Closing the Gap between TD Learning and Supervised Learning – A Generalisation Point of View

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Abstract

Some reinforcement learning (RL) algorithms have the capability of recombining together pieces of previously seen experience to solve a task never seen before during training. This oft-sought property is one of the few ways in which dynamic programming based RL algorithms are considered different from supervised learning (SL) based RL algorithms. Yet, recent RL methods based on off-the-shelf SL algorithms achieve excellent results without an explicit mechanism for stitching; it remains unclear whether those methods forgo this important stitching property. This paper studies this question in the setting of goal-reaching problems. We show that the desirable stitching property corresponds to a form of generalization: after training on a distribution of (state, goal) pairs, one would like to evaluate on (state, goal) pairs not seen *together* in the training data. Our analysis shows that this sort of generalization is different from *i.i.d.* generalization. This connection between stitching and generalization reveals why we should not expect existing RL methods based on SL to perform stitching, even in the limit of large datasets and models. We experimentally validate this result on carefully constructed datasets. This connection also suggests a simple remedy, the same remedy for improving generalization in SL: data augmentation. We propose a naive *temporal* data augmentation approach and demonstrate that adding it to RL methods based on SL enables them to stitch together experience so that they succeed in navigating between states and goals unseen together during training.

Recent methods that view RL as a purely SL problem of mapping input states and desired goals, to optimal actions [1, 2] have gained a lot of attention due to their simplicity, scalability [3]. These methods sample a goal g from the dataset $\mathcal{D}$, previously encountered after taking an action a from a state s, and then imitate a by treating it as an optimal label for reaching g from s.

$$\max_{\pi(\cdot|\cdot,\cdot)} \mathbb{E}_{(s,a,g)\sim\mathcal{D}}\left[\log \pi(a \mid s, g)\right]. \tag{1}$$

This simple recipe achieves excellent results on common benchmarks [4]. However, at a deeper and fundamental level, there are some important differences between RL and SL. This paper studies one of those differences: the capability of some RL algorithms to stitch together pieces of experience to solve a task never seen during training. This stitching property [5] is common among RL algorithms that perform dynamic programming (e.g., DQN [6], DDPG [7], TD3 [8], IQL [9]). While some papers have claimed that some SL approaches already have this stitching property [2], both our theoretical and empirical analyses suggest some important limitations of these prior claims.

Our primary contribution is to formally relate this stitching property to a form of generalization (Section 1). Taking a cue from SL, if generalization is the problem, then data augmentation is likely an effective approach. The second contribution of this paper is a way of applying data augmentation to these same SL methods such that they acquire this stitching property and succeed in navigating between unseen (start, goal) pairs. Section 2 provide a naive data-augmentation method that can perform stitching and Section 3 verifies our claims experimentally. We also provide a summary of related work in Appendix F.

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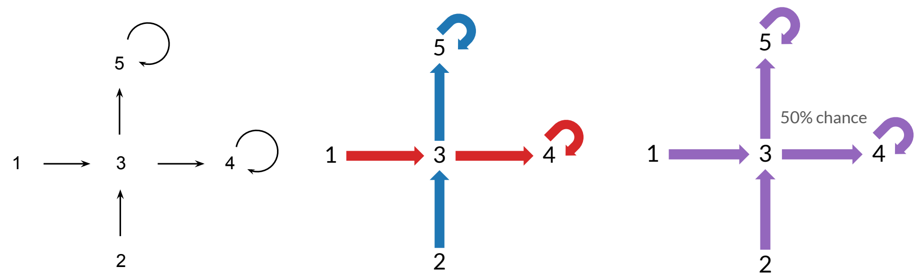

(a) MDP. (b) $\beta_{h=1}$, $\beta_{h=2}$. (c) β.

Figure 1: *(a)* The MDP has 5 states and two actions (up and right). Prior work [15, 16] also have similar counterexamples, though they have not related this to generalization to the best of our knowledge.*(b)* Data is collected using two contexts conditioned policies shown in blue and red. *(c)* The average policy is shown in purple to indicate that it is obtained from combining data from both blue and red policies. The distribution of the purple policy is used to test stitching. This distribution is different from the training distribution. During training *(b)*, state 2 and goal 4 will never be sampled together, but during testing *(c)*, state 2 and goal 4 have a non-zero probability of being sampled.

1 STITCHING

In this section, we intuitively introduce *stitching* as a form of generalization and provide a pictorial version of the proof. See Appendix A for a formal definition and proof. *Stitching* is a form of generalisation because the distribution to test for stitching is different from the training distribution. Intuitively, *stitching* looks at navigating between states and goals which are never seen together in the same trajectory, but can be navigated using the information present in different trajectories. It therefore tests a form of "stitching" [10, 11], and is akin to "compositional generalization" [12–14]. To understand why this is the case, look at the MDP described in 1. The offline datasets is collected by two policies ($h = 1$ and $h = 2$), which navigate upward from state two to state five, and rightward from state one to state four respectively. At test time, an agent that perform stitching should be able to navigate from state 2 to goal 4. But during training, OCBC methods always sample state-goal pairs from the *same* trajectory. Hence, the goal 4 will never be sampled with 2. But this is precisely the state-goal pair that is sampled to from the testing distribution of stitching. QED.

