

Characterizing hydrostatic mass bias with Mock-X

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ABSTRACT

Surveys in the next decade will deliver large samples of galaxy clusters that transform our understanding of their formation. Cluster astrophysics and cosmology studies will become systematics limited with samples of this magnitude. With known properties, hydrodynamical simulations of clusters provide a vital resource for investigating potential systematics. However, this is only realized if we compare simulations to observations in the correct way. Here we introduce the MOCK-X analysis framework, a multiwavelength tool that generates synthetic images from cosmological simulations and derives halo properties via observational methods. We detail our methods for generating optical, Compton- γ and X-ray images. Outlining our synthetic X-ray image analysis method, we demonstrate the capabilities of the framework by exploring hydrostatic mass bias for the IllustrisTNG, BAHAMAS and MACSIS simulations. Using simulation derived profiles we find an approximately constant bias $b \approx 0.13$ with cluster mass, independent of hydrodynamical method or subgrid physics. However, the hydrostatic bias derived from synthetic observations is mass-dependent, increasing to $b = 0.3$ for the most massive clusters. This result is driven by a single temperature fit to a spectrum produced by gas with a wide temperature distribution in quasi-pressure equilibrium. The spectroscopic temperature and mass estimate are biased low by cooler gas dominating the emission, due to its quadratic density dependence. The bias and the scatter in estimated mass remain independent of the numerical method and subgrid physics. Our results are consistent with current observations and future surveys will contain sufficient samples of massive clusters to confirm the mass dependence of the hydrostatic bias.

Key words: methods: numerical – galaxies: clusters: general – galaxies: clusters: intracluster medium – X-rays: galaxies: clusters

1 INTRODUCTION

Galaxy clusters form from the largest amplitude fluctuations present in the early Universe. Growing hierarchically over cosmic time as gravity draws in gas, stars, dark matter and other collapsed structures, galaxy clusters are the most massive collapsed objects we encounter at the current epoch. The distribution of galaxy clusters observed as a function of mass and redshift depends strongly on the initial spectrum and growth of the primordial fluctuations (e.g. Davis et al. 1985; Peacock & Heavens 1985; Bardeen et al. 1986). Therefore, clusters have the potential to place stringent constraints on the fundamental cosmological pa-

rameters that describe the Universe (e.g. Allen et al. 2011; Kravtsov & Borgani 2012), including the nature of dark energy (e.g. Weinberg et al. 2013). However, a prerequisite for cluster cosmology studies is well characterized observable-mass relations (e.g. Reiprich & Böhringer 2002; Vikhlinin et al. 2003; Giodini et al. 2013). Given that dark matter comprises ~ 85 per cent of most clusters, mass estimates are difficult and require very high-quality data. Additionally, as galaxy clusters collapse the galaxies residing within them grow and evolve via a range of astrophysical processes, such as radiative cooling, star formation, supernovae and the energetic outbursts of supermassive black holes. These processes continually shape the cluster’s baryonic components, making many observables a complex interplay of both cosmology and astrophysics. Therefore, for cluster cosmology

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studies a detailed understanding of both the mass estimate and observed properties are critical.

Galaxy clusters have proven to be an invaluable tool in placing relatively competitive constraints on fundamental cosmological parameters (e.g. Mantz et al. 2014; de Haan et al. 2016; Bocquet et al. 2019), which includes dark energy, the summed neutrino masses (e.g. Mantz et al. 2015a; Madhavacheril et al. 2017) and modifications to gravity (e.g. Okabe et al. 2013; Wilcox et al. 2015). Additionally, the constraints provided by clusters are often orthogonal to those produced via other methods. Due to the high-quality data required for reliable mass estimation, many studies are currently limited by statistical errors due to small sample sizes. However, the current decade will see a transformation in galaxy cluster observations. Surveys from facilities such as *Euclid* (Laureijs et al. 2011), LSST (LSST Science Collaboration et al. 2009), *e-Rosita* (Merloni et al. 2012), SPT-3G (Benson et al. 2014) and the Simons Observatory (Ade et al. 2019) will yield samples with $> 10^5$ objects, orders of magnitude larger than currently available. Combined with extensive follow-up programs, these surveys will provide a detailed, multiwavelength 10 Gyr picture of the growth and evolution of clusters. In this new regime of precision cluster cosmology, systematic uncertainties will dominate statistical errors. Therefore, to realise the potential of galaxy clusters as precision probes of cosmology and to maximise the scientific return from future surveys we require a thorough examination of all potential sources of systematics.

Numerical simulations provide a vital resource in this regard: galaxy clusters with exactly known properties. Cosmological simulations have now matured to the point that many independent groups have simulated either a sufficiently large volume or performed targeted zoom simulations that yield large samples of realistic galaxy clusters, i.e. their properties are broadly matched with observed clusters (e.g. Planelles et al. 2013; Le Brun et al. 2014; Pike et al. 2014; McCarthy et al. 2017; Barnes et al. 2017a,b; Springel et al. 2018; Henden et al. 2018; Cui et al. 2018; Tremmel et al. 2019). Lacking both the computational power and a detailed understanding of the physics, simulations model the astrophysical processes known to be important in structure formation in a subgrid manner and calibrate the relevant free numerical parameters on key observational scaling relations, such as the stellar mass-halo mass or gas mass-halo mass relations (e.g. Vogelsberger et al. 2014a; Schaye et al. 2015; Pillepich et al. 2018a; Davé et al. 2019). Therefore, independent models may produce similar global effective behaviours, such as the formation of a quiescent galaxy population or the expulsion of baryons from massive haloes, but they can vary dramatically in the details of how this behaviour is realised. Although simulations may not capture the full picture of galaxy and structure formation, the plethora of statistically large simulated galaxy cluster samples provide an excellent resource for exploring systematics that have the potential to impact cosmological constraints from future surveys.

However, it is well known that the direct comparison of simulations to observational data is intrinsically flawed, a significant challenge at X-ray wavelengths (e.g. Nagai et al. 2007; Khedekar et al. 2013). The impact of membership, projection effects, multi-temperature structures, and contaminating foregrounds and backgrounds complicate any insights gained from direct comparisons. To overcome these

difficulties, especially at X-ray wavelengths, there have been efforts to create synthetic observations of simulation data (e.g. Rasia et al. 2008; Heinz et al. 2011; Chluba et al. 2012; Biffi et al. 2013; ZuHone et al. 2014; Torrey et al. 2015). Synthetic images are produced by projecting the calculated emission spectrum of a source along a chosen line of sight, convolving with the instrumental response and adding any required backgrounds. However, many synthetic observation codes simply produce the image. Rather, to quantify any bias or scatter introduced during an observation we require a self-consistent framework that produces images and then derives quantities from them in a manner consistent with observational techniques. A comparison of synthetic-image-derived quantities to those derived directly from the simulation has the potential to highlight systematics and quantify the scatter introduced by analysis techniques (e.g. Ruppen et al. 2019b).

In this paper we introduce MOCK-X, an analysis framework designed to derive observed properties from multi-wavelength synthetic images of numerical simulations. Designed to explore the systematics that may impact cosmological constraints from future surveys, it yields optical, Compton- γ and X-ray images of simulated galaxy clusters and analyses them in a manner consistent with observational techniques. We will detail how synthetic images are generated from a numerical simulation and how we derive directly observable and reconstructed properties from these images. Given that one of the most significant systematics is the modelling of baryonic astrophysical processes, the framework is designed to be agnostic to the numerical simulation used as input. To demonstrate the capabilities of MOCK-X, we then present a study of bias introduced by measuring cluster masses from synthetic X-ray observations under the assumption of hydrostatic equilibrium using the IllustrisTNG, BAHAMAS and MACSIS simulations. We explore the impact of projection on recovered mass and examine how the bias and scatter in estimated mass varies as a function of cluster mass. In future work, we will explore the redshift evolution of the mass bias (Kannan et al. in prep.), study the multitude of criteria used to define relaxed clusters (Cao et al. in prep.), examine the scatter and covariance of cluster observables (Jorgensen et al. in prep.), and analyze the impact of choosing a cluster centre (Barnes et al. in prep.).

The rest of this paper is structured as follows. In Section 2 we provide a brief description of the simulations used throughout this work, outline our sample selection criteria, present how the synthetic images are produced and detail our approach to deriving the thermodynamic profiles and mass estimates of galaxy clusters from synthetic X-ray images. We then examine the bias introduced by estimating cluster masses assuming that they are in hydrostatic equilibrium in Section 3, comparing to bias found in recent observational studies, exploring the scatter in estimated mass, and examining the impact of cluster projection. In Section 4, we study how the assumption of spherical symmetry impacts the recovered mass estimate and how selecting relaxed cluster subsets impacts the result. Finally, in Section 5 we examine why the profiles from synthetic X-ray images yield a mass-dependent hydrostatic bias and we present our conclusions in Section 6.

2 METHODS

In this Section, we outline our numerical approach. First, we briefly detail the simulations used throughout this work and the halo selection method. We then define quantities that are computed directly from the simulations, before outlining the methods used for generating synthetic images for each halo. Finally, we summarize the analysis method for estimating cluster masses from synthetic X-ray images, including the derivation of the thermodynamic profiles from the images, the chosen deprojection method and the mass estimation method. We note that the chosen X-ray image analysis method is not unique and is one combination of many possible choices (e.g. [Ettori et al. 2013](#)). However, the method outlined below is sufficiently computationally efficient to determine masses for the $\sim 14,500$ cluster projections used in this work.

2.1 Simulated cluster samples

One of the most significant systematics is our incomplete knowledge of the physical processes that drive galaxy formation. Combined with the limited dynamic range afforded by finite computational resources, this necessitates the “subgrid” approach adopted by all numerical simulations that model galaxy formation on cosmological scales. A significant step forward in the last decade has been the development of calibrated subgrid galaxy formation models by independent groups. Calibrated subgrid models adjust their numerical free parameters until they match a limited set of key observational relations, such as the stellar mass function or gas mass-halo mass relation. Although most models reproduce the overall effects of key astrophysical processes, like galactic winds or the formation of the quiescent galaxy population, the detailed implementation can vary dramatically, such as the feedback channels for stars and active galactic nuclei (AGN). Therefore, throughout this work, we make use of three numerical simulations to explore the impact of different subgrid models and other associated numerical choices, such as hydrodynamic scheme and numerical resolution. Below, we briefly describe the subgrid models employed by these simulations but refer the reader to the relevant papers for a more detailed explanation. Key properties of the simulations used in this work are summarised in Table 1. We note that the small differences in adopted cosmologies between the simulations have a negligible impact on the results presented in this paper. We then outline the sample selection method applied to all simulations.

2.1.1 IllustrisTNG

The IllustrisTNG project ([Marinacci et al. 2018](#); [Naiman et al. 2018](#); [Nelson et al. 2018](#); [Pillepich et al. 2018b](#); [Springel et al. 2018](#)) is the successor of the Illustris project ([Vogelsberger et al. 2014b,a](#); [Genel et al. 2014](#); [Sijacki et al. 2015](#)). The updated galaxy formation model ([Pillepich et al. 2018a](#); [Weinberger et al. 2017](#)) has been shown to reproduce a wide range of observable properties from dwarf galaxies to cluster scales. In particular, for galaxy clusters, it has been shown to reproduce the metal content of the ICM ([Vogelsberger et al. 2018](#)) and yield reasonable cool-core fractions at low-redshift ([Barnes et al. 2018](#)). In this work, we make use of all

Table 1. Table summarising key properties of the different numerical simulations used throughout this work. From left to right the columns present the simulation, (target) gas mass, gas softening length, dark matter mass, dark matter softening length and the number of clusters (projections) selected at $z = 0.1$, respectively.

Simulation	m_{gas} [M_{\odot}]	ϵ_{gas} [kpc]	m_{DM} [M_{\odot}]	ϵ_{DM} [kpc]	$N_{\text{clusters}} (N_{\text{proj}})$ [$M_{200} \geq 10^{14} M_{\odot}$]
TNG300- l1	1.1×10^7	0.37	5.9×10^7	1.48	250 (1500)
TNG300- l2	8.8×10^7	0.74	4.7×10^8	2.95	250 (1500)
TNG300- l3	7.0×10^8	1.48	3.8×10^9	5.90	242 (1452)
BAHAMAS	1.2×10^9	5.96	6.6×10^9	5.96	1994 (11964)
MACSIS	1.2×10^9	5.96	6.6×10^9	5.96	390 (2340)

three resolution levels (L1, L2, and L3) of the largest volume simulation - TNG300, a periodic cubic volume with a side length of 302.6 Mpc. All IllustrisTNG simulations use a cosmological model whose parameters are chosen in accordance with the [Planck Collaboration et al. \(2016a\)](#) constraints: $\Omega_b = 0.0486$, $\Omega_M = 0.3089$, $\Omega_{\Lambda} = 0.6911$, $\sigma_8 = 0.8159$, $H_0 = 100 h = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and $n_s = 0.9667$.

Employing an updated version of the Illustris galaxy formation model ([Vogelsberger et al. 2013](#); [Torrey et al. 2014](#)), the IllustrisTNG subgrid model includes a new radio mode AGN feedback scheme ([Weinberger et al. 2017](#)), a re-calibrated SN wind model and an extended chemical evolution scheme ([Pillepich et al. 2018a](#)), magnetic fields ([Pakmor & Springel 2013](#)) and refinements to the numerical scheme that improve its convergence properties ([Pakmor et al. 2016](#)). The magneto-hydrodynamics equations are evolved with the moving-mesh code AREPO ([Springel 2010](#)). The dark matter particles have a mass, m_{DM} , of $5.9 \times 10^7 M_{\odot}$, $4.7 \times 10^8 M_{\odot}$ and $3.8 \times 10^9 M_{\odot}$ for the level 1, 2 and 3 resolutions, respectively. The collisionless particles, i.e. dark matter and stars, have softening lengths, ϵ_{DM} , of 1.48 kpc, 2.95 kpc and 5.90 kpc for levels 1, 2 and 3 respectively, which is a fixed physical length for $z \leq 1$ and comoving for $z > 1$. The gas cells have a target mass, m_{gas} , of $1.1 \times 10^7 M_{\odot}$, $8.8 \times 10^7 M_{\odot}$ and $7.0 \times 10^8 M_{\odot}$, and an adaptive comoving softening length, ϵ_{gas} , that reaches a minimum of 0.37 kpc, 0.74 kpc and 1.48 kpc for levels 1, 2 and 3, respectively. We note that the subgrid model for IllustrisTNG is calibrated once, for the highest resolution TNG100 volume, and remains fixed across the three different resolution levels, enabling a convergence study with chosen numerical resolution.

