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ABSTRACT

With the advent of DeepSeek-R1, a new wave of reinforcement learning (RL) methods has emerged that seem to unlock stronger mathematical reasoning. However, a closer look at the open-source ecosystem reveals a critical limitation: with sufficiently many draws (e.g., pass@1024), many existing base models already solve nearly all questions on widely used math benchmarks such as MATH-500 and AIME 2024. This suggests that the RL fine-tuning methods prevalent in the LLM reasoning literature largely sharpen existing solution modes rather than discovering entirely new ones. Such sharpening stands in contrast to the broader promise of RL: to foster exploration and to acquire new skills. To move beyond this plateau, we introduce MATH-Beyond (MATH-B), a benchmark deliberately constructed to defeat common open-source models of up to 8B parameters even under large sampling budgets. Improving performance on our benchmark via RL requires methods that learn to reason in ways that go beyond base model capabilities in repeated sampling. Since the problems are drawn from subsets of DAPO-Math-17K and DeepScaleR datasets, they remain topically equivalent to standard high-school math. Validating our premise, RL fine-tuned models such as Nemotron-Research-Reasoning-Qwen-1.5B and DeepScaleR-1.5B-Preview perform poorly on MATH-B at pass@1024, showing how existing approaches fall short on tackling harder instances. We hope MATH-B will catalyze exploration-driven RL approaches that elicit deeper reasoning capabilities. We release MATH-B at <https://huggingface.co/datasets/brendel-group/MATH-Beyond>.

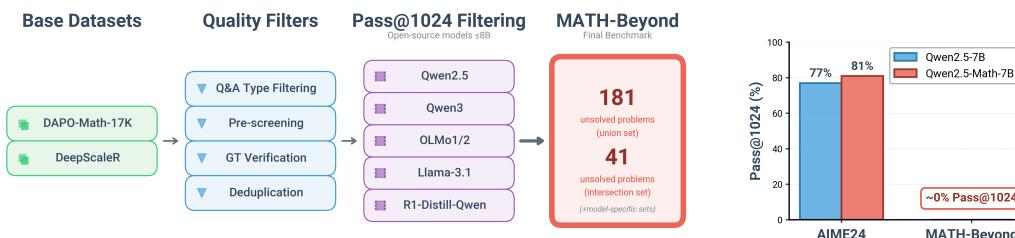


Figure 1: **MATH-Beyond: Benchmark Construction and Difficulty.** *Left:* Schematic of the MATH-B creation process. A large set of problems from DAPO-Math-17K and DeepScaleR is first refined through quality filters to ensure answer correctness and verifiability. This is followed by evaluation against a gauntlet of open-source base models ($\leq 8B$, e.g., Qwen3, Qwen2.5 (-Math), DeepSeek-R1-Distill) at a pass@1024 budget to isolate problems that lie beyond their limits. The filtering yields the MATH-B suite of benchmarks: a 41-problem intersection set (unsolved by all base models) for evaluating universal difficulty, and a larger 181-problem union set (unsolved by at least one model) with model-specific splits for targeted analysis. This suite provides a rigorous testbed to drive the development of exploration methods for RL. *Right:* An illustration of MATH-B’s significant difficulty compared to common test sets like AIME24. Representative open-source models like Qwen2.5 achieve near-zero pass@1024 scores on MATH-B, highlighting its difficulty. Qwen2.5 results are from [Yue et al. \(2025\)](#).

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In the 2010s, deep reinforcement learning showcased its power through striking demonstrations
of exploration and skill acquisition (Mnih et al., 2013). Atari agents, starting from random play,
mastered complex games by discovering strategies unreachable to base policies, guided by exploration
incentives and intrinsic rewards (Ladosz et al., 2022; Amin et al., 2021). Methods such as count-based
exploration (Bellemare et al., 2016) and later Go-Explore (Ecoffet et al., 2021) drove dramatic jumps
from inept play to expertise, highlighting RL’s ability to uncover new capabilities. Around the same
time, AlphaGo (Silver et al., 2016) and AlphaZero (Silver et al., 2017) extended this promise to board
games like Go, Chess, and Shogi, where self-play took agents from scratch to superhuman mastery,
revealing novel strategies along the way.065
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Against this backdrop, academic progress in reasoning-focused LLMs has taken a very different
path. Community-trained models often show improved accuracy on popular benchmarks such as
MATH or AIME24 (Liu et al., 2025b; Song et al., 2025; Chen et al., 2025; Cheng et al., 2025; Cui
et al., 2025; Shao et al., 2025; Wang et al., 2025; Yu et al., 2025b). However, these RL models
typically succeed only on problems that their corresponding base models could already solve given
realistic sampling budgets (Wu et al., 2025; Yue et al., 2025). This is a far cry from earlier RL
successes, where base policies were incapable of solving tasks outright and progress required genuine
exploration and skill acquisition. This disconnect between RL’s exploratory promise and its current
application reflects a substantial blindspot in the current open-source evaluation ecosystem. Because
several open-source base models already achieve nearly 100% pass@1024 on several popular
benchmarks (Yue et al., 2025), test sets in their current form are fundamentally inadequate for
measuring—or encouraging—genuine progress in reasoning beyond the base model’s reach.077
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To address this gap, we introduce **MATH-Beyond (MATH-B)**, a new benchmark of high-
school-level competition math problems, specifically constructed so that popular open-weight base
models are unlikely to solve even with 1024 attempts. As a result, progress on MATH-B necessarily
requires expanding the reasoning capabilities of base models, making it an ideal target for academic
research.082
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MATH-B is constructed by filtering mathematical reasoning datasets (DAPO-Math-17K (Yu et al.,
2025b) and DeepScaleR (Luo et al., 2025)), resulting in problems that are topically indistinguishable
from those in standard benchmarks. While constructing the dataset, we also uncovered and addressed
several non-obvious failure modes in programmatic verification, which informed our final benchmark
design. To ensure correctness, all problems are additionally verified against stronger reasoning models
such as GPT-5-Mini or o4-mini-high, which reliably solve them. We further confirm that leading
community RL models, including Nemotron-Research-Reasoning-Qwen-1.5B (Liu et al., 2025a) and
DeepScaleR-1.5B, perform poorly on MATH-B, underscoring the limitations of current approaches
and the need for methods that extend reasoning capabilities. In summary, our contributions are as
follows:092
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• **New benchmark suite:** We construct MATH-Beyond, a benchmark suite derived from the
failures of a large and diverse set of base models ($\text{pass}@1024 \approx 0$). This suite includes:
a union set of 181 problems unsolved by at least one of these models; model-specific
benchmarks for targeted analysis; and a highly challenging intersection set of 41 problems
that proved unsolvable for the entire considered set. To ensure quality, all problems are
annotated for topic and difficulty following the procedure from Omni-MATH (Gao et al.,
2024a), and undergo answer verification by frontier models (o4-mini-high & GPT5-Mini).
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• **Verification pitfalls:** During benchmark construction, we observed several pitfalls in
standard RLVR verification. We take these into account in our benchmark design and also
document them, highlighting subtle edge cases for the community to be aware of.
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• **Model evaluation:** We evaluate RL-finetuned models such as Nemotron-Research-
Reasoning-Qwen-1.5B, DeepScaleR-1.5B, and Skywork-OR1 (He et al., 2025a) on MATH-
B and find that they do not substantially expand reasoning boundaries. In contrast, we find
that newer model families like Qwen3-4B and Qwen3-8B (Yang et al., 2025) perform better,
presumably due to better distributional overlap with our dataset.