In summary, while the average policy $\beta(a \mid s)$ will visit the same states as the mixture of data collecting policies *on average*, conditioned on some state, the BC policy $\beta(a \mid s)$ may visit a different distribution of future states (goals) than the mixture of policies. The important implication of this negative result is that *stitching generalization is not the same as i.i.d. generalization*. Even in the limit of huge datasets, there will be pairs of states that will never co-occur in a single trajectory. For OCBC based on SL (e.g., DT [2], RvS [4], GCSL [17], PCHID [18], URL [1]), it is not clear apriori why these methods should have the stitching generalization property, leading to the following hypothesis:

Hypothesis 1. *Conditional imitation learning methods do not have the stitching generalization property.*

We will test this hypothesis empirically in our experiments. In Appendix C, we discuss connections between stitching generalisation and spurious correlations.

2 TEMPORAL AUGMENTATION FACILITATES GENERALIZATION

Casting stitching as a form of generalization allows us to employ a standard tool from SL: data augmentation. When the computer vision expert wants a model that can generalize to random crops, they train their model on randomly-cropped images. Indeed, prior work has applied data augmentation to RL to achieve *various* notions of generalization [14, 19]. However, we use a different type of data augmentation to facilitate stitching. In this section, we describe a data augmentation approach that allows OCBC methods to improve their stitching capabilities.

Algorithm 1 OCBC + Temporal Data Augmentation

Input: Dataset : $D = (\{s_0, a_0, \dots\})$.
Initialize OCBC policy $\pi_\theta(a|s,g)$ with parameters θ.
Set ϵ = augmentation probability, m = mini-batch size.
$(\{d_0, d_1, \dots\})$ = CLUSTER($\{s_0, s_1, \dots\}$). ▷ Group all states in the dataset.
while not converged **do**
  for $t = 1, \cdots, m$ **do**
    Sample $(s_t, a_t, g_{t+}) \sim D$. ▷ Equation (8)
    if $u \sim unif[0,1] \leq \epsilon$ **then**
      Get the group of the goal: $k = d_{t+}$.
      Sample waypoint states from the same group: $w \sim \{s_i; \forall i \text{ such that } d_i = k\}$.
      Sample augmented goal $\tilde{g}$ from the future of w, from the same trajectory as w.
      Augment the goal $g_{t+} = \tilde{g}$.
      Collect the loss $\mathcal{L}_t(\theta) = -\log \pi_\theta(a_t \mid s_t, g_{t+})$.
    Update θ using gradient descent on the mini-batch loss $\frac{1}{m}\sum_{t=1}^{m} \mathcal{L}_t(\theta)$
  Return : $\pi_\theta(a|s,g)$

Recall that OCBC policies are trained on (s, a, g) triplets. To perform data augmentation, we will replace g with a different goal $\tilde{g}$. To sample these new goals $\tilde{g}$, we first take the original goal g and identify states from the offline dataset which are nearby to this goal (Section 2). Let w denote one of these nearby "waypoint" states. Looking at the trajectory that contains w, the new goal $\tilde{g}$ is a random state that occurs after w in this trajectory. We visualize this data augmentation in Fig. 2.

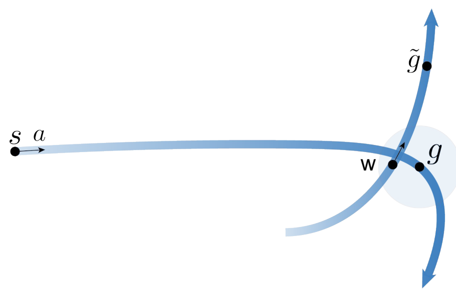

Figure 2: DATA AUGMENTATION FOR STITCHING: After sampling an initial training example (s, a, g) (Eq. (8)), we look for a *waypoint* state w in the light blue region around the original goal g, and then sample a new *augmented* goal $\tilde{g}$ from later in that trajectory. This is a simple approach to sample cross trajectory goals $\tilde{g}$ such that the action a is still an optimal action at state g.

Identifying nearby states. The problem of sampling nearby states can be solved by clustering all the states from the offline dataset before training. This assumes a distance metric in the state space. Using this distance metric, every state can be assigned a discrete label from k different categories using a clustering algorithm [20, 21]. Finding a good distance metric is difficult, especially in high-dimensional settings [22].

Method summary. Algorithm 1 summarizes how our data augmentation can be applied on top of existing OCBC algorithms. Given a method to group all the states in the offline dataset, we can add our data-augmentation to existing OCBC algorithms by adding about 5 lines of code (marked in blue). In our experiments, we use the k-means algorithm [20] to group states together. We use two types of inputs to the k-means to group states together (1) Entire state vector, and (2) Only the goal coordinates of the state vector. In all our experiments, goals are specified by the desired x, y coordinates. The first approach does not assume separate access to the goal coordinates and is therefore generally applicable.

3 Experiments

The experiments aim *(1)* to verify our theoretical claim that OCBC methods do not always exhibit stitching generalisation; and *(2)* to evaluate how adding temporal augmentation to OCBC methods improves stitching.

OCBC methods. **RvS** [4] is an OCBC algorithm that uses a fully connected neural network policy and often achieves results comparable to TD-learning algorithms on various offline RL benchmarks [4]. **DT** [2] treats RL as a sequential SL problem and uses the transformer architecture as a policy. DT outputs an action, conditioning not only on the current state, but a history of states, actions and goals. We use all the original hyperparameters of the baselines. See Appendix B.2 for implementation details.

