LAYERMIX LAW: SCALING LAW FOR LARGE LANGUAGE MODELS ON QUALITY-WEIGHTED MIXTURE DATA WITH REPETITION

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ABSTRACT

Upweighting high-quality data in large language model (LLM) pretraining typically improves performance. However, the limited availability of high-quality data—particularly in overtrained regimes—means that stronger upweighting often increases repetition, which can degrade performance. This creates a fundamental trade-off between data quality and data repetition. In this paper, we systematically investigate how varying data quality and repetition affects models across different scales. Concretely, we partition the source corpus into buckets based on quality scores and sample from each bucket with different weights, thereby constructing training sets with diverse scales, quality distributions, and repetition levels. We then train a family of models on these datasets to measure performance across conditions. Building on these observations, we introduce a theoretical framework analogous to scaling laws, which we call **LayerMix Law**. LayerMix Law predicts model loss as a function of consumed tokens, model size, sampling weights, and repetition levels. The key intuition is to view training as the accumulation of information from data, where the amount of information is governed by data quality, while model scale and repetition determine the information gained per training step. We show that LayerMix Law accurately predicts the model performance on unseen data recipes at larger computation scale (up to 7B parameter run with 425B token, each x2 invest compute), with 0.15% average absolute error and 0.96% maximum absolute error, which enables efficient search for optimal data recipes without costly additional experiments. Moreover, LayerMix Law extrapolates reliably to different degrees of overtraining, providing a efficient tool for selecting data recipes under varying computational budgets.

1 INTRODUCTION

Training large language models (LLMs) requires access to high-quality data (Brown et al., 2020a; Chowdhery et al., 2023). However, the availability of high-quality data is severely limited (Villalobos et al., 2024), and in the data-constrained settings, upweighting higher-quality data inevitably increases repetition, which has been shown to impair performance when excessive (Muennighoff et al., 2023). This issue is further exacerbated by the widespread adoption of overtraining (Touvron et al., 2023; Yang et al., 2025)—a strategy that reduces inference costs compared to the compute-optimal regime (Hoffmann et al., 2022).

To address the shortage of high-quality data as model scale increases, a common compromise is to incorporate lower-quality data, thereby reducing the repetition of high-quality samples. Intuitively, high-quality data provides greater performance gains than low-quality data upon first exposure, but as repetition increases, the marginal benefit decays—eventually approaching that of unseen low-quality data. However, the optimal balance between quality and repetition remains unclear. A standard approach for identifying optimal mixing strategies is to run smaller-scale experiments and extrapolate performance to larger compute budgets using scaling laws (OpenAI et al., 2024; Hoffmann et al., 2022; Chowdhery et al., 2023). Yet, as shown in Figure 1, under conditions of data repetition, standard scaling laws fail to reliably predict model performance at scale (Hernandez et al., 2022; Muennighoff et al., 2023). Moreover, they do not generalize across different mixing

strategies , necessitating grid searches over data recipes—an approach that is costly even at small scales.

In this paper, we study the problem of scaling large language models in a data-aware regime, where training data consists of a heterogeneous mixture with varying quality levels, and each quality level is repeated to different extents. We introduce a theoretical framework, the LayerMix Law, which accounts for both the scaling effects of mixture weights and the impact of repetition. Our formulation views training as a process of accumulating information from the dataset, with model performance determined by the total information gained by the end of training. At each step, the information gain is modeled as the sum of contributions from different quality ranges. Within each quality range, the gain depends on two factors: an information density function, parameterized by quality (with higher quality assigned higher density), and an exponential decay term that captures the interactions between model scale, data scale, and repetition level.

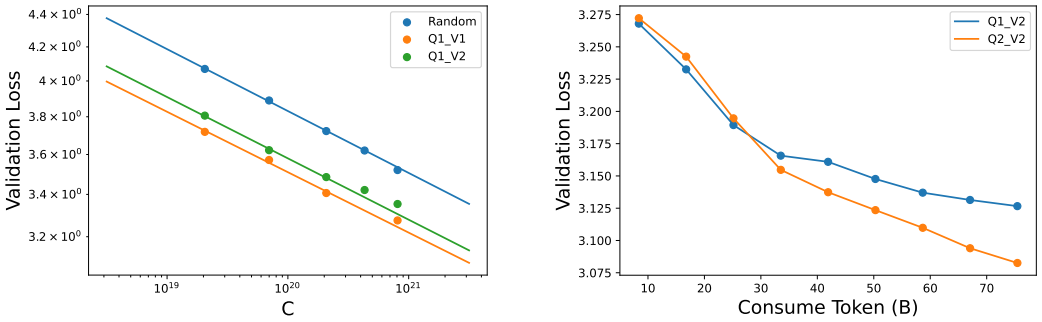
To fit the parameters of the LayerMix Law, we construct a suite of datasets that vary along three axes: scale, quality, and repetition level. Specifically, we partition the source dataset into buckets according to quality scores, and then sample from each bucket with different weights, a procedure we refer to as LayerMix sampling. Following the data-constrained setting, the source dataset is first downsampled to the target scale to ensure stable repetition effects. We then train 9 models ranging from 252M to 1.2B parameters from scratch, each under the same 3.6x over-trained ratio (Gadre et al., 2024). For each model, we construct three datasets with distinct LayerMix sampling configurations, resulting in 27 total training runs. Model performance is evaluated as the average perplexity across five downstream tasks. Finally, we fit the LayerMix Law to these results, estimating the parameters that best capture the relationship between information gain and observed performance.