2.1.2 BAHAMAS

The BAHAMAS project ([McCarthy et al. 2017](#)) is a suite of periodic cubic volumes with a side length of 596 Mpc that vary both their cosmological parameters and the free parameters of the subgrid galaxy formation model. An evolution of the Cosmo-OWLS project ([Le Brun et al. 2014](#)), the project was designed to yield large samples of realistic simulated clusters for cluster cosmology studies. The galaxy formation model was calibrated to ensure that the baryonic content of galaxy clusters is well matched to observed clusters, and

the reference calibration of the model reproduces a wide range of observed scaling relations. In the work, we make use of the reference *Planck* cosmology run. This assumes a flat Λ CDM cosmology constrained by [Planck Collaboration et al. \(2014\)](#): $\Omega_b = 0.0490$, $\Omega_M = 0.3175$, $\Omega_\Lambda = 0.6825$, $\sigma_8 = 0.834$, $H_0 = 100 h = 67.11 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and $n_s = 0.9624$.

With its origins in the OWLS ([Schaye et al. 2010](#)) and GIMIC ([Crain et al. 2009](#)) projects, BAHAMAS evolves the hydrodynamics equations using traditional SPH. The galaxy formation model includes radiative cooling ([Wiersma et al. 2009a](#)), stochastic, metallicity independent star formation that by construction reproduces the Kennicutt-Schmidt relation ([Schaye & Dalla Vecchia 2008](#)), mass and metal return due to stellar evolution ([Wiersma et al. 2009b](#)), kinetic wind supernovae feedback ([Dalla Vecchia & Schaye 2008](#)), and the seeding, growth and feedback from supermassive black holes ([Springel et al. 2005](#); [Booth & Schaye 2009](#)). The initial gas particle mass is $1.21 \times 10^9 M_\odot$ and the dark matter particle mass is $6.63 \times 10^9 M_\odot$. The gravitational softening length is set to 5.96 comoving kpc for $z > 3$ and to the same value in physical kpc for $z \leq 3$. The minimum smoothing length of the SPH kernel is set to a tenth of the gravitational softening.

2.1.3 MACSIS

The rarity of massive galaxy clusters is such that even a volume of $\sim 600^3 \text{ Mpc}^3$ is too small to contain the largest clusters expected to form in a Λ CDM cosmology. The MACSIS project ([Barnes et al. 2017a](#)) was designed to simulate these missing clusters. From a dark matter only periodic cube with a side length of 3.2 Gpc, all haloes identified by a standard *Friend-of-Friends* (FoF) percolation algorithm whose mass $M_{\text{FoF}} > 10^{15} M_\odot$ were grouped in logarithmically spaced bins with a width $\Delta \log_{10}(M_{\text{FoF}}) = 0.2$. All haloes in a bin were selected if the number of objects in the bin was less than 100, otherwise, 100 haloes were selected from the bin at random. The end result of this process is a sample of 390 massive galaxy clusters.

Each cluster in the sample was resimulated at increased resolution via the zoomed simulation technique ([Katz & White 1993](#); [Tormen et al. 1997](#)), with the high-resolution region free of contaminating tidal particles out to $5 r_{500}^1$. The BAHAMAS galaxy formation model was used for the full physics resimulations and the mass and spatial resolution was chosen to be identical to the original BAHAMAS volume. The cosmology of the MACSIS simulations was not altered from the parent dark matter volume, which assumes a marginally different flat Λ CDM cosmology constrained by [Planck Collaboration et al. \(2014\)](#): $\Omega_b = 0.04825$, $\Omega_M = 0.307$, $\Omega_\Lambda = 0.693$, $\sigma_8 = 0.8288$, $H_0 = 100 h = 67.77 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and $n_s = 0.9611$. The combination of MACSIS and BAHAMAS spans the complete mass range of galaxy clusters expected to form in a Λ CDM cosmology. We note that the MACSIS sample suffers from selection effects, due to haloes being selected by *Friends-of-Friends* mass and the analysis below using spherical over-

density masses. We highlight where this potentially impacts the results presented.

2.1.4 Sample selection

All simulations used in this work identified haloes via a standard *Friend-of-Friends* algorithm run on the dark matter particles, with a typical value of the linking length in units of the mean interparticle separation ($b = 0.2$). Baryonic particles/cells are attached to haloes by locating their nearest dark matter particle. Self-bound structures were then identified by via SUBFIND ([Springel et al. 2001](#); [Dolag et al. 2009](#)), with the most massive subhalo in each FoF group defined as the central object, and the remaining self-bound structures being defined as subhaloes. For all simulations, clusters were selected from the snapshot closest to $z = 0.1$ via the mass cut $M_{500} \geq 10^{14} M_\odot$. With this threshold, the simulated cluster samples contain 250, 250, 242, 1994, and 390 clusters for TNG300-1, TNG300-2, TNG300-3, BAHAMAS and MACSIS, respectively. Throughout this work, we define the cluster centre as the potential minimum, which is chosen to be the most bound particle identified by the SUBFIND algorithm.

2.1.5 Properties derived from the simulation

In this work, we use or compare to some cluster properties derived directly from the simulations. Three-dimensional cluster centric radial density and mass-weighted temperature profiles were computed in the range $0 - 1.5 r_{500, \text{sim}}$ by binning the gas cells/particles in 50 linearly spaced radial bins, where the subscript sim denotes that the value was produced by the SUBFIND algorithm. The mass-weighted temperature is defined as

$$T_{\text{mw}} = \frac{\sum_k m_k T_k}{\sum_k m_k}, \quad (1)$$

where m_k is the mass of the k th cell/particle, T is the temperature and the sum runs over all particles in a given radial bin. Additionally, to ensure that the fitting of APEC templates to synthetic X-ray spectra is computationally efficient we compute 2D density, temperature and metallicity cluster centric radial profiles by projecting the cells/particles along the axis of interest and then binning them into 50 linearly spaced radial bins over $0 - 1.5 r_{500, \text{sim}}$.

We compute a theoretical criterion for defining a cluster as relaxed: the energy ratio E_{rat} . The energy ratio ([Barnes et al. 2017b](#)) is the kinetic energy of the gas within the cluster divided by its thermal energy. When clusters collapse or undergo significant mergers there will be substantial energy stored in the random kinetic motions of the gas. Over time, this energy is converted to thermal energy via weak shocks (e.g. [Kunz et al. 2011](#)) and potentially turbulent cascades ([Zhuravleva et al. 2014](#)) as the cluster relaxes. Therefore, relaxed clusters should have a lower ratio of kinetic to thermal energy relative to disturbed objects. We measure this ratio within $r_{500, \text{sim}}$ by removing the bulk motion of the cluster and then computing the ratio of the sum of kinetic energy to the sum of thermal energy for all gas cells/particles that lie within the 3D aperture

$$E_{\text{rat}} = E_{\text{kin}, 500} / E_{\text{thm}, 500}. \quad (2)$$

¹ The radius r_{500} denotes the radius of sphere that encloses a mass M_{500} and has a mean density equal to 500 times the critical density of the Universe.

In this work, we define a cluster as relaxed if it $E_{\text{ratio}} < 0.1$. Unless otherwise stated, all other cluster properties used in this work are derived from the synthetic images.

2.2 Synthetic image generation

For every cluster in the simulation samples, we generate synthetic images that form the basis of our analysis. The centre of every image is taken to be the projection of the potential minimum. We note that how to define the centre of a cluster and how derived quantities depend on this choice (e.g. [Ruppin et al. 2019a](#)) is still an open question, but we leave this issue to future work.

To investigate the impact of projection effects we create 6 projections of every halo. We create 3 projections along x , y and z . In addition, we characterize the shape of every cluster via the mass distribution tensor \mathcal{M} , or equivalently the inertia tensor \mathcal{I} (e.g. [Bett et al. 2007](#)). Modelling the cluster as a uniform ellipsoid, the mass distribution tensor is a square matrix with components:

$$\mathcal{M}_{ij} = \sum_{k=1}^{N_{200}} m_k r_{k,i} r_{k,j}, \quad (3)$$

where m_k is the mass of the k th cell/particle, $r_{k,i}$ is the i th component of the position vector \mathbf{r}_k in cluster centric coordinates and the sum is over the number of particles, N_{200} , within $r_{200,\text{sim}}$. The square roots of the eigenvalues A , B and C of matrix \mathcal{M} are the lengths of the semiprincipal axes, with the convention that $A \geq B \geq C$. Via the mass tensor, we rotate the particle/cell distribution for every cluster and produce a further 3 projections along the semi-principal axes A , B and C . Additionally, we parameterize how spherical every cluster is by computing its sphericity $s = (C/A)^{\sqrt{3}}$, where a perfectly spherical cluster would have $s = 1$.

A schematic of our approach is shown in Fig. 1 for the IllustrisTNG level 1 simulation, with the left panel showing a 1 Mpc depth slice through the gas column density at $z = 0.1$. In the centre of the image is the most massive cluster in the simulation volume, with the red square denoting $3r_{500,\text{sim}}$. The right-hand column of three images shows a synthetic bolometric optical luminosity image, the Compton- y image and a smoothed soft-band (0.5–2.0 keV) X-ray photon counts image of this cluster projected down the z axis. The bottom left and bottom middle panels show the X-ray counts projected down the longest (A) and shortest (C) axes through the cluster.

We note that in this work we assume perfect signal to noise, i.e. the synthetic images have no background. For the X-ray images explored in this work the lack of background produces a minor reduction in the scatter but does not bias the results (e.g. [Rasia et al. 2008](#)). Therefore, we leave the description of how we generate background noise to future work where it is relevant.

2.2.1 Optical

Following previous work (e.g. [Torrey et al. 2015](#)), we generate optical images by treating every star particle as a stellar population with a single metallicity and age given by the simulation and assuming a [Chabrier \(2003\)](#) initial mass function. For each star particle its bolometric luminosity and its

luminosity in the Sloan Digital Sky Survey u, g, r, i, z bands is computed. In principle, the light from every star particle should be attenuated by intervening dust and gas. To accurately model this obscuration we require detailed knowledge of the gas and dust distributions on sub-parsec scales and should perform detailed radiative transfer calculations. However, all of the simulations used in this work lack the spatial resolution and the self-consistent dust physics to perform such calculations. Instead, we estimate the impact of dust obscuration by calculating the column density of dust along the line of sight from every star (e.g. [Hopkins et al. 2005](#); [Wuyts et al. 2009](#)). We note that, given the limited spatial resolution of the simulations and the treatment of the interstellar medium as an effective single-phase medium, this is still a relatively naive calculation of the optical depths to star particles.

For all star particles with the FoF group, the stellar light is then projected along the relevant axis and smoothed to create maps in all six bands. The smoothing length of every star particle of interest was computed using a k-d tree. Throughout this project we determine a star's smoothing length by computing the distance to the 32nd nearest gas neighbour. Potentially, the choice of smoothing length can subtly alter the shapes of galaxies. However, for the cluster scale maps generated here, we find that the choice smoothing length has a negligible effect. For our fiducial maps, we assume an SDSS-like angular pixel resolution of 0.24 arcsec and use a square field of view of length $3r_{500}$.

2.2.2 Compton- y

The free electrons in the hot plasma of the ICM interact with the photons of the cosmic microwave background (CMB) via inverse Compton scattering, shifting the energy distribution of the CMB from its black-body spectrum and producing the Sunyaev-Zel'dovich (SZ) effect. In the Mock-X framework we focus on the non-relativistic, thermal SZ effect. Proposed in 1970 ([Sunyaev & Zeldovich 1972](#)), the first cluster was discovered via the SZ effect was presented in [Staniszewski et al. \(2009\)](#) and it is now routinely used to build large catalogues of galaxy clusters (e.g. [Reichardt et al. 2013](#); [Hasselfield et al. 2013](#); [Planck Collaboration et al. 2016b](#); [Bleem et al. 2019](#)). An advantage of SZ selected cluster samples is that they are relatively unbiased to the presence of a cool-core ([Lin et al. 2015](#)). To compute realistic images of the SZ effect, in principle one should compute the response of every gas cell/particle at several frequencies and extract the Compton- y signal using a match filtering approach. However, for this initial set of projects, we only desire the integrated Compton- y signal and centre derived from it. Therefore, we directly compute the expected Compton- y signal expected for every gas cell/particle. The projected Compton- y signal along a given line of sight, l , is proportional to the projected electron pressure

$$y = \frac{\sigma_T}{m_e c^2} \int P_e(l) dl, \quad (4)$$

where σ_T is the Thompson scattering cross-section, m_e is the mass of an electron, c is the speed of light, $P_e = n_e k_B T$ is the electron pressure, n_e is the electron number density, k_B is the Boltzmann constant and T is the temperature. The Compton- y parameter is computed for all gas cells/particles

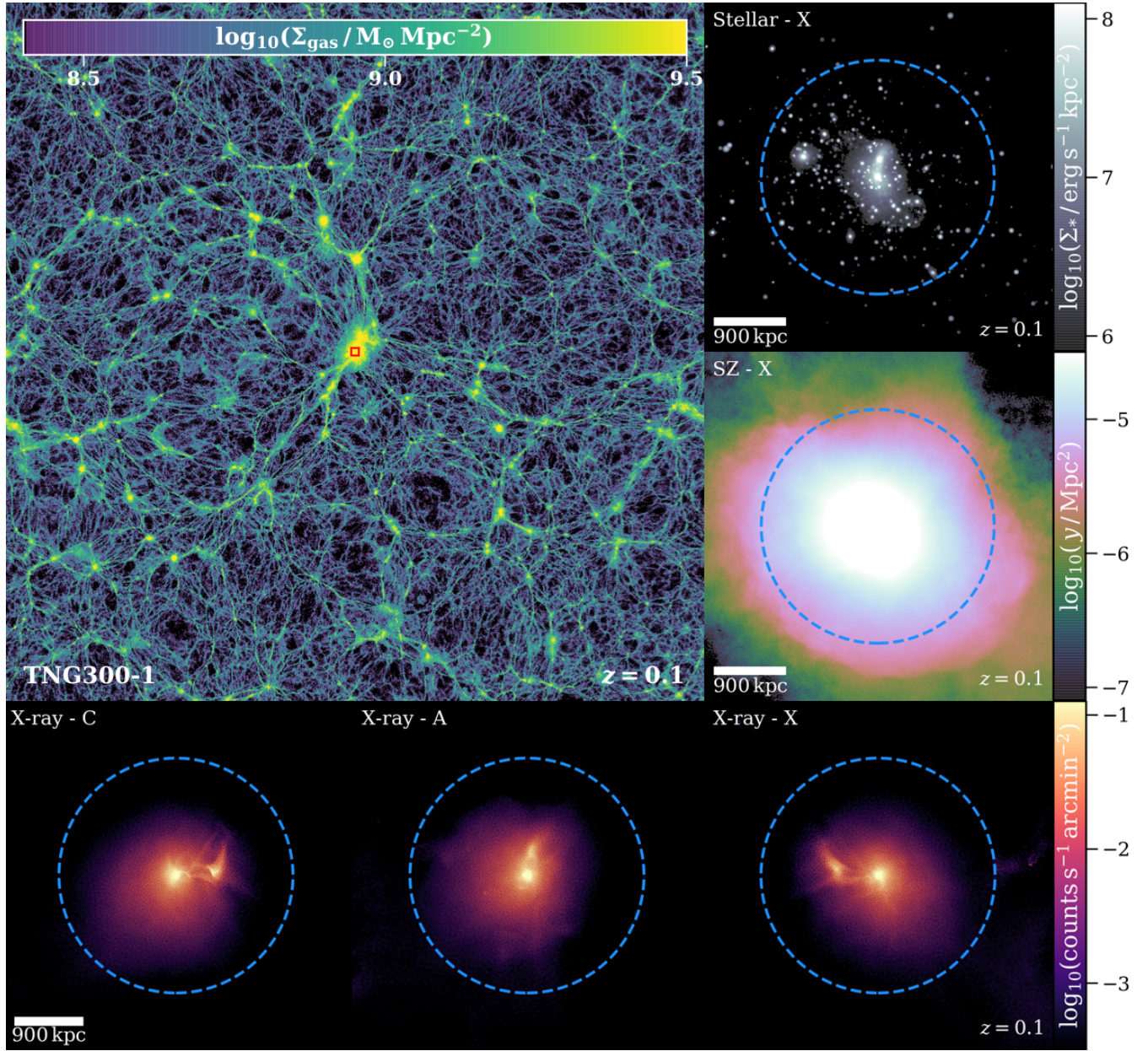


Figure 1. Mock-X schematic from the IllustrisTNG 300^3 Mpc^3 volume. *Top left:* Gas density slice of 1 Mpc width through the simulation volume centred on the most massive cluster at $z = 0.1$. The red square denotes the $3r_{500,\text{sim}}$ square field-of-view used for synthetic image generation. *Top right:* Synthetic optical image of the cluster showing the bolometric luminosity of the star particles. *Middle right:* Synthetic Compton-y image for a NIKA2-like facility. *Bottom row:* Synthetic X-ray images for a Chandra-like telescope projected along the shortest axis (C, left), longest axis (A, middle) and the x axis (right) of the simulation volume. The dashed blue line in the synthetic image panels denotes $r_{500,\text{sim}}$.

within the FoF group and then projected down the relevant axis to yield Compton-y maps. We create square maps with a physical side length of $3r_{500}$ for all clusters with a resolution of 11 arcsec, similar to the spatial resolution of current SZ facilities like NIKA2 (Adam et al. 2018) and MUSTANG-2 (Dicker et al. 2014).