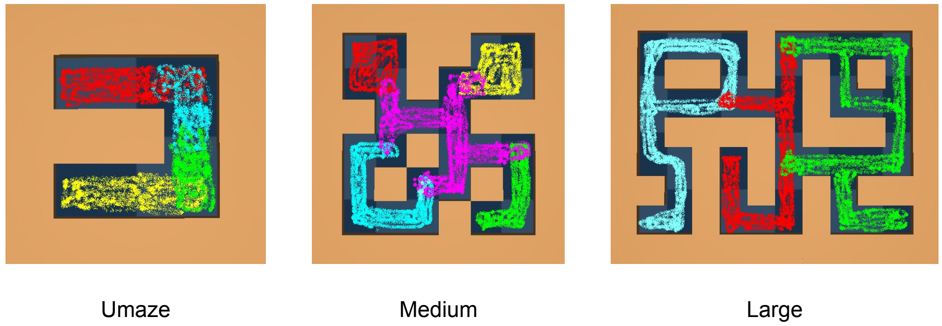

Figure 3: **Offline datasets that do require stitching.** Different colors represent the navigation regions of different data collecting policies. These visualisations are for the "point" mazes. The "ant" maze datasets are similar to these ones. Appendix Fig. 6 shows the "ant" maze datasets.

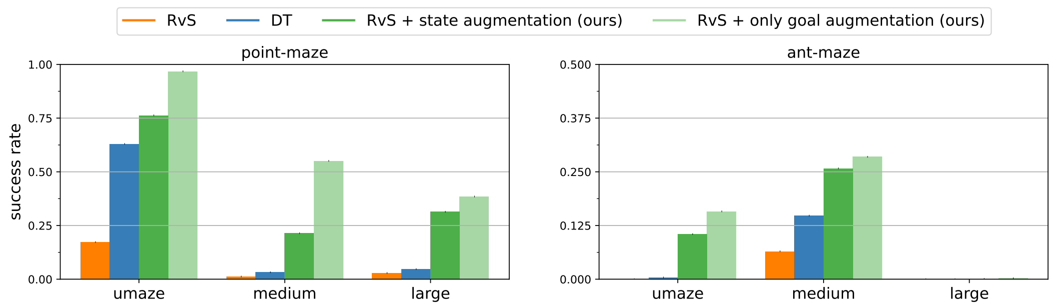

Figure 4: Adding data augmentation outperforms the OCBC baselines on most tasks. "Only goal augmentation" refers to an oracle version of our augmentation that uses priviledged information (x, y coordinates) when performing augmentation.

Tasks. We use the "point" and "ant" mazes (umaze, medium and large) from D4RL [10]. We carefully collect our new offline datasets to test for stitching Appendix C.1 (see Fig. 3 and Fig. 6 for visualisation).

Testing for stitching. We command the agent to navigate between (state, goal) pairs previously unseen *together*, and measure the success rate. Each task chooses states and goals randomly from 2-6 different regions in the maze. In Fig. 4, we can see that both DT and RvS struggle to solve unseen tasks at test-time. However, applying temporal data-augmentation to RvS improves the goal-reaching success rate on $^5/_6$ tasks, likely because the augmentation results in sampling (state, goal) pairs otherwise *unseen together*. While temporal augmentation boosts the success rates, there remains room for more sophisticated methods to achieve even better performance.

Additional experiments. In Appendix D, we perform additional experiments that show increasing the size of the data or the size of the models does not seem to improve stitching abilities of OCBC algorithms. In Appendix C.1 we mention the differences between the D4RL datasets (which do not test for stitching) and our datasets in detail.

Limitations and future work. While we gave an intuitive argument for why augmentation facilitates stitching, there is more work to be done to provide a formal proof that this augmentation always results in stitching. Our proposed augmentation also assumes an access to a good distance metric in the state space. Lifting these assumptions and developing scalable OCBC algorithms that generalise is a promising direction for future work.

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A Proof that stitching is a form of generalisation.

A.1 Preliminaries

Controlled Markov processes. We will study the problem of goal-conditioned RL in a controlled Markov process with states $s \in \mathcal{S}$ and actions $a \in \mathcal{A}$. The dynamics are $p(s' \mid s, a)$, the initial state distribution is $p_0(s_0)$, the discount factor is γ. The policy $\pi(a, \mid s, g)$ is conditioned on a pair of state and goal $s, g \in \mathcal{S}$. For a policy π, define $p_t^\pi(s_t \mid s_0)$ as the distribution over states visited after exactly t steps. We can then define the discounted state occupancy distribution and its conditional counterpart as

$$p_+^\pi(s_{t+} = g) \triangleq \mathbb{E}_{s \sim p_0(s_0)} \left[p_+^\pi(s_{t+} = g \mid s_0 = s) \right], \tag{2}$$

$$p_+^\pi(s_{t+} = g \mid s_0 = s) \triangleq (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t p_t^\pi(s_t = g \mid s_0 = s), \tag{3}$$

where s_{t+} is the variable that specifies a future state corresponding to the discounted state occupancy distribution. Given a state-goal pair $s, g \sim p_{\text{test}}(s, g)$ at test time, the task of the policy is to maximise the probability of reaching the goal g in the future

$$\max_\pi J(\pi; s, g), \quad \text{where } \; J(\pi; s, g) = \mathbb{E}_{s,g \sim p_{\text{test}}(s,g)} \left[p_+^\pi(s_{t+} = g \mid s_0 = s) \right]. \tag{4}$$

Data collection. Our work focuses on the offline RL setting where the agent has access to a fixed dataset of N trajectories $D = (\{s_0^i, a_0^i, ..\})_{i=1}^N$. Our theoretical analysis will assume that the dataset is collected by a set of policies $\{\beta(a \mid s, h)\}$, where h specifies some *hidden* context. For example, h could reflect different goals, different language instructions, different users or even different start state distributions. Precisely, we assume that the data was collected by first sampling a context from a distribution $p(h)$, and then sampling a trajectory from the corresponding policy $\{\beta(a \mid s, h)\}$. We will use the shorthand notation $\beta_h(\cdot \mid \cdot) = \beta(\cdot \mid \cdot, h)$ to denote the data collecting policy conditioned on context h. Trajectories are assumed to be stored without h, hence the context denotes all hidden information that the true data collection policies used to collect the data. This construction covers real-world cases where data is collected from various sources depending on various hidden factors.