To verify the generalization ability of LayerMix Law, we conduct three extrapolate experiments. Firstly we create datasets with two other unseen LayerMix sampling parameters to test the extrapolation on unseen data recipe. Secondly, we conduct experiments with larger computation scale on both seen and unseen LayerMix sampling parameters to test the scaling ability. Thirdly, we conduct experiments with larger over-trained ratio (25x), to test the generalization ability of LayerMix Law parameters. We found that LayerMix Law is able to predict loss on unseen data recipes at different scales (up to 7B parameter run with 425B token, each x2 invest compute), with 0.15% average absolute error and 0.96% maximum absolute error. We search for the optimal data recipe on 2.5B model with LayerMix Law with no additional experiments, and the result achieves the best among 4 other random ablations. Also on over-trained ratio, directly apply LayerMix Law yields good scaling results.

2 Related Work

Scaling Laws Scaling of transformer language models (Vaswani et al., 2017) and training data has been shown to provide consistent performance improvements (Chowdhery et al., 2023; Radford et al., 2019). This has led to the development of many large-scale models, including both dense architectures (Brown et al., 2020b; Rae et al., 2021; Grattafiori et al., 2024) and mixture-of-experts (MoE) variants (DeepSeek-AI et al., 2025; Yang et al., 2025; Fedus et al., 2021). Early empirical studies observed that neural networks exhibit predictable power-law scaling behavior (Hestness et al., 2017). Building on this, Hoffmann et al. (2022) investigated the compute-optimal setting, suggesting that model size and training data should be scaled in roughly equal proportions, whereas Kaplan et al. (2020) proposed a different allocation strategy emphasizing alternative trade-offs. More recent work, such as DeepSeek-AI et al. (2024), further explored how compute budget C interacts with optimization hyperparameters, including the choice of batch size and learning rate.

Recently, there has been a growing trend of over-training smaller models on large datasets (Touvron et al., 2023; Yang et al., 2025), motivated by both efficiency considerations and deployment constraints. Sardana et al. (2024) extended the Chinchilla framework by incorporating data quality and inference requirements, deriving optimal allocations between model size and dataset scale under these additional factors. Complementary to this, Gadre et al. (2024) demonstrated that scaling laws remain reliable even in the over-trained regime, where models are trained significantly beyond the compute-optimal point. These findings highlight the importance of revisiting scaling strategies in regimes constrained by data availability, quality variation, or inference efficiency demands.



(a) *Loss-C* Curve of model trained with random data and Layermix packed data

(b) The validation loss of different LayerMix experiments in training process.

Figure 1: Effects of quality selection and data repetition

For predicting downstream performance, Isik et al. (2025) demonstrated that downstream metrics also exhibit predictable scaling effects after fine-tuning, extending the scope of scaling laws beyond pre-training loss. Schaeffer et al. (2023) further established a connection between non-linear evaluation metrics and model perplexity, providing a more stable predictor of performance compared to prior approaches such as Wei et al. (2022), which observed instability in emergent metrics.

Data-Aware Scaling Traditional scaling laws typically assume that training data is both fixed and unlimited. In practice, however, high-quality data is scarce and often upsampled to improve model performance (Lin et al., 2022). Under such data-constrained settings, classical scaling laws fail to accurately predict performance at larger scales. For example, Hernandez et al. (2022) demonstrated that upsampling a small fraction (1%) of the dataset up to 100 times leads to substantial performance degradation. Similarly, Muennighoff et al. (2023) showed that repeating the entire dataset across multiple epochs causes performance to deteriorate as training progresses. More recently, Chen et al. (2025) studied how sub-scaling laws interact with data density, providing a finer-grained understanding of scaling under limited-data regimes.

Other lines of work have applied scaling laws to guide the design of optimal data recipes. For instance, Ye et al. (2025) incorporated data mixture weights into the functional form of loss prediction, while Kang et al. (2025) suggested that optimal mixing strategies themselves depend on model scale. Gu et al. (2024); Que et al. (2024) investigated scaling laws in the context of continued pre-training, using them to inform domain mixture strategies. In addition, Chang et al. (2024) analyzed how scaling interacts with data quality. In contrast to these approaches, our work aims to predict model loss on mixtures of varying quality and repetition levels, thereby providing a more general framework for data-aware scaling.

3 Limitations of Conventional Scaling Laws

In this section, we reveal and substantiate a critical limitation of conventional scaling laws in the context of data repetition and quality selection. First, we introduce the LayerMix sampling function in section 3.1, to imitate real scenario where the data is a mixture of different quality and repetition degrees. Next, we compare the relationship between the model's loss L and amount of compute C in cases with and without repetition in section 3.2, and the results show that the traditional scaling law performs well on data without repetition

3.1 LayerMix Sampling Function

Source Data We obtain our training corpora from Common Crawl (Common Crawl Foundation), following similar filtering process as Penedo et al. (2023) and obtain 15T English tokens. We ran global fuzzy deduplication across all snapshots to ensure there is no repeat data in the corpora. The final dataset contains 3.7T token. The details are in Appendix A.

Training Data Sampling We first define the quality score for each document following Liu et al. (2025), where two quality classifiers (Penedo et al., 2024; Li et al., 2025) are applied and the final quality score is averaging the normalized score from each classifier. With the quality score, We then rank all documents based on quality score and empirically split the training corpora into six quality buckets based on their quality rankings, which are 0-5%, 5%-20%, 20%-40%, 40%-60%, 60%-80%, 80%-100%.