108 2 FRAMEWORK FOR EVALUATING MODEL EXPANSION 109

110 Our goal is to quantify whether a post-trained model (e.g., after RLV) *expands* its reasoning
111 capabilities beyond its base model. While post-training can affect many aspects (e.g., robustness,
112 exploration, or transfer), our framework is designed to isolate the specific phenomenon of *boundary*
113 *expansion*—what new problems the post-trained model can solve relative to its base model. This
114 section formalizes this evaluation framework, adapting the nomenclature and definitions from Wu
115 et al. (2025) into a slightly simplified, empirical version. We apply this framework to our benchmark
116 MATH-B, which is a “*zero-baseline*” test. We define this as a setting where a benchmark is
117 specifically constructed such that the base model empirically fails on all problems (i.e., its observed
118 pass@k is zero) given a realistic sampling budget.

119 2.1 THE EMPIRICAL PASS@K METRIC 120

121 We evaluate a post-trained policy π against its base model q on a dataset D . Our evaluation is based
122 on the empirical pass@k metric. For a given problem $x \in D$ and a policy $p \in \{\pi, q\}$ (usually an
123 LLM), we draw k i.i.d. samples $\{y_1, \dots, y_k\}$. Let $\mathcal{C}(x)$ be the set of all correct completions for x .
124 The empirical success is:

$$125 \quad \text{pass@k}(p; x) = \begin{cases} 1 & \text{if } \exists i \in \{1, \dots, k\} \text{ such that } y_i \in \mathcal{C}(x), \\ 126 & \\ 127 0 & \text{otherwise.} \end{cases}$$

128 This binary value—1 for a “pass” and 0 for a “fail”—is the ground-truth measure of success for a
129 single problem.

130 With a slight abuse of notation, the central metric reported in our paper is the average success rate
131 across the entire dataset D :

$$132 \quad \text{pass@k}(p) = \frac{1}{|D|} \sum_{x \in D} \text{pass@k}(p; x),$$

133 the **ratio of problems in D solved by policy p** using a k -sample budget.

134 2.2 DECOMPOSITION AND KEY METRICS 135

136 To understand π ’s performance differs from q ’s, we first define the **Reachable Set** $\mathcal{R}_k(p, D)$ as
137 the set of problems p solves:

$$138 \quad \mathcal{R}_k(p, D) = \{x \in D : \text{pass@k}(p; x) = 1\}.$$

139 Comparing the reachable sets of π and q allows us to isolate our primary metric and contextualize it.

140 **Expansion (Primary Metric).** Our *primary focus* and sole reported metric is the **Expansion Rate**.
141 This measures genuine boundary expansion—new problems π solves that q could not. It is defined
142 based on the **Expansion Set** $\mathcal{E}_k = \mathcal{R}_k(\pi, D) \setminus \mathcal{R}_k(q, D)$:

$$143 \quad \text{Expansion Rate} = \frac{|\mathcal{E}_k|}{|D|}.$$

144 **Shrinkage.** For diagnostic context, we also define *Shrinkage* (or “forgetting”). This measures
145 problems q could solve that π now fails, defined by the *Shrinkage Set* $\mathcal{S}_k = \mathcal{R}_k(q, D) \setminus \mathcal{R}_k(\pi, D)$. It
146 can be quantified as:

$$147 \quad \text{Shrinkage Rate} = \frac{|\mathcal{S}_k|}{|D|}.$$

148 **Preservation.** Similarly, *Preservation* measures the fraction of q ’s capabilities that π retains, defined
149 by the *Preservation Set* $\mathcal{P}_k = \mathcal{R}_k(\pi, D) \cap \mathcal{R}_k(q, D)$. It is quantified as:

$$150 \quad \text{Preservation Rate} = \frac{|\mathcal{P}_k|}{|\mathcal{R}_k(q, D)|}.$$

162 **Consolidation.** Finally, *Consolidation* is a concept for measuring if preserved solutions become
 163 more robust (i.e., solvable at $\text{pass}@1$). It is defined as:

$$165 \quad C_k(\pi, q) = \frac{|\mathcal{P}_k \cap \mathcal{R}_1(\pi, D)|}{|\mathcal{P}_k|}.$$

167 **Interpretation.** The overall pass rate of π decomposes as $\text{pass}@k(\pi) = (|\mathcal{E}_k| + |\mathcal{P}_k|)/|D|$. While
 168 Shrinkage, Preservation, and Consolidation are crucial concepts for a full diagnosis, our target for
 169 *expanding the reasoning boundary* is squarely the **Expansion Rate**. The other concepts serve as a
 170 theoretical guardrail to ensure that measured gains are from genuine expansion, not mere reallocation.
 171

172 2.3 SPECIAL CASE: THE MATH-B BENCHMARK

174 The metrics framework in Section 2.2 simplifies cleanly for MATH-B. This benchmark is constructed
 175 as a “zero-baseline” meaning it is composed of problems where the base model q was empirically
 176 observed to fail within the sampling budget k (i.e. $\text{pass}@k \approx 0$). In our evaluation, this means the
 177 base model’s reachable set is empty::

$$178 \quad \mathcal{R}_k(q, D) = \emptyset.$$

179 Under this premise, the decomposition from Section 2.2 collapses:

$$181 \quad \mathcal{S}_k = \emptyset, \quad \mathcal{P}_k = \emptyset, \quad \mathcal{E}_k = \mathcal{R}_k(\pi, D).$$

182 Shrinkage and preservation are thus not applicable. Every solve by π is by definition an expansion,
 183 and the single relevant metric becomes:

$$185 \quad \text{Expansion Rate} = \frac{|\mathcal{R}_k(\pi, D)|}{|D|} = \text{pass}@k(\pi).$$

187 A “win” on MATH-B is simply a positive Expansion Rate, providing an unambiguous readout of
 188 genuine boundary expansion.

190 3 CREATING THE BENCHMARK

192 3.1 ON THE PITFALLS OF VERIFICATION

194 The goal of our work is to identify questions that models fail at high $\text{pass}@1024$ due to genuine
 195 methodological or calculation errors, not because of artifacts in the verification pipeline. Current
 196 rule-based verification methods for math, used both in training and evaluation, are themselves prone
 197 to systematic failures. These failures can mask true performance by penalizing answers where the
 198 reasoning or ground truth is technically correct but formatted poorly, making it appear as though the
 199 model failed when the verifier simply could not handle the formatting or parsing.

200 To guide our evaluation and benchmark design, and to alert the community to these pitfalls, we sketch
 201 seven distinct failure modes observed across common RL finetuning and evaluation frameworks. We
 202 uncovered these issues through an analysis of reasoning traces produced by DeepSeek-R1-Distill-
 203 Qwen2.5-7B (DeepSeek-AI et al., 2025) when applied to a subset of NuminaMath-1.5 (LI et al.,
 204 2024). Table 1 provides concise but complete examples (problem, ground truth, model output snippet,
 205 and the precise mismatch), and Table 3 in the Appendix summarizes how different frameworks are
 206 affected.

