ARTIFICIAL DESIGN: MODELING ARTIFICIAL SUPER INTELLIGENCE WITH EXTENDED GENERAL RELATIVITY AND UNIVERSAL DARWINISM VIA GEOMETRIZATION FOR UNIVERSAL DESIGN AUTOMATION (FAR FROM THE MADDING CROWD: HOW TO BE SHALLOW TO AVOID GOT LOST IN DEEP FOREST?)

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ABSTRACT

Let us share the joy of our unprecedented discovery with you: Physical Geometry and Biological Geometry are the outcome of the physical laws and biological laws respectively, and Artificial Super Intelligence (ASI) has to be a combination of both design and learning instead of learning alone, with that we propose Artificial Design, a bio-physical inspired mathematical model for Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning (HMMMPDRL) based ASI by reusing and extending General Relativity and Universal Darwinism with Geometrization. With Artificial Design we solve Deep Reinforcement Learning blackbox puzzle in AI and ASI. By treating HMMMPDRL as multiverse regardless the mutual exclusiveness between Multi-Agent and Multi-Environment, we reuse General Relativity's 4-Dimensional Pseudo-Riemannian Manifold based SpaceTime Model for Reinforcement Learning part of HMMMPDRL, we also make a T-symmetry extension to General Relativity, replace N-Dimensional space with N-Dimensional GeneSpace, and formulate a N-Dimensional Riemannian Manifold based GeneSpace Model for Deep Learning part of HMMMPDRL, whereas Deep Learning architecture is adopted to approximate very complex state-action space composed environments in HMMMPDRL. By modeling ASI with Artificial Design rigorously in this way, we claim that intelligence, whether natural, artificial, or super-artificial like ASI, is just the geometry effect of N-Dimensional GeneSpace caused by Geometrization, and that paves the way in achieving ASI through Universal Design Automation of Artificial Design in theory. Of course, our Multiversal endeavor won't stop from there, endorse us to artificially co-accelerate human civilization in every possible way you might imagine.

Index terms— Artificial Design, Artificial Super Intelligence, General Relativity, Universal Darwinism, Geometrization, Gene, Manifold, Tensor, SpaceTime, GeneSpace, Deep Reinforcement Learning, Universal Approximation, Geometry Effect, Universal Design Automation, Universal Paradox

1 INTRODUCTION

Sage Laozi, in his book titled Tao Te Ching wrote in 6th century BC ago, with its ink on bamboo version of the Warring States Era and unearthed in 1993, along with its ink on silk version of 2nd century BC and unearthed in 1973 as both illustrated, said: Tao begets One, One begets Two, Two begets Three, Three begets Everything. That is how the universe began, and here One refers to Taiji as illustrated. It of course makes sense philosophically given the fact of such claim were made 2,500 years ago. Around the similar time frame when Laozi was alive, Greek Philosopher Pythagoras believed in multiverse (musica universalis) governed by mathematical equations, and metempsychosis (transmigration of souls) holding that soul is immortal as cycling among different bodies, unfortu-

Figure 1: Taiji

Figure 2: Tao Te Ching Ink on Bamboo

nately no excavation evidence so far yet. Anyway all make sense philosophically. Most recently, German Philosopher Hegel said: What is rational is actual and what is actual is rational, which also makes sense philosophically. So far so good, so where are we now?

Software is eating the world, yet AI is increasingly eating software and software-defined everything. The latest AI research now can generate and synthesize algorithms (hence software) right from requirements as opposed to from just algorithms to software as before. What does it mean to us as a software developer when 500,000 lines of source code via programming gets replaced by 500 lines of TensorFlow[17] (even much less if in Keras) code via learning? Please note, Apollo 11 mission controller's source code has only 130,000 lines of assembly code, while modern autopilot has millions of line of source code, and Tesla's Autopilot has 400,000,000 lines of source code. Even with the capability of processing PB-scale big data pipeline in real-time, and EB-scale big data lake in batch-mode, all powered by elastic public/private/hybrid cloud computing/edge-computing in bare metal/Virtual Machine/Container through Infrastructure-as-a-Service (IaaS)/Platform-as-a-Service (PaaS)/Software-as-a-Service (SaaS) leveraging Software-Defined-Compute (SDC), Software-Defined-Network (SDN), Software-Defined-Storage (SDS). Such mighty low cost high performance cloud computing equipped with millions of CPU/GPU cores spanning multiple data centers in different data centers of same/different availability zone(s) is making traditional HPC quickly obsolete nowadays, furthermore the upcoming 5G makes both cloud computing/edge computing and edge-computing/edge computing seamless coordination in real-time possible, please note in theory 5G has only 1ms delay, way below human's 100ms response time. Meanwhile Software Process, especially development, testing, delivery, deployment, operation, has been significantly advanced due to the mature of Continuous Integration, Continuous Delivery/Deployment (CI/CD), that yields multiple stable releases a day as de facto industry standard as opposed to monthly/quarterly/yearly release in old non-elastic data center computing days. The same true technically agile methodology can also applied to AI model training and serving seamlessly as well. Last but not the least, most recently an AI-chip startup in Silicon Valley released a Wafer-Scale Chip with its die size bigger than an iPad, integrating 12 trillion transistors with 40K AI cores, 18G on-chip memory, 9PB/s memory bandwidth, 100PB/s fabric bandwidth, in TSMC 16nm process in attempting to overcome the downside of distributed processing brought by SDC, SDN and SDS. With such paradigm shift in both hardware and software, now we are able to

Figure 3: Tao Te Ching Ink on Silk

model Deep Learning Neural Networks (DNN)[5] with billions of connections, millions of parameters, hundreds of layers for real-life applications. However, even with such significant progress in both software and hardware, the matter of fact is the degree of intelligence demonstrated by AI still falls far behind of human intelligence in most cases.