Then we define a LayerMix sampling function $H(w, K, S, B)$, where K represents the token for training data, and we perform 1 epoch training to avoid additional repetition factors, $w = [w_0, w_1, ..., w_5]$ and $\sum w_d = 1$, representing the token proportion of the data in the d-th bucket in the training data. $B = [B_0, B_1, ..., B_5]$ is the range of each data bucket. Then for each document in bucket d, the expectation appearance times in the training data is $E_d = \frac{w_d K}{B_d S}$. If $E_d < 1$, the document will appear in the training data with probability E_d. The detailed algorithm is shown in Appendix B.

With different setting of w, K, S, the LayerMix sampling function generate datasets with different scales, quality and degrees of repetition. We ensure w_d is greater than w_{d+1} to guarantee higher quality buckets have a larger proportion in the training data. And w_5 is set to 0 so that we drop the bottom 20% of data ranked by quality score. Throughout work, we set $K = S$ to reduced-complexity unless mentioned, so that we only focus on the repetition change caused by w.

3.2 Traditional Scaling Law Between Loss and Amount of Compute

We compare the relationship between the model's loss L and amount of compute C in cases with and without repetition in this section. We conduct experiments under a 3.6x over-trained settings following the definition in Gadre et al. (2024), where $K = \sqrt{m}K^*$ is the consume token, $N = \frac{1}{\sqrt{m}}N*$ is the non-embedding FLOPs per token as defined in DeepSeek-AI et al. (2024), K^*, N^* is the compute-optimal point (Hoffmann et al., 2022) related with $C = N^*K^*$, and $m = 3.6$. Experiments are run on different compute C on three kinds of dataset.

Random The training data is randomly sampled from sufficient large source data, where $S \gg K$, meaning there is rarely no repetition in the training data.

Q1_V1 We use the layermix sampling function and w is set to mainly focus on high quality data, namely Q1, see details in Appendix B. Then we set $S \gg K$, denoting as V1, meaning there is no repetition in train data

Q1_V2 We use the layermix sampling function and set $S = K$, denoting as V2, where there exists repetition in the training data.

We plot the log-log plot between C and model loss in Figure 1a. Note that the loss L mentioned here, including later references, is the average loss of the model on the following five downstream tasks: HellaSwag (Zellers et al., 2019), ARC-E, ARC-C (Clark et al., 2018), MMLU (Hendrycks et al., 2021), TriviaQA (Joshi et al., 2017). Following Schaeffer et al. (2023), we transfer the accuracy on downstream tasks into perplexity for better scaling effect. The results show that for random dataset, the scaling law loss-C curve can fit all of the data points. But for Q1_V1 and Q1_V2, the performance decay as compute C scales.

The above observation indicating that traditional scaling law only holds for completely random and non-repeated data. The change of data quality distribution or repeate time undermines its predictive accuracy. So we need a modified scaling law that incorporates both data quality distrubution and the degree of data repetition as core variables.

4 LayerMix Law

Scaling laws for large language models traditionally rely on compute, but often fail to account for effects of data quality and repetition. In this section, we introduce the design of LayerMix Law. We treat the training process as gaining information from the dataset and propose to calculate Information Quantity as accumulation of information gain throughout the training process, which

synthesizes the impacts of data quality, repetition level, model scales and total training tokens, and then build power-law relationship with the model's final validation loss.

4.1 Information Quantity

We first show the evaluation loss in training process of two 850M models trained on two datasets packed with different LayerMix sampling weights in Figure 1b. The dataset $Q1\_V2$ repeates top 5% quality data for about 16 times and $Q2\_V2$ for 10 times. Initially, the $Q1$ and $Q2$ experiments achieves almost same evaluation loss. However, the loss of the more repetitive $Q1$ experiment decreases much more slowly in the later stages, resulting in a worse final performance compared to the $Q2$ experiment, indicating that repeatedly learning from the same data provides progressively smaller gains.

Based on this observation, we propose an exponential decay function to model the decreasing information gain of repeated data. Assuming the Information Quantity a document i contains is I_i, then the information a language model gets at t-th learning from the document i is:

$$I_{i\_\text{part}}(t, \lambda_N) = I_i \cdot \lambda_N e^{-\lambda_N t} \tag{1}$$

where λ is a hyperparameter which is related to the non-embedding FLOPs/token N.

When a languge model learning the document for total T times, the Information Quantity learned from the document is:

$$I_{i\_\text{total}}(T, \lambda_N) = \int_0^T I_{i\_\text{part}}(t, \lambda_N) dt = I_i \cdot (1 - e^{-\lambda_N T}) \tag{2}$$

Equation 2 captures the principle of diminishing returns in learning: repeated exposure to a document yields progressively smaller gains, causing the total acquired information to saturate and asymptotically approach the document's full information content I_i.