207 3.2 MATH-B CONSTRUCTION PIPELINE

209 Our benchmark is the result of a multi-stage filtering pipeline designed to isolate problems that are
 210 (a) correct and unambiguous, (b) novel, and (c) demonstrably unsolvable by a suite of open-source
 211 base models even at a high sampling budget. The process is detailed below and in Figure 1.

213 3.2.1 BASE DATASET SOURCING

215 Our construction begins by sourcing a large pool of candidate problems. The primary goal, as stated,
 is to find problems that defeat open-source models at high sampling budgets ($\text{pass}@1024$). This

216
 217 **Table 1: Common failure modes in rule-based math answer verification.** These failures often
 218 stem from rigid heuristics, such as reading only the first or last boxed answer, requiring specific text
 219 anchors (e.g., "Answer:"), or other parsing failures. Each row shows the ground truth (GT), a model
 220 snippet, and the resulting verifier error.

Mode	GT	Model snippet	Verifier behavior
F1: Multiple valid	5 or 13	... $x = \boxed{5}$ or $\boxed{13}$	<i>Expected:</i> accept both values. <i>Actual:</i> only first/last boxed read; one valid ignored.
F2: Late correct	150	... $\boxed{50}$; ... $\boxed{100}$; ... $\boxed{150}$	<i>Expected:</i> final value only. <i>Actual:</i> early intermediate captured.
F3: Early correct	42	... $\boxed{42}$... later $\boxed{84}$	<i>Expected:</i> accept any correct boxed answer. <i>Actual:</i> later wrong overrides early correct.
F4: Corrections	25	... $\boxed{20}$, then correcting $\boxed{25}$	<i>Expected:</i> use corrected value. <i>Actual:</i> first match persists; "last-only" works, "first-only" fails.
F5: Unordered tuple	3, 4, 5	... $\boxed{5}, \boxed{3}, \boxed{4}$	<i>Expected:</i> order-agnostic match. <i>Actual:</i> order-sensitive; heuristics misgrade.
F6: Missing anchors	4	... $2 + 2 = \boxed{4}$ (no "Answer:" prefix)	<i>Expected:</i> accept boxed numeral alone. <i>Actual:</i> anchor required; otherwise wrong.
F7: MCQ partial	C) 1000	... "The answer is \boxed{C} ."	<i>Expected:</i> accept label-only (C). <i>Actual:</i> requires exact "C)1000".

244 requirement immediately disqualifies many common datasets. For instance, DeepMath103K (He
 245 et al., 2025b) and Big-Math (Albalak et al., 2025) are explicitly designed to be solvable (e.g., for
 246 GRPO training), and indeed, models like Llama-3.1-8B solve them with few attempts. We also
 247 sought to minimize data contamination, ruling out datasets like NuminaMath which have been seen
 248 in pretraining by models like Qwen2.5-Math (Yang et al., 2024) and consequently DeepSeek-R1.

249 We therefore selected two base datasets: **DAPO-Math-17K** (Yu et al., 2025a), which satisfies our
 250 criteria for both high difficulty (not verified by open-source models) and novelty (released after
 251 DeepSeek-R1), and **DeepScaleR** (Luo et al., 2025), which, while potentially seen, provides a large
 252 corpus of problems that also lack open-source verification. This combined set forms our initial
 253 candidate pool of 53,682 problems.

254 3.2.2 QUALITY FILTERS

255 We then apply an array of quality filters as described in the following paragraphs.

256 **Question and Answer-Type Filtering** To mitigate the verification pitfalls detailed in Section 3.1,
 257 we apply several deterministic filters. First, to avoid ambiguity from failure modes like F1 (Multiple
 258 valid answers), and F5 (Unordered Tuples), we filter DeepScaleR to include only problems with
 259 integer-based ground truths (DAPO-Math-17K already meets this criterion). Second, using regex-
 260 based filters, we remove all multiple-choice questions (to prevent F7 parsing failures) and any
 261 questions containing Chinese characters. Finally, to ensure all problems are self-contained, we
 262 remove any questions referencing external figures or images. This filtering process reduced the pool
 263 to 34,515 datapoints.

264 **Pre-screening and Random Sampling** The problems from the previous step still represent a
 265 computationally prohibitively large set for large-scale evaluation. To reduce this pool, we first perform
 266 a difficulty pre-screening step. We evaluate Deepseek-R1-Distill-Qwen2.5-7B on all problems,

270 keeping only those that it *could not solve* within a `pass@16` budget. In addition to the screening, we
 271 randomly sample a portion of this dataset for further processing.
 272

273 **Ground-Truth Answer Verification** A critical step is to ensure that these problems are unsolved
 274 due to their intrinsic challenge, not because their provided ground-truth answers are incorrect. While
 275 our source datasets are generally reliable (e.g., DeepScaleR is derived from AIME (MAA) and
 276 AMC (MAA, 2023) competitions (Luo et al., 2025)), we took an additional step to ensure the ground-
 277 truth answers are correct. We evaluate (`pass@2`) this pre-screened subset using two frontier-class
 278 models, o4-mini-high and GPT-5-Mini(Balunović et al., 2025). For each problem, we prompt the
 279 models to "*Think step-by-step and put your final answer in \boxed{}*." and check if the extracted
 280 answer matches the dataset's ground truth. We retain only the problems where *at least one* of these
 281 frontier models successfully reproduced the ground-truth answer.
 282

283 **Deduplication Against Standard Benchmarks** To ensure the novelty of our dataset, we first
 284 perform an exact string-match deduplication of our candidate problems against several common
 285 test sets, including MATH-500 (Hendrycks et al., 2021), MinervaMath (Lewkowycz et al., 2022),
 286 OlympiadBench (He et al., 2024), AMC23 (MAA, 2023), AIME-2024, and AIME-2025 (MAA). We
 287 confirmed that no question from our set is present in these benchmarks. At the end of this stage, a
 288 collection of 184 problems remain.
 289

3.2.3 FINAL BENCHMARK CONSTRUCTION (`PASS@1024` FILTERING)

290 The final step in our benchmark's construction is to filter the candidate problems using a diverse suite
 291 of representative open-source models. The definition of a '*base model*' is contextual; it typically
 292 refers to a model intended for further fine-tuning. Keeping in mind the models the community
 293 often uses for post-training research, we select a suite of models categorized as either '*base*' or
 294 '*supplementary*' models. As we will detail, these two groups are used to construct different subsets of
 295 our final benchmark.
 296

297 **Base Models** This set is used to define the most challenging subset of our benchmark. It includes:
 298 Qwen2.5-1.5B, Qwen2.5-7B (Qwen et al., 2025), Qwen2.5-Math-1.5B, Qwen2.5-Math-7B (Yang
 299 et al., 2024), Qwen3-4B-Base, Qwen3-8B-Base (Yang et al., 2025), DeepSeek-R1-Distill-Qwen2.5-
 300 1.5B, DeepSeek-R1-Distill-Qwen2.5-7B (DeepSeek-AI et al., 2025), OLMo-7B (Groeneveld et al.,
 301 2024), OLMo-2-7B (OLMo et al., 2025), and Llama-3.1-8B (Grattafiori et al., 2024).
 302

