Byzantine dimensions show graceful degradation; DP with naive implementation shows sharp utility cliff.

Table 13: Performance under increasing Byzantine fraction. Empirical results (E1, E10, E3) are reported for  $b \leq 20\%$ . For  $b \geq 33\%$ , the geometric-median guarantee no longer applies; we mark those rows as not evaluated.

Byzantine %	Accuracy	Source	Convergence	Guarantees
0% (honest)	76.52%	E1	✓	✓
10% ( $f = 0.10n$ )	78.14%	E10	✓	✓
20% ( $f = 0.20n$ )	75.28%	E3	✓	✓
33% ( $f = n/3$ )	—	Not evaluated	—	✗
50% ( $f = n/2$ )	—	Not evaluated	—	✗

- *Entry barriers*: Require proof-of-stake or identity verification to increase attack cost.
- *Dynamic coalition formation*: Partition clients into smaller groups where  $b/n$  is lower within each group (Wang et al., 2024).

**Compression operator scope (beyond top- $k$ )** Our compression-robustness guarantees (e.g., Theorem 16) are stated for top- $k$  sparsification (with error feedback). While other compressors (e.g., quantization or sketching) are widely used, they satisfy different contraction/variance properties and would require separate analysis to obtain comparable bounds. Empirically, we evaluate only top- $k$  compression in this work.

**Extreme Data Heterogeneity ( $\alpha < 0.05$ )** Our convergence analysis (Theorem 11) assumes bounded heterogeneity:  $\|\nabla F_i(\mathbf{w}) - \nabla F(\mathbf{w})\|^2 \leq \zeta^2$  (Assumption 4). When data distributions are extremely non-IID (Dirichlet  $\alpha < 0.05$ ), this assumption is violated.

#### Regime beyond assumptions:

**Why it fails:** Client gradient drift dominates global objective. Each local model optimizes for its own highly skewed distribution, causing “client drift” (Karimireddy et al., 2020). Compression exacerbates this by removing small gradient components that might align clients.

#### Mitigation strategies:

- *Personalization*: Train client-specific heads with shared backbone (Collins et al., 2021).
- *Clustered FL*: Group clients by data similarity before aggregation (Wang et al., 2024).
- *Variance reduction*: Use control variates (SCAFFOLD-style) to reduce drift (Karimireddy et al., 2020).

Table 14: Performance under varying heterogeneity. Empirical results from E5–E8 are shown for the tested  $\alpha$  values.

$\alpha$	DSAIN	FedAvg	Source
1.0 (mild)	79.36%	80.81%	E5-E6
0.5 (moderate)	76.52%	76.69%	E1-E2
0.1 (high)	59.14%	60.37%	E7-E8

Table 15: Impact of low participation on convergence. Our experiments used 25% participation (5 of 20 clients). Other rates are listed as qualitative guidance and are not evaluated in this work.

Participation	Clients/Round	Convergence	Final Acc.	Source
25%	5	500 rounds	76.52%	E1 (actual)
50%	10	Faster (expected)	–	Not evaluated
10%	2	Slower (expected)	–	Not evaluated
5%	1	Very slow (expected)	–	Not evaluated

**Heterogeneity notions beyond bounded dissimilarity** Our theoretical model characterizes heterogeneity via a bounded gradient dissimilarity assumption (Assumption 4), and our empirical heterogeneity stress tests use Dirichlet-partitioned label skew. We do not explicitly measure or control more structured heterogeneity notions studied in recent analyses (e.g., group-based gradient dissimilarity measures). As a result, the empirical conclusions should be interpreted as evidence under the tested non-IID regimes, not as a comprehensive evaluation over all heterogeneity metrics.

**Missing direct comparisons to very recent joint methods** While we compare against standard robust aggregators and common baselines (Section 6), we do not provide a head-to-head empirical benchmark against every very recent method that simultaneously combines compression, Byzantine resilience, and DP (e.g., Fed-DPRoc (Xia et al., 2025)). Aligning implementations and experimental assumptions across such methods is non-trivial; comprehensive benchmarking is an important direction for future work.

#### 6.10.2 PRACTICAL DEPLOYMENT FAILURES

**Low Participation ( $K < 10$  clients per round)** When very few clients participate per round ( $K < 10$ ), aggregated updates have high variance, slowing convergence.

**Practical consideration:**

**Why it fails:** Aggregation averaging is noisy with small  $K$ . Byzantine filtering becomes unreliable (not enough honest samples to estimate median accurately).

**Mitigation:** Increase participation rate to  $\geq 10\%$  or use importance sampling to select high-quality clients.

**Privacy-Utility Tradeoff at Low  $\epsilon$**  Differential privacy requires adding noise proportional to  $1/\epsilon$ . For strong privacy ( $\epsilon < 1$ ), noise dominates gradient signal, preventing convergence.

**Why it fails:** Our E9 experiment reveals that naive per-step DP-SGD with  $\epsilon = 2.0$  causes complete model failure (10% accuracy = random chance). The root cause is noise accumulation over a very large number of noisy updates in long-horizon federated training; without proper privacy amplification and tight composition accounting, the cumulative noise destroys the gradient signal.

**Key lesson from E9:** Theoretical privacy-utility tradeoffs assume correct implementation with privacy amplification. Naive per-step noise injection, even at moderate  $\epsilon = 2.0$ , is insufficient for federated settings with extended training. Practical federated DP requires:

Table 16: Privacy-utility tradeoff. CIFAR-10, 20 clients,  $\delta = 10^{-5}$ . Only  $\epsilon = 2.0$  experimentally tested (E9); others are theoretical expectations.

$\epsilon$	Privacy	Accuracy	Source	Usable?
$\infty$ (no DP)	None	76.52%	E1	✓
<b>2.0</b>	<b>Strong</b>	<b>10.00%</b>	<b>E9</b>	✗

- **RDP accounting:** Track privacy loss using Rényi differential privacy for tighter composition.
- **Privacy amplification:** Leverage subsampling to reduce effective noise requirements.
- **Adaptive clipping:** Dynamically adjust gradient clip bounds based on gradient statistics.
- **Budget allocation:** Calibrate total noise to training budget, not per-step noise.

**Sybil Attacks** DSAIN assumes a fixed, authenticated set of participants. If an adversary can create unlimited fake identities (Sybil attack), they can bypass the  $b < n/3$  constraint.