2.2.3 X-ray

The impact of multi-temperature structure, gas clumping and projection effects are particularly relevant for X-ray observations and synthetic observations are a vital tool in the faithful comparison of simulations to observations. Our method for generating synthetic X-ray observations mirrors many previous works in this area (e.g. Gardini et al. 2004; Nagai et al. 2007; Rasia et al. 2008; Heinz & Brüggen 2009; Biffi et al. 2012; ZuHone et al. 2014, Pop et al. in prep.). However, we have optimised it for the large sample sizes

used in this work and the high resolution of some clusters, where a single cluster can contain $> 10^7$ resolution elements within r_{500} .

Similar to previous work (e.g. Le Brun et al. 2014; Barnes et al. 2017a), we begin by generating an X-ray spectrum for every gas particle/cell within the FoF group via a lookup table of spectral templates. We generate the table using the Astrophysical Plasma Emission Code (APEC; Smith et al. 2001) via the PYATOMDB module with atomic data from ATOMDB v3.0.9 (last described in Foster et al. 2012). For the 11 chemical elements tracked by the simulations we create a spectral table that spans the temperature range $10^6 - 10^9$ K with a spacing $\Delta \log_{10}(T/\text{K}) = 0.02$. The energy range and resolution of the spectra depend on the desired instrument, for example, the table of an instrument similar to *Chandra* ACIS-I has an energy range of 0.5 – 10.0 keV with an energy resolution of 150 eV. Improving on previous work, the spectra are convolved with the corresponding response matrix and the effective area for the desired energy bins is taken from the instrument’s ancillary response file. We account for galactic absorption using the WABS model (Morrison & McCammon 1983), assuming a fixed column density of $n_{\text{H}} = 2 \times 10^{20} \text{ cm}^{-2}$. Via this method, it is trivial to precompute spectra lookup tables for current, e.g. *Chandra*, *XMM*, *eRosita*, and proposed, e.g. *Athena*, *Lynx*, *AXIS*, facilities and we demonstrate this by computing the expected emission for a $1.7 \times 10^{14} M_{\odot}$ cluster at $z = 1$ for current and (potential) future missions in Fig. 2. We have deliberately selected a cluster below the detection threshold of the *eRosita* mission, predicting it would see 7 counts within r_{500} for a 100 ksec exposure, and the power of future missions is clear, with a significant number photons expected out to r_{500} for *Athena*, *AXIS* and *Lynx*. Unless otherwise stated, throughout the remainder of this work we assume an instrument similar to *Chandra* ACIS-I.

For every particle/cell we then compute a total spectrum using its temperature, density and metal abundance by summing over the chemical elements tracked by the simulations, which all track the same elements. We note that we discard the final “element” tracked by TNG that ensures the total mass of the elements is equal to the mass of the gas cell. If a particle/cell breaches any of the following conditions we exclude it from the analysis: i) its temperature is less than 10^6 K, ii) its star formation rate is non-zero (i.e. it is following an enforced equation of state), or iii) its net cooling rate is positive (i.e. it is increasing in temperature). These criteria typically remove a few per cent of the total cells/particles within a cluster, ensuring that gas that would not be X-ray emitting or is following an enforced equation of state (i.e. eligible to form stars) is excluded from the image generating process. Substructures are removed at this stage of image production via SUBFIND, with any gas cells/particles bound to any structure other than the central object removed.

The spectrum for every particle is then projected down the relevant axis and smoothed onto a square grid with a physical side length of $3r_{500}$. In this work, we neglect issues such as chip gaps, the requirement of stitching multiple pointings together, or that the instrument response can vary across the focal plane. The pixel resolution of the synthetic image is set by the angular resolution of the chosen instrument, which we set to 0.5 arcsec for a *Chandra*-like instrument. Each image pixel stores the combined spectrum of

all particles smoothed onto it. This creates a 3D datacube, where the first two dimensions are the pixel locations on the image map and the third dimension is the X-ray spectrum of the pixel. We then assume an exposure time of 100 ksec to generate the expected photons in each pixel, though with perfect signal to noise this step is academic for this study.

The final result of the synthetic image generation process is a set of images for every simulated cluster: 6 optical images, a Compton-y image and an X-ray image for each of the 6 chosen projection axes. For the remainder of this work, we focus on the estimation of cluster masses from synthetic X-ray images and spectra, assuming hydrostatic equilibrium. We now outline our analysis method for extracting the thermodynamic profiles of clusters from synthetic X-ray images and how they are used to estimate the mass of the cluster.

2.3 Hydrostatic masses from Synthetic X-ray images

We now detail how cluster masses are estimated from the synthetic X-ray images. We begin by noting that the approach presented below is by no means a unique approach to extracting estimated cluster masses from thermodynamic profiles (e.g. Pointecouteau et al. 2005; Nulsen et al. 2010; Sanders et al. 2018). We direct the interested reader to Ettori et al. (2013) for a review of advantages and disadvantages of different approaches. The approach taken in this work is designed to yield stable mass estimates in a manner that is sufficiently computationally efficient to enable the analysis of $> 10^4$ mock X-ray datacubes.

2.3.1 Gas density profiles

When computing the thermodynamic radial profiles we centre on the potential minimum of the cluster, defined as the most bound particle identified by the SUBFIND algorithm. We extract the gas density profile of each datacube from its surface brightness map in the energy band 0.7 – 1.2 keV. In this band, the emission is relatively insensitive to the gas temperature and proportional to the square of the density. The pixels are binned into 25 linearly spaced annuli in the range $0.0 - 1.5 r_{500}$. We then compute the median surface brightness within each annulus. In agreement with previous work (e.g. Zhuravleva et al. 2013), we find the median, relative to the mean, to be more robust to the presence of accreting material and the inhomogeneities found in the gas distribution at larger cluster centric radii (Nagai & Lau 2011; Vazza et al. 2013; Roncarelli et al. 2013). We use the same APEC lookup table used to generate the datacubes, with the same galactic absorption model and instrument response and effective area, to model the emission in each annulus as a thin plasma. The emission measure is then related via

$$Norm = \frac{10^{-14}}{4\pi [d_A(1+z)]^2} \int_V n_e n_H dV, \quad (5)$$

where d_A is the angular diameter distance, n_e is the electron number density, and n_H is the ion number density, where we assume $n_e = 1.17 n_H$ (Anders & Grevesse 1989). The projected, 2D radial gas density profile is then computed from the derived emission measure.

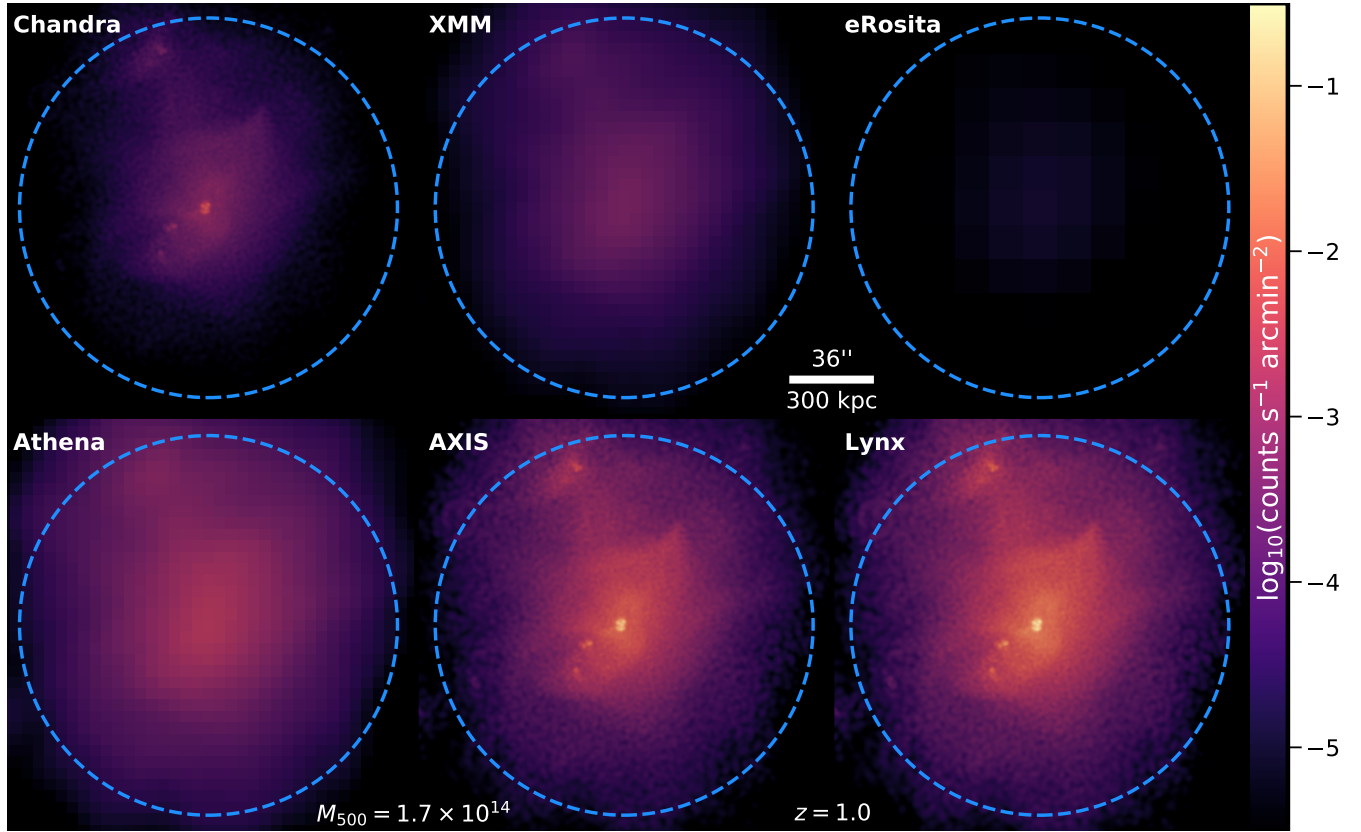


Figure 2. Synthetic X-ray images of a cluster in the TNG300 level 1 volume at $z = 1$ for facilities like those currently available and planned. We demonstrate the expected images from *Chandra* (top left), *XMM* (top middle), *eRosita* (top right), *Athena* (bottom left), *AXIS* (bottom middle) and *Lynx* (bottom right) like instruments. The blue circle denotes $r_{500,\text{sim}}$. We note the chosen cluster is below the expected detection threshold of *eRosita* at this redshift, with 7 total photons predicted within $r_{500,\text{sim}}$ for an exposure of 100 ksec. The power of future facilities is demonstrated by the significant increase in photons collected at larger radii.

2.3.2 Deprojection

The modelling of the surface brightness profile yields the 2D emission measure and density profiles that require deprojection. Under the assumption of spherical symmetry, the emission measure and density profiles can be deprojected by computing the projected volume V of each spherical shell onto each 2D annulus. The profiles are deprojected using the $L1$ regularization method, a non-parametric method that is built upon the work of Croston et al. (2006) and Ameglio et al. (2007). For a given 2D emission measure profile EMP , the values of the 3D emissivity profile ϵ are then given by maximising the likelihood function

$$-2 \log \mathcal{L} = \chi^2 = \sum (\mathcal{V} \times \epsilon - EMP)^2 + \lambda \sum \left| \frac{\partial^2 \log \epsilon}{\partial \log r^2} \right|, \quad (6)$$

where $\mathcal{V}_{i,j}$ is the geometrical matrix volume of the j th shell intercepted by the i th annulus, \times denotes a matrix product, and the sum is performed over all annuli. The second derivative of the emissivity is computed numerically as the derivative of $\epsilon(r)$. The first term on the right-hand side of equation 6 for observations is typically divided by the uncertainties, however a perfect knowledge of the 2D emission measure profile is assumed throughout this work. The second term on the right-hand side is a penalty term introduced to kill spurious small-scale fluctuations in the recovered 3D

profile (Diaz-Rodriguez et al. 2017). The parameter λ determines the degree of regularisation of the recovered profile and setting it zero makes the method equivalent to the onion-peeling technique (introduced in Kriss et al. 1983; Ettori et al. 2002, 2010).

We have compared this approach to the multiscale fitting deprojection method presented in Eckert et al. (2016) and Ghirardini et al. (2019). In general, we find good agreement between the recovered 3D gas density profiles when the two methods are applied to synthetic images of clusters with more regular morphologies. However, we found that for disturbed images the multiscale approach led to significantly more smoothing of features in the density profile relative to the 3D density profile produced directly from the simulation. Additionally, the multiscale fitting approach for disturbed clusters was found to be more computationally expensive for disturbed clusters. Therefore, we adopt the $L1$ approach for deprojection throughout this work.