303 **Supplementary Models** This group is combined with the base models to define the full benchmark.
 304 It includes: Qwen2.5-1.5B-Instruct, Qwen2.5-7B-Instruct (Qwen et al., 2025), Qwen2.5-Math-1.5B-
 305 Instruct, Qwen2.5-Math-7B-Instruct (Yang et al., 2024), Qwen3-4B, Qwen3-8B (Yang et al., 2025),
 306 DeepScaler-1.5B (Luo et al., 2025), Nemotron-Research-Reasoning-Qwen-1.5B(v1 and v2) (Liu
 307 et al., 2025a), and Skywork-OR1-7B (He et al., 2025a).
 308

309 The deduplicated and pre-screened candidate set is then subjected to our final filtering stage. We
 310 evaluate the problems against *all listed models* generating 1024 samples for each. During this
 311 evaluation, we apply our robust verification logic (see Table 1 for failure modes) to ensure we were
 312 measuring genuine reasoning failures. Our experiments and analyses required over 20 000 A100
 313 GPU hours. Please refer to Appendix A.2 for more details on the evaluation parameters.
 314

315 This comprehensive evaluation allows us to define our final benchmark, **MATH-B**, which comprises
 316 three broad subsets for targeted analysis:
 317

- 318 • **MATH-Beyond-Union Set (MATH-B-U):** The full benchmark of **181 problems**, containing
 319 any problem that at least one model from our *entire suite* (both base and post-trained) failed
 320 to solve within 1024 attempts.
- 321 • **MATH-Beyond-Intersection Set (MATH-B-I):** A more challenging core subset of **41**
 322 **problems**, that *all* of our considered **base models** failed to solve. This is a hard subset of
 323 MATH-B-U (see Table 5 for the QA pairs).
- 324 • **Model-Specific Sets:** For any given model in our suite, this is the collection of all problems
 325 in the Union Set that it failed to solve (see Table 4). These sets enable fine-grained testing
 326 of RL finetuned models derived from one of the evaluated base models.

324 Overall, our MATH-B datasets serve as benchmarks of reasoning capability expansion (see Section 2).
 325

326 **3.3 DATA CHARACTERISTICS AND KEY REMARKS**
 327

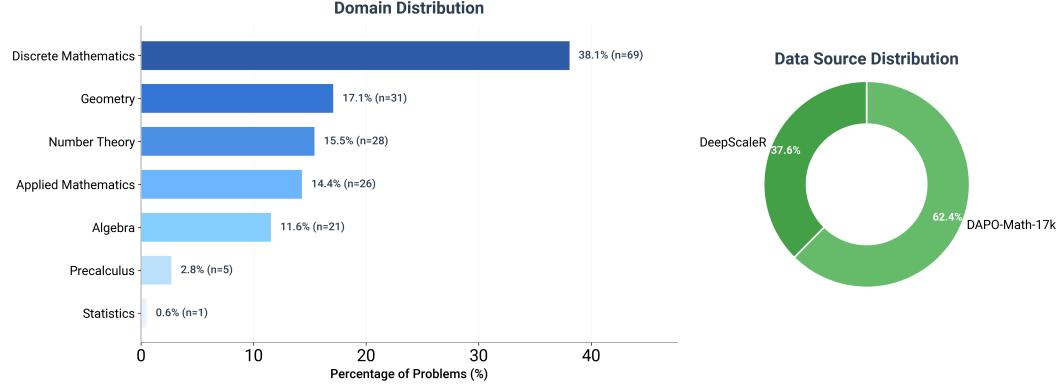


Figure 2: **Characteristics of the MATH-B-U dataset.** Left subplot shows the distribution of math domains. Right subplot show the distribution of source datasets.

346 **Domain and Difficulty Annotation** To analyze the characteristics of the MATH-B dataset, we
 347 annotate each problem for its mathematical domain and human-perceived difficulty. We adapt the
 348 procedure from Omni-MATH (Gao et al., 2024a), using GPT-5 to perform the labeling based on
 349 a contrastive prompting strategy (see supplementary material for the full prompt). This method
 350 leverages labeled examples from various math datasets to contextualize and assign scores to new
 351 problems and has shown to be aligned well with human judgment (Gao et al., 2024a). A selection of
 352 annotated problems from MATH-B-I is provided in Table 5. We also plot the distribution of ground
 353 truth answers in Figure 4.

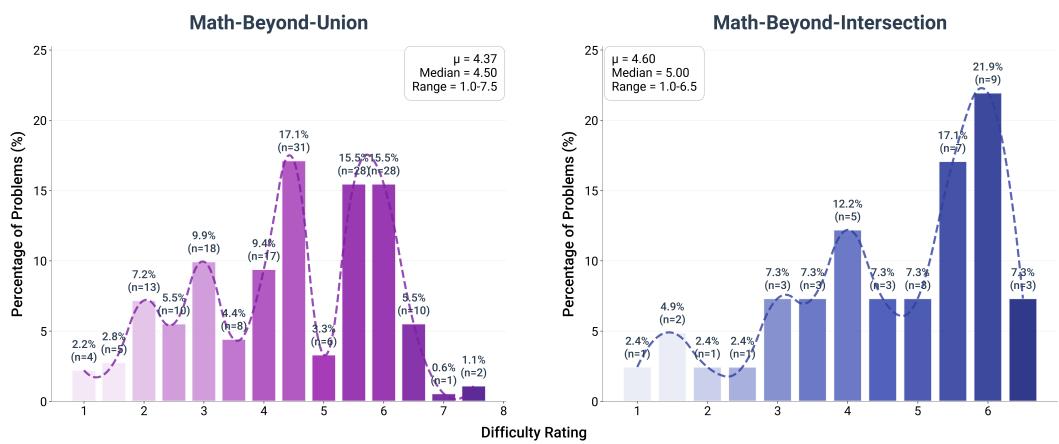


Figure 3: **Difficulty distribution.** The left subplot shows the difficulty distribution of the MATH-Beyond-Union set, while the right subplot shows that of the MATH-Beyond-Intersection set. The wide spread of difficulty levels highlights a key mismatch: the problems that models find challenging are not necessarily those that humans typically struggle with.

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374 **Distribution Analysis** As shown in Figure 2, the topic distribution of MATH-B consists entirely of
 375 standard high-school mathematics subjects, ensuring topical relevance. The difficulty distribution,
 376 plotted in Figure 3, reveals a wide range for both the Union and Intersection sets, with a median
 377 human-difficulty rating of 4 out of 10. Notably, even for the challenging MATH-B Intersection set,
 the maximum difficulty score is only 6.5. This suggests a significant disconnect between human-

378 perceived difficulty and model failure modes; problems that are not considered exceptionally hard for
 379 humans can still be robustly unsolvable by current models.
 380

381 **Benchmark Justification** A strong benchmark should be realistic, difficult, and efficient (Nie et al.,
 382 2025). MATH-B meets these with (i) *realism*: problems drawn from standard high-school curricula;
 383 (ii) *difficulty*: base models perform poorly even with a large sampling budget (i.e. `pass@1024`);
 384 and (iii) *efficiency*: the small suite enables potentially rapid, low-cost evaluation, and—as shown in
 385 Section 4—post-trained models exhibit only marginal gains at `pass@1024`, making MATH-B a
 386 perfect test bed for research.
 387

