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ABSTRACT

Most state-of-the-art probabilistic time series forecasting models rely on sampling to represent future uncertainty. However, this paradigm suffers from inherent limitations, such as lacking explicit probabilities, inadequate coverage, and high computational costs. In this work, we introduce **Probabilistic Scenarios**, an alternative paradigm designed to address the limitations of sampling. It operates by directly producing a finite set of {Scenario, Probability} pairs, thus avoiding Monte Carlo-like approximation. To validate this paradigm, we propose **TimePrism**, a simple model composed of only three parallel linear layers. Surprisingly, TimePrism achieves 9 out of 10 state-of-the-art results across five benchmark datasets on two metrics. The effectiveness of our paradigm comes from a fundamental reframing of the learning objective. Instead of modeling an entire continuous probability space, the model learns to represent a set of plausible scenarios and corresponding probabilities. Our work demonstrates the potential of the Probabilistic Scenarios paradigm, opening a promising research direction in forecasting beyond sampling.

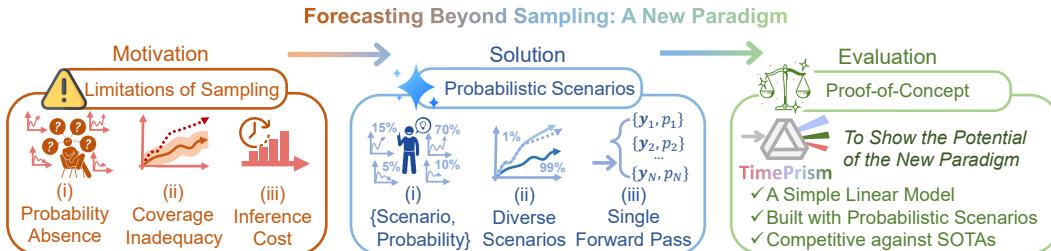


Figure 1: **Motivation, solution, and evaluation of this work.** We illustrate the limitations of the prevailing sampling-based paradigm for probabilistic forecasting. In response, we introduce Probabilistic Scenarios, a new paradigm that directly produces a set of {Scenario, Probability} pairs, and validate its potential with a simple proof-of-concept model, TimePrism.

1 INTRODUCTION

Probabilistic time series forecasting is fundamental to optimal decision-making under uncertainty, as it describes the likelihood of future outcomes (Gneiting & Katzfuss, 2014; Hyndman & Athanasopoulos, 2021). Although this problem has been studied extensively within the machine learning community, current approaches tend to rely on a predefined predictive distribution or sampling approximation (Kong et al., 2025; Fang & Wang, 2020; Lim & Zohren, 2021). These strategies have led to three main categories of models: (i) *Parametric Distribution Models*, assumes that the predictive distribution conforms to a predefined parametric family, such as a Gaussian (Salinas et al., 2020); (ii) *Generative Models*, such as diffusion-based models (Rasul et al., 2021), which learn an iterative process to generate samples from the latent distribution without explicitly defining its density function; and (iii) *Structured Probabilistic Models*, such as Flow-based Models (Rasul et al., 2020; Ashok et al., 2023), which learns a continuous probability density field, from which trajectories are then sampled.

054 However, the reliance on sampling introduces challenges (Cortés et al., 2025). While the alternative of
 055 a predefined distribution is not widely discussed by state-of-the-art methods for its evident inflexibility
 056 (Zhang et al., 2024; Ashok et al., 2023), the sampling paradigm suffers from three primary limitations,
 057 as shown in Figure 1: **(i) Probability Absence.** The most significant issue is that the generated
 058 trajectories are not paired with their probability of occurrence. Although confidence intervals can be
 059 inferred from a large set of samples, this process is indirect, computationally intensive, and lacks the
 060 intuitiveness of a direct scenario-probability mapping. **(ii) Coverage Inadequacy.** A finite set of
 061 samples may fail to represent low-probability, high-impact tail events. This is a critical failure for
 062 applications where preparing for rare occurrences is paramount, such as extreme weather or stock
 063 market volatility (Cortés et al., 2025). **(iii) Inference Cost.** The process of generating multiple
 064 samples is often expensive, with costs scaling to the number of samples required. This expense
 065 exacerbates the two preceding issues in practical applications, limiting the reliability and utility of
 066 such forecasts (Chen & Boccelli, 2018; Ashok et al., 2023).

067 To address the limitations of sampling-based forecasting, we introduce a new paradigm for proba-
 068 bilistic forecasting that we term **Probabilistic Scenarios**. The objective is to produce, in a single
 069 forward pass and without reliance on sampling, a finite set of **{Future Scenario, Probability}** pairs
 070 that explicitly represents the predictive distribution. While some state-of-the-art deep learning models
 071 have made progress toward this goal, none have fully achieved it. Structured probabilistic models like
 072 TempFlow and TACTiS/TACTiS2 can compute probabilities, but only as a continuous density field
 073 (Rasul et al., 2020; Ashok et al., 2023; Drouin et al., 2022), not as discrete, interpretable scenarios;
 074 obtaining explicit trajectories still requires reverting to the expensive sampling. Meanwhile, al-
 075 though TimeMCL (Cortés et al., 2025) produces a set of discrete scenarios, its optimization objective
 prioritizes scenario fidelity over probability matching.

076 To realize and validate the concept of Probabilistic Scenarios, we designed a proof-of-concept model,
 077 **TimePrism**. As its name suggests, TimePrism processes the input history to generate a discrete set of
 078 distinct future trajectories, which we term Scenarios, and concurrently estimates their likelihood to
 079 yield a set of **{Future Scenario, Probability}** pairs. Specifically, to validate the effectiveness of our
 080 paradigm, the architecture is intentionally kept simple. TimePrism is composed of only three parallel
 081 linear layers, designed end-to-end to directly produce Probabilistic Scenarios. Despite its simplicity,
 082 TimePrism achieves 9 out of 10 state-of-the-art (SOTA) results and one second-best result across five
 083 benchmark datasets on our two primary metrics.

084 Contributions:

- 086 • We introduce a new paradigm for probabilistic time series forecasting. This paradigm addresses
 087 the limitations of sampling by reframing the learning objective from continuous probability space
 088 estimation to a more structured task of learning a distribution over a set of scenarios
- 089 • For quantitative measurements, we establish an evaluation framework with two complementary
 090 metrics and provide distinct but comparable formulations for both sampling-based models and our
 091 paradigm, serving as a fair standard for future research on Probabilistic Scenarios.
- 092 • To show the potential of our paradigm, we introduce TimePrism, a simple linear model built
 093 within the Probabilistic Scenarios paradigm. Despite its simple structure, TimePrism still achieves
 094 competitive performance against SOTA sampling-based models, indicating a promising research
 095 direction for forecasting with Probabilistic Scenarios.

096 2 RELATED WORKS

099 **Parametric Distribution Models** employ a neural network to output the parameters of a prespecified
 100 probability distribution (Wu et al., 2020), with examples including DeepAR (Salinas et al., 2020),
 101 GPVar (Salinas et al., 2019), and Mixture Density Networks (Li et al., 2024). The primary limitation
 102 of this approach is its reliance on a strong distributional assumption. Due to this inherent inflexibility,
 103 this approach is less commonly adopted in recent SOTA methods (Zhang et al., 2024).

104 **Generative Models** represent the predictive distribution implicitly through a learned sampling
 105 process (Ho et al., 2020). For instance, Rasul et al. (2021) (TimeGrad) and Alcaraz & Strodthoff
 106 (2022) (SSSD) pioneered the use of conditional diffusion models for probabilistic forecasting and
 107 imputation. Recent advancements have focused on adapting the diffusion process to the sequential
 108 nature of time series (Gao et al., 2025; Biloš et al., 2023). Another direction focuses on enhancing

108 the conditioning mechanism (Kolloviev et al., 2023; Liu et al., 2025) and adding cross-modal visual
 109 information (Ruan et al., 2025). Further adaptations include designing non-stationary processes (Ye
 110 et al., 2025) and specializing models for tasks like imputation, such as CSDI (Tashiro et al., 2021).
 111 Related work also employs Variational Autoencoders (VAEs), such as GP-VAE (Fortuin et al., 2020).
 112 While powerful, all these methods produce forecasts via an iterative sampling procedure and do not
 113 provide an explicit probability density for any given trajectory.

114 **Structured Probabilistic Models** are a class of methods that learn an explicit, continuous probability
 115 density field over the forecast horizon, primarily including flow-based models and copula-based
 116 models. Flow-based models learn a distribution transformation (Papamakarios et al., 2021), with
 117 recent applications in time series forecasting, such as TempFlow, using techniques like conditioned
 118 normalizing flows and flow matching (Rasul et al., 2020; Kolloviev et al., 2024). Copula-based
 119 models, which have a long history in econometrics and finance (Patton, 2012; Größer & Okhrin,
 120 2022), construct a joint distribution by a copula (Salmon & Bouyé, 2008; Bouyé et al., 2008; Wang &
 121 Tao, 2020). With neural networks involved to model the copula (Wen & Torkkola, 2019; Krupskii
 122 & Joe, 2020; Mayer & Wied, 2023; Toubeau et al., 2019), recent research leads to fully neural,
 123 non-parametric approaches like TACTiS (Drouin et al., 2022) and its successor, TACTiS-2 (Ashok
 124 et al., 2023). Despite their different formulations, models in this category still rely on a sampling
 125 procedure, drawing from a continuous probability density field to obtain future trajectories.

126 **Multiple Choice Learning** (MCL) framework offers a practical path toward realizing our Probabilistic
 127 Scenarios paradigm (Cortés et al., 2025). MCL, introduced by Guzmán-rivera et al. (2012), uses a
 128 Winner-Takes-All (WTA) loss to train a multi-head network, where each head specializes in a different
 129 mode of the data. This approach has been successfully applied and extended in various domains,
 130 particularly computer vision and reinforcement learning (Lee et al., 2016; Rupprecht et al., 2017;
 131 Tian et al., 2019; Seo et al., 2020; Garcia et al., 2021). Recent work has further analyzed its geometric
 132 properties and variants (Letzelter et al., 2024; 2023; Perera et al., 2024). In probabilistic time series
 133 forecasting, TimeMCL (Cortés et al., 2025) produces a discrete set of scenarios. However, its score
 134 heads do not directly model a probability distribution. Consequently, to compute probabilistic metrics
 135 like Continuous Ranked Probability Score (CRPS), the original work resamples from its finite set of
 136 scenarios. The authors acknowledge their prioritization of scenario fidelity over probability matching.
 137 This design addresses **Coverage Inadequacy** to some extent, but fails to solve **Probability Absence**.
 138 As reported in their work, this trade-off results in CRPS scores less competitive than SOTA models
 139 such as TACTiS-2 (Ashok et al., 2023) and TimeGrad (Rasul et al., 2021).

140 To transcend this trade-off, our Probabilistic Scenarios paradigm unifies scenario fidelity and probability
 141 matching, designed to address all three limitations of sampling coherently: **Probability Absence**,
 142 **Coverage Inadequacy** and **Inference Cost**.

3 THE PROBABILISTIC SCENARIOS PARADIGM

3.1 CONVENTIONAL FORECASTING PARADIGM WITH SAMPLING

143 We begin by formalizing the objective of probabilistic time series forecasting. Given a historical
 144 context window of length L , denoted as $\mathbf{x} = (x_1, \dots, x_L) \in \mathbb{R}^{L \times D}$, where D is the number of
 145 variates, the goal is to predict the distribution of the future trajectory over a horizon T , denoted as
 146 $\mathbf{y} = (y_1, \dots, y_T) \in \mathbb{R}^{T \times D}$. The objective is to learn a model that captures the conditional probability
 147 distribution over all possible future trajectories:

$$P(\mathbf{y}|\mathbf{x}) \tag{1}$$

148 Directly modeling this high-dimensional distribution is often intractable. Consequently, state-of-
 149 the-art sampling-based methods learn a model, parameterized by θ , that represents this distribution,
 150 denoted as $P_\theta(\mathbf{y}|\mathbf{x})$. A single forecast sample, $\hat{\mathbf{y}}$, is generated by sampling from this learned
 151 distribution in equation 2. The final probabilistic forecast is then represented by a set of S such
 152 samples, $\mathcal{Y}_{\text{samples}} = \{\hat{\mathbf{y}}_i\}_{i=1}^S$, where S is the number of samples. This set serves as an empirical
 153 Monte Carlo approximation of the true conditional distribution in equation 1. This workflow leads to
 154 the mentioned limitations: the **Probability Absence** for any given sample $\hat{\mathbf{y}}_i$, the risk of **Coverage
 155 Inadequacy** when the set $\mathcal{Y}_{\text{samples}}$ fails to capture rare but critical events, and the high **Inference Cost**

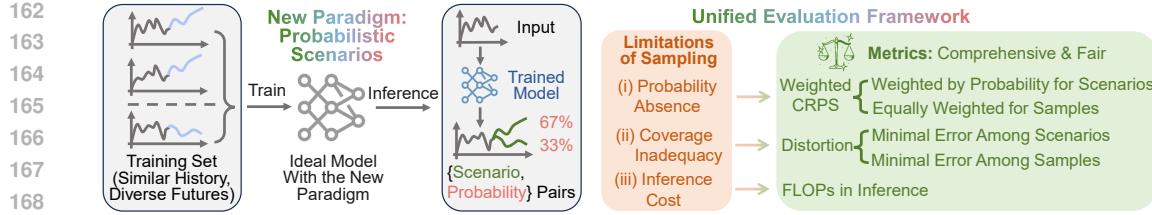


Figure 2: **Probabilistic Scenarios Paradigm and Unified Evaluation Framework.** The left panel illustrates an ideal behavior: a model trained on a dataset where similar histories lead to diverse futures should learn to output $\{\text{Scenario, Probability}\}$ pairs that reflect the empirical frequency of those futures. The right panel details our evaluation framework, which links the limitations of sampling to adapted metrics and provides distinct yet comparable formulations for both paradigms.

of generating a sufficient number of samples.

$$\hat{\mathbf{y}} \sim P_{\theta}(\mathbf{y}|\mathbf{x}) \quad (2)$$

3.2 NEW PARADIGM: PROBABILISTIC SCENARIOS

In light of the challenges in sampling, we explore an alternative paradigm that reframes the forecasting objective, as illustrated in Figure 2. Instead of learning a generative process to approximate a continuous distribution, our paradigm aims to learn a direct mapping from the historical context to a discrete, finite probability space of future scenarios. Formally, we define a model under this paradigm as a function, f , that maps the history \mathbf{x} to a tuple containing both the set of all future scenarios and their corresponding probabilities:

$$f(\mathbf{x}) = (\mathcal{Y}_{\text{pred}}, \mathbf{p}) \quad (3)$$

where: $\mathcal{Y}_{\text{pred}} = \{\mathbf{y}_n\}_{n=1}^N$ is the finite set of N predicted future scenarios, with each scenario $\mathbf{y}_n \in \mathbb{R}^{T \times D}$. $\mathbf{p} = (p_1, \dots, p_N)$ is the vector of probabilities associated with the scenarios in $\mathcal{Y}_{\text{pred}}$. The probabilities must satisfy the axioms $p_n \geq 0$ for all n and $\sum_{n=1}^N p_n = 1$.