General Relativity[1] and Darwinism[2] are cornerstones of universe and life respectively. General Relativity generalizes Special Relativity and refines Newton's Law of Universal Gravitation, providing a unified description of gravity as a geometric effect of Space and Time, or Spatial-Termporal (SpaceTime). The curvature of SpaceTime models gravity, and is directly related to the energy and momentum of whatever matter and radiation are present. The relationship is specified by the Einstein Filed Equations. Darwinism is a theory of biological evolution developed by Darwin and others, claiming that all species of organisms arise and develop through the natural selection of small, inherited variations that increase the individual's ability to compete, survive, and reproduce. However, when Darwinism was developed, there was no concept of gene yet. Neo-Darwinism[6], also called the Modern Evolutionary Synthesis, just like synthesis in Electronic Design Automation (EDA), generally denotes the integration of Charles Darwin's theory of evolution by natural selection, Gregor Mendel's theory of genetics as the basis for biological inheritance, and mathematical population genetics. Furthermore Universal Darwinism, a variety of approaches that extend Darwinism and Neo-Darwinism beyond its original domain of biological evolution on Earth, was formulated in a generalized version to apply to explain evolution in a wide variety of other domains, including physics, psychology, economics, culture, medicine, computer science.

Among universe, multiverse, genes, brains, economies, games, blockchains, AI and ASI, even fluid dynamics, one thing in common is their non-linearity as complex dynamical systems. Since the claims on the similarity between man/brain and machine/computer made by those visionaries like Turing, von Neumann, Wiener, Schrodinger, Shannon, McCarthy, and Minsky, followed by the discovery of DNA structure by Watson and Crick[12], AI has been experiencing several boom and bust. However even with the recent progress on Deep Learning, Reinforcement Learning[4], Meta Learning (AutoML and AutoDL), Learning to Learn, Transfer Learning, Never-end Learning, in mimicking they way how human and other living system learning by interacting with environment, and the way how human's and other living system's brain operating based on ANN in the form of DNN, implemented in both software and hardware, as well as the most recent encouraging results on AI's application on gaming , such as model-free/model-based board games, card games, and video games with perfect/imperfect information such as Go (AlphaGo[13], AlphaZero[14]), StarCraft (AlphaStar), DeepStack, No-Limit Poker (Libratus)[15], ATARI, Dota2, a clear sign of ascending from AI to ASI, is definitely emerging, which might lead to a new kind of Grand Unification in the context of intelligence. Actually the latest endeavor is Yang-Mills-Higgs Equations, which attempts to tackle Grand Unification in physics by unifying gravitational theory such as General Relativity with Quantum Electrodynamics, the electroweak theory, the standard model of particle physics, as Higgs field is a theory of interactions of spin zero particles, and General Relativity is for spin two particles and a symmetric rank-2 Tensor, the metric Tensor. Despite of impressive success achieved by the state-of-the-art of Deep Reinforcement Learning[8] via various heuristics, there are a lot of cases where it does not work at all or does not work well with current practice in real-life[19], for example, in non-stationary partially observable dynamic safety-critical environment where exploration/training and exploitation/serving cannot be separated in real-time. We believe that the root cause of such weakness actually lies in not only the failure of establishing a consistent mathematical foundation for ASI, but also the fiasco of understanding the nature and sources of intelligence in ASI from bio-physical perspective.

EDA aims to fully automate/synthesize the whole production process ranging from specification, design, manufacture, and even operation of semiconductor chips and its composed printed circuit boards and electronic systems, in other words, EDA is all about synthesis, from Algorithm-level, Hardware-system-level/Software-system-level, RTL/Source code-level, Gate-level/Assembly-level, to Layout-level/Executable-level. Technically ASI through Artificial Design can be implemented in either Life Synthesis or Robot Synthesis. Life Synthesis means living system, can be specified and transformed step by step from gene to, protein, organelle, cell, tissue, organ, till body. While Life Synthesis is still far away, biosynthesis such as gene synthesis and protein synthesis do work. Robot Synthesis means intelligent robot, regardless of size and shape, can self-reproduce or self-replicate, self-organize, and self-improve itself. Robot Synthesis is the more viable way as opposed to Life Synthesis if we can leverage how the way the mature EDA industry works on synthesizing trillions of transistors on a single System-On-a-Chip (SoC) possible nowadays. Similar to EDA for SoC, ASI can be realized by Universal Design Automation of Artificial Design.

2 Complex System as AI and Vice Verse

Universes, multiverse, genes, brains, social network, economies, and blockchains among other systems are all complex systems for sure, but how complex are they? Complexity has different interpretations in different contexts, such as system complexity, network complexity, and computational complexity, etc. The key difference between complex system and complex network is, originally the former focuses more on the external property, while the latter focuses more on the internal property. However nowadays their difference has vanished.

Original complex network research focused on random graphs such as Erdos-Renyi model before the rising of World Wide Web and the Internet. It arose with the discovery of small-world model, followed by scale-free, power law model. Artificial Neural Networks, molecular networks including gene regulatory networks for computational neurogentic modeling, protein-protein interaction networks in molecular kinetics, cell signaling networks with neural networks as its subset, metabolic networks, food networks in ecosystems, and blockchains, are all complex networks and nonlinear dynamical systems. The function of a dynamical system may be deterministic or stochastic depending on whether the mapping between an initial state and a final state is deterministic or stochastic.

