REJUVENATING CROSS-ENTROPY LOSS IN KNOWLEDGE DISTILLATION FOR RECOMMENDER SYSTEMS

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ABSTRACT

This paper analyzes Cross-Entropy (CE) loss in knowledge distillation (KD) for recommender systems. KD for recommender systems targets at distilling rankings, especially among items most likely to be preferred, and can only be computed on a small subset of items. Considering these features, we reveal the connection between CE loss and NDCG in the field of KD. We prove that when performing KD on an item subset, minimizing CE loss maximizes the lower bound of NDCG, only if an assumption of closure is satisfied. It requires that the item subset consists of the student's top items. However, this contradicts our goal of distilling rankings of the teacher's top items. We empirically demonstrate the vast gap between these two kinds of top items. To bridge the gap between our goal and theoretical support, we propose **R**ejuvenated **C**ross-**E**ntropy for **K**nowledge **D**istillation (RCE-KD). It splits the top items given by the teacher into two subsets based on whether they are highly ranked by the student. For the subset that defies the condition, a sampling strategy is devised to use teacher-student collaboration to approximate our assumption of closure. We also combine the losses on the two subsets adaptively. Extensive experiments demonstrate the effectiveness of our method. Our code is available at `https://anonymous.4open.science/r/RCE-KD`.

1 INTRODUCTION

Recently, with the scaling law in recommender systems Zhai et al. (2024) being gradually discovered, many researchers have proposed extremely large models Ohsaka & Togashi (2023); Zhai et al. (2024) to pursue better recommendation accuracy. However, the increase in model size inevitably incurs high storage costs and inference latency, causing higher maintenance costs and lower user satisfaction.

To improve the inference efficiency and decrease the storage cost of recommendation models without sacrificing their recommendation accuracy, knowledge distillation (KD) for recommender systems Kang et al. (2020); Sun et al. (2024) has attracted attention. KD Hinton et al. (2015) is an approach for model compression. It aims to transfer knowledge from a pre-trained large teacher to a small student. Once training is complete, only the small student is used for inference. Among existing works on KD, response-based KD Hinton et al. (2015) encourages students to mimic the teacher's predictions and has gained extreme attention due to its excellent performance. As a popular loss for response-based KD methods, Cross-Entropy (CE) loss is very important. Most response-based KD methods Huang et al. (2022); Cui et al. (2023) in Computer Vision (CV) and Natural Language Processing (NLP) are based on CE loss. However, little work has been done to use or analyze CE loss in KD for recommender systems. Note that KD for recommender systems has two unique features: 1) It focuses more on rankings than specific scores, especially among the teacher's top items Kang et al. (2020). 2) KD can only be conducted on a small subset of items since the quantity of all the items is very large. These features make the compatibility of CE loss in KD for recommender systems questionable. To obtain an initial insight into the performance of CE loss, we present the results of vanilla CE loss and several response-based KD methods in Figure 1. To cover as many types of loss functions as possible, we consider the point-wise loss (i.e., CD Lee et al. (2019)), pair-wise loss (i.e., UnKD Chen et al. (2023)), and RRD-based losses Kang et al. (2020) (a list-wise loss, i.e., RRD Kang et al. (2020) and HetComp Kang et al. (2023)). In vanilla CE loss, we compute CE loss using the teacher's top items. We find that vanilla CE loss is often inferior to all baselines. This result contrasts with the extensive use of CE loss for KD in other fields.

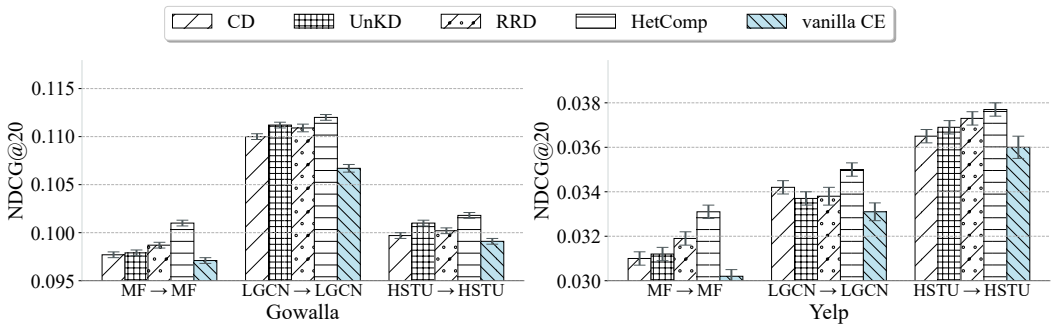

Figure 1: Performance comparison of different KD methods. We report the results in three homogeneous Teacher → Student settings.