388 **Benchmark Usage** We expect researchers to use MATH-B with the base models and their cor-
 389 responding splits specified in Section 3.2.3: (1) evaluate the base model to estimate its `pass@k`
 390 for reasonably large k (e.g., $k = 1024$, expected to be nearly 0); (2) apply some RL method of
 391 interest; (3) re-estimate `pass@k` for the post-trained policy, which indicates the *Expansion Rate* per
 392 Section 2. The benchmark is intended specifically for methods that aim to *expand* the listed base
 393 model’s reasoning boundary. While this is a research benchmark, we hope the methods that people
 394 come up with scales to larger scales and other scenarios
 395

4 EVALUATING EXPANSION ACROSS FINETUNING METHODS

398 We evaluate several post-trained models on MATH-B to measure their ability to expand beyond
 399 their base model’s reasoning capabilities (i.e. Expansion Rate Section 2). Our analysis of mod-
 400 els finetuned with reinforcement learning (RL) reveals that current methods achieve only modest
 401 expansion. As shown in Table 2, the three RL models based on DeepSeek-R1-Distill-Qwen1.5B
 402 (r1-1.5b) (DeepSeek-AI et al., 2025) solve fewer than 10% of the test problems. In contrast, Skywork-
 403 OR1-7B (skywork_or1) (He et al., 2025a) reaches a more promising 21% expansion; notably, its
 404 training involves adaptive entropy control and a higher temperature, likely affording a greater scope
 405 for exploration. Keep in mind that these are `pass@1024` evaluations. This result suggests that
 406 RL techniques designed to explicitly encourage exploration may indeed result in higher Expansion
 407 Rates. We also observe that while prolonged training (i.e. more RL compute) can offer marginal
 408 gains—as seen in the slight improvement from Nemotron-Research-Reasoning-Qwen-1.5B ver-
 409 sion 1 (nemotron_v1) to Nemotron-Research-Reasoning-Qwen-1.5B version 2 (nemotron_v2) (Liu
 410 et al., 2025a)—the rather small 1.5% increase underscores the need for more efficient and effective
 411 exploration methods. See also the evolution of Expansion Rate in Figure 5.

412 **Table 2: Expansion Rates of post-trained models** using either RL or SFT/Distillation. The
 413 Expansion Rate measures the percentage of previously unsolvable problems (from the base model’s
 414 perspective) that the post-trained model can now solve. We additionally add AIME24 (pass@1)
 415 numbers of the post-trained models to illustrate the difficulty of our dataset (He et al., 2025a; Yang
 416 et al., 2025; Liu et al., 2025a).

417 Base model	418 Post-trained	419 Method	420 Base	421 Expansion Rate	422 AIME24
			423 unsolved	424 (%, pass@1024)	425 (pass@1)
<i>Reinforcement Learning (RL) models</i>					
420 r1-1.5b	421 nemotron_v1	422 RL	423 115	424 7.83	425 48.13
421 r1-1.5b	422 nemotron_v2	423 RL	424 115	425 9.57	426 49.58
422 r1-1.5b	423 DeepScaleR	424 RL	425 115	426 5.22	427 40.21
423 r1-7b	424 skywork_or1	425 RL	426 99	427 21.2	428 70.2
<i>Supervised Fine-Tuning (SFT) models</i>					
425 Qwen3-4B-base	426 Qwen3-4B	427 Long CoT Dist.	428 112	429 58.93	430 73.3
426 Qwen3-8B-base	427 Qwen3-8B	428 Long CoT Dist.	429 116	430 66.38	431 76.0

428 As an illustrative contrast, we evaluated Qwen3-4B and Qwen3-8B. These models are the result of
 429 finetuning their respective base versions (Qwen3-4B-Base and Qwen3-8B-Base) by distilling long
 430 Chain-of-Thought (CoT) reasoning trajectories from a more capable teacher model. They demon-
 431 strate substantially higher Expansion Rates of 58.93% and 66.38%, respectively. While this is not a direct
 432 apples-to-apples comparison due to differing training setups, the result is highly informative: it

432 shows that significant expansion is achievable when a model is exposed to the correct distribution of
 433 reasoning steps, an overlap that long CoT distillation provides (Yang et al., 2025).
 434

435 This contrast highlights that the primary limitation of current RL techniques may not be an inherent
 436 inability of the base models to learn, but rather the failure of the exploration process to find these
 437 effective reasoning pathways on its own. Developing RL methods that can discover these pathways
 438 without a teacher model remains a key challenge for reaching frontier capabilities and is the central
 439 motivation for our work.

440 5 DISCUSSION AND RELATED WORK

441 **On the Choice of $k = 1024$** Our selection of $k = 1024$ for all $\text{pass}@k$ evaluations is a deliberate
 442 choice to ensure our benchmark is challenging, stable, and efficient. Firstly, a large sampling budget
 443 is necessary to probe the true reasoning boundary of a model. Many popular benchmarks become
 444 saturated at this scale, with base models solving a high percentage of problems and leaving no room to
 445 measure improvement (Yue et al., 2025). We chose $k = 1024$ precisely because it represents a budget
 446 where MATH-B remains difficult, creating a meaningful testbed for genuine expansion. Secondly,
 447 this choice is strongly supported by our empirical analysis. While overall $\text{pass}@k$ performance
 448 shows a consistent log-linear increase with the sampling budget (Figure 6), the marginal gains for
 449 each additional sample diminish considerably (Figure 7). Most importantly, the Expansion Rate for
 450 RL-finetuned models—our core metric for progress—largely plateaus as the budget approaches 1024
 451 (Figure 5). Therefore, $k = 1024$ represents a principled trade-off: it is large enough to push models
 452 beyond their comfort zone on a challenging benchmark, yet provides a stable and computationally
 453 feasible point to reliably measure the expansion of the reasoning boundary.

454 **Limitations of Existing Benchmarks and Metrics** In mathematical reasoning, progress is often
 455 measured by $\text{pass}@k$. For $k > 1$, this metric is taken to be indicative of a model’s exploratory
 456 potential. However, $\text{pass}@k$ is an incomplete measure of exploratory potential, as it conflates the
 457 sharpening of existing solutions (Consolidation) with the discovery of entirely new ones (Expansion)
 458 (Wu et al., 2025). As detailed in Section 2, our work is concerned with Expansion. Many
 459 existing benchmarks are now saturated by strong open-source base models, making it impossible
 460 to measure new boundary expansion (Yue et al., 2025; Balunović et al., 2025; Wu et al., 2025).
 461 Furthermore, these benchmarks are targets of hyper-optimization, potentially rewarding spurious
 462 correlations (Shao et al., 2025). MATH-B addresses this by serving as a diagnostic tool. It comprises
 463 problems that are topically standard high-school math but are constructed to expose the subtle failures
 464 and brittleness of the dominant open-source research paradigm.