2.3.3 Temperature profile extraction

To extract a temperature profile from a synthetic image, the pixels were first binned into 25 linearly spaced annuli in the range $0.0 - 1.5 r_{500}$. In each annulus, the pixel spectra were summed to yield a single spectrum for each annulus. The

spectrum was then modelled as a single-temperature plasma with the temperature, emission measure and metallicity free to vary. We leverage the 2D mass-weighted temperature, emission measure and metallicity profiles extracted directly from the simulation to provide an initial starting point for the fit, leading to a significant increase in the computational efficiency of our approach. The solar abundance of [Anders & Grevesse \(1989\)](#) was assumed. To mimic the observations more closely, each fit assumes an integration time of 100 ksec to convert the spectrum from erg s^{-1} to photon count in each energy bin, using the midpoint of the energy as the assumed photon energy. The fit was then performed in the energy range 0.5 – 10.0 keV for those energy bins that contained at least 1 count, making use of spectra interpolated from the APEC lookup table.

The method yields a spectroscopic temperature measurement in each annulus, the combined result of fitting all annuli produces the 2D temperature profile. This profile is then deprojected using the projection matrix \mathcal{V} weighted by the emissivity produced when deprojecting the gas density profile. Following [Mazzotta et al. \(2004\)](#) and [Ghirardini et al. \(2019\)](#), the 2D spectroscopic temperature profile is weighted in a spectroscopic-like fashion during the deprojection, i.e.

$$T_{\text{sl}} = \frac{\sum n_e^2 T^{-1/2}}{\sum n_e^2 T^{-3/2}}. \quad (7)$$

This yields a 3D spectroscopic temperature profile for each synthetic image. The analysis produces spectroscopic measurements of the thermodynamic profiles for every projection. Under the assumption that the cluster is in hydrostatic equilibrium, we then compute an estimated mass for each projection using the 3D profiles derived both directly from the simulation and those computed from the synthetic X-ray images.

2.4 Hydrostatic mass estimation

Deriving a cluster’s mass assuming spherical symmetry and that it is in hydrostatic equilibrium is a well-established technique. There have been many observational (e.g. [Miyatake et al. 2019](#), and references therein) and theoretical (e.g. [Nelson et al. 2014](#); [Biffi et al. 2016](#)) studies that have explored the bias induced by the required assumptions. Following [Vikhlinin et al. \(2006\)](#), the 3D gas density profile is modelled via a modified β -model profile

$$n_e n_H = n_0^2 \frac{(r/r_c)^{-\alpha}}{(1 + r^2/r_c^2)^{3\beta-\alpha/2}} \frac{1}{(1 + r\gamma/r_s^\gamma)^{\xi/\gamma}}, \quad (8)$$

where the value of γ is fixed such that $\gamma = 3$ and unphysically sharp density breaks are excluded via the condition $\xi < 5$. The best-fit values obtained are then fed into the analytic derivative of eq. (8) to yield a smoothly varying estimate of the density gradient.

Additionally, the 3D temperature profiles are fit with the general model

$$T_{3D}(r) = T_0 t_{\text{cool}}(r) t(r), \quad (9)$$

where T_0 is a model parameter and

$$t_{\text{cool}}(r) = \frac{x + T_{\text{min}} T_0}{(x + 1)}, \quad (10)$$

where $x = (r/r_{\text{cool}})^{a_{\text{cool}}}$ and

$$t(r) = \frac{r/r_t}{[1 + (r/r_t)^b]^{c/b}}. \quad (11)$$

The best fitting model parameters are again used in the analytic derivative of eq. (9) to provide a smoothing varying estimate of the gradient of the temperature.

The cumulative total mass profile of the projection is then computed assuming hydrostatic equilibrium via

$$M_{\text{tot}}(< r) = \frac{rk_B T(r)}{G\mu m_p} \left[\frac{d \log \rho_g}{d \log r} + \frac{d \log T}{d \log r} \right], \quad (12)$$

where r is the 3D radial cluster centric distance in Mpc, G is the gravitational constant, $\mu = 0.59$ is the mean molecular weight and m_p is proton mass. Given the cumulative mass as a function of cluster centric radial distance, the density profile of a given projection can then be computed. For a given redshift and cosmology, the density is divided by the critical density and $r_{500,\text{est}}$ is found by interpolating the profile to the radius at which it crosses a value of 500. The estimated cluster mass, $M_{500,\text{est}}$, is then given by the summing the cumulative mass profile to the value of $r_{500,\text{est}}$. Depending on the profiles used for the estimate, we label the recovered mass and radius as follows. For thermodynamic profiles extracted directly from the simulation, we denote them as $M_{500,\text{hse}}$ and $r_{500,\text{hse}}$, respectively, while for spectroscopic profiles extracted from the synthetic X-ray images we label them as $M_{500,\text{x-ray}}$ and $r_{500,\text{x-ray}}$, respectively. For each cluster, we compute 6 mass estimate using the deprojected spectroscopic profiles and 1 estimate from the mass-weighted profiles.

2.5 Centroid shift criterion

The centroid shift criterion is used to classify clusters as dynamically relaxed and is computed from the surface brightness maps of galaxy clusters. It has been used extensively in the literature (e.g. [Mohr et al. 1993](#); [Thomas et al. 1998](#); [Poole et al. 2006](#); [Kay et al. 2007](#); [Maughan et al. 2008](#); [Böhringer et al. 2010](#); [Rasia et al. 2013](#)). The criterion measures the deviation of the X-ray centroid as the radius of the chosen aperture reduces. The centroid shift measures the circular symmetry of the X-ray emission and tests for the presence of significant X-ray emission associated with larger substructures. We compute the centroid shift via

$$w = \frac{1}{r_{500,\text{sim}}} \sqrt{\frac{\sum \Delta_i - \langle \Delta \rangle}{M - 1}}, \quad (13)$$

where Δ is the separation of the centroids, the angle brackets denote the average, and we compute the centroids in circular apertures with radii in the range 0.15 – 1.0 $r_{500,\text{sim}}$ with steps of 0.05 $r_{500,\text{sim}}$. A cluster is classified as relaxed if $w < 0.01$ (e.g. [Weißmann et al. 2013](#)).

3 HYDROSTATIC MASS BIAS

We begin by exploring the bias induced by estimating cluster masses assuming that they are in hydrostatic equilibrium. Throughout this work we define the bias as $b = 1 - M_{500,\text{est}}/M_{500,\text{sim}}$, and we denote the SUBFIND mass, $M_{500,\text{sim}}$, as the “true” cluster mass. Using density and temperature

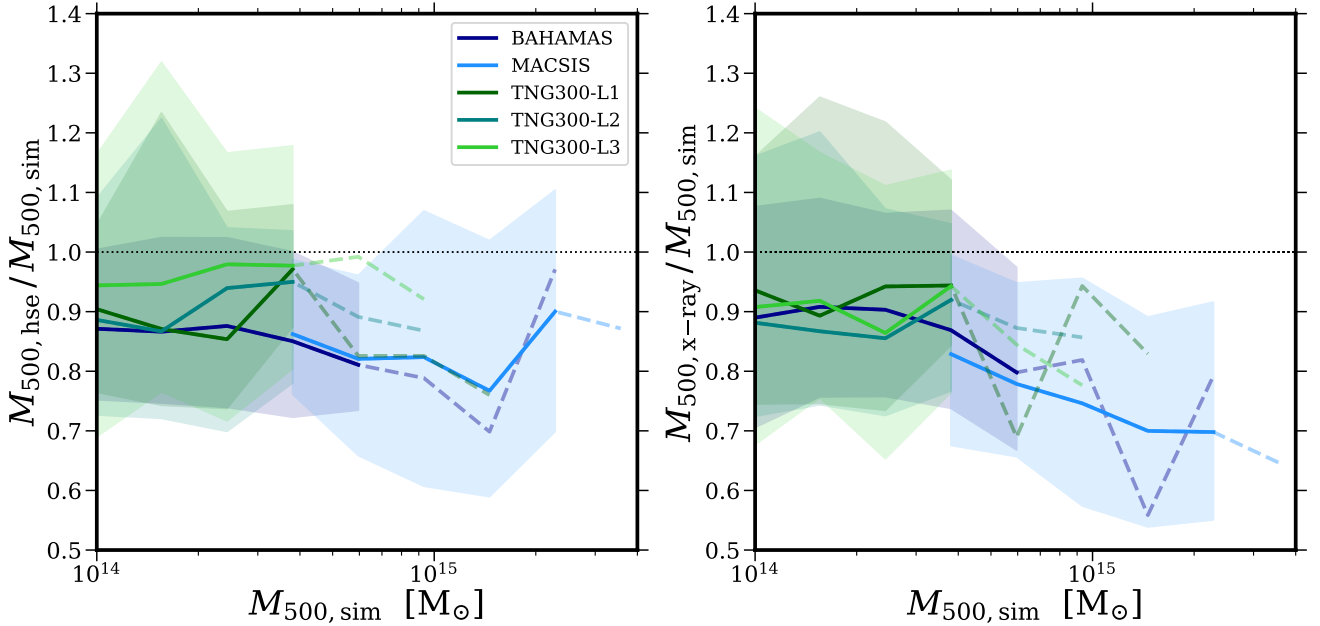


Figure 3. The median hydrostatic mass estimate to true mass ($M_{500,\text{sim}}$) ratio as a function of true mass at $z = 0.1$ for estimates derived from simulation (left) and synthetic X-ray image (right) profiles. We plot simulated samples from BAHAMAS (dark blue), MACSIS (light blue), TNG300 level 1 (dark green), level 2 (medium green), and level 3 (light green). The shaded area denotes the 1σ scatter and the dashed lines denote the median ratio where the number of clusters in a bin of width $\Delta \log_{10}(M_{500,\text{sim}}/M_{\odot}) = 0.1$ is less than 10. The hydrostatic bias is relatively independent of true mass for profiles derived directly from the simulation but is clearly mass-dependent for synthetic X-ray profiles.

profiles extracted from the simulation, we find that for IllustrisTNG there is an excellent agreement in the amplitude of the median bias for the highest resolution runs with $b = 0.11 \pm 0.01$ and $b = 0.11 \pm 0.01$ for levels 1 and 2, respectively. We find a decrease in the median bias for the lowest resolution, with $b = 0.05 \pm 0.01$ for level 3, but this difference is significantly smaller than the scatter in the population. The uncertainty on the bias is computed via bootstrap resampling the cluster populations 10,000 times. The BAHAMAS sample yields a median bias of $b = 0.13 \pm 0.002$, in good agreement with the IllustrisTNG level 1 result. Finally, the MACSIS sample yields a median bias of $b = 0.15 \pm 0.003$. In the left panel of Fig. 3, we plot the ratio of the estimated mass from simulation derived profiles to true mass as a function of true mass. All simulations show a significant scatter in the estimated to true mass ratio, with TNG yielding a slightly larger scatter relative to BAHAMAS at fixed mass. The MACSIS sample yields a larger scatter relative to BAHAMAS, which may be due to the selection function of the MACSIS haloes. We find no obvious trend of hydrostatic bias with mass when using the thermodynamic profiles derived directly from the simulation. The choice of numerical resolution, hydrodynamic scheme and subgrid physics reassuringly appears to have minimal impact on the hydrostatic bias recovered, under the assumption that the simulation is being performed at, or near, the numerical resolution level at which the subgrid model was calibrated.

If the spectroscopic density and temperature profiles, estimated from synthetic X-ray projections, are used to compute an estimated mass, we find that the IllustrisTNG

samples yield similar results, with a median bias of $b = 0.10 \pm 0.01$, $b = 0.12 \pm 0.01$ and $b = 0.08 \pm 0.01$ for levels 1, 2 and 3, respectively. The BAHAMAS sample yields a bias of $b = 0.11 \pm 0.003$, consistent with the value from simulation derived profiles. However, the median bias of the MACSIS sample increases to 0.25 ± 0.005 . In the right panel of Fig. 3, we plot the ratio mass estimated via spectroscopic profiles to true mass as a function of true mass and we find that hydrostatic bias is now mass-dependent. For low mass clusters ($M_{500,\text{sim}} < 3 \times 10^{14} M_{\odot}$) the bias is consistent between the different samples, with TNG again yielding slightly larger scatter. However, above this mass the hydrostatic bias begins to increase and for the most massive clusters ($M_{500,\text{sim}} > 2 \times 10^{15} M_{\odot}$) the bias is as large as $b = 0.3$. Although lacking the statistics for a rigorous comparison, all of the simulation samples show some evidence that the bias is increasing with cluster mass. The mass dependence of the hydrostatic mass bias is consistent with previous numerical work that computed estimated cluster masses from spectroscopic profiles (Henson et al. 2017; Barnes et al. 2017b; Pearce et al. 2020), although these works did not account for the impact of projection. We now compare our results to those of previous work.

3.1 Comparison to previous results

Observationally, there are many approaches to estimating a cluster's mass. Relative mass calibration is very common and derives a cluster's mass by equating an observable property, such as X-ray temperature or integrated Compton-y signal,

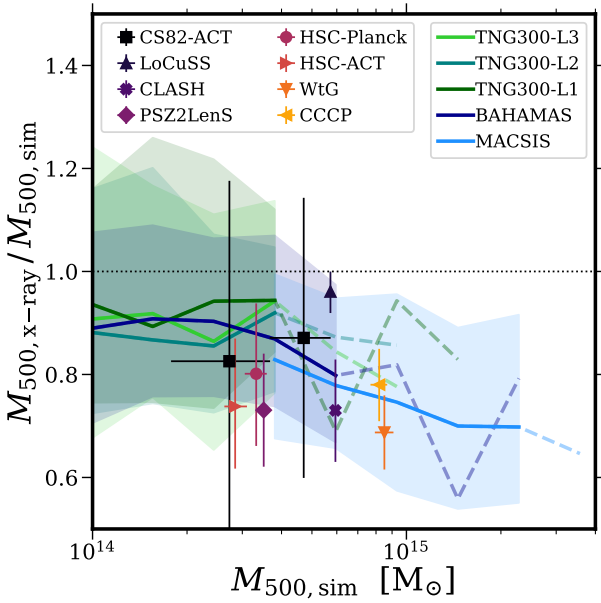


Figure 4. The median hydrostatic mass derived from X-ray images to true mass ratio as a function of true mass at $z = 0.1$ against a collection of observed biases. The simulation lines styles are the same as Fig. 3. The collection of observational points compare hydrostatic mass estimates to weak lensing derived mass estimates and are taken from Weighing the Giants (WtG) (von der Linden et al. 2014), CCCP (Hoekstra et al. 2015), CS82-ACT (Battaglia et al. 2016), LoCuSS (Smith et al. 2016), CLASH (Penna-Lima et al. 2017), PSZ2LenS (Serenio et al. 2017), HSC-Planck (Medezinski et al. 2018) and HSC-ACT (Miyatake et al. 2019). We find excellent agreement in the magnitude of hydrostatic bias between the simulated and observed samples. Additionally, the sample variance of the observations is well matched to the scatter of the simulated sample.

to an observable-mass scaling relation. However, this method is only possible once a scaling relation is calibrated via absolute mass measurements. Absolute mass calibration measurements typically require very high-quality observational data. The member galaxies of a cluster can be used to estimate its mass, either through measuring the richness (e.g. Yee & Ellingson 2003; Simet et al. 2017), i.e. the number of galaxies present, or by using them as kinematic tracers of the underlying potential (e.g. Diaferio & Geller 1997; Zhang et al. 2011; Bocquet et al. 2015; Sereno & Ettori 2015; Gifford et al. 2017). However, this approach is fundamentally limited by determining cluster membership (e.g. Old et al. 2014) and the fact that theoretical studies have shown galaxies are biased tracers of the underlying potential, agreeing on the size of the bias but not its sign (Munari et al. 2013; Armitage et al. 2018). A study of caustic mass estimates to hydrostatic mass estimates by Maughan et al. (2016) favoured a hydrostatic bias of less than 10 per cent, though we caution that a recent detailed study of the X-COP clusters found caustic masses underestimate hydrostatic mass significantly (Ettori et al. 2019).

