Large Average Subtensor Problem: Ground-State, Algorithms, and Algorithmic Barriers

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Abstract

We introduce the large average subtensor problem: given an order-p tensor over $\mathbb{R}^{N \times \cdots \times N}$ with i.i.d. standard normal entries and a $k \in \mathbb{N}$, algorithmically find a $k \times \cdots \times k$ subtensor with a large average entry. This generalizes the large average submatrix problem, a key model closely related to biclustering and high-dimensional data analysis (Madeira and Oliveira, 2004; Shabalin et al., 2009), to tensors. For the submatrix case, rigorous guarantees regarding the optimal value, algorithms and algorithmic barriers were sought by Bhamidi, Dey, and Nobel (2017) and Gamarnik and Li (2018). In particular, Bhamidi, Dey, and Nobel (2017) explicitly highlights the regime $k = \Theta(N)$ as an intriguing open question.

Addressing the regime $k = \Theta(N)$ for tensors, we establish that the largest average entry concentrates around an explicit value

$$E_{\max} = \sqrt{\frac{2p}{k^p} \log \binom{N}{k}}$$

provided that the tensor order p is sufficiently large, formally corresponding to the regime $N \to \infty$ followed by $p \to \infty$. We establish this result using concentration-enhanced second moment method, which augments the classical second moment method (ineffective in our setting) with Borell-TIS inequality through an argument of Frieze (1990) from random graphs.

Furthermore, we prove that for any $\gamma > 0$ and large p, this model exhibits multi Overlap Gap Property (m-OGP) above the threshold $\gamma E_{\max}$. The m-OGP is a rigorous barrier for a broad class of algorithms exhibiting input stability. These results hold for both $k = \Theta(N)$ and $k = o(N)$. For smaller values of k, we also propose a polynomial-time algorithm that finds a subtensor with an average entry of order $\Theta_p(\frac{1}{\sqrt{p}})E_{\max}$. Our algorithm extends that of Gamarnik and Li (2018), originally devised for the matrix case, to tensors. Our results show in particular that m-OGP is asymptotically sharp: onset of the m-OGP and the algorithmic threshold match as p grows. A recent follow-up work by Bhamidi, Gamarnik, and Gong (2025) established that our algorithmic guarantee is in fact sharp for all p, using a branching OGP argument analogous to Du, Gong, and Huang (2025).

Our results show that while the case $k = \Theta(N)$ remains open for submatrices, it can be rigorously analyzed for tensors in the large p regime. This is achieved by interpreting the model as a Boolean spin glass and drawing on insights from recent advances in the Ising p-spin glasses, in particular those regarding the shattering phenomenon (Gamarnik et al., 2025; El Alaoui et al., 2025b; El Alaoui, 2025; El Alaoui et al., 2025a) and quantum spin glasses (Anschuetz et al., 2025), alongside an earlier work by Talagrand (2000).

Keywords: Large average subtensor, algorithmic lower bounds, spin glasses, overlap gap property.

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