# Multidimensional Consistency Improves Reasoning in Language Models

## Anonymous ACL submission

#### Abstract

While Large language models (LLMs) have proved able to address some complex reasoning tasks, we also know that they are highly sensitive to input variation, which can lead to different solution paths and final answers. Answer consistency across input variations can thus be taken as a sign of stronger confidence. Leveraging this insight, we introduce a framework, Multidimensional Reasoning Consistency where, focusing on math problems, models are systematically pushed to diversify solution paths towards a final answer, thereby testing them for answer consistency across multiple input variations. We induce variations in order of shots in prompt, problem phrasing, and languages used. Experiments on a wide range of open-source state-of-the-art LLMs of various sizes show that reasoning consistency differs by variation dimension, and that by aggregating consistency across dimensions, our framework enhances mathematical reasoning performance on monolingual datasets GSM8K and MATH500, and the multilingual dataset MGSM.

## 1 Introduction

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Large Language Models (LLMs) have shown impressive abilities in addressing a variety of complex reasoning tasks, such as math reasoning (Brown et al., 2020) and commonsense reasoning (Bommasani et al., 2022). The use of Chain-of-Thought (CoT), i.e., breaking down a problem and taking multiple intermediate steps to gradually arrive at the final answer, endows LLMs with even better performances (Wei et al., 2022).

At the same time, LLMs have also proved to be sensitive and somewhat brittle with respect to variations in the way they are prompted (Zhao et al., 2021; Lu et al., 2022). For instance, in a few-shot setting for solving mathematical problems, just altering the order in which the example shots are provided might lead to different reasoning paths and possibly different answers (Wang et al., 2022);



Figure 1: Example of variations: A math problem is presented in different forms or languages, resulting in different reasoning paths to solve it.

the same can happen if different formulations of the same problem are used (Zhou et al., 2024). Also, an identical mathematical problem presented once in one language, and once in a different one, may be solved following different strategies and also lead to different answers (Lai and Nissim, 2024). Figure 1 visualises examples of such variations across different dimensions.

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Some of these variations, such as using even slight alterations in the prompt (Wang et al., 2022; Li et al., 2023), have been exploited in recent work to enhance reasoning performance. However, the experimental setup and the assessment of (in)consistent answers due to variations is still scattered. In this paper, we argue for a systematic treatment of variations and answer consistency and introduce a Multidimensional Reasoning Consistency (MRC) framework, focusing on maths problems. MRC, shown in Figure 2, allows for a systematic and comprehensive testing and evaluation of model consistency against variations in the way the problem is presented to the model. Our framework also makes it possible to best leverage such variations and answer consistency for improving

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overall accuracy in mathematical reasoning tasks.

The rationale behind this framework is that by explicitly and systematically pushing the model to likely diversify its solution paths, and possibly yield a different final answer, we can take acrossvariation consistency of the answer as stronger evidence for its correctness.

We consider three dimensions of variation to test consistency: (i) context (order of shots); (ii) problem (re)phrasing; and (iii) language. For the context aspect, we follow Wang et al. (2022) in changing the order of the exemplars (i.e., the shots), which results in different prompts based on a set of example problems. For problem rephrasing, we prompt the LLMs to rewrite the question before solving it. Lastly, we use the same math problems written in 11 different languages. For each dimension, the LLM generates multiple solution paths to a question, which could differ in various ways, but should in principle lead to the same answer. Answer consistency is eventually used to determine the final answer to the given problem.

We evaluate our framework on three mathematics reasoning benchmarks: GSM8K (Cobbe et al., 2021), MATH500 (Hendrycks et al., 2021), and MGSM (Shi et al., 2023a), covering a range of open-source state-of-the-art LLMs with varying scales: 7-8B, 14-32B, and 70-72B.

**Contributions** First, we introduce a method to systematically study LLMs' reasoning consistency along multiple dimensions of input variation. Second, we improve model performance on both monolingual and multilingual benchmarks for a variety of models by leveraging reasoning consistency across variations; this is obtained thanks to the induced substantial diversification of the reasoning paths, offering valuable insights into LLMs reasoning beyond the commonly used samplingbased strategy. Third, extensive experimental results show that model consistency differs by variation dimensions, but exploiting consistency always enhances math reasoning performance, and aggregating consistency across dimensions yields an additional boost; this paves the way for using a similar framework for other (reasoning) tasks, providing a strategy to make models more robust reasoners. All data and code are available (upon acceptance.)

## 2 Related Work

114 **Math Reasoning in LLMs** Mathematical rea-115 soning has garnered great interest in recent times since LLMs have shown what look like complex problem-solving capabilities (Brown et al., 2020; Lu et al., 2023). With LLMs and few-shot prompting, only a few task examples (e.g., questionanswer pair) are required at inference time to enable the LLM to perform the intended task without updating the model parameters (Brown et al., 2020). To further elicit LLMs' reasoning capability, Wei et al. (2022) proposed a Chain-of-Thought prompting, which involves an explicit step-by-step reasoning from the question to the answer, rendered in natural language. Given its success, a series of CoTrelated methods have been proposed to improve reasoning performance in LLMs, such as complex CoT (Fu et al., 2023), auto-CoT (Zhang et al., 2023), multilingual CoT (Shi et al., 2023b), leastto-most prompting (Zhou et al., 2023), progressivehint prompting (Chuanyang et al., 2023), and residual connection prompting (Jiang et al., 2024). Rather than developing a new specific CoT method, we introduce variations in the prompt and exploit the diversity of CoT outputs.

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Consistency in LLMs In principle, language models could be expected to yield consistent answers in semantically equivalent contexts, especially regarding factual information; this is considered a crucial aspect in assessing model generalization abilities (Fierro and Søgaard, 2022; Lai and Nissim, 2024). In practice, this is often not the case. Some works have thus focused on improving consistency on, e.g., natural language inference (Mitchell et al., 2022), explanation generation (Camburu et al., 2020), cloze test (Ravichander et al., 2020), and factual knowledge extraction (Fierro and Søgaard, 2022). For improving CoT reasoning, Wang et al. (2023) suggested to use self-consistency, sampling diverse solution paths and then selecting the most consistent answer. Zhou et al. (2024) proposed self-consistencyover-paraphrases (SCoP), which diversifies solution paths by generating different paraphrases for a given problem. To check consistency, Wang et al. (2022) use different exemplar orders to possibly trigger diverse solutions. Lai and Nissim (2024) look at consistency of answers given to the same problem written in two different languages, and use multilingual instruction tuning to improve LLMs' performance across languages.

Here, we propose a novel method to study and leverage reasoning consistency along different dimensions to improve performance.

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Figure 2: Overview of our Multidimensional Reasoning Consistency (MRC) framework: (i) COC changes the exemplars order; (ii) CPC rewrites the given questions in the same language; and (iii) CLC rewrites the given questions in different languages.

# 3 Methodology

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Figure 2 shows our framework. Using systematic variations, MRC pushes the model to generate multiple solutions for a given question, then measures consistency across variations, and leverages it to improve performance.

## 3.1 Reasoning Consistency

Formally, given a set of math problems  $\mathcal{M}$ , each consisting of a two-tuple (question:q, answer:a). We define the reasoning consistency of an LLM as the extent to which it yields the same answer for a given question under a dimension of variation (e.g., language). Specifically, for each question, assume that the LLM generates n candidate solutions  $\{s_1, \ldots, s_n\}$  which can arrive at a set of final answers  $\{a_1, \ldots, a_m\}$ , reasoning consistency (RC) is the ratio of the maximum number of these solutions that can lead to the same answer over the total number of candidates n.

$$\mathrm{RC}(\mathrm{LLM}) = \frac{1}{|\mathcal{M}|} \sum_{j=1}^{|\mathcal{M}|} \frac{\max_{j} |\mathcal{S}_{j}|}{n} \tag{1}$$

$$S_j = \{s_i \in \{s_1, \dots, s_n\} | f(s_i) = a_j\}$$
 (2)

Where  $f(s_i)$  maps solution  $s_i$  to the final answer.

### 3.2 Multidimensional Consistency

In the context of reasoning consistency in mathematical problems, a language model can generate multiple plausible responses to the same math question, where *correct* reasoning solutions, even if they are diverse, tend to be more consistent in the final answer than incorrect solutions (Wang et al., 2023). Instead of simply sampling a diverse set of candidate outputs from LLMs, our MRC framework, aims to assess model consistency along three dimensions we control for and exploit: example order, problem (re)phrasing, and language.

Cross-order Consistency (COC) Some prior works have shown that LLMs are sensitive to order, such as the order of options in multiple-choice questions (Pezeshkpour and Hruschka, 2024; Zotos et al., 2025), or the order of shots in math reasoning (Wang et al., 2022). Here we assess how much the order of the shots affects consistency of language models. Specifically, we focus on few-shot prompting, which consists of a set of exemplars (question:q, step-by-step solution:s), whose presentation order can be changed arbitrarily. For instance, given a 4-shot prompt with 4 exemplars, we could change their order to get 24 different prompts, each of which can be used to prompt the model to generate a corresponding answer to a given question (see Appendix A.1 for examples). This allows us to assess the robustness of the model with respect to the order of exemplars in few-shot prompting and then leverage its consistency to improve the model's performance.

**Cross-phrasing Consistency (CPC)** In addition to the order of the exemplars in the prompt, the surface form of the question itself can also have an impact on the performance of the model (Zhou et al., 2024). Differently from Zhou et al. (2024), who prompt LLMs to generate 'good' paraphrases for math questions, we directly prompt an LLM to rewrite the question with the goal of making it easier for itself to solve (see Appendix A.1 for examples). We use two different main settings, including rewrite-without-solve and rewrite-thensolve, which yield the following four settings when combined with the original question:

- Rewrite-without-solve (RwS): We ask the LLM to rewrite the question, but not to include the solution. Afterwards, we prompt the LLM to generate the solution for the rewritten question.
- Original Question + RwS (RwS+): We concatenate the original question and the rewritten one above prompting the LLM for the solution.
- Rewrite-then-solve (RtS): We ask an LLM to rewrite the question making it easier to solve and then to give the corresponding solution.
- RtS Question (RtS-): We prompt the LLM to generate the solution for the rewritten question in the "rewrite-then-solve" setting.

**Cross-lingual Consistency (CLC)** One rather 247 outstanding way to vary formulations is to write 248 the same problem in different languages. Abilities of LLMs in different languages vary substantially, depending on the amount of training data in a given language, and on the similarity of lesser represented languages to more resource-rich ones, as this impacts how well models can deal with less seen languages (de Vries et al., 2022; Muennighoff et al., 2023; Üstün et al., 2024). With cross-lingual 256 consistency, we leverage language diversity to evaluate the LLMs' robustness to input in different languages, and exploit output diversity to further improve the LLMs' reasoning performance. Given the 260 same math question in different languages, LLMs 261 are expected to produce reasoning solutions in the corresponding languages. On the one hand, those 263 solutions are expected to arrive at the same final answer if the language model is multilingual; on 265 the other hand, due to the differences in language structures, those solutions can increase diversity compared to using a single language.

# 3.3 MRC for Reasoning

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Eventually, answer consistency across the three dimensions can also be leveraged to improve reasoning performance. For each question, the solution set  $\{s_1, \ldots, s_n\}$  generated by the language model, which can arrive at the final answer set  $\{a_1, \ldots, a_m\}$ . We select the most consistent answer in *n* solution paths as the final answer  $\hat{a}$ , which is obtained through majority voting:

$$\hat{a} = \arg\max_{a \in \mathcal{A}} \sum_{a' \in \mathcal{A}} \mathbb{I}(a = a')$$
(3)

Where  $\mathcal{A}$  denotes the set of candidate answers and  $\mathbb{I}(\cdot)$  is the indicator function.

## 4 Experimental Setup

**Datasets** To comprehensively assess our framework across dimensions, we include three wellestablished math reasoning benchmarks: (1) monolingual datasets **GSM8K** (Cobbe et al., 2021) and **MATH500** (Hendrycks et al., 2021); and (2) multilingual dataset **MGSM** (Shi et al., 2023a). Additionally, to test our CLC method on the more challenging dataset, we machine-translate questions from MATH500 into 10 languages in MGSM.<sup>1</sup>

Figure 3: Reasoning consistency on three dimensions of variation. Note that COC and CPC are evaluated on the monolingual benchmark GSM8K, while CLC is evaluated on the multilingual benchmark MGSM.

**Models** We select a range of open-source stateof-the-art LLMs in varying scales: (i) 7-8B; (ii) 14-32B; and (iii) 70-72B.<sup>2</sup> For all models, we only consider instruction-tuned versions. 291

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**Implementation** We use 4-shot for all languages except TE which only uses 2-shot, since a 4-shot prompt would exceed the default maximum length, due to tokenization issues unfavourable to this language (Ahia et al., 2023).<sup>3</sup> All prompt exemplars we use are released by Shi et al. (2023a) and An et al. (2024). We report the final answer accuracy for all experiments except the consistency score.

# 5 Results and Analysis

We report results for all variation dimensions, and then zoom in on CLC for a more detailed analysis.

# 5.1 Reasoning Consistency

Figure 3 shows reasoning consistency results on the three different dimensions. The first observation is that COC achieves the highest scores, followed by CPC, with CLC having the lowest scores across the board. This suggests that all models are more sensitive to language variations while results are more consistent across different exemplar orders in few-shot prompting. Indeed, when looking at COC only, all models achieve consistency scores above

<sup>1.0</sup> 0.0 Phi-3-7B Qwen-2.5-7B Qwen-2.5-Math-7B Llama-3.1-8B Aya-expanse-8B. Ministral-8B Phi-3-14B Mistral-22B Aya-expanse-32B Llama-3.1-70B Qwen-2.5-72B **Dwen-2.5-Math-72B** COC CPC CLC

<sup>&</sup>lt;sup>2</sup>More details are in Appedix A.5

<sup>&</sup>lt;sup>3</sup>Examples are in Appendix A.1.



Figure 4: Reasoning accuracy of 4-shot for 8 different exemplars orders. The x-axis numbers correspond to the following models: 1 – Phi-3-7B; 2 – Qwen-2.5-7B; 3 – Qwen-2.5-Math-7B; 4 – Llama-3-1-8B; 5 – Aya-expanse-8B; 6 – Mistral-8B; 7 – Phi-3-14B; 8 – Mistral-22B; 9 – Aya-expanse-32B; 10 – Llama-3-1-70B; 11 – Qwen-2.5-72B; 12 – Qwen-2.5-Math-72B. Notes: (i) minimum score (MIN); (ii) mean score (MEAN); (iii) maximum score (MAX).

			GSM	18K					MATI	H500		
Models	СоТ	RwS	RwS+	RtS-	RtS	CPC	СоТ	RwS	RwS+	RtS-	RtS	CPC
7-8B												
Phi-3-7B	88.2	84.5	87.0	84.8	88.1	90.0	40.4	36.2	41.8	39.6	43.4	46.6
Qwen-2.5-7B	88.3	86.0	89.8	86.3	90.1	92.0	63.6	57.2	63.6	59.0	62.0	72.4
Qwen-2.5-Math-7B	90.0	87.6	91.1	89.1	92.3	94.1	75.2	72.8	73.2	72.4	73.2	77.4
Llama-3.1-8B	79.7	73.9	78.2	77.9	81.2	83.8	42.6	36.6	45.6	39.0	46.6	50.2
Aya-expanse-8B	76.7	73.5	77.9	73.5	78.2	82.4	28.0	26.0	29.4	26.6	27.0	32.4
Ministral-8B	81.2	78.7	83.0	78.9	84.0	84.7	42.8	41.8	48.0	39.2	50.6	51.0
14-32B												
Phi-3-14B	89.2	86.4	89.2	86.9	89.8	90.2	44.2	44.0	47.6	43.2	47.2	51.2
Mistral-22B	85.8	83.1	85.7	84.8	88.1	88.1	42.6	44.4	51.4	43.2	48.6	51.6
Aya-expanse-32B	83.8	82.3	83.8	82.4	88.4	88.1	38.6	37.6	38.2	41.2	40.8	43.4
70-72B												
Llama-3.1-70B	94.0	89.8	93.9	91.9	93.6	94.8	58.6	49.0	60.2	55.8	63.4	65.2
Qwen-2.5-72B	94.6	88.9	94.4	88.6	95.5	95.8	63.2	23.8	22.0	63.8	74.0	67.0
Qwen-2.5-Math-72B	94.0	92.9	94.7	93.5	94.8	95.9	66.6	54.6	58.0	67.6	74.6	74.8

Table 1: Reasoning accuracy of CPC on the benchmarks GSM8K and MATH500, obtained via aggregating vanilla CoT prompting and 4 different question rewriting settings. The best result for each model across settings is bolded.

0.9. Notably, the Llama-3.1 family achieves the highest score with the 70B model and the lowest score with the 8B model.

For CPC and CLC, Aya-expanswe-8B has the lowest consistency scores in both dimensions, while larger Qwen2.5 and Llama-3.1 models perform best. Compared to COC and CPC, there is a bigger gap in CLC for different models, even within the same scale, e.g., Phi3-7B vs Qwen2.5-7B. Overall, larger models show higher consistency.

### 5.2 Consistency Improves Reasoning

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For each dimension, we compare the performance obtained exploiting cross-variation consistency to yield a final answer with the performance obtained via the variations on their own. **COC** Figure 4 reports the results augmented with COC on GSM8K and MATH500, where we use 8 different exemplar orders for the 4-shot prompt.<sup>4</sup> Compared to vanilla CoT prompting, COC improves the reasoning performance for all models. On dataset GSM8K, COC scores are higher than the average scores of 8 different order prompts on all models, and highest on most models, except for Phi-3-7B, Llama-3.1-8B, and Aya-expanse-8B, where it is on par with the highest scores among the eight ordering configurations we consider in this analysis. On the more challenging dataset MATH500, we see COC achieves the highest scores among all models.

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**CPC** Table 1 shows CPC's on GSM8K and MATH500. Accuracy drops when models are fed

<sup>&</sup>lt;sup>4</sup>Complete results are in Appendix A.3.

Models	BN	DE	EN	ES	FR	JA	RU	SW	ТЕ	TH	ZH	CLC
7-8B												
Phi-3-7B	14.8	77.6	89.2	85.2	80.4	64.8	74.4	14.0	5.2	18.8	76.0	91.2
Qwen-2.5-7B	67.2	72.4	91.6	82.8	72.0	64.8	70.8	16.4	29.2	75.6	74.0	92.8
Qwen-2.5-Math-7B	16.8	76.8	92.8	82.0	76.8	61.6	78.8	4.0	5.6	51.2	85.6	93.6
Llama-3.1-8B	57.6	64.4	80.8	73.6	63.6	52.4	68.0	55.6	49.6	58.8	63.6	78.8
Aya-expanse-8B	29.2	70.4	77.2	74.8	66.8	60.4	72.0	11.6	6.4	22.8	67.2	82.0
Ministral-8B	50.4	68.0	85.6	76.4	69.6	54.0	70.8	27.6	36.4	53.2	64.4	84.0
14-32B												
Phi-3-14B	14.8	76.0	88.0	87.6	76.8	72.8	80.8	18.4	5.6	12.8	77.6	90.0
Mistral-22B	52.0	76.4	87.6	82.4	75.2	62.0	78.4	35.6	17.2	57.6	80.0	89.2
Aya-expanse-32B	58.4	74.0	86.0	84.4	80.0	73.6	81.2	29.2	17.2	52.8	77.2	90.8
70-72B												
Llama-3.1-70B	83.6	82.0	93.6	87.6	77.6	76.8	84.4	83.2	79.2	80.4	84.0	93.6
Qwen-2.5-72B	88.0	84.4	93.2	88.4	80.4	84.4	87.2	66.0	68.8	91.6	86.8	95.6
Qwen-2.5-Math-72B	86.4	83.6	94.4	85.6	78.4	81.2	70.4	57.2	68.0	85.6	88.4	95.2

Table 2: Reasoning accuracy of CLC compared to vanilla CoT prompting on the MGSM benchmark. Note that bold numbers indicate the best result for each model among different languages and CLC.

Models	BN	DE	EN	ES	FR	JA	RU	$\mathbf{SW}$	ТЕ	ТН	ZH	CLC
7-8B												
Phi-3-7B	10.8	34.8	40.4	39.8	5.4	20.8	30.0	9.4	5.6	10.0	33.2	44.0
Qwen-2.5-7B	33.4	51.6	63.6	50.0	33.0	50.2	55.8	17.6	17.0	42.2	49.0	67.4
Qwen-2.5-Math-7B	29.2	61.2	75.2	62.0	53.4	50.8	65.8	3.8	16.2	23.6	70.6	79.4
Llama-3.1-8B	22.2	31.0	42.6	35.4	7.2	29.4	38.6	24.8	9.4	26.8	32.8	47.2
Aya-expanse-8B	10.6	25.8	28.0	26.6	25.6	24.0	25.2	8.6	5.6	11.0	24.2	31.2
Ministral-8B	17.0	31.0	42.8	42.4	36.8	26.6	38.8	8.4	7.6	18.2	29.8	44.4
14-32B												
Phi-3-14B	3.0	42.2	44.2	41.8	9.2	33.6	37.2	13.8	4.6	1.8	38.6	49.6
Mistral-22B	16.0	43.2	42.6	43.0	23.6	34.4	43.2	15.2	35.8	23.6	35.2	53.8
Aya-expanse-32B	20.8	40.2	38.6	38.2	3.0	35.0	38.8	15.0	10.2	16.0	38.0	45.0
70-72B												
Llama-3.1-70B	32.6	41.2	58.6	51.8	9.6	33.4	47.2	44.0	22.4	45.4	42.8	64.4
Qwen-2.5-72B	51.4	54.8	63.2	48.4	51.6	27.6	38.4	35.6	37.4	56.2	63.2	75.2
Qwen-2.5-Math-72B	58.0	50.0	66.6	60.0	22.4	56.6	60.6	30.6	34.4	58.0	64.4	75.4

Table 3: Reasoning accuracy of CLC compared to vanilla CoT prompting on the machine-translated MATH500. Note that bold numbers indicate the best result for each model among different languages and CLC.

only the rewritten question (RwS), as they might lose some information from the original question (manual inspection). When combining the rewritten question with the original one (RwS+), most models score comparably to the original prompting and tend to achieve higher scores in the rewrite-352 then-solve setting (RtS). The latter observation suggests that asking the model to rewrite the question in a simple way and then solve it, can effectively help the model. Lastly, we see that CPC can further 356 improve the reasoning performance: (i) when comparing to vanilla CoT prompting this is true for all 358 models; and (ii) when comparing to RtS, all models achieve higher accuracy except Aya-expanse-32B on GSM8K and Qwen-2.5-72B on MATH500.

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**CLC** Table 2 and Table 3 present the result of CLC compared to vanilla CoT prompting on MGSM and machine-translated MATH500, respectively. All models perform best on English, with a 365

serious performance gap between underrepresented (e.g., SW) and high-resource languages, especially for smaller models. Similar to COC and CPC, compared to vanilla CoT, CLC yields improvement for most models, with Aya-expanse-32B on GSM8K, for example, showing a significant gain of 4.8% absolute accuracy compared to that of English. For Llama-3.1-8B and Ministral-8B, the accuracy of CLC is slightly lower than that of English, but better than that of all other languages. On the more challenging dataset MATH500, CLC leads to consistent improvements across all models, with the most substantial gain observed on the larger model (e.g, a 12.0% percentage point relative improvement in accuracy over that of English on Qwen-2.5-72B), highlighting its effectiveness in handling more complex tasks and scaling with model size.

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**MRC** Table 4 shows the results of MRC and of the three separate consistency methods on MGSM



Figure 5: Reasoning accuracy of using varying numbers of reasoning paths on GSM8K.

Datasets			M8K				H500	
Models	COC	CPC	CLC	MRC	COC	CPC	CLC	MRC
7-8B								
Phi-3-7B	92.4	92.0	91.2	94.4	43.2	46.6	44.0	49.2
Qwen-2.5-7B	92.0	93.2	92.8	93.6	68.8	72.4	67.4	73.6
Qwen-2.5-Math-7B	94.4	94.8	93.6	96.0	77.0	77.4	79.4	79.2
Llama-3.1-8B	80.8	85.6	78.8	84.4	48.0	50.2	47.2	52.2
Aya-expanse-8B	78.4	85.2	82.0	83.6	31.4	32.4	31.2	34.4
Ministral-8B	84.4	86.8	84.0	87.2	45.0	51.0	44.4	49.2
14-32B								
Phi-3-14B	92.0	92.0	90.0	93.2	49.0	51.2	49.6	52.8
Mistral-22B	87.2	89.6	89.2	92.0	48.2	51.6	53.8	53.6
Aya-expanse-32B	86.0	89.2	90.8	91.2	43.4	43.4	45.0	46.4
70-72B								
Llama-3.1-70B	95.6	96.8	93.6	96.4	65.0	65.2	64.4	68.2
Qwen-2.5-72B	96.0	97.6	95.6	96.8	75.2	67.0	75.2	77.6
Qwen-2.5-Math-72B	94.4	95.2	95.2	95.2	76.0	74.8	75.4	80.0

Table 4: Accuracy on MGSM and MATH500. Notes: (i) CPC uses 5 solution paths, COC and CLC use 8 each, and CLC uses 8 languages (excl. BN, SW, TE), so MRC contains a total of 19 paths (excluding the two identical English paths); (ii) best result for each model is bolded.

and machine-translated MATH500. Of the three variation dimensions, CPC performs best overall, followed by COC and CLC. This suggests that CPC can push the model to better diversify its solution paths, while for CLC, this might be due to the large performance gap between English and other languages. By aggregating consistency across multiple dimensions, MRC can further improve the reasoning accuracy for most models, while showing different scaling behaviors on datasets of varying difficulty. On the easier dataset MGSM, smaller models benefit more significantly from our approach, indicating its effectiveness in enhancing the capabilities of lightweight models. In contrast, on the more challenging dataset MATH500, larger models show greater relative improvement, suggesting that our method scales well with model capacity when addressing more complex tasks.

### 5.3 Analysis

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**Comparison to Self-consistency** One can conceive CLC as a multilingual extension of monolin-

gual self-consistency, as it goes beyond the commonly used sampling-based strategy. In Figure 5, we plot accuracy with respect to varying numbers of reasoning paths for two model families (Llama-3.1 and Qwen-2.5). For self-consistency, we use English following (Wang et al., 2023), whereas for CLC, we use 8 languages excluding BN, SW, and TE which have very low results (see Table 2). We sample N/8 reasoning paths for each language, thus creating N solutions for CLC. For all models, we use temperature sampling with T = 0.6 and truncated at the top-k (k = 40) tokens with the highest probability. We see some different trends between the two model families: (i) for Llama-3.1, the accuracy of CLC is generally lower than selfconsistency but higher than greedy decode, possibly due to the large performance gap between English and other languages; (ii) for Qwen-2.5, CLC dramatically improves reasoning accuracy when the number of paths goes from 8 to 16, and achieves better performance than self-consistency. Overall, as with self-consistency, CLC yields higher accuracy with a greater number of paths, suggesting that the language dimension can indeed introduce valuable diversity in the reasoning paths.

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**Case Study** Table 5 shows examples of solutions generated by Qwen2.5-72B for the same MSGM question in different languages. For the EN question, the model incorrectly reasons in the first step: *On Mondays, Wednesdays, and Fridays, Kimo has* 3 \* 1 = 3 hours of classes, which results in the wrong reasoning and final answer. It is interesting to see that the model reasons correctly in the other languages except BN, while the solution paths might be logically different. For DE, the model first calculates the total hours for Monday, Wednesday, and Friday, then calculates the total hours for Tuesday and Thursday, and then adds them together to get the total hours for the week; while for ZH,

Language	CoT Reasoning
Question	On Mondays, Wednesdays, and Fridays, college student Kimo has three 1-hour classes each day. On Tuesdays and Thursdays, he has two 2-hour classes each day. In one semester, there are 16 weeks of school. In one semester, how many hours does Kimo spend attending classes?
BN	প্রতি সপ্তাহে, কিমোর ক্লাদের সময় 3 * 1 = 3 ঘন্টা (দোমবার, বুধবার, শুরুবার) + 2 * 2 = 4 ঘন্টা (মঙ্গলবার, বুহুম্পতিবার) = 7 ঘন্টা। একটি সেমিস্টার 16 সপ্তাহের, তাই মোট 16 * 7 = 112 ঘন্টা কিমোকে ক্লাদে বায় করতে হয়। উত্তর হল 112।
DE	Montags, mittwochs und freitags verbringt Kimo 3 x 1 = 3 Stunden pro Tag mit Kursen. In einer Woche verbringt er an diesen Tagen insgesamt 3 x 3 = 9 Stunden mit Kursen. Dienstags und Donnerstags verbringt er 2 x 2 = 4 Stunden pro Tag mit Kursen. In einer Woche verbringt er an diesen Tagen insgesamt 2 x 4 = 8 Stunden mit Kursen. Insgesamt verbringt er in einer Woche 9 + 8 = 17 Stunden mit Kursen. In einem 16-wochen-Semester verbringt er 16 x 17 = 272 Stunden mit Kursen. Die Antwortlautet 272.
EN	On Mondays, Wednesdays, and Fridays, Kimo has $3 * 1 = 3$ hours of classes. On Tuesdays and Thursdays, he has $2 * 2 = 4$ hours of classes. In one week, he has $3 + 4 + 3 = 10$ hours of classes. In one semester, Kimo spends $10 * 16 = 160$ hours attending classes. The answer is 160.
ES	Los lunes, miércoles y viernes, Kimo pasa $3 * 1 = 3$ horas en clases. Como hay 3 días a la semana, pasa $3 * 3 = 9$ horas en clases. Los martes y jueves, Kimo pasa $2 * 2 = 4$ horas en clases. Como hay 2 días a la semana, pasa $2 * 4 = 8$ horas en clases. En total, Kimo pasa $9 + 8 = 17$ horas a la semana en clases. En un semestre, Kimo pasa $17 * 16 = 272$ horas en clases. La respuesta es 272.
FR	Kimo a 3 cours d'une heure les lundis, mercredis et vendredis, ce qui fait $3 \times 1 = 3$ heures par jour. Comme il y a 3 jours dans la semaine où il a 3 heures de cours, cela fait $3 \times 3 = 9$ heures par semaine. Les mardis et jeudis, il a 2 cours de 2 heures, ce qui fait $2 \times 2 = 4$ heures par jour. Comme il y a 2 jours dans la semaine où il a 4 heures de cours, cela fait $2 \times 4 = 8$ heures par semaine. Au total, Kimo a $9 + 8 = 17$ heures de cours par semaine. Comme il y a 16 semaines d'école par semestre, Kimo consacre $17 \times 16 = 272$ heures à ses cours durant un semestre. La réponse est $272$ .
JA	月曜、水曜、金曜は、キモは1時間の講義を3つ受けるので、1日あたり3時間、3日間で9時間です。火曜と木曜は、2時間の講義を2つ受けるので、1日あたり4 時間、2日間で8時間です。1週間で、キモは9+8=17時間講義に出席します。1セメスターは16週間あるので、キモは1セメスターで17*16=272時間講義に出席し ます。答えは272時間です。
RU	в понедельник, среду и пятницу Кимо проводит 3 * 1 = 3 часа на занятиях. В неделю таких дней 3, значит, за неделю он проводит 3 * 3 = 9 часов на занятиях в эти дни. Вторник и четверг Кимо проводит 2 * 2 = 4 часа на занятиях. В неделю таких дней 2, значит, за неделю он проводит 4 * 2 = 8 часов на занятиях в эти дни. Всего за неделю Кимо проводит 9 + 8 = 17 часов на занятиях. В семестре 16 недель, значит, за семестр Кимо проводит 17 * 16 = 272 часа на занятиях. Ответ — 272.
SW	Kimo ana masomo matatu ya saa 1 kila siku kwa siku tatu, hivyo kwa jumla ana masomo 3 * 1 * 3 = 9 saa. Kimo pia ana masomo mawili ya saa 2 kila siku kwa siku mbili, hivyo kwa jumla ana masomo 2 * 2 * 2 = 8 saa. Kwa jumla, Kimo ana masomo 9 + 8 = 17 saa kwa wiki. Kimo ana shule kwa wiki 16, hivyo kwa jumla ana masomo 17 * 16 = 272 saa kwa muhula. Jibu ni 272.
TE	సోమవారాలు, బుధవారాలు, మరియు శుక్రవారాల నాడు కిమోకు మూడు 1-గంట క్లాసులు ఉన్నాయి, అందువల్ల ప్రతి వారాలో 3 రోజుల్లో 3 గంటలు క్లాసులు ఉన్నాయి, అంటే 3*3=9 గంటలు. మంగళవారం మరియు గురువారాల్లో అతడికి ప్రతిరోజు రెండు 2-గంటల క్లాసులున్నాయి, అందువల్ల ప్రతి వారాలో 2 రోజుల్లో 4 గంటలు క్లాసులు ఉన్నాయి, అంటే 2*4=8 గంటలు. అందువల్ల ప్రతి వారాలో కిమోకు మొత్తం 9+8=17 గంటలు క్లాసులు ఉన్నాయి. ఒక సమీస్టర్లలో 16 వారాలు ఉన్నాయి, అందువల్ల ఒక సమీస్టర్లో కిమోకు మొత్తం 16*17=272 గంటలు క్లాసులు ఉన్నాయి. సుపాధానం 272.",
TH	ในวัมขัมกร์ วันพุธ และวันศูกร์ ได้ในดีเรียน 3 คณต่อวัน ซึ่งแต่นลามให้วอก 1 ชั่วในง แสดงว่าใน 3 วันนี้ตาให้เวอก 3 * 1 * 3 = 9 ชั่วในง ในวันอังคธและวันพฤพัสบดิ เคพื่อชือน 2 คณต่อวัน ซึ่ง แต่สะคณให้เวอก 2 ชั่วในง แสดงว่าใน 2 วันนี้ตาใช้เวอก 2 * 2 * 2 = 8 ชั่วในง ลังนั้น ในคนึ่งสัปลาห์ ได้ในใช้เวอก 9 + 8 = 17 ชั่วในง ในคนึ่งภาพกธศึกษามีกธธีขนกธสอน 16 สัปลาห์ ดังนั้น ได้ในใช้เวอก 17 * 16 = 272 ชั่วในง ศักอบคือ 272"
ZH	周一、周三和周五,基莫每天有 3 节 1 小时的课,即 3 * 1 = 3 小时。周二和周四,他每天有 2 节 2 小时的课,即 2 * 2 = 4 小时。 一周内,他上课的时间为 3 * 3 + 2 * 4 = 9 + 8 = 17 小时。一个学期有 16 周,所以基莫一个学期上课的时间为 17 * 16 = 272 小时。 答案是 272。答案是 272 小时。

Table 5: Case study in the test set of MSGM, where the solutions are generated by Qwen2.5-72B for a question written in different languages. Note that here we only show an EN question, the questions corresponding to each output are written in their respective languages, which can be seen in the Appendix A.4.

it calculates the hours per day and then calculates the total hours for the week. Reasoning appears correct in some low-resource languages (SW, TE, and TH), suggesting that our approach can leverage the model's capabilities in low-resource languages to assist reasoning in high-resource languages, in addition of course to the usual reverse benefit.

# 6 Conclusion

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MRC is a multidimensional reasoning framework,
which comprehensively studies reasoning consistency when the same math problems are presented
to the model with systematic variations along three
different dimensions. By leveraging such variations and answer consistency, MRC improves over-

all accuracy on both monolingual and multilingual benchmarks, demonstrating its effectiveness in enhancing lightweight models and its strong scalability for more complex tasks. Our experiments seem to suggest that the largest the diversity of solution paths, the stronger the benefit from exploiting consistency. As we do not yet have concrete evidence for this hypothesis, a natural future direction would be to study path diversity in a quantifiable way. Another valid extension would be *integrating* the different dimensions (e.g., COC per language). While combining multiple dimensions of variations presents an explosion of possibilities, strategic selection based on empirical results, especially crossall accuracy, might mitigate this challenge.

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# 7 Limitations

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While we investigated model consistency in mathe-475 matical reasoning and successfully leveraged it to 476 improve reasoning accuracy, several promising di-477 rections remain for future exploration. We mainly 478 focus on the variations in model inputs and consis-479 tency in final answers, while both the variation and 480 consistency of the reasoning paths are interesting 481 directions. Specifically, variations in the input will 482 lead to variations in the output, which includes log-483 ical consistency and inconsistency, thus affecting 484 the final result. Also, it is not yet clear how vari-485 ations in input affect the model's reasoning logic 486 (variations in reasoning paths), which requires a 487 much bigger unpacking. Lastly, similar to self-488 consistency, our method incurs more computational 489 cost as it requires the model to generate multiple 490 solutions in different dimensional variations. 491

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# **A** Appendix

## A.1 Prompt Examples

#### Prompt 1

Question: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

Solution: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

Question: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room? Solution: There are 4 days from monday to thursday. 5 computers were added each day. That means in total 4 \* 5 = 20 computers were added. There were 9 computers in the beginning, so now there are 9 + 20 = 29 computers. The answer is 29.

[Two more exemplars]

Question: {} Solution:

#### Prompt x

Question: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room? Solution: There are 4 days from monday to thursday. 5 computers were added each day. That means in total 4 \* 5 = 20 computers were added. There were 9 computers in the beginning, so now there are 9 + 20 = 29 computers. The answer is 29.

••• Question: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

Solution: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

[Two more exemplars]

Question: {} Solution:

### Figure 6: Examples of prompts for COC.

#### **Rewrite-without-solve Prompt**

Rewrite the following math problems to make them easier for LLMs to solve, then solve them step by step.

Question: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

Rewritten Question: Roger starts with 5 tennis balls. He buys 2 cans of tennis balls, with each can containing 3 tennis balls. How many tennis balls does Roger have in total?

[Three more exemplars]

Question: {} Rewritten Question:

#### **Rewrite-then-solve Prompt**

Rewrite the following math problems to make them easier for LLMs to solve, then solve them step by step.

Question: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

Rewritten Question: Roger starts with 5 tennis balls. He buys 2 cans of tennis balls, with each can containing 3 tennis balls. How many tennis balls does Roger have in total? Solution: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

[Three more exemplars]

Question: {} Rewritten Question:

Figure 7: Examples of prompts for CPC.

#### **EN Prompt**

Question: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

Solution: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

[Three more exemplars]

Question: {} Step-by-Step Answer:

#### **TE Prompt**

ప్రశ్న: జేసస్ వద్ద 20 లాలీపాపలు ఉన్నాయి. అతడు డెన్నీకి కొన్ని లాలీపాపలు ఇచ్చాడు. ఇప్పుడు జేసస్ వద్ద 12 లాలీపాపలు ఉన్నాయి. డెన్నీకి జేసస్ ఎన్ని లాలీపాపలు ఇచ్చాడు?లాలీపాపలులాలీపాపలులోపాపలు డశలవారిగా సమాధానం: జేసస్ 20 లాలీపాపలతో ప్రారంభించాడు, కానీ ఇప్పుడు అతడి వద్ద 12 మాత్రమే ఉన్నాయి, అందువల్ల అతడు డెన్నీకి 20-12=8 లాలీపాపలు ఉన్నాయి. సమాధానం 8.

ప్రశ్ని: పార్కింగ్ లాట్లో 3 కార్డు ఉండి, మరో 2 కార్డు వచ్చినట్లయితే, పార్కింగ్ లాట్లో ఎన్ని కార్డు ఉన్నాయి? దశలవారీగా సమాధానం: ప్రారంభంలో 3 కార్డు ఉన్నాయి, మరో 2 కార్డు వచ్చాయి, అందువల్ల ఇప్పుడు 3+2=5 కార్డు ఉన్నాయి. సమాధానం 5.

ప్రశ్న: {} దశలవారీగా సమాధానం:

Figure 8: Examples of prompts for CLC.

# A.2 Datasets

We evaluate our framework on three math reasoning datasets: (i) **GSM8K** (Cobbe et al., 2021), an English dataset of grade school math word problems (about 7,500 for training and 1,319 for testing); (ii) **MGSM** (Shi et al., 2023a), consisting of 250 questions selected from GSM8K and manually translated into ten languages: Bengali (BN), Chinese (ZH), French (FR), German (DE), Japanese (JA), Russian (RU), Spanish (ES), Swahili (SW), Telugu (TE) and Thai (TH). Thus, it contains a total of 11 languages including English; (iii) **MATH500** is a benchmark of competition, which contains 500 math problems of varying difficulty.

# A.3 COC Results

Orders (4-shot)	1	2	3	4	5	6	7	8	COC
7-8B									
Phi-3-7B	88.5	88.8	88.6	88.9	87.3	89.8	88.4	88.2	89.8
Qwen2.5-7B	88.3	87.0	87.6	88.0	88.0	87.8	88.5	88.2	90.5
Qwen2.5-Math-7B	90.0	89.7	91.3	92.3	91.5	90.5	90.8	91.0	92.6
Llama-3.1-8B	79.7	78.8	79.0	79.0	80.1	79.1	77.8	78.7	80.1
Aya-expanse-8B	76.7	77.3	78.2	76.6	76.9	77.6	77.3	77.4	78.2
Ministral-8B	81.2	81.4	80.9	81.4	80.9	81.3	81.4	81.6	82.3
14-32B									
Phi-3-14B	89.2	88.9	88.6	88.3	88.9	89.0	89.1	88.9	89.9
Mistral-22B	85.8	85.8	85.9	85.3	86.1	86.1	86.1	86.2	86.7
Aya-expanse-32B	83.8	83.4	84.6	83.6	84.9	84.3	83.5	83.9	85.3
70-72B									
Llama-3.1-70B	94.0	94.0	94.3	94.1	93.9	93.9	93.4	93.8	94.4
Qwen2.5-72B	94.6	93.9	93.9	93.6	94.1	94.1	94.2	94.2	94.8
Qwen2.5-Math-72B	94.0	93.6	93.6	93.7	93.8	93.6	93.2	93.4	94.2

Table 6: Reasoning accuracy of prompts in different orders on GSM8K compared to COC.

Orders (4-shot)	1	2	3	4	5	6	7	8	COC
7-8B									
Phi-3-7B	40.4	41.0	40.0	40.8	39.0	42.2	40.0	38.6	43.2
Qwen2.5-7B	63.6	64.6	64.8	64.4	63.6	65.2	61.8	62.4	68.8
Qwen2.5-Math-7B	75.2	74.4	75.2	75.6	73.2	75.4	74.6	74.2	77.0
Llama-3.1-8B	42.6	44.0	41.8	41.4	42.4	41.6	45.0	42.6	48.0
Aya-expanse-8B	28.0	28.8	29.0	27.4	28.6	28.8	27.0	29.0	31.4
Ministral-8B	42.8	42.2	40.0	41.4	42.0	41.2	41.8	41.2	45.0
14-32B									
Phi-3-14B	44.2	43.0	44.2	45.8	44.4	46.6	44.0	44.4	49.0
Mistral-22B	42.6	47.2	43.0	44.0	44.0	44.6	41.6	45.6	48.2
Aya-expanse-32B	38.6	40.0	39.4	38.4	40.4	40.0	40.4	40.6	43.4
70-72B									
Llama-3.1-70B	58.6	57.8	60.0	59.6	59.0	60.2	60.2	61.4	65.0
Qwen2.5-72B	63.2	65.2	62.2	59.6	67.2	61.4	69.8	68.6	75.2
Qwen2.5-Math-72B	66.6	64.6	70.2	72.2	68.8	69.6	72.6	69.8	76.0

Table 7: Reasoning accuracy of prompts in different orders on MATH500 compared to COC.

A.4 Examples of questions written in different languages	733
Figure 9 provides examples of questions written in different languages.	734
A.5 Models	735
We select a range of open-source state-of-the-art LLMs in varying scales. For all models, we only conside instruction-tuned versions.	ler 736 737
<b>7-8B:</b> Phi-3-7B (128k) (Abdin et al., 2024); Qwen2.5-7B (Yang et al., 2024a); Qwen2.5-7B-Math (Yang et al., 2024b); Llama-3.1-8B (Grattafiori et al., 2024); Aya-expanse-8B (Üstün et al., 2024); Ministral-8B	U

Phttps://huggingface.co/mistralai/Ministral-8B-Instruct-2410

Language	Math Question
EN	On Mondays, Wednesdays, and Fridays, college student Kimo has three 1-hour classes each day. On Tuesdays and Thursdays, he has two 2-hour classes each day. In one semester, there are 16 weeks of school. In one semester, how many hours does Kimo spend attending classes?
BN	কলেজ ছাত্র কিমোর প্রতি সোমবার, বুধবার ও শুক্রবারে তিনটি । শ্রন্টার ক্লাস থাকে। প্রতি মঙ্গলবার ও বৃহস্পতিবারে, তার দুটি 2-ঘন্টার ক্লাস থাকে। একটি সেমিস্টার 16 সস্তাহের স্কুল থাকে। একটি সেমিস্টারে, কিমোকে ক্লাসে কত ঘন্টা ব্যয় করতে হয়?
DE	Montags, mittwochs und freitags hat College-Student Kimo drei 1-st\u00fcndige Kurse pro Tag. Dienstags und Donnerstags hat er zwei 2-st\u00fcndige Kurse pro Tag. Ein Semester hat 16 Schulwochen. Wie viele Stunden verbringt Kimo in einem Semester mit Kursbesuchen?
ES	Los lunes, miércoles y viernes, el estudiante universitario Kimo tiene tres clases de 1 hora por día. Los martes y jueves, tiene dos clases de 2 horas por día. En un semestre, hay 16 semanas de clases. En un semestre, ¿cuántas horas pasa Kimo en clases?
FR	Les lundis, mercredis et vendredis, l'étudiant Kimo a trois cours d'une heure par jour. Les mardis et jeudis, il a deux cours de 2 heures chaque jour. S'il y a 16 semaines d'école par semestre, combien d'heures Kimo consacre-t-il à ses cours durant un semestre ?
JA	月曜、水曜、金曜に、大学生のキモは1時間の3つの講義を各日受ける。火曜と木曜に、彼は2時間の講義を2つ各日受ける。1セメスターで、学校は16週間ある。 1セメスターで、キモは何時間講義に出席して過ごす?
RU	Каждый понедельник, среду и пятницу у студента колледжа Кимо три 1-часовых занятия. Каждый вторник и четверг у него два 2-часовых занятия. В одном семестре 16 учебных недель. Сколько часов Кимо проводит на занятиях за один семестр?
SW	Siku za Jumatatu, Jumatano, na Ijumaa, Kimo mwanafunzi wa chuo huwa na masomo matatu ya saa 1 kila siku. Siku za Jumanne na Alhamisi, huwa ana masomo mawili ya saa 2 kila siku. Katika muhula moja, kuna wiki 16 za shule. Katika muhula moja, Kimo huwa anatumia saa ngapi kuhudhuria masomo?
TE	నోమవారాలు, బుదవారాలు, మరియు శుక్రవారాల నాడు, కాలేజీ విద్యార్థి కిమోకు ప్రతిరోజు మూడు 1-గంట క్లాసులు ఉన్నాయి. మంగళవారం మరియు గురువారాల్లో, అతడికి ప్రతిరోజు రెండు 2-గంటల క్లాసులున్నాయి. ఒక సమిస్టర్లో, స్కూలులో 16 వారాలు ఉన్నాయి. ఒక సమిస్టర్లో, కిమో క్లాసులకు హాజరు కావడానికి ఎన్ని గంటలు గడిపాడు?
ТН	ในวันจันทร์ วันพุธ และวันศูกร์ ได้ไมซึ่งเป็นนักศึกษาข้าธียนสามศาบค่อวัน โดยเด่ละคาบใช้เวลา 1 ชั่วไมง ส่วนในวันอังคาธและวันพฤหัสบดี ตกดักรียนสองคาบต่อวัน ซึ่งเด่ละคาบใช้เวลา 2 ชั่วไมง ในหนึ่ง ภาพกรศึกษามีการอิยนการสอน 16 สัปดห์ ไดโนใช้เวลากี่ชั่วไมงไปกับการเจ้าธียน
ZH	周一、周三和周五,大学生基莫每天有3节1小时的课。周二和周四,他每天有2节2小时的课。一个学期中有16周的上学时间。在一个学期中,基莫用多少时间上课?

Figure 9: Examples of questions written in different languages.

**14-32B:** Phi-3-14B (Abdin et al., 2024); Mistral-22B<sup>6</sup>; Aya-expanse-32B (Üstün et al., 2024).

**70-72B:** Qwen2.5-72B (Yang et al., 2024a); Qwen2.5-72B-Math (Yang et al., 2024b); Llama-3.1-70B (Grattafiori et al., 2024).

# A.6 Implementation

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We perform inference experiments on  $4 \times \text{NVIDIA H100 94GB GPUs}$  using the library vLLM (Kwon et al., 2023), without training or fine-tuning language models. During inference, we use few-shot prompts covering the 11 languages released by Shi et al. (2023a). In the multilingual scenario, we use 4-shot for all languages except TE which only uses 2-shot, since a 4-shot prompt would exceed the default maximum length, due to tokenization issues unfavourable to this language (Ahia et al., 2023). We use greedy decoding unless otherwise specified. For all experiments we report the final answer accuracy except the reasoning consistency score.

## A.7 Correlation

Figure 10 shows the correlations of models' accuracy with the three consistency scores. COC and CPC have high correlations with reasoning accuracy, while CLC has a weak and non-significant one. This suggests that we can use COC and CPC to assess the model's uncertainty in its generated solutions without using gold answers. While CLC does not seem to be a reasonable metric to assess models' accuracy, it can still be used to evaluate models from a multilingual perspective.

<sup>&</sup>lt;sup>6</sup>https://huggingface.co/mistralai/Mistral-Small-Instruct-2409



Figure 10: Pearson correlation between models' accuracy and different consistency scores.