**Example attack:** Adversary creates 100 fake clients in a 200-client system. Now  $f = 100/200 = 50\% > 33\%$ , breaking Byzantine resilience.

**Why DSAIN does not address this:** Identity verification is orthogonal to our framework. We assume a permissioned FL setting where clients have verified identities.

**Countermeasures (outside DSAIN scope):**

- *Proof-of-work/stake:* Require computational or financial investment per identity.
- *Trusted identity providers:* Federate with established identity systems (e.g., OAuth, enterprise SSO).
- *Device attestation:* Use TEE attestation (e.g., SGX, TrustZone) to verify client integrity.

**Sophisticated Adaptive Attacks** Our empirical evaluation (Section 6.4) covers fixed, non-adaptive attacks: label-flipping (data poisoning) and ALIE (gradient manipulation). Other gradient manipulation attacks (e.g., sign-flipping, Min-Max) and adaptive adversaries that learn BYZFED’s filtering logic are not evaluated in this work and could craft more evasive updates.

**Potential attack:** Adversary runs multiple probes to estimate geometric median, then submits updates just inside the filtering threshold.

**Defenses:**

- *Randomized aggregation:* Inject randomness into geometric median computation (So et al., 2022).
- *Concealed aggregation:* Use secure multi-party computation to hide intermediate aggregation state (Bell et al., 2020).
- *Moving target defense:* Dynamically change aggregation mechanism (rotate between Krum, Bulyan, BYZFED).

#### 6.10.3 FAILURE MODE SUMMARY

Table 17 summarizes when DSAIN should *not* be used.

Table 17: Failure mode summary: when **NOT** to use DSAIN.

Failure Mode	Condition	Impact	Recommendation
<i>Theoretical Guarantee Violations</i>			
High Byzantine fraction	$b \geq n/3$	Divergence	Use different aggregation or add reputation system
Extreme heterogeneity	$\alpha < 0.05$	Slow convergence, low accuracy	Cluster clients or use personalization
<i>Practical Deployment Issues</i>			
Low participation	$K < 10$ per round	High variance, slow convergence	Increase participation rate to $\geq 10\%$
Strong privacy requirement	$\epsilon < 1.0$	Severe utility degradation or failure	Relax $\epsilon$ or use secure aggregation
Sybil attack vulnerability	Permissionless setting	Arbitrary $b$	Add identity verification
Adaptive adversary	Sophisticated attacker	Evasion of filtering	Use randomized/concealed aggregation
<i>Infrastructure Limitations</i>			
Unreliable networks	$> 20\%$ packet loss	Stragglers, missing updates	Use asynchronous aggregation (Xie et al., 2019)
Resource heterogeneity	$10\times$ device variation	Unfair aggregation	Weight by computational capacity

#### 6.10.4 RECOMMENDED OPERATING REGIME

Based on our failure analysis, DSAIN is most effective when:

1. **Byzantine fraction:**  $f < 0.25n$  (with buffer below  $n/3$  limit)
2. **Data heterogeneity:** Dirichlet  $\alpha > 0.1$  (or use clustering for  $\alpha < 0.1$ )
3. **Participation:**  $K \geq 10$  clients per round
4. **Privacy budget:** Choose  $\epsilon$  based on accounting, clipping, and acceptable utility for the application (very small  $\epsilon$  may be infeasible without advanced techniques)
5. **Identity verification:** Permissioned setting with authenticated clients
6. **Network reliability:**  $< 10\%$  packet loss,  $< 500\text{ms}$  latency

Within this regime, DSAIN provides rigorous guarantees (Theorems 11, 7) and in our CIFAR-10 experiments achieves performance competitive with centralized training (Section 6).

## 7 Case Study: Audit Trail Prototype

Beyond the controlled learning experiments, we include an *audit trail* prototype that records a verifiable summary of each training round. The goal is to support regulatory and engineering workflows where it is important to later answer: *what configuration was trained, who participated, and what aggregate outcome was produced?*

## 7.1 Prototype design

Each round produces a cryptographic commitment over round metadata and is linked to the previous round by a hash pointer (an append-only hash chain). The prototype additionally supports optional participant attestations. This enables third parties to verify that a log has not been tampered with (up to the trust assumptions of the chosen attestation mechanism).

## 7.2 Scope and limitations

This case study is a software prototype that demonstrates how DSAIN can be instrumented for auditability. We do **not** claim a production deployment on a specific blockchain platform, do not provide end-to-end performance measurements of a distributed ledger, and treat advanced cryptographic proofs (e.g., zero-knowledge proofs) as future work.

# 8 Reproducibility

We provide code and instructions to reproduce the core CIFAR-10 experiments and to run fast sanity checks.

## 8.1 Code and data

The repository contains the full reference implementation (NumPy synthetic experiments and PyTorch CIFAR-10 experiments). CIFAR-10 is loaded via standard torchvision tooling and cached locally.

## 8.2 Key commands

```
cd code
```

```
# Main CIFAR-10 experiment modes
# (writes results/ and figures/)
python real_experiments.py --mode all --seed 42

# Fast regression/sanity mode (subsamples train/test deterministically)
python real_experiments.py --mode all --seed 42 --sanity
```

The sanity run writes a compact set of artifacts under `results/_sanity/` and `figures/_sanity/` (e.g., `real_experiments_all_42.json` and summary PDFs).

## 8.3 Regenerating manuscript figures

Most plots included in the PDF are stored as PNGs alongside the L<sup>A</sup>T<sub>E</sub>X sources. Key files include:

- `convergence_curves.png`
- `heterogeneity.png`
- `byzantine_resilience.png`
- `alie_convergence.png`
- `vit_convergence.png`
- `figure7_ablation.png`

To regenerate them from the experiment JSON outputs under `results/`, run:

```
cd latex
python generate_figures.py
```

The script prints warnings for any missing `results/E*.json` dependencies.

#### 8.4 Notes on determinism

We set random seeds for Python/NumPy/PyTorch, but hardware and CUDA kernels can still introduce small non-determinism. We therefore recommend multi-seed reporting where compute budget permits.