This formulation directly yields an explicit set of $\{\text{Scenario, Probability}\}$ pairs in a single forward pass. It differs from the Monte Carlo approximation of equation 2 by providing a discrete probability distribution that is both interpretable and computationally efficient. In essence, this paradigm neither assumes a parametric distributional form nor requires sampling, but instead learns to end-to-end generate $\{\text{Scenario, Probability}\}$ pairs. This reframing of the objective simplifies the learning problem. A detailed discussion and theoretical analysis are provided in the Appendix A.1.

3.3 UNIFIED EVALUATION FRAMEWORK

To quantitatively measure the limitations of sampling-based methods, we establish an evaluation framework by adapting two complementary metrics. We use the standard metric for overall forecast quality (Zhang et al., 2024; Zheng & Sun, 2025), the **CRPS**, to assess **Probability Absence**. Concurrently, we use **Distortion**, defined as the error of the best single trajectory in a set, to assess **Coverage Inadequacy** (Cortés et al., 2025). For both metrics, we provide distinct but comparable formulations for the sampling-based and Probabilistic Scenarios paradigms.

Weighted CRPS for Probability Absence We employ the energy score formulation of CRPS, which is defined for a single ground truth observation \mathbf{y}_{gt} and a set of forecasts as $\mathbb{E}[\|\mathbf{y} - \mathbf{y}_{\text{gt}}\|] - \frac{1}{2}\mathbb{E}[\|\mathbf{y} - \mathbf{y}'\|]$, where \mathbf{y} and \mathbf{y}' are independent samples from the forecast distribution. We generalize this to our discrete, weighted scenario set. Given a set of N scenarios $\mathcal{Y}_{\text{pred}} = \{\mathbf{y}_n\}_{n=1}^N$ and a corresponding probability vector $\mathbf{p} = (p_1, \dots, p_N)$, the Weighted CRPS is defined as:

$$\text{CRPS}(\mathcal{Y}_{\text{pred}}, \mathbf{p}, \mathbf{y}_{\text{gt}}) = \sum_{n=1}^N p_n \|\mathbf{y}_n - \mathbf{y}_{\text{gt}}\|_1 - \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^N p_n p_j \|\mathbf{y}_n - \mathbf{y}_j\|_1 \quad (4)$$

where $\|\cdot\|_1$ denotes the L1 norm summed over all elements of the trajectory. We apply this formulation to both paradigms:

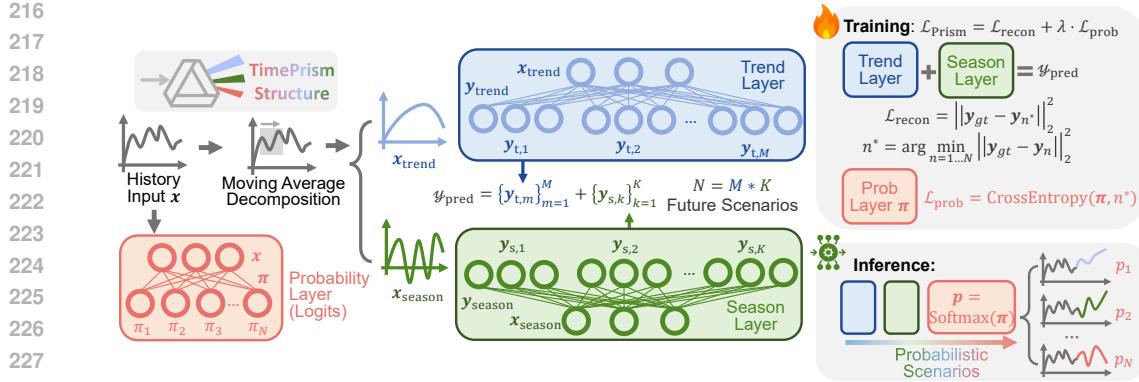


Figure 3: **Structure of TimePrism**, a linear model to demonstrate the potential of the Probabilistic Scenarios paradigm. The model operates in three parallel streams: after an initial decomposition, separate linear layers generate a basis of M trend and K seasonal forecasts. Simultaneously, a third linear layer produces the $N = M * K$ logits from the undecomposed history. This architecture, built within the Probabilistic Scenarios paradigm, achieves competitive performance despite its simplicity, demonstrating the potential of the new paradigm.

- **For Probabilistic Scenarios**, the scenarios $\{y_n\}_{n=1}^N$ and probabilities $\{p_n\}_{n=1}^N$ are taken directly from the model’s output ($\mathcal{Y}_{\text{pred}}$, \mathbf{p}).
- **For sampling-based models**, the evaluation is performed on the generated set of S samples, $\mathcal{Y}_{\text{samples}} = \{\hat{y}_i\}_{i=1}^S$. Each sample is assigned a uniform probability, i.e., $p_i = 1/S$.

The CRPS directly rewards models that assign higher probabilities to scenarios that are closer to the ground truth, thus quantitatively measuring the impact of *Probability Absence*.

Distortion for Coverage Inadequacy. Distortion measures the best-case performance of a forecast, quantifying how well the generated set of trajectories covers the true outcome (Cortés et al., 2025). It is defined as the minimum error between any single trajectory in the set and the ground truth. Following the implementation in our evaluation code, we define it as the minimum Root Mean Squared Error (RMSE) over the set of trajectories:

$$\text{Distortion}(\mathcal{Y}, \mathbf{y}_{\text{gt}}) = \min_{\mathbf{y}_n \in \mathcal{Y}} \sqrt{\frac{1}{T \cdot D} \|\mathbf{y}_n - \mathbf{y}_{\text{gt}}\|_F^2} \quad (5)$$

where $\|\cdot\|_F$ is the Frobenius norm. We apply this formulation as follows:

- **For Probabilistic Scenarios**, the minimization is performed over the complete set of N scenarios generated by the model, $\mathcal{Y} = \mathcal{Y}_{\text{pred}}$.
- **For sampling-based models**, the minimization is performed over the set of S generated samples, $\mathcal{Y} = \mathcal{Y}_{\text{samples}}$.

This metric directly assesses the diversity and reach of the generated set of futures. A lower Distortion score indicates better coverage, particularly for tail events that may be missed by a limited number of samples, thus measuring *Coverage Inadequacy*.

4 TIMEPRISM: A PROOF-OF-CONCEPT MODEL

4.1 DESIGN PHILOSOPHY

The primary goal of TimePrism is not to introduce a new complex architecture, but to serve as a clear proof-of-concept for the Probabilistic Scenarios paradigm. We intentionally adopt a minimalist design to test a core hypothesis: that the Probabilistic Scenarios paradigm can prove effective even when implemented with a simple model architecture.

To this end, we construct TimePrism using only three parallel linear layers as its core learnable components, devoid of any non-linear activation functions or deep, stacked layers. This deliberate

270 simplicity acts as a controlled experiment. By stripping away architectural complexity, we ensure
 271 that the model's strong performance can be directly attributed to the strengths of the paradigm itself.
 272

273 **4.2 TIMEPRISM ARCHITECTURE**
 274

275 The architecture of TimePrism is illustrated in Figure 3. It consists of three parallel streams that
 276 process the input history to generate the final set of Probabilistic Scenarios. Inspired by recent works
 277 such as DLinear and FITS (Zeng et al., 2023; Xu et al., 2023), which showed that simple architectures
 278 can effectively validate a new paradigm, we use a backbone based on decomposition and linear layers.
 279 The model operates as follows.

280 *1) Decomposition:* First, the input history $\mathbf{x} \in \mathbb{R}^{L \times D}$ is separated into a trend component $\mathbf{x}_{\text{trend}}$ and a
 281 seasonal component $\mathbf{x}_{\text{season}}$ using a moving average filter. This is a standard decomposition technique
 282 where:

$$283 \mathbf{x}_{\text{season}} = \mathbf{x} - \mathbf{x}_{\text{trend}}, \quad \text{with} \quad \mathbf{x}_{\text{trend}} = \text{AvgPool}(\text{Padding}(\mathbf{x})) \quad (6)$$

284 *2) Trend and Season Layers:* The two decomposed components are then fed into two independent
 285 linear layers. The trend layer maps the trend component $\mathbf{x}_{\text{trend}}$ to a set of M distinct trend forecasts,
 286 $\mathcal{M} = \{\mathbf{y}_{t,m}\}_{m=1}^M$. Concurrently, the season layer maps the seasonal component $\mathbf{x}_{\text{season}}$ to a set of K
 287 distinct seasonal forecasts, $\mathcal{K} = \{\mathbf{y}_{s,k}\}_{k=1}^K$. The complete set of $N = M \cdot K$ future scenarios, $\mathcal{Y}_{\text{pred}}$,
 288 is constructed by combining these two sets:

$$289 \mathcal{Y}_{\text{pred}} = \mathcal{M} + \mathcal{K} = \{\mathbf{y}_{t,m} + \mathbf{y}_{s,k} \mid \mathbf{y}_{t,m} \in \mathcal{M}, \mathbf{y}_{s,k} \in \mathcal{K}\} \quad (7)$$

291 *3) Probability Layer:* Operating in parallel to the scenario generation, a third linear layer acts as
 292 the probability module. This layer takes the original, undecomposed history \mathbf{x} as input and directly
 293 produces a logits vector $\boldsymbol{\pi} \in \mathbb{R}^N$. Each element π_n in this vector corresponds to one of the N
 294 scenarios generated via the combinatorial process in Eq. equation 7.

295 **4.3 TRAINING AND INFERENCE**
 296

297 **Loss Function and Training.** The design of the loss function is directly guided by the Probabilistic
 298 Scenarios paradigm. Specifically, the loss function, $\mathcal{L}_{\text{Prism}}$, is composed of two terms, each designed to
 299 supervise one component of the target **{Scenario, Probability}** output. The reconstruction loss, $\mathcal{L}_{\text{recon}}$,
 300 is responsible for optimizing the fidelity of the generated Scenarios. Concurrently, the probability
 301 loss, $\mathcal{L}_{\text{prob}}$, supervises the learning of a meaningful probability distribution over these scenarios. The
 302 coefficient of the probability term, λ , is set to 1 in this work. Given the ground truth future trajectory
 303 \mathbf{y}_{gt} , the total loss is:

$$304 \mathcal{L}_{\text{Prism}} = \mathcal{L}_{\text{recon}} + \lambda \cdot \mathcal{L}_{\text{prob}} \quad (8)$$

305 *For Scenarios:* The reconstruction loss, $\mathcal{L}_{\text{recon}}$, uses the Winner-Takes-All (WTA) principle. It first
 306 identifies the index n^* of the scenario in $\mathcal{Y}_{\text{pred}}$ that has the lowest Mean Squared Error (MSE) with
 307 the ground truth. The loss is then the MSE of this single "winner" scenario:

$$308 \mathcal{L}_{\text{recon}} = \|\mathbf{y}_{\text{gt}} - \mathbf{y}_{n^*}\|_2^2, n^* = \arg \min_{n=1 \dots N} \|\mathbf{y}_{\text{gt}} - \mathbf{y}_n\|_2^2 \quad (9)$$

310 *For Probability:* The probability loss, $\mathcal{L}_{\text{prob}}$, trains the probability layer to assign the highest probability
 311 to this winner. It is the Cross-Entropy loss between logits vector $\boldsymbol{\pi}$ and winner index n^* :

$$313 \mathcal{L}_{\text{prob}} = \text{CrossEntropy}(\boldsymbol{\pi}, n^*) = -\log \left(\frac{\exp(\pi_{n^*})}{\sum_{j=1}^N \exp(\pi_j)} \right) \quad (10)$$

315 In our experiments, we employ a relaxed variant of the WTA loss (Rupprecht et al., 2017) to further
 316 stabilize training. The complete formulation of this loss, including its specific implementation for the
 317 multivariate case, is provided in the Appendix C.2.