A dynamical system's time and state space can be either discrete and modeled by difference equations traditionally, or continuous and modeled by differential equations traditionally, with examples like Navier-Stokes Equations in fluid dynamics, the Lotka-Volterra Equations in ecology, and Michaelis-Menten Equations in enzyme kinetics. A nonlinear system is a system in which the change of the output is not proportional to the change of the input. Nonlinear dynamical systems, describing changes in variables over time, may appear chaotic, unpredictable, or counter-intuitive, contrasting with much simpler linear systems. A lot on nonlinear dynamical systems exhibit extreme sensitivity to small perturbations in initial conditions, e.g., Lorenz system, can produce a phenomenon known as chaos demonstrating butterfly effect. Actually chaos is temporal fractal, and fractal is spatial chaos, both of them stem from dynamical systems. Initially chaos was recognized as in the three-body problem in celestial mechanics. For a dynamical system to display chaotic behavior, it must be either nonlinear with three or more dimensions or infinite-Dimensional if linear. The biggest challenge for complex heterogeneous networks right now is the combinatorial explosion due to the size, non-linearity, and heterogeneous nature, and traditional models and related algorithms do not help in tackling such challenges.

Universes, as complex systems, is all of space, time, matter, and energy, the latter can be either inorganism or organism, which forms the basis of life. So far General Relativity is the de facto physical law in governing universe, as both Newtonian Theory and Special Relativity only work with linear systems representing flat space or flat SpaceTime, while universe is actually nonlinear representing curved SpaceTime.

Multiverse is a hypothetical group of multiple universes, and it can be hierarchical. Together, these universes comprise everything that exists: the entirety of space, time, matter, energy, and the physical laws and constants that describe them. The different universes within the multiverse are called parallel universes. Multiverse has been hypothesized in cosmology, physics, astronomy among other domains. The physics community has debated the various multiverse theories over time, and is divided about whether any other universes exist outside of our own, however that does not prevent us from adopting the concept of multiverse in modeling Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI.

Genes are DNA sequences that encode biological instructions for the synthesis of proteins, are complex systems. The total amount of DNA in a cell is referred to as genome. There are approximately 60 trillion cells in human being. At any moment, human genome, which consists of 6.4 billion letters encoded in A (Adenine), C (Cytosine), G (Guanine), and T (Thymine), is being decoded to produce 20 possible amino acids for protein synthesis. Human genome contains the ensemble of the genetic heredity, and the instructions for both construction and operation. Through heredity, variations between individuals can accumulate and cause species to evolve by natural selection, so-called Natural evolution as opposed to Accelerated Spontaneous Artificial Genetic Evolution, namely Artificial

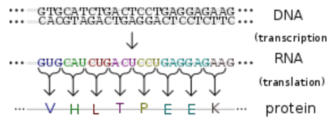

Figure 4: Central Dogma Simplified

Design. The phenomena of mirror neutron was hard to explain, quantum entanglement that can be used for teleportation might be one of the possible of explanations. Despite of that, gene is still a much better explanation at biological level. As of today, the roles of most functional sequences in human genome still remain not completely decoded, hence unknown. There are three types of RNA, namely messenger RNA or mRNA, ribosomal or rRNA, and transfer RNA or tRNA, they serve different roles in inter-playing with both DNA and protein during gene expression. The way it roughly works through gene regulatory network is called Central Dogma. The transmission of genes to organism's offspring is the basis of the heredity of phenotypic traits. These genes make up different DNA sequences called genotypes. Genotypes combined with ecology-based environmental and evolution-based developmental factors determine what the phenotypes will be. Gene regulatory network is a collection of molecular regulators, which can be DNA, RNA, protein and complexes of these, that interact with each other and with other substances in the cell to govern the gene expression levels of mRNA and proteins. Again we still not sure how exactly it works. DNA Computing is a branch of computing which uses DNA, biochemistry, and molecular biology hardware, instead of the traditional silicon-based microelectronics technologies. Latest research on DNA Computing can perform reversible DNA Computing bringing it one step closer to the silicon-based microelectronics technology, which is called molectronics. DNA computing is very suited as a medium for data processing. According to different calculations a DNA-computer with one liter of fluid containing six grams of DNA could potentially have a memory capacity of 3072EB. The theoretical maximum data transfer speed would also be enormous due to the massive parallelism of the calculations far beyond today's most powerful computers can reach. DAN computing, just like Quantum Computing, which uses quantum-mechanical phenomena such as superposition and entanglement too perform computation, is still in its infancy.

Human brain, as a complex system, has a cellular structure, it is composed primarily of two broad classes of cells: neurons and neuroglia. Neurons as opposed to neuroglia, are usually considered the most important cells in the brain. The property that makes neurons unique is their ability to send signals to specific target cells over long distances. Brain, specifically cerebral cortex, used to be regarded as the sole source of intelligence, it has 16 billion neurons in contrast to cerebellum's 69 billion neurons. These neurons are connected to each other in a complex, recurrent fashion. It is still far away to completely understand how human brain exactly functions in demonstrating intelligence through brain science at present. So far ANN vaguely inspired by the biological neural networks in brain has been successfully adopted in AI with known limitations. However current brain-inspired AI take neither evolution nor development into consideration, and for brain itself, it does not distinguish the role of different parts of brain inter-plays. Today, ANN-based Deep Learning in modeling complex nonlinear relationships between inputs and outputs is widely adopted in AI. Deep Learning uses a lot of techniques on artificial neural networks in the form of Deep Neural Networks (DNN) such as Feedforward Neural Networks (FNN), Convolutional Neural Networks (CNN) for image recognition, Recurrent Neural Networks (RNN) for speech recognition and Natural Language Processing (NLP), auto-encoder and Long Short-Term Memory (LSTM) for machine translation, which can have billions of nodes and billions of parameters in a ultra-high-dimensional feature space, in either manual or fully automatic neural network design way and optimization techniques such as stochastic gradient descent with back propagation and automatic differentiation for inference, in attempt to converge quickly instead of being stuck at saddle points in non-convex Manifold in a heuristic way.

Social network as online society as a complex systems is a social structure made up of a set of social actors (such as individuals or organizations), sets of dyadic ties, and other social interactions between actors. The social network perspective provides a set of methods for analyzing the structure of whole social entities as well as a variety of theories explaining the patterns observed in these structures. Nowadays with major social networks worldwide are producing social data in PB-scale individually, and in EB-scale combined everyday.