Considering in large-scale training, due to the forgetting effect: as the total train token K grows, the average Information Quantity language model acqiure from a singel sample decreases. We introduce train token K to the Equation 1 to accommodate this phenomenon:

$$I_{i\_\text{part}}(t, \lambda_N, K) = I_i \cdot \lambda_N e^{-\lambda_N t / \log(K)} \tag{3}$$

Then the Equation 2 becomes to:

$$I_{i\_\text{total}}(t, \lambda_N, K) = \int_0^T I_{i\_\text{part}}(t, \lambda_N, K) dt = I_i \cdot \log(K)(1 - e^{-\lambda_N T / \log(K)}) \tag{4}$$

For all the training data, we sum them together as the final Information Quantity the language model learned from the training corpora, denoting as $info$:

$$\begin{aligned} info(w, K, S, f, \lambda_N) &= \sum_d I_d \cdot \log(K)(1 - e^{-\lambda_N R_d / \log(K)}) \\ &= \sum_d f_d M_d \log(K) \cdot (1 - e^{-\lambda_N R_d / \log(K)}) \end{aligned} \tag{5}$$

where d is the quality bucket number from 0 to 5, $M_d = min(w_d K, B_d S)$ is the packed unique token from the d-th quality bucket, f_d refers to the quality density of the d-th quality bucket, and $R_d = \frac{w_d K}{M_d}$ is the average repeat times for the data from the d-th bucket. f_d is parameterized quality density function and λ_N is related with N, which are to be fitted from the data.

Equation 5 can be divided into two parts: the first term is $I_d = f_d M_d \log(K)$, it represents the total Information Quantity contained in the packed data of the d-th bucket, and the second term is

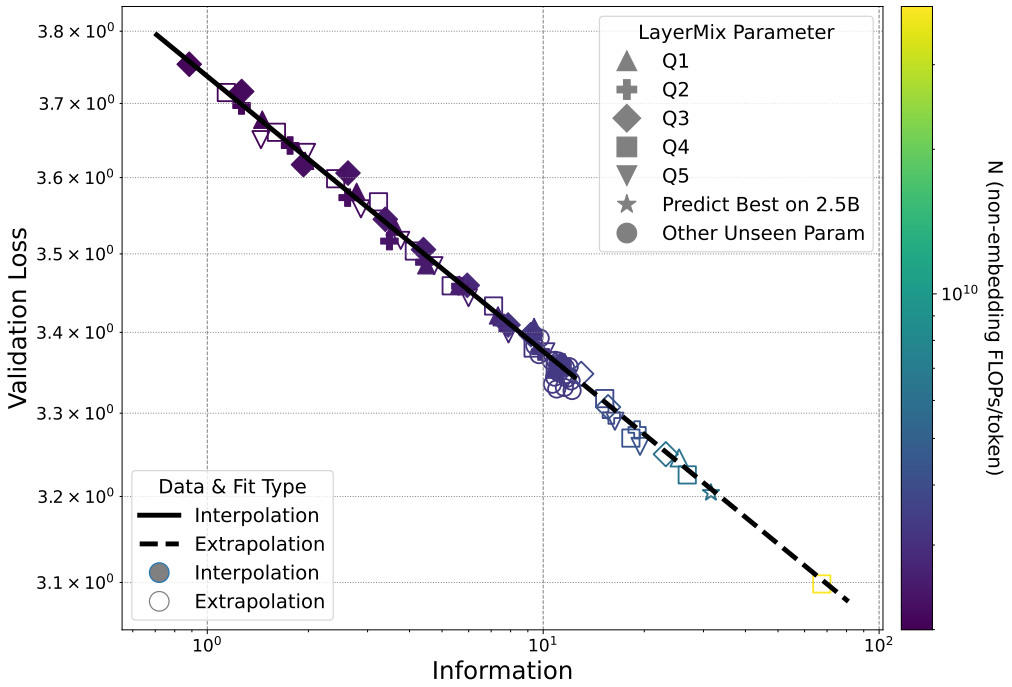


Figure 2: The Unified Information-Loss Scaling Law. Experimental results from diverse configurations (different LayerMix strategies distinguished by marker shape, model non-embedding FLOPs/token indicated by color) collapse onto a single power-law curve when plotted against our proposed Information metric. The scaling law is fitted only on the interpolation data (solid markers), yet it accurately predicts the performance of held-out extrapolation data (hollow markers).

$1 - e^{-\lambda_N R_d / \log(K)}$, it represents the language model's learning ability on this data when repeated an average of R_d times. And the total Information Quantity learned by the language model is the product of these two terms.

We propose Information Quantity, a metric computed from LayerMix sampling weights w, train token K and two fitted functions (f_d, λ_N, to quantify the knowledge learned during training. Since it is designed to be monotonic with model performance, it enables loss prediction for various training configurations prior to any actual runs. The fitting of f_d and λ_N is described in Section 5.2.

4.2 Information-Loss Scaling Law on Repeated Data

As illustrated in Figure 1a, the evaluation loss of the model trained with repeated data is higher than that of the model trained without repeated data, and the traditional scaling law fails to predict language model's performance under this circumstance.

We use the Information Quantity proposed in Section 4.1 and plot the $L\text{-}info$ figure. As illustrated in Figure 2, when we replace the traditional computation axis C with our novel metric: Information Quantity, the experimental points with different LayerMix sampling weights w, Model non-embedding FLOPs/token N and Train Token K now collapse perfectly onto a single, unified power-law curve, where they were previously scattered and separated.

The power-law relationship between the loss L and $info$ can be formulated as:

$$L = \alpha \cdot info^{-\beta} \tag{6}$$

In our experiment, $\alpha = 3.7373$ and $\beta = 0.0441$. We show them in a log-log plot, so it appears as a straight line with a slope of $-\beta$ and an intercept of $\log(\alpha)$.