465 **Our Empirical Contribution in Context** Building on the framework of Wu et al. (2025), we
 466 *stantiate* these ideas in a concrete, reusable benchmark. their work introduces and analyzes
 467 Expansion within a specific setting, whereas we put forth a zero-baseline dataset (i.e., $\text{pass}@1024 \approx$
 468 0 for the base models) that operationalizes the concept and scales evaluation across a wide range of
 469 open-weight models. This offers a practical path to shift community focus from merely improving
 470 $\text{pass}@k$ on saturated suites to *demonstrably expanding the reasoning boundary*. Further, while
 471 MATH-B is instantiated for a specific set of open-weight base models, we expect a subset of items
 472 to remain zero-baseline for larger open-weight models. We did not evaluate those due to resource
 473 constraints. Since MATH-B is intended to drive RL methods that *expand* a given base model’s
 474 reasoning boundary (rather than chase model-specific quirks), we expect the resulting methods and
 475 insights to transfer across model families and scales.

476 6 CONCLUSION

477 Our findings indicate that current post-training methods in the open-source ecosystem, particularly
 478 for models up to 8B parameters, primarily refine pre-existing reasoning abilities rather than creating
 479 new ones. The poor performance of these post-trained models on MATH-B empirically confirms that
 480 they struggle to expand their capabilities to problems that lie just beyond their base model’s reach.
 481 We introduce MATH-B as a precise diagnostic tool to address this issue. Its purpose is to catalyze
 482 research into post-training methods that achieve genuine exploration, providing a clear and reliable
 483 signal for when a model has truly expanded the boundaries of machine reasoning.

486 7 REPRODUCIBILITY STATEMENT
487488 We are committed to ensuring the reproducibility of our work. All code required to reproduce the
489 experiments is provided in the supplementary material. Detailed derivations are included in the
490 main text (Section 2). Experimental settings and hyperparameters are described in the main paper
491 (Section 3.2) and supplementary sections (Appendix A.2).
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702 **A APPENDIX**
703704 **A.1 VERIFICATION ISSUES IN DIFFERENT FRAMEWORKS**
705706 **Table 3: Verification failure mode vulnerabilities across frameworks.** ✓ = vulnerable, – = resilient,
707 ✓/– = partial. *Frameworks*: TRL (Transformers RL) (von Werra et al., 2024), VERL (Sheng et al.,
708 2024), LM-Eval (hendrycks/minerva) (Gao et al., 2024b), LightEval (Habib et al., 2023), SCORE
709 (LM-Eval SCORE math) (Gao et al., 2024b), evalchemy (ZeroEval) (Raoof et al., 2025), HMMT
710 (evalchemy matharena) (Raoof et al., 2025), Math-V (Math-Verify) (Kydlíček, 2024). *Methods*:
711 First# = first number in string; Right = rightmost priority; Ltd. = limited support (≤ 5 expressions).
712 *F1–F7*: See Table 1 for detailed descriptions.

Failure Mode	TRL	VERL	LightEval	LM-Eval	SCORE	evalchemy	HMMT	Math-V
F1: Multiple solutions (OR)	✓	✓	✓/–	✓	✓	✓	✓/–	✓/–
F2: Late correct	✓	✓/–	✓/–	✓/–	–	✓	–	✓/–
F3: Early correct	–	✓	✓	✓	–	–	✓	✓
F4: Answer corrections	✓	–	✓	–	–	✓	–	✓
F5: Unordered sets	✓	✓	✓	✓	✓	✓	–	✓
F6: Missing anchors	✓	–	–	–	–	–	✓	–
F7: MCQ partial match	✓	✓	✓	✓	✓	✓	✓	✓
Method	First	Last	Right	Last	All	First#	Last/All	Right
Anchors	Yes	No	No	No	No	No	No	Yes
Multi-Answer	No	No	Ltd.	No	Partial	No	Yes	Ltd.

725 **A.2 INFERENCE PARAMETERS FOR EVALUATION**
726727 All models were run with nucleus sampling at a `top_p` of 0.95. Other parameters were set according
728 to model-specific recommendations:729

- **OLMo Models:** Temperature of 1.0, with `max_tokens` set to 2048 (v1) and 4096 (v2).
- **Qwen2.5-Math Models:** Temperature of 0.6 and `max_tokens` of 4096.
- **All Other Models:** Temperature of 0.6 and `max_tokens` of 32,768.

734 **A.3 MATH-B DETAILS**
735736 In Figure 4, we show the distribution of final answers for MATH-B-U, which spans a broad range of
737 integer values.
738739 **A.3.1 LOOKING AT SOME SAMPLES FROM MATH-B**
740741 In Tables 5 and 6, we list ten randomly sampled MATH-B-I questions—the base split and the full
742 set, respectively.743 **A.4 ANALYSIS OF `PASS@K` PERFORMANCE SCALING**
744745 To further justify our evaluation methodology, we analyze the performance scaling of all 21 models
746 from Table 4 on the MATH-B Union set.
747748 In Figure 6, we plot the complete `pass@k` evolution as the sampling budget k increases up to 1024.
749 The performance curves for nearly all models exhibit a characteristic log-linear growth, indicating
750 that improvement is consistent but not linear with computational effort. While this trend suggests
751 continued gains with more sampling, it also shows initial signs of plateauing at higher values of k .752 To better quantify this observation, Figure 7 visualizes the marginal gain in performance. Specifically,
753 it plots the average increase in the `pass@k` rate for each successive 64-sample increment. This plot
754 clearly illustrates the principle of diminishing returns: the most significant gains are concentrated at
755 lower sampling budgets, and the rate of improvement slows considerably as the budget approaches
1024. Together, these figures provide strong empirical support for our choice of $k = 1024$ as a

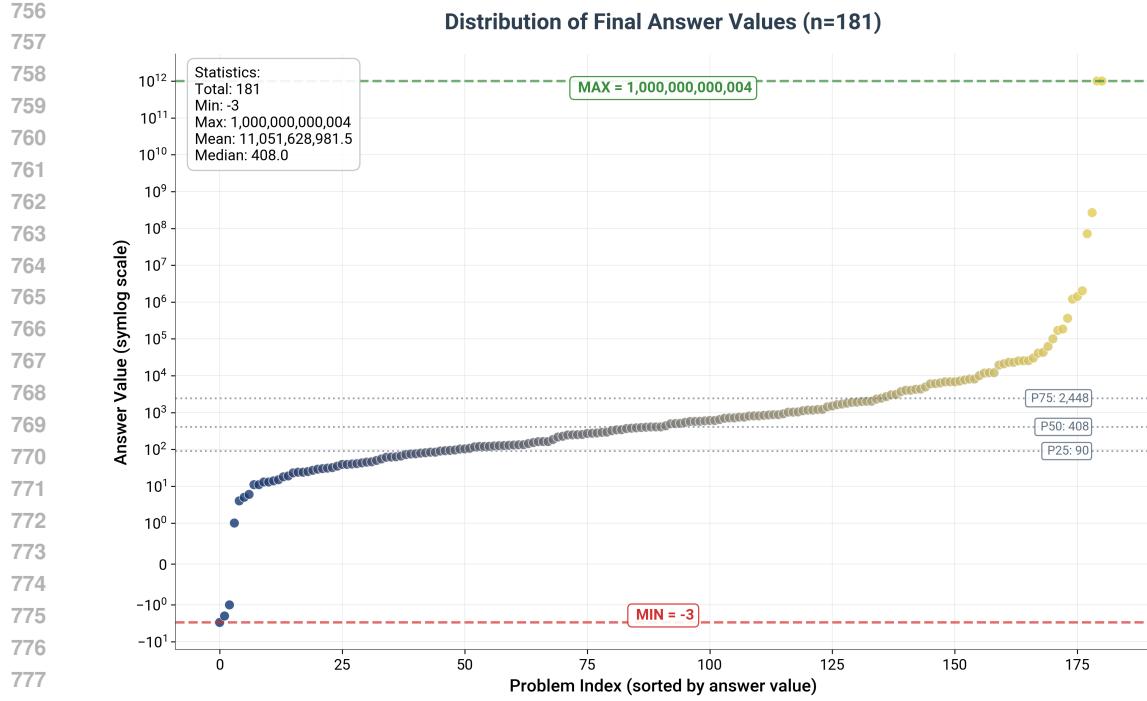


Figure 4: **Distribution of ground truth (final-answers) in MATH-B-U.** We use the log-scale for better readability.

practical and stable point for evaluation, beyond which brute-force sampling becomes an increasingly inefficient path to solving the remaining hard problems.