318 **Inference.** During inference, the model performs a single forward pass to generate the set of N
 319 scenarios, $\mathcal{Y}_{\text{pred}}$, and the logits vector, $\boldsymbol{\pi}$. The logits are then converted into a valid probability vector,
 320 \mathbf{p} , using the Softmax function as in equation 11. The final output of TimePrism is the complete set of
 321 Probabilistic Scenarios, $\{(\mathbf{y}_n, p_n)\}_{n=1}^N$.

$$323 \mathbf{p} = \text{Softmax}(\boldsymbol{\pi}), \quad \text{where } p_n = \frac{\exp(\pi_n)}{\sum_{j=1}^N \exp(\pi_j)} \quad (11)$$

324

5 EXPERIMENTS

325

5.1 BASIC SETUP

326 **Data.** Following the recent benchmark for probabilistic time series forecasting, ProTS (Zhang et al.,
 327 2024), we evaluate our model on five datasets: Electricity (Elec.), Exchange (Exch.), Solar(Sol.),
 328 Traffic(Traf.), and Wikipedia (Wiki.). These are benchmark datasets taken from the GluonTS
 329 library (Alexandrov et al., 2020), preprocessed exactly as in prior works (Gasthaus et al., 2019). A
 330 detailed analysis of dataset properties cited from previous work is provided in the Appendix C.1.
 331 Following prior work, we set the forecast horizon to 24 (hours) for the hourly datasets (Electricity,
 332 Solar, Traffic) and 30 (days) for the daily datasets (Exchange, Wikipedia) (Zhang et al., 2024). For
 333 TimePrism, the input length is set equal to the forecast horizon. Other baselines may use longer
 334 context lengths as lagged features, for which we adhere to their configurations in prior work (Cortés
 335 et al., 2025). Notably, TimePrism achieves strong performance even with less information input. For
 336 a comprehensive comparison, we also provide results in the Appendix D.2, where TimePrism uses an
 337 input length comparable to that of the baselines.
 338

339 **Metrics.** As established in our framework, the primary metrics are **Weighted CRPS** and **Distortion**.
 340 To measure the *Inference Cost*, we report inference Floating Point Operations (FLOPs). While MSE
 341 and Mean Absolute Error (MAE) are not primary indicators for probabilistic forecasting, we include
 342 their definitions and normalized results in the Appendix B.3 and D.1 for a comprehensive comparison.
 343

344 **Baselines.** For a comprehensive comparison, we select seven models covering all three categories
 345 discussed in our related work. ETS (Hyndman et al., 2008) serves as a non-neural baseline. DeepAR
 346 (Salinas et al., 2020) represents parametric distribution models. TimeGrad (Rasul et al., 2021) is a
 347 diffusion-based generative model. TempFlow (Rasul et al., 2020), Transformer TempFlow (Trf.Flow),
 348 and TACTiS-2 (Ashok et al., 2023) are structured probabilistic models. Tempflow is implemented
 349 with Long Short-Term Memory (LSTM) backbone (Hochreiter & Schmidhuber, 1997) and Trf.Flow
 350 is implemented with a Transformer backbone (Vaswani et al., 2017), as in Rasul et al. (2020). Finally,
 351 TimeMCL (Cortés et al., 2025) represents multi-choice learning models.
 352

353 **Training Details.** All models are trained using the Adam optimizer with an initial learning rate of
 354 10^{-3} for 200 epochs. Given its lack of hidden layers, the number of scenarios N is the primary tunable
 355 hyperparameter for TimePrism. In this section, TimePrism uses $N = 625$ scenarios, composed
 356 of $M = 25$ trend and $K = 25$ seasonal components. In practice, if the number of distinct future
 357 scenarios is known a priori, N can be set to match this number; otherwise, as in our benchmark
 358 datasets, N should be set to a value large enough to allow the model to learn on its own. Further
 359 training details and analysis on N are included in the Appendix C.3. To ensure a fair comparison,
 360 considering that TimeMCL employs 16 hypotheses in its original implementation, we specifically
 361 include a variant with $N = 16$, denoted as **TimePrism-16**. This serves to validate the effectiveness
 362 of the new paradigm, demonstrating that it functions effectively with a simple structure and without
 363 requiring a large number of parameters.
 364

365

5.2 MAIN RESULTS

366 Table 1: **CRPS for Probability Absence.** Results on 5 benchmark datasets. We report the mean \pm
 367 standard deviation over 3 random seeds. The best and second results are in **bold** and underlined.
 368

Model	Elec.	Exch.	Sol.	Traf.	Wiki.
ETS	0.376 ± 0.00	1.22 ± 0.02	0.375 ± 0.00	0.813 ± 0.00	4.88 ± 0.01
DeepAR	0.997 ± 0.03	0.701 ± 0.00	0.583 ± 0.02	0.826 ± 0.01	1.75 ± 0.30
TimeGrad	0.232 ± 0.00	0.845 ± 0.24	0.241 ± 0.00	0.162 ± 0.00	0.517 ± 0.02
TempFlow	0.316 ± 0.00	0.669 ± 0.01	0.272 ± 0.00	0.601 ± 0.01	1.26 ± 0.06
Trf.Flow	0.396 ± 0.08	1.07 ± 0.17	0.280 ± 0.02	0.607 ± 0.01	1.71 ± 0.12
TACTiS-2	0.299 ± 0.01	0.648 ± 0.03	0.236 ± 0.03	0.257 ± 0.01	0.484 ± 0.00
TimeMCL	0.370 ± 0.01	1.12 ± 0.15	0.290 ± 0.03	0.262 ± 0.01	0.640 ± 0.03
TimePrism-16	0.414 ± 0.12	<u>0.611 ± 0.06</u>	<u>0.137 ± 0.00</u>	<u>0.159 ± 0.02</u>	0.654 ± 0.01
TimePrism	0.133 ± 0.02	0.468 ± 0.01	0.0852 ± 0.00	0.111 ± 0.00	<u>0.506 ± 0.00</u>

378 **Probability Absence and Weighted CRPS.** The limitation of *Probability Absence* means that
 379 decision-makers cannot directly assess the likelihood of specific outcomes from sampling-based
 380 models. To quantitatively measure the benefit of providing explicit probabilities and evaluate the
 381 overall quality of the forecast distribution, we use Weighted CRPS. Table 1 presents the results on
 382 five datasets, reported as the mean and standard deviation over three random seeds (3141, 3142,
 383 3143), following prior work. TimePrism achieves state-of-the-art performance on four of the five
 384 datasets and secures the second-best result on Wikipedia. Furthermore, a detailed discussion on the
 385 applicability of TimePrism is provided in the Appendix C.1.2.

386
 387 **Table 2: Distortion for Coverage Inadequacy.** Results on 5 benchmark datasets. We report the mean
 388 \pm standard deviation over 3 random seeds. The best and second results are in **bold** and underlined.
 389

Model	Elec.	Exch.	Sol.	Traf.	Wiki.
ETS	1.24 ± 0.02	1.92 ± 0.06	1.03 ± 0.00	2.69 ± 0.01	142 ± 0.71
DeepAR	2.82 ± 0.11	1.87 ± 0.03	1.09 ± 0.02	1.86 ± 0.09	5.36 ± 0.42
TimeGrad	0.731 ± 0.02	1.37 ± 0.17	0.550 ± 0.03	0.561 ± 0.02	1.64 ± 0.03
TempFlow	1.41 ± 0.04	1.32 ± 0.02	0.515 ± 0.03	0.981 ± 0.00	37.8 ± 6.12
Trf.Flow	1.70 ± 0.28	1.70 ± 0.22	0.552 ± 0.04	1.02 ± 0.01	63.7 ± 8.02
TACTiS-2	0.674 ± 0.04	<u>0.873 ± 0.04</u>	0.586 ± 0.02	0.592 ± 0.05	1.26 ± 0.10
TimeMCL	0.607 ± 0.01	1.08 ± 0.08	0.462 ± 0.04	0.454 ± 0.00	1.49 ± 0.30
TimePrism-16	0.911 ± 0.27	0.920 ± 0.04	<u>0.307 ± 0.04</u>	<u>0.346 ± 0.09</u>	<u>1.16 ± 0.15</u>
TimePrism	0.211 ± 0.04	0.595 ± 0.01	0.101 ± 0.03	0.144 ± 0.00	1.04 ± 0.03

400
 401 **Coverage Inadequacy and Distortion.** To assess *Coverage Inadequacy*, we use the Distortion
 402 metric, with results presented in Table 2. TimePrism achieves the state-of-the-art result across all
 403 five datasets, demonstrating its superior ability to generate a diverse set of scenarios that covers the
 404 ground truth. This is because our reconstruction loss, $\mathcal{L}_{\text{recon}}$, allows the model not to be heavily
 405 penalized for predicting a plausible but non-realized future, in datasets containing similar histories
 406 but diverse futures.
 407

408
 409 **Table 3: Inference FLOPs.** FLOPs required to generate S forecast samples on the Exchange dataset
 410 with batch size = 1. The cost for TimeMCL and TimePrism is constant as they produce all scenarios
 411 in a single forward pass.

Sampling S	DeepAR	TimeGrad	TempFlow	Trf.Flow	TACTiS-2	TimeMCL	TimePrism
1	2.9×10^4	1.9×10^8	5.8×10^6	1.4×10^7	2.5×10^7		
10	2.9×10^5	1.9×10^9	5.8×10^7	1.3×10^8	1.2×10^8	8.8×10^6	5.1×10^5
100	2.9×10^6	1.9×10^{10}	5.8×10^8	1.3×10^9	1.1×10^9		

412
 413 **Inference Cost and FLOPs.** To evaluate the *Inference Cost*, we compare the FLOPs required by
 414 each model to generate a set of S samples, with results shown in Table 3. As demonstrated, the
 415 inference cost of TimePrism is constant regardless of the number of samples required, as it generates
 416 all N scenarios and their probabilities in a single forward pass. In contrast, the cost for sampling-
 417 based models scales with S , forcing a direct trade-off between forecast quality and computational
 418 efficiency. TimeMCL also generates its full set of hypotheses in a single pass. However, lacking
 419 explicit probabilities, its original implementation for CRPS evaluation relies on resampling from this
 420 fixed set. For a fair comparison, we also only report the single-pass FLOPs of TimeMCL.
 421

422 **Overall Comparison.** The CRPS and Distortion results in Tables 1 and 2 are based on $S = 100$
 423 samples for all baselines. At this sampling level, TimePrism is more efficient by one to five orders
 424 of magnitude than its most competitive counterparts (TimeGrad, TACTiS-2, and TimeMCL). The
 425 results confirm that TimePrism is more efficient than sampling-based models, especially when a large
 426 number of samples is needed, highlighting the efficiency of the Probabilistic Scenarios paradigm.
 427 This analysis details the trade-off between inference cost and forecast quality, and how our paradigm
 428 transcends it.

432 **Table 4: Impact of Scenario Count (N) on Performance and Complexity.** This table presents a
 433 systematic ablation study across Electricity, Exchange, and Solar datasets, illustrating how model
 434 complexity and forecasting error (CRPS, Distortion) scale with the number of scenarios N .
 435

436 N	437 FLOPs	438 Solar		439 Electricity		440 Exchange	
		441 CRPS	442 Distortion	443 CRPS	444 Distortion	445 CRPS	446 Distortion
1	1.0x	0.199 \pm 0.00	0.266 \pm 0.00	0.409 \pm 0.01	0.733 \pm 0.03	0.596 \pm 0.00	0.803 \pm 0.00
16	4.2x	0.137 \pm 0.00	0.307 \pm 0.03	0.414 \pm 0.10	0.911 \pm 0.22	0.611 \pm 0.04	0.920 \pm 0.04
256	19.9x	0.0927 \pm 0.000	0.158 \pm 0.01	0.162 \pm 0.01	0.267 \pm 0.03	0.486 \pm 0.00	0.666 \pm 0.02
625	34.8x	0.0852 \pm 0.000	0.101 \pm 0.03	0.133 \pm 0.01	0.211 \pm 0.03	0.468 \pm 0.01	0.595 \pm 0.01
1024	48.3x	0.0822 \pm 0.000	0.0917 \pm 0.010	0.139 \pm 0.01	0.212 \pm 0.02	0.452 \pm 0.00	0.583 \pm 0.02

447 5.3 IMPACT OF SCENARIO SET SIZE

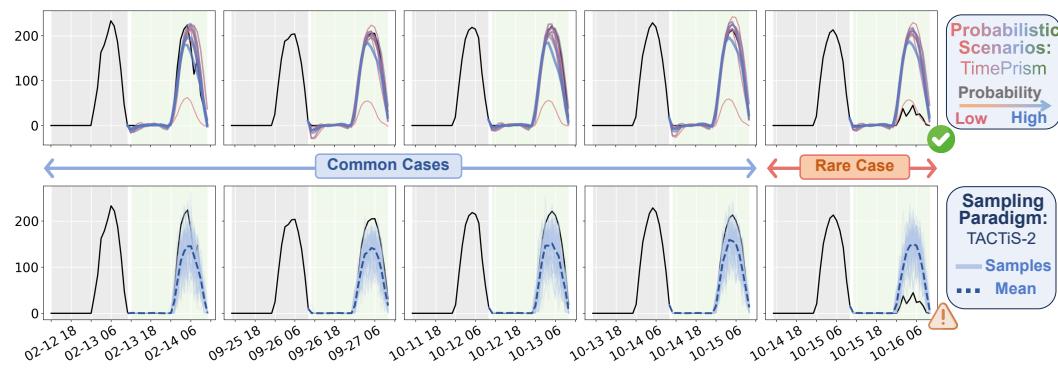
448 We conduct a systematic analysis to investigate the trade-off between the scenario set size N and
 449 model performance. Table 4 summarizes the results across three representative datasets (Electricity,
 450 Exchange, and Solar) with N ranging from 1 to 1024.

451 **Complexity Scaling.** A key advantage of our combinatorial architecture ($N = M \times K$) is its
 452 efficiency. The parameter complexity of the shared basis layers (Trend and Season) scales as $\mathcal{O}(\sqrt{N})$,
 453 while only the probability head scales linearly as $\mathcal{O}(N)$. Consequently, the overall model complexity
 454 grows favorably between $\mathcal{O}(N^{1/2})$ and $\mathcal{O}(N)$, allowing for large scenario sets.

455 **Performance Trends.** Increasing N generally leads to lower CRPS and Distortion errors, as a larger
 456 discrete set can approximate the continuous probability space with higher fidelity. However, we
 457 observe **diminishing returns**: the performance gains tend to plateau around $N = 625$. Beyond this
 458 point, the marginal benefit of adding scenarios decreases while the computational cost continues to
 459 rise. Based on this equilibrium, we adopted $N = 625$ as the unified setting for our main experiments.