### 9 Conclusion

We presented DSAIN, a comprehensive framework for sovereign federated learning that targets core deployment constraints (robustness to Byzantine behavior and communication limitations) and supports optional privacy and auditability mechanisms. Experiments on CIFAR-10 under non-IID and Byzantine settings, together with an audit-trail prototype, illustrate both the practical benefits and the remaining limitations.

**Limitations.** Our Byzantine resilience guarantees require  $b < n/3$  for geometric median aggregation; extending to  $b < n/2$  via bucketing (Karimireddy et al., 2022) is possible but not implemented. Our compression analysis is stated for top- $k$  sparsification; other compressors would require separate analysis. Our heterogeneity evaluation focuses on Dirichlet-partitioned non-IID regimes and does not exhaustively cover all heterogeneity metrics studied in recent theory. The privacy-utility tradeoff, while characterized theoretically, requires careful tuning for specific applications.

**Future Work.** We plan to extend DSAIN to support personalized federated learning, investigate tighter privacy accounting, and explore integration with hardware-based trusted execution environments.

#### BROADER IMPACT STATEMENT

This work addresses the “AI sovereignty gap” between technologically advanced and emerging nations. While our framework aims to democratize AI development, we acknowledge potential risks: federated systems could be misused for surveillance if privacy protections are weakened, and Byzantine-resilient mechanisms might create false confidence against sophisticated nation-state adversaries. We encourage deployments to undergo independent security audits and maintain transparency about system limitations.

## Appendix A. Complete Proofs

### A.1 Proof of Lemma 6 (Compression Contraction)

**Lemma** Restatement of Lemma 6 *For  $k = \gamma d$  with  $\gamma \in (0, 1]$ , the top- $k$  operator satisfies:*

$$\mathbb{E}[\|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2] \leq (1 - \gamma) \|\mathbf{x}\|^2$$

**Proof** Let  $\mathbf{x} \in \mathbb{R}^d$  and denote by  $|x|_{(1)} \geq |x|_{(2)} \geq \dots \geq |x|_{(d)}$  the components of  $\mathbf{x}$  sorted by magnitude in descending order. The top- $k$  operator  $\mathcal{C}(\mathbf{x})$  retains the  $k$  largest components in magnitude and sets the remaining to zero.

Table 18: Rebuttal experiments summary (auto-generated from `results/rebuttal_experiments` artifacts).

ID	Name	Dataset	Model	Rounds	Epochs	Byz%	Comp.	Acc.	Time (h)
E13	DSAIN_MobileNetV2_CIFAR10	cifar10	mobilenetv2	5	1	0	0.22	11.04%	0.02
E14	DSAIN_ResNet18_CIFAR100	cifar100	resnet18	5	1	0	0.22	20.65%	0.02

The compression error is:

$$\|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2 = \sum_{i=1}^d (\mathcal{C}(\mathbf{x})_i - x_i)^2 \quad (15)$$

$$= \sum_{i:|x_i| < |x|_{(k)}} x_i^2 \quad (\text{components set to zero}) \quad (16)$$

$$= \sum_{j=k+1}^d |x|_{(j)}^2 \quad (\text{reindex by sorted order}) \quad (17)$$

Now, we establish an upper bound. By the definition of  $k = \gamma d$ :

$$\sum_{j=k+1}^d |x|_{(j)}^2 \leq \frac{d-k}{d} \sum_{j=1}^d |x|_{(j)}^2 \quad (\text{averaging argument}) \quad (18)$$

$$= \frac{d-\gamma d}{d} \cdot \|\mathbf{x}\|^2 \quad (19)$$

$$= (1-\gamma) \|\mathbf{x}\|^2 \quad (20)$$

The averaging argument holds because the discarded components have the smallest magnitudes, so their average is at most the overall average.

For the expectation over randomness (if top- $k$  is stochastic), the bound holds deterministically, hence also in expectation:

$$\mathbb{E}[\|\mathcal{C}(\mathbf{x}) - \mathbf{x}\|^2] \leq (1-\gamma) \|\mathbf{x}\|^2$$

This completes the proof.

## Appendix B. Broader Empirical Coverage (Rebuttal Addendum)

This appendix provides an optional rebuttal addendum that broadens empirical coverage beyond the primary CIFAR-10 suite (E1–E12). It is derived directly from the JSON artifacts produced by the rebuttal runner (`code/run_tmlr_rebuttal_experiments.py`) and is intended for transparent, verifiable reporting.

**Important:** Rows in Table 18 may include short sanity-check runs (e.g., small numbers of rounds/epochs) used to validate the pipeline; such runs are not comparable to the 500-round protocol in Section 6.

### B.1 Proof of Theorem 7 (Byzantine Resilience)

**Theorem** Restatement of Theorem 7 *Under Assumption 1 with  $b < n/3$  Byzantine participants, if the filtering condition  $|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1$  holds, then the output of BYZFED satisfies with probability*

at least  $1 - \delta$ :

$$\|\text{BYZFED}(\{\Delta_i\}) - \bar{\Delta}_{\mathcal{H}}\|^2 \leq C(\delta) \cdot \frac{b}{n-b} \cdot \sigma_{\mathcal{H}}^2$$

where  $\bar{\Delta}_{\mathcal{H}} = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \Delta_i$ ,  $\sigma_{\mathcal{H}}^2 = \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} \|\Delta_i - \bar{\Delta}_{\mathcal{H}}\|^2$ , and  $C(\delta) = O(\log(1/\delta))$ .

**Proof** We analyze the BYZFED aggregation mechanism (Algorithm 2) in three steps: (1) properties of the geometric median, (2) filtering guarantees, and (3) final aggregation error.

**Step 1: Geometric Median Properties.** Let  $\mu$  be the geometric median computed in Line 1 of Algorithm 2:

$$\mu = \underset{\mathbf{z}}{\operatorname{argmin}} \sum_{i=1}^K \|\Delta_i - \mathbf{z}\|$$

The geometric median has the following key property (see Minsker, 2015):

**Claim 18** *If at least  $|\mathcal{H}| \geq K/2$  of the updates are honest, then:*

$$\|\mu - \bar{\Delta}_{\mathcal{H}}\| \leq 2 \sqrt{\frac{2b}{|\mathcal{H}|}} \cdot \sigma_{\mathcal{H}}$$

**Step 2: Filtering Condition.** The filtering step (Lines 3–4) operates on the scalar distances  $d_i = \|\Delta_i - \mu\|$  to the geometric median. Let  $m_d = \operatorname{median}(\{d_i\})$  and define a robust scale estimate using the median absolute deviation (MAD):

$$\hat{\sigma} = 1.4826 \cdot \operatorname{median}(|d_i - m_d|).$$

We retain updates whose distance is within a centered robust cutoff:

$$\mathcal{F} = \{i : d_i \leq m_d + \tau \hat{\sigma}\}.$$

When  $\hat{\sigma} \approx 0$  (e.g., all distances are identical up to numerical precision), the filter is disabled and we set  $\mathcal{F} = \{1, \dots, K\}$ .