Like the traditional scaling law (Hoffmann et al., 2022), we can now conduct experiments on small models to compare the advantages and disadvantages of different experimental configurations, and then use our proposed information scaling law to extrapolate the performance of larger models under larger training tokens.

5 EXPERIMENTS

5.1 TRAINING SETUP

We train 9 models ranging from 252M to 1.2B on 3 layermix sampling weights ($Q1$, $Q2$, and $Q3$), with 3.6x over-trained ratio, resulting in 27 experiment runs in total to collect data for fitting the LayerMix Law parameters. We use transformer architecture (Vaswani et al., 2017), SwiGLU (Shazeer, 2020) as the activation function and RoPE embeddings (Su et al., 2024). We use a tokenizer with 250k vocabulary. See Appendix B and Appendix D for details about LayerMix sampling weights, model structure, learnign rate and optimizer.

5.2 FITTING THE CURVE

In this section, we introduce how to fit the parameters in LayerMix Law to predict the model performance collected in Section 5.1. Since Information Quantity $info$ indicates the knowledge learned by the model, we expect larger $info$ to correspond to lower evaluation loss L. Considering that there may exist scale difference between $info$ and model loss L, we choose Spearman correlation ρ_s as the fitting metric, i.e., the object is to find the optimal quality density f and λ_N such that the Spearman correlation between evaluation loss L and $info$ is minimized for all the experiments over N, w :

$$(f^*, \lambda^*) = \underset{f,\lambda}{\operatorname{argmin}} \sum^{N} \sum^{w} \left(\rho_s \left(L_N, info\left(w, K_N, S_N, f, \lambda_N\right)\right)\right) \tag{7}$$

To prevent from over-fitting, we make some assumption based on naive intuition. For f, as it indicates the quality density, the higher-quality bucket should have larger f. As smaller d corresponds to higher-quality buckets, we define f in the following form to ensure it is a decreasing function:

$$f_d(\theta) = e^{-\theta * d} \tag{8}$$

where θ is a hyperparameter and $\theta > 0$.

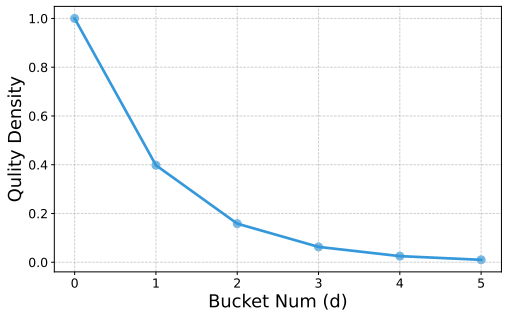
λ_N is related to the model's learning capacity, so λ_N should increase as N increases. But we need to find the formula for λ_N related with N so that it can scale to larger N. To do this we first sample 100,000 combinations of θ and λ_N from the parameter space, then select optimal θ^* and λ_N^* based on Equation 7. The fitted quality density $f(\theta^*)$ is shown in Figure 3a with fitted $\theta^* = -0.922$.

Having the λ_N^* values of different models, as is shown in Figure 3b, we try to fit the λ_N-N curve. The relationship between λ_N and N is observed to be non-linear, exhibiting rapid growth for smaller N and gradually saturating as N increases. This trend is well-approximated by a logarithmic function. Therefore, we choose the λ_N-N curve using following formula:

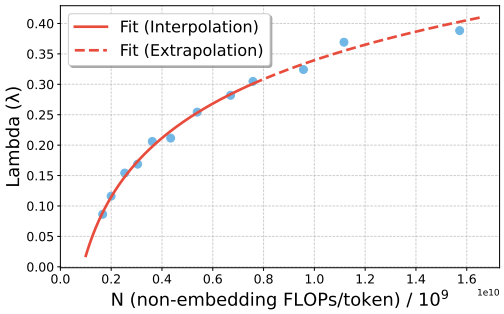
$$\lambda_N(a, b) = a \cdot \ln(N) + b \tag{9}$$

Using existing λ_N^*, we fit the λ_N-N curve in Figure 3b with fitted $a^* = 0.140$, $b^* = 0.018$. To validate this fit, we compute λ_N^* for larger N under the fixed θ^*, and examine whether these values lie on the predicted λ_N-N curve. As illustrated in Figure 3b, the results demonstrate strong extrapolation performance, supporting the correctness of our formulation.

Finally, with $f(\theta^*)$ and $\lambda_N(a^*, b^*)$, we can calculate the Information Quantity for arbitrary layermix sampling weights w, train token K, source token S and model non-embedding FLOPs/token N.



(a) The fitted quality density function f_d. The quality density is a monotonically decreasing function of the bucket index, meaning buckets with higher-quality data are assigned a higher density value.



(b) The relationship between λ and N with a fitted curve. The blue scattered points represent the observed data. The solid red line shows the fit within the data range, while the dashed line represents the extrapolation.

Figure 3: The fitted function of quality density function and relationship between λ_N and N

Table 1: Results of four distinct LayerMix experiments on 2.5B model.

Name	$Q1$	$Q3$	$Q4$	Pred Best
Validation Loss	3.246	3.250	3.226	**3.204**

6 EXTRAPOLATION

The LayerMix Law achieves a strong fit on the training data after parameter optimization, and we subsequently employ it to predict loss under unseen conditions to assess its robustness. To rigorously evaluate its extrapolation capability, we design experiments along three key axes of generalization: (i) novel LayerMix sampling weights, (ii) larger computational scales, and (iii) varying degrees of over-training.

