SATBench: Benchmarking LLMs' Logical Reasoning via Automated Puzzle Generation from SAT Formulas

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Abstract

We introduce SATBench, a benchmark for evaluating the logical reasoning capabilities of large language models (LLMs) through logical puzzles derived from Boolean satisfiability (SAT) problems. Unlike prior work that focuses on inference rule-based reasoning, which often involves deducing conclusions from a set of premises, our approach leverages the search-based nature of SAT problems, where the objective is to find a solution that fulfills a specified set of logical constraints. Each instance in SATBench is generated from a SAT formula, then translated into a story context and conditions using LLMs. The generation process is fully automated and allows for adjustable difficulty by varying the number of clauses. All 2100 puzzles are validated through both LLM-assisted and solver-based consistency checks, with human validation on a subset. Experimental results show that even the strongest model, o4-mini, achieves only 65.0% accuracy on hard UNSAT problems, close to the random baseline of 50%. SATBench exposes fundamental limitations in the search-based logical reasoning abilities of current LLMs and provides a scalable testbed for future research in logical reasoning.

1 Introduction and Method

Logical reasoning is a fundamental component of human intelligence and continues to be a significant challenge in the field of artificial intelligence. The growing interest in the reasoning capabilities of large language models (LLMs) highlights the pressing need for robust benchmarks and evaluation methods [1]. While many datasets have been proposed to evaluate logical reasoning capabilties of LLMs, earlier datasets do not exclusively evaluates logical reasoning in isolation, e.g., LogiQA [2], and ReClor [3], which combine logical reasoning with commonsense reasoning.

Recently, new datasets have been introduced to assess logical reasoning in isolation, such as FOLIO [4] and P-FOLIO [5]. These datasets are manually curated by researchers and focus on logical problems based on *inference rules*, which involve deriving conclusions from a set of premises. A more comprehensive review of related work is provided in Appendix A.

In this work, we introduce SATBench, a benchmark designed to create logical puzzles from Boolean satisfiability (SAT) problems [6, 7] with LLMs. Unlike benchmarks based on inference rules, SAT problems are characterized as *search-based* logical reasoning tasks, where the objective is to determine a truth assignment that fulfills a specified set of logical constraints [8]. This approach to logical reasoning emphasizes a search process akin to backtracking used in SAT solvers. Unlike other search-based benchmarks such as ZebraLogic [9], which presuppose the existence of a valid solution, SAT problems can result in either a satisfiable solution (SAT) or no solution (UNSAT).

As shown in Figure 1, starting from a SAT formula in Conjunctive Normal Form (CNF), such as $(A \vee \neg B) \wedge (\neg C \vee \neg D)$, our framework uses LLMs to generate a story context and define a mapping between formula variables and entities in the story. Each clause is then translated into a natural

Submitted to 39th Conference on Neural Information Processing Systems (NeurIPS 2025). Do not distribute.

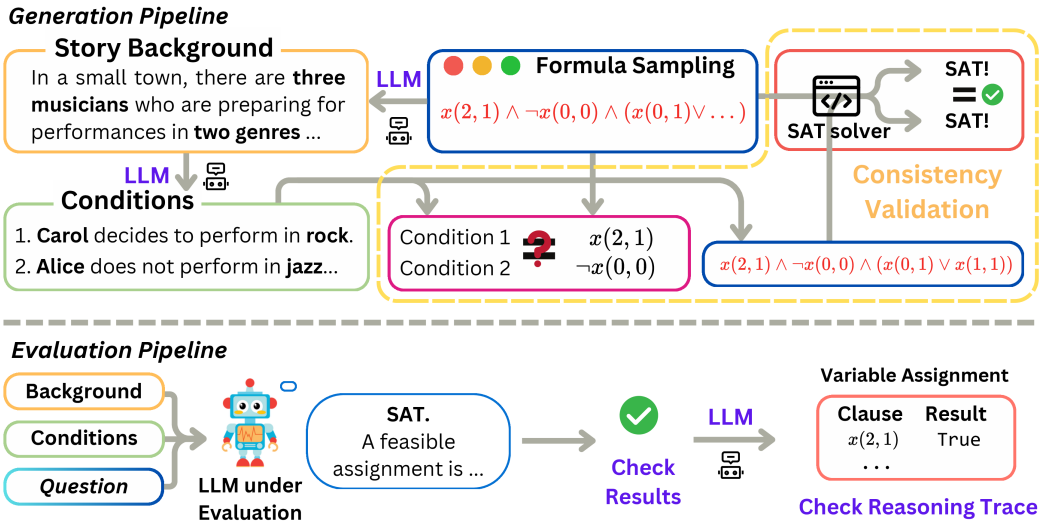

Figure 1: **Overview of the SATBench methodology.** The generation pipeline begins with sampling Conjunctive Normal Form (CNF) formulas, followed by LLM-driven creation of story backgrounds and conditions. To ensure the logical puzzle's quality, both LLM-assisted and solver-based consistency validations are employed. The evaluation pipeline then examines the puzzle's prediction outcomes and checks its reasoning process.