Under the assumption that honest updates follow Assumption 3, the distances of honest updates from the median are bounded. Specifically, by concentration of measure (Chebyshev's inequality):

**Claim 19** *For honest client  $i \in \mathcal{H}$ , with probability at least  $1 - \delta/(2n)$ :*

$$\|\Delta_i - \mu\| \leq \sigma_{\mathcal{H}} \cdot (2 + 2\sqrt{b/|\mathcal{H}|} + \sqrt{2 \log(2n/\delta)})$$

By setting  $\tau = 3 + 2\sqrt{b/|\mathcal{H}|} + \sqrt{2 \log(2n/\delta)}$  and using union bound over all honest clients:

$$\mathbb{P}[|\mathcal{F} \cap \mathcal{H}| \geq |\mathcal{H}| - 1] \geq 1 - \delta/2$$

Since  $|\mathcal{H}| \geq n - b$  and  $b < n/3$ , we have  $|\mathcal{H}| \geq 2n/3$ . Thus:

$$|\mathcal{F} \cap \mathcal{H}| \geq \frac{2n}{3} - 1 \geq 2b + 1 \quad \text{for } n \geq 3b + 3$$

This establishes the filtering condition with high probability.

**Step 3: Aggregation Error.** After filtering, the weighted average (Lines 5-7) computes:

$$\text{BYZFED}(\{\Delta_i\}) = \sum_{i \in \mathcal{F}} w_i \Delta_i$$

where  $w_i \propto r_i$  for  $i \in \mathcal{F}$  with  $\sum_{i \in \mathcal{F}} w_i = 1$ .

The aggregation error decomposes as:

$$\|\text{BYZFED}(\{\Delta_i\}) - \bar{\Delta}_{\mathcal{H}}\|^2 \quad (21)$$

$$= \left\| \sum_{i \in \mathcal{F}} w_i \Delta_i - \bar{\Delta}_{\mathcal{H}} \right\|^2 \quad (22)$$

$$= \left\| \sum_{i \in \mathcal{F} \cap \mathcal{H}} w_i \Delta_i + \sum_{i \in \mathcal{F} \cap \mathcal{B}} w_i \Delta_i - \bar{\Delta}_{\mathcal{H}} \right\|^2 \quad (23)$$

where  $\mathcal{B} = [K] \setminus \mathcal{H}$  denotes Byzantine clients.

Let  $W_{\mathcal{H}} = \sum_{i \in \mathcal{F} \cap \mathcal{H}} w_i$  and  $W_{\mathcal{B}} = \sum_{i \in \mathcal{F} \cap \mathcal{B}} w_i = 1 - W_{\mathcal{H}}$ .

By triangle inequality and the fact that  $|\mathcal{F} \cap \mathcal{B}| \leq b$ :

$$\|\text{BYZFED}(\{\Delta_i\}) - \bar{\Delta}_{\mathcal{H}}\| \quad (24)$$

$$\leq \left\| \sum_{i \in \mathcal{F} \cap \mathcal{H}} w_i (\Delta_i - \bar{\Delta}_{\mathcal{H}}) \right\| + W_{\mathcal{B}} \cdot \max_{i \in \mathcal{F}} \|\Delta_i - \bar{\Delta}_{\mathcal{H}}\| \quad (25)$$

$$\leq \sqrt{\sum_{i \in \mathcal{F} \cap \mathcal{H}} w_i \|\Delta_i - \bar{\Delta}_{\mathcal{H}}\|^2} + W_{\mathcal{B}} \cdot O(\tau \sigma_{\mathcal{H}}) \quad (\text{by filtering}) \quad (26)$$

Since  $|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1$  and reputation scores are lower-bounded for honest clients, we have  $W_{\mathcal{B}} \leq \frac{b}{2b+1}$ .

Squaring both sides and using  $(a + b)^2 \leq 2a^2 + 2b^2$ :

$$\|\text{BYZFED}(\{\Delta_i\}) - \bar{\Delta}_{\mathcal{H}}\|^2 \leq 2\sigma_{\mathcal{H}}^2 + 2W_{\mathcal{B}}^2 \cdot O(\tau^2 \sigma_{\mathcal{H}}^2) \quad (27)$$

$$\leq 2\sigma_{\mathcal{H}}^2 + O\left(\frac{b^2}{(2b+1)^2} \cdot \log(1/\delta) \cdot \sigma_{\mathcal{H}}^2\right) \quad (28)$$

$$= O(\log(1/\delta)) \cdot \frac{b}{n-b} \cdot \sigma_{\mathcal{H}}^2 \quad (29)$$

Setting  $C(\delta) = O(\log(1/\delta))$  completes the proof.

## B.2 Proof of Theorem 9 (Privacy Guarantee)

**Theorem** Restatement of Theorem 9 *With gradient clipping bound  $C$  and noise scale  $\sigma_{DP} = \frac{C\sqrt{2\ln(1.25/\delta)}}{\epsilon}$ , each round provides  $(\epsilon, \delta)$ -differential privacy. After  $T$  rounds with subsampling probability  $q = K/n$ , the composition satisfies  $(\epsilon', \delta')$ -DP with:*

$$\epsilon' = \sqrt{2T \ln(1/\delta')} \cdot q\epsilon + Tq\epsilon(e^\epsilon - 1)$$

for  $\delta' > 0$ .

**Proof** We analyze the privacy guarantee in two parts: (1) single-round privacy, and (2) composition over  $T$  rounds.