460 **Dataset Dependence.** The results also indicate that the "saturation point" varies slightly by dataset.
 461 For instance, the Solar dataset benefits more from a larger N compared to the Exchange dataset. This
 462 suggests that the optimal N is determined by the intrinsic complexity of the data, highlighting the
 463 potential for future work on adaptive mechanisms that dynamically adjust N .
 464

465 5.4 VISUALIZATION AND QUALITATIVE ANALYSIS



480 **Figure 4: Qualitative Analysis of the New Paradigm.** A visual comparison between the Probabilistic
 481 Scenarios paradigm (TimePrism) and the Sampling Paradigm (TACTIS-2). The figure highlights their
 482 distinct behaviors in both common high-peak cases and a rare low-peak case, on the Solar dataset.
 483

484 To visually compare the two paradigms, we conduct a qualitative analysis on the Solar energy dataset,
 485 selecting the last variate ($D = 137$) and identifying instances with similar histories but diverse
 486 futures. As shown in Figure 4, these instances include four *Common Cases* of high-peak futures

486 and one *Rare Case* of a low-peak future. We compare TimePrism against TACTiS-2, the strongest
 487 baseline. The top panel displays the top 10 scenarios from TimePrism, with line color and thickness
 488 representing probability from low (red, thin) to high (blue, thick). TimePrism successfully captures
 489 both types of cases, assigning high probabilities to the common cases while also identifying a rare
 490 case with low probability. In contrast, the bottom panel shows that the $S = 100$ samples from
 491 TACTiS-2 cluster around their mean (dashed line). While the envelope of TACTiS-2 samples may
 492 loosely cover both high-peak and low-peak, its forecast suffers from **Probability Absence**. Without
 493 explicit probabilities, the common high peak case can not be distinguished from the rare low peak
 494 case, rendering the entire set of samples uninformative for assessment.

495 6 CONCLUSIONS AND DISCUSSION

496 6.1 DISCUSSION

500 **Reason of Effectiveness.** The strong performance of TimePrism stems from the paradigm’s reframing
 501 of the learning objective. Instead of learning to model an entire continuous probability space, the
 502 model is learning a more structured problem: a probability distribution over a discrete set of scenarios.
 503 This concept parallels Vector Quantization (VQ) techniques in representation learning, most notably
 504 VQ-VAE (van den Oord et al., 2017), but applies the discretization directly to the output trajectory
 505 space rather than a latent space (see Appendix A.3 for a detailed discussion). This shift reduces the
 506 required model capacity, allowing a simple linear architecture to achieve strong results.

507 Limitations:

- 508 • **Dataset Applicability.** The intentionally simple structure of TimePrism, while effective for
 509 validating the paradigm, may have limitations in more complex scenarios, such as those with
 510 extremely high dimensionality, or series lacking trend or seasonal patterns.
- 511 • **Structural Rigidity.** As a linear model, the current version of TimePrism requires fixed-length
 512 inputs and prediction horizons, limiting its flexibility in scenarios where variable-length contexts
 513 are available during inference.
- 514 • **Simplified Multivariate Modeling.** Our current implementation utilizes a weight-sharing strategy.
 515 We believe there is significant room for improvement by incorporating more sophisticated channel-
 516 mixing mechanisms to model cross-variate relationships.

517 Future Works:

- 518 • **Models within the new Paradigm.** TimePrism serves only as a proof-of-concept. The true
 519 potential of Probabilistic Scenarios lies in its application to more powerful backbones. Future work
 520 could integrate this paradigm with state-of-the-art architectures like Transformers, Diffusion, or
 521 Flow Matching models to unlock new levels of multivariate performance.
- 522 • **Refinements of the new Paradigm.** The paradigm itself can be further enhanced. For instance,
 523 developing methods to adaptively determine the number of scenarios based on data complexity
 524 could improve its practical utility.
- 525 • **Decision-Centric Assessment.** Metrics like CRPS and Distortion may not fully reflect the down-
 526 stream utility of probabilistic forecasts in real-world environments. In future work, decision-centric
 527 metrics can be incorporated, such as tail-risk assessment and utility-based scores. Furthermore,
 528 we plan to explore the direct integration of our Probabilistic Scenarios paradigm into real-world
 529 decision-making to demonstrate its practical value beyond pure forecasting accuracy.

530 6.2 CONCLUSION

532 Probabilistic time series forecasting is crucial for reliable decision-making. While powerful, current
 533 SOTA methods predominantly rely on sampling, a paradigm that faces limitations of *Probability*
 534 *Absence*, *Coverage Inadequacy*, and *Inference Cost*. To address these challenges, we introduced the
 535 **Probabilistic Scenarios** paradigm. This paradigm operates by directly producing a set of { Scenario,
 536 Probability } pairs in a single forward pass, without reliance on sampling. We validated this new
 537 paradigm with TimePrism, a simple linear model. Evaluated under our unified framework, TimePrism
 538 addresses these challenges and demonstrates the potential of the new paradigm. In summary, our work
 539 provides a practical alternative to sampling and broadens the conceptual landscape of probabilistic
 forecasting, establishing a promising foundation for future research.

540 ETHIC STATEMENT
541542 This paper presents work whose goal is to advance the field of Machine Learning. There are many
543 potential societal consequences of our work, none of which we feel must be specifically highlighted
544 here.545 We comply with intellectual property agreements for all data sources. Data are properly anonymized
546 with no concerns regarding sensitive or illegal activity in our dataset.548 549 REPRODUCIBILITY STATEMENT
550551 The code of this work is available at: [https://anonymous.4open.science/r/
552 probabilistic-scenarios-submission-550A](https://anonymous.4open.science/r/probabilistic-scenarios-submission-550A).553 554 LLM STATEMENT
555

556 We utilized a large language model to assist in polishing the grammar and phrasing of our manuscript.

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810 811 Appendix

812 A THEORETICAL ANALYSIS

815 A.1 EFFECTIVENESS OF THE PROBABILISTIC SCENARIOS PARADIGM

817 The empirical success of the Probabilistic Scenarios paradigm is rooted in its fundamental reframing
818 of the learning objective. This section provides a theoretical perspective on why this reframing leads
819 to a more tractable and effective learning problem.

820 The conventional sampling-based paradigm requires a model to learn a complex, high-dimensional
821 conditional probability distribution, $P_\theta(\mathbf{y}|\mathbf{x})$, over the continuous space $\mathbb{R}^{T \times D}$. Optimizing this
822 objective, often by maximizing the log-likelihood $\log P_\theta(\mathbf{y}_{\text{gt}}|\mathbf{x})$, is difficult. It requires the model to
823 correctly assign a probability density to every possible point in an infinite space, a task that demands
824 immense model capacity.

825 In contrast, our Probabilistic Scenarios paradigm transforms this intractable density estimation
826 problem into a more structured, two-part learning task:

- 827 **1. Scenario Representation:** The first task is to learn a finite set of N discrete points, $\mathcal{Y}_{\text{pred}} =$
828 $\{\mathbf{y}_n\}_{n=1}^N$, that effectively represent the most meaningful regions of the true conditional distribution.
829 This simplifies the problem from modeling the entire continuous space to finding a good discrete
830 basis for it.
- 831 **2. Probability Assignment:** The second task is to learn a categorical distribution, \mathbf{p} , over this
832 finite set of N scenarios. The objective shifts from computing a density $P_\theta(\mathbf{y}_{\text{gt}}|\mathbf{x})$ to solving a
833 large-scale classification problem: determining which of the N representative regions the ground
834 truth \mathbf{y}_{gt} is most likely to fall into.

835 In essence, the paradigm decouples the problem of "what" can happen (the scenarios) from "how
836 likely" it is to happen (the probabilities). This structured decomposition significantly reduces the
837 complexity of the learning problem, allowing even simple models to allocate their limited capacity
838 efficiently and achieve strong performance.

840 A.2 THEORETICAL FOUNDATIONS OF TIMEPRISM

842 The theoretical analysis of the Winner-Takes-All principle in this section is inspired by the framework
843 presented in Cortés et al. (2025) and Letzelter et al. (2024). However, we adapt and extend this
844 analysis to our specific non-autoregressive, combinatorial architecture and our probabilistic objective,
845 which, as we will show, provides stronger theoretical guarantees.

846 A.2.1 OPTIMAL SCENARIOS VIA RECONSTRUCTION LOSS

848 The goal of our reconstruction loss is to find a set of scenarios that provides the best discrete
849 approximation of the continuous space of all possible future trajectories. We formalize this in the
850 following proposition.

851 **Proposition 1.** *Assuming that the model parameters reach a local minimum of the reconstruction loss,
852 a necessary condition is that the set of $N = M \cdot K$ combined scenarios forms a Centroidal Voronoi
853 Tessellation (CVT) of the space of future trajectories, conditioned on the input history. Specifically,
854 each combined scenario $\mathbf{y}_{t,m} + \mathbf{y}_{s,k}$ converges to the conditional mean of its corresponding Voronoi
855 region.*

856 *Proof.* The objective is to find the model parameters (which in turn define the scenarios) that minimize
857 the expected reconstruction loss over the data distribution $P(\mathbf{x}, \mathbf{y}_{\text{gt}})$. Our model is non-autoregressive,
858 so the generated scenarios $\{\mathbf{y}_n(\mathbf{x})\}$ are a direct function of the input history \mathbf{x} . The expected loss is:
859

$$860 \mathbb{E}[\mathcal{L}_{\text{recon}}] = \mathbb{E}_{\mathbf{x}} \left[\mathbb{E}_{\mathbf{y}_{\text{gt}}|\mathbf{x}} \left[\min_{n=1 \dots N} \|\mathbf{y}_{\text{gt}} - \mathbf{y}_n(\mathbf{x})\|_2^2 \right] \right] \quad (12)$$

862 The \min operator partitions the space of future trajectories, for a given \mathbf{x} , into N Voronoi regions,
863 $\{R_n(\mathbf{x})\}_{n=1}^N$. We formally define the Voronoi region for the n -th scenario as $R_n(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}^{T \times D} \mid$
 $\|\mathbf{y} - \mathbf{y}_n(\mathbf{x})\|_2 \leq \|\mathbf{y} - \mathbf{y}_j(\mathbf{x})\|_2, \forall j\}$. Each region $R_n(\mathbf{x})$ contains all trajectories \mathbf{y}_{gt} for which the

864 n -th scenario is the winner. The inner expectation can then be rewritten as a sum of integrals over
 865 these regions:

$$867 \quad \mathbb{E}_{\mathbf{y}_{\text{gt}}|\mathbf{x}} \left[\min_{n=1 \dots N} \|\mathbf{y}_{\text{gt}} - \mathbf{y}_n(\mathbf{x})\|_2^2 \right] = \sum_{n=1}^N \int_{R_n(\mathbf{x})} \|\mathbf{y}_{\text{gt}} - \mathbf{y}_n(\mathbf{x})\|_2^2 p(\mathbf{y}_{\text{gt}}|\mathbf{x}) d\mathbf{y}_{\text{gt}} \quad (13)$$

870 To find the optimal scenarios $\{\mathbf{y}_n(\mathbf{x})\}$, we take the functional derivative of the expected loss with
 871 respect to each $\mathbf{y}_n(\mathbf{x})$ and set it to zero. The derivative only affects one term in the summation.
 872 Following the derivation in Cortés et al. (2025), the minimum is achieved when:

$$873 \quad \mathbf{y}_n(\mathbf{x}) = \frac{\int_{R_n(\mathbf{x})} \mathbf{y}_{\text{gt}} p(\mathbf{y}_{\text{gt}}|\mathbf{x}) d\mathbf{y}_{\text{gt}}}{\int_{R_n(\mathbf{x})} p(\mathbf{y}_{\text{gt}}|\mathbf{x}) d\mathbf{y}_{\text{gt}}} = \mathbb{E}[\mathbf{y}_{\text{gt}} \mid \mathbf{y}_{\text{gt}} \in R_n(\mathbf{x})] \quad (14)$$

876 This derivation holds provided that the Voronoi region has non-zero probability mass, that is, when
 877 $\int_{R_n(\mathbf{x})} p(\mathbf{y}_{\text{gt}}|\mathbf{x}) d\mathbf{y}_{\text{gt}} \neq 0$. This demonstrates that for any given history \mathbf{x} , the scenarios generated by
 878 an optimal model must be the conditional means of their respective **Voronoi regions** (Du et al., 1999).
 879 In geometric terms, the set of N scenarios acts as a set of centers that partition the high-dimensional
 880 space of all possible futures into N distinct regions, known as a Voronoi tessellation. Each region
 881 consists of all future trajectories that are closer to one specific scenario than to any other. Our result
 882 shows that the WTA training objective effectively drives the model to find an optimal set of "cluster
 883 centers" (our scenarios) that best represent the underlying structure of the data, where "best" is defined
 884 in the sense of minimizing the expected squared error, akin to the objective in k-means clustering
 (Cortés et al., 2025; Arthur & Vassilvitskii, 2007).

886 A.2.2 SCENARIO REPRESENTATION AND DISTORTION

887 Our reconstruction loss is designed to optimize for scenario fidelity, which directly contributes to
 888 the model's ability to achieve a low Distortion score. The core mechanism lies in how the Winner-
 889 Takes-All (WTA) objective interacts with datasets exhibiting diverse potential futures from similar
 890 histories.

891 Consider the gradient of the reconstruction loss, $\mathcal{L}_{\text{recon}}$, with respect to the model's parameters θ .
 892 The parameters θ define the mapping from the input \mathbf{x} to the entire set of scenarios $\mathcal{Y}_{\text{pred}}(\mathbf{x}; \theta) =$
 893 $\{\mathbf{y}_n(\mathbf{x}; \theta)\}_{n=1}^N$. The loss for a single data instance $(\mathbf{x}, \mathbf{y}_{\text{gt}})$ is:

$$894 \quad \mathcal{L}_{\text{recon}}(\theta) = \|\mathbf{y}_{\text{gt}} - \mathbf{y}_{n^*}(\mathbf{x}; \theta)\|_2^2 \quad (15)$$

895 where the winner index n^* is itself a function of θ :

$$896 \quad n^*(\theta) = \arg \min_{n=1 \dots N} \|\mathbf{y}_{\text{gt}} - \mathbf{y}_n(\mathbf{x}; \theta)\|_2^2 \quad (16)$$

897 Assuming the winner index n^* is locally constant with respect to small changes in θ , the gradient of
 898 the loss is given by the chain rule:

$$902 \quad \nabla_{\theta} \mathcal{L}_{\text{recon}} = \frac{\partial \mathcal{L}_{\text{recon}}}{\partial \mathbf{y}_{n^*}} \cdot \frac{\partial \mathbf{y}_{n^*}(\mathbf{x}; \theta)}{\partial \theta} \quad (17)$$

904 Crucially, for all non-winning scenarios where $n \neq n^*$, the partial derivative of the loss with respect
 905 to their outputs is zero:

$$906 \quad \frac{\partial \mathcal{L}_{\text{recon}}}{\partial \mathbf{y}_n} = \mathbf{0} \quad \forall n \neq n^* \quad (18)$$

908 This implies that the gradients for the parameters governing the non-winning scenarios are also zero
 909 for this specific training instance.