With the advent of deep and wide optical surveys, weak lensing, the statistical distortion of background galaxies due to the intervening mass, has become the absolute cluster

mass estimator of choice. Using the weak lensing cluster mass estimate as a measure of the true cluster mass, it is possible to infer the bias induced by measuring a cluster’s mass assuming it is in hydrostatic equilibrium. Though we caution that theoretical studies have shown that weak lensing mass estimates themselves typically underestimate cluster masses by 5 – 10 per cent (e.g. Becker & Kravtsov 2011; Bahé et al. 2012; Rasia et al. 2014; Henson et al. 2017).

There is mild disagreement in the magnitude of the hydrostatic bias between observational studies. Some have found that the bias is less than 10 per cent, with Applegate et al. (2014), Israel et al. (2014) and Smith et al. (2016) finding values of $b = 0.04$, $b = 0.08$ and $b = 0.05$, respectively. However, others have found biases greater than 20 per cent, with studies by Mahdavi et al. (2008), Mahdavi et al. (2013), von der Linden et al. (2014), Hoekstra et al. (2015), Simet et al. (2015), Battaglia et al. (2016), Penna-Lima et al. (2017), Sereno et al. (2017), Medezinski et al. (2018) and Miyatake et al. (2019) all finding significantly larger hydrostatic bias, on the order of 20 – 30 per cent. A potential explanation for part of the differing bias estimates may be attributed to the varying overdensities at which the masses are estimated and the different mass and redshift distributions of the various samples.

In Fig. 4 we compare our hydrostatic bias measurement from spectroscopically measured thermodynamic profiles as a function of true cluster mass against a collection of observational results taken from Miyatake et al. (2019), many of which were also presented in Medezinski et al. (2018). We find excellent agreement between the observed hydrostatic bias measurements and those recovered from the synthetic images of simulated samples. Additionally, we note that the sample variance of the observational results is well matched to the scatter in the simulated results. We note that scatter in the simulated ratios will increase slightly if lensing masses were used, rather than the true mass, due to the intrinsic scatter in the mass estimate. However, we delay a detailed study of lensing mass estimates to a future study. The observed amplitude of the hydrostatic bias is reproduced by the simulations and the presence of a mass-dependent mass bias is certainly not ruled out by the observations, with Medezinski et al. (2018) stating that the observations may suggest a mass-dependent bias.

Our results are also consistent with previous numerical studies. Those that derive a hydrostatic mass estimates directly from simulation data, typically mass-weighted profiles, find they are biased low relative to true masses by 10 – 20 per cent (e.g. Lau et al. 2009; Kay et al. 2012; Rasia et al. 2014; Nelson et al. 2014; Biffi et al. 2016; Angelinelli et al. 2019; Ansarifard et al. 2019). Additionally, for those studies that compute a cluster’s thermodynamic profiles from synthetic X-ray data also find good agreement, biased low at the level of 10 – 20 per cent for low mass ($M_{500, \text{sim}} < 8 \times 10^{14} M_{\odot}$) clusters (e.g. Nagai et al. 2007; Rasia et al. 2012; Le Brun et al. 2014) with a bias that increases with increasing mass (Henson et al. 2017; Pearce et al. 2020). We note that Biffi et al. (2016) also compute the bias using a spectroscopic-like weighted profiles (Mazzotta et al. 2004) and find that the hydrostatic bias in their relatively massive ($M_{200, \text{sim}} > 8 \times 10^{14} h^{-1} M_{\odot}$) sample increased. Given the large scatter in estimated mass, the small sample size of many of these studies, typically < 50 , is a limiting

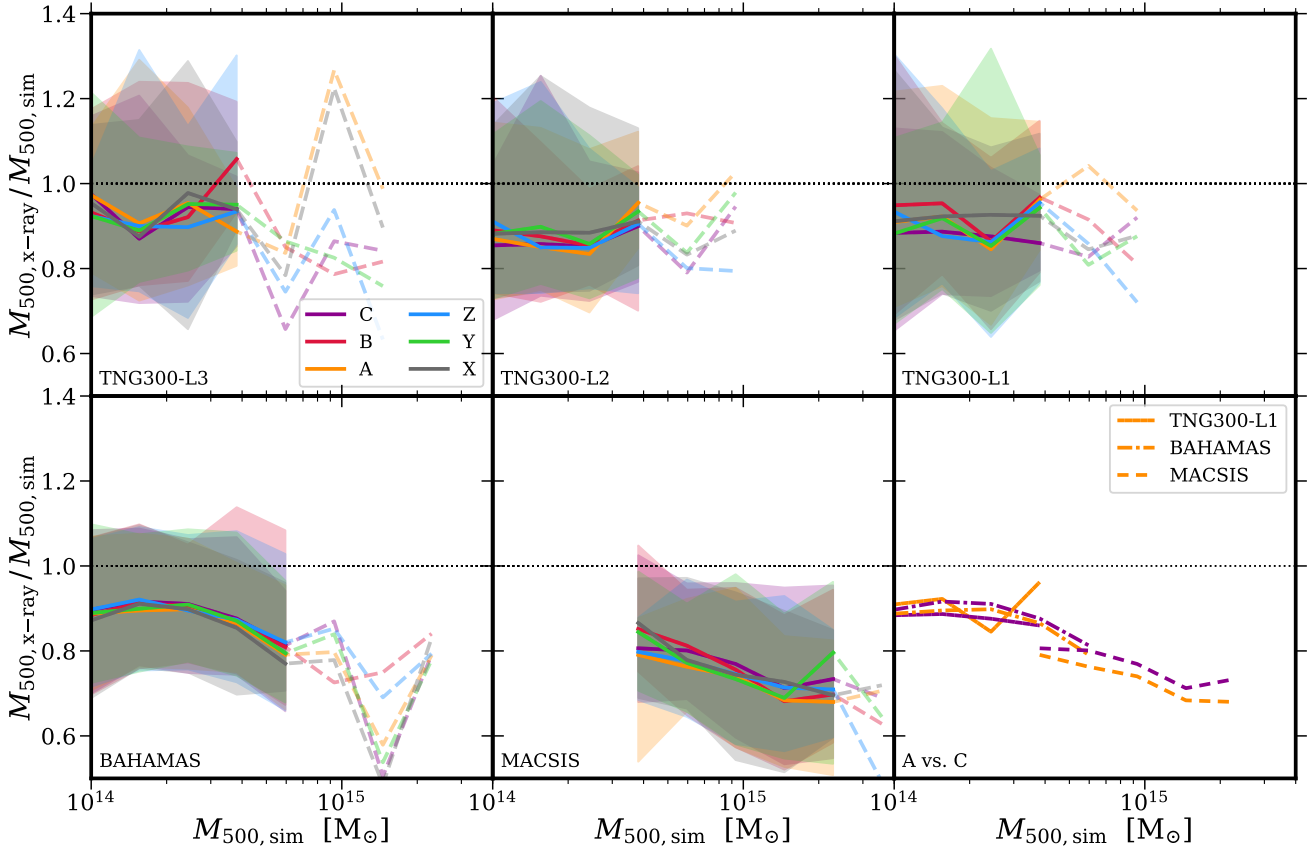


Figure 5. The median X-ray derived hydrostatic mass to true mass ratio as a function of true mass at $z = 0.1$, split by simulation and projection axis. For TNG300 levels 3 (top left), 2 (top middle), 1 (top right), BAHAMAS (bottom left) and MACSIS (bottom middle) we plot the median ratio along axes x (grey), y (green), z (blue), A (orange), B (red) and C (purple). The dashed lines denote the median ratio where the number of clusters in a bin of width $\Delta \log_{10}(M_{500, \text{sim}}/M_{\odot}) = 0.1$ is less than 10 and the shaded regions denote the 1σ scatter in the ratio. In the bottom right panel, we compare projections along the longest (A) and shortest (C) axes of the cluster and find minor differences in the median ratio that are significantly smaller than the scatter in any given sample.

factor and the synthetic X-ray approach is typically limited to the direct computation of 3D profiles, ignoring issues such as projection effects and the presence of clumpy gas.

3.2 The impact of projection

All clusters to varying degrees are triaxial in nature and determining the projection angle is very difficult (e.g. Morandi et al. 2012; Sereno et al. 2017), although there have been interesting recent works relating the orientation of the brightest cluster galaxy to the cluster triaxiality (e.g. Wittman et al. 2019; Herbonnet et al. 2019). This is potentially an issue for hydrostatic mass estimates due to the assumption that the cluster is spherically symmetric. The triaxial nature of clusters can be removed from analyses by stacking many clusters into a single profile. However, in numerical simulations, we know the orientation of the cluster. Therefore, we now examine the impact of projection on the estimated cluster masses by splitting the synthetic images for the different simulations into their projection axes. To assess the median hydrostatic bias of randomly orientated haloes we use the Cartesian projection axes (x , y , z) as samples of randomly

projected clusters, making the fair assumption that a cluster’s triaxiality should be independent of its orientation in the simulation volume. The projections along the eigenvectors of the mass tensor (A , B , C) enable an examination of the impact of cluster triaxiality on the estimated mass.

In Fig. 5 we plot the ratio of the estimated mass from synthetic X-ray observations to true mass as a function of true mass for the different samples. Although somewhat noisy, we find that the median of the ratio of estimated mass to true mass projected along either the A or C axes is consistent with values recovered from projecting along the x , y , z axes for all simulations. Additionally, the scatter in the ratio is also consistent for all the chosen projection axes. For clarity, in the bottom right panel of Fig. 5 we plot the median ratios for the BAHAMAS, MACSIS and IllustrisTNG level 1 simulations computed for synthetic images projected along the A and C axes. The average change in the bias from projecting along C rather than A is $\Delta b = -0.01$, 0.02 and 0.03 for TNG100-L1, BAHAMAS and MACSIS, respectively. However, the difference in hydrostatic bias for any of the simulated samples is significantly smaller than the scatter in the population of the cluster sample. Therefore,

we conclude that for a sufficiently large sample of clusters the projection angle has minimal impact on the hydrostatic mass estimate. On a cluster-by-cluster basis, the scatter in estimated mass will dominate over the projection angle.

3.3 Estimated mass scatter

To study the scatter in estimated mass, we first compute a kernel-weighted moving average of the mass estimated from synthetic X-ray observations as a function of true mass for all simulated cluster samples. For the i th cluster of mass M_i , we average over the neighbouring clusters using of a Gaussian weight function of the form

$$w_k = \frac{1}{\sqrt{2\pi}\tau} \exp\left[-\frac{\mu_k^2}{2\tau^2}\right], \quad (14)$$

where $\mu_k = \log(M_k / M_i)$ for the k th neighbour. The parameter τ is a hyperparameter that specifies the width of the kernel, with a small value leading to a noisy moving average and a large value over-smoothing. After testing, we use a value of $\tau = 0.1$ to compute the moving average. We compute the moving average in log-log space.

We then compute the scatter about the moving average for a set of 17 points in log mass that are linearly spaced in the range $14.0 < \log_{10}(M / M_\odot) < 15.7$ with $\Delta \log(M) = 0.1$ via

$$\sigma = \sqrt{\frac{1}{N-2} \sum_k^N [\log_{10}(Y_k) - \log_{10}(Y_{\text{MA}}(M_k))]^2}, \quad (15)$$

where the sum runs over the N clusters within a top-hat window function of width $\Delta \log_{10}(M) = 0.2$, Y_k is the estimated mass for the k th cluster and Y_{MA} is the moving average of the estimated mass at true mass $M_k \equiv M_{k,500,\text{sim}}$ of the cluster. We compute the uncertainty on the estimated scatter by bootstrap resampling the samples 10,000 times.

In Fig. 6, we plot the scatter in estimated mass as a function of true mass for all simulated samples. For IllustrisTNG, we find that the scatter in estimated mass reduces with increasing cluster mass, independent of numerical resolution, but increases in the final mass as the sample size reduces. The amplitude of the scatter appears to increase with decreasing numerical resolution. At $M_{500,\text{sim}} = 10^{14} M_\odot$, there is small increase in scatter from level 1 ($\sigma = 0.112$) to level 2 ($\sigma = 0.114$), but a significantly larger increase in scatter for IllustrisTNG level 3 ($\sigma = 0.147$). The normalization of the estimated mass scatter is slightly larger for the BAHAMAS sample relative to IllustrisTNG level 1, with $\sigma = 0.120$ at $M_{500,\text{sim}} = 10^{14} M_\odot$. However, we find the same trend of decreasing scatter with increasing cluster mass, before increasing again as the sample statistics become small. The MACSIS sample has greater scatter at fixed mass relative to the BAHAMAS sample, however, this is likely due to the way the sample was selected. These lower mass MACSIS clusters have a significantly larger FoF mass relative to $M_{500,\text{sim}}$, as they were selected via $M_{\text{FoF}} \geq 10^{15} M_\odot$, compared to the BAHAMAS sample. Using the BAHAMAS sample, if we restrict our analysis to clusters with similar values of $M_{500,\text{sim}} / M_{\text{FoF}}$ we find that these systems are just forming and very dynamically disturbed and that the amplitude of the scatter in estimated mass increases. Therefore, the selection of the MACSIS sample likely explains why the

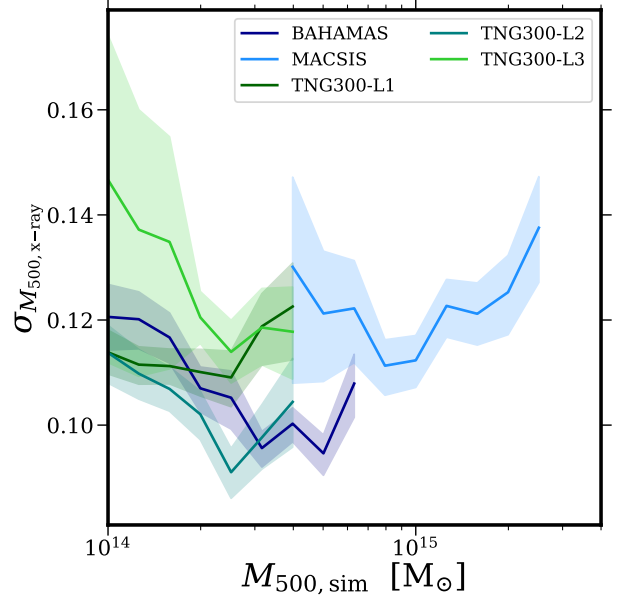


Figure 6. Hydrostatic mass estimate scatter as a function of true mass at $z = 0.1$ for the simulation samples. Line styles are the same as Fig. 3. The scatter does not change significantly with mass, but there is evidence of a minor trend of decreasing scatter with increasing mass before the sample statistics become small and the scatter increases. Selection effects are likely the cause of the increased amplitude of the scatter at fixed mass for the MACSIS sample relative to the BAHAMAS sample.

scatter in estimated mass is larger at fixed mass relative to the BAHAMAS sample. We find that the scatter for the MACSIS clusters is generally flat, though there is evidence that the scatter decreases with mass before increasing again as the sample statistics become small.