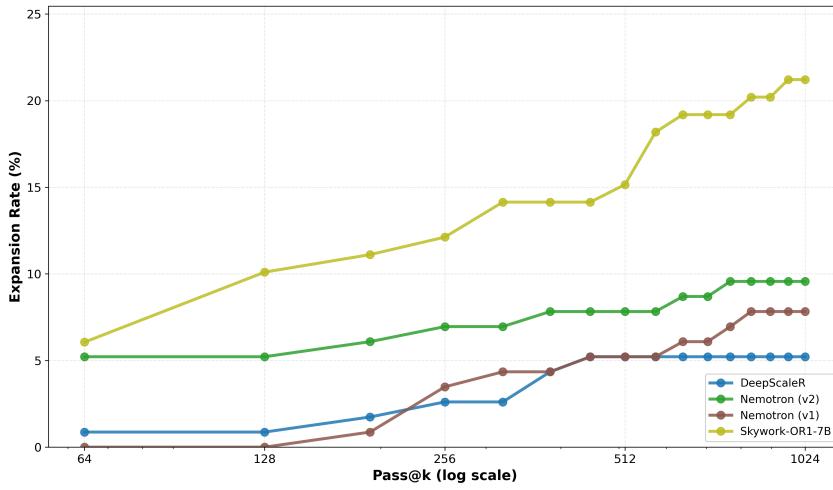


Figure 5: **Evolution of Expansion Rate for RL Models.** Models are evaluated on the MATH-B problems failed by their respective base models (115 for R1-Qwen2.5-1.5B; 99 for R1-Qwen2.5-7B).

A.5 LLM USAGE

The authors of this submission use IDEs with built-in LLM support, so LLMs have been used to help with menial coding tasks. Further, we used models like GPT-5, Gemini for rephrasing several

Table 4: **Models' number of unsolved questions at pass@1024** . Models are grouped into Base vs. Supplementary. “Intersection (base)” and “Intersection (all)” indicate the overlap of unsolved problems across models.

Model	# Unsolved	Type	Notes
Qwen2.5-1.5B	145	Base	
Qwen2.5-7B	120	Base	
Qwen2.5-Math-1.5B	115	Base	
Qwen2.5-Math-7B	101	Base	
Qwen3-4B-Base	112	Base	
Qwen3-8B-Base	116	Base	
DeepSeek-R1-Qwen2.5-1.5B	115	Base	Distill
DeepSeek-R1-Qwen2.5-7B	99	Base	Distill
OLMo-7B	158	Base	
OLMo-2-7B	132	Base	
Llama-3.1-8B	151	Base	
Qwen2.5-1.5B-Instruct	124	Supplementary	
Qwen2.5-7B-Instruct	131	Supplementary	
Qwen2.5-Math-1.5B-Instruct	139	Supplementary	
Qwen2.5-Math-7B-Instruct	117	Supplementary	
Qwen3-4B	67	Supplementary	
Qwen3-8B	52	Supplementary	
DeepScaler-1.5B	142	Supplementary	
Nemotron-1.5B-v1	137	Supplementary	
Nemotron-1.5B-v2	142	Supplementary	
Skywork-OR1-7B	103	Supplementary	
Intersection (base)	41	—	Shared failures across all base models
Intersection (all)	13	—	Shared failures across all models

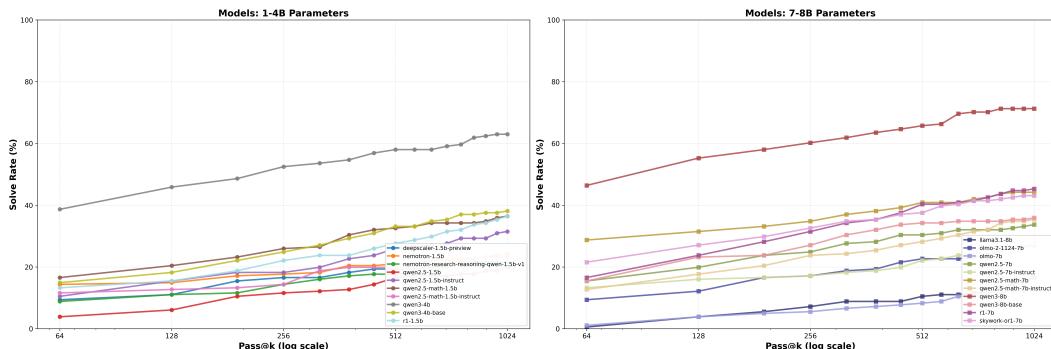


Figure 6: Evolution of $\text{pass}@k$ for all models on MATH-B-U.

paragraphs in this manuscript. In addition to that, we used GPT-5-mini and o4-mini-high to label the difficulty and topics of the benchmark we create (Section 3.2).

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870 Table 5: **Question texts (verbatim) from MATH-B-I (base).** Each question includes the sentence
871 “Let’s think step by step and output the final answer within `\boxed{ }.`” appended.