910 The direct consequence of Eq. equation 18 is that the model is not explicitly penalized for generating
 911 a plausible but non-realized scenario. In a dataset containing instances of "similar histories, diverse
 912 futures," this property allows different scenarios within the set $\mathcal{Y}_{\text{pred}}$ to specialize in representing
 913 different potential outcomes without interfering with one another during training. For one training
 914 instance, only the parameters responsible for the winning scenario are updated to better match the
 915 ground truth. For another instance with a similar history but a different future, a different scenario
 916 may become the winner, and its corresponding parameters will be updated. This dynamic encourages
 917 the model to maintain a diverse and comprehensive set of scenarios to cover the full spectrum of
 918 possibilities observed in the training data, directly leading to a lower expected Distortion.

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A.2.3 OPTIMAL PROBABILITIES VIA PROBABILITY LOSS

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The goal of our probability loss is to ensure that the learned probability vector \mathbf{p} accurately reflects the true probability mass over the Voronoi regions defined by the optimal scenarios.

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Proposition 2. *At the global minimum of the expected probability loss, the predicted probability vector \mathbf{p} matches the true conditional probability mass function over the Voronoi regions. That is, $p_n = P(\mathbf{y}_{gt} \in R_n | \mathbf{x})$, where R_n is the Voronoi region of the n -th scenario.*

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Proof. The optimization objective for the probability loss is to minimize the expected Cross-Entropy loss. We denote the cross-entropy between two discrete distributions \mathbf{q} and \mathbf{p} as $H(\mathbf{q}, \mathbf{p}) = -\sum_n q_n \log p_n$.

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$$\mathbb{E}[\mathcal{L}_{\text{prob}}] = \mathbb{E}_{\mathbf{x}} [\mathbb{E}_{\mathbf{y}_{gt} | \mathbf{x}} [-\log p_{n^*}(\mathbf{x}, \mathbf{y}_{gt})(\mathbf{x})]] = \mathbb{E}_{\mathbf{x}} [H(\mathbf{q}(\mathbf{x}), \mathbf{p}(\mathbf{x}))] \quad (19)$$

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Let $q(n | \mathbf{x}) = P(\mathbf{y}_{gt} \in R_n | \mathbf{x})$ be the true, unknown probability that the n -th scenario is the winner for a given history \mathbf{x} . The inner expectation corresponds to the cross-entropy between this true distribution $\mathbf{q}(\mathbf{x})$ and the model's predicted distribution $\mathbf{p}(\mathbf{x}) = \text{Softmax}(\boldsymbol{\pi}(\mathbf{x}))$. By the properties of cross-entropy:

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$$\mathbb{E}_{\mathbf{x}} [H(\mathbf{q}(\mathbf{x}), \mathbf{p}(\mathbf{x}))] = \mathbb{E}_{\mathbf{x}} [D_{KL}(\mathbf{q}(\mathbf{x}) \| \mathbf{p}(\mathbf{x}))] + \mathbb{E}_{\mathbf{x}} [H(\mathbf{q}(\mathbf{x}))] \quad (20)$$

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Since the entropy of the true distribution $H(\mathbf{q}(\mathbf{x}))$ is a constant with respect to our model's parameters, minimizing the expected cross-entropy is equivalent to minimizing the expected KL divergence. The KL divergence is non-negative and is minimized at zero if and only if $\mathbf{p}(\mathbf{x}) = \mathbf{q}(\mathbf{x})$ for all \mathbf{x} . Thus, the optimal solution for our probability output is the true probability distribution over the discrete set of winner outcomes.

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A.2.4 PROBABILITY MATCHING AND CRPS

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Our paradigm's ability to achieve strong performance on the Weighted CRPS metric is rooted in its direct optimization of a true probability distribution. As established in Proposition 2, the Cross-Entropy loss drives the model's output probability vector, $\mathbf{p} = \text{Softmax}(\boldsymbol{\pi})$, to match the true conditional probability mass function over the set of optimal scenarios. The objective is to minimize the Kullback-Leibler (KL) divergence between the predicted and true discrete distributions, $D_{KL}(\mathbf{q}(\mathbf{x}) \| \mathbf{p}(\mathbf{x}))$, where $\mathbf{q}(\mathbf{x})$ is the true distribution of winner outcomes. Since the Weighted CRPS directly incorporates the probability vector \mathbf{p} (equation 4), a model that learns a more accurate probability distribution is expected to achieve a lower (better) score.

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This approach provides a strong theoretical foundation for probabilistic modeling. The probability p_n for a scenario \mathbf{y}_n in our framework represents a holistic assessment of the entire trajectory, conditioned on the initial history. In contrast, autoregressive multi-hypothesis models like TimeMCL (Cortés et al., 2025), where scenarios, termed hypotheses in the original work, are generated step-by-step, face a challenge in aggregating pointwise confidences into a valid trajectory-level probability. For instance, consider two scenarios over a horizon of $T = 2$. Scenario A might have pointwise confidences of $(0.2, 0.2)$, while Scenario B has $(0.1, 0.3)$. Averaging these values, as is done for evaluation in TimeMCL, would assign both scenarios an identical score of 0.2. However, under the principles of conditional probability, their joint probabilities would be different (0.04 vs. 0.03), a distinction that simple averaging fails to capture. Furthermore, the set of scores produced by TimeMCL does not constitute a valid probability distribution as their sum is not constrained to be one.

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A.3 CONNECTION TO DISCRETE REPRESENTATION LEARNING

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As noted in our discussion on the model's effectiveness, the *Probabilistic Scenarios* paradigm shares conceptual roots with discrete representation learning techniques, most notably Vector Quantized Variational AutoEncoders (VQ-VAE) (van den Oord et al., 2017). Both approaches posit that continuous spaces can be effectively approximated by a finite set of discrete vectors. However, TimePrism distinguishes itself from VQ-VAE in three fundamental aspects, tailored for the forecasting task:

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- **Discretization Target:** VQ-VAE discretizes latent features, which serve as intermediate representations. In contrast, TimePrism directly discretizes future trajectories, operating within the final output space.

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- **Nature of Codebook/Scenarios:** VQ-VAE utilizes a static, global codebook shared across all inputs, where codes are fixed parameters learned from the entire dataset. Conversely, TimePrism generates a dynamic set of scenarios in real-time based on the input. These scenarios function as a conditional codebook that adapts to the specific history of each time series.
- **Probability Modeling:** VQ-VAE typically employs an implicit, two-stage approach that requires training a separate prior model over discrete codes to perform sampling and probability estimation. TimePrism, however, uses an explicit, end-to-end approach featuring a built-in probability head that directly outputs the probability distribution p over the generated scenarios in a single forward pass.

B METRICS

B.1 IMPLEMENTATION DETAILS

This section provides detailed formulations for our primary metrics, Weighted CRPS and Distortion, clarifying how they are applied to the outputs of both the Probabilistic Scenarios and sampling-based paradigms.

Weighted CRPS. Our implementation of the Continuous Ranked Probability Score is computed on a per-channel basis. For each variate $d \in \{1, \dots, D\}$, we calculate the score using the energy score formulation. Given the normalized ground truth for a single channel, $\mathbf{y}'_{\text{gt},d} \in \mathbb{R}^T$, a set of N normalized scenarios for that channel, $\{\mathbf{y}'_{n,d}\}_{n=1}^N$, and a corresponding probability vector for that channel, $\mathbf{p}_d = (p_{1,d}, \dots, p_{N,d})$, the per-channel Weighted CRPS is:

$$\text{CRPS}_d = \sum_{n=1}^N p_{n,d} \|\mathbf{y}'_{n,d} - \mathbf{y}'_{\text{gt},d}\|_1 - \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^N p_{n,d} p_{j,d} \|\mathbf{y}'_{n,d} - \mathbf{y}'_{j,d}\|_1 \quad (21)$$

where $\|\cdot\|_1$ denotes the L1 norm. The final reported CRPS score is the average of these per-channel scores. For sampling-based models, each of the S samples is assigned a uniform probability $p_{i,d} = 1/S$ for all channels.

Distortion. In contrast to CRPS, our Distortion metric is computed jointly across all dimensions to assess the quality of the entire multivariate trajectory. This aligns with its purpose of evaluating the coverage of the joint distribution. It is defined as the minimum Root Mean Squared Error (RMSE) over the set of complete multivariate scenarios:

$$\text{Distortion}(\mathcal{Y}, \mathbf{y}_{\text{gt}}) = \min_{\mathbf{y}_n \in \mathcal{Y}} \sqrt{\frac{1}{T \cdot D} \|\mathbf{y}_n - \mathbf{y}_{\text{gt}}\|_F^2} \quad (22)$$

where \mathcal{Y} represents the set of scenarios and $\|\cdot\|_F$ is the Frobenius norm. Note that the calculation is performed on normalized data as described above. For Probabilistic Scenarios, the minimization is performed over the complete set of N scenarios, $\mathcal{Y} = \mathcal{Y}_{\text{pred}}$. For sampling-based models, it is performed over the set of S generated samples, $\mathcal{Y} = \mathcal{Y}_{\text{samples}}$.

B.2 COMPREHENSIVENESS AND FAIRNESS

Scenarios and Probabilities. Our evaluation framework is comprehensive because its two primary metrics are complementary, addressing the two core components of a probabilistic scenario. The Weighted CRPS evaluates the quality of the entire predictive distribution, considering both the accuracy of the *scenarios* and the correctness of their assigned *probabilities*. Distortion, on the other hand, isolates the quality of the scenario set itself by focusing solely on its best-case coverage, irrespective of probability assignments.

Per-channel and Joint Evaluation. While our per-channel CRPS formulation is a standard approach (Zhang et al., 2024), it is known to be insensitive to errors in the correlation structure of a multivariate forecast (Marcotte et al., 2023). We specifically complement this with a jointly computed Distortion metric. Because Distortion evaluates the error over the entire $T \times D$ space for each scenario, it is

1026 sensitive to the quality of the multivariate structure, thus compensating for the limitations of the
 1027 per-channel CRPS.
 1028

1029 **L1 and L2 Norms.** The use of different norms for our two primary metrics is a deliberate design
 1030 choice. For Weighted CRPS, we use the L1 norm, which is standard for this metric and provides
 1031 robustness against outliers (Zhang et al., 2024). This is appropriate for a metric assessing the overall
 1032 distributional quality, where the influence of single extreme errors should be contained. For Distortion,
 1033 whose sole purpose is to measure the fidelity of the best available scenario, we use the L2 norm (via
 1034 RMSE), aligned with related work (Cortés et al., 2025). Its higher sensitivity to large deviations is a
 1035 feature, as it more strictly penalizes a model whose best-case scenario is still far from the ground
 1036 truth.
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1038 **Fairness.** Our evaluation framework is designed to be fair. The Continuous Ranked Probability Score
 1039 is a strictly proper scoring rule, meaning it is minimized in expectation if and only if the predicted
 1040 distribution coincides with the true data-generating distribution (Zhang et al., 2024). Our Weighted
 1041 CRPS, as an average of these strictly proper rules applied to the marginal distributions, inherits this
 1042 property for the set of marginals. Distortion, however, is not a strictly proper scoring rule as it only
 1043 considers the single best scenario. For this reason, it serves as a complementary, auxiliary metric
 1044 focused specifically on coverage, not as a complete measure of probabilistic quality.
 1045

1044 B.3 COMPLEMENTARY METRICS

1045 For a more comprehensive comparison, we also report on two complementary metrics: Mean Squared
 1046 Error (MSE) and Mean Absolute Error (MAE). These metrics are computed on the same per-channel
 1047 normalized data as our primary metrics to ensure a consistent evaluation scale. While these are
 1048 typically used for deterministic forecasting, we include their definitions and results in the Appendix
 1049 to align with standard practices in recent benchmarks (Zhang et al., 2024; Cortés et al., 2025).
 1050

1051 For our Probabilistic Scenarios paradigm, we derive a single representative forecast from the set of
 1052 scenarios by weighting them by their learned probabilities. For sampling-based models, this is the
 1053 standard mean or median of the samples.
 1054

Mean Squared Error (MSE). Following standard practice, the MSE is calculated based on the
 1055 **mean forecast**, $\hat{\mathbf{y}}_{\text{mean}}$. For a set of scenarios $\mathcal{Y}_{\text{pred}}$ with probabilities \mathbf{p} , this is the expectation of the
 1056 predictive distribution:
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$$\hat{\mathbf{y}}_{\text{mean}} = \sum_{n=1}^N p_n \mathbf{y}_n \quad (23)$$

1058 The MSE score is then the average of the per-channel Mean Squared Errors:
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$$\text{MSE} = \frac{1}{D} \sum_{d=1}^D \left(\frac{1}{T} \|\mathbf{y}_{\text{gt},d} - \hat{\mathbf{y}}_{\text{mean},d}\|_2^2 \right) \quad (24)$$

1060 **Mean Absolute Error (MAE).** The MAE is calculated based on the **median forecast**, $\hat{\mathbf{y}}_{\text{median}}$, which
 1061 is the 0.5-quantile of the predictive distribution. For a set of scenarios $\mathcal{Y}_{\text{pred}}$ with probabilities \mathbf{p} , the
 1062 weighted median is computed for each point in the trajectory. The MAE score is then the average of
 1063 the per-channel Mean Absolute Errors, where $\|\cdot\|_1$ denotes the L1 norm:
 1064

$$\text{MAE} = \frac{1}{D} \sum_{d=1}^D \left(\frac{1}{T} \|\mathbf{y}_{\text{gt},d} - \hat{\mathbf{y}}_{\text{median},d}\|_1 \right) \quad (25)$$