Economy being defined as a social domain that emphasizes the practices, discourses, and material expressions associated with the production, use, and management of resources, is also a nonlinear dynamical system. The widely adopted Arrow-Debreu model, which suggests that under certain economic assumptions such as convex preferences, perfect competition , and demand independence , there must be a set of prices such that aggregate supplies will equal aggregate demands for every commodity in the economy. The assumption of convexity precluded many applications before the proof of the existence of economic equilibrium when some consumer preferences need not be convex . A convexified economy has general equilibrium that are closely approximated by quasi-equilibrium of the original non-equilibrium economy based on the Shapley-Folkman theorem. Of course, Nash Equilibrium, part of Game Theory, also plays significant role in tackling real economical challenges, and yet not quite fruitful.

The frenzy on Bitcoin, as a complex system, has been lasting for a while, while experiencing boom and bust, it is surging again most recently. The foundation of bitcoin along with other crypt-currencies is blockchain. Blockchain is a peer-to-peer decentralized, distributed and public digital ledger, that is used to record transactions across many computers, so that the record cannot be altered retroactively without the alteration of all subsequent blocks and the collusion of the network. This allows the participants to verify and audit transactions inexpensively. A blockchain database is managed autonomously using a peer-to-peer self-organizing network and a distributed time-stamping server. They are authenticated by mass collaboration powered by collective self-interests. Hence blockchain network is a system both complex and self-organizing, and it serves the technical foundation of Decentralized Autonomous Organization (DAO). Blockchain makes it possible for any organization to adopt DAO in conducting business without the need of a trusted authority or broker, that is called Trustless Trust. Blockchain can be either public or private.

3 Gene and Environment as Source of Intelligence

There is no strict definition of intelligence so far yet. Typically intelligence can be defined as the ability of learning, as well as instinct such as spatial and temporal reasoning capability. Our way of interpreting intelligence is a brute-force way, we claim that intelligence is opposite to brute-force. Leverage of intelligence is called exploitation, and accumulation of intelligence is called exploration. Exploration typically replies on search, which is environment-dependent and resource-constraint , hence may or may not succeed. Intelligence is most often studied in humans but has also been observed in other living systems. Intelligence in machines, whether in software or hardware, or combined, is called AI, and ASI means machines' cognizance supersedes that of human.

Any living system who has genes, regardless with or without brain, possesses certain degree of intelligence. A lot of research on AI claims its inspiration comes from human brain, the fact is most of them are only related to visual cortex in cerebral instead. As opposed to cerebral, cerebellum modulates the outputs of other brain systems, whether motor related or thought related, to make them certain and precise. Removal of the cerebellum does not prevent a living system from doing anything in particular, but it makes actions hesitant and clumsy. Such kind of precision is not built-in, but learned by trial and error. Only 10% of the brain's total volume consists of the cerebellum and yet 50% of all neurons are held within its structure. For human being, the cerebellum plays an important role in motor control, and it may also be involved in some cognitive functions such as attention and language as well as for emotion regulation. The human cerebellum does not initiate movement, but contributes to coordination, precision, and accurate timing: it receives input from sensory systems of the spinal cord and from other parts of the brain, and integrates these inputs to fine-tune motor activity. Hence as opposed to just cerebral, both cerebral and cerebellum contribute to intelligence, partially due to brain. Furthermore, current research on AI, ASI and ASI ignores intelligent living systems' brain-body seamless coordination almost completely.

Is human intelligence as well as other natural intelligence inborn by design via gene or developed through environment by learning? The answer is both, and that also applies to all living systems. According to one of latest research on human brain, human hippocampal neurogenesis drops sharply in children at age 13 to undetectable levels in adults, that explains exactly why children older than 13 cannot easily adapt to new languages. Here we divide intelligence into intrinsic Agent Intelligence/Computing Intelligence/Planning Intelligence/Exploitation Intelligence, and extrinsic Environment Intelligence/Learning Intelligence/Trial and Error Intelligence/Exploration Intelligence.

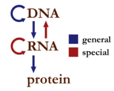

Figure 5: Alternative View of Central Dogma

For example, epistemology is the branch of philosophy concerned with the theory of knowledge. Ontology in philosophy means the combination of subject and object, hence we can divide epistemology into intrinsic epistemology and extrinsic epistemology, as well as ontology into intrinsic ontology and extrinsic ontology. Both Intrinsic Epistemology and Intrinsic Ontology are pre-determined by gene as soul, and both Extrinsic Epistemology and Extrinsic Ontology are acquired after living system's birth by learning from environment. Being intrinsic means via heredity, and both evolution-dependent and development-independent, and being extrinsic means both environment-dependent and development-dependent. Actually in information theory, Shannon entropy [21] measures the number of internal arrangements of a system that result in the same outward appearance, and it rises because , a system evolves toward states that have many internal arrangements or statistical reasons. Surprisingly Shannon also had a less well-known PhD. thesis on genetics [18] that might blow your mind.

Creativity is the generation of novelty as a result of inspiration. Like intelligence, creativity is determined by gene, development, and environment. Creativity can also be divided into intrinsic creativity and extrinsic creativity.