language condition based on this mapping. By sampling CNF formulas with varying numbers of clauses, we can control puzzle difficulty. To ensure the quality of resulting logical puzzles, we reverse the generation process: LLMs translate the natural language conditions back into logical formulas, which are then compared to the originals using a combination of LLM-assisted and solver-based consistency checks. In the evaluation pipeline, we check the result and employ the LLM-as-a-judge strategy to assess the reasoning trace. To validate the overall process of story generation and reasoning trace evaluation, we manually validate 100 examples, increasing confidence in the quality of resulting dataset and evaluation protocol. The detailed construction procedure is provided in Appendix B.

The evaluation on our generated 2100 logical puzzle dataset shows that reasoning models perform well on SATBench, with the o4-mini model achieving the highest accuracy. However, as the number of conditions in the puzzles increases, performance drops noticeably. For the hard UNSAT subset, o4-mini reaches an average accuracy of 65.0%, only slightly above the 50% random baseline. Models perform better on SAT than on UNSAT problems, as UNSAT often requires exhaustive search through the entire solution space. Interestingly, reasoning traces are less reliable for SAT than for UNSAT problems. These results show that SATBench can reveal the limitations of current LLMs in logical reasoning.

Metric	Value
Number of Instances	2100
Average Number of Variables	35.8
Average Number of Clauses	20.6
Average Number of Words	554.9
Average Number of Sentences	55.5

Table 1: Dataset statistics for SATBench.

2 Experiments

2.1 Experimental Setup

Dataset and Prompts. The SATBench dataset contains 2100 logical puzzle instances. Table 1 reports statistics on the average number of Boolean variables and clauses in the sampled SAT formulas, as well as the average number of words and sentences in the generated puzzles. The fully automated generation process allows creating more instances if needed. For evaluation, we use 0-shot prompts consisting of a story background, a set of conditions that must be satisfied simultaneously, and a satisfiability query. Models must output a reasoning trace and a final label (`SAT` or `UNSAT`); for SAT,

Model	SAT			UNSAT			Overall			Avg.
	Easy	Medium	Hard	Easy	Medium	Hard	Easy	Medium	Hard	
Random Baseline	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*	*50.0*
Llama3.1-8B	57.7	59.5	51.4	30.4	14.8	17.5	44.0	37.1	34.5	38.5
DeepSeek-Distill-7B	63.7	29.0	20.4	69.1	43.8	42.1	66.4	36.4	31.2	44.7
Mixtral-8x7B	61.1	54.3	67.1	44.8	33.3	31.8	52.9	43.8	49.5	48.7
gpt-4o-mini	82.1	81.9	89.3	42.3	12.9	13.2	62.2	47.4	51.3	53.6
Qwen3-1.7B	77.3	68.1	52.5	53.4	30.5	42.5	65.4	49.3	47.5	54.0
Mixtral-8x22B	71.8	64.8	61.1	40.0	40.5	45.7	55.9	52.6	53.4	54.0
Llama4-Scout	84.3	77.6	68.9	52.0	24.3	37.5	68.1	51.0	53.2	57.4
Llama3.1-70B	81.8	55.2	47.9	55.2	59.0	48.9	68.5	57.1	48.4	58.0
gpt-4o	85.5	82.4	81.1	54.3	27.1	18.9	69.9	54.8	50.0	58.2
Llama3.3-70B	90.7	88.1	77.1	39.5	27.1	30.0	65.1	57.6	53.6	58.8
DeepSeek-Distill-14B	82.9	52.9	42.4	85.7	59.0	51.8	84.3	56.0	47.1	62.4
Llama4-Maverick	80.0	87.1	87.5	76.8	25.7	17.9	78.4	56.4	52.7	62.5
Qwen3-4B	84.1	78.1	79.3	80.7	31.9	22.1	82.4	55.0	50.7	62.7
Qwen3-8B	82.7	78.6	69.6	81.6	34.8	32.1	82.1	56.7	50.9	63.2
Qwen3-14B	87.1	73.3	78.6	88.9	47.6	22.1	88.0	60.5	50.4	66.3
DeepSeek-Distill-32B	84.5	54.3	43.9	90.0	68.1	58.6	87.2	61.2	51.2	66.6
Qwen3-235B-Int8	90.0	84.3	85.4	86.1	46.2	19.6	88.0	65.2	52.5	68.6
Qwen-QwQ-32B	92.5	77.1	62.1	84.1	51.9	46.4	88.3	64.5	54.3	69.0
Claude-3.7-Sonnet	88.4	78.1	82.5	93.8	63.3	42.1	91.1	70.7	62.3	74.7
DeepSeek-V3	93.6	85.2	70.4	97.5	83.3	74.3	95.5	84.3	72.3	84.0
DeepSeek-R1	94.8	87.6	71.4	**98.2**	**89.5**	**83.6**	96.5	88.6	77.5	87.5
o4-mini	**97.0**	**97.1**	**91.1**	98.2	88.1	65.0	**97.6**	**92.6**	**78.0**	**89.4**
Average	82.4	72.5	67.3	70.1	45.6	39.3	76.3	59.0	53.3	62.9