1073 C DATA AND EXPERIMENT DETAILS

1074 C.1 DATA ANALYSIS

1075 C.1.1 DATASET PROPERTIES

1076 We evaluate our approach on five widely-used benchmark datasets sourced from the GluonTS library
 1077 (Alexandrov et al., 2020), with preprocessing consistent with recent work (Cortés et al., 2025). As
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1082 Table 5: Dataset characteristics and properties.
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Dataset	Dim. D	Domain \mathcal{X}	Freq.	Time Steps	T	Trend	Seasonality	Non-Gaussianity
Sol.	137	\mathbb{R}^+	Hour	7,009	24	0.1688	0.8592	0.5004
Elec.	370	\mathbb{R}^+	Hour	5,833	24	0.6443	0.8323	0.3579
Exch.	8	\mathbb{R}^+	Day	6,071	30	0.9982	0.1256	0.2967
Traf.	963	(0, 1)	Hour	4,001	24	0.2880	0.6656	0.2991
Wiki.	2,000	\mathbb{N}	Day	792	30	0.5253	0.2234	0.2751

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1091 summarized in Table 5, these datasets span multiple domains and exhibit diverse characteristics in
1092 terms of dimensionality (Dim. D), data domain (\mathcal{X}), frequency, and length. To further characterize the
1093 data within the forecast horizon (T), we include three quantitative indicators from a recent benchmark,
1094 ProbTS (Zhang et al., 2024): trend strength (F_T), seasonality strength (F_S), and Non-Gaussianity.
1095 This selection allows for a comprehensive evaluation across a spectrum of time series properties,
1096 from low to high dimensionality and from strong periodicity to trend-dominated behavior.
1097

- 1098 • **Electricity (Elec.)** contains the hourly power consumption of 370 clients. It exhibits strong
1099 seasonality ($F_S = 0.83$) due to daily and weekly human activity patterns, along with a noticeable
1100 trend ($F_T = 0.64$).
- 1101 • **Exchange (Exch.)** records the daily exchange rates of eight currencies. As is common with
1102 financial data, it is heavily dominated by trend ($F_T = 0.99$) and shows very weak seasonality
1103 ($F_S = 0.13$).
- 1104 • **Solar (Sol.)** consists of the hourly solar power output from 137 locations. It has the strongest
1105 seasonality ($F_S = 0.86$) in our benchmark due to the clear day-night cycle, but a very weak
1106 underlying trend ($F_T = 0.17$). It also displays the highest non-Gaussianity.
- 1107 • **Traffic (Traf.)** measures the hourly occupancy rates of 963 road sensors. It shows moderate
1108 seasonality ($F_S = 0.67$) driven by daily rush-hour patterns, coupled with a relatively weak trend
1109 ($F_T = 0.29$).
- 1110 • **Wiki (Wiki.)** contains the daily page views for 2000 Wikipedia articles. As the most
1111 high-dimensional dataset, its series are characterized by a moderate trend ($F_T = 0.53$) but weak
1112 seasonality ($F_S = 0.22$).

1112 C.1.2 APPLICABILITY OF THE PROPOSED TIMEPRISM
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1114 Our proof-of-concept model, TimePrism, is built upon a backbone that decomposes the time series into
1115 trend and seasonal components. As shown in Table 5, all five benchmark datasets exhibit a significant
1116 presence of either trend or seasonality, providing a solid foundation for this decomposition-based
1117 architecture to perform well.

1118 However, it is crucial to distinguish the contributions of the paradigm from those of the specific
1119 backbone. The remarkable performance of TimePrism, achieving 9 out of 10 state-of-the-art results, is
1120 primarily attributable to the fundamental shift in the learning objective introduced by the Probabilistic
1121 Scenarios paradigm. By transforming the complex task of continuous density estimation into a
1122 more structured problem of learning a discrete distribution over a combinatorial scenario space, the
1123 paradigm itself simplifies the learning challenge. The decomposition backbone merely provides a
1124 simple yet effective way to generate the initial candidate scenarios for this paradigm.

1125 Consequently, while the current implementation of TimePrism might be less suitable for datasets
1126 where both trend and seasonality are weak, this does not diminish the validity of the underlying
1127 paradigm. The Probabilistic Scenarios framework itself makes no assumptions about the data's
1128 characteristics and can be integrated with more advanced backbones better suited for different data
1129 characteristics in future work.

1130 C.2 IMPLEMENTATION DETAILS OF PROPOSED TIMEPRISM
1131

1132 This section provides the exact formulations for the loss functions used to train TimePrism in the
1133 multivariate setting. The total loss, $\mathcal{L}_{\text{Prism}}$, is the sum of a reconstruction loss and a probability loss.

1134 For the multivariate case, the loss is computed on a per-channel basis and then averaged across all D
 1135 channels.

1136 For each channel $d \in \{1, \dots, D\}$, we first identify the channel-specific winner index, n_d^* :

$$1138 \quad n_d^* = \arg \min_{n=1 \dots N} \|\mathbf{y}_{\text{gt},d} - \mathbf{y}_{n,d}\|_2^2 \quad (26)$$

1140 The total reconstruction loss, incorporating the Relaxed-WTA mechanism, is the average of the
 1141 per-channel relaxed losses:

$$1142 \quad \mathcal{L}_{\text{recon}} = \frac{1}{D} \sum_{d=1}^D \left[(1 - \epsilon) \cdot \mathcal{L}_{n_d^*,d} + \frac{\epsilon}{N-1} \sum_{n \neq n_d^*} \mathcal{L}_{n,d} \right] \quad (27)$$

1146 where $\mathcal{L}_{n,d} = \|\mathbf{y}_{\text{gt},d} - \mathbf{y}_{n,d}\|_2^2$ is the MSE for the n -th scenario on the d -th channel.

1147 Similarly, the total probability loss is the average of the per-channel Cross-Entropy losses, where
 1148 each channel's probability distribution is optimized against its own winner:

$$1150 \quad \mathcal{L}_{\text{prob}} = \frac{1}{D} \sum_{d=1}^D \text{CrossEntropy}(\boldsymbol{\pi}_d, n_d^*) \quad (28)$$

1152 The following subsections provide a detailed motivation for the two key components of these loss
 1153 functions: the Relaxed-WTA mechanism and the per-channel, weight-sharing design.

1155 C.2.1 RELAXED WINNER-TAKES-ALL LOSS

1157 The motivation for the relaxed variant in Eq. equation 27 addresses a potential issue in the standard
 1158 WTA objective. In the standard formulation ($\epsilon = 0$), non-winning scenarios receive zero gradient for
 1159 a given training instance. This can lead to parameter stagnation if certain scenarios are consistently
 1160 not selected as winners across the dataset. By providing a small, non-zero gradient to all non-winning
 1161 scenarios (controlled by the hyperparameter $\epsilon = 0.01$ in our work), the relaxed loss ensures that all
 1162 parameters in the scenario-generating layers receive continuous updates, promoting more robust and
 1163 stable optimization (Rupprecht et al., 2017).

1164 C.2.2 WEIGHT SHARING

1166 To maintain the structural simplicity and lightweight nature of TimePrism, we adopt a weight-sharing
 1167 strategy for handling multivariate time series. Instead of learning a separate set of parameters for
 1168 each of the D variates, the three linear layers in our model (Trend, Season, and Probability layers)
 1169 share their weights across all variates. This design significantly reduces the total parameter count
 1170 (Zeng et al., 2023).

1171 As detailed in Eq. equation 28, TimePrism learns a separate probability distribution (parameterized
 1172 by $\boldsymbol{\pi}_d$) over the shared set of scenarios for each channel, rather than explicitly modeling the joint
 1173 probability distribution. However, the use of weight sharing allows the model to *implicitly* learn
 1174 cross-channel relationships during training. Because the weights of the linear layers are shared,
 1175 the gradient used to update them is an aggregation of the gradients from all D channels. This
 1176 forces the model to learn a basis of trend and seasonal components, along with their probabilistic
 1177 mappings, that is collectively useful for the entire multivariate system. Thus, while the model is fully
 1178 decoupled across channels during inference, the training process is coupled, enabling the simple
 1179 architecture to capture implicit cross-channel structures. This design choice directly explains the
 1180 model's performance on the high-dimensional (2000 variates) Wikipedia dataset. The weight-sharing
 1181 assumption is less likely to hold in datasets with high channel heterogeneity, where each series may
 1182 follow a distinct pattern. The observed lower performance on this specific dataset is therefore an
 1183 expected consequence of our intentionally simple, weight-sharing design, rather than a flaw in the
 1184 Probabilistic Scenarios paradigm itself.

1185 C.3 TRAINING PROCEDURE

1186 **Baseline Configurations.** The configurations for all baseline models, including DeepAR, TimeGrad,
 1187 TempFlow, Transformer TempFlow, TACTiS-2, and TimeMCL, adhere to the experimental setups

1188
 1189 Table 6: **MAE**. Results on five benchmark datasets, reported as the mean \pm standard deviation over
 1190 three random seeds. Lower is better. The best result is in **bold**, and the second best is underlined.
 1191

Model	Elec.	Exch.	Sol.	Traf.	Wiki.
ETS	0.577 ± 0.00	1.90 ± 0.05	0.558 ± 0.00	1.21 ± 0.00	4.81 ± 0.13
DeepAR	1.12 ± 0.02	1.09 ± 0.01	0.921 ± 0.04	1.36 ± 0.05	3.83 ± 0.96
TimeGrad	<u>0.369 ± 0.00</u>	1.33 ± 0.35	<u>0.383 ± 0.00</u>	<u>0.278 ± 0.00</u>	1.03 ± 0.03
TempFlow	0.633 ± 0.12	1.73 ± 0.27	0.451 ± 0.03	1.02 ± 0.00	2.74 ± 0.19
Trf.Flow	0.633 ± 0.12	1.73 ± 0.27	0.451 ± 0.03	1.02 ± 0.00	2.74 ± 0.19
Tactis2	0.467 ± 0.03	<u>1.02 ± 0.03</u>	0.388 ± 0.03	0.420 ± 0.02	0.944 ± 0.01
TimeMCL	0.519 ± 0.00	1.38 ± 0.21	0.431 ± 0.04	0.438 ± 0.03	1.23 ± 0.09
TimePrism	0.171 ± 0.03	0.666 ± 0.01	0.0832 ± 0.01	0.144 ± 0.00	0.995 ± 0.01

1203
 1204
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 1209
 1210 established in prior work (Cortés et al., 2025), encompassing model architecture, hyperparameters,
 1211 and other training details. In this work, TimeMCL is configured with $N = 16$ scenarios, consistent
 1212 with its original implementation (Cortés et al., 2025). We deem this a fair comparison because
 1213 TimeMCL’s autoregressive structure is computationally intensive. Even with only 16 scenarios, its
 1214 inference FLOPs (8.8×10^6) are an order of magnitude higher than TimePrism’s with 625 scenarios
 1215 (5.1×10^5). The original work presents two variants, relaxed-WTA (r-WTA) and annealed-WTA (a-
 1216 WTA). Based on their reported results in Table 1 of their work, the r-WTA variant achieved stronger
 1217 performance (3 first-place and 2 second-place results versus 2 second-place results for a-WTA).
 1218 Therefore, we use the more competitive r-WTA variant as our baseline. All other configurations for
 1219 TimeMCL are kept identical to the original work.

1220 **Batch Size and Scaler.** Following the setup in Cortés et al. (2025), all baselines are trained with a
 1221 batch size of 200, with the exception of TimeGrad, which uses a batch size of 100 on the Wikipedia
 1222 dataset due to memory constraints. For TimePrism, we use a batch size of 100 for all datasets except
 1223 Wikipedia, for which a batch size of 50 is used. While TimePrism has very low inference FLOPs, our
 1224 intentionally simple implementation is not optimized for memory efficiency, necessitating a slightly
 1225 smaller batch size on high-dimensional datasets. The data scaler configurations for all baseline
 1226 models are identical to those used in Cortés et al. (2025). For TimePrism, we use the ‘mean_std’
 1227 scaler for the Exchange dataset and the ‘mean’ scaler for all other datasets.

1228 **Proposed TimePrism Configuration.** The number of scenarios N in TimePrism is automatically
 1229 factorized into the two closest integers for the number of trend (M) and seasonal (K) components.
 1230 In our main experiments, N is set to 625, corresponding to a configuration of $M = 25$ and $K = 25$.
 1231 Given the hourly (24) and daily (30) frequencies of our datasets, we set the decomposition kernel size
 1232 to 7. An analysis of the effect of different values of N on performance is provided in a subsequent
 1233 appendix.

1234 **Historical Context Length.** Nominally, for datasets sourced from GluonTS, the input look-back
 1235 length is often set equal to the prediction horizon T (Zhang et al., 2024; Cortés et al., 2025; Alexandrov
 1236 et al., 2020). However, in practice, some models, like TimeMCL, are designed to use a longer history
 1237 by incorporating lagged features. Modifying these structural designs to only use an input of length
 1238 T would be complex and potentially unfair. We therefore adhere to their established configurations.
 1239 In contrast, our implementation of TimePrism requires only a look-back window of length T . It is
 1240 noteworthy that TimePrism achieves strong results even with less historical information, highlighting
 1241 the potential of the new paradigm. For a comprehensive comparison, we also provide results in a
 1242 subsequent appendix where TimePrism uses the full available history as input, which we term “Full
 1243 History”.