This setup of collecting data corresponds to a mixture of Markovian policies [1]. There is a classic result saying that, for every such *mixture* of Markovian policies, there exists a Markovian policy that has the same discounted state occupancy measure.

Lemma A.1 (Rephrased from Theorem 2.8 of [23], Theorem 6.1 of [24])**.** *Let a set of context-conditioned policies $\{\beta_h(a \mid s)\}$ and distribution over contexts $p(h)$ be given. There exists a Markovian policy $\beta(a \mid s)$ such that it has the same discounted state occupancy measure as the mixture of policies:*

$$p_+^\beta(s_{t+}) = \mathbb{E}_{p(h)} \left[p_+^{\beta_h}(s_{t+}) \right]. \tag{5}$$

The policy $\beta(a \mid s)$ is simple to construct mathematically as follows. For data collected from the mixture of context conditioned policies, let $p^\beta(h \mid s)$ be the distribution over the context given that the policy arrived in state s.

$$\beta(a \mid s) \triangleq \sum_h \beta(a \mid s, h) p^\beta(h \mid s). \tag{6}$$

Theorem 6.1 [24] proves the correctness of this construction. The policy $\beta(a \mid s)$ is also easy to construct empirically – simply perform behavioral cloning (BC) on data aggregated from the set of policies. We will hence call this policy the BC policy.

Outcome Conditional behavioral cloning (OCBC). While our theoretical analysis will consider generalization abstracted away from any particular RL algorithm, we will present empirical results using a simple and popular class of goal-conditioned RL methods: Outcome conditional behavioral cloning [25]. These methods go by a number of names, including Decision Transformer (DT) [2],

[1] Note that the mixture is at the level of trajectories, not at the level of individual actions.

Upside down RL [1], RL via Supervised Learning (RvS) [4], Goal Conditioned Supervised Learning (GCSL) [17] and many others [18, 26]. These methods take as input a dataset of trajectories $\mathcal{D} = \{(s_0, a_0, s_1, a_1)\}$ and learn a goal-conditioned policy $\pi(a \mid s, g)$ using a maximum likelihood objective:

$$\max_{\pi(\cdot|\cdot,\cdot)} \mathbb{E}_{(s,a,g)\sim\mathcal{D}} \left[\log \pi(a \mid s, g)\right]. \tag{7}$$

The sampling above can be done by first sampling a trajectory from the dataset (uniformly at random), then sampling a (state, action) pair from that trajectory, and setting the goal to be a random state that occurred later in that same trajectory. If we incorporate our data collecting assumptions, then this sampling can be written as

$$\max_{\pi(\cdot|\cdot,\cdot)} \mathbb{E}_{h\sim p(h)} \Big[\mathbb{E}_{\substack{s,a\sim p_+^{\beta_h}(s,a) \\ s_{t+}\sim p_+^{\beta_h}(s_{t+}|s,a)}} \left[\log \pi(a \mid s, s_{t+})\right]\Big]. \tag{8}$$

A.2 Stitching

To define this generalization, we will specify a training distribution and testing distribution. The training distribution corresponds to sampling a context $h \sim p(h)$ and then sampling an (s, g) pair from the corresponding policy β_h. This is similar to how OCBC methods are trained in practice (Appendix A.1). The testing distribution corresponds to sampling an (s, g) pair from the BC policy $\beta(a \mid s)$ defined in Equation (6). For each distribution, we will measure the performance $f^{\pi(\cdot|\cdot,g)}(s, g)$ of goal-conditioned policy $\pi(a \mid s, g)$.

Definition 1 (Stitching generalization)**.** *Let a set of context-conditioned policies $\{\beta_h(a \mid s)\}$ be given, along with a prior over contexts $p(h)$. Let $\beta(a \mid s)$ be the policy constructed via Eq.* (6)*. Let $\pi(a \mid s, g)$ be a policy for evaluation. The stitching generalization of a policy $\pi(a \mid s, g)$ measures the differences in goal-reaching performance for goals sampled $g \sim p_+^{\beta}(s_{t+} \mid s)$ versus goals sampled from $g \sim \mathbb{E}_{p(h)}[p_+^{\beta_h}(s_{t+} \mid s)]$:*

$$\underbrace{\mathbb{E}_{\substack{s\sim p_+^{\beta}(s) \\ g\sim p_+^{\beta}(s_{t+}|s)}} \left[f^{\pi(\cdot|s,g)}(s, g)\right]}_{\text{test performance}} - \underbrace{\mathbb{E}_{\substack{h\sim p(h),\, s\sim p_+^{\beta_h}(s) \\ g\sim p_+^{\beta_h}(s_{t+}|s)}} \left[f^{\pi(\cdot|s,g)}(s, g)\right]}_{\text{train performance}}.$$