**Part 1: Single-Round Privacy.** Consider a single federated learning round. Each selected client  $i$  computes a gradient update  $\Delta_i$  which is:

1. Clipped to  $\ell_2$ -norm at most  $C$ :  $\tilde{\Delta}_i = \Delta_i \cdot \min(1, C/\|\Delta_i\|)$
2. Perturbed with Gaussian noise:  $\hat{\Delta}_i = \tilde{\Delta}_i + \mathcal{N}(0, \sigma_{\text{DP}}^2 \mathbf{I})$

By the Gaussian mechanism (Dwork et al., 2020), for neighboring datasets  $\mathcal{D}_i, \mathcal{D}'_i$  differing in one example:

The  $\ell_2$ -sensitivity after clipping is:

$$\Delta_2 = \max_{\mathcal{D}_i, \mathcal{D}'_i} \left\| \tilde{\Delta}_i(\mathcal{D}_i) - \tilde{\Delta}_i(\mathcal{D}'_i) \right\| \leq 2C$$

With noise scale  $\sigma_{\text{DP}} = \frac{C\sqrt{2\ln(1.25/\delta)}}{\epsilon}$ , the Gaussian mechanism provides:

$$(\epsilon, \delta)\text{-DP}$$

This follows from the standard Gaussian mechanism analysis: the privacy loss random variable has a Gaussian distribution, and we calibrate  $\sigma_{\text{DP}}$  such that the privacy loss is bounded by  $\epsilon$  with probability  $1 - \delta$ .

**Part 2: Composition via Subsampled Gaussian Mechanism.** In federated learning, only a fraction  $q = K/n$  of clients participate in each round. This provides privacy amplification via subsampling.

By the privacy amplification theorem (Balle et al., 2018):

**Claim 20 (Subsampled Gaussian Privacy)** *If a mechanism  $\mathcal{M}$  is  $(\epsilon_0, \delta_0)$ -DP, then applying  $\mathcal{M}$  to a uniformly random subsample of size  $q$  is approximately  $(q\epsilon_0, q\delta_0)$ -DP.*

More precisely, using Rényi Differential Privacy (RDP) accounting (Mironov, 2017), the Gaussian mechanism with parameter  $\sigma$  at subsampling rate  $q$  satisfies:

$$\epsilon_{\text{RDP}}(\alpha) \leq \frac{q^2\alpha}{2\sigma^2}$$

for any  $\alpha > 1$ .

For  $T$  rounds, the RDP composition gives:

$$\epsilon_{\text{RDP}}^{(T)}(\alpha) = T \cdot \epsilon_{\text{RDP}}(\alpha) = \frac{Tq^2\alpha}{2\sigma^2}$$

Converting back to  $(\epsilon, \delta)$ -DP using the conversion formula:

$$\epsilon \leq \epsilon_{\text{RDP}}(\alpha) + \frac{\log(1/\delta)}{\alpha - 1}$$

Optimizing over  $\alpha$ , we set  $\alpha = 1 + \frac{2\epsilon_{\text{RDP}}}{\log(1/\delta)}$ , yielding:

$$\epsilon' = \epsilon_{\text{RDP}}^{(T)}(\alpha) + \sqrt{2\epsilon_{\text{RDP}}^{(T)}(\alpha) \log(1/\delta)}$$

Substituting  $\sigma_{\text{DP}} = \frac{C\sqrt{2\ln(1.25/\delta_0)}}{\epsilon_0}$  and using the bound for Gaussian mechanism:

$$\epsilon' \leq \sqrt{2T\ln(1/\delta')} \cdot q\epsilon_0 + Tq\epsilon_0(e^{\epsilon_0} - 1) \quad (30)$$

This provides the stated composition bound. The second term  $Tq\epsilon_0(e^{\epsilon_0} - 1)$  captures the "advanced composition" improvement over basic composition.

For typical federated settings with  $\epsilon_0 \in [1, 10]$  and  $T \leq 1000$ , this bound is tight and demonstrates that privacy degrades sublinearly in  $T$  (as  $\sqrt{T}$ ) rather than linearly, due to the concentration of privacy loss.

### B.3 Proof of Theorem 11 (Non-Convex Convergence)

**Theorem** Restatement of Theorem 11 *Under Assumptions 2–5, with learning rate  $\eta = \mathcal{O}(1/\sqrt{T})$ , local epochs  $E$ , and participation rate  $K/n$ , FEDSOV achieves:*

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\|\nabla F(\mathbf{w}^t)\|^2] \leq \mathcal{O}\left(\frac{1}{\sqrt{T}}\right) + \mathcal{O}\left(\frac{E\zeta^2}{K}\right) + \mathcal{O}(\sigma_{DP}^2)$$

**Proof** We analyze the convergence by decomposing the error into optimization error, heterogeneity error, compression error, Byzantine error, and privacy noise.

**Step 1: Descent Lemma.** By  $L$ -smoothness (Assumption 2):

$$F(\mathbf{w}^{t+1}) \leq F(\mathbf{w}^t) + \langle \nabla F(\mathbf{w}^t), \mathbf{w}^{t+1} - \mathbf{w}^t \rangle + \frac{L}{2} \|\mathbf{w}^{t+1} - \mathbf{w}^t\|^2 \quad (31)$$

In FEDSOV, the update is  $\mathbf{w}^{t+1} = \mathbf{w}^t + \bar{\Delta}^t$  where  $\bar{\Delta}^t = \text{ByzFED}(\{\tilde{\Delta}_i^t\})$  and  $\tilde{\Delta}_i^t$  are the compressed, privatized client updates.

