

000 001 002 003 004 005 LONG-ATTENTION WEAVING: SYNTHESIZING LONG- 006 CONTEXT DATA WITH HIGH-QUALITY SHORT DATA 007 008 009

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ABSTRACT

012 The advent of long-context Large Language Models (LLMs) has been hindered
013 by a critical bottleneck: the scarcity of high-quality training data. Standard data
014 synthesis methods, which typically concatenate short documents, often fail to create
015 the challenging, long-range dependencies essential for robust learning. In this
016 work, we introduce **Long-Attention Weaving (LAW)**, a novel framework that
017 leverages a model’s own self-attention mechanism to synthesize a superior long-
018 context training curriculum. LAW operates in two stages: first, it employs a multi-
019 scale attention-based score to identify short documents that are inherently rich in
020 long-range dependencies. Second, it utilizes a novel *interleaving* strategy to weave
021 these selected documents into complex sequences, compelling the model to estab-
022 lish non-trivial, long-distance relationships. We demonstrate that continually pre-
023 training LLaMA-2 7B on data synthesized by LAW extends its effective context
024 window to 64k and significantly outperforms strong baselines on a suite of long-
025 context benchmarks, LongBench. Our findings highlight the efficacy of attention-
026 guided data engineering for unlocking the full potential of long-context LLMs.
027 All code and data are available at <https://anonymous.4open.science/r/LAW-B056>.

028 1 INTRODUCTION 029

030 The capacity to process extensive contexts is a critical frontier in the advancement of Large Lan-
031 guage Models (LLMs), underpinning their application in complex, real-world domains such as com-
032 prehending entire codebases, summarizing lengthy legal documents, or engaging in multi-turn dia-
033 logues over long histories (Vaswani et al., 2017). While proprietary models like GPT-4 and Claude
034 3 have demonstrated remarkable long-context capabilities (OpenAI, 2023; Anthropic, 2024), the
035 open-source community’s efforts to replicate this success are often hampered by a fundamental bot-
036 tleneck: the scarcity of high-quality, naturally occurring long-text data (Liu et al., 2023; Bai et al.,
037 2023).

038 To circumvent this data shortage, a common practice is to synthesize long training samples by con-
039 catenating shorter documents from large corpora (Raffel et al., 2020; Gao et al., 2020), a strategy
040 employed during the pre-training of many foundational models (Brown et al., 2020; Touvron et al.,
041 2023). However, this approach often creates an “illusion” of long context—sequences that are long
042 in token count but lack the deep, interwoven dependencies that characterize genuine long-form text
043 (Bai et al., 2023). Models trained on such data can often achieve low perplexity without developing
044 the sophisticated, long-range reasoning skills they are intended to acquire, as the necessary informa-
045 tion is frequently found within local, more easily accessible context windows (Liu et al., 2023; Xiao
046 et al., 2023).

047 This challenge has spurred two main lines of research, both of which have significant limitations.
048 The first, which we term **brute-force synthesis**, attempts to create more challenging data by inter-
049 leaving multiple short documents, as seen in methods like LongSkywork (Zhao et al., 2024). While
050 an improvement over simple concatenation, this strategy treats all source documents as equally val-
051 uable, inevitably diluting the training corpus with material that lacks strong internal dependencies and
052 thus offers a weak learning signal (Swayamdipta et al., 2020).

053 The second line of work focuses on **misaligned filtering**. These methods aim to select higher-quality
054 data but rely on flawed proxies. Linguistic-based approaches like ProLong (Cheng et al., 2024) use
055

054 metrics such as perplexity to filter data, a technique which is widely used in web-scale data cleaning
 055 (Wenzek et al., 2019; Raffel et al., 2020), yet one which is computationally intensive and poorly
 056 aligned with the token-level attention mechanisms that govern LLM processing. More model-aware
 057 methods, such as LongAttn (Wu et al., 2025), cleverly use self-attention scores to find dependency-
 058 rich texts, drawing on the idea that models can report on their own internal states (Kadavath et al.,
 059 2022). However, their framework is designed to filter a corpus of *already-long* documents, not to
 060 guide the synthesis of new long documents from a pool of short ones, making it ill-suited for the
 061 data scarcity problem.

062 In this paper, we introduce **Long-Attention Weaving (LAW)**, a new framework that moves beyond
 063 these limitations by directly using an LLM’s internal self-attention scores to both select high-quality
 064 source material and synthesize a challenging long-context curriculum. Our core insight is that the
 065 most effective data forces the model to learn relationships that it currently struggles with, a signal
 066 best captured by its own attention mechanism. LAW implements this via a two-stage process:

- 068 • We first design a multi-scale, attention-based scoring metric to efficiently identify short
 069 documents that are rich in internal, long-range dependencies, ensuring that every compo-
 070 nent of our synthetic data is of high quality.
- 071 • We then employ a novel *interleaving* synthesis strategy that weaves these selected docu-
 072 ments into complex sequences, constructing complex sequences that explicitly require the
 073 model to connect semantically related but distant segments of text.

074 We demonstrate through extensive experiments that continually pre-training LLaMA-2 7B on data
 075 synthesized by LAW significantly enhances its long-context capabilities on a diverse array of down-
 076 stream benchmarks. Our contributions are not just a new method, but a new perspective: treating
 077 long-context data synthesis as a data engineering problem, guided by the model’s own perception of
 078 contextual dependency.

079 2 RELATED WORK

080 The pursuit of longer context windows in LLMs has advanced along two primary axes: architectural
 081 innovations and data-centric strategies. Our work falls into the latter, focusing on the generation of
 082 high-quality training data, a component we argue is critical for any architecture to realize its full
 083 potential.

084 **Architectural Innovations** A significant body of research has focused on modifying model ar-
 085 chitectures to handle longer sequences. Key efforts include developing more efficient attention
 086 mechanisms to mitigate the quadratic complexity of standard self-attention (Zhuang et al., 2023).
 087 These “X-formers” encompass a wide range of techniques, such as sparse attention patterns (Belt-
 088 agy et al., 2020; Zaheer et al., 2020), hardware-aware optimizations like FlashAttention (Dao et al.,
 089 2022), and alternative architectures like state-space models that offer linear-time complexity (Gu &
 090 Dao, 2023). Another critical area is the adaptation of positional encodings to extrapolate beyond
 091 their original training length. Techniques such as Positional Interpolation (PI) (Chen et al., 2023)
 092 and YaRN (Peng et al., 2023) have become standard practices for extending the context window
 093 of existing models like LLaMA. However, these architectural modifications only provide the *ca-
 094 pacity* for longer contexts; the model must still learn to *utilize* this capacity through exposure to
 095 appropriate data, a challenge that remains significant. Our work is orthogonal and complementary to
 096 these efforts, providing the high-quality data needed to make such architectural extensions effective.

097 **Data-Centric Strategies** The performance of long-context models is intrinsically linked to the
 098 data they are trained on (Touvron et al., 2023). Given the scarcity of naturally long, high-quality
 099 documents, research in this area has broadly bifurcated into data synthesis and data selection.

100 **Data Synthesis via Document Combination.** A common and scalable strategy for creating long-
 101 text data is to combine shorter documents from large-scale corpora like The Pile (Gao et al., 2020),
 102 C4 (Raffel et al., 2020), and BookCorpus (Zhu et al., 2015). The simplest form is sequential concate-
 103 nation, a method used in the pre-training of many foundational models (Brown et al., 2020; Touvron
 104 et al., 2023). A more advanced technique, employed by LongSkywork (Zhao et al., 2024), uses a
 105 chunk-interleaving strategy that weaves segments from multiple documents together. This forces

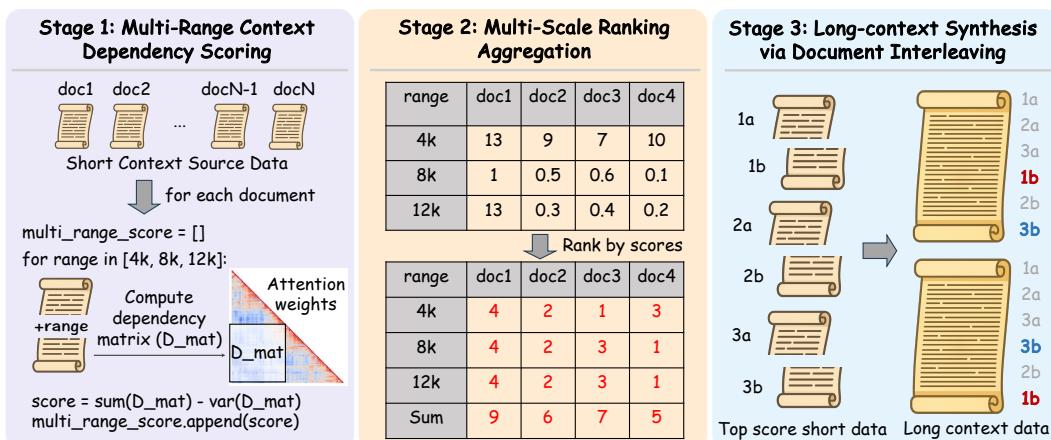
108 the model to track information from different sources simultaneously, creating a more challenging
 109 training task than simple concatenation. However, by treating all source documents as equally valuable,
 110 these synthesis-only approaches risk diluting the training data with samples that lack strong
 111 internal dependencies, potentially leading to inefficient or ineffective learning.

112 **Data Selection via Quality Scoring.** To address the issue of data quality, other methods focus
 113 on selecting the most valuable data from a corpus, a practice philosophically rooted in ideas like
 114 dataset cartography (Swayamdipta et al., 2020) and influence functions (Koh & Liang, 2017). Early
 115 approaches like ProLong (Cheng et al., 2024) used linguistic metrics like perplexity to score and
 116 select coherent document sequences, a technique also common in large-scale data cleaning pipelines
 117 (Wenzek et al., 2019). While intuitive, such metrics are often computationally expensive and are not
 118 directly aligned with the internal mechanisms of the transformer architecture. A more direct and
 119 aligned approach was pioneered by LongAttn (Wu et al., 2025), which leverages the model’s own
 120 self-attention scores to identify existing long documents rich in long-range dependencies, building
 121 on the insight that models can possess self-knowledge about their own internal states (Kadavath
 122 et al., 2022). This was a key insight, but its methodology is designed as a *filter* for an existing
 123 corpus of long texts. It does not address the core problem of how to *synthesize* new long documents
 124 when a large, high-quality long-text corpus is not readily available.

125 **Positioning of Long-Attention Weaving** Our work, Long-Attention Weaving (LAW), synergizes
 126 and advances these two paradigms. We adopt the synthesis-centric approach of methods like
 127 LongSkywork, but address its core limitation by introducing a critical data selection stage. Our
 128 selection mechanism is inspired by the model-centric philosophy of LongAttn, but we make two
 129 crucial adaptations: (1) we apply it to score *short documents* to assess their suitability as building
 130 blocks for synthesis, and (2) we enhance the scoring with a multi-scale analysis and a robust rank-
 131 aggregation scheme. By first selecting for dependency-rich short documents and then weaving them
 132 together using a challenging interleaving strategy, LAW creates a more potent and efficient training
 133 curriculum specifically designed to foster long-range reasoning.

135 3 METHODOLOGY

137 The core of our framework is a three-stage pipeline designed to generate a high-quality, long-context
 138 training curriculum from a large corpus of short documents. The stages are: (1) multi-range context
 139 dependency scoring of short documents, (2) multi-scale ranking aggregation for robust selection,
 140 and (3) long-context synthesis via document interleaving.



156 Figure 1: The overall framework of Long-Attention Weaving (LAW) consists of three stages. Stage
 157 1: Short-context documents are scored for long-range dependency richness at multiple ranges (e.g.,
 158 4k, 8k, 12k) using the first-layer self-attention matrix to compute multi-range scores. Stage 2:
 159 Scores from each range are converted to ranks and aggregated via summation to yield a final score
 160 for robust document selection. Stage 3: Top-scoring short documents are bisected and interleaved
 161 using ordered and reverse-ordered strategies to synthesize challenging long-context training data.

162 3.1 STAGE 1: MULTI-RANGE CONTEXT DEPENDENCY SCORING
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164 Our process begins with a large-scale corpus of short documents, primarily sourced from open-
165 source code repositories and ArXiv papers due to their rich semantic content. Raw documents are
166 segmented into fixed-length chunks (e.g., 4k tokens) using a sliding-window strategy designed to
167 maximize data utilization while preserving informational integrity. For a document of n tokens and
168 a target length L , we extract chunks from the beginning, end, and middle, ensuring that no content
169 is wasted and that each chunk represents a contiguous block of text. This forms the initial pool of
170 short-context documents, \mathcal{D} .

171 To identify the most valuable documents for synthesis, we introduce a scoring mechanism that quan-
172 tifies long-range dependencies from the model’s own perspective. Our approach extends the core
173 idea of LongAttn (Wu et al., 2025) by adapting it to score short documents and incorporating a
174 multi-range analysis for robustness.

175 The process begins by computing a Long-Range Dependency Score (LDS) for each document $s \in \mathcal{D}$
176 at multiple distance thresholds $k \in K$. The score is derived from the self-attention matrix \mathbf{M} of a
177 pre-trained LLM, extracted specifically from its first layer. This choice is deliberate: the first layer
178 is computationally efficient and its attention patterns are less influenced by task-specific heads or
179 the “attention sink” phenomenon, providing a purer signal of fundamental token-level relationships
180 (Xiao et al., 2023). The score is defined as:

$$181 \text{LDS}(s, k) = \text{Mean}(\mathbf{M}_{i,j|j-i>k}) - \alpha \cdot \text{Var}(\mathbf{M}_{i,j|j-i>k}) \quad (1)$$

183 where the first term is the average attention strength beyond distance k , and the second term penal-
184 izes non-uniform dependency distributions with a hyperparameter α .

186 3.2 STAGE 2: MULTI-SCALE RANKING AGGREGATION
187

188 A critical challenge in utilizing our multi-range dependency scores is that the raw LDS values are
189 not directly comparable across different scales k . The distribution and magnitude of attention scores
190 can vary significantly between short-range and long-range dependencies; a naive summation would
191 be susceptible to biases, allowing a single scale to dominate the final score. To mitigate this issue of
192 incommensurability, we adopt a robust, non-parametric approach by converting the raw scores into
193 ranks. For each distance scale $k \in K$, we rank all documents $s \in \mathcal{D}$ according to their $\text{LDS}(s, k)$
194 values, yielding a set of rank lists, $R(s, k)$.

195 These individual rank lists are then aggregated into a single, unified score using a method inspired
196 by the Borda Count (Emerson, 2013) consensus system. In this formulation, each scale k acts as a
197 “voter,” and the final score for a document reflects its overall standing across all voters. This is
198 achieved by summing the ranks a document received across all scales:

$$199 \text{Score}_{\text{final}}(s) = \sum_{k \in K} R(s, k) \quad (2)$$

201 This rank-aggregation strategy is inherently robust, as it prioritizes documents that achieve a strong
202 consensus of high performance across the entire spectrum of dependency ranges. It rewards docu-
203 ments that are consistently ranked favorably, rather than those that may have an exceptionally high
204 score at one scale but perform poorly at others.

206 Finally, we perform a principled filtering step by selecting the top p -th percentile (e.g., top 50%) of
207 documents based on their $\text{Score}_{\text{final}}(s)$. This process yields the high-quality subset \mathcal{D}^* , which serves
208 as the source material for our subsequent synthesis stage.

210 3.3 STAGE 3: LONG-CONTEXT SYNTHESIS VIA DOCUMENT INTERLEAVING
211

212 Using the high-quality document set \mathcal{D}^* , we synthesize long-context samples designed to be chal-
213 lenging for the model. The process begins by sampling a batch of N documents (e.g., $N = 8$). Each
214 document D_i is bisected into two halves, $P_{i,1}$ and $P_{i,2}$.

215 We employ two interleaving strategies. The first, **Ordered-Interleaving**, concatenates all first-half
parts followed by all second-half parts in their original order, creating a structure that requires the

216 model to connect related but now distant halves of the same documents:

$$D_{\text{synth,ordered}} = [P_{1,1} \circ \dots \circ P_{N,1} \circ P_{1,2} \circ \dots \circ P_{N,2}] \quad (3)$$

219 Our primary strategy, **Reverse-Ordered Interleaving**, introduces a greater challenge by reversing
220 the order of the second-half parts. This breaks simple positional heuristics the model might learn and
221 forces a more robust, content-based understanding of semantic links to solve the complex “semantic
222 binding” task of reconnecting the document halves:

$$D_{\text{synth,reversed}} = [P_{1,1} \circ \dots \circ P_{N,1} \circ P_{N,2} \circ \dots \circ P_{1,2}] \quad (4)$$

224 where \circ denotes concatenation. The final training set is composed of synthetic documents generated
225 using a mix of these strategies, creating a diverse and challenging curriculum for long-context
226 learning.

227 The final stage of our framework utilizes the synthesized long-context corpus for the continual pre-
228 training of a base LLM. The primary training objective is to extend its effective context window.
229 The central logic of our data generation process is formalized in Algorithm 1.

231 4 EXPERIMENTS

233 To validate the efficacy of our proposed framework, Long-Attention Weaving (LAW), we conduct
234 a comprehensive suite of experiments. We first detail the experimental setup, then present the main
235 results comparing LAW against strong baselines, and conclude with in-depth ablation studies and
236 qualitative analyses to deconstruct the sources of its performance gains.

238 4.1 EXPERIMENTAL SETUP

240 **Training Details** All models were initialized from the LLaMA-2 7B checkpoint and continually
241 pre-trained on a corpus of 1 billion (1B) tokens. This corpus was synthesized by sampling from a
242 curated dataset of ArXiv papers and source code at a 1:1 ratio. We extended the model’s context
243 window from its native 4k to 64k tokens. Following established best practices for long-context
244 adaptation (Lu et al., 2024), we employed a learning rate of 2×10^{-5} with a linear warmup schedule
245 and no weight decay. All experiments were conducted on a cluster of 8 NVIDIA H800 GPUs.

246 **Baselines** We benchmark LAW against three strong baselines designed to isolate the contributions
247 of our framework’s core components:

- 249 • **Base Model:** The original LLaMA-2 7B checkpoint, without any long-context continual
250 pre-training, serving as a reference for pre-adaptation performance.
- 251 • **Random Concatenation:** A naive baseline that bypasses our attention-based filtering
252 stage. Short documents are randomly sampled and concatenated to form 64k-token se-
253 quences, representing a common but simplistic approach to long-context data construction.
- 254 • **Ordered-Interleaving w/o Filter:** This baseline ablates our filtering mechanism by apply-
255 ing the ordered-interleaving strategy to randomly selected documents. It serves to isolate
256 the performance gains attributable specifically to our attention-guided document selection.

257 **Evaluation Tasks** We employ a diverse set of intrinsic and extrinsic evaluation tasks to provide a
258 holistic assessment of model capabilities.

- 260 • **Intrinsic Evaluation:** We measure perplexity (PPL) on the PG19 and Proof-pile datasets
261 to assess fundamental language modeling quality across various context lengths.
- 262 • **Extrinsic Evaluation:** We report performance on LongBench (Bai et al., 2023), a standard
263 multi-task benchmark for long-context understanding. Additionally, we assess in-context
264 learning ability on the Trec News dataset using a “ManyShots” learning paradigm.

266 4.2 MAIN RESULTS

268 **Language Modeling Perplexity** Table 1 presents the perplexity scores on the PG19 and Proof-
269 pile datasets. At the target context length of 64k, LAW achieves superior language modeling per-
270 formance, confirming its effectiveness in modeling long-range dependencies.

270 A noteworthy observation is LAW’s slightly elevated perplexity at shorter context lengths (e.g., 2k-
 271 8k) compared to the baselines. We posit that this is not a deficiency but rather an expected artifact
 272 and a positive indicator of our method’s success. The data synthesized by LAW is intentionally
 273 structured to be complex and non-locally redundant, compelling the model to resolve dependencies
 274 that span beyond short evaluation windows. Consequently, when evaluated on these shorter win-
 275 dows, the model’s attempts to predict tokens based on unavailable long-range context may result
 276 in a marginal PPL increase. In contrast, baseline models trained on locally coherent but globally
 277 simplistic concatenated data excel at local prediction but fail to develop the mechanisms for genuine
 278 long-range reasoning, as evidenced by their degraded performance at the full 64k context length.

279
 280 Table 1: Perplexity (PPL) on PG19 and Proof-pile datasets at various context lengths. All models are
 281 trained with a 64k context window. Lower PPL indicates better performance. LAW (Ours) achieves
 282 the best performance at the longest context lengths, validating its long-range modeling capabilities.

Dataset	Model	2k	4k	8k	16k	32k	64k
PG19	Random Concatenation	7.11	6.73	6.50	6.36	6.25	6.20
	Ordered-Interleaving w/o Filter	7.25	6.84	6.57	6.40	6.27	6.19
	LAW (Ours)	7.41	7.29	6.62	6.45	6.31	6.18
Proof-pile	Random Concatenation	3.28	3.00	2.82	2.68	2.59	2.53
	Ordered-Interleaving w/o Filter	3.29	3.01	2.83	2.67	2.57	2.50
	LAW (Ours)	4.63	4.67	2.83	2.67	2.57	2.49

293 **Downstream Task Performance** As shown in Table 2, LAW consistently and significantly outper-
 294 forms all baselines on the LongBench benchmark average score. This result underscores the tangible
 295 benefits of our attention-guided data synthesis strategy for downstream applications. The substantial
 296 gains over both Random Concatenation and Ordered-Interleaving without Filter highlight that both
 297 components of our framework—the principled selection of documents with high dependency po-
 298 tential and the challenging interleaving synthesis strategy—are critical for achieving state-of-the-art
 299 performance.

300 Crucially, the superior performance is not merely in information retrieval but extends to tasks re-
 301 quiring deeper reasoning, such as Question Answering and Summarization. We attribute this to the
 302 nature of the training signal provided by LAW. The interleaving process forces the model to disentan-
 303 gle, track, and integrate information from multiple, non-contiguous sources within a single context,
 304 thereby directly training the cognitive primitives necessary for complex, multi-hop reasoning.

306 4.3 ABLATION STUDIES AND ANALYSIS

308 **Impact of Framework Components** To deconstruct the contributions of LAW’s key components,
 309 we conducted targeted ablation studies, with results presented in Table 3.

- 311 • **Ablation on Document Filtering:** Isolating the effect of our attention-based filtering by
 312 comparing the full LAW model to the “Ordered-Interleaving w/o Filter” baseline reveals
 313 its criticality. The performance degradation (from 31.26 to 30.60 on LongBench) confirms
 314 that our scoring mechanism is highly effective at identifying documents that provide a rich
 315 training signal for learning long-range dependencies.
- 316 • **Ablation on Synthesis Strategy:** The performance gap between LAW and Random Con-
 317 catenation (31.26 vs. 31.05) underscores the efficacy of the interleaving synthesis method.
 318 To further probe this, we trained a variant, ‘LAW w/o Reverse-Order’, which omits the
 319 reverse-ordered interleaving component. As shown in Table 3, this variant underperforms
 320 the full method, validating that the increased complexity introduced by the bidirectional
 321 interleaving strategy is a key contributor to the model’s final performance.

322 **Sensitivity and Robustness Analysis** We analyze the robustness of our method with respect to
 323 key design choices.

324
 325 Table 2: Main results on the LongBench benchmark, averaged across task categories. LAW (Ours)
 326 demonstrates substantial improvements over baselines, highlighting the effectiveness of our data
 327 synthesis framework in fostering advanced reasoning capabilities.

328 Category	329 Task	330 Base Model	331 Rand. Concat.	332 Ord. Interleaving w/o Filter	333 LAW (Ours)
334 Question Answering	NQA	20.46	22.73	22.31	22.43
	QAP	28.08	28.56	29.22	28.53
	MQA	37.72	36.44	39.27	40.94
	HQA	42.03	41.61	39.56	40.58
	WQA	30.11	34.25	31.60	31.92
	TQA	85.31	87.27	86.92	86.76
335 Summarization	MSQ	14.40	14.75	18.27	17.74
	QSM	20.93	20.02	20.37	20.47
	MWS	14.47	15.62	13.12	17.13
	SMS	41.20	40.74	41.90	41.75
336 Code	PSC	2.68	1.18	1.45	2.55
	PSR	8.75	6.05	5.91	5.55
	LCC	23.10	32.98	16.67	18.07
	REP	26.54	29.54	28.31	29.79
337 Other	GR	21.66	15.61	24.73	25.96
	TRE	70.50	69.50	70.00	70.00
Average		30.50	31.05	30.60	31.26

344
 345 Table 3: Ablation and data scaling results on LongBench (average score). These results demonstrate
 346 the positive impact of scaling training data and confirm the benefit of our full reverse-ordered inter-
 347 leaving strategy.

348 Model	349 LAW (Ours, 1B)	350 LAW (2B Tokens)	351 LAW (w/o Reverse-Order)
Avg. Score	31.26	32.14	31.06

352
 353 • **Impact of Training Data Scale:** By training a model on a 2B-token corpus synthesized by
 354 LAW, we observe a consistent performance improvement (from 31.26 to 32.14 on Long-
 355 Bench), demonstrating the scalability and positive data-scaling properties of our frame-
 356 work.

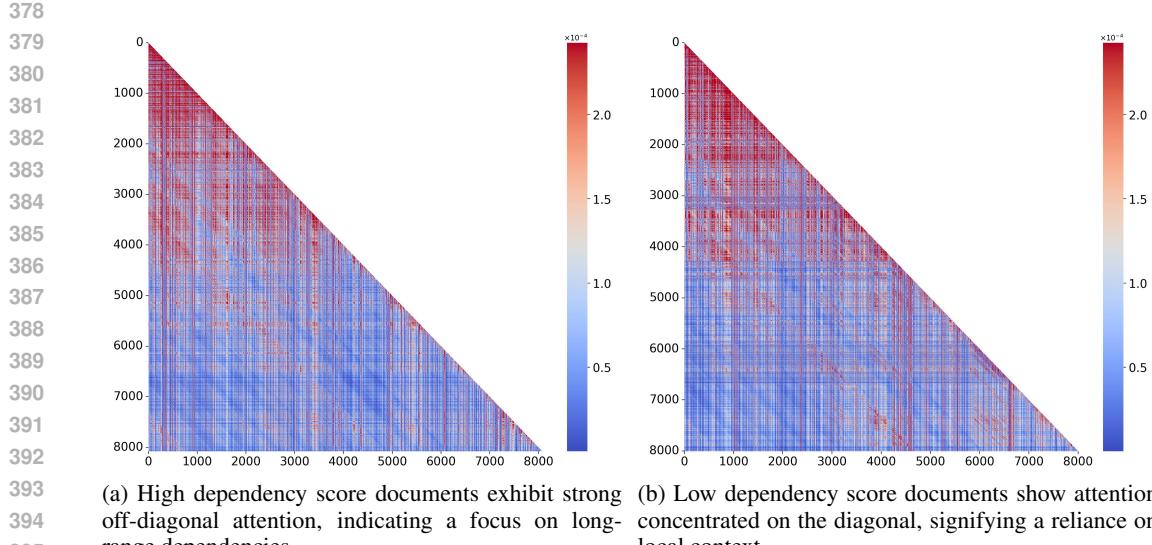
357 • **Generalization Across Context Lengths:** The perplexity results in Table 1 show a grace-
 358 ful degradation as the evaluation context shortens (e.g., 6.18 at 64k vs. 6.31 at 32k on
 359 PG19). This suggests the learned long-context capabilities are robust and not pathologi-
 360 cally tied to the maximum training length.

361 • **Orthogonality to Position Extrapolation Method:** To ensure our data-centric im-
 362 provements are not conflated with architectural choices, we trained models using LAW-
 363 synthesized data with two alternative RoPE extrapolation techniques: Positional Interpo-
 364 lation (PI) and YaRN. In all configurations, training on LAW data yielded significant im-
 365 provements over baselines. This confirms that our synthesis method provides benefits that
 366 are largely orthogonal to and complementary with advances in positional encoding strate-
 367 gies.

368 4.4 QUALITATIVE ANALYSIS OF ATTENTION MECHANISMS

370 To provide qualitative evidence for how LAW shapes the model’s reasoning, we visualize its atten-
 371 tion patterns. Figure 2 compares heatmaps from documents with high and low dependency scores,
 372 as determined by our filtering mechanism.

373 As illustrated in Figure 2a, documents with high dependency scores elicit dense, non-local attention
 374 patterns. The strong off-diagonal signals, particularly in the lower-left quadrant, indicate that tokens
 375 late in the sequence are actively attending to foundational concepts introduced much earlier. This is
 376 a hallmark of sophisticated, long-range reasoning and validates that our scoring metric successfully
 377 identifies texts that demand such behavior.



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(a) High dependency score documents exhibit strong off-diagonal attention, indicating a focus on long-range dependencies.
 (b) Low dependency score documents show attention concentrated on the diagonal, signifying a reliance on local context.

Figure 2: A comparison of attention patterns on the LAW dataset. The model dynamically adjusts its focus from long-range (a) to short-range (b) dependencies based on the document’s structural complexity.

Conversely, for documents with low dependency scores (Figure 2b), the attention pattern converges to a band-diagonal structure. This signifies a reliance on local context, where tokens primarily attend to their immediate neighbors, reflecting a more sequential and less integrative processing mode.

These visualizations provide compelling evidence that our training framework does not merely extend the context window but instills a **dynamic, input-dependent attentional strategy**. The model learns to allocate its cognitive resources efficiently, shifting from a global, integrative focus for complex documents to a local, sequential focus for simpler ones. This adaptive capability is a direct result of being trained on a curriculum curated and structured by LAW.

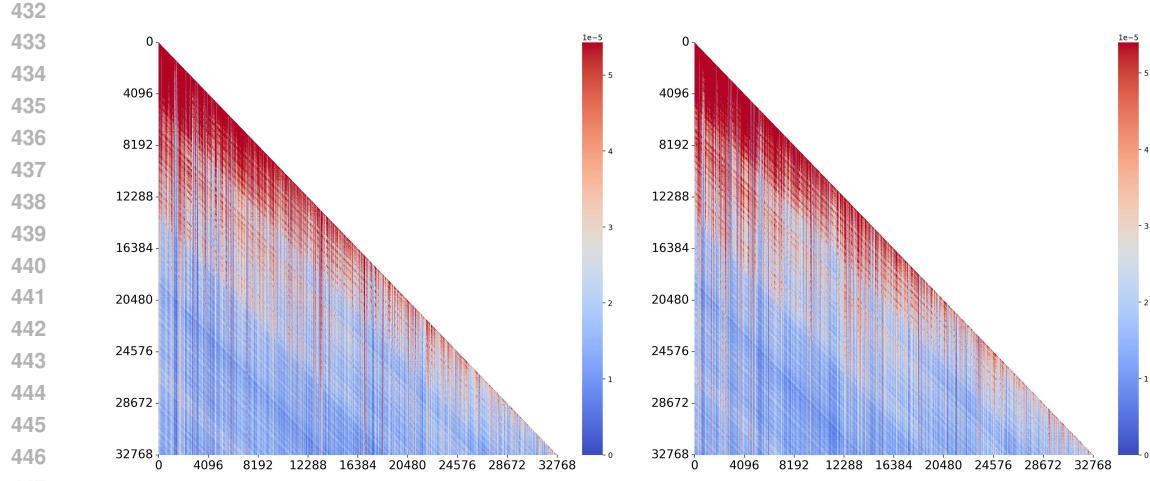
4.5 ABLATION STUDY ON DATA SYNTHESIS STRATEGY

To further dissect the contribution of our proposed data synthesis method, we conduct an ablation study focusing on the ‘reverse’ operation. We visualize the attention patterns of models trained on data synthesized with our full methodology versus a variant where the ‘reverse’ step is omitted. This comparison, presented in Figure 3, serves to highlight the impact of this specific component on the model’s ability to capture long-range dependencies.

Figure 3a displays the attention heatmap from a model trained on data synthesized using our complete method. The heatmap is characterized by a pronounced and vibrant signal in the lower-left quadrant. This indicates that tokens appearing later in the sequence (represented by the y-axis) are assigning high attention weights to tokens from the very beginning of the sequence (x-axis). Such a pattern is a definitive indicator of robust long-range dependency modeling, as it shows the model’s capacity to maintain and access context over extended distances.

In contrast, Figure 3b illustrates the attention pattern from a model trained on data synthesized without the ‘reverse’ operation. While off-diagonal attention is still present, the intensity of the signal in the lower-left quadrant is visibly diminished compared to Figure 3a. The reduced attention scores in this critical region suggest a comparative weakening in the model’s ability to form connections between distant tokens. The ‘reverse’ operation, by forcing the model to predict the beginning of a sequence from its end, explicitly trains the model to integrate information across the entire context length, thereby strengthening these long-range dependencies.

This qualitative comparison underscores the efficacy of our full data synthesis approach. The heightened attention scores in the lower-left quadrant of Figure 3a are not merely an artifact but a direct visualization of the model’s enhanced capacity for long-range reasoning—a capacity specifically



(a) Full synthesis method. The pronounced attention in the lower-left quadrant demonstrates strong long-range dependencies.

(b) Synthesis method without ‘reverse’ operation. The attenuated signal in the lower-left indicates a reduced focus on long-range dependencies.

Figure 3: Ablation study of attention patterns. A comparison of our full data synthesis method (a) with a variant lacking the ‘reverse’ operation (b). The full method results in markedly stronger long-range attention.

cultivated by the inclusion of the ‘reverse’ operation in our data synthesis pipeline. This provides strong evidence that our complete method is superior for instilling the desired long-range dependency capabilities in the model.

5 CONCLUSION

In this work, we introduced Long-Attention Weaving (LAW), a new framework for synthesizing a high-quality, long-context training curriculum. Our core contribution is a model-centric approach to data engineering: we leverage an LLM’s own self-attention mechanism to both identify short documents rich in long-range dependencies and weave them into complex synthetic sequences that promote robust learning. Through extensive experiments, we demonstrated that training a LLaMA-2 7B model on data generated by LAW leads to significant improvements in perplexity and downstream performance on a wide range of long-context benchmarks. The success of LAW underscores a critical principle: for long-context learning, the structure and quality of data are as important as architectural innovations. Our findings suggest that future progress in this domain will increasingly rely on sophisticated, model-aware data synthesis strategies. This work represents a step in that direction, opening up promising avenues for research into attention-guided curriculum learning, the interplay between synthetic and natural data distributions, and the automated creation of challenging training regimes that push the boundaries of what LLMs can achieve.

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A APPENDIX

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A.1 LONG-ATTENTION WEAVING (LAW) DATA SYNTHESIS PROCESS

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599 **Algorithm 1** Long-Attention Weaving (LAW) Data Synthesis Process

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```

1: Input: Short document corpus  $\mathcal{D}$ , scoring model  $\mathcal{M}$ , distance scales  $K = \{k_1, \dots, k_m\}$ , selec-
 601   tion percentile  $p$ , synthesis batch size  $N$ .
2: Output: A batch of synthetic long documents  $\mathcal{B}_{\text{synth}}$ .
3:
4: ▷ Stage 1: Multi-Range Context Dependency Scoring
5: Initialize score list  $\mathcal{S}_k$  for each  $k \in K$ .
6: for each document  $s \in \mathcal{D}$  do
7:   Extract first-layer attention matrix  $\mathbf{M}$  from  $\mathcal{M}(s)$ .
8:   for each scale  $k \in K$  do
9:     Compute  $\text{LDS}(s, k)$  from  $\mathbf{M}$  using Equation 1.
10:    Append score to  $\mathcal{S}_k$ .
11:   end for
12: end for
13:
14: ▷ Stage 2: Multi-Scale Ranking Aggregation
15: for each scale  $k \in K$  do
16:   Convert scores  $\mathcal{S}_k$  to ranks  $R(s, k)$  for all  $s \in \mathcal{D}$ .
17: end for
18: for each document  $s \in \mathcal{D}$  do
19:   Calculate  $\text{Score}_{\text{final}}(s)$  using Equation 2.
20: end for
21: Create high-quality set  $\mathcal{D}^*$  by selecting top  $p\%$  of documents from  $\mathcal{D}$  based on  $\text{Score}_{\text{final}}$ .
22:
23: ▷ Stage 3: Long-Context Synthesis via Document Interleaving
24: Initialize batch  $\mathcal{B}_{\text{synth}} = []$ .
25: Sample  $N$  documents  $\{D_1, \dots, D_N\}$  from  $\mathcal{D}^*$ .
26: Bisect each document  $D_i$  into parts  $P_{i,1}$  and  $P_{i,2}$ .
27: Construct  $D_{\text{synth,ordered}}$  and  $D_{\text{synth,reversed}}$  (Eq. 3 and 4).
28: Add synthesized documents to  $\mathcal{B}_{\text{synth}}$ .
29: return  $\mathcal{B}_{\text{synth}}$ 

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630 A.2 LIMITATIONS

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632 While Long-Attention Weaving demonstrates significant efficacy, we acknowledge several limita-
 633 tions that offer avenues for future research. First, our framework relies on a pre-trained model’s
 634 self-attention scores as a proxy for semantic dependency. While we argue this is more aligned
 635 than external linguistic metrics, these attention scores can be noisy and may not perfectly capture
 636 all forms of long-range relationships, particularly abstract or inferential ones. The quality of the
 637 synthesized data is therefore inherently tied to the capabilities of the initial scoring model.

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639 Second, the data synthesis process, particularly the interleaving strategy, creates a distribution of
 640 text that is structurally different from naturally occurring long documents. While we have shown
 641 this to be a powerful training signal, it may introduce a subtle domain mismatch, potentially leading
 642 the model to develop biases or heuristics optimized for this synthetic structure. An important future
 643 direction is to investigate methods for gradually annealing the training curriculum from synthetic,
 644 interleaved data towards more natural long-form text.

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648 Finally, our multi-scale scoring approach, while more robust than single-scale methods, introduces
 649 hyperparameters related to the choice of distance thresholds (K) and the selection percentile (p). Al-
 650 though our experiments show strong performance with a standard configuration, a more systematic
 651 exploration of these hyperparameters could yield further improvements and a better understanding
 652 of their impact on different data modalities.

648 A.3 USAGE OF LLMs FOR WRITING
649650 We use LLMs to aid or polish writing the whole paper.
651652 A.4 RESPONSE EXAMPLES OF THE TRAINED MODEL ON LONGBENCH
653654 In this subsection, we present illustrative response examples from our trained model on the LAW
655 dataset. These cases demonstrate the model’s proficiency in managing long-range dependencies,
656 synthesizing coherent narratives from interleaved document segments, and adapting to complex
657 semantic structures. Each example highlights different aspects of the model’s long-context capabili-
658 ties, such as multi-hop reasoning, information retrieval across distant contexts, and robust handling
659 of reversed interleaving patterns.
660661 Example 1
662664 **NarrativeQA:** You are given a story, which can be either a novel or a
665 movie script, and a question. Answer the question as concisely as you
666 can, using a single phrase if possible. Do not provide any explanation.
667668 Story: {Here is the story.}
669670 Now, answer the question based on the story as concisely as you can,
671 using a single phrase if possible. Do not provide any explanation.
672673 Question: {input}
674675 **Ours:** A security guard.
676677 **Ground truth:** A security guard.
678679 **Prompt length:** 29630
680681
682683
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685 Figure 4: Model response in the first case from the LongBench dataset, showcasing effective reso-
686 lution of long-range semantic dependencies in an interleaved context.
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717 Example 2

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720 **NarrativeQA:** You are given a story, which can be either a novel or a
721 movie script, and a question. Answer the question as concisely as you
722 can, using a single phrase if possible. Do not provide any explanation.
723

724 Story: {context}
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727 Now, answer the question based on the story as concisely as you can,
728 using a single phrase if possible. Do not provide any explanation.
729

730 Question: {input}
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733 **Ours:** Blue Lou Boyle.
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735 **Ground truth:** A mobster named "Blue Lou Boyle"
736

737 **Prompt length:** 29633
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741 Figure 5: Model response in the second case from the LongBench dataset, illustrating multi-hop
742 reasoning across non-contiguous document segments.
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772 Example 3

773 **RepoBench-P:** Please complete the code given below.

774 {Context: here are some examples for the code completion task.}

775 {Input: the user provides the following code to be completed}

776 Next line of code:

777 **Ours:** public static Frameset frameset(Attribute... attributes)

778 {

779 return new Frameset(attributes);

780 }

781 public static Head head(Attribute... attributes) {

782 return new Head(attributes);

783 }

784 **Ground truth:** public static Frameset frameset(Attribute...

785 attributes) {

786 **Prompt length:** 8158

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791 Figure 6: Model response in the third case from the LongBench dataset, demonstrating adaptation
792 to reverse-ordered interleaving and maintenance of narrative coherence.

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826 Example 4

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828 **SAMSum:** Summarize the dialogue into a few short sentences. The
829 following are some examples.
830

831 {Context: examples of dialogue summarization are provided here.}
832

Dialogue: Ana: You sleeping?

Catherine: Not yet.

Ana: Wanna go visit grandma tomorrow? I miss her.

Catherine: Yeah that would be nice :) I'll call you when I wake up

Ana: Oki :) sleep well, good night.

Catherine: Good night, u too.

Ours: Ana wants to visit her grandma tomorrow. Catherine will call her when she wakes up.

Ground truth: Ana wants to visit grandma tomorrow. Catherine will go with her. She will call Anna when she wakes up.

Prompt length: 9352

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849 Figure 7: Model response in the fourth case from the LongBench dataset, highlighting robust infor-
850 mation synthesis in challenging long-context scenarios.

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