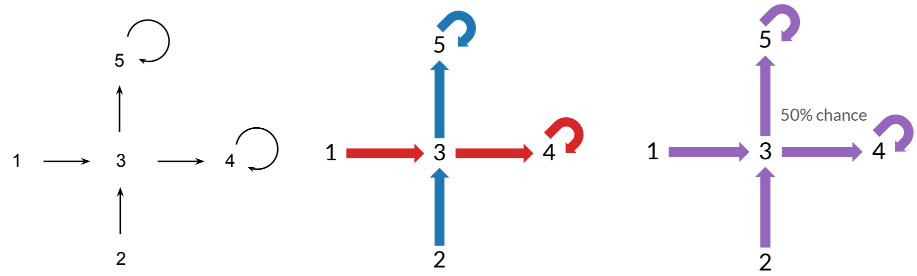
The precise way performance f is measured is not important for our analysis: "generalization" simply means that the performance under one distribution is similar to the performance under another. In our experiments, we will look at performance measured by the success rate at reaching the commanded goal.

On the surface, it may seem like stitching generalization is the same as the standard i.i.d. generalization studied in SL. It may seem that both the test and train distributions are the same. Indeed, Lemma A.1 about reducing mixtures of policies to a single Markovian policy seems to hint that this might be true. *However, stitching generalization is not the same as i.i.d. generalization, and is more akin to combinatorial generalization.* Indeed, this distinction has not been made before while analysing OCBC methods [16, 25]. This misconception is demonstrated by the following lemma:

Lemma A.2. *There exist a collection of policies $\{\beta_h\}$ and context distribution $p(h)$ such that,* conditioned on a state*, the distribution of states and goals for the data collecting policies (training) is different from the distribution of states and goals (testing) for BC policy β.*

$$\mathbb{E}_{p(h)} \left[p_+^{\beta_h}(s_{t+} \mid s) p_+^{\beta_h}(s)\right] \neq p_+^{\beta}(s_{t+} \mid s) p_+^{\beta}(s) \quad \text{for some states } s, s_{t+}. \tag{9}$$

Proof. We prove this Lemma by providing a simple counterexample. Consider the deterministic MDP shown in Figure 5 which has five states $[1, 5]$ and two actions $\{\text{right, up}\}$. The states four and five are absorbing states; once the agent enters one of these states, it will stay there for eternity. There are two data collecting agents β_h with contexts $h = 1$ and $h = 2$, which navigate upward from state two to state five, and rightward from state one to state four respectively. Both policies collect equal amount of data $p(h = 1) = p(h = 2) = 0.5$. We will prove that the training and testing distribution (LHS and RHS of Eq. (9)) for the state-goal pair $\{s = 2, s_{t+} = 4\}$ are not equal.



(a) MDP. (b) $\beta_{h=1}$, $\beta_{h=2}$. (c) β.

Figure 5: *(a)* The MDP has 5 states and two actions (up and right). Prior work [15, 16] also have similar counterexamples, though they have not related this to generalization to the best of our knowledge.*(b)* Data is collected using two contexts conditioned policies shown in blue and red. Both policies collect an equal amount of data $p(h = 1) = p(h = 2) = 0.5$. *(c)* The behavior cloned policy (equation 6) is shown in purple to indicate that it is obtained from combining data from both blue and red policies. The conditional occupancy distribution of the purple policy is used to test stitching. This distribution is different from the training distribution. During training *(b)*, state 2 and goal 4 will never be sampled together, but during testing *(c)*, state 2 and goal 4 have a non-zero probability of being sampled.

$$
\begin{aligned}
&\mathbb{E}_{p(h)}\left[p_+^{\beta_h}(s_{t+} = 4 \mid s = 2)p_+^{\beta_h}(s = 2)\right] \\
&= \frac{p_+^{\beta_{h=1}}(4 \mid 2)p_+^{\beta_{h=1}}(2)}{2} + \frac{p_+^{\beta_{h=2}}(4 \mid 2)p_+^{\beta_{h=2}}(2)}{2} \\
&= \frac{p_+^{\beta_{h=1}}(4 \mid 2)(1-\gamma)}{2} + \frac{0 \times (1-\gamma)}{2} \\
&= \frac{(1-\gamma)^2}{2}\sum_{t=0}^{\infty}\gamma^t p_t^{\beta_{h=1}}(4 \mid 2) \\
&= \frac{(1-\gamma)^2}{2} \times 0 = 0
\end{aligned}
$$

$$
\begin{aligned}
&p_+^{\beta}(s_{t+} = 4 \mid s = 2)p_+^{\beta}(s = 2) \\
&= \frac{p_+^{\beta}(s_{t+} = 4 \mid s = 2)(1-\gamma)}{2} \\
&= \frac{(1-\gamma)^2}{2}\sum_{t=0}^{\infty}\gamma^t p_t^{\beta}(4 \mid 2) \\
&= \frac{(1-\gamma)^2}{2}\left(\frac{\gamma^2}{2} + \frac{\gamma^3}{2} + \dots\right) \\
&= \frac{(1-\gamma)\gamma^2}{4}
\end{aligned}
$$

The LHS and RHS are unequal for all values of $\gamma \in (0, 1)$. Hence proved. □

B Experimental Details

B.1 Environments

We use the "point" and "ant" mazes (umaze, medium and large) from D4RL [10]. As discussed in Appendix C.1, we carefully collect our new offline datasets to test for stitching generalisation (see Fig. 3 for visualisation). In the "point" maze, the task is to navigate a ball with 2 degrees of freedom that is force-actuated in the cartesian directions x and y. In the "ant" maze task, the agent is a 3-d ant from Farama Foundation [27]. To collect data for the "point" maze, we use a PID controller. To collect data for the "ant" maze, we use the same pre-trained policy from D4RL [10]. In Fig. 6, we provide a visualisation of the offline dataset in all "ant" mazes.