Extrapolation to other LayerMix Sampling Weights

We first test the ability to generalize to an unseen LayerMix sampling weights. We generate dataset with two more sampling weights $Q4$, $Q5$ on model scales ranging from $252M$ to $1.2B$, which are within the range of training data. Also we random sample 25 more sampling weights and run experiments on $1.2B$ model only.

The result is shown in Figure 2 marked by hollow squares (for $Q4$) and hollow downward-pointing triangles (for $Q5$). As can be seen, these points align remarkably well with the scaling law curve established by the initial $Q1 - Q3$ data, demonstrating the predictive power of our model on unseen LayerMix sampling weights.

Extrapolation to Larger Models

To test the extrapolation ability on model scale, we use the same Layermix sampling weights $Q2$,$Q3$ to train models ranging from $1.5B$ to $2.5B$ and $Q1$, $Q3$ to train model with 2.5B parameters, which are out of the range of training data. The experimental results of larger models are shown in Figure 2, we can see LayerMix Law predict the loss on larger scale accurately for all three sampling weights, proving the ability of scaling on model size.

Combination of Extrapolation

Further more, we combine the two extrapolation above and test the effectiveness on both unseen LayerMix sampling weights and unseen scales. We run experiments with $Q4$,$Q5$ on models ranging from $1.5B$ to $7B$. As shown in Figure 2, LayerMix Law also generate well on these combined extrapolation condition. This enables us to search for best data recipe on unseen scale without additional experiments. To verify this, we conduct experiments on $2.5B$ model with optimal data recipe and 3 other layermix sampling weights. As shown in Table 1, our optimal recipe achieves the best validation loss. On all the unseen data points, including unseen LayerMix sampling weights

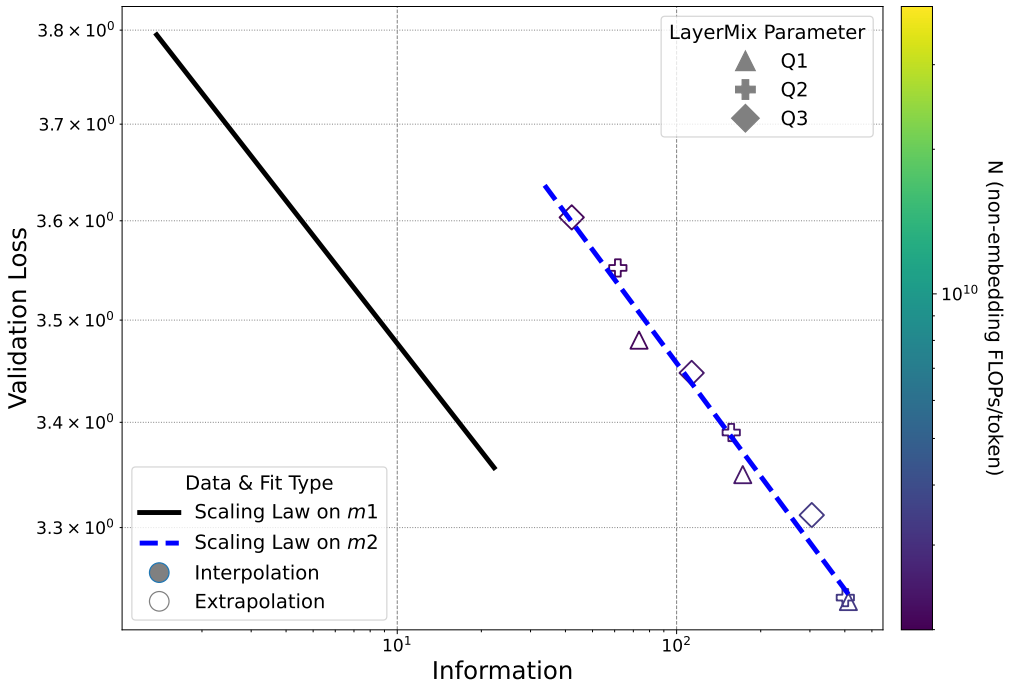


Figure 4: Cross-Regime Prediction of the Scaling Law. The blue line (m_2) is a pure prediction, generated using parameters fitted only on the m_1 data (black line). The fit for the m_2 points demonstrates our LayerMix Scaling Law's power to extrapolate across different overtrain degrees.

and model scales, LayerMix Law predict the validation loss with 0.15% average absolute error and maximum error is 0.96%. This proves that our proposed information scaling law has reliable extrapolation capability.

Extrapolation to Larger Overtrain Degree

To explore the model's reliability under varying sub-optimality, we conducted a second series of experiments at a higher overtrain degree, $m_2 = 25$. This new regime was anchored by a 1.2B model trained on 640B tokens (the m_2-experiment), contrasting with our initial $m_1$experiment anchored at 106B tokens.

For the m_2-experiment, we calculated the Information Quantity using the same quality density $f(\theta^*)$ and $\lambda_N(a^*, b^*)$ fitted previously on the m_1 data. As shown in Figure 4, the new experimental points align with a new scaling law curve. The resulting curves for m_1 and m_2 appear nearly parallel, suggesting the overtrain degree m primarily shifts the curve's intercept. This confirms that our proposed Information Scaling Law is effective across different overtrain degrees.