1242 Table 7: **MSE**. Results on five benchmark datasets, reported as the mean \pm standard deviation over
 1243 three random seeds. Lower is better. The best result is in **bold**, and the second best is underlined.
 1244

Model	Elec.	Exch.	Sol.	Traf.	Wiki.
ETS	0.519 ± 0.01	3.96 ± 0.30	0.455 ± 0.00	2.09 ± 0.01	550 ± 47.70
DeepAR	1.42 ± 0.07	1.47 ± 0.04	1.19 ± 0.07	1.87 ± 0.06	11.5 ± 4.00
TimeGrad	<u>0.278 ± 0.01</u>	2.43 ± 1.28	<u>0.361 ± 0.00</u>	<u>0.190 ± 0.00</u>	1.84 ± 0.10
TempFlow	3.84 ± 3.21	4.73 ± 2.14	0.463 ± 0.07	1.04 ± 0.01	676 ± 302.55
Trf.Flow	3.84 ± 3.21	4.73 ± 2.14	0.463 ± 0.07	1.04 ± 0.01	676 ± 302.55
Tactis2	0.366 ± 0.03	<u>1.23 ± 0.07</u>	0.365 ± 0.06	0.368 ± 0.02	<u>1.34 ± 0.19</u>
TimeMCL	0.393 ± 0.01	2.46 ± 1.11	0.542 ± 0.13	0.319 ± 0.02	13.7 ± 18.85
TimePrism	0.104 ± 0.02	0.712 ± 0.09	0.0769 ± 0.01	0.0983 ± 0.01	1.28 ± 0.02

D ADDITIONAL EXPERIMENTS

D.1 RESULTS OF COMPLEMENTARY METRICS

MAE. Table 6 presents the results for the Mean Absolute Error, reported as the mean \pm standard deviation over three random seeds (3141, 3142, 3143). As both MAE and our primary metric, CRPS, are based on the L1 norm, the overall ranking of the models shows a similar pattern. TimePrism achieves the best performance on four out of five datasets and the second-best on Wikipedia, reinforcing the conclusions from our main results and demonstrating its strong performance in terms of the median forecast.

MSE. The Mean Squared Error results are presented in Table 7, reported as the mean \pm standard deviation over three random seeds (3141, 3142, 3143). As a metric based on the L2 norm, the MSE is more sensitive to large errors or outliers. The results show a consistent pattern where TimePrism outperforms all baselines across all five datasets, demonstrating the robustness of its mean forecast even under a stricter, squared-error evaluation.

D.2 EXPERIMENTS ON HISTORY LENGTH CONFIGURATION

As discussed in the main text, some baseline models, such as TimeMCL (Cortés et al., 2025), are structurally designed to utilize a historical context longer than the nominal forecast horizon T by incorporating lagged features. Modifying these established architectures to only use an input of length T would be complex and potentially unfair. It is noteworthy that the main results for TimePrism are achieved using only this nominal input length T , demonstrating the potential of the new paradigm even with less information.

For a more direct comparison, we present an additional experiment in Table 8 where TimePrism uses the full available history, a variant we term "Full History" (Full His.). The length of this history is set to be comparable to the total context available to the baselines' data processing modules as in Cortés et al. (2025). The results show that using a longer history does not consistently improve TimePrism's performance; in some cases, the scores are similar or slightly worse, though still highly competitive. This is not a perfectly fair comparison, as other models are designed with feature engineering capabilities to extract value from long lagged inputs, while our simple linear model uses the full history directly. For such a simple architecture, a much longer input sequence can introduce noise without a sophisticated mechanism to filter it, which explains why more data does not necessarily lead to better performance.

This highlights a potential direction for future work, where more advanced feature engineering or model structures could be integrated within our paradigm to better leverage longer historical contexts.

1296 Table 8: **Main Results on Primary Metrics with Full History.** Comparison of Weighted CRPS and
 1297 Distortion on five benchmark datasets. Lower is better. The best result is in **bold**, and the second best
 1298 is underlined. TimePrism (Full His.) refers to our model using the full historical context for a more
 1299 direct comparison with baselines.

Model	Elec.		Exch.		Sol.		Traf.		Wiki.	
	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.
ETS	0.376	1.23	1.23	1.98	0.374	1.03	0.815	2.69	4.88	142
DeepAR	0.993	2.79	0.698	1.89	0.607	1.11	0.829	1.82	1.41	4.88
TimeGrad	0.230	0.720	0.739	1.33	0.237	0.587	<u>0.163</u>	0.540	0.516	1.62
TempFlow	0.449	1.73	0.988	1.55	0.278	0.555	0.613	1.01	1.81	71.6
Trf.Flow	0.449	1.73	0.988	1.55	0.278	0.555	0.613	1.01	1.81	71.6
Tactis2	0.285	0.637	0.641	0.919	<u>0.222</u>	0.567	0.243	0.55	0.481	1.37
TimeMCL	0.375	0.603	1.30	1.10	0.301	0.485	0.251	0.455	0.624	1.32
TimePrism	0.148	0.237	0.456	0.588	0.0835	0.140	0.109	0.140	0.508	1.01
TimePrism (Full His.)	<u>0.210</u>	<u>0.475</u>	<u>0.461</u>	<u>0.594</u>	0.224	<u>0.295</u>	0.184	<u>0.346</u>	<u>0.505</u>	<u>1.025</u>

1317 Table 9: **Generalizability Analysis with Transformer Backbone.** Comparison of Primary Metrics
 1318 (CRPS and Distortion) across five datasets. **TimePrism-iT** represents the iTransformer (Liu et al.,
 1319 2023) architecture adapted to our Probabilistic Scenarios paradigm. All experiments use Seed 3141.

Model	Elec.		Exch.		Sol.		Traf.		Wiki.	
	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.	CRPS	Dis.
DeepAR	0.993	2.79	0.698	1.89	0.607	1.11	0.829	1.82	1.41	4.88
TimeGrad	<u>0.230</u>	0.720	0.739	1.33	0.237	0.587	<u>0.163</u>	0.540	0.516	1.62
TempFlow	0.449	1.73	0.988	1.55	0.278	0.555	0.613	1.01	1.81	71.6
Trf.Flow	0.449	1.73	0.988	1.55	0.278	0.555	0.613	1.01	1.81	71.6
Tactis2	0.285	0.637	0.641	0.919	0.222	0.567	0.243	0.55	0.481	1.37
TimeMCL	0.375	0.603	1.30	1.10	0.301	0.485	0.251	0.455	0.624	1.32
TimePrism	0.148	0.237	<u>0.456</u>	0.588	0.0835	0.140	0.109	0.140	0.508	1.01
TimePrism-iT	0.330	0.600	0.454	0.681	0.164	<u>0.245</u>	0.201	<u>0.371</u>	0.756	1.425

D.3 PARADIGM GENERALIZABILITY: ADAPTING TO TRANSFORMER ARCHITECTURES

1338 To more rigorously validate that the superior performance of our method stems from the proposed
 1339 **Probabilistic Scenarios** paradigm rather than solely the specific linear architecture of TimePrism, we
 1340 conducted a controlled study adapting a distinct, complex architecture to our framework. We selected
 1341 one of the state-of-the-art Transformer-based time series models, iTransformer (Liu et al., 2023), as
 1342 the backbone.

1343 **Experimental Setup.** We developed a variant named TimePrism-iT, where the linear encoder of
 1344 TimePrism is replaced by the inverted Transformer structure from Liu et al. (2023). Crucially, to
 1345 demonstrate the "out-of-the-box" applicability and robustness of our paradigm, we *did not* perform
 1346 extensive hyperparameter tuning for TimePrism-iT. Instead, we applied a generally consistent config-
 1347 uration across all datasets. This setup serves as a rigorous stress test to verify if the paradigm can
 1348 yield performance gains without relying on architecture-specific optimization.

1349 **Results Analysis.** The comparative results are presented in Table 9. Despite being an unoptimized
 implementation, TimePrism-iT demonstrates remarkable performance. It outperforms standard

1350
1351 Table 10: The results of models in datasets from GIFT-Eval (Aksu et al., 2024) and fev-bench Shchur
1352 et al. (2025). The best result in each column is in **bold**.

1353 1354 1355 1356 1357 1358 1359 1360	Dataset		UCI		Hosp.		Hier.		M-Den.	
	Metrics	CRPS	Distortion	CRPS	Distortion	CRPS	Distortion	CRPS	Distortion	
1361	ETS	0.450	0.811	0.585	1.30	0.990	4.03	0.782	2.23	
1362	Tactis2	0.605	0.787	0.583	1.20	0.623	1.58	0.614	1.13	
1363	TimeMCL	0.359	0.449	0.722	1.13	1.08	1.33	0.771	0.597	
1364	TimePrism	0.261	0.394	0.565	1.06	0.602	1.03	0.907	1.11	

1365 baselines on 6 out of 10 metrics across the five datasets. Notably, on the **Exchange** dataset, TimePrism-
1366 iT achieves a CRPS of **0.454**, slightly surpassing even the original linear TimePrism (0.456).

1367 D.4 EXTENDED EVALUATION ON ADDITIONAL BENCHMARKS

1368 To provide a more comprehensive evaluation of our proposed paradigm, we extended our experiments
1369 to include four datasets selected from two latest benchmarks: **Gift-Eval** (Aksu et al., 2024) and
1370 **fev-bench** (Shchur et al., 2025). These datasets were chosen to cover diverse domains: **Hierarchical**
1371 **Sales** (Retail, abbr. Hier.), **M-DENSE** (Mobility, abbr. M-Den.), **Hospital Admissions** (Healthcare,
1372 abbr. Hosp.), and **UCI Air Quality** (Nature, abbr. UCI).

1373 **Experimental Setup.** For these experiments, we selected the numerical baseline **ETS** and the two
1374 most competitive neural models from Table 1 and Table 2, namely **TimeMCL** and **Tactis2**, for
1375 comparison. All models were evaluated using a random seed of 3141 to ensure reproducibility.

1376 **Results Analysis.** As shown in the additional results in Table 10, TimePrism maintains its strong
1377 performance across these new domains. Regarding the M-DENSE dataset, we observed that TimePrism
1378 exhibits relatively higher distortion. We hypothesize that the nature of this dataset may be more
1379 suitable for RNN backbones, as both Tactis-2 and TimePrism perform suboptimally on this dataset,
1380 while TimeMCL remains competitive. This is not a limitation of our new paradigm, but rather a
1381 consequence of TimePrism’s simple structure. Nevertheless, achieving SOTA results in 15 out of 18
1382 metrics across 9 datasets still demonstrates the effectiveness of the TimePrism model and highlights
1383 the potential of the new paradigm.

1384 D.5 PROBABILITY CALIBRATION DIAGNOSTICS

1385 To rigorously assess the reliability of the probabilities assigned by TimePrism, we employ two
1386 standard diagnostic tools: the **Reliability Diagram** (Coverage vs. Nominal Confidence) and the
1387 **Probability Integral Transform (PIT) Histogram**.

1388 **Methodology.** Since TimePrism outputs a tuple $(\mathcal{Y}_{\text{pred}}, \mathbf{p})$ consisting of a finite set of scenarios
1389 $\mathcal{Y}_{\text{pred}} = \{\mathbf{y}_n\}_{n=1}^N$ and their associated probabilities $\mathbf{p} = (p_1, \dots, p_N)$, we compute these metrics as
1390 follows:

- 1391 • **PIT Histogram:** For a ground truth observation \mathbf{y}_{gt} , the PIT value is the cumulative
1392 probability of scenarios that are less than or equal to the observation: $\text{PIT} = \sum_{n=1}^N p_n \cdot \mathbb{I}(\mathbf{y}_n \leq \mathbf{y}_{\text{gt}})$. For a perfectly calibrated model, the distribution of PIT values over the test
1393 set should approach a Uniform distribution $U[0, 1]$, resulting in a flat histogram.
- 1394 • **Reliability Diagram:** We calculate the empirical coverage for varying nominal confidence
1395 levels $\alpha \in [0, 1]$. The prediction interval for a level α is constructed by aggregating the
1396 scenarios \mathbf{y}_n with the highest probabilities until their cumulative sum reaches α . If the model
1397 is well-calibrated, the curve should align with the diagonal $y = x$. Curves above the diagonal
1398 indicate under-confidence (conservative), while curves below indicate over-confidence.

1399 **Analysis.** We performed these diagnostics on two representative datasets: Exchange and Solar. The
1400 results are visualized in Figure 5.

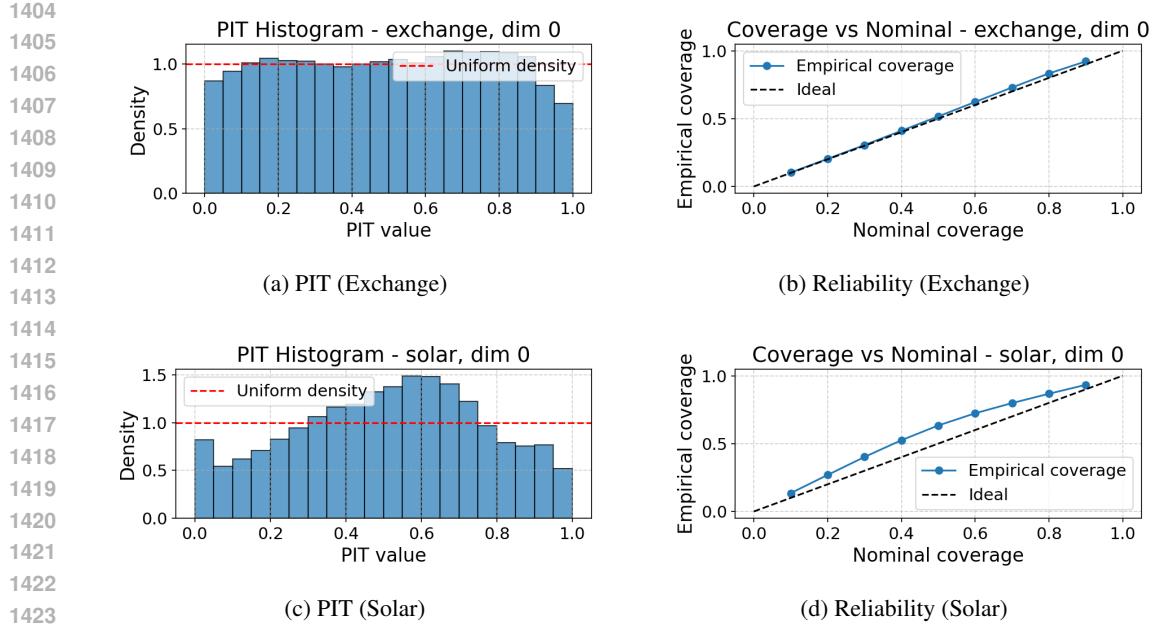


Figure 5: **Calibration Diagnostics.** The diagnostics show different behaviors across datasets: (a)(b) Exchange dataset demonstrates near-perfect calibration; (c)(d) Solar dataset exhibits a slightly conservative profile to ensure robust tail coverage.