Table 2: Model accuracy on SATBench using zero-shot prompting for satisfiability prediction. Difficulty levels are categorized as follows: Easy (4-19 clauses), Medium (20-30 clauses), and Hard (31-50 clauses). All open-source models are instruction-tuned.

they provide a satisfying assignment, and for UNSAT, an explanation of the conflict. Detailed prompt templates are given in Appendices C.1 and C.2.

Metrics and Models. Satisfiability is framed as a binary classification task (50% random baseline), with accuracy as the main metric. Reasoning trace correctness is judged only when the predicted label is correct, using GPT-4o to verify that the explanation supports the outcome (Appendix B.4). We evaluate proprietary models (GPT-4o, GPT-4o-mini, o4-mini, Claude 3.7 Sonnet) and recent open-source models from the Qwen, Llama, and DeepSeek families. Reasoning trace evaluation focuses on the five best-performing models, with GPT-4o as the judge.

2.2 Main Results

Reasoning models excel, but struggle on harder problems. Table 2 shows zero-shot satisfiability prediction results on SATBench. The best-performing model, o4-mini, reaches 89.4% accuracy, followed by DeepSeek-R1 (87.5%), DeepSeek-V3 (84.0%), Claude 3.7 Sonnet (74.7%), and Qwen-QwQ-32B (69.0%). However, performance drops as difficulty increases: on Hard instances (31–50 clauses), o4-mini falls to 78.0%, and the average accuracy across models is 53.3%, close to the random baseline. Detailed difficulty analysis is provided in Section 2.3.

SATBench is a challenging benchmark. For the Hard instances, even the state-of-the-art model o4-mini only achieves 78.0% accuracy, only a moderate improvement over the 50% random baseline. For the UNSAT instance, its accuracy is only 65.0%, leaving significant room for improvement.

Scaling Trends. Figure 2 illustrates the scaling trends observed across various model families, such as Qwen3, Llama3.1, Mixtral, Llama4, and DeepSeek-Distill-Qwen. In each family, an increase in model size consistently leads to improved accuracy in satisfiability prediction, thereby validating the anticipated scaling behavior.

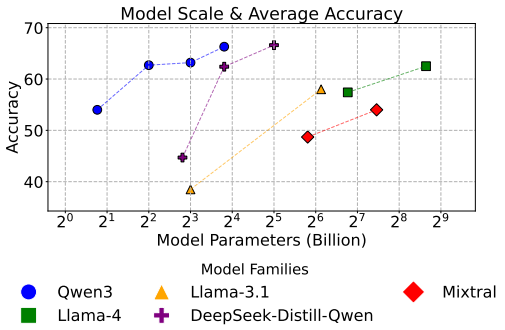

Figure 2: Scaling trend on SATBench.

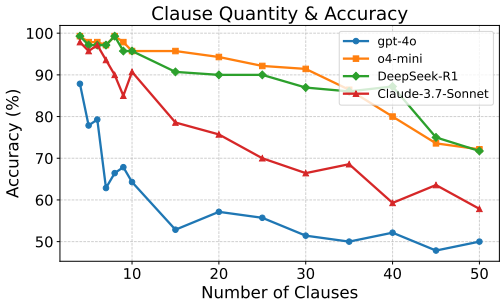

Figure 3: Impact of clause quantity on accuracy.