7 Conclusion

In this paper, we propose a refined scaling law modeling **LayerMix Law**, which focus on predicting model performance on downstream tasks under data-constrained settings with weighted-quality mixing. The LayerMix Law provides accurate predictions of model performance on unseen data recipes at larger computational scales, achieving an average absolute error of only 0.15% and a maximum error of 0.96%. This enables efficient discovery of optimal data recipes without the need for extensive additional experiments. Furthermore, the LayerMix Law extrapolates reliably across varying degrees of over-training, offering an effective tool for selecting data recipes under different computational budgets.

8 ETHICS STATEMENT

Our research is based on the publicly available Common Crawl dataset. We do not foresee any direct negative societal impacts stemming from our methodology or the resulting models.

9 REPRODUCIBILITY STATEMENT

Our experiments are based on the open-source Common Crawl dataset. All experimental settings, model architectures, hyperparameters, and implementation details have been thoroughly described in the main body and the appendix to ensure that other researchers can independently reproduce our results based on this information.

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A TRAINING DATASET

We use the English portion of the Common Crawl Dataset (Common Crawl Foundation), utilizing 96 of the snapshots, from CC-MAIN-2013-20 to CC-MAIN-2024-18. Following Bi et al. (2024), we ran a global fuzzy deduplication across all snapshots, resulting in a total dataset with 3.7T tokens.

B LAYERMIX SAMPLING FUNCTION

We show the detail of LayerMix sampling function in Algorithm 1. And the 5 sets LayerMix sampling weights $Q1$ to $Q5$ are illustrated in Table 2.

Algorithm 1 LayerMix Sampling Function $H(w, K, S, B)$

function H(w, K, S, B)
Require:
w: A list of target proportions for six buckets, $w = [w_0, ..., w_5]$, where $\sum w_d = 1$.
K: The total number of tokens for the final training dataset.
S: The total number of tokens in the entire source corpora.
B: A list of six data buckets, $B = [B_0, ..., B_5]$, partitioned by quality.
Ensure:
D_{train}: The final packed training dataset.
M: A list of unique token counts for each layer, $M = [M_0, ..., M_5]$.
R: A list of average repetition counts for each layer, $R = [R_0, ..., R_5]$.

Initialize an empty training dataset $D_{train} \leftarrow \emptyset$.
Initialize empty lists for statistics: $M \leftarrow []$, $R \leftarrow []$.
Define the source distribution proportions $P_{source} \leftarrow [0.05, 0.15, 0.2, 0.2, 0.2, 0.2]$.

for $d \leftarrow 0$ **to** 5 **do** ▷ Iterate through each quality bucket
  $K_{needed} \leftarrow K \times w_d$ ▷ Calculate tokens needed from bucket B_d for the target mix
  $S_d \leftarrow S \times P_{source}[d]$ ▷ Calculate source tokens available in bucket B_d
  $Ratio_d \leftarrow K_{needed}/S_d$ ▷ Calculate the sampling ratio for the current bucket
  ▷ — Detailed sampling process for bucket B_d —
  Initialize an empty temporary set $D_{sampled\_d} \leftarrow \emptyset$.
  for all data point x in bucket B_d **do**
    ▷ 1. Deterministic copy for the integer part of the ratio
    for $i \leftarrow 1$ to $\lfloor Ratio_d \rfloor$ **do**
      Add x to $D_{sampled\_d}$
    end for
    ▷ 2. Probabilistic sampling for the fractional part
    if $Ratio_d - \lfloor Ratio_d \rfloor > 0$ and $random() < (Ratio_d - \lfloor Ratio_d \rfloor)$ **then**
      Add x to $D_{sampled\_d}$
    end if
  end for
  Append all data from $D_{sampled\_d}$ to D_{train}.
  $M_d \leftarrow \min(K_{needed}, S_d)$ ▷ Calculate unique tokens based on the new formula
  Append M_d to M.
  $R_d \leftarrow K_{needed}/M_d$ ▷ Calculate average repetition count
  Append R_d to R.
end for

return D_{train}, M, R ▷ Return dataset and statistics
end function

Table 2: LayerMix sampling weights $Q1$ to $Q5$.

Name	$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
Q1	0.80	0.10	0.03	0.03	0.02	0.0
Q2	0.48	0.23	0.13	0.07	0.07	0.0
Q3	0.24	0.20	0.19	0.18	0.17	0.0
Q4	0.38	0.21	0.20	0.11	0.08	0.0
Q5	0.66	0.22	0.05	0.03	0.02	0.0
Pred Best of 2.5B model with m_1	0.50	0.49	0.01	0.0	0.0	0.0

Table 3: Structure of models used in LayerMix.

Model	**Hidden dim. (C)**	**MLP dim. (D)**	**Layers (L)**	**Heads**
252M	1024	2752	20	16
302M	1024	2752	24	16
392M	1280	3392	20	20
470M	1280	3392	24	20
566M	1536	4096	20	24
680M	1536	4096	24	24
850M	1792	4800	22	28
1B	1920	5120	24	30
1.2B	2048	5440	24	16
1.5B	2304	6144	24	36
1.8B	2304	6144	28	36
2.5B	2560	6848	32	40
7.7B	4096	14336	32	32

C Scaling Law for Overtraining

Following Gadre et al. (2024), we define a overtrain degree factor m, to quantify the deviation of a given training configuration from the Chinchilla-optimal point. For a training run with model non-embedding FLOPs/token N and D tokens, we first calculate the total compute $C = ND$ and find the corresponding optimal model non-embedding FLOPs/token N^* and tokens D^* according to the Chinchilla scaling law. The overtrain degree m is then defined as:

$$\sqrt{m} = \frac{N^*}{N} = \frac{D}{D^*} \tag{10}$$

A value of $m = 1$ indicates a compute-optimal training run, while $m > 1$ signifies that the model is overtrained relative to its compute budget.