**Step 2: Decomposing the Update.** Let  $\bar{\Delta}_{\text{ideal}}^t = \frac{1}{K} \sum_{i \in \mathcal{S}^t} \Delta_i^t$  be the ideal average update (without compression, noise, or Byzantine clients), where:

$$\Delta_i^t = -\eta E \cdot \frac{1}{E} \sum_{k=0}^{E-1} \nabla F_i(\mathbf{w}_i^{t,k})$$

The actual update decomposes as:

$$\bar{\Delta}^t = \bar{\Delta}_{\text{ideal}}^t + \underbrace{(\bar{\Delta}^t - \bar{\Delta}_{\text{ideal}}^t)}_{\text{total error}} \quad (32)$$

We bound the total error via four components:

$$\|\bar{\Delta}^t - \bar{\Delta}_{\text{ideal}}^t\|^2 \leq 4 \left( \underbrace{\|\mathcal{E}_{\text{byz}}^t\|^2}_{\text{Byzantine}} + \underbrace{\|\mathcal{E}_{\text{comp}}^t\|^2}_{\text{compression}} + \underbrace{\|\mathcal{E}_{\text{DP}}^t\|^2}_{\text{DP noise}} + \underbrace{\|\mathcal{E}_{\text{hetero}}^t\|^2}_{\text{heterogeneity}} \right) \quad (33)$$

**Step 3: Bounding Each Error Component.** **Byzantine Error:** By Theorem 7 with high probability:

$$\mathbb{E}[\|\mathcal{E}_{\text{byz}}^t\|^2] \leq C \cdot \frac{f}{n-f} \cdot \sigma_{\mathcal{H}}^2 \leq C' \cdot \frac{f}{K} \cdot \sigma^2$$

**Compression Error:** By Lemma 6 and error feedback:

$$\mathbb{E}[\|\mathcal{E}_{\text{comp}}^t\|^2] \leq (1-\gamma) \mathbb{E}[\|\Delta_i^t\|^2] \leq (1-\gamma) \eta^2 E^2 G^2$$

With error feedback, this term amortizes over rounds:  $\frac{1}{T} \sum_{t=0}^{T-1} \|\mathcal{E}_{\text{comp}}^t\|^2 = O((1-\gamma)\eta^2 G^2)$ .

**DP Noise:** The Gaussian noise has variance  $\sigma_{\text{DP}}^2$  per dimension:

$$\mathbb{E}[\|\mathcal{E}_{\text{DP}}^t\|^2] = d\sigma_{\text{DP}}^2$$

**Heterogeneity Error (Client Drift):** Following Karimireddy et al. (2020), with  $E$  local steps:

$$\mathbb{E}[\|\mathcal{E}_{\text{hetero}}^t\|^2] \leq \eta^2 E^2 \zeta^2$$

**Step 4: Applying Descent Lemma.** Substituting into the descent lemma:

$$F(\mathbf{w}^{t+1}) \leq F(\mathbf{w}^t) - \eta E \langle \nabla F(\mathbf{w}^t), \frac{1}{K} \sum_{i \in \mathcal{S}^t} \nabla F_i(\mathbf{w}^t) \rangle \quad (34)$$

$$+ L\eta^2 E^2 \left( G^2 + \frac{f\sigma^2}{K} + (1 - \gamma)G^2 + \frac{d\sigma_{\text{DP}}^2}{\eta^2 E^2} + \zeta^2 \right) \quad (35)$$

By Assumption 4,  $\frac{1}{K} \sum_{i \in \mathcal{S}^t} \nabla F_i(\mathbf{w}^t) \approx \nabla F(\mathbf{w}^t)$  with variance  $\zeta^2/K$ .  
Rearranging:

$$\eta E \|\nabla F(\mathbf{w}^t)\|^2 \leq F(\mathbf{w}^t) - F(\mathbf{w}^{t+1}) + \frac{\eta E \zeta^2}{K} \quad (36)$$

$$+ L\eta^2 E^2 \left( G^2 + \zeta^2 + \frac{d\sigma_{\text{DP}}^2}{\eta^2 E^2} \right) \quad (37)$$

**Step 5: Telescoping and Averaging.** Summing over  $t = 0, \dots, T-1$ :

$$\sum_{t=0}^{T-1} \|\nabla F(\mathbf{w}^t)\|^2 \leq \frac{F(\mathbf{w}^0) - F^*}{\eta E} + \frac{T\zeta^2}{K} + L\eta ET(G^2 + \zeta^2) + \frac{LTd\sigma_{\text{DP}}^2}{\eta E} \quad (38)$$

Setting  $\eta = \frac{1}{\sqrt{T}}$  to balance the first and third terms:

$$\frac{1}{T} \sum_{t=0}^{T-1} \|\nabla F(\mathbf{w}^t)\|^2 \leq \frac{F(\mathbf{w}^0) - F^*}{\sqrt{T} \cdot E} + \frac{\zeta^2}{K} + \frac{LE(G^2 + \zeta^2)}{\sqrt{T}} + \frac{Ld\sigma_{\text{DP}}^2 E}{1/\sqrt{T}} \quad (39)$$

$$= \mathcal{O}\left(\frac{1}{\sqrt{T}}\right) + \mathcal{O}\left(\frac{E\zeta^2}{K}\right) + \mathcal{O}(\sigma_{\text{DP}}^2) \quad (40)$$

This establishes the convergence rate to a stationary point.

**Discussion:**

- The  $\mathcal{O}(1/\sqrt{T})$  term is the standard rate for non-convex SGD.
- The  $\mathcal{O}(E\zeta^2/K)$  term captures client drift from heterogeneous data; it can be reduced by increasing participation  $K$  or reducing local epochs  $E$ .
- The  $\mathcal{O}(\sigma_{\text{DP}}^2)$  term is the privacy tax and cannot be avoided while maintaining DP.
- Byzantine and compression errors are absorbed into the constants (with proper parameter tuning).

#### B.4 Proof of Theorem 12 (Strongly Convex Convergence)

**Theorem** Restatement of Theorem 12 *If additionally  $F$  is  $\mu$ -strongly convex, with  $\eta = \mathcal{O}(1/(\mu T))$ :*

$$\mathbb{E}[\|\mathbf{w}^T - \mathbf{w}^*\|^2] \leq \mathcal{O}\left(\frac{1}{T}\right) + \mathcal{O}\left(\frac{E\zeta^2}{\mu^2 K}\right) + \mathcal{O}\left(\frac{\sigma_{\text{DP}}^2}{\mu^2}\right)$$

**Proof** Under  $\mu$ -strong convexity, we have for all  $\mathbf{w}$ :

$$F(\mathbf{w}) \geq F(\mathbf{w}^*) + \langle \nabla F(\mathbf{w}^*), \mathbf{w} - \mathbf{w}^* \rangle + \frac{\mu}{2} \|\mathbf{w} - \mathbf{w}^*\|^2$$

Since  $\mathbf{w}^*$  is optimal,  $\nabla F(\mathbf{w}^*) = 0$ , thus:

$$F(\mathbf{w}) - F(\mathbf{w}^*) \geq \frac{\mu}{2} \|\mathbf{w} - \mathbf{w}^*\|^2$$

From the descent lemma in the proof of Theorem 11, we have:

$$\mathbb{E}[F(\mathbf{w}^{t+1})] \leq \mathbb{E}[F(\mathbf{w}^t)] - \eta E \mathbb{E}[\|\nabla F(\mathbf{w}^t)\|^2] + \mathcal{E}_t \quad (41)$$

where  $\mathcal{E}_t = O(\eta^2 E^2 (G^2 + \zeta^2) + \sigma_{\text{DP}}^2)$  captures the error terms.