872

ID	Question (wrapped to fit)	Difficulty	Final Answer
1	Fisica and Ritmo discovered a piece of Notanium shaped like a rectangular box, and wanted to find its volume. To do so, Fisica measured its three dimensions using a ruler with infinite precision, multiplied the results and rounded the product to the nearest cubic centimeter, getting a result of 2017 cubic centimeters. Ritmo, on the other hand, measured each dimension to the nearest centimeter and multiplied the rounded measurements, getting a result of V cubic centimeters. Find the positive difference between the least and greatest possible positive values for V .	4.5	7174
2	Let \mathbb{N} denote the natural numbers. Compute the number of functions $f : \mathbb{N} \rightarrow \{0, 1, \dots, 16\}$ such that $f(x + 17) = f(x)$ and $f(x^2) \equiv f(x)^2 + 15 \pmod{17}$ for all integers $x \geq 1$.	6.0	12066
3	Each of the distinct letters in the following subtraction problem represents a different digit. Find the number represented by the word TEAM. PURPLE – COMET = TEAM.	3.5	6852
4	Find the sum of all positive integers n such that there exists an integer b with $ b \neq 4$ such that the base -4 representation of n is the same as the base b representation of n .	4.0	1026
5	Let S be a set of size 11. A random 12-tuple (s_1, \dots, s_{12}) of elements of S is chosen uniformly at random. Moreover, let $\pi : S \rightarrow S$ be a permutation of S chosen uniformly at random. The probability that $s_{i+1} \neq \pi(s_i)$ for all $1 \leq i \leq 12$ (where $s_{13} = s_1$) can be written as $\frac{a}{b}$ where a and b are relatively prime positive integers. Compute a .	6.0	1000000000004
6	Let \mathbb{N} denote the natural numbers. Compute the number of functions $f : \mathbb{N} \rightarrow \{0, 1, \dots, 16\}$ such that $f(x + 17) = f(x)$ and $f(x^2) \equiv f(x)^2 + 15 \pmod{17}$ for all integers $x \geq 1$.	4.5	12066
7	A unicorn is tethered by a 20-foot rope to the base of a cylindrical tower of radius 8 feet. The rope is attached to the tower at ground level and to the unicorn at a height of 4 feet. The rope is taut, its end is 4 feet from the nearest point on the tower, and the length of rope touching the tower is $\frac{a-\sqrt{b}}{c}$ feet, where a, b, c are positive integers and c is prime. Find $a + b + c$.	3.5	813
8	For which maximal N does there exist an N -digit number such that among any sequence of consecutive decimal digits some digit is present only once?	6.0	1023
9	Let $P = \{(x, y) \mid 0 \leq x, y \leq 25, x, y \in \mathbb{Z}\}$. Let T be the set of triangles formed by picking three distinct points in P (rotations, reflections, and translations count as distinct). Compute the number of triangles in T with area larger than 300.	1.5	436
10	Katie has a chocolate bar that is a 5-by-5 grid of square pieces, but she only wants to eat the center piece. She repeatedly (i) chooses a gridline and splits the bar, (ii) discards the part not containing the center, (iii) repeats until only the center piece remains. Compute the number of possible sequences of operations.	3.0	6384

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 919 **Table 6: Question texts (verbatim) from MATH-B-I (all) set of all considered models in Section 3.2.3.** Each question includes the sentence “Let’s think step by step and output the final answer
 920 within `\boxed{ }.`” appended.
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ID	Question (wrapped to fit)	Difficulty	Final Answer
1	Bob is writing a sequence of letters of the alphabet, each of which can be either uppercase or lowercase, according to the following two rules: If he had just written an uppercase letter, he can either write the same letter in lowercase after it, or the next letter of the alphabet in uppercase. If he had just written a lowercase letter, he can either write the same letter in uppercase after it, or the preceding letter of the alphabet in lowercase. For instance, one such sequence is $aAaABCDDcbBC$. How many sequences of 32 letters can he write that start at (lowercase) a and end at (lowercase) z ?	1.5	376
2	Let $V = \{1, \dots, 8\}$. How many permutations $\sigma : V \rightarrow V$ are automorphisms of some tree? (A graph consists of some set of vertices and some edges between pairs of distinct vertices. It is connected if every two vertices in it are connected by some path of one or more edges. A tree G on V is a connected graph with vertex set V and exactly $ V - 1$ edges, and an automorphism of G is a permutation $\sigma : V \rightarrow V$ such that vertices $i, j \in V$ are connected by an edge iff $\sigma(i)$ and $\sigma(j)$ are.)	5.5	30212
3	Find the sum of all positive integers n such that there exists an integer b with $ b \neq 4$ such that the base -4 representation of n is the same as the base b representation of n .	4.0	1026
4	A unicorn is tethered by a 20-foot silver rope to the base of a magician’s cylindrical tower whose radius is 8 feet. The rope is attached to the tower at ground level and to the unicorn at a height of 4 feet. The rope is taut, the end of the rope is 4 feet from the nearest point on the tower, and the length of the rope touching the tower is $\frac{a-\sqrt{b}}{c}$ feet, where a, b, c are positive integers and c is prime. Find $a + b + c$.	3.5	813
5	A 16x16 square sheet of paper is folded once in half horizontally and once in half vertically to make an 8x8 square. This square is again folded in half twice to make a 4x4 square. This square is folded in half twice to make a 2x2 square. This square is folded in half twice to make a 1x1 square. Finally, a scissor is used to make cuts through both diagonals of all the layers of the 1x1 square. How many pieces of paper result?	6.0	544
6	Matt writes a permutation of $\{1, 2, 3, \dots, 10\}$ across his paper with the leftmost number equal to 1 and the rightmost equal to 10. Exactly one interior number (not including 1 or 10) is less than both its immediate left and right neighbors. How many such permutations are there?	6.0	1636
7	Let \mathbb{N} denote the natural numbers. Compute the number of functions $f : \mathbb{N} \rightarrow \{0, 1, \dots, 16\}$ such that $f(x + 17) = f(x)$ and $f(x^2) \equiv f(x)^2 + 15 \pmod{17}$ for all integers $x \geq 1$.	4.5	12066
8	Sir Alex plays the following game on a row of 9 cells. Initially all cells are empty. In each move he either (1) chooses any number of the form 2^j (nonnegative j) and puts it into an empty cell, or (2) chooses two cells with the same number 2^j , replaces the number in one cell with 2^{j+1} and erases the number in the other cell. At the end, one cell contains 2^n and the others are empty. Determine the maximum number of moves possible in terms of n . Provide the value when $n = 10$.	5.0	2025
9	Alice and Bob play on a board of one row of 2022 consecutive squares. They take turns placing domino tiles that cover two adjacent squares; Alice goes first. A tile must not cover a square already covered. The game ends when no tile can be placed. Alice wants to maximize the number of uncovered squares at the end; Bob wants to minimize it. What is the greatest number of uncovered squares Alice can ensure, no matter how Bob plays?	4.0	290
10	(Caos) A cao [sic] has 6 legs, 3 on each side. A walking pattern is an ordered sequence of raising and lowering each of the legs exactly once (total 12 actions), starting and ending with all legs on the ground. The pattern is safe if at any point he has at least 3 legs on the ground and not all three legs are on the same side. Estimate N , the number of safe patterns.	4.0	1416528

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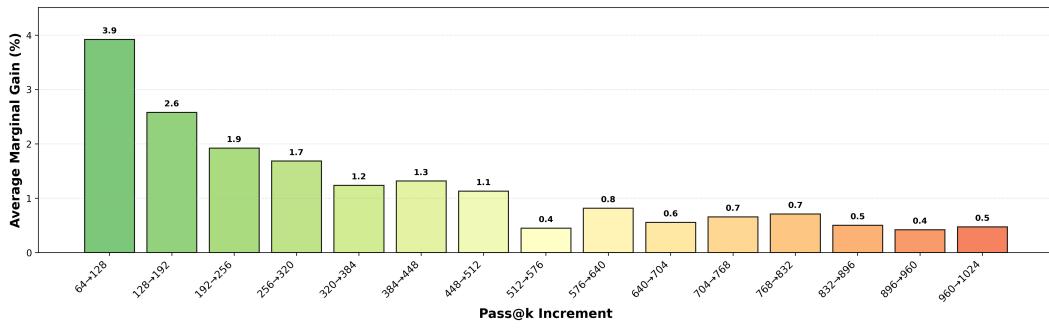


Figure 7: **Average gains in pass@k relative to the size of MATH-B-U.** Averaged over 21 models, the rate of solving new problems per 64-sample increment decreases as the total budget k grows, demonstrating diminishing returns.