Research Article

Interference Between Number Magnitude and Parity

Discrete Representation in Number Processing

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Abstract: Interference between number magnitude and other properties can be explained by either an analogue magnitude system interfering with a continuous representation of the other properties or by discrete, categorical representations in which the corresponding number and property categories interfere. In this study, we investigated whether parity, a discrete property which supposedly cannot be stored on an analogue representation, could interfere with number magnitude. We found that in a parity decision task the magnitude interfered with the parity, highlighting the role of discrete representations in numerical interference. Additionally, some participants associated evenness with large values, while others associated evenness with small values, therefore, a new interference index, the dual index was introduced to detect this heterogeneous interference. The dual index can be used to reveal any heterogeneous interference that were missed in previous studies. Finally, the magnitude-parity interference did not correlate with the magnitude-response side interference (Spatial-Numerical Association of Response Codes [SNARC] effect) or with the parity-response side interference (Markedness Association of Response Codes [MARC] effect), suggesting that at least some of the interference effects are not the result of the stimulus property markedness.

Keywords: analogue magnitude system, discrete number representation, numerical interference, homogeneous interference, heterogeneous interference, SNARC effect, MARC effect, PNARC effect

Highlights

- Numbers interfere with discrete parity property, supporting discrete number representation models in numerical interference effects.
- Numerical interference effects do not correlate, contradicting the polarity and the markedness interference models.
- Heterogeneous interference can be revealed with the new dual index method.

Number Magnitude Interference Effects

Number magnitude can interfere with other properties. A salient example is the SNARC (Spatial-Numerical Association of Response Codes) effect. In the SNARC effect, typically in a parity task, participants respond faster for small values with the left response button than with the right response button, and they respond faster for large values with the right response button than with the left response button (Dehaene, Bossini, & Giraux, 1993). Thus, small numbers are associated with the left side, and large numbers with the right side, although the association depends in part on cultural background, for example, Iranian participants associate small numbers with the right side, and large numbers with the left side (Dehaene et al., 1993). Another example of numerical interference is the size congruity effect, in which the physical size of the stimuli interferes with the numerical value (Henik & Tzelgov, 1982). As another example, number magnitude also interferes with duration (Oliveri et al., 2008).

There are various explanations for such interference effects. In the present introduction we take the SNARC effect as an example, but the explanations could be generalized to several other symbolic numerical interferences. To explain the SNARC effect some of the explanations have presupposed analogue (i.e., continuous) representations in the background (Figures 1A and 1B), while other accounts have supposed discrete (i.e., categorical) representations (Figures 1C and 1D).

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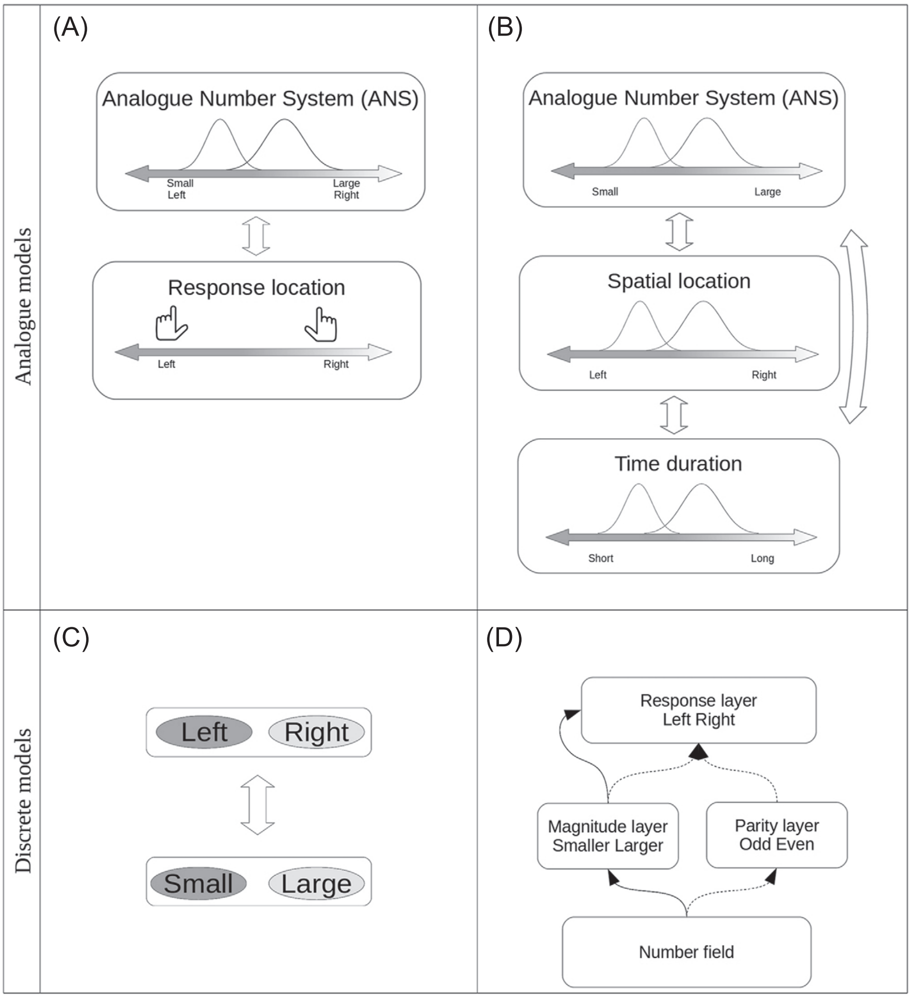

Figure 1. Four explanations of the SNARC effect. Light and dark arrows and light and dark nodes represent the same measurement direction and the same polarity/markedness. Solid lines in the delta-rule model represent automatic connections. (A) Analogue number system (ANS) model, (B) generalized magnitude system (GMS) model, (C) polarity/markedness, and (D) delta-rule connectionist model.

Analogue Explanations

As a first explanation, a continuous, noisy representation, the Analogue Number System (ANS) is proposed, which works according to the Weber's law, similar to many other representations processing simple perceptual properties (Moyer & Landauer, 1967). Dehaene and his colleagues propose that this ANS is what causes the interference: it might have a spatial property (values are connected to spatial locations), and this spatial property interferes with the spatial representation of the response locations (Dehaene et al., 1993).

According to a related, second explanation, many non-numerical properties and the numerical values can be stored in similar analogue systems. While these systems process different inputs, all of them adhere to Weber's law (Figure 1). This Generalized Magnitude System (GMS) model supposes that partly because of the similar mechanisms, these representations might interfere with each other (Cantlon, Platt, & Brannon, 2009). In line with this reasoning, many interference effects were described between continuous properties, for example, pitch interfering with response location (Rusconi, Kwan, Giordano, Umiltà, & Butterworth, 2006), luminance interfering with response location (Fumarola et al., 2014), or duration interfering with number magnitude (Oliveri et al., 2008). Although based on different motivations, similar additional models predicting interference of continuous representations have been proposed (Bueti & Walsh, 2009; Cohen Kadosh, Lammertyn, & Izard, 2008; Henik, Leibovich, Naparstek, Diesendruck, & Rubinsten, 2012; Walsh, 2003).

Discrete Explanations

A next group of models explaining the SNARC effect suppose discrete representation in the background

(Figures 1C and 1D). According to a third model, the SNARC effect is rooted in the interference of discrete concept pairs (Proctor & Cho, 2006). In this polarity model, antonyms, like small-large, left-right or true-false, have polarity properties: one of the items is positive, the other one is negative (*positive* and *negative* are arbitrary labels of the items, and they are not directly related to the sign of the numbers). Concepts with the same polarity can enhance the processing, while concepts with opposing polarity might inhibit it, resulting in interference. In the case of the SNARC effect the small-large concepts interfere with the left-right concepts, supposing that in Western culture *small* and *left* have the same polarity. The polarity model was successfully applied to simulate the SNARC effect (Leth-Steensen, Lucas, & Petrusic, 2011; although see a study where an extension of the polarity model in interference effects was not confirmed: Santiago & Lakens, 2013).

The markedness model offers a similar explanation as the polarity model (Hines, 1990). The markedness model proposes that one member of an opposing pair is marked, for example, in the case of parity, odd is marked. As originally proposed in linguistics, markedness of a word has many related properties, for example, marked members of the pairs are usually slower to process, children learn them later, and sometimes they are linguistically formed from the unmarked member, and so forth. Critically, markedness can explain interference as well: items with same markedness can be processed faster, while items with opposing markedness are processed slower (Nuerk, Iversen, & Willmes, 2004). The markedness model is similar to the polarity model with a few important differences: while markedness is related to language and is a relatively stable property of a pair, polarity is more flexible and can be task dependent (Cho & Proctor, 2007). Also, in the polarity model it is not always clear why positive or negative values are set for a specific property (Huber et al., 2015). Because polarity and markedness models have very similar predictions about the potential interference effects, we handle these two models together in several sections of the paper (but see the discussion of the critical differences in the PNARC Effect section and in the General Discussion). Although in the numerical interference context the markedness model was proposed only to explain the parity-response interference (Markedness Association of Response Codes [MARC] effect[1]), but not the SNARC effect (Nuerk et al., 2004), it is reasonable to extend to several other interference effects, as discussed in the work of Proctor and Cho (2006), or as it was noted later (Patro, Nuerk, Cress, & Haman, 2014).

A fourth model also proposes a discrete explanation for the SNARC effect. A formerly introduced delta-rule connectionist model of numerical effects (Leth-Steensen & Marley, 2000; Verguts & Fias, 2004, 2008; Verguts, Fias, & Stevens, 2005; Verguts & Van Opstal, 2014) could explain and model the SNARC effect (Gevers, Verguts, Reynvoet, Caessens, & Fias, 2006). In this model the number layer (number field) is connected to additional layers that are representing parity or categorical magnitude (large or small), finally, a response layer is used to represent left and right responses of a task (Figure 1). Based on cultural experiences, association is formed between the magnitude and the response layers. In a task, the magnitude is automatically activated, independent of whether the layer is relevant (e.g., in a comparison task) or not (e.g., in a parity task), then this categorical magnitude layer automatically activates the response layer, which will interfere with the task-relevant response layer activation, resulting in the SNARC effect. The model successfully explains some additional effects regarding the SNARC interference, for example, why slower responses show a stronger effect, or why the SNARC effect shows a categorical pattern instead of a continuous pattern in a comparison task (Gevers et al., 2006). Because the model supposes that the key for the explanation is the categorization of the values (large vs. small in the magnitude layer), the model can be considered as a discrete model.[2]

Some of the models introduced above propose an analogue explanation for the SNARC effect, while others propose discrete representations. One cannot contrast the two model types based solely on the presence of the SNARC effect, because all of the models give satisfactory explanations for it. In the present work, we contrast the current analogue and the current discrete explanations with a new type of numerical interference.

[1] The term MARC is used inconsistently in the literature. Nuerk et al. (2004) originally used the term MARC for both the parity-response interference, and for the markedness model, and the literature uses the term in both meanings. Here, we use MARC to denote the interference effect, in line with other similarly named interference effects, and we term the model as markedness, as the term has already been used in the linguistic and numerical cognition literature.

[2] Although the number field of the model was originally interpreted as an implementation of the ANS (Verguts & Fias, 2004; Verguts, Fias, & Stevens, 2005), the model is unable to produce Weber's law, since the size effect can be seen only after introducing the frequency of the numbers to the model (Verguts et al., 2005). Instead of the original interpretation, it might be more appropriate to consider the number field as an implementation of a discrete model, in which the "noise" of the number field is not a noise based on Weber's law, but the spreading activation of the discrete units (Krajcsi, Lengyel, & Kojouharova, 2016). Thus, the delta-rule connectionist model is a discrete model not only because of the magnitude layer, but also because of the number field.

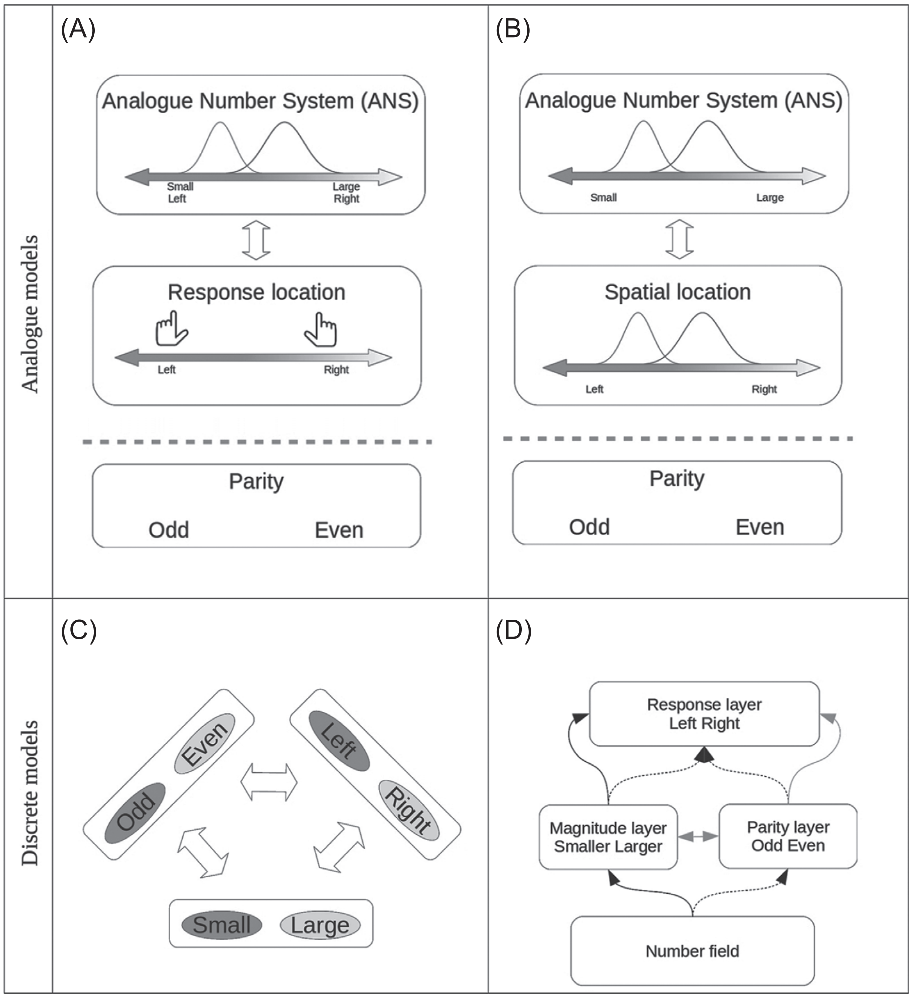

Figure 2. Prediction of the four models for the MARC and PNARC effects. In the delta-rule connection model the new connections are denoted with light gray arrows. (A) Analogue number system (ANS) model, (B) generalized magnitude system (GMS) model, (C) polarity/markedness, and (D) delta-rule connectionist model.

Does Numerical Information Interfere With a Discrete Property? The Number Magnitude-Parity Interference

One reason why the presence of the SNARC effect cannot contrast the analogue and the discrete models is that both properties of the interference (number and space) could be represented both continuously or discretely: either on analogue representations as proposed by the ANS or the GMS models, or on discrete representations as suggested by the polarity/markedness and the delta-rule connectionist models. This problem also applies to other known numerical interference effects, for example, in the size congruity effect (Henik & Tzelgov, 1982), the physical size could also be stored on both continuous and discrete representations.

The present work investigates whether number magnitude interferes with a discrete property that is supposedly not stored on an analogue representation. Here, we investigate the presence of the magnitude-parity interference. Following the tradition of the literature we term it the PNARC (Parity-Numerical Association of Representational Codes) effect.[3]

Existence of the PNARC effect would be in line with the discrete models above, but cannot be explained by the presented analogue models (Figure 2). More specifically, (1) extending the ANS model could imply that the ANS has a parity feature (similar to the spatial feature), which does not seem to be a feasible assumption. Consequently, the ANS model does not suppose the presence of the PNARC effect. (2) Because most probably parity is not

[3] Note that unlike in the term SNARC (Spatial-Numerical Association of Response Codes) effect, in the term PNARC effect, R stands for Representational and not Response, because in a parity-number association response is probably not a key component.

stored in an analogue representation, the GMS model does not presuppose the PNARC effect either. While in a strict sense neither the ANS nor the GMS model excludes the presence of the PNARC effect (they might interpret the potential PNARC effect as the result of an unspecified additional numerical representation), the PNARC effect would question the general statement that the ANS is the only source of the magnitude-related symbolic numerical effects. (3) According to the polarity/markedness models, the PNARC effect can be present, because any property pairs that have polarity/markedness can interfere. Actually, the models have an even stronger prediction. In our parity task, there are three properties that are relevant in the interference effects we investigate (Figure 3): parity, number and response location, and all of them can interfere with each other. It is known that numbers and response location interfere (the SNARC effect: Dehaene et al., 1993), and response location and parity also interfere (the MARC effect: Nuerk et al., 2004). Based on these results the models suppose that all three relevant properties have polarity, consequently the models also predict that the interference of parity and magnitude should also exist. Thus, the polarity/markedness models not only allow the appearance of the PNARC effect, but expect it. (4) In the case of the SNARC effect the delta-rule connectionist model relies on the dual-route model of interference, in which both the relevant and the irrelevant information is processed simultaneously, and the identical or differing results of the two routes generate the interference (Gevers et al., 2006). The architecture of the model displayed above is deliberately an initial version, and additional details, for example, connections can be added (Verguts & Fias, 2008). It is easy to imagine that an automatic connection between the magnitude and the parity layers can produce the PNARC effect. Although it is not trivial what environmental experiences would form such a connection, the possibility cannot be excluded. Overall, based on our current knowledge of the model, the connectionist model does not predict the appearance of the PNARC effect, although the model allows for its presence (see the summary of the four models about the appearance of the PNARC effect in Table 1).

Interestingly, the presence of the PNARC effect has already been reported in the literature, although its importance or even its presence was not discussed. While investigating the MARC and SNARC effect in various conditions (positive and negative Indo-Arabic numbers and number words) in a parity decision task, Nuerk et al. (2004) analyzed the data with ANOVA, where the number magnitude-parity interaction was also reported. They found that in positive Indo-Arabic numbers both error rate and reaction time showed a number magnitude-parity interaction. In number words, only the error rate showed this interaction, while in negative Indo-Arabic numbers no interaction was

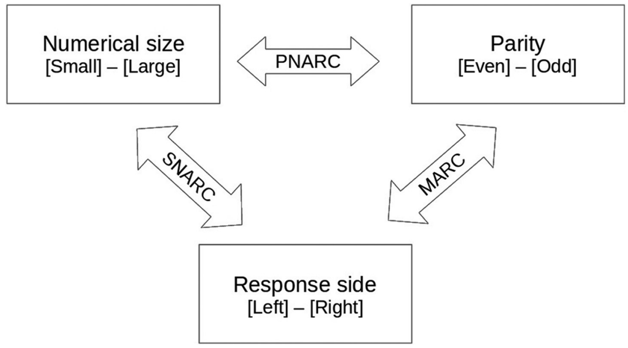

Figure 3. Three critical properties in our parity task, and the interference effects between them.

observed. While the significant results were reported, the presence or the importance of these interactions as a PNARC effect was not discussed either in the results part or in the discussion. Here, we try to replicate those former findings, and in contrast with the Nuerk et al. (2004) study we discuss the theoretical consequences of the presence of that interference, and we also investigate the relations of the interference effects measured in parity task (see the Possible Correlation of the Interference Effects section).

To summarize, the suggested PNARC effect cannot be explained by the current analogue models, while it would be in line with the current discrete models. The main question of the present study is whether the PNARC effect exists, supporting the discrete models, or does not exist, supporting the analogue models.

Possible Correlation of the Interference Effects

As already noted above, the planned parity task includes three properties that might interfere with each other: parity, number and response location (Figure 3). The three properties might form three interference effects:

(a) the SNARC effect between the number magnitude and response code (Dehaene et al., 1993),
(b) the MARC effect between the response code and the parity (Nuerk et al., 2004), and
(c) the potential PNARC effect between the parity and number magnitude.

Beyond the presence of the PNARC effect, it might be of interest whether the interference effects correlate (Table 1). A precondition of such analysis is the appropriate variability of the interference effects. Accordingly, several SNARC studies revealed individual differences in the interference indices (see a summary in Hoffmann, Mussolin, Martin, & Schiltz, 2014). Regarding the predictions of the models (1 and 2) both ANS and GMS models argue that

Table 1. The prediction of the four models for the two critical results

	Analogue models	Discrete models	
	(1) ANS and (2) GMS models	(3) Polarity/markedness models	(4) Delta-rule connectionist model
Appearance of the PNARC effect	Strictly, independent of the ANS/GMS models. More generally, PNARC should not exist.	PNARC should exist	PNARC might exist
Correlation of the interference effects	SNARC does not correlate with MARC or PNARC. Correlation between MARC and PNARC is unspecified.	The three interference effects should correlate	Either lack or presence of correlation is possible

Notes. ANS = analogue number system; GMS = generalized magnitude system; PNARC = parity-numerical association of representational codes; SNARC = spatial-numerical association of response codes; MARC = markedness association of response codes.

the SNARC effect originates in the ANS, while the MARC and supposedly the potential PNARC effects are independent of the ANS. Thus, the SNARC effect probably does not correlate with the other two interference effects. The models do not have any predictions about the correlation of MARC and PNARC effects. (3) The polarity/markedness models predict that interference effects that contain overlapping property might correlate (e.g., SNARC and PNARC effects might correlate, because both contain the magnitude), supposing that individual differences of the indices originate in the individual differences of the overlapping property. Thus, the models predict correlation, if sufficient statistical power is given. (4) According to the delta-rule connectionist model the interference effects root in the connections between specific layers, and because technically all connections between layers are independent of other connections, in a strict sense the interference effects should not correlate. However, considering the environmental sources of those connections, some correlation might be expected. Due to the flexibility of this model and because only a few relevant details are known about it, it is not trivial to propose a straightforward prediction about the correlation of the interference effects.

There are two general methodological constrains in this test. First, it is possible that there are general components (e.g., the conflict resolution), that are independent of the property-specific processing, and which might generate correlation. Second, because the sizes of the potential correlations are not known, it is possible that our test will lack statistical power. Thus, either we can observe correlations even if the overlapping representations would not predict them, or we might miss observing the correlations even if they are present. Therefore, it is possible that the presence or the absence of the correlations cannot contrast the models. However, a further aspect of the results will help us to clarify these issues, which we are going to discuss in the relevant parts of the Results section.

To summarize, in this study we investigate (a) whether parity interferes with number magnitude, and (b) whether the three interference effects of the three relevant properties (SNARC, MARC, and PNARC) correlate. The predictions of the four models are summarized in Table 1.

Methods

Participants

Fifty-seven university students took part in the experiment for partial course credit. Participants with higher error rate than the mean +2 *SD* of the group, which was 14.9%, were excluded (3 participants). Data of 54 participants were analyzed: 39 women, with mean age of 21.4 years, 2.4 years *SD*. Six of the participants were left-handed, and because this is only a very small part of the sample, right- and left-handed participants were not analyzed separately.

Stimuli and Procedure

Participants made parity decisions. Numbers were between 1 and 9, and the instruction was explicit about this range.[4] In a trial, a single Indo-Arabic digit appeared on the screen, and the participants decided whether the digit was even or odd, by pressing one of the two response buttons. The digit was visible until response. In case of erroneous response an auditory feedback was given. After the response, a blank screen was shown for 700 ms. Two hand conditions were used to ensure that all numbers and parities can be assigned to both response sides. In one of the hand conditions even numbers were responded to with the left Control key with the left hand, and odd numbers were responded to with the decimal sign on the numeric keypad with the right hand (the leftmost and the next to rightmost keys

[4] Unlike in some similar studies, number 5 was also used here, resulting in more odd than even numbers, therefore more “odd” than “even” responses in the parity decision task. However, this inequality in the responses does not cause any bias in the interference analyses applied in the present study.

on the keyboard), while in the other hand condition the stimulus-response association was reversed. In both conditions all numbers between 1 and 9 were presented 40 times. The order of the stimuli was randomized. All participants took part in both hand conditions, and the order of the conditions was counterbalanced across participants. The instruction stressed both speed and accuracy. Running the whole experiment required approximately 30 min for a participant. Stimulus presentation and data collection were performed with PsychoPy software (Peirce, 2009).

Analysis Methods

Unified Index Calculation

First, the interference effects were computed with the regression analysis method that is frequently applied for testing the SNARC effects (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996; Lorch & Myers, 1990). For example, to compute the PNARC effect, median reaction time of the correct responses was calculated for all digits for all participants. Then, the slope of the reaction time change across digits was calculated for both the odd and the even numbers. Finally, the difference of the two slopes was calculated (even slope subtracted from odd slope), and the deviation of the PNARC index from zero was tested.[5] Similar methods were applied to the other interference effects.

Dual Index Calculation

However, the interference effects can be investigated with an alternative method. While analyzing the data, we observed that in the case of the PNARC effect, the slopes showed a systematic relation: most of the participants, who had positive slope for the odd numbers (i.e., larger odd numbers are responded slower than smaller odd numbers), had negative slope for even numbers (i.e., larger even numbers are responded faster than smaller even numbers). In the same time, most of the participants showing negative slope for the odd numbers, have shown positive slope for even numbers. This means that some participants associated oddness with small numbers and evenness with large numbers, and some other participants associated these properties in a reversed way. We term this type of interference a *heterogeneous interference*, because although there are associations between the properties, the direction of the association is not uniform within the group. To analyze this relation in a more statistical way, one can plot the even and odd slopes on a scatter plot (as in Figure 4 later in the Results section) and look for the correlation. If there is a systematic relation between the two slopes as described above, one should observe a negative correlation. This negative correlation means that the magnitude information influences the parity decision time, but the direction of this influence shows individual differences, forming a heterogeneous interference.[6]

This heterogeneous interference in which different part of the sample shows different direction of the association is a known phenomenon in the literature. A similar heterogeneous interference was observed in Dehaene et al. (1993) where the direction of the numerical-spatial association depended on reading habits: while Iranian participants, moving recently to France, associated large numbers with the left side (in line with the right-to-left Persian writing system), Iranian participants, living in France for a longer time, associated large numbers with the right side (in line with the left-to-right Western writing system; see various additional factors that may change the direction of the SNARC effect in Shaki & Gevers, 2011). It is not the presence of the heterogeneous interference that is new in our analysis, but the method with which the interference can be detected: while in the cited work the time of movement from Iran to France is necessary to reveal the interference, with the current method this additional information is not necessary to discover a heterogeneous interference (we discuss this example in more details in the Interference Effects paragraph provided in ESM 1).

While we have discussed the method in the context of the PNARC effect, this correlational method can be applied to any interference effects investigated in the present study, or to any other interference effects.

See more details about the relation of the unified index and the dual index, and what information they are sensitive to in the Electronic Supplementary Material, ESM 1.

[5] Although this procedure is slightly different from the classic calculation procedure of regression analysis (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996; Lorch & Myers, 1990), it gives the exact same result. In the case of the SNARC effect, first, the reaction times for all digits and separately for both hands are calculated, then the differences of the two hands for all digits are computed, and finally, the slopes of the reaction time change across digits are calculated. However, in the case of the PNARC effect, one cannot use a similar procedure, because analogously one should subtract the data of the even digits from the data of the odd digits, but a digit cannot be both even and odd. Consequently, the classic SNARC-like calculation procedure is impossible for the PNARC effect. Nevertheless, in the SNARC procedure, the slope of the hand differences gives mathematically the same result as the difference of the single hand slopes. Importantly, the latter version is the calculation procedure that we applied here for the PNARC effect. Thus, the procedure applied here gives the very same result as if the classic SNARC computation procedure were used.

[6] Importantly, the correlation reflecting a heterogeneous interference cannot be a mathematical artifact, because the slopes of the even and the odd numbers are independent, they are calculated based on different trials, thus, they could have been uncorrelated, that is, participants might show increasing or decreasing reaction time for larger numbers independent of the parity of the values.

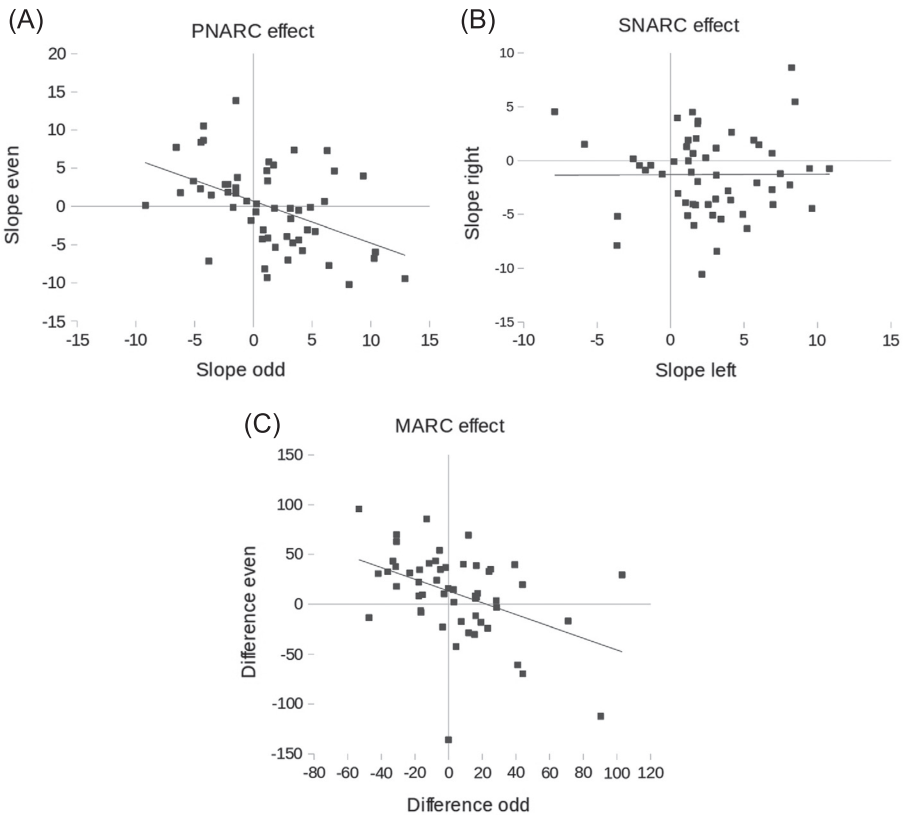

Figure 4. Scatterplot of (A) the slopes of even and odd numbers (PNARC), (B) the slopes of the right and left hands (SNARC), and (C) the hand differences of the even and odd numbers (MARC).

Results and Discussion

The raw data are available at https://osf.io/g7t2q/.

Interference Effects

PNARC Effect

PNARC effect was calculated as the reaction time change (slope) across the even or the odd numbers. For the unified index the slopes of the two parity were subtracted (even slope subtracted from odd slope). The PNARC index did not differ from zero with the unified index, $M_{\text{slope}} = 1.33$, 95% CI [−1.03, 3.69], $t(53) = 1.13$, $p = .265$, showing that homogeneous PNARC effect is not observable. However, measured with the dual index, the two slopes show a clear negative correlation, $r(52) = -0.461$, $p < .001$ (Figure 4).

The present results reveal the PNARC effect, although the direction of parity and number magnitude association is not homogeneous in our sample, similar to the already mentioned SNARC effect of Experiment 7 in Dehaene et al. (1993). On one hand, the present results replicated the findings of Nuerk et al. (2004), because both studies found PNARC effect. On the other hand, while in the present study the PNARC effect is heterogeneous, in the Nuerk et al. (2004) study the interference is homogeneous. The difference most probably cannot be explained by different analysis methods, because both analysis of variance (ANOVA) and the slope measurement methods rely on similar information, and they should give similar results (Pinhas, Tzelgov, & Ganor-Stern, 2011). At the moment it is not clear what could cause the difference, if this difference is reliable. Additionally, it is not possible to reveal the cause of the different parity-number associations based on the present results. Different associations can be formed by different individual numerical experiences (find some influencing factors in the case of the SNARC effect in Shaki & Gevers, 2011), or the differences can be ad hoc associations applied only in a single session (as similarly observed in the case of the SNARC effect in Fischer, Mills, & Shaki, 2010). Still, for the aim of the present work, it is not an important issue to know why the interference is heterogeneous, or what the sources of these individual differences are, but the important result is that the interference between parity and number magnitude is clearly observable. The appearance of the PNARC effect is not in line with the current analogue models, while it is more coherent with the presented discrete models.

SNARC Effect

SNARC effect was calculated as the reaction time change (slope) across the values for both hands. For the unified index the slopes of the two hands were subtracted (left slope subtracted from right slope). SNARC effect was observable with the unified index, $M_{\text{slope}} = -3.94$, 95% CI [−2.48, −5.41], $t(53) = -5.4$, $p < .001$. Measured with the dual index, there was no correlation (Figure 4), $r(52) = 0.004$, $p = .975$. These results replicated many studies, confirming that the SNARC effect is predominantly homogeneous in the published studies.

MARC Effect

MARC effect was calculated as the reaction time difference between left-hand and right-hand responses (left hand subtracted from right hand) for both even and odd numbers. For the unified index the two hand-differences values were subtracted (even subtracted from odd). MARC effect was not shown with the unified index, $M_{\text{slope}} = 6.54$, 95% CI [23.72, −10.63], $t(53) = 0.764$, $p = .448$, while it was significant with the dual index, $r(52) = -0.432$, $p = .001$ (Figure 4). These results show that MARC effect is mainly heterogeneous in our sample.

Lack of the MARC effect with the unified index is not surprising, since previous studies reported mixed results. Some studies found significant MARC effect with typically 20–30 participants (Berch, Foley, Hill, & Ryan, 1999; Cho & Proctor, 2007; Landy, Jones, & Hummel, 2008; Nuerk et al., 2004), while some others did not find the effect (Nuerk, Bauer, Krummenacher, Heller, & Willmes, 2005; Roettger & Domahs, 2015), and some additional studies found the effect in some tasks and/or conditions, and they could not find it in some other tasks or conditions, while the critical variable that makes the effect appear and disappear could not be identified (Berch et al., 1999; Cho & Proctor, 2007; Nuerk et al., 2004).

These previous studies might suggest that the size of the MARC effect is much smaller than the size of the SNARC effect, which is easier to replicate. However, our result also indicates that the MARC effect might be stronger than it has been shown in previous studies, because the heterogeneous nature of the effect in these samples could not be unveiled formerly.

To summarize, all three interference effects which were investigated here are observable, and in our sample the SNARC effect is homogeneous, while the PNARC and MARC effects are heterogeneous.

The present sample included six left-handed participants. It has been demonstrated that the MARC effect might depend on the handedness of the participant, while in the SNARC effect no handedness effect was observed (Huber et al., 2015). To investigate whether the present results are stable independent of the handedness of the participants, the data were reanalyzed without the six left-handed participants (left-handed participants were not analyzed separately or in contrast with the right-handed participants because of the very small sample size). Removing the six participants hardly changed the statistical results, and the very same significance pattern was found as with the whole sample. See the detailed results and the scatterplots as a function of handedness in ESM 1.

See a replication study that ensures the reliability of these findings in ESM 1.

While setting the aims of this study, we did not consider the possibility of a heterogeneous interference. Does the existence of the heterogeneous interference influence the predictions of the models? While in several models it is possible that the direction of the association is flexible, the sources of the directions may set some constrains. First, the markedness model supposed that the direction of the association is rooted in language use. However, language use in a linguistically and culturally homogeneous sample, as in our study, would predict a homogeneous interference. Therefore, markedness model seems to be an improbable explanation for the observed heterogeneous interference. Similarly, because the SNARC effect has been demonstrated to be dependent on the writing system (Dehaene et al., 1993), it is supposed that the ANS and the GMS models suppose homogeneous interference effects, unless the non-numerical component of the interference could explain the heterogeneous direction. Importantly, as discussed above, ANS/GMS models cannot explain either the MARC or the PNARC effects, because the MARC effect does not rely on the number magnitude information, and the PNARC effect relies on a discrete property. Finally, the more flexible polarity and the delta-rule models can be in line with heterogeneous effects.

Critically, our results show the presence of the PNARC effect. This result can be explained by two of the presented discrete models, while the introduced analogue models cannot account for it. Additionally, the PNARC effect is an interference between number magnitude and another property, as in the case of the SNARC effect, and as in the cases of other symbolic numerical interference effects. These similarities raise the possibility that if the PNARC effect cannot be explained by an analogue model, then maybe the analogue explanations are also incorrect in the cases of other similar symbolic numerical interference effects.

Correlation of the Three Interference Effects

To repeat the predictions of the models, according to the analogue models the SNARC effect does not correlate with the other two interference effects, according to the polarity/markedness models the three interference effects correlate, while the delta-rule connectionist model is underspecified to have straightforward predictions about the correlations (Table 1). Note that unified indices are used for these correlations.

However, appearance of the heterogeneous interference modified the prediction of the polarity/markedness model. Note that in the case of heterogeneous interference, the correlation of the interference effects means that a specific direction in one of the interference effects is related to another specific direction in the other interference. For example, correlation between the MARC and the PNARC effect means that if, for example, someone associates evenness with large numbers, then the same person associates evenness with the right side (direction of the associations depend on the sign of the correlation). Regarding the modified predictions, first, heterogeneous interference ensures the statistical power in the correlation of heterogeneous interference effects if the polarity/markedness model is correct. In the polarity/markedness model, a heterogeneous interference signals that one of the properties shows variance in the polarity (i.e., it changes across participants). Because SNARC is homogeneous, it means that in our sample the polarity of the number magnitude and the response side is fixed (i.e., these properties do not change across participants), therefore it is the parity that should vary across participants. Additionally, a significant correlation of the dual index shows that the data have sufficient variance for the given sample size, and a similar variance will be used in the interference correlation study: instead of the negative correlation seen in the scatterplot of the dual index, the variation along the $y = -x$ axis of the unified index will be used (see Figure 2 in ESM 1). Thus, if the polarity/markedness model is correct, in the present sample the PNARC and the MARC effects should correlate significantly. Second, according to the polarity/markedness model the heterogeneous interference effects will not correlate strongly with the homogeneous interference (i.e., the SNARC effect), because in the heterogeneous polarity the unified index changes its sign, and while on one side (i.e., with one sign) it creates positive correlation with the homogeneous interference, it creates negative correlation with the other sign, and the two correlations extinguish each other in the whole sample. Finally, the appearance of the heterogeneous interference does not change the predictions of the other models about the correlations.

Results did not show a significant correlation between the MARC and the PNARC effects, $r(52) = -0.101$, $p = .469$. This result is at odds with the polarity/markedness model. As it was discussed, the lack of the correlation cannot be caused by the lack of the statistical power, because the dual index correlation reflected sufficient statistical power in an analysis based on similar variance. This single result can be in line with the analogue models and the delta-rule connectionist model.

Neither the SNARC and the PNARC effects correlated, $r(52) = -0.099$, $p = .476$, nor the SNARC and the MARC effects, $r(52) = 0.251$, $p = .067$ (correlation of the SNARC and MARC effects were also investigated in Huber et al., 2015, and in line with our result, no significant correlation was found). The lack of these correlations can be in line with all models, thus these correlations cannot contrast the models. Specifically, (1 and 2) according to the analogue models an analogue system caused SNARC effect should not correlate with the other systems related MARC or PNARC effects. (3) According to the polarity/markedness model, because SNARC was a homogeneous interference, and PNARC and MARC were heterogeneous interference effects, these interference effects should not correlate. (4) The delta-rule connectionist model is flexible and for our purposes it is underspecified at the moment, thus any result can be consistent with it, although it is not trivial to specify what environmental experiences can form connection weights that could cause uncorrelated interference effects.

Among the three correlations, the PNARC-MARC correlation was critical from the viewpoint of model testing, because polarity/markedness models clearly predicted a correlation. Nonetheless, the correlation was not observed. This result does not entirely invalidate the polarity/markedness model, however. It is possible that while not all interference effects are caused by the mechanisms described by the model, some of them are. Although in this case the polarity/markedness model should specify why some of the interference effects can be explained by the model and why some of them cannot.

General Discussion

The main aim of the present study was to contrast the current models explaining numerical interference, in the frame of the analogue and discrete representation types. Our first specific question was whether number magnitude can interfere with a discrete property (in this case, parity), which is not likely to be handled by an analogue representation. Our results show that parity interferes with number magnitude, in this case supporting the introduced discrete representation models, and opposing the current analogue models.

After running the present study we found a conference publication describing a study with a different motivation, but with results relevant to our inquiry (Landy et al., 2008). In this work, the spatial feature of the ANS was contrasted with the non-spatial polarity model. To contrast whether a spatial or a non-spatial model can explain the SNARC effect, the parity task was modified to avoid the two-side spatial responses, and participants used verbal yes-no answers to decide whether a number was even or not. If in this paradigm there is still a number-response interference, then it cannot be explained by a spatial account. On the other hand if the interference cannot be observed in this paradigm, it means that the spatial response (and a spatial representation) was responsible for the interference. The SNARC-like number-response effect was observed even with these yes-no responses (i.e., faster "yes" response with larger numbers than with smaller numbers, and faster "no" response with smaller numbers than with larger numbers), showing that it is not the spatial feature of the ANS that causes the interference. From our viewpoint this study has another relevant aspect: the utilized yes-no responses are discrete categories, similar to the parity, unlikely to be stored in an analogue representation. This serves as another example that discrete and continuous properties might interfere, again supporting the discrete explanations against the current analogue models. While our work shows an interference between two properties of the stimuli, the verbal response parity task shows an interference between a property of the stimuli and the responses. The parity-number magnitude interference demonstrated in our study and the yes-no – number magnitude interference shown by Landy et al. (2008) converge to the same direction, supporting the role of the discrete representation in numerical interference effects.

Additionally, we found that the PNARC interference was heterogeneous. This property cannot be explained by the markedness model, because in the present sample with homogeneous linguistic and cultural background one would have expected a homogeneous interference. Markedness model seems to be an improbable explanation for the numerical interference effects for other reasons, as well. For example, Cho and Proctor (2007) have demonstrated that response side interferes not only with the parity of the number, but also with the property whether a number can be divided by 3. Because it is unlikely that the category whether a number can be divided by 3 is already stored as a linguistic category, linguistic markedness does not seem to be a likely explanation for that interference. More generally, flexibility of these interferences (Shaki & Gevers, 2011) makes the markedness model a less probable account for the numerical interference effects.

Interpreting these results more generally, different options are possible. (1) On one hand, we can keep the suppositions of the current models, suggesting that (a) all number magnitude related interference effects are handled by the same number representation and (b) the interference effects are rooted in the same types of representations (i.e., both representations are either continuous or discrete). If these suppositions are correct, then number representation should be a discrete one, because while some numerical interference effects can be explained by both types of representations, the PNARC effect and the yes-no – number magnitude interference can be supported only by a discrete system. This supposed discrete number representation would be in line with other approaches proposing that other symbolic number processing effects can also be explained with discrete number representations (Krajcsi, Lengyel, & Kojouharova, 2016). (2) However, alternatively, it is possible that different numerical interference effects are supported by different number representations (Patro et al., 2014), and therefore, in some interference effects potentially an analogue number representation is involved, while in some other interference effects a discrete system is utilized. Various homogeneous and heterogeneous numerical interference effects found in the present study may also be in line with this supposition. If this is the case, further studies are needed to investigate what type of representations are involved in the specific numerical interference effects, and it cannot be taken for granted that interference effects of continuous properties necessarily mean analogue representations in the background. (3) Finally, one might also question the second supposition of the current models, and imagine that an analogue representation could interfere with a discrete representation, which possibly cannot be excluded at the moment. Importantly, this hypothesis should introduce an entirely new type of model to account for the numerical interference effects.

The second main question of the present study was whether the interference effects correlate, since this result may test the polarity and markedness models. We found no correlation between the interference effects (in line with the finding of Huber et al., 2015). These results may be consistent with other findings showing differing properties of different interference effects (e.g., Huber et al., 2015 found that while handedness modulated the MARC effect, it did not modulate the SNARC effect), suggesting that different interference effects may rely on different processes. This result shows that the interference effects cannot be rooted exclusively in the polarity or markedness of the categories. Finally, the architecture of the delta-rule connectionist model could explain for non-correlating interference effects, because the connections between the properties can be changed independently between different property pairs. However, at the moment it is not clear what environmental input would set the connections independently for all property pairs. Overall, in its current form none of the current models can readily explain the present findings. Additional convergent data and further modifications of the model seem to be necessary to ensure that the models are appropriate to describe the symbolic numerical effects.

More generally, it is possible that the lack of the correlations between the interference effects reflects that the interference effects do not root in the labeling of the properties (e.g., markedness or polarity of the number magnitude, parity, etc.) themselves, but in the relation or connection of those properties, as for example, in a delta-rule model. Alternatively, it is also possible that different interference effects may be supported by different number representations, and that is why the interference effects do not correlate.

A third, methodological result of the present work is the introduction of the method for revealing heterogeneous interference. The former, unified index was only appropriate to show homogeneous interference, and occasional methodological extensions were required to unveil heterogeneous interference (as in Dehaene et al., 1993). With the newly presented dual index heterogeneous interference can also be shown. We argue that the two methods mostly complement each other, and interference studies would benefit from using both indices to discover both homogeneous and heterogeneous interference effects.

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Electronic Supplementary Material

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ESM 1. Text and Figures (.pdf)

Interference between number magnitude and parity: Discrete representation in number processing.

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