Accounting for selection effects, the simulated samples overall produce a similar level of scatter in estimated mass at fixed mass, suggesting that it is insensitive to the implementation of subgrid physics. As the numerical resolution decreases from that at which the model was calibrated, we find that the scatter in estimated mass increases marginally. Overall, the amplitude of the hydrostatic bias we recover appears insensitive to the numerical choices, yielding a similar trend with mass and level of scatter. Additionally, the bias recovered shows good broad agreement with the bias measured observationally using weak lensing masses as a proxy for true mass.

4 SPHERICALLY SYMMETRIC HALOES

The other assumption required for many cluster mass estimation techniques is that galaxy clusters are spherically symmetric. Often in cluster studies (e.g. Mantz et al. 2015b) the most regular systems are selected because it is believed their three-dimensional properties can be recovered with reduced systematic uncertainty, which reduces the systematic uncertainty on the mass estimate. We now explore this assumption using the mass tensor. Specifically, we measure how spherical a cluster is via its sphericity, with an idealized

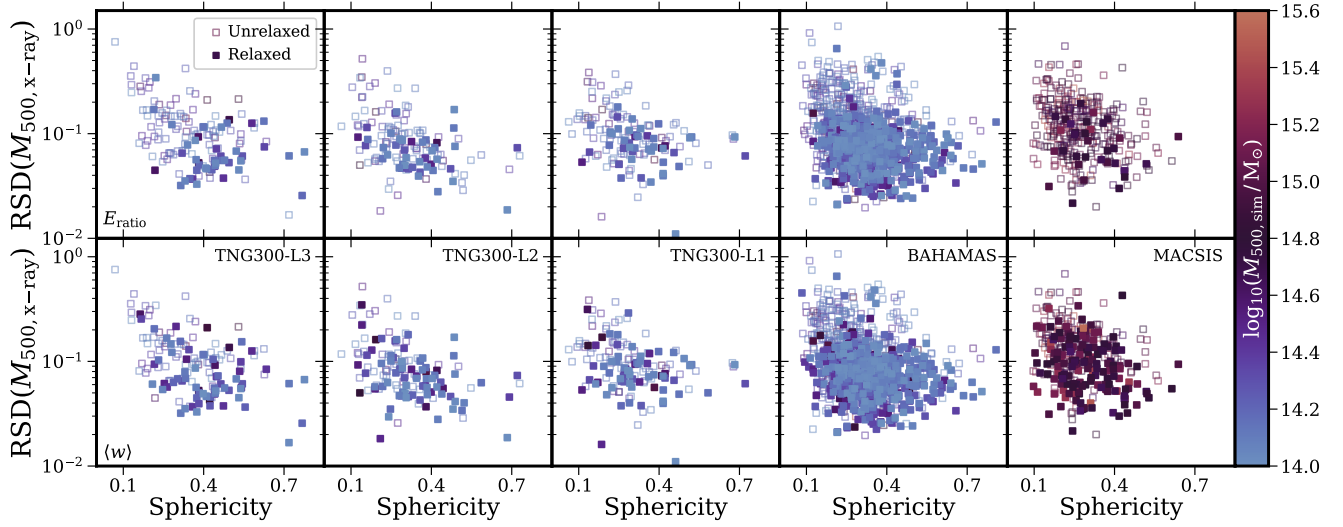


Figure 7. Relative standard deviation (RSD) of the estimated mass between cluster projections as a function of cluster sphericity at $z = 0.1$. From left to right we plot the TNG300 level 3, 2, 1, BAHAMAS and MACSIS samples, respectively. The points are coloured by the log of the true mass. The filled (open) points are classified as relaxed (unrelaxed) by the energy ratio (top) and centroid shift (bottom) criteria. We find the expected result that the RSD in the estimated mass reduces as the cluster becomes more spherical. Overall, the relaxation criteria remove those clusters with the largest estimated mass variations, but they do not select more spherical haloes on average nor do they classify the same subset of clusters as relaxed.

spherical halo yielding $s = 1$. As a measure of how reliable the mass estimate is we compute the relative standard deviation (RSD), also known as the coefficient of variation, for the different projections of each cluster

$$RSD = \frac{1}{\langle M_{500,x-ray} \rangle} \sqrt{\frac{\sum (M_{500,x-ray} - \langle M_{500,x-ray} \rangle)^2}{N - 1}}, \quad (16)$$

where the sum runs over all of the projections and the angle brackets denote the mean value of the estimated mass. If the profiles of more regular clusters can be recovered with less systematic uncertainty, then one would expect the RSD value to decrease for clusters whose sphericity values are larger.

In Fig. 7, we plot the estimated mass RSD as a function of sphericity for the different simulated samples. Additionally, to explore any trend with mass we colour the symbol based on the true mass of the cluster. Overall, all of the simulated samples in this work show the same trend: a reduction in the value of the RSD as the sphericity of the cluster increases. The median estimated mass difference is $\Delta M_{500,x-ray} \approx 0.1$. Therefore, our result confirms that more spherical clusters yield estimated masses with reduced systematic uncertainty. We note that the MACSIS sample has a lower median sphericity value relative to the other simulated samples. This is driven by the fact that the sample is more massive and, therefore, has formed more recently. These clusters have had less time to relax since their formation and the most spherical clusters are absent from the sample. Finally, there is no discernible trend with cluster mass for any of the samples, which likely reflects a large range of formation histories possible for a cluster at fixed mass.

Additionally, we split the simulated samples plotted in Fig. 7 into relaxed and unrelaxed via two criteria. In the top

row, we use the energy ratio E_{ratio} , a theoretical, 3D aperture criterion that measures the ratio of kinetic energy to thermal energy for gas cells/particles within a spherical aperture, to define a relaxed subset. In the bottom row, we classify the clusters using the centroid shift, an observational 2D aperture criterion that measures how much the centroid of the X-ray surface brightness shifts when the aperture used to compute it changes. The definitions of the energy ratio and centroid shift and how we compute them are presented in Sections 2.1.5 and 2.5, respectively. Relaxed (unrelaxed) clusters are denoted by filled (open) symbols. Observationally, relaxed clusters play a prominent role in studies of cluster astrophysics, scaling relations and cosmology because they are thought to be the most regular systems with minimal systematic uncertainties. For both criteria used in this work we find the same result, selecting relaxed clusters removes those objects with the largest RSD values. Interestingly, the selection of relaxed clusters does not lead to any significant change in the median sphericity value of the sample. Finally, we note that although both relaxation criteria remove the largest RSD values they do not select the same clusters, with 48, 49, 54, 72 and 19 per cent of those clusters classified as relaxed by the energy ratio appearing in the subset defined as relaxed by the centroid shift for TNG300 level 1, level 2, level 3, BAHAMAS and MACSIS, respectively. A detailed study of many common relaxation criteria, both theoretical and observational, is presented in Cao et al. (in prep.).

5 ORIGIN OF MASS DEPENDENT BIAS

We now seek to understand why the hydrostatic bias becomes mass-dependent for profiles derived from the synthetic X-ray images. As shown in eq. 12, the hydrostatic mass estimate depends linearly on the temperature mea-

sured at the radius of interest, i.e. $r_{500, \text{x-ray}}$. Therefore, we use the estimated mass to derive the radial point of interest and then extract both the mass-weighted and spectroscopic temperatures. In practice, we interpolate the radial temperature profiles to obtain the temperature estimates at the radius of interest.

In Fig. 8, we plot the median bias of the spectroscopic temperature relative to the mass-weighted temperature as a function of the cluster’s true mass for all simulated samples. For low-mass clusters ($M_{500, \text{sim}} < 4 \times 10^{14} M_{\odot}$), the spectroscopic temperature is consistent with the mass-weighted temperature, though there is significant scatter in the samples. We find that the numerical resolution and the subgrid physics implementation does not affect the recovered temperatures, with the TNG300 level 1, 2, 3 and BAHAMAS samples all yielding similar median values around zero. The scatter in the bias is slightly larger for all TNG samples. However, as the mass of the cluster increases, we find that the spectroscopic temperature is increasingly biased low relative to the mass-weighted estimate. This result is dominated by the MACSIS cluster sample but the other simulated samples also show the spectroscopic measurement is biased low, though we are limited by small sample statistics. For the most massive clusters ($M_{500, \text{sim}} \geq 2 \times 10^{15} M_{\odot}$) the median bias reaches 0.2, i.e. the spectroscopic temperature is only 80 per cent of the mass-weighted temperature, and the scatter reaches values close to 0.3. Given the linear dependence of the hydrostatic mass on the temperature at the radius at which the estimate is made, the increasingly bias spectroscopic measurement is sufficient to explain the apparent mass dependence. For massive clusters, the mass bias can be explained as the sum of the bias induced by assuming the cluster is in hydrostatic equilibrium ($b = 0.1$) and the bias produced by the spectroscopic temperature being lower than the mass-weighted temperature ($b = 0.2$), yielding a total bias of $b = 0.3$. Those clusters with the largest scatter would agree with the hydrostatic bias found for the most massive clusters in [Pearce et al. \(2020\)](#).

To understand why the spectroscopic temperature estimate is biased low relative to the mass-weighted temperature, we now explore the distribution of gas temperatures and pressures as a function of radius for two subsets of simulated clusters. We select the entire BAHAMAS sample, noting that similar results are obtained for all resolution levels of IllustrisTNG, and all MACSIS clusters with a mass $M_{500, \text{sim}} \geq \times 10^{15} M_{\odot}$. The MACSIS cluster subset ensures that any impact of the sample selection function is minimized. For each cluster, the gas cells/particles are binned into 50 linearly spaced radial bins in the range $0-1.25 r_{500, \text{sim}}$. Within each radial bin, the mass-weighted cumulative distribution function of the temperature is computed and stored at 200 percentile points, i.e. increments of 0.5. Additionally, the volume-weighted pressure distribution is computed in each radial bin for the cells/particles still ordered by their temperature. For both samples, the value of the median percentile in every pixel is then computed. At this stage, it is important to account for the fact the more massive clusters are hotter and have larger pressures due to their deep potential wells. The mass dependence is removed from the temperature and pressure distributions by dividing them by the virial temperature, $k_B T_{500}$, and pressure, P_{500} , expected

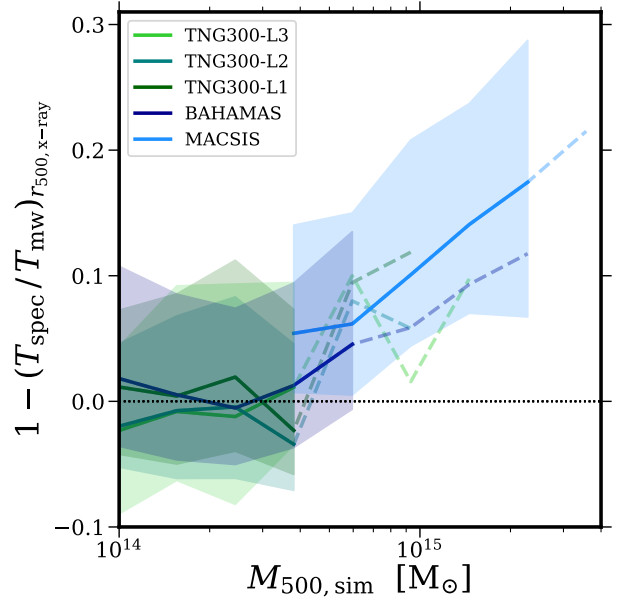


Figure 8. Median spectroscopic temperature bias relative to the mass-weighted temperature measured at $r_{500, \text{x-ray}}$ as a function of true mass. Line styles are the same as Fig. 3. For low-mass clusters ($M_{500, \text{sim}} < 4 \times 10^{14} M_{\odot}$) the two temperature estimates are consistent within the scatter of the samples. However, at higher masses, the spectroscopic temperature estimate consistently underpredicts the mass-weighted estimate for all simulation samples. The amplitude of this bias is sufficient to account for the mass dependence of the hydrostatic mass bias.

for a system with mass $M_{500, \text{sim}}$. These are calculated via

$$k_B T_{500} = \frac{GM_{500, \text{sim}} \mu m_p}{2r_{500, \text{sim}}}, \quad (17)$$

and

$$P_{500} = 500 f_b k_B T_{500} \frac{\rho_{\text{crit}}}{\mu m_p}, \quad (18)$$

where f_b is the universal baryon fraction and ρ_{crit} is the critical density of the Universe at $z = 0.1$. For both samples, the median value in every pixel is then computed and we reintroduce the mass dependence by multiplying back through by the median values of $k_B T_{500}$ and P_{500} .