D Training

The model structures used in LayerMix are illustrated in Table 3. We train all the model with 2048 as the max sequence length, we use a cosine decay schedular and the initial learning rate calculated by $lr = round(0.3118 \cdot C^{-0.1250}, 8)$, the warm up ratio is set 0.5%. We use AdamW optimizer with $\beta_1 = 0.9$, $\beta_2 = 0.95$, weight decay$= 0.1$.

E Deviation of Traditional Scaling Law

We show all $Loss$-C curve of different LayerMix sampling weights with V1 and V2 in Figure 5 and Figure 6, they all exhibit a clear deviation from the traditional scaling law, which is fitted from the first three data points.

Algorithm 2 Calculation of Overtrain Degree and Optimal Tokens

function CALCULATEOVERTRAINEXTRAPOLATION($model_{curr}, D_{curr}, models_{target}$)

Require:

$model_{curr}$: The size of the current model configuration.
D_{curr}: The number of tokens used to train the current model.
$model_{target}$: The size of the target model configuration.

Ensure:

m: The calculated overtrain degree for the current configuration.
D_{target}: The train token of target model under same overtrain degree.

// Part 1: Calculate overtrain degree m from the current configuration

$N_{curr} \leftarrow$ Get_N($model_{curr}$) ▷ Get N (non-embedding FLOPs/token) for the current model
$C \leftarrow N_{curr} \times D_{curr}$ ▷ Calculate the total compute budget

$N_{opt} \leftarrow 0.06085 \times C^{0.5445}$ ▷ Calculate Chinchilla-optimal model non-embedding FLOPs/token for budget C
$D_{opt} \leftarrow 16.4326 \times C^{0.4555}$ ▷ Calculate Chinchilla-optimal tokens for budget C

$\sqrt{m} \leftarrow N_{opt}/N_{curr}$ ▷ Calculate the overtrain degree m
▷ This is equivalent to $\sqrt{m} = D_{curr}/D_{opt}$

// Part 2: Extrapolate to target model while keeping m constant

for each $model_t$ in $[model_{curr}] + models_{target}$ **do**
$N_t \leftarrow$ Get_N($model_t$) ▷ Get N (non-embedding FLOPs/token) for the target model

$N'_{opt} \leftarrow N_t \times \sqrt{m}$ ▷ Find the corresponding optimal model non-embedding FLOPs/token for the target
$C_{new} \leftarrow (N'_{opt}/0.06085)^{1/0.5445}$ ▷ Derive the new compute budget

$D'_{opt} \leftarrow 16.4326 \times C_{new}^{0.4555}$ ▷ Find optimal tokens for the new budget
$D_{target} \leftarrow D'_{opt} \times \sqrt{m}$ ▷ Calculate the required tokens for the target model
end for

return m, D_{target} ▷ Return the overtrain degree and the train token of target model under same m.
end function

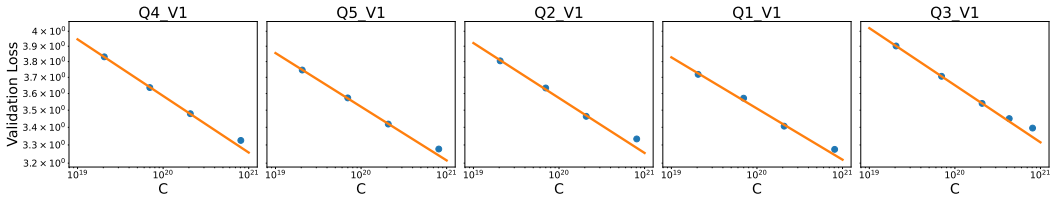

Figure 5: $Loss$ and C Curve of different LayerMix $V1$ experiments

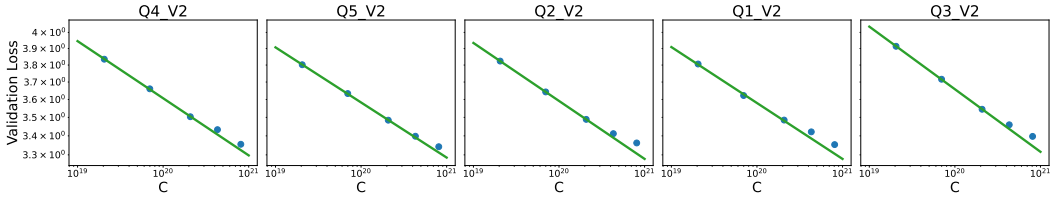

Figure 6: $Loss$ and C Curve of different LayerMix $V2$ experiments

F LIMITATION

We note several limitations of our work. Our data bucketing is based on a fixed, empirical heuristic. We have not performed ablation studies to determine the optimal number or boundaries of these quality tiers. A more systematic approach to data partitioning could further improve the model's predictive accuracy. And while we observe that the overtrain degree m systematically shifts the scaling law curve, a theoretical explanation for this behavior is still needed. These areas present clear avenues for future work.

G USAGE OF LLM

During the preparation of this paper, large language models (LLMs) were utilized. The use of these tools was only for polishing the language, improving grammatical structure, and performing spell checks.