Intelligence cannot work without the help of memory. For any learning used in AI and ASI, all need memory. Memory, whether physical or virtual, plays pivotal role in AI, both from software perspective and hardware perspective. There is radical difference between GPU memory architecture and CPU's memory architecture, in terms of clock frequency, memory bandwidth, and controller complexity among others. In AI and ASI, memory is modeled by both short-term memory and long-term memory, we believe cache-memory should be served as intermediate layer between short-term memory and long-term memory, just like the function of cache in computer architecture. Soul is the mental abilities of living system, traditionally it is regarded as a unscientific concept. According to Aristotle, there are three kinds of soul: the vegetative soul, the sensitive soul, and the rational soul, and human beings possess all of three souls together. Georg Wilhelm Friedrich Hegel once said, the heart is everywhere. While it sounds ridiculous, actually those sages' views inspired us in linking soul with gene, which is also ubiquitous. Instinct is the inherent inclination of a living system towards a particular complex behavior, such as Fixed Action Pattern (FAP). Any behavior is instinctive if it is performed without learning to be experienced, and is therefore based upon gene through heredity.

In computer science, gene is well-known for its conceptional adoption in genetic algorithm (GA), which is a meta-heuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA) for evolution simulation of natural evolution strategies. However it does not leverage too much potential of the role of gene. Schrodinger regarded gene as hereditary code-script in his book [16], just like software in a computer. Contrary to Darwin, Schrodinger also claimed that mutation instead of variation, is the driver of natural selection. Francis Crick, once co-authored a paper claiming gene as ultimate parasite, selfish and immortal. Actually gene is not only the source of intelligence, but also the source of consciousness, and existence of consciousness is the key difference between AI and ASI. The alternative view of Central Dogma is illustrated below. There is no strict definition of consciousness so far. Roughly consciousness can be defined as the change of state in being aware of both self and environment. Similarly self-Consciousness can be defined as the change of state in being aware of self. No state change, no consciousness and self-Consciousness. Both consciousness and self-Consciousness have not been major research topics since the inception of AI [3], partly because they are too hard to tackle, partly because it is located at where both science and philosophy can hardly reach. Any living system, whether animal such as human being or plant, possesses Consciousness and self-Consciousness, because all of them are being defined by genes. consciousness can be demonstrated anywhere in such living system as a peer-to-peer based distributed system besides brain if there is one.

Tononi proposed Integrated Information Theory (IIT) in attempt to explain what consciousness is, and why it might be associated with certain physical systems. His theory predicts whether that sys-

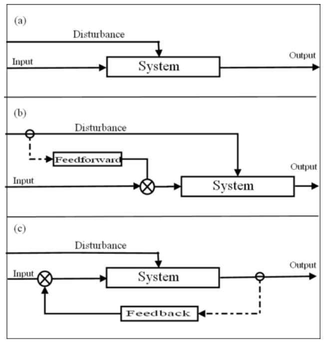

Figure 6: Feedforward Control and Feedback Control

tem is conscious, to what degree it is conscious, and what particular experience it is having. According to IIT, a system's consciousness is determined by its causal properties like cause-effect generated by the relationships among different states the system demonstrates, and is therefore an intrinsic, fundamental property of any physical system. In contrast to IIT, we interpret and model consciousness with environment in consideration by Cybernetics pioneered by Weiner[11] via feed-forward mechanism and feedback (both positive and negative) mechanisms, from engineering perspective instead of philosophical perspective. Cybernetics is a interdisciplinary approach for exploring regulatory systems, their structures, constraints, and possibilities. Wiener defined cybernetics as, the scientific study of control and communication in the animal and the machine. The word cybernetics coincides with Cogito ergo sum in Latin by Rene Descartes, which means I think, therefore I am. Schrodinger had a similar vision, he attempted to check whether one's body functions as a pure mechanism according to Laws of Nature; and one takes full responsibility for it by controlling so-called the motion of the atoms according to Laws of Nature as consciousness is everywhere, not just in brain.

Here we divide artificial consciousness into deterministic consciousness and stochastic consciousness. The advantage of modeling consciousness with Cybernetics facilitates the computation of consciousness by leveraging both algorithm-based symbolic computing, and learning-based neural computing such as Deep Learning, which has shown to be very powerful and promising in extracting pertinent features especially for complex nonlinear dynamical systems, where conventional techniques are unable to tackle. Contrary to linear systems that classical physics governs, complex systems like living systems have to rely on correlation-based learning approach if causality-based approach does not work well alone.

Inspired by the works of Gibbs, both Schrodinger and Prigogine attempted to unify living system and physical system via thermodynamics, Prigogine claimed that any system's, whether living system or physical system , self-organization relies on its recursive self-improvement, butterfly effect can happen under certain circumstances via feed-forward mechanism. The most recent emerging Darwinian Dynamics claims that, the evolution of order, or the value of entropy, in both living systems and physical systems obeys the same fundamental principle.

von Neumann defined artificial life in the following way when he proposed Universal Constructor, a self-reproducing machine in a cellular automata[9] , firstly it can reproduce itself, and secondly it can simulate the Turing Machine, unfortunately Turing Machine has its modeling limitation as we mentioned before. While Quantum Darwinism[20] aspires to explain the emergence of the classical world from quantum world as the superposition, entanglement, and environment as witness, due to a process of Darwinian natural selection induced by the environment interacting with the quantum system, where the many possible quantum states are selected against in favor of a possible pointer state, from living system-inspired AI's and ASI's point of view, the long-standing foe of Quantum Electrodynamics, General Relativity is more suitable to be combined with Universal Darwinism in Macro-world. Accelerated Spontaneous Artificial Genetic Evolution is achievable through the self-improvement capability of ASI, just like directed evolution through either recombinant DNA or polymerase chain reaction (PCR) in an artificially or unusually naturally changing environment, that paves the way for Artificial Design of ASI.

4 MODEL ASI BY EXTENDING GENERAL RELATIVITY AND UNIVERSAL DARWINISM

Since human intelligence as well as other natural intelligence is inborn by design via gene and developed through environment by learning, hence ASI should also be inborn by design via Multi-Agent and developed through Multi-Environment by learning, that is why we model ASI with Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning by adopting the concept of Multiverse.