B.2 Implementation Details

In this section we provide all the implementation details as well as hyper-parameters used for all the algorithms in our experiments – DT, RvS, and RvS + temporal data augmentation.

DT. We used the exact same hyper-parameters that the original DT paper [2] used for their mujoco experiments. The original DT paper [2] conditioned the transformer on future returns rather than future goals. For our experiments, we switch this conditioning to goals instead. At every time-step

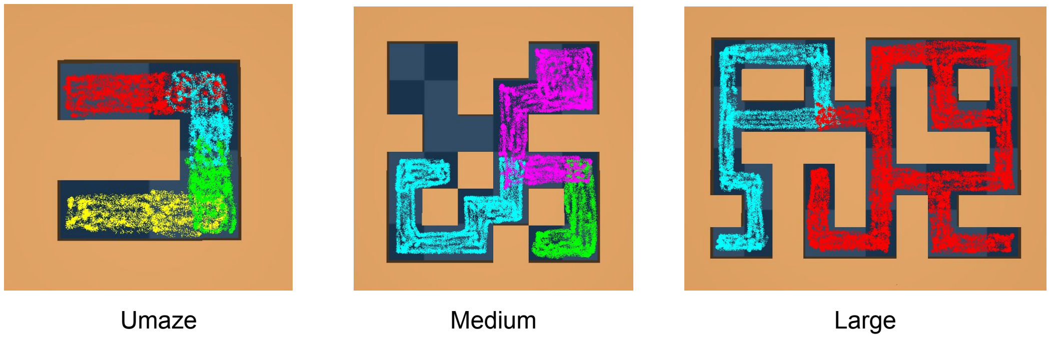


Figure 6: Offline datasets that we collect for the "ant" mazes. Different colors represent the navigation regions of different data collecting policies. See Fig. 3 for a similar visualisation of the "point" maze datasets.

the transformer takes in as input the previous action, current state, and desired goal. The desired goal remains constant throughout the episode, but is still fed to the transformer at every timestep. All hyperparameters used for DT are mentioned in Table 1.

Table 1: Hyperparameters for DT.

hyperparameter	value
training steps	1×10^5
batch size	256
context len	20
optimizer	AdamW
learning rate	1×10^{-3}
warmup steps	5000
weight decay	1×10^{-4}
dropout	0.1
hidden layers (self attention layers)	3
embedding dimension	128
number of attention heads	1

RvS. RvS is an OCBC algorithm which uses a fully connected neural network policy. We use the hyperparameters as prescribed by the original paper [4]. All hyperparameters used for RvS are mentioned in Table 2.

Table 2: Hyperparameters for RvS.

hyperparameter	value
training steps	1×10^6
batch size	256
optimizer	Adam
learning rate	1×10^{-3}
hidden layers	2
hidden layer dimension	1024

RvS + temporal data augmentation. As mentioned in Algorithm 1, given a method to cluster states together, it only requires 5 lines of code to add the temporal data augmentation on top of an OCBC method. We use the k-means algorithm from scikit-learn [28] with the default parameters to group states together. Adding data augmentation on top of RvS introduces 2 extra hyperparameters, which we mention in Table 3. We *do not* tune both of these hyperparameters in our paper.

Table 3: Hyperparameters for temporal data augmentation.

hyperparameter	value
K (number of clusters for k-means) :	
umaze	20
medium	40
large	80
ϵ (probability of augmenting a goal)	0.5

C Relationship with Spurious Correlations.

Handling stitching is somewhat akin to handling spurious correlations studied in the SL community. In the RL setting, we want to navigate from A to B given a dataset that contains some trajectories with A and some with B but none with both A and B. This is somewhat analogous to common settings in object detection in computer vision, where the object background is highly indicative of the object class. For example, the common waterbirds dataset [30] aims to classify images of birds into two classes, "land birds" and "water birds," but the image backgrounds are correlated with the class: water birds are usually depicted on top of a background with water. For evaluation, the classifier is shown an image of a "water bird" on top of a land background (and vice versa). Similar to the RL setting, SL evaluation is done using pairs of inputs that are rarely seen together during training. However, whereas the SL setting aims to learn a classifier that ignores certain aspects of the input, the RL setting is different because the aim is to learn a policy that can reason about both inputs.

There is another connection between the RL setting and spurious correlations, a connection that makes the RL setting look the opposite of the SL setting. For some goal-conditioned RL datasets, the current state is sufficient for predicting which action leads to the goal – the policy does not need to look at the goal. In other datasets, the goal is sufficient for predicting the correct action. However, for navigating between pairs of states unseen together in the dataset, a policy must look at both the state and the goal inputs.