Considering the features of recommender systems and the surprisingly poor performance of CE Loss, we analyze CE loss in KD for recommender systems. Firstly, we extend the connection between CE loss and NDCG to full-item KD, where CE loss is computed using all items. We theoretically prove that minimizing CE loss maximizes the lower bound of NDCG with the relevance scores proportional to the teacher's predicted scores. This suggests a strong motivation for using CE loss in KD.

However, full-item KD is not practical due to the extremely large number of items. In real-world scenarios, CE loss could only be computed on a subset of items (i.e., the partial-item KD), such as the teacher's top items in vanilla CE. For this case, we define partial NDCG, which only considers rankings within a subset of items. Then, we prove that CE loss bounds partial NDCG. However, it holds only if the item subset satisfies our assumption of closure (Assumption 4.3). It requires that all items that the student ranks higher than any item in the subset are also in the subset. This assumption emphasizes the effect of the student's top items. Recall that our goal is to distill rankings among the teacher's top items Reddi et al. (2021). Unfortunately, we observe that the top items given by the teacher are usually ranked low by the student. This makes it difficult for the teacher's top items to satisfy our assumption of closure. Thus, vanilla CE cannot bound partial NDCG and performs poorly.

To fully unleash the potential of CE loss by re-establishing its connection with NDCG, we propose Rejuvenated Cross-Entropy for Knowledge Distillation (RCE-KD), which consists of four key points: 1) It divides the teacher's top items into two subsets: the subset that consists of items also ranked high by the student and the one that consists of the rest of the items. 2) For the first subset, we distill rankings among these items by using CE loss directly on the student's top items. 3) For the second subset, we design a sampling strategy to sample from the student's top items and compute CE loss on a new item set that approximately satisfies Assumption 4.3. 4) The fusion weights of the losses on these two subsets are adaptively updated based on their size. With the above improvements, we can nearly completely unleash the potential of CE loss while ensuring high training efficiency.

To sum up, the key contributions of our work are as follows:

• We theoretically extend the connection between CE and NDCG to the field of KD for recommender systems in real scenarios, where KD is performed on an item subset. Specifically, we first define partial NDCG, which measures ranking ability on a subset of items. Then, we prove that minimizing CE loss on a given item subset maximizes the partial NDCG on it. We also give the critical assumption made on the item subset for the conclusion to hold.

• Based on the analysis, we propose RCE-KD to unleash the potential of CE loss fully while ensuring high training efficiency. It splits the top items of teachers and calculates the loss separately. A dynamic weighting method is devised to adaptively fuse the losses on all subsets.

• Extensive experiments are conducted on three public datasets and both homogeneous and heterogeneous KD settings to demonstrate the superiority of the proposed approach.

2 Related Work

2.1 Knowledge Distillation for Recommender Systems

Existing KD methods fall into three categories: response-based, feature-based, and relation-based.

Response-based methods focus on teachers' predictions. CD Lee et al. (2019) samples unobserved items from a distribution associated with their rankings predicted by students, and distills with a point-wise loss. RankDistil Reddi et al. (2021) enables students to mimic teachers by sampling high-ranking items predicted by teachers and calculating multiple forms of loss functions on them. RRD Kang et al. (2020) adopts a list-wise loss to maximize the likelihood of the teacher's recommendation list. Note that RRD could be regarded as the extension of ListMLE loss Xia et al. (2008) to the top-K setting Xia et al. (2009). Based on RRD, DCD Lee & Kim (2021) uses the discrepancy between the teacher and student model predictions to decide which knowledge to distill. HetComp Kang et al. (2023) transfers the ensemble knowledge of heterogeneous teachers by constructing easy-to-hard knowledge sequences from the teachers' trajectories.

Feature-based methods focus on the intermediate representations of the teacher. FreqD Zhu & Zhang (2024) defines knowledge as different frequency components of the features and proposes emphasizing important knowledge by graph filtering. PCKD Zhu & Zhang (2025) observes that projectors in feature-based KD interrupt user preference contained in the features and designs two regularization terms to restrict the projectors.

Relation-based methods focus on the relationships between different items. HTD Kang et al. (2021) distills the sample relation hierarchically to alleviate the capacity gap between the student and teacher.