Fig. 9 shows the temperature and pressure distributions as a function of radius for the two samples. As expected, for BAHAMAS the average temperature decreases with radius. A similar result is seen for the IllustrisTNG samples, as seen in [Barnes et al. \(e.g. 2018, 2019\)](#). At $r_{500, \text{x-ray}}$, all of the gas resides within a small temperature range (1.1–2.8 keV). The MACSIS samples yields the same radial decline in temperature, but, as expected for a more massive sample, the temperature normalization is larger. At any radius the MACSIS sample has a significantly wider temperature distribution, with the gas temperatures in the 3.1–13.1 keV at $r_{500, \text{x-ray}}$. Examining the gas pressure we find both samples yield distributions with a similar width, the pressure varies by a factor 2.5 at $r_{500, \text{x-ray}}$. For the MACSIS sample the width of the pressure distribution is significantly smaller than the temperature distribution. Given that $P = nk_B T$, the wider tem-

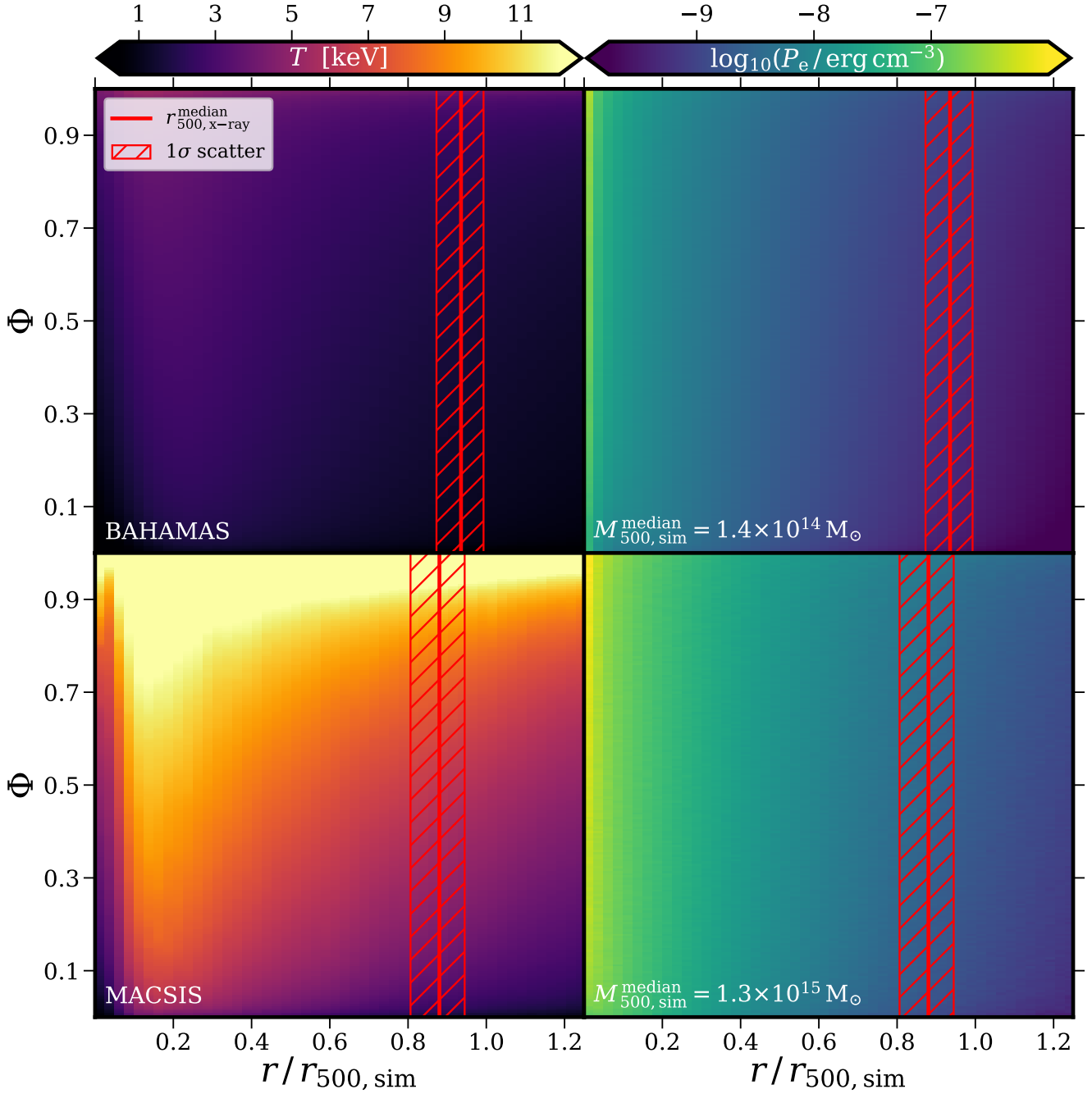


Figure 9. Temperature (left) and pressure (right) distributions for the BAHAMAS sample (top) and those clusters in the MACSIS sample with $M_{500,\text{sim}} \geq 10^{15} M_{\odot}$ (bottom). The red line denotes the median $r_{500,\text{x-ray}} / r_{500,\text{sim}}$ value and the hashed region shows the 1σ scatter in the ratio. The MACSIS selection removes those clusters impacted by sample selection effects. The pressure distribution is ordered by the temperature of the gas. Despite the large temperature range in the MACSIS sample, the gas has a significantly smaller range in pressure. Therefore, the cooler gas must have a higher density and a larger relative contribution to the spectroscopic temperature, due to the quadratic density weighting of the emission process.

perature distribution in the MACSIS sample implies a wider distribution of densities, with cooler gas having a linearly higher relative density. This difference in density is further amplified by the n^2 dependency of the X-ray emission mechanism. Therefore, the cooler gas is disproportionately represented in the X-ray emission and the spectroscopic fitting

yields a lower temperature than the mass-weighted temperature estimate for the most massive clusters.

This leads to the question of why this lower entropy gas has not sunk to centre of these massive clusters. The likely answer is that these haloes are only just collapsing by the current epoch, and this infalling, colder gas has not had the opportunity to settle closer to the cluster centre. However,

examining the history and fate of this gas is beyond the scope of this study and we leave it to future work.

The mass dependence of the hydrostatic bias found in this study is driven by the result of fitting a single temperature plasma model to an X-ray spectrum that is the sum of gas at a range of temperatures. Given the unimodal nature of temperature distribution for the massive clusters present in the MACSIS sample, it is not obvious that fitting a two (or more) component model to the spectrum is the correct approach to removing this bias. The recent X-COP study found that the pressure profiles derived from X-ray data were in good agreement with the pressure profiles from SZ observations (Ghirardini et al. 2019). Due to the different emission mechanism, observations of the SZ signal should yield a temperature estimate that is more closely aligned with the expected mass-weighted temperature. Therefore, they found that the mass-weighted and spectroscopic temperature estimates were in good agreement with each other. This study agrees with the X-COP results. The median mass of the X-COP sample is $5 \times 10^{14} M_{\odot}$ (Eckert et al. 2017), where we find that the median spectroscopic temperature is biased 3 per cent lower than the mass-weighted temperature and the scatter encompasses an unbiased result. Therefore, the simulated spectroscopic and mass-weighted pressure profiles will agree with each other. The limitation of the X-COP study is the small number of low-mass objects (12) studied. The next generation of detailed surveys that sample a larger number of clusters, with significant statistics for very massive clusters (like the *XMM* Heritage cluster project), have the potential to fully characterize the amplitude of the hydrostatic bias as a function of cluster mass.

6 CONCLUSIONS

In this work, we have introduced the multiwavelength analysis framework MOCK-X. The framework is agnostic to the chosen simulation, generating synthetic multiwavelength images of simulated haloes. We have generated synthetic images for 18,756 projections of mass-limited ($M_{500,\text{sim}} > 10^{14} M_{\odot}$) samples from all three resolution levels of the IllustrisTNG (300)³ Mpc³ volume, the reference *Planck* BAHAMAS volume and the MACSIS simulations. We have used the synthetic Chandra-like X-ray images to explore the bias induced by estimating cluster masses assuming that they are in hydrostatic equilibrium. The mass estimates are derived from the thermodynamic profiles extracted directly from the synthetic images, with our chosen approach following common observational methods (Section 2). Our main results are as follows:

- Hydrostatic mass estimates derived from mass-weighted simulation profiles yield a bias of $b = 0.11 - 0.15$ with significant scatter in the population (Fig. 3). The choice of hydrodynamical method or subgrid physics implementation has a negligible impact on the recovered bias. However, when masses are estimated using the thermodynamic profiles derived from synthetic X-ray images we find that the bias shows a significantly stronger mass dependence. The bias increases from $b = 0.1$ at $10^{14} M_{\odot}$ to $b = 0.3$ at $2 \times 10^{15} M_{\odot}$.
- We compare the bias recovered from synthetic X-ray images to a collection of hydrostatic mass bias estimates

from Miyatake et al. (2019) and references therein. The observations use mass estimates derived from weak lensing as a proxy for the true cluster mass. We find that the amplitude of the hydrostatic bias yielded by the simulations is in excellent agreement with observed bias amplitude (Fig. 4). Additionally, the sample variance of the observations is in good agreement with the scatter in mass bias found for the simulated samples.

- We examine the impact of projection on the recovered hydrostatic mass estimate (Fig. 5) and find it has a negligible impact on the recovered mass. The amplitude and scatter in recovered mass is consistent for all projections and numerical samples.

- The scatter in estimated mass is roughly constant as a function of mass (Fig. 6). There is some evidence it decreases slightly with increasing cluster mass, but the scatter increases again as the sample statistics become small. Additionally, we find some evidence the mass scatter increases as the resolution of the TNG simulation decreases. The BAHAMAS and TNG300 level 1 samples yield consistent scatter, suggesting it is independent of chosen numerical method or subgrid physics. The MACSIS sample has a higher scatter at fixed mass, but this is driven by the method used to select the sample (i.e. via M_{FoF}).

- We examined the relative standard deviation of the estimated mass as a function of halo sphericity (Fig. 7), finding the expected result that the mass is more reliably estimated for those clusters that are more spherical. We then select relaxed clusters using the theoretical energy ratio criterion and the observational centroid shift criterion. Although the two criteria select those clusters whose mass is more consistently recovered, neither preferentially select more spherical haloes nor do they select consistent subsets of relaxed clusters.

- We then explored the origin of the mass dependence of the hydrostatic bias by exploring the bias induced by measuring temperatures spectroscopically, rather than mass-weighted, at the radius of interest $r_{500,\text{x-ray}}$ (Fig. 8). At low mass ($M_{500,\text{sim}} = 10^{14} M_{\odot}$) we find that the spectroscopically estimated temperature is consistent with the mass-weighted temperature. However, for high-mass clusters ($M_{500,\text{sim}} = 2 \times 10^{15} M_{\odot}$) the spectroscopic temperature is only 80 per cent of the mass-weighted temperature.

- We then examined the temperature distribution of the gas as a function of radius (Fig. 9) and find that for low-mass clusters there is a narrow range (~ 2 keV) of temperatures at a given radius. For high-mass clusters, the width of the temperature distribution increases to > 10 keV. However, at any given radius we find that the width of the pressure distribution is significantly smaller. Given the $P = nk_{\text{B}}T$, this implies that the cooler gas in massive clusters is denser than the hot gas. Due to the quadratic density dependence of the X-ray emission, this implies that the cold gas has a greater weighting in the spectroscopic temperature fit that decreases the recovered temperature for more massive clusters. Given the hydrostatic mass estimate depends linearly on temperature (equation 12), this result explains why the hydrostatic bias is mass-dependent for thermodynamic profiles derived from synthetic X-ray images. We left the origin and fate of this colder gas to a future study.

Despite the mass dependent bias found in this study, the results presented in this work are consistent with current ob-

servational results. For example, at the median mass of the X-COP sample we find that the spectroscopic temperature is statistically consistent with the mass-weighted temperature and the X-ray and SZ pressure profiles should agree. Both future simulations and observations will be important tests of this result. Larger volume numerical simulations with a different hydrostatic method (e.g. Cui et al. 2018) that varies the subgrid physics implementation (e.g. Kannan et al. 2017; Barnes et al. 2019) will be important to further assess the impact of numerical choices on the hydrostatic bias. Future observational projects, such as the X-COP successor the XMM heritage cluster project, that will yield detailed X-ray observations of more massive clusters that can confirm the presence of a mass-dependent hydrostatic bias.

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REFERENCES

- Adam R., et al., 2018, *A&A*, **609**, A115
- Ade P., et al., 2019, *J. Cosmology Astropart. Phys.*, **2**, 056
- Allen S. W., Evrard A. E., Mantz A. B., 2011, *ARA&A*, **49**, 409
- Ameglio S., Borgani S., Pierpaoli E., Dolag K., 2007, *MNRAS*, **382**, 397
- Anders E., Grevesse N., 1989, *Geochimica Cosmochimica Acta*, **53**, 197
- Angelinelli M., Vazza F., Giocoli C., Jones T. W., Ettori S., Brunetti G., Brüggén M., Eckert D., 2019, arXiv e-prints, p. [arXiv:1905.04896](#)
- Ansarifard S., et al., 2019, arXiv e-prints, p. [arXiv:1911.07878](#)
- Applegate D. E., et al., 2014, *MNRAS*, **439**, 48
- Armitage T. J., Barnes D. J., Kay S. T., Bahé Y. M., Dalla Vecchia C., Crain R. A., Theuns T., 2018, *MNRAS*, **474**, 3746
- Bahé Y. M., McCarthy I. G., King L. J., 2012, *MNRAS*, **421**, 1073
- Bardeen J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, *ApJ*, **304**, 15
- Barnes D. J., Kay S. T., Henson M. A., McCarthy I. G., Schaye J., Jenkins A., 2017a, *MNRAS*, **465**, 213
- Barnes D. J., et al., 2017b, *MNRAS*, **471**, 1088
- Barnes D. J., et al., 2018, *MNRAS*, **481**, 1809
- Barnes D. J., et al., 2019, *MNRAS*, **488**, 3003
- Battaglia N., et al., 2016, *J. Cosmology Astropart. Phys.*, **2016**, 013
- Becker M. R., Kravtsov A. V., 2011, *ApJ*, **740**, 25
- Benson B. A., et al., 2014, in *Millimeter, Submillimeter, and Far-Infrared Detectors and Instrumentation for Astronomy VII*, p. 91531P ([arXiv:1407.2973](#)), doi:10.1117/12.2057305
- Bett P., Eke V., Frenk C. S., Jenkins A., Helly J., Navarro J., 2007, *MNRAS*, **376**, 215
- Biffi V., Dolag K., Böhringer H., Lemson G., 2012, *MNRAS*, **420**, 3545
- Biffi V., Dolag K., Böhringer H., 2013, *MNRAS*, **428**, 1395
- Biffi V., et al., 2016, *ApJ*, **827**, 112
- Bleem L. E., et al., 2019, arXiv e-prints, p. [arXiv:1910.04121](#)
- Bocquet S., et al., 2015, *ApJ*, **799**, 214
- Bocquet S., et al., 2019, *ApJ*, **878**, 55
- Böhringer H., et al., 2010, *A&A*, **514**, A32
- Booth C. M., Schaye J., 2009, *MNRAS*, **398**, 53
- Chabrier G., 2003, *PASP*, **115**, 763
- Chluba J., Nagai D., Sazonov S., Nelson K., 2012, *MNRAS*, **426**, 510
- Crain R. A., et al., 2009, *MNRAS*, **399**, 1773
- Croston J. H., Arnaud M., Pointecouteau E., Pratt G. W., 2006, *A&A*, **459**, 1007
- Cui W., et al., 2018, *MNRAS*, **480**, 2898
- Dalla Vecchia C., Schaye J., 2008, *MNRAS*, **387**, 1431
- Davé R., Anglés-Alcázar D., Narayanan D., Li Q., Rafieferantsoa M. H., Appleby S., 2019, *MNRAS*, **486**, 2827
- Davis M., Efstathiou G., Frenk C. S., White S. D. M., 1985, *ApJ*, **292**, 371
- Diaferio A., Geller M. J., 1997, *ApJ*, **481**, 633
- Diaz-Rodriguez J., Eckert D., Monajemi H., Paltani S., Sardy S., 2017, arXiv e-prints,
- Dicker S. R., et al., 2014, *Journal of Low Temperature Physics*, **176**, 808
- Dolag K., Borgani S., Murante G., Springel V., 2009, *MNRAS*, **399**, 497
- Eckert D., et al., 2016, *A&A*, **592**, A12
- Eckert D., Ettori S., Pointecouteau E., Molendi S., Paltani S., Tchernin C., 2017, *Astronomische Nachrichten*, **338**, 293
- Ettori S., De Grandi S., Molendi S., 2002, *A&A*, **391**, 841
- Ettori S., Gastaldello F., Leccardi A., Molendi S., Rossetti M., Buote D., Meneghetti M., 2010, *A&A*, **524**, A68
- Ettori S., Donnarumma A., Pointecouteau E., Reiprich T. H., Giodini S., Lovisari L., Schmidt R. W., 2013, *Space Sci. Rev.*, **177**, 119
- Ettori S., et al., 2019, *A&A*, **621**, A39
- Foster A. R., Ji L., Smith R. K., Brickhouse N. S., 2012, *ApJ*, **756**, 128
- Gardini A., Rasia E., Mazzotta P., Tormen G., De Grandi S., Moscardini L., 2004, *MNRAS*, **351**, 505
- Genel S., et al., 2014, *MNRAS*, **445**, 175
- Ghirardini V., et al., 2019, *A&A*, **621**, A41
- Gifford D., Kern N., Miller C. J., 2017, *ApJ*, **834**, 204
- Giodini S., Lovisari L., Pointecouteau E., Ettori S., Reiprich T. H., Hoekstra H., 2013, *Space Sci. Rev.*, **177**, 247
- Hasselfield M., et al., 2013, *J. Cosmology Astropart. Phys.*, **2013**, 008
- Heinz S., Brüggén M., 2009, arXiv e-prints,
- Heinz S., Brüggén M., Friedman S., 2011, *ApJS*, **194**, 21
- Henden N. A., Puchwein E., Shen S., Sijacki D., 2018, *MNRAS*, **479**, 5385
- Henson M. A., Barnes D. J., Kay S. T., McCarthy I. G., Schaye J., 2017, *MNRAS*, **465**, 3361
- Herbonnet R., von der Linden A., Allen S. W., Mantz A. B., Modumudi P., Morris R. G., Kelly P. L., 2019, *MNRAS*, **490**, 4889
- Hoekstra H., Herbonnet R., Muzzin A., Babul A., Mahdavi A., Viola M., Cacciato M., 2015, *MNRAS*, **449**, 685
- Hopkins P. F., Hernquist L., Martini P., Cox T. J., Robertson B., Di Matteo T., Springel V., 2005, *ApJ*, **625**, L71