In the current practice of AI, both Reinforcement Learning and Deep Learning play significant roles. Today Deep Learning has been applied to fields including computer vision, speech recognition, natural language processing, audio recognition, social network filtering, machine translation, bioinformatics, drug design, medical image analysis, material inspection and gaming producing results comparable to and in some cases superior to human experts. Despite of such success, how Deep Learning exactly works remains unknown as a blackbox, Various efforts have been attempted to understand how Deep Learning works mathematically, one significant of them is the Universal Approximation Theorem. The theorem states that simple neural networks such as FNN can represent a wide variety of continuous functions approximately when given appropriate parameters and proper activity function; however, it does not explain why those neural networks can demonstrate learnability and hence intelligence mathematically.

Reinforcement Learning enables an agent to self-navigate an environment using rewards. The environment is typically formulated as a Markov Decision Process (MDP) or Multi-Armed Bandits, for the former Bootstrapping through Dynamic Programming technique is used. The main difference between the classical Dynamic Programming methods and Reinforcement Learning algorithms is that the latter do not assume knowledge of an exact mathematical model of the MDP, and they target large MDPs through function approximation by leveraging Deep Learning where exact methods become infeasible. The exploration vs. exploitation trade-off in Reinforcement Learning has been most thoroughly studied through the single-state Multi-Armed Bandit problem and finite state space MDPs. Reinforcement Learning requires clever exploration mechanisms. Randomly selecting actions, without reference to an estimated probability distribution, shows poor performance. The case of (small) finite Markov Decision Processes is relatively well understood. However, due to the lack of algorithms that scale well with the number of states (or scale to state space exploration problem), simple exploration methods are the most practical. One way to tackle such challenge is adopting greedy algorithms. While whether exact/optimal or approximate/greedy, all of those algorithms have their pros and cons, how to model them along as other undiscovered counter-parts mathematically in a unified way remains a challenge.

Deep Reinforcement Learning as illustrated uses Deep Learning and Reinforcement Learning principles in order to create efficient algorithms that can be applied on both games and real-life applications. Instead of using actual state-value pairs, which is often too large in environments where the State-Action/Spatial-Temporal space (SpaceTime) for Q-learning to converge in short time, by using Deep Learning architecture such as DNN in approximating the action-value function such as Q-function with Reinforcement Learning algorithms such as Q-learning, actor critic or etc, a powerful Deep Reinforcement Learning model can be created that is capable to scale to problems with very large state-action spaces that were previously unsolvable. While Reinforcement Learning itself can be explained mathematically, Deep Reinforcement Learning is not due to the blackbox nature of Deep Learning used for function approximation.

A Hierarchical Multi-Agent Multi-Environment system (Self-Organized System) is a complex hierarchical system composed of multiple interacting intelligent agents interacting with multiple environments. Hierarchical Multi-Agent Multi-Environment systems can solve problems that are difficult or impossible for an individual agent or a monolithic system to solve. Intelligence in hierarchy including strategy, tactics and reactive control may include methodical, functional, procedural approaches, algorithmic search. General Relativity, provides a unified description of gravity as a geometric property of space and time (SpaceTime), by generalizing Special Relativity and Newtonian Theory. It leads to spectacular predictions as black holes, gravitational waves, and the big bang in early universe. In General Relativity, space and time are not modeled as separate entities, instead they are modeled as 4-Dimensional SpaceTime, three spatial dimensions and one time dimension, and gravity is viewed as a consequence of the curved geometry of such 4-Dimensional SpaceTime.

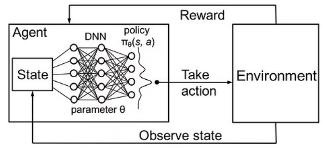

Figure 7: Deep Reinforcement Learning

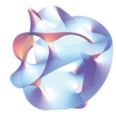

Figure 8: Projection of a Calabi-Yau Manifold

The curvature of SpaceTime is directly related to the energy and momentum. Differential Operators, Lie Groups that provide a natural framework for analyzing the continuous symmetries of differential equations, and Metric Manifolds (Manifolds with Tensor Fields), as special Abelian Groups, are three mathematical foundations of General Relativity; whereas for Quantum Mechanics, they are Differential Operators, and Hilbert Spaces, which are special Banach Spaces, and special Metric Spaces, and special Topological Spaces. Einstein Field Equation, a system of Nonlinear Partial Differential Equations, as the core of General Relativity, describes the relation between the geometry of a 4-Dimensional, Pseudo-Riemannian Manifold in SpaceTime topology, and the energy-momentum contained in that SpaceTime. Being nonlinear in nature, it is very difficult to solve, except several known exact solutions under restricted conditions, such as the Schwarzschild Solution, the Reissner-Nordstrom Solution, the Kerr Solution, most of those solutions are approximations relying on computing-based numerical methods such as perturbation or function approximation adopting Deep Learning based approach. Among them one solution is the Expansion of Universe.

A Pseudo-Riemannian Manifold is a generalization of a Riemannian Manifold in which the requirement of positive-definiteness is relaxed, the metric Tensor need not be positive-definite, but only need be a non-degenerate bi-linear form. Einstein Manifold is a Riemannian or Pseudo-Riemannian Manifold whose Ricci Tensor is proportional to the metric, which is a solution of the vacuum Einstein Field Equations with cosmological constant. A projection of a Calabi-Yau Manifold yielding applications in theoretical physics, particularly in Superstring Theory, is a special Kahler Manifold , who is further a Riemannian Manifold. The Riemann curvature Tensor is given in terms of the Levi-Civita connection ∇ by the following formula: $R(u, v)w = \nabla_u \nabla_v w - \nabla_v \nabla_u w - \nabla_{[u,v]} w$, where [u,v] is the Lie bracket of vector fields in Lie algebra. For each pair of tangent vectors $u, v, R(u, v)$ is a linear transformation of the tangent space of the Manifold. It is linear in u and v, and so defines a Tensor. For metric g on Manifold M, the tangent vectors at each point in the Manifold M can be classed into three different types, timelike if $g(X, X) < 0$ null or lightlike if $g(X, X) = 0$ spacelike if $g(X, X) > 0$

The Einstein Field Equation can be used in modeling the Reinforcement Learning part of Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI, and it is listed as follows:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \frac{G}{C^4} T_{\mu\nu} \tag{1}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{2}$$

In the above equations, $\mu, \nu = 1, 2, 3, 4$ in 4-Dimensional Pseudo-Riemannian Manifold of SpaceTime Model SO(1,3), which adopts SpaceTime Algebra (also as Clifford Algebra) $Cl_{1,3}(R)$, it is built up from an orthogonal basis of one time-like vector γ_0, and three space-like vectors, $\gamma_1, \gamma_2, \gamma_3$, with the multiplication rule $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski Metric with signature (- + + +). Thus, $\gamma_0^2 = +1$, $\gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -1$ Otherwise $\gamma_\mu \gamma_\nu = -\gamma_\nu \gamma \mu$, Einstein Tensor $G_{\mu\nu}$ is symmetric, hence $G_{\mu\nu} = G_{\nu\mu}$

Furthermore in the above equations, $R_{\mu\nu}$ is the Ricci curvature Tensor, $g_{\mu\nu}$ is the metric Tensor, R the scalar curvature. $T_{\mu\nu}$ is the Energy-Momentum (also as stress-energy) Tensor, Λ is the Cosmological Constant explaining the existence of Dark Energy related to the Expansion of Universe, G the Newton's Gravitational Constant, c the Speed of Light, and $8\pi\frac{G}{C^4}$ the proportionality constant.

In the above Einstein Equation for the Reinforcement Learning part of Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI, it states that SpaceTime (True/Quasi-Spatial Action-Temporal State Space), tells Reinforcement Learning of ASI how to demonstrate Agent Intelligence/Computing Intelligence/Planning Intelligence/Exploitation Intelligence, and Reinforcement Learning of ASI tells SpaceTime how to curve. Hence Agent Intelligence/Computing Intelligence/Planning Intelligence/Exploitation Intelligence is just the geometry effect of 4-Dimensional SpaceTime caused by Geometrization, in other words, Agent Intelligence/Computing Intelligence/Planning Intelligence/Exploitation Intelligence is modeled as curvature of 4-Dimensional SpaceTime.

When a system behaves no difference when time is reversed, it is said to show T-symmetry. The second law of thermodynamics explains the phenomenon of irreversibility of any isolated system with its entropy keeping on increasing in nature. However since intelligence originally only belongs to living systems, also is part of life, whether artificial life or real life, and life is an open system which can demonstrate reversibility that can operate continuously at non-equilibrium. Hence we can make a T-Symmetry extension for intelligence. Based on CPT (Charge, Parity, Time) symmetry, which is one of the most fundamental symmetry in physics. Such extension leverages Riemannian Manifold's curvature symmetry, as part of space symmetries, which Pseudo-Riemannian Manifold lacks, because it is curvature collineation that preserves the Riemann Tensor.

$$g_{\mu\nu}(x) = \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) & g_{14}(x) \\ g_{21}(x) & g_{22}(x) & g_{23}(x) & g_{24}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) & g_{34}(x) \\ g_{41}(x) & g_{42}(x) & g_{43}(x) & g_{44}(x) \end{bmatrix} \tag{3}$$

$\Longrightarrow$

$$g_{\alpha\beta}(x) = \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x)... & g_{1N}(x) & \\ g_{21}(x) & g_{22}(x) & g_{23}(x)... & g_{2N}(x) & \\ g_{31}(x) & g_{32}(x) & g_{33}(x)... & g_{3N}(x) & \\ ... & ... & ... & ... & ... \\ g_{(N1}(x) & g_{N1}(x) & g_{N3}(x)... & g_{NN}(x) & \end{bmatrix} \tag{4}$$

The Extended Einstein Field Equation on Manifold (M_N, g) in modeling the Deep Learning part of Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI is formulated as follows:

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi\frac{G}{C^4}T_{\alpha\beta} \tag{5}$$

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} \tag{6}$$

Where $\alpha, \beta = 1, 2, ..., N$ in N-Dimensional T-symmetry Extended Riemannian Manifold of GeneSpace Model by replacing physical space with GeneSpace as source of intelligence. The model adopts Clifford algebra $Cl_{1,N}(R)$, it is built up from an orthogonal basis of N GeneSpace-like vector, N can be as big as infinite in terms of GeneSpace dimensionality. $\gamma_1, \gamma_2, \gamma_3, , \gamma_N$, with the multiplication rule $\gamma_\alpha\gamma_\beta + \gamma_\alpha\gamma_\beta = 2\eta_{\alpha\beta}$ where $\eta_{\alpha\beta}$ is the Riemannian metric with signature (+ + + ... +). with $\gamma_\alpha\gamma_\beta = \gamma_\beta\gamma\alpha$

In the above Extended Einstein Equation for the Deep Learning part of Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI, it states that GeneSpace, tells Deep Learning of ASI how to demonstrate Environment Intelligence/Learning Intelligence/Trial and Error Intelligence/Exploration Intelligence, and Deep Learning of ASI tells GeneSpace how to curve. Hence Environment Intelligence/Learning Intelligence/Trial and Error Intelligence/Exploration Intelligence is just the geometry effect of

N-Dimensional GeneSpace caused by Geometrization, in other words, Environment Intelligence/Learning Intelligence/Trial and Error Intelligence/Exploration Intelligence is modeled as curvature of N-Dimensional GeneSpace.

Even though our Extended General Relativity does not reduce the computational complexity of ASI, as the Extended Einstein Field Equations are still higher-order nonlinear partial differential equations, and whether continuous optimization, or differential geometry, or differential equations, are functionally equivalent in solving nonlinear problems algorithmically, geometrically, algebraically respectively. Actually there is a chicken-and-egg paradox here: one way of solving both the Extended Einstein Field Equations is through function approximation adopting Deep Learning based approach, and Deep Learning can be formulated in the Extended Einstein Field Equations themselves, we call it a typical Problem/Solution Paradox, or more universally as Universal Paradox, actually all paradoxes can be classified as Universal Paradox, where solutions rely on problems, and vice verse. Anyway our extension not only lays a consistent mathematical foundation and makes solution space visible and transparent for ASI, but also paves the way for our ongoing work on complexity reduction for ASI.

Complexity reduction in computation and learning, whether discrete or continuous, deterministic or stochastic, online or off-line, exact or approximate with/without tight (upper and/or lower) bounds to optimal solution, remains big challenges for AI and ASI. Optimization can be classified as different ways: Discrete optimization (such as integer programming and Combinatorial Optimization) vs. Continuous Optimization; Gradient-based (First Order as opposed to Second Order Optimization) vs. Constrained Optimization; Convex Optimization vs. Non-Convex Optimization. All problems can be classified as P, NP, NP-complete, and NP-hard. There are very few optimal/exact yet greedy algorithms which can take the advantage of special structures such as matroid in geometric space (independent sets) with the following properties: heredity and augmentation/exchange, whereas all of the remaining, such as non-convex optimization[7] for geodesics in high-dimensional nonlinear Manifold frequently encountered in Deep Learning, have to resort to various heuristics for approximation with or without bounded complexity. In the meantime, in the era of AI and ASI, as correlation dominating causality, the challenges of NP-complete and NP-hard have been mitigated due to the inherent difference between Computing Complexity and Learning Complexity is increasing. Nevertheless, there is no free lunch in computational complexity reduction, all are trade-off among (running) time and space (memory and disk). In the mean time, technology advance such as all-memory computing, quantum computing also does not change the nature of computational complexity. However, fortunately achieving ASI does not require 100% computational tractability: for hard problems, you just need a viable solution instead of the best solution.

5 Discussion

New scientific theory and new technological practice are two pillars of any industrial revolution. It is hard to deny that while the latter has to be accepted by the majority in the market from the beginning, the former seems always is being embraced by the minority in the academia since its inception. The matter of fact is only dictatorship instead of Democracy works in this world, while revolutionary theory always gets more resistance than evolutionary theory because politics exists everywhere and human nature tends to follow the intuition instead of the counter-intuition. Since the renaissance, causality-based Reductionism, such as Precipice Reductionism in physics and Step-by-Step Reductionism in biology, instead of correlation-based Anti-Reductionism, has been in dominance, and quite successful. However, when it comes to fundamental problems beyond basic science and technology, such as the origin of universe and life, the nature of intelligence and consciousness, and how brain and genes work among, just to name a few, Reductionism experiences difficulty in offering decent answers.

Now in modern physics, one of biggest challenges is the incompatibility of General Relativity and Quantum Electrodynamics, part of the Grand Unification challenge. If we don't quantize General Relativity, then we run into contradictions with Quantum Electrodynamics. The easiest one to understand is the infinite universe problem: Observations indicate that the universe is probably infinite, even though the observed universe is finite. Quantum Electrodynamics tells us the universe must be finite, and General Relativity tells us it can be infinite. Resolving such incompatibility is very important as Multi-Agent in Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI, has different interpretations in General Relativity and Quantum Electrodynamics, which can be a huddle.

In Abiogenesis, part of the roots of Artificial Design, one of the most fundamental questions is the chicken-and-egg paradox for DNA and protein, or the DNA/Protein Paradox, the origin of life puzzle, the dilemma of causality, it belongs to Problem/Solution Paradox, and is just another kind of Universal Paradox. While there are different answers for that, such as proximate causation, ultimate causation, and reciprocal causation, or even RNA first, however, nothing is convincing. Our answer to this DNA-and-Protein paradox is that, both DNA and protein were created by someone who depends on our faith, just like matter and curved-SpaceTime (field), life and intelligence, were created simultaneously, just like our formulation of Artificial Design model of Hierarchical Multi-Agent Multi-Environment Model-agnostic Policy-agnostic Deep Reinforcement Learning based ASI for Artificial Design. In other words, initial conditions for any complex system in conventional thinking, whether living or physical, does not work any more. Simply put, Artificial Design, which states that that intelligence, whether natural, artificial, or super-artificial like ASI, is just a combination of the geometry effect of both 4-Dimensional SpaceTime and N-Dimensional GeneSpace caused by Geometrization, just like both General Relativity and Universal Darwinism, is also counter-intuitive.

6 Appendix

Here we list a few existing works we never know till finalizing this paper at the last minute as follows, we are glad there are already a few existing interesting works aside from pioneering works done by both Einstein and Darwin, yet none of them offers a global universal mathematical foundation for bio-physical inspired ASI associated with both General Relativity and Universal Darwinism in our humble opinion, use your own judgment please. By the way, the following list may not be complete, we are sure it can be expanded as we continue in working on this challenging yet exciting frontier. The problem is being in industry, we do not possesses the luxury in conducting extensive if not exhaustive literature search (let alone reading), especially given its interdisciplinary nature, so please bear with us. In case your important work has not included yet, please let us know at your earliest convenience.

- Hornik on Universal Approximation Theorem [22]
- Rashevsky on Geometrization of Biology [23]
- Palais on Geometrization of Physics [24]
- Btyson on Intelligence by Design [25]
- Campbell on Bayesian methods and Universal Darwinism [26]
- Pellionisz and Llinas on Geometrization of Brain Function through Tensor Network Theory [27]
- Lawrence on Manifold Learning for Dimension Reduction [28]

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