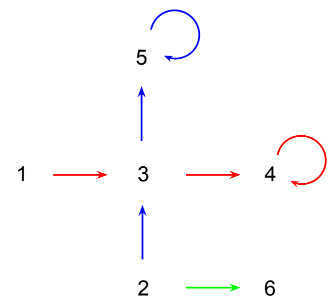


Figure 7: Spurious Correlations: To understand how spurious correlations are related to stitching, let's look at this simple determinisitc MDP in which three data collecting policies (red, blue and green) collect the offline dataset. Note that an OCBC algorithm which ignores the state, can also achieve a zero training loss Eq. (7) on this offline dataset. Whenever 4 is the desired goal in the dataset, action right is always optimal irrespective of the current state. Any SL algorithm that learns a minimal decision rule will in fact learn to ignore the state to reduce the training loss to zero in this case [29]. But during test time, starting at state 2 and conditioned on goal 4 such an SL algorithm will ignore the current state and move towards right which is clearly suboptimal.

C.1 Differences between the original D4RL and our datasets.

Differences between original D4RL and our datasets. We made two main changes in the way our datasets (Fig. 3, Fig. 6) were collected compared to the original D4RL datasets (Fig. 8). *First*, we ensure that different data collecting policies have distinct navigation regions, with only a small overlapping region. This change helps to clearly distinguish between algorithms that can and cannot perform stitching generalisation. *Second*, the agent in the original D4RL datasets often moves in a direction that is largely dependent on its current location in the maze. For example in the topmost row of the umaze, the D4RL policy always moves towards the right. To reduce such spurious relations, we randomize the start-state and goal sampling, for the data collecting policies. That is, in the topmost row of our umaze datasets, the data collecting policy moves both towards the right or left, depending on its start-state and goal. Details about how such spurious relations can hamper stitching generalization are discussed in Appendix C.

Popular offline datasets do not evaluate stitching. While the maze datasets from D4RL [10] were originally motivated to test the stitching capabilities of RL algorithms, we find that most test state-goal pairs are already present in the dataset. Thus, a good success rate on these datasets does not necessarily mean that a method performs stitching. This may explain why OCBC methods have achieved excellent results on the original maze tasks [4], despite the fact that our theory suggests that these methods do not perform stitching. In our experiments, we collect new offline datasets that explicitly test for stitching. To evaluate stitching, we need to test the policy on state-goal pairs which satisfy two conditions: 1) No individual data collection policy (β_h) should navigate between the state-goal pairs; 2) The BC policy (β) should have some chance of navigating between the state-goal pairs. Formally this means that we should test using (s, g) pairs such that

$$p_+^{\beta}(s_{t+} = g \mid s) > 0 \quad \text{and} \quad p_+^{\beta_h}(s_{t+} = g \mid s) = 0 \;\forall\; h.$$

Figure 3 visualises the datasets that we collect for our experiments. Different data collecting policies (shown in different colors) have different regions of navigation. During data collection, these policies navigate between random state-goal pairs chosen from their region of navigation. In Appendix C.1, we contrast our datasets with the original D4RL datasets (see Fig. 8 for a similar visualisation of the original D4RL datasets) and discuss the differences that are necessary to test for stitching generalisation.

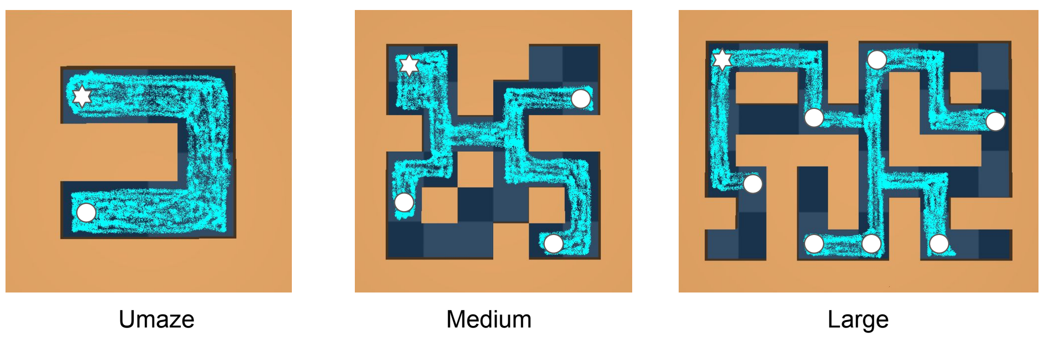


Figure 8: Similar to Fig. 3 and Fig. 6, we create a visualisation of the original d4rl dataset. This is not an exact visualisation of the actual trajectories that are present inside those datasets, but a visualisation of the data collecting policies that those datasets used [10]. During data collection, the policy starts from one starting region, which is marked by a white star. The policy navigates to a goal selected from one of the goal regions, which are marked by white circles. During test time, start-states and goals are selected from similar starting and goal regions, making it difficult to evaluate the stitching generalisation of offline RL algorithms.

D Additional experiments.

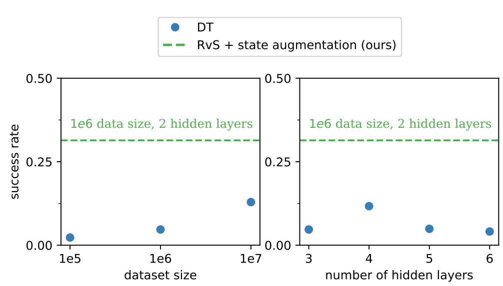


Figure 9: Comparison of DT trained on different sizes of offline dataset (left) and using a different number of hidden layers (right). We also add RvS + data augmentation on the complete state. Even with larger datasets or model sizes, the generalisation of DT is worse than RvS + data augmentation.