Our work compensates for the lack of theoretical analysis of CE loss in response-based methods. Based on theoretical analysis, we design a split-and-fusion paradigm with a novel sampling strategy and adaptive loss fusion mechanism to enhance vanilla CE loss, thereby unlocking its full potential.

2.2 Connection between CE Loss and NDCG

Recently, many studies Cao et al. (2007); Ravikumar et al. (2011); Bruch et al. (2019); Wu et al. (2024); Yang et al. (2024); Xu et al. (2024) on learning-to-rank (LTR) have focused on the impact of different surrogate loss functions on NDCG. Among them, CE loss is of particular interest due to its wide range of applications. As a pioneer, ListNet Cao et al. (2007) introduces CE loss into LTR by defining the top-one probability. Then, Bruch et al. (2019) for the first time proves that CE loss is a bound on NDCG when considering binary ground-truth labels. Subsequently, work has been done to improve CE loss based on this conclusion. For example, PSL Yang et al. (2024) changes the surrogate activations, and SCE Xu et al. (2024) increases the weight of negative samples in CE loss to achieve a tighter bound of NDCG. Another work relevant to us is Wu et al. (2024). It reveals the pros and cons of sampled CE loss for item recommendation and also relates it to NDCG. However, these methods mentioned above hardly address the case of non-binary ground-truth labels. Moreover, they either do not focus on the scenarios that need item sampling or simply use uniform sampling without making any assumptions about the items being sampled. This makes them entirely inapplicable for KD, where we take the teacher's predictions as labels and emphasize the top-ranked items.

3 Preliminary

3.1 Top-N Recommendation

This work focuses on the top-N recommendation with implicit feedback. Let $\mathcal{U}$ and $\mathcal{I}$ denote the user and item sets, respectively. Then, $|\mathcal{U}|$ and $|\mathcal{I}|$ are the number of users and items, respectively. A recommendation model scores the items not interacted with by the user and recommends N items with the largest scores. We use r_{ui} to denote the score of interaction (u, i) predicted by the recommendation model and use $\boldsymbol{r}_u \in \mathbb{R}^{|\mathcal{I}|}$ to denote the predicted scores of all items for user u. In this paper, we use superscripts S and T to denote the student and the teacher, respectively. In the following sections, we default our analysis to any $u \in \mathcal{U}$ if not specified.

3.2 Cross-Entropy Loss for Knowledge Distillation

Given an item set $\mathcal{J}^u$ for each user $u \in \mathcal{U}$, CE loss in KD for recommender systems is computed as:

$$\mathcal{L}_{CE} = -\frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \sum_{i \in \mathcal{J}^u} \sigma(r^T_{ui}, \mathcal{J}^u) \log \sigma(r^S_{ui}, \mathcal{J}^u), \tag{1}$$

where r_{ui}^T and r_{ui}^S denote the scores predicted by the teacher and the student, respectively. $\sigma(r_{ui}^T, \mathcal{J}^u) = \exp(r_{ui}^T)/\sum_{j\in\mathcal{J}^u}\exp(r_{uj}^T)$ denotes the softmax over item set $\mathcal{J}^u$. Similarly $\sigma(r_{ui}^S, \mathcal{J}^u) = \exp(r_{ui}^S)/\sum_{j\in\mathcal{J}^u}\exp(r_{uj}^S)$. Note that for each user u, we only have access to a sampled item subset since it is computationally intractable over the entire item set $\mathcal{I}$ Sun et al. (2024).

4 Connection between CE Loss and Ranking Imitation in KD

This section reveals the connection between CE loss and ranking imitation in the field of KD. As a starting point, we extend the connection between CE and NDCG to the **full-item KD**, where the CE loss is computed using all items. Note that although the conclusion is promising, the full-item KD is not practical due to the extremely large number of items. Therefore, we further analyze the connection between CE loss and partial NDCG in **partial-item KD**, where CE loss is computed using only a subset of items. Finally, we demonstrate the challenges when using CE loss as distillation loss by showing the large differences in the student's and teacher's top items.