- **Exchange Dataset (Fig. 5a & 5b):** The diagnostics indicate near-perfect calibration. The PIT histogram is remarkably flat, and the Reliability Diagram closely follows the ideal diagonal line. This suggests that for stable financial data, TimePrism accurately estimates the true uncertainty distribution.
- **Solar Dataset (Fig. 5c & 5d):** The diagnostics exhibit a slightly conservative profile. The PIT histogram shows a mild hump shape, and the Reliability curve lies slightly above the diagonal. This behavior is expected and often desirable for highly stochastic, multimodal data like Solar energy. It indicates that TimePrism tends to widen its predicted scenario distribution to safely encompass multimodality and potential outliers. This "conservative" strategy ensures robust coverage of low-probability, high-impact tail events without becoming over-confident, aligning with our design goal of prioritizing coverage adequacy.

D.6 ADDITIONAL VISUALIZATION AND QUALITATIVE ANALYSIS

Window Selection Rule. To provide a fair and insightful qualitative comparison, we developed a systematic rule for selecting the windows to be visualized. For a given dataset and variate, we first select a query window from a recent part of the historical data. We then search through the entire history to find the five past windows that are most similar to this query window, based on Euclidean distance. To ensure that the selected windows represent distinct, non-overlapping events, we enforce a minimum temporal separation between them. This greedy, iterative process allows us to identify a set of instances where the model is repeatedly faced with a similar historical context, providing a controlled setting to analyze its predictive behavior.

D.6.1 VISUALIZATIONS ON OTHER DATASETS

To further demonstrate the applicability of our paradigm, we provide additional qualitative results for TimePrism on the Electricity and Traffic datasets in Figure 6. The top panel showcases forecasts for the Electricity dataset. Across the selected windows, the model successfully generates a diverse set of scenarios that cover the volatile and complex patterns of power consumption, assigning higher probabilities (thicker, blue lines) to the most plausible outcomes. The bottom panel of Figure 6 displays the results for the Traffic dataset. Here, the model also produces a sharp and well-calibrated set of scenarios that effectively captures the distinct peaks and troughs characteristic of traffic flow

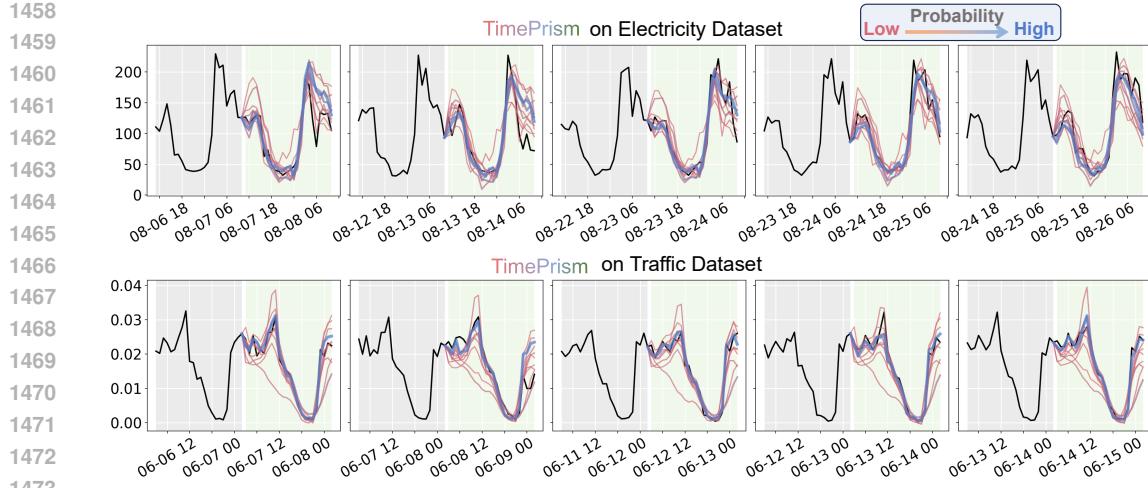


Figure 6: **Qualitative Analysis on Electricity and Traffic Datasets.** Visualization of TimePrism’s probabilistic scenarios on two additional benchmark datasets. The top row shows forecasts for the Electricity dataset, and the bottom row shows forecasts for the Traffic dataset.

data. These visualizations further confirm that the Probabilistic Scenarios paradigm can generate meaningful forecasts across different domains and data characteristics.

D.6.2 FULL COMPARISON ON SOLAR

We now present a full visual comparison of all neural network-based baselines against TimePrism on the Solar dataset. We select two representative variates for this analysis: the first ($D = 1$) and the last ($D = 137$). The figures display the top 10 scenarios from TimePrism, with line color and thickness representing probability from low (red, thin) to high (blue, thick), and 100 samples from each baseline model. The historical context is shown with a gray background, while the future prediction horizon has a light green background.

Figure 7 shows the results for the first variate of the Solar dataset. Across the five selected windows, we observe several *Common Cases* of high-peak solar generation, along with two *Rare Cases* (third and fourth from the left) that exhibit more volatile or lower-peak behavior. For the common cases, TimePrism correctly assigns high probabilities (thicker, blue lines) to scenarios that accurately match the ground truth. Crucially, for the rare cases, it successfully identifies and covers these less frequent patterns while correctly assigning them lower probabilities (thinner, redder lines). In contrast, the sampling-based models, including the strong baseline TACTiS-2, tend to produce a cloud of samples centered around an average forecast. This often results in a mean forecast that matches neither the common nor the rare cases well, and the sample envelope may fail to adequately cover the true outcome in the rare cases, demonstrating the limitations of **Probability Absence** and **Coverage Inadequacy**.

Figure 8 presents the analysis for the last variate of the dataset. This example provides a clear distinction between four *Common Cases* and one *Rare Case* (far right). TimePrism again demonstrates the strength of the Probabilistic Scenarios paradigm: it allocates the majority of its probability mass to accurately predict the common high-peak cases, while still generating a low-probability scenario that correctly captures the rare low-peak future. The sampling-based models, however, struggle with this scenario. Their samples tend to cluster around a mean that represents an uninformative compromise between the high and low peaks. This visually exemplifies how a forecast lacking explicit probabilities can fail to provide actionable insights for decision-making, especially when preparing for rare but critical outcomes.

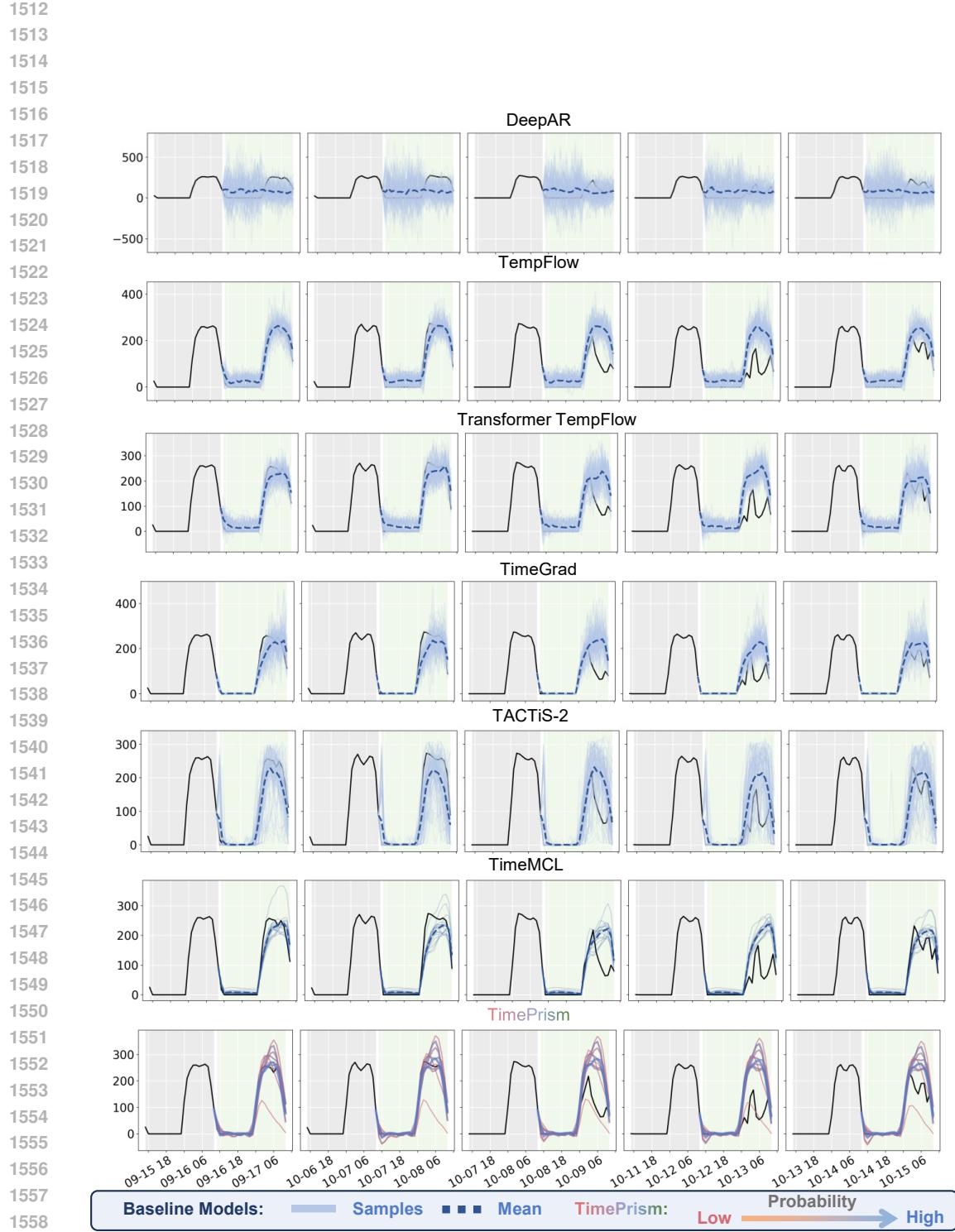


Figure 7: **Qualitative Analysis on Solar (D=1).** A visual comparison of forecasts from all neural network-based models on the first variate of the Solar dataset. The figure highlights performance on both common high-peak cases and two rare cases.

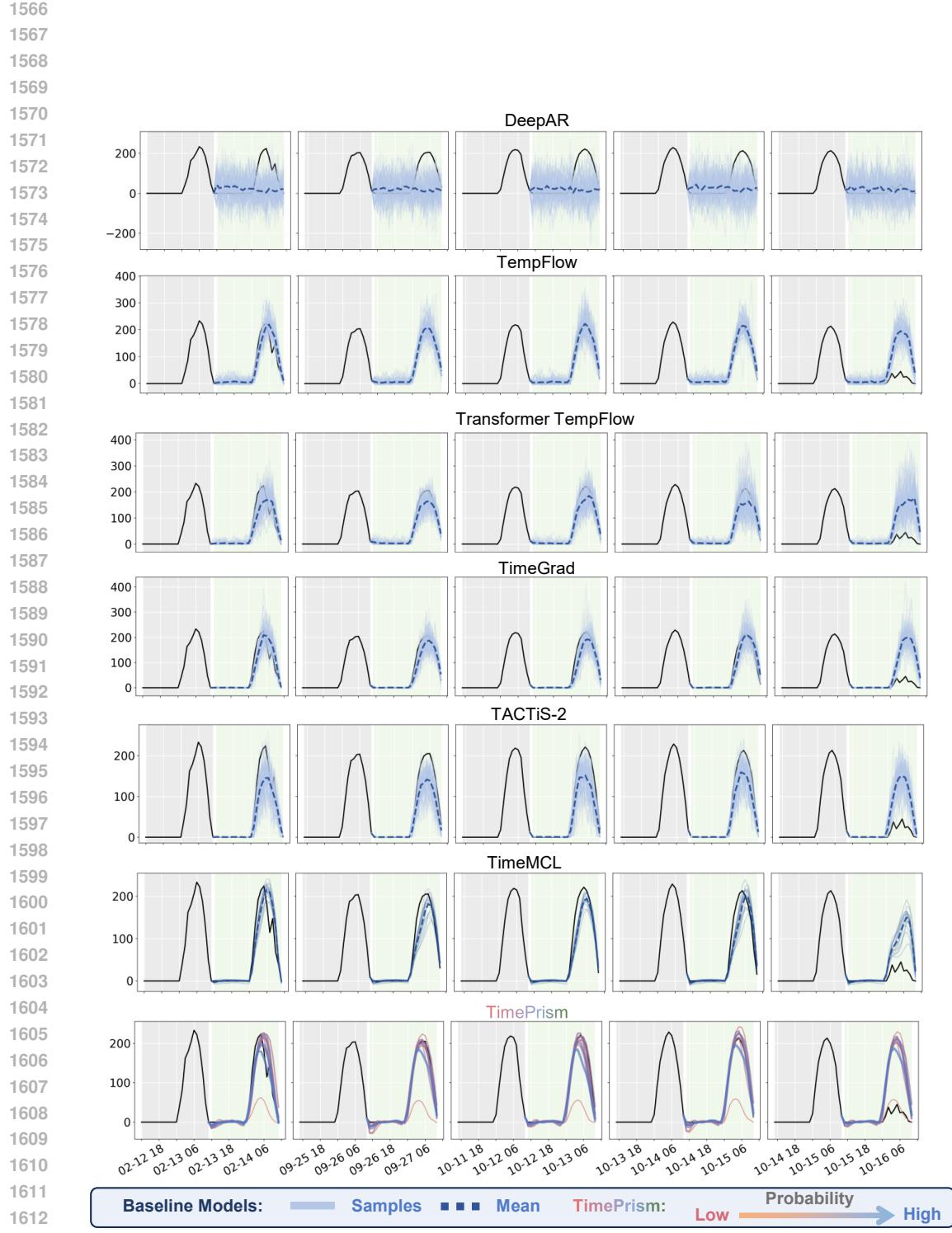


Figure 8: **Qualitative Analysis on Solar (D=137).** A visual comparison on the last variate of the Solar dataset. This case clearly distinguishes between four common high-peak cases and one rare low-peak case.