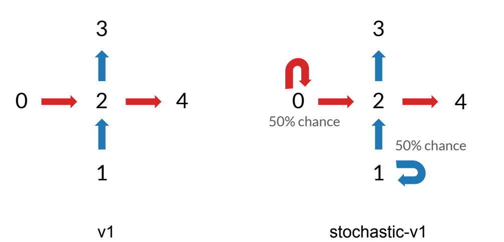


Figure 10: MDP with 5 states and 2 actions (up and right). All episodes end after taking two actions. Data is collected using two policies (red and blue). The only difference between v1 and v1-stochastic, is the data collecting policies are stochastic at states 0 and 1. The stitching task is to navigate from states 1 to 4.

Does more data remove the need for augmentation? Although our theory (Lemma A.2) suggests that generalisation is required because of a change in distribution and is not a problem due to limited data, conventional wisdom says that larger datasets generally result in better generalisation. To empirically test whether this is the case, we train DT on 10 million transitions (10 times more than Fig. 4) on the point-maze large taks. In Fig. 9 (left), we see that even with more data, the stitching generalisation of DT does not improve much. Lastly, scaling the size of transformer models [31] is known to perform better in many SL problems. To understand whether this can have an effect on stitching capabilities, we increased the number of layers in the original DT model. In Fig. 9 (right), we can see that increasing the number of layers does not have an effect on DT's stitching capabilities.

Can OCBC algorithms ever exhibit stitching generalisation? In Fig. 4, we can see that OCBC algorithms do not always achieve zero performance. To understand why OCBC algorithms seldom perform some amount of stitching, we use two offline datasets collected from a simple didactic MDP (see Fig. 10). In $\mathrm{v1}$, $^3/_{10}$ random seeds of DT are successfully able to navigate from states 1 to 4, while $^7/_{10}$ fail. We show that in the both cases, the DT model picks up on one of two spurious relations present in the dataset. In the *success* seeds, the DT model learns to output action up and ignore the goal, whenever it is in state 1. In the *failure* seeds, the DT model learns to output action right, and ignore the state, whenever the goal is 4. In stochastic-$\mathrm{v1}$, we deliberately remove the spurious relation that leads to success – the model can no longer ignore the goal, as the optimal action depends on it, i.e., if the goal is 1, then the optimal action is right , and if the goal is 3, the optimal action is up. In stochastic-$\mathrm{v1}$, we see that all 10 seeds of DT fail. To be certain that models ignore either states or goals, we check that the model's outputs remain invariant them. The affirms the hypothesis that OCBC algorithms can succeed if the spurious relations in the offline datasets set them up for success. In Appendix E, we show an experiment with the same results on a different didactic MDP and in Appendix C we discuss the relation of stitching generalisation with spurious relations.

E Additional didactic experiments.

Similar to Appendix C.1, we perform more experiments on two offline datasets v2 and stochastic-v2 (see Fig. 11). In v2, $10/10$ random seeds of DT are able to successfully navigate from states 0 to 4. The DT model in all cases picks up on the only spurious relation present in v2 – the optimal action depends only on the current state and the goal can be ignored. We ensure that the model actually ignores the goal at state 0, by checking that its outputs remain invariant to changing goals. In stochastic-v2, we remove this spurious relation; the optimal action at state 0 depends on the goal as well, i.e., if the goal is 0, then the optimal action is up, and if the goal is 6, the optimal action is right. After removing the *success* spurious relation, we observe that the success of DT drops to $0/10$ seeds. This result also algins with the same hypothesis that OCBC algorithms can succeed if the spurious relations in the offline datasets set them up for success at test time.

Figure 11: MDP with 7 states and 2 actions (up and right). All episodes end after taking three actions. Data is collected using two policies (red and blue). The only difference between v2 and v2-stochastic, is the data collecting policies are stochastic at states 0 and 1. The stitching task is to navigate from states 0 to 4.

F Related Work

Generalization in RL is generally associated with making correct predictions for unseen but similar states and actions [32–34], ignoring irrelevant details [35–37], or robustness towards changes in the reward or transition dynamics [38–40]. A main thrust of RL research over the last few years seems to be to look at how large models [41, 42] can provide generalization [43–45] with impressive results. We cast stitching as a distinct form of generalisation and show that such large models, trained only with simple SL objectives, cannot perform stitching [9]. We acknowledge that using a learned model of the world might facilitate stitching [46–48] because the model can be used to generate trajectories that navigate between state-goal pairs unseen together in the training data.

Data augmentation in RL has been proposed as a remedy to improve generalisation in RL [19, 49–54], akin to SL [55]. Our proposed data augmentation requires augmenting the true goal, such that the optimal action remains the same even for the augmented goals. Perhaps the most similar prior work are the ones which use dynamic programming to augment existing trajectories to improve the performance properties of SL algorithms [56–58]. However, because these methods still require dynamic programming, they don't have the same simplicity that make SL algorithms appealing in the first place. Our method is conceptually similar, but draws an important connection between generalization and stitching, and our empirical results show that good performance can be achieved without any dynamic programming.

Prior methods that do some form of explicit stitching. Previous work on stitching abilities of SL algorithms have conflicting claims. The DT paper [2] shows experiments where their method performs stitching, suggesting that transformer-based SL algorithms generalise on out-of-distribution goals. On the contrary, [16] provide a simple example where SL algorithms do not perform stitching, regardless of whether a transformer is used. RvS [4] shows that a simple SL algorithm with a fully connected architecture can match the performance of TD algorithms, especially on D4RL's antmaze environment, which supposedly requires stitching to achieve high performance. We provide a formal definition of stitching, and show (both empirically and theoretically) that SL-based RL algorithms do not exhibit such generalisation.