4.1 Analysis in Full-Item KD

This section studies the full-item KD, where CE loss is computed on the entire item set, i.e., $\mathcal{J}^u = \mathcal{I}$. Given a ground-truth relevance scores vector $\boldsymbol{y} \in \mathbb{R}^{|\mathcal{I}|}$ with y_i denoting the score of item i, and the predicted permutation $\boldsymbol{\pi}$, NDCG is defined as:

$$\text{NDCG}(\boldsymbol{\pi}, \boldsymbol{y}) = \frac{\text{DCG}(\boldsymbol{\pi}, \boldsymbol{y})}{\text{DCG}(\widetilde{\boldsymbol{\pi}}, \boldsymbol{y})}, \tag{2}$$

where $\widetilde{\boldsymbol{\pi}}$ is the ideal ranked list (where items are sorted according to $\boldsymbol{y}$). DCG is defined as follows:

$$\text{DCG}(\boldsymbol{\pi}, \boldsymbol{y}) = \sum_{i=1}^{|\mathcal{I}|} \frac{2^{y_i} - 1}{\log_2(1 + \pi^{-1}(i))}, \tag{3}$$

where $\pi^{-1}(i)$ is the rank of item i.

In the following theorem, we show that minimizing CE loss maximizes the lower bound of NDCG, where the relevance scores of items are proportional to the scores predicted by the teacher.

Theorem 4.1. *Suppose that we compute CE loss on the entire item set $\mathcal{I}$ and take the teacher's predicted scores (i.e., $\boldsymbol{r}_u^T$) as the target. In that case, we maximize a lower bound of NDCG, with the teacher's transformed predictive scores $\boldsymbol{y} = \log_2(\sigma(\boldsymbol{r}_u^T) + 1)$ being the relevance scores. Here $\sigma(\cdot)$ denotes the softmax function and $\sigma(\boldsymbol{r}_u^T)_i = \exp(r_{ui}^T)/\sum_{j\in\mathcal{I}}\exp(r_{uj}^T)$.*

The proof is provided in Appendix B.1. Theorem 4.1 demonstrates that when we minimize CE loss over the entire item set, the student can imitate the teacher in terms of NDCG. This theorem gives an intuitive explanation of the rationality of using CE loss as a distillation loss.

4.2 Analysis in Partial-Item KD

Although the above conclusion is promising, we can only afford CE loss with a sampled item subset in real-world scenarios. This section shows that CE loss must involve both the teacher's and the student's predicted top items to make the student benefit from the teacher's ranking ability.

Firstly, we define the partial NDCG to describe NDCG in the partial-item KD scenario. It only focuses on the rankings within the item subset.

Definition 4.2 (Partial NDCG)**.** *Given an item set $\mathcal{J}^u$, the partial NDCG on $\mathcal{J}^u$ (denoted as $NDCG_{\mathcal{J}^u}$) is defined as follows:*

$$NDCG_{\mathcal{J}^u}(\boldsymbol{\pi}, \boldsymbol{y}) \triangleq \frac{DCG(\boldsymbol{\pi}, \boldsymbol{y}_{\mathcal{J}^u})}{DCG(\widetilde{\boldsymbol{\pi}}_{\mathcal{J}^u}, \boldsymbol{y}_{\mathcal{J}^u})}, \tag{4}$$

where

$$(\boldsymbol{y}_{\mathcal{J}^u})_i = \begin{cases} y_i & \text{if } i \in \mathcal{J}^u, \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

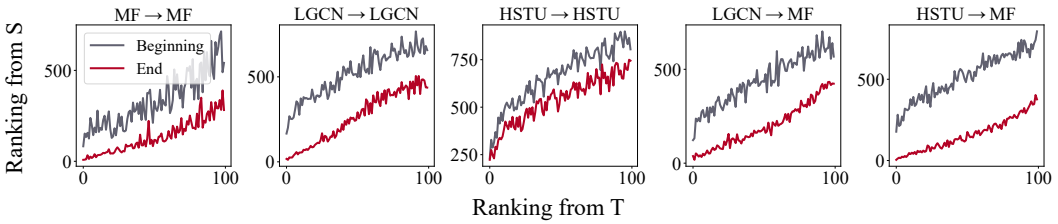

Figure 2: Relationship between rankings given by the teacher (shown in x-axis) and the student (shown in y-axis). Items are sorted in decreasing order according to the teacher's rankings.

denotes the truncated $\boldsymbol{y}$ that only retains the scores corresponding to the items in $\mathcal{J}^u$, and $\widetilde{\boldsymbol{\pi}}_{\mathcal{J}^u}$ is the corresponding ideal ranked list.