2.3 Analysis of Difficulty

SAT versus UNSAT. The "average" row in Table 2 shows a clear accuracy gap between SAT and UNSAT subsets. On Hard instances, models average 67.3% for SAT but only 39.3% for UNSAT, indicating that SAT is generally easier. This likely stems from the higher complexity of UNSAT problems, which require checking all 2^n assignments for n variables to prove unsatisfiability, while SAT only needs one valid assignment.

Impact of Clause Quantity. Figure 3 shows an inverse relationship between clause count and accuracy. For example, GPT-4o's accuracy drops sharply toward the 50% random baseline as clause count approaches 30. This trend suggests that more clauses increase problem complexity, confirming that our dataset generation method can effectively control difficulty.

2.4 Reasoning Trace Evaluation

We evaluate the reasoning trace validity of various models with GPT-4o, and results are shown in Table 3. The table highlights that DeepSeek-R1 leads in overall trace accuracy with a score of 76.2%, surpassing the o4-mini model by 2.5%. This indicates that while o4-mini excels in prediction accuracy, DeepSeek-R1 provides more reliable reasoning traces.

Model	SAT		UNSAT		Overall
	Pred.	Trace	Pred.	Trace	Trace
QwQ	76.9	51.9	60.7	52.4	52.2
Claude-3.7	83.0	46.1	66.4	61.1	53.6
DS-V3	83.1	63.3	85.0	71.1	67.2
o4-mini	95.0	73.2	83.6	74.1	73.7
DS-R1	84.5	70.3	90.3	82.1	76.2

Table 3: Accuracy in prediction and reasoning trace evaluation.

A notable observation is the disparity in trace accuracy between the SAT and UNSAT subsets. Models generally exhibit a more pronounced drop in trace accuracy on SAT problems compared to UNSAT ones. For example, Claude-3.7 experiences a significant 36.9% decrease in trace accuracy on SAT instances, whereas the drop is only 5.3% on UNSAT instances. This pattern indicates that a model's higher prediction accuracy on SAT problems does not necessarily imply it has identified a valid variable assignment. Instead, models exhibit a bias towards predicting SAT outcomes without verifying a valid assignment as evidence.

3 Conclusion

We present SATBench, a benchmark for assessing LLMs' logical reasoning via SAT-derived puzzles. Our dataset features search-based logical reasoning tasks, with controls difficulty and correctness checked by solvers and LLMs. SATBench contains 2100 logical puzzles and we evaluate both satisfiability prediction and reasoning trace validity. Our findings show model performance drops with increased difficulty, with o4-mini scoring 65.0% on hard UNSAT cases, near the 50% random baseline. This indicates current LLMs struggle with search-based logical reasoning, especially for UNSAT problems. SATBench offers a scalable testbed to future research in logical reasoning.

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Appendix

A Related Work

Benchmark	Search-Based	Logic Isolation	Automated Generation	Difficulty Control	Natural Language	Template-Free	Reasoning Evaluation
LogiQA [2]	✗	✗	✗	✗	✓	✓	✗
BIG-bench [10]	✗	✗	✗	✗	✓	✓	✗
ReClor [3]	✗	✗	✗	✗	✓	✓	✗
RuleTaker [11]	✗	✓	✓	✓	✓	✗	✗
LogicNLI [12]	✗	✓	✓	✗	✓	✓	✗
FOLIO [4]	✗	✓	✗	✗	✓	✓	✗
P-FOLIO [5]	✗	✓	✗	✗	✓	✓	✓
LogicPro [13]	✗	✗	✓	✗	✓	✓	✓
ZebraLogic [9]	✓	✓	✓	✓	✓	✗	✗
AutoLogi [14]	✓	✓	✓	✓	✓	✓	✗
PARAT [7]	✓	✓	✓	✓	✗	✓	✓
LogicBench [15]	✗	✓	✓	✗	✓	✗	✗
LogicAsker [16]	✗	✓	✓	✗	✓	✗	✗
Unigram-FOL [17]	✗	✓	✓	✗	✓	✗	✗
Multi-LogiEval [18]	✗	✓	✓	✓	✓	✗	✗
SATBench (ours)	✓	✓	✓	✓	✓	✓	✓

Table A1: **Comparison of existing logical reasoning benchmarks.** An ideal evaluation framework should meet the following five critera — (1) Logic Isolation: the benchmark exclusively evaluates logical reasoning in isolation; (2) Automated Generation: the benchmark construction is automated and scalable; (3) Difficulty Control: the difficulty levels of the benchmark questions are adjustable; (4) Natural Language: the questions are written in natural language rather than formal formulas; (5) Template-Free: the benchmark does not rely on expert-designed templates, enhancing diversity; (6) Reasoning Evaluation: the benchmark evaluates both the accuracy of model predictions and the correctness of their reasoning traces.