- Israel H., Reiprich T. H., Erben T., Massey R. J., Sarazin C. L., Schneider P., Vikhlinin A., 2014, *A&A*, **564**, [A129](#)
- Kannan R., Vogelsberger M., Pfrommer C., Weinberger R., Springel V., Hernquist L., Puchwein E., Pakmor R., 2017, *ApJ*, **837**, [L18](#)
- Katz N., White S. D. M., 1993, *ApJ*, **412**, [455](#)
- Kay S. T., da Silva A. C., Aghanim N., Blanchard A., Liddle A. R., Puget J.-L., Sadat R., Thomas P. A., 2007, *MNRAS*, **377**, [317](#)
- Kay S. T., Peel M. W., Short C. J., Thomas P. A., Young O. E., Battye R. A., Liddle A. R., Pearce F. R., 2012, *MNRAS*, **422**, [1999](#)
- Khedekar S., Churazov E., Kravtsov A., Zhuravleva I., Lau E. T., Nagai D., Sunyaev R., 2013, *MNRAS*, **431**, [954](#)
- Kravtsov A. V., Borgani S., 2012, *ARA&A*, **50**, [353](#)
- Kriss G. A., Cioffi D. F., Canizares C. R., 1983, *ApJ*, **272**, [439](#)
- Kunz M. W., Schekochihin A. A., Cowley S. C., Binney J. J., Sanders J. S., 2011, *MNRAS*, **410**, [2446](#)
- LSST Science Collaboration et al., 2009, arXiv e-prints,
- Lau E. T., Kravtsov A. V., Nagai D., 2009, *ApJ*, **705**, [1129](#)
- Laureijs R., et al., 2011, arXiv e-prints,
- Le Brun A. M. C., McCarthy I. G., Schaye J., Ponman T. J., 2014, *MNRAS*, **441**, [1270](#)
- Lin H. W., McDonald M., Benson B., Miller E., 2015, *ApJ*, **802**, [34](#)
- Madhavacheril M. S., Battaglia N., Miyatake H., 2017, *Phys. Rev. D*, **96**, [103525](#)
- Mahdavi A., Hoekstra H., Babul A., Henry J. P., 2008, *MNRAS*, **384**, [1567](#)
- Mahdavi A., Hoekstra H., Babul A., Bildfell C., Jeltama T., Henry J. P., 2013, *ApJ*, **767**, [116](#)
- Mantz A. B., Allen S. W., Morris R. G., Rapetti D. A., Applegate D. E., Kelly P. L., von der Linden A., Schmidt R. W., 2014, *MNRAS*, **440**, [2077](#)
- Mantz A. B., et al., 2015a, *MNRAS*, **446**, [2205](#)
- Mantz A. B., Allen S. W., Morris R. G., Schmidt R. W., von der Linden A., Urban O., 2015b, *MNRAS*, **449**, [199](#)
- Marinacci F., et al., 2018, *MNRAS*, **480**, [5113](#)
- Maughan B. J., et al., 2008, *MNRAS*, **387**, [998](#)
- Maughan B. J., Giles P. A., Rines K. J., Diaferio A., Geller M. J., Van Der Pyl N., Bonamente M., 2016, *MNRAS*, **461**, [4182](#)
- Mazzotta P., Rasia E., Moscardini L., Tormen G., 2004, *MNRAS*, **354**, [10](#)
- McCarthy I. G., Schaye J., Bird S., Le Brun A. M. C., 2017, *MNRAS*, **465**, [2936](#)
- Medezinski E., et al., 2018, *PASJ*, **70**, [S28](#)
- Merloni A., et al., 2012, arXiv e-prints,
- Miyatake H., et al., 2019, *ApJ*, **875**, [63](#)
- Mohr J. J., Fabricant D. G., Geller M. J., 1993, *ApJ*, **413**, [492](#)
- Morandi A., et al., 2012, *MNRAS*, **425**, [2069](#)
- Morrison R., McCammon D., 1983, *ApJ*, **270**, [119](#)
- Munari E., Biviano A., Borgani S., Murante G., Fabjan D., 2013, *MNRAS*, **430**, [2638](#)
- Nagai D., Lau E. T., 2011, *ApJ*, **731**, [L10](#)
- Nagai D., Vikhlinin A., Kravtsov A. V., 2007, *ApJ*, **655**, [98](#)
- Naiman J. P., et al., 2018, *MNRAS*, **477**, [1206](#)
- Nelson K., Lau E. T., Nagai D., Rudd D. H., Yu L., 2014, *ApJ*, **782**, [107](#)
- Nelson D., et al., 2018, *MNRAS*, **475**, [624](#)
- Nulsen P. E. J., Powell S. L., Vikhlinin A., 2010, *ApJ*, **722**, [55](#)
- Okabe N., Smith G. P., Umetsu K., Takada M., Futamase T., 2013, *ApJ*, **769**, [L35](#)
- Old L., et al., 2014, *MNRAS*, **441**, [1513](#)
- Pakmor R., Springel V., 2013, *MNRAS*, **432**, [176](#)
- Pakmor R., Springel V., Bauer A., Mocz P., Munoz D. J., Ohlmann S. T., Schaaf K., Zhu C., 2016, *MNRAS*, **455**, [1134](#)
- Peacock J. A., Heavens A. F., 1985, *MNRAS*, **217**, [805](#)
- Pearce F. A., Kay S. T., Barnes D. J., Bower R. G., Schaller M., 2020, *MNRAS*, **491**, [1622](#)
- Penna-Lima M., Bartlett J. G., Rozo E., Melin J. B., Merten J., Evrard A. E., Postman M., Rykoff E., 2017, *A&A*, **604**, [A89](#)
- Pike S. R., Kay S. T., Newton R. D. A., Thomas P. A., Jenkins A., 2014, *MNRAS*, **445**, [1774](#)
- Pillepich A., et al., 2018a, *MNRAS*, **473**, [4077](#)
- Pillepich A., et al., 2018b, *MNRAS*, **475**, [648](#)
- Planck Collaboration et al., 2014, *A&A*, **571**, [A16](#)
- Planck Collaboration et al., 2016a, *A&A*, **594**, [A13](#)
- Planck Collaboration et al., 2016b, *A&A*, **594**, [A24](#)
- Planelles S., Borgani S., Dolag K., Ettori S., Fabjan D., Murante G., Tornatore L., 2013, *MNRAS*, **431**, [1487](#)
- Pointecouteau E., Arnaud M., Pratt G. W., 2005, *A&A*, **435**, [1](#)
- Poole G. B., Fardal M. A., Babul A., McCarthy I. G., Quinn T., Wadsley J., 2006, *MNRAS*, **373**, [881](#)
- Rasia E., Mazzotta P., Bourdin H., Borgani S., Tornatore L., Ettori S., Dolag K., Moscardini L., 2008, *ApJ*, **674**, [728](#)
- Rasia E., et al., 2012, *New Journal of Physics*, **14**, [055018](#)
- Rasia E., Meneghetti M., Ettori S., 2013, *The Astronomical Review*, **8**, [40](#)
- Rasia E., et al., 2014, *ApJ*, **791**, [96](#)
- Reichardt C. L., et al., 2013, *ApJ*, **763**, [127](#)
- Reiprich T. H., Böhringer H., 2002, *ApJ*, **567**, [716](#)
- Roncarelli M., Ettori S., Borgani S., Dolag K., Fabjan D., Moscardini L., 2013, *MNRAS*, **432**, [3030](#)
- Ruppin F., et al., 2019a, arXiv e-prints, [p. arXiv:1911.00560](#)
- Ruppin F., et al., 2019b, *A&A*, **631**, [A21](#)
- Sanders J. S., Fabian A. C., Russell H. R., Walker S. A., 2018, *MNRAS*, **474**, [1065](#)
- Schaye J., Dalla Vecchia C., 2008, *MNRAS*, **383**, [1210](#)
- Schaye J., et al., 2010, *MNRAS*, **402**, [1536](#)
- Schaye J., et al., 2015, *MNRAS*, **446**, [521](#)
- Sereno M., Ettori S., 2015, *MNRAS*, **450**, [3633](#)
- Sereno M., Covone G., Izzo L., Ettori S., Coupon J., Lieu M., 2017, *MNRAS*, **472**, [1946](#)
- Sijacki D., Vogelsberger M., Genel S., Springel V., Torrey P., Snyder G. F., Nelson D., Hernquist L., 2015, *MNRAS*, **452**, [575](#)
- Simet M., Battaglia N., Mandelbaum R., Seljak U., 2015, in *American Astronomical Society Meeting Abstracts #225*. p. [443.04](#)
- Simet M., McClintock T., Mandelbaum R., Rozo E., Rykoff E., Sheldon E., Wechsler R. H., 2017, *MNRAS*, **466**, [3103](#)
- Smith R. K., Brickhouse N. S., Liedahl D. A., Raymond J. C., 2001, *ApJ*, **556**, [L91](#)
- Smith G. P., et al., 2016, *MNRAS*, **456**, [L74](#)
- Springel V., 2010, *MNRAS*, **401**, [791](#)
- Springel V., White S. D. M., Tormen G., Kauffmann G., 2001, *MNRAS*, **328**, [726](#)
- Springel V., Di Matteo T., Hernquist L., 2005, *MNRAS*, **361**, [776](#)
- Springel V., et al., 2018, *MNRAS*, **475**, [676](#)
- Staniszewski Z., et al., 2009, *ApJ*, **701**, [32](#)
- Sunyaev R. A., Zeldovich Y. B., 1972, *Comments on Astrophysics and Space Physics*, **4**, [173](#)
- Thomas P. A., et al., 1998, *MNRAS*, **296**, [1061](#)
- Tormen G., Bouchet F. R., White S. D. M., 1997, *MNRAS*, **286**, [865](#)
- Torrey P., Vogelsberger M., Genel S., Sijacki D., Springel V., Hernquist L., 2014, *MNRAS*, **438**, [1985](#)
- Torrey P., et al., 2015, *MNRAS*, **447**, [2753](#)
- Tremmel M., et al., 2019, *MNRAS*, **483**, [3336](#)
- Vazza F., Eckert D., Simionescu A., Brüggen M., Ettori S., 2013, *MNRAS*, **429**, [799](#)
- Vikhlinin A., et al., 2003, *ApJ*, **590**, [15](#)
- Vikhlinin A., Kravtsov A., Forman W., Jones C., Markevitch M., Murray S. S., Van Speybroeck L., 2006, *ApJ*, **640**, [691](#)
- Vogelsberger M., Genel S., Sijacki D., Torrey P., Springel V., Hernquist L., 2013, *MNRAS*, **436**, [3031](#)

- Vogelsberger M., et al., 2014a, [MNRAS](#), **444**, 1518
- Vogelsberger M., et al., 2014b, [Nature](#), **509**, 177
- Vogelsberger M., et al., 2018, [MNRAS](#), **474**, 2073
- Weinberg D. H., Mortonson M. J., Eisenstein D. J., Hirata C., Riess A. G., Rozo E., 2013, [Phys. Rep.](#), **530**, 87
- Weinberger R., et al., 2017, [MNRAS](#), **465**, 3291
- Weißmann A., Böhringer H., Šuhada R., Ameglio S., 2013, [A&A](#), **549**, A19
- Wiersma R. P. C., Schaye J., Smith B. D., 2009a, [MNRAS](#), **393**, 99
- Wiersma R. P. C., Schaye J., Theuns T., Dalla Vecchia C., Tornatore L., 2009b, [MNRAS](#), **399**, 574
- Wilcox H., et al., 2015, [MNRAS](#), **452**, 1171
- Wittman D., Foote D., Golovich N., 2019, [ApJ](#), **874**, 84
- Wuyts S., Franx M., Cox T. J., Hernquist L., Hopkins P. F., Robertson B. E., van Dokkum P. G., 2009, [ApJ](#), **696**, 348
- Yee H. K. C., Ellingson E., 2003, [ApJ](#), **585**, 215
- Zhang Y. Y., Andernach H., Caretta C. A., Reiprich T. H., Böhringer H., Puchwein E., Sijacki D., Girardi M., 2011, [A&A](#), **526**, A105
- Zhuravleva I., Churazov E., Kravtsov A., Lau E. T., Nagai D., Sunyaev R., 2013, [MNRAS](#), **428**, 3274
- Zhuravleva I., et al., 2014, [Nature](#), **515**, 85
- ZuHone J. A., Biffi V., Hallman E. J., Randall S. W., Foster A. R., Schmid C., 2014, arXiv e-prints,
- de Haan T., et al., 2016, [ApJ](#), **832**, 95
- von der Linden A., et al., 2014, [MNRAS](#), **439**, 2

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