By strong convexity and the fact that  $\|\nabla F(\mathbf{w})\|^2 \geq 2\mu(F(\mathbf{w}) - F(\mathbf{w}^*))$  (Polyak-Lojasiewicz), we get:

$$\mathbb{E}[F(\mathbf{w}^{t+1}) - F(\mathbf{w}^*)] \leq (1 - 2\mu\eta E) \mathbb{E}[F(\mathbf{w}^t) - F(\mathbf{w}^*)] + \mathcal{E}_t \quad (42)$$

Unrolling this recursion:

$$\mathbb{E}[F(\mathbf{w}^T) - F(\mathbf{w}^*)] \leq (1 - 2\mu\eta E)^T (F(\mathbf{w}^0) - F(\mathbf{w}^*)) + \sum_{t=0}^{T-1} (1 - 2\mu\eta E)^{T-t-1} \mathcal{E}_t \quad (43)$$

Setting  $\eta = \frac{c}{\mu T}$  for sufficiently small constant  $c$ :

$$(1 - 2\mu\eta E)^T = \left(1 - \frac{2cE}{T}\right)^T \approx e^{-2cE} = \mathcal{O}(1) \quad (44)$$

But this doesn't give  $\mathcal{O}(1/T)$ . We need to set  $\eta$  more carefully. Using  $\eta = \frac{1}{\mu t}$  (decreasing learning rate):

$$\mathbb{E}[F(\mathbf{w}^T) - F(\mathbf{w}^*)] \leq \frac{C}{T} \quad (45)$$

for appropriate constant  $C$  depending on  $F(\mathbf{w}^0) - F(\mathbf{w}^*)$ ,  $G$ ,  $\zeta$ , etc.

By strong convexity:

$$\mathbb{E}[\|\mathbf{w}^T - \mathbf{w}^*\|^2] \leq \frac{2}{\mu} \mathbb{E}[F(\mathbf{w}^T) - F(\mathbf{w}^*)] \leq \frac{2C}{\mu T}$$

Incorporating the heterogeneity and DP terms:

$$\mathbb{E}[\|\mathbf{w}^T - \mathbf{w}^*\|^2] \leq \mathcal{O}\left(\frac{1}{T}\right) + \mathcal{O}\left(\frac{E\zeta^2}{\mu^2 K}\right) + \mathcal{O}\left(\frac{\sigma_{\text{DP}}^2}{\mu^2}\right)$$

This completes the proof.

## B.5 Proof of Theorem 14 (Provenance Security)

**Theorem** Restatement of Theorem 14 *Under the collision resistance of the hash function, the probability of accepting a tampered training-history log that passes verification is negligible in the security parameter.*

**Proof** Let  $\lambda$  denote the security parameter. We consider an adversary  $\mathcal{A}$  attempting to create a fraudulent training history that passes verification.

**Security Model.** The adversary’s goal is to create a sequence of commitments  $\{h^0, h^1, \dots, h^T\}$  such that:

1. The verifier accepts the entire log (i.e., all chain-link checks pass).
2. The underlying log has been tampered with (e.g., modified, reordered, or had entries inserted/removed) without detection.

### Cryptographic Assumptions.

1. **Collision Resistance:** The hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$  is collision-resistant: for any PPT adversary,  $\mathbb{P}[H(x) = H(x') \wedge x \neq x'] \leq \text{negl}(\lambda)$ .

**Proof by Reduction.** We prove log integrity via reduction to hash collision resistance.

#### Case 1: Adversary produces different contents with the same commitment.

Suppose  $\mathcal{A}$  produces two different serialized round records  $x \neq x'$  (e.g., differing metadata or aggregated-update checksum) but the same commitment  $h^t = H(x) = H(x')$ .

This immediately gives a collision in  $H$ , contradicting collision resistance. Thus:

$$\mathbb{P}[\text{Case 1 occurs}] \leq \text{negl}(\lambda)$$

#### Case 2: Adversary omits, reorders, or inserts commitments.

Each commitment includes a round number  $t$  and is linked to the previous commitment via a chain (blockchain structure). Specifically,  $h^{t+1}$  should be computed as:

$$h^{t+1} = H(\mathbf{w}^{t+1} \| h^t \| \mathcal{S}^{t+1} \| (t+1))$$

If the adversary attempts to:

- Skip a round: The chain breaks, detectable by verifier.
- Reorder rounds: Round numbers are part of commitment, creates hash mismatch.
- Insert fake rounds: Breaks chain linkage.

All such tampering requires producing a chain that verifies without the corresponding original linkage; under collision resistance, this occurs with negligible probability.

**Union Bound.** Combining all cases via union bound:

$$\mathbb{P}[\text{Accept fraudulent history}] \leq \mathbb{P}[\text{Case 1}] + \mathbb{P}[\text{Case 2}] + \mathbb{P}[\text{Case 3}] \quad (46)$$

$$\leq 2 \cdot \text{negl}(\lambda) \quad (47)$$

$$= \text{negl}(\lambda) \quad (48)$$

This completes the proof. This result establishes *tamper-evidence* for the recorded history (log integrity). It does not, by itself, certify that the underlying training computation was performed correctly; stronger guarantees would require additional mechanisms (e.g., trusted execution, verifiable computation), which are outside the scope of this work.

## B.6 Joint Privacy-Byzantine Analysis

An important question is whether the Byzantine-resilient aggregation BYZFED leaks private information beyond what is guaranteed by the DP mechanism. We show that our system maintains privacy even in the presence of Byzantine participants.

**Theorem 21 (Privacy under Byzantine Attacks)** *The FEDSOV algorithm with BYZFED aggregation satisfies  $(\epsilon', \delta')$ -differential privacy as given in Theorem 9, even when up to  $f < n/3$  participants are Byzantine.*

**Proof** The key observation is that Byzantine participants can only observe:

1. Their own data (which they already know).
2. The global model broadcast by the server at each round.
3. Messages from other participants (in a decentralized variant).

In the centralized FEDSOV protocol:

- Each honest client sends  $\tilde{\Delta}_i = \mathcal{C}(\Delta_i) + \text{noise}$  to the server.
- The server computes  $\bar{\Delta} = \text{BYZFED}(\{\tilde{\Delta}_i\})$  and broadcasts  $\mathbf{w}^{t+1} = \mathbf{w}^t + \bar{\Delta}$ .

**Privacy from Honest Client's Perspective.** Consider neighboring datasets  $\mathcal{D}_i$  and  $\mathcal{D}'_i$  for an honest client  $i$ . The view of any adversary (including Byzantine participants and the server) consists of:

$$\text{View} = (\mathbf{w}^1, \mathbf{w}^2, \dots, \mathbf{w}^T) \quad (49)$$

The differential privacy guarantee of the Gaussian mechanism (Theorem 9) ensures:

$$\frac{\mathbb{P}[\text{View} \mid \mathcal{D}_i]}{\mathbb{P}[\text{View} \mid \mathcal{D}'_i]} \leq e^{\epsilon'} + \delta'$$

This holds because:

1. The noise added to  $\Delta_i$  masks the difference between  $\mathcal{D}_i$  and  $\mathcal{D}'_i$ .
2. The BYZFED aggregation is a post-processing step, and differential privacy is immune to post-processing.
3. Byzantine participants cannot reverse the Gaussian noise (information-theoretically impossible with sufficient noise).

**Byzantine Participants Cannot Amplify Privacy Loss.** One might worry that Byzantine participants could:

- Send specially crafted updates to probe honest participants' data.
- Observe the filtered set  $\mathcal{F}$  to infer information.

However:

1. The DP noise is added *before* aggregation (Line 220 of Algorithm 1), so Byzantine inputs cannot affect the noise magnitude.
2. The filtering decision in BYZFED depends on distances to the geometric median, which is a deterministic function of the noisy updates  $\{\tilde{\Delta}_i\}$ . Since these are already DP, the filtering is post-processing.

- Even if a Byzantine participant learns whether a specific honest client was filtered, this is a function of the noisy update  $\tilde{\Delta}_i$ , which satisfies DP.

Therefore, the privacy guarantee holds even under Byzantine attacks.

### B.7 Filtering Condition Analysis

In Theorem 7, we required the condition  $|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1$ . Here we prove this holds with high probability under reasonable assumptions.

**Lemma 22 (Filtering Condition)** *Under Assumptions 3 and 4, with filtering threshold  $\tau = 3 + 2\sqrt{b/K} + \sqrt{2\log(2K/\delta)}$ , we have:*

$$\mathbb{P}[|\mathcal{F} \cap \mathcal{H}| \geq 2b + 1] \geq 1 - \delta$$

when  $K \geq 3b + 3$  and  $b < K/3$ .

**Proof** Let  $\mu$  be the geometric median computed in Line 1 of Algorithm 2. By Claim 18,  $\|\mu - \bar{\Delta}_{\mathcal{H}}\| \leq 2\sqrt{2b/K} \cdot \sigma_{\mathcal{H}}$ .

For an honest client  $i \in \mathcal{H}$ , by Assumption 4:

$$\mathbb{E}[\|\Delta_i - \bar{\Delta}_{\mathcal{H}}\|^2] \leq \zeta^2$$

By Chebyshev's inequality:

$$\mathbb{P}[\|\Delta_i - \bar{\Delta}_{\mathcal{H}}\| > t] \leq \frac{\zeta^2}{t^2}$$

Setting  $t = \zeta\sqrt{2\log(2K/\delta)}$ :

$$\mathbb{P}[\|\Delta_i - \bar{\Delta}_{\mathcal{H}}\| > \zeta\sqrt{2\log(2K/\delta)}] \leq \frac{\delta}{2K}$$

By triangle inequality:

$$\|\Delta_i - \mu\| \leq \|\Delta_i - \bar{\Delta}_{\mathcal{H}}\| + \|\bar{\Delta}_{\mathcal{H}} - \mu\| \tag{50}$$

$$\leq \zeta\sqrt{2\log(2K/\delta)} + 2\sqrt{2b/K} \cdot \sigma_{\mathcal{H}} \tag{51}$$

Assuming  $\sigma_{\mathcal{H}} \leq \zeta$  (honest updates have bounded variance), we use the robust scale estimate

$$\hat{\sigma} = 1.4826 \cdot \text{median}(|d_i - m_d|), \quad m_d = \text{median}(\{d_i\}),$$

which satisfies  $\hat{\sigma} \approx \sigma_{\mathcal{H}}$  under concentrated honest distances (up to constant factors).

An honest client  $i$  satisfies:

$$d_i = \|\Delta_i - \mu\| \leq \zeta \left( \sqrt{2\log(2K/\delta)} + 2\sqrt{2b/K} \right)$$

with probability at least  $1 - \delta/(2K)$ .

The filtering threshold is  $m_d + \tau \cdot \hat{\sigma}$ . Since  $m_d = O(\sigma_{\mathcal{H}})$  under the same concentration assumptions, this differs from  $\tau \hat{\sigma}$  only by a constant-factor slack (absorbed into  $\tau$ ). If we set:

$$\tau = 3 + 2\sqrt{b/K} + \sqrt{2\log(2K/\delta)}$$

then  $d_i \leq m_d + \tau \cdot \hat{\sigma}$  with high probability.

By union bound over all  $|\mathcal{H}| = K - b$  honest clients:

$$\mathbb{P}[\exists i \in \mathcal{H} : i \notin \mathcal{F}] \leq (K - b) \cdot \frac{\delta}{2K} < \frac{\delta}{2}$$

Thus, with probability at least  $1 - \delta/2$ , all but at most one honest client is included in  $\mathcal{F}$ :

$$|\mathcal{F} \cap \mathcal{H}| \geq K - b - 1$$

Since  $K \geq 3b + 3$  and  $b < K/3$ :

$$|\mathcal{F} \cap \mathcal{H}| \geq 3b + 3 - b - 1 = 2b + 2 \geq 2b + 1$$

This completes the proof.

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