Then, to draw a promising conclusion analogous to the full-item KD, we must make a not mild but critical assumption about the item subset $\mathcal{J}^u$.

Assumption 4.3 (Closure of $\mathcal{J}^u$)**.** *For each item i in $\mathcal{J}^u$, we assume that all items that are considered by the student to be ranked higher than i are also in $\mathcal{J}^u$. Formally,*

$$\left(\bigcup_{i \in \mathcal{J}^u} \{ j | \pi^{-1}(j) \leq \pi^{-1}(i) \} \right) \subseteq \mathcal{J}^u, \tag{6}$$

where $\pi^{-1}(i)$ is the rank of item i predicted by the student.

Finally, we have the following theorem that connects CE loss and partial NDCG.

Theorem 4.4. *Given an item set $\mathcal{J}^u \subseteq \mathcal{I}$ that satisfies Assumption 4.3, minimizing CE loss on $\mathcal{J}^u$ maximizes a lower bound of NDCG$_{\mathcal{J}^u}$, where the relevance scores are $\boldsymbol{y}_{\mathcal{J}^u} = \left(\log_2(\sigma(\boldsymbol{r}_u^T) + 1)\right)_{\mathcal{J}^u}$.*

The proof is provided in Appendix B.2. Note that Theorem 4.1 can be regarded as a special case of Theorem 4.4 when $\mathcal{J}^u = \mathcal{I}$. Previous works Kang et al. (2020); Reddi et al. (2021) find that if the student can learn the rankings of top items given by the teacher, it benefits from the teacher's ranking ability. In other words, they expect to connect their distillation losses with NDCG$_{\mathcal{J}^u}$ where $\mathcal{J}^u$ involves the teacher's top items. Our theorem gives a method with theoretical support for accomplishing that purpose. That is, $\mathcal{J}^u$ must also involve enough top items provided by the student.

4.3 Challenge in Partial-Item KD

According to our analysis, $\mathcal{J}^u$ must satisfy Assumption 4.3 for the connection between CE loss and partial NDCG to hold. However, it is difficult to satisfy this assumption if we do not explicitly consider the student's top items. Specifically, in Figure 2, we report the relationship between the student's and the teacher's rankings at the beginning and end of the training. The student is trained with vanilla CE loss, which is computed using the teacher's top items. The dataset is CiteULike. Detailed analysis and results on all datasets are given in Appendix A. From the results, we find that:

Observation 4.5. *The teacher's top items are very likely to be ranked low by the student, especially at the beginning of the training.*

As a result, if we compute CE loss only on the teacher's or the student's top items, we cannot bound partial NDCG on the teacher's top items. Moreover, if we simply add the student's top items to an item subset that initially contains the teacher's top items to make it satisfy the assumption of closure, it will result in a very large item subset.

5 Rejuvenated Cross-Entropy for Knowledge Distillation

5.1 Overview of RCE-KD

To unleash the potential of CE loss of distilling rankings among the teacher's top items, we propose RCE-KD, a novel approach involving both the teacher's and the student's top items in KD. The key is

to split the teacher's top items into two subsets based on whether or not an item is ranked high by the student. Then, we try to make both item subset satisfy Assumption 4.3 exactly or approximately.

Let $\mathcal{Q}_u^T \triangleq \arg\text{top}K(\boldsymbol{r}_u^T)$ and $\mathcal{Q}_u^S \triangleq \arg\text{top}K(\boldsymbol{r}_u^S)$ denote the sets of top-K items predicted by the teacher and the student, respectively. We aim to transfer the teacher's ranking ability over $\mathcal{Q}_u^T$ Kang et al. (2020); Lee et al. (2019); Tang & Wang (2018). In RCE-KD, we propose to separate $\mathcal{Q}_u^T$ into two subsets. The first subset is the interaction between $\mathcal{Q}_u^T$ and $\mathcal{Q}_u^S$. The second subset contains the remaining items in $\mathcal{Q}_u^T$. Formally, $(\mathcal{Q}_u^T)_1 \triangleq \mathcal{Q}_u^T \cap \mathcal{Q}_u^S \quad \text{and} \quad (\mathcal{Q}_u^T)_2 \triangleq \mathcal{Q}_u^T \backslash (\mathcal{Q}_u^T)_1$.

5.2 Loss for $(\mathcal{Q}_u^T)_1$

For the first subset, we transfer the knowledge within it by computing CE loss on $\mathcal{Q}_u^S$. Formally,

$$\mathcal{