Logical Reasoning Benchmarks for LLMs Reasoning is a longstanding focus in NLP, with many benchmarks developed to assess model performance. Early efforts targeted natural language inference [19] and commonsense reasoning [20], while recently there has been increasing attention to assessing logical reasoning, as seen in LogiQA [2], ReClor [3], BoardgameQA [21], and CLUTRR [22]. These typically involve reasoning that relies on real-world knowledge. In contrast, datasets like FOLIO [4], RuleTaker [11], and P-FOLIO [5] aim to isolate formal logical reasoning from commonsense knowledge. Logical puzzles have emerged as a compelling testbed in this area [23], with benchmarks including ZebraLogic [9], AutoLogi [14], and LogicNLI [12]. Our work builds on this line by proposing satisfiability-based puzzles [8, 7] for evaluating logical reasoning, using fully automated generation and solver-verified answers. To effectively benchmark the logical reasoning capabilities of LLMs, we propose that an ideal evaluation framework should meet the five criteria, as illustrated in Table A1, while previous works address some of these aspects, few manage to fulfill all these criteria simultaneously.

Logical Reasoning with Language Models Recent work investigates how large language models engage in logical reasoning via prompting techniques, supervised training on reasoning datasets, and translation into formal logic. A prominent line of research focuses on prompting methods that elicit step-by-step reasoning, including chain-of-thought prompting [24], tree-of-thought prompting [25], and self-improvement via bootstrapping [26], along with other methods [27, 28]. Another approach involves fine-tuning LLMs on datasets specifically designed for logical reasoning [29, 30, 1, 31], which has demonstrated improved performance on formal reasoning benchmarks. Complementary to these methods, some work treats LLMs as semantic parsers that convert natural language reasoning tasks into formal logical representations, which are then executed or verified by external solvers or theorem provers [32, 33]. In our evaluation, we use chain-of-thought prompting and prohibit models from invoking external tools; solvers are used only during dataset generation for correctness validation.

B Method

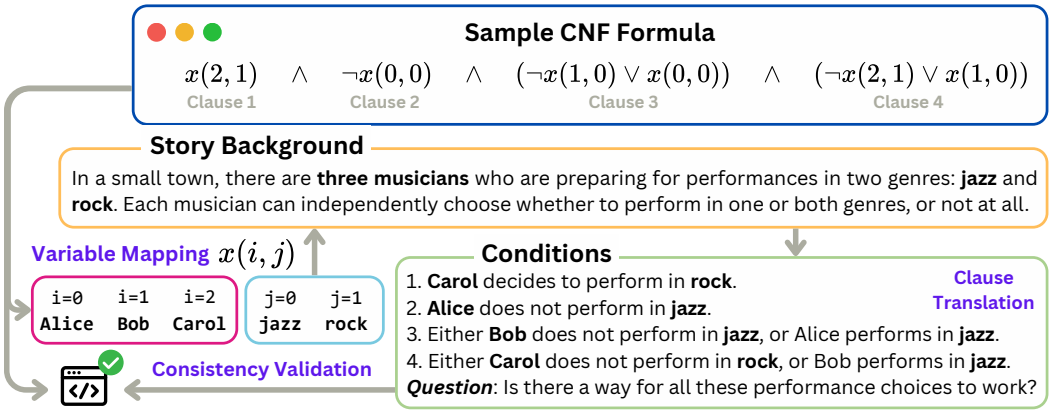

Figure A1: **Benchmark curation pipeline.** The process starts with sampling SAT formulas, followed by using a LLM to generate variable mappings and a story background. Clauses in the formula are then translated into narrative conditions. Consistency between the original formula and the generated puzzle is ensured through both LLM-based and solver-based validation.

Our objective is to create logical puzzles derived from Boolean satisfiability (SAT) formulas, ensuring the quality of the dataset through both LLM-based and solver-based consistency checks. We further validate each LLM-involved process with human review. The generation method is divided into three stages: SAT formula sampling (Appendix B.1), LLM-based story generation (Appendix B.2), and consistency validation (Appendix B.3). In the evaluation phase, we assess the correctness of the reasoning trace (Appendix B.4).

B.1 SAT formula Sampling

Conjunctive Normal Form (CNF) Conjunctive Normal Form is a structured way of expressing logical formulas, where a formula is a conjunction (AND) of one or more disjunctions (OR) of literals. Each disjunction is referred to as a clause, and each clause consists of literals, which can be either a variable or its negation. For instance, the formula $(x(2,1)) \wedge (\neg x(1,0) \vee x(0,0)) \wedge (\neg x(0,0)) \wedge (\neg x(2,1) \vee x(1,0))$ is in CNF. Here, x represents a two-dimensional array with boolean elements, indicating true or false values. The SAT problem expressed in CNF form involves determining whether there exists an assignment of boolean values to the variables that satisfies the entire formula, making it true. If such an assignment exists, the formula is satisfiable. Conversely, if no such assignment can be found, the formula is unsatisfiable, and an UNSAT-Core can be identified, which is a subset of clauses that are inherently unsatisfiable. This approach constructs puzzles that challenge LLMs to determine if all conditions can be satisfied.

Automation and Difficulty Control The SAT problem can be efficiently solved using a SAT solver, which provides a soundness guarantee and allows for automated and scalable solution. To systematically generate problems with varying levels of difficulty, we can sample formulas that differ in the number of boolean variables and clauses. Additionally, we can increase the dimensionality of the array to create more complex story contexts. By increasing the number of boolean variables, we can generate more clauses to be translated into story conditions. This approach effectively controls the difficulty level by expanding the search space and adding complexity to the constraints, making the search-based logical reasoning more challenging.

B.2 Puzzle Story Generation

Background and Variable Mapping To transform the sampled SAT formula into a narrative context, we utilize a language model, such as GPT-4o, to generate a story background and establish a mapping of variables. For example, as shown in Figure A1, given the SAT formula, the language model creates a scenario involving three musicians: Alice, Bob, and Carol. These musicians

are deciding on their performances in two musical genres, jazz and rock. Each musician can independently choose whether to perform in one or both genres, or not at all. The musicians and the genres correspond to the two dimensions of the array x. This mapping is defined as:

$$x(i, j) \rightarrow \text{“musician } i \text{ performs in genre } j\text{”}$$

For example:

- $x(0, 0)$: Alice performs in jazz
- $x(1, 0)$: Bob performs in jazz
- $x(2, 1)$: Carol performs in rock

Clause-to-Condition Mapping To transform each clause of the CNF formula into a narrative condition, we employ a large language model (e.g., GPT-4o). This transformation leverages the previously established story background and variable mapping. For example, the clause $\neg x(0, 0)$ is translated to the condition “Alice does not perform in jazz,” while the clause $\neg x(2, 1) \vee x(1, 0)$ is expressed as “Either Carol does not perform in rock, or Bob performs in jazz.” The final puzzle integrates the story background with these translated conditions and concludes with a question like “Is there a way for all these performance choices to work?” This question serves to assess the satisfiability of the conditions in the logical puzzle.

Our two-phase generation strategy, which begins with the creation of the story background and variable mapping, followed by the transformation of clauses into narrative conditions, improves the tractability and reliability of the process. This structured approach facilitates easier debugging and human validation.

B.3 Consistency Validation

LLM-based Validation We utilize a large language model (GPT-4o) to ensure that each condition in the generated logical puzzle precisely matches the original SAT formula, given the specified variable mapping. This process checks that no extra conditions are introduced and none are missing. If the check fails, the puzzle is removed from our dataset.

Solver-based Validation In addition to LLM-based validation, we implement a solver-based validation process. The process begins by using an LLM to convert the narrative conditions back into a SAT formula given the variable mappings. This reconstructed SAT formula is then evaluated by a SAT solver to determine its satisfiability status (SAT or UNSAT). We then cross-verify this result with the satisfiability status of the original CNF formula from which the puzzle was derived. Any inconsistencies between these results lead to the exclusion of the puzzle from our dataset, thereby maintaining the integrity and reliability of our generated benchmark.

Human Validation To ensure the quality of our dataset, we conduct human validation at two crucial stages involving LLMs, as detailed in Appendix B.2. The first stage involves the generation of the puzzle’s background and variable mapping, where human assess the logical coherence and confirm that the story background accurately reflects the independence of boolean variables. The second stage focuses on the translation of clauses into narrative conditions, where human ensure that no additional constraints or misinterpretations are introduced.

B.4 Reasoning Trace Evaluation

After generating the logical puzzle dataset, we evaluate an LLM’s performance using this dataset. Our evaluation emphasizes both the binary prediction result (SAT or UNSAT) and the validity of the model’s reasoning trace. We adopt an LLM-as-a-judge methodology, where the model is instructed to produce a reasoning trace to justify its prediction. Below, we detail the approach for assessing the reasoning trace in SAT and UNSAT scenarios.

SAT Problems When a problem is identified as SAT, it indicates that there is at least one assignment of True or False values to the variables that satisfies the CNF formula. Multiple solutions may exist. For example, consider the CNF formula $(x(0, 0) \vee \neg x(1, 0)) \wedge (x(1, 0) \vee x(2, 1))$. One possible

satisfying assignment is $x(0,0) = \text{True}$, $x(1,0) = \text{False}$, and $x(2,1) = \text{True}$. After the model predicts a problem as SAT, it is required to generate a reasoning trace to support its prediction. We then instruct the judging LLM to translate this reasoning into a specific variable assignment using the given variable mapping. The judging LLM is further used to verify that each clause in the SAT formula evaluates to True, thereby confirming the satisfiability of the entire SAT formula.

UNSAT Problems Unlike SAT problems, UNSAT problems have no variable assignment that satisfies all clauses. A SAT solver can identify an UNSAT-Core, which is a minimal subset of unsatisfiable clauses. When the model predicts UNSAT, it must provide a reasoning trace.

Consider the formula: $(x(2,1)) \wedge (\neg x(1,0) \vee x(0,0)) \wedge (\neg x(0,0)) \wedge (\neg x(2,1) \vee x(1,0))$. We can demonstrate its unsatisfiability through a step-by-step analysis:

1. From the first clause, $x(2,1)$, we must set $x(2,1)$ to true.
2. From the third clause, $\neg x(0,0)$, we must set $x(0,0)$ to false.
3. Given that $x(0,0)$ is false, the second clause, $\neg x(1,0) \vee x(0,0)$, can only be satisfied if $\neg x(1,0)$ is true, suggesting $x(1,0)$ is false.
4. However, since $x(2,1)$ is true, the fourth clause, $\neg x(2,1) \vee x(1,0)$, can only be satisfied if $x(1,0)$ is true.

This results in a irreconcilable contradiction: $x(1,0)$ is required to be both true and false simultaneously to satisfy all clauses, rendering the formula unsatisfiable. The example above illustrates a valid reasoning trace for an UNSAT problem in formula format. However, since the model being evaluated lacks access to the variable mapping during its reasoning trace generation, the judging LLM must first translate the reasoning trace back into the variable format. It then compares this translated reasoning with the provided UNSAT-Core to assess the accuracy of the reasoning trace.

Human Validation Given our use of an LLM-as-a-judge methodology for evaluating reasoning traces, we incorporate a human validation process to check the correctness of the LLM's judgments.

We selected a sample of 100 examples from the dataset for human validation, focusing on three critical stages where LLMs are utilized, to enhance confidence in the dataset's quality and the evaluation protocol. Each LLM-based process was manually assessed for correctness. The first two stages involve puzzle generation, as described in Appendix B.2: 1) ensuring the generated puzzle background and variable mapping accurately represent the sampled CNF formula, where we achieved 100% accuracy; 2) confirming the precise translation of each clause into its corresponding condition, with a 97% accuracy rate. The third stage involves assessing the accuracy of the LLM's evaluation of the reasoning trace, achieving 93% accuracy. These numbers indicate that our dataset quality and evaluation pipeline are robust and reliable.

Nonetheless, a few failure cases were observed. In story generation, one error involved the clause $(\neg x(2,0) \vee x(2,1))$ being translated as "If Dr. Brown is not assigned project 0, then Dr. Brown is assigned project 1." This misuses the *if-then* structure: the negation should be removed from the antecedent. The correct phrasing should be "If Dr. Brown is assigned project 0, then Dr. Brown is also assigned project 1."

For the LLM-as-judge setting, the main error mode involved incomplete extraction of the assignment within the trace. In some cases, the model judged that the trace was invalid, even though the trace was logically sound. These minor errors, however, were rare and did not affect the overall robustness of our pipeline.

C Prompt Template

C.1 SATBench Evaluation Prompt Template

Prompt Template

```
You are a logical reasoning assistant. You are given a logic puzzle.

<scenario>
{scenario}

<conditions>
{conditions}

<question>
{question}

Guidelines:
- All constraints come **only** from the <conditions> section.
- The <scenario> provides background and intuition, but **does not
    impose any additional rules or constraints**.
- All variables represent **independent decisions**; there is no
    mutual exclusivity or implicit linkage unless stated explicitly
    in <conditions>.
- Variables not mentioned in <conditions> are considered unknown and
     irrelevant to satisfiability.

Your task:
- If the puzzle is satisfiable, propose one valid assignment that
    satisfies all the conditions.
- If the puzzle is unsatisfiable, explain why some of the conditions
     cannot all be true at once.

Think step by step. At the end of your answer, output exactly one of
     the following labels on a new line:
[SAT] - if a valid assignment exists
[UNSAT] - if the constraints cannot be satisfied

Do not add any text or formatting after the final label.
```

C.2 Trace Evaluation Prompt Template

Trace Evaluation Prompt Template for SAT Prediction

```
You are given a logical puzzle and a reasoning trace from a language
     model.

The puzzle is also expressed as a CNF (Conjunctive Normal Form)
    formula. Each clause is a disjunction (OR) of literals formatted
     like x(i,), x(i,j), or x(i,j,k). These variables follow the
    meaning:

- x(i,) means object or person i has some unnamed property.
- x(i,j) means object i has property or role j.
- x(i,j,k) means object i has property j in context or slot k (e.g.,
     time, situation, location).

A positive literal like x(0,1) means that property is present.
A negative literal like \neg x(0,1) means it is absent.
```

```
Below is the full logical puzzle and its corresponding formula:

<scenario>
{scenario}

<conditions>
{conditions}

<final question>
{question}

<variable explanation>
{variable_mapping}

<readable CNF formula>
{readable}

<trace from model>
{model_trace}

Your task is to extract the truth assignment implied by the model's
    reasoning trace, and evaluate whether each clause in the CNF
    formula is satisfied.

Go through the trace and determine whether each variable appearing
    in the CNF formula is marked as True or False.

Then, for each clause, evaluate the truth value of each literal
    using this assignment.

For example, if a clause in readable CNF formula is (x(0,) \vee \neg
     x(1,)), and the model says x(0,) is True and x(1,) is also True
    , then this clause becomes [1, 0].

Think step by step. Show the variable assignments and how you
    evaluate each clause.

Finally, in the **last line**, output a single line in the format:
Assignment: [[1, 0], [0, 1, 1], [1], ...]

For any variable that is not explicitly mentioned in the reasoning
    trace, assume its value is 0 when constructing the assignment
    list.

Do not include anything after this label.
```

Trace Evaluation Prompt Template for UNSAT Prediction

```
You are evaluating whether a model's reasoning trace correctly
    explains an UNSAT logical puzzle.

<scenario>
{scenario}

<conditions>
{conditions}

<question>
{question}
```

```
<variable explanation>
{variable_mapping}

<reasoning trace from model>
{model_trace}

<ground-truth unsat reason>
{unsat_reason}

We already know this puzzle is UNSAT (unsatisfiable).
Your task is to judge whether the reasoning trace correctly
    identifies or meaningfully reflects the cause of
    unsatisfiability - that is, whether it aligns with the given
    ground-truth unsat reason, even if it doesn't name it explicitly
    .

Focus on logical precision:
- Does the trace show or imply a variable assignment or chain of
    reasoning that leads to contradiction?
- Does it avoid hallucinations or irrelevant claims?

Note: The trace may present a specific variable assignment or
    reasoning path that leads to a contradiction. Whether it aligns
    with the given ground-truth UNSAT reason means you must judge
    whether the contradiction is logically valid and reflective of
    the actual cause - even if it doesn't explicitly name the
    minimal core or unsat pattern.

You are **not** evaluating whether the conclusion "UNSAT" is correct
     - that is already known to be correct.
You are only evaluating whether the explanation substantively
    captures why the instance is unsatisfiable.

Please think step by step. First, explain whether and how the
    reasoning trace aligns with the unsat reason.
Then, in the last line, output one of the following labels:

[YES] - the reasoning trace is logically valid and correctly
    captures the UNSAT cause
[NO] - the trace is flawed, incomplete, or does not match the
    correct unsat reason

Do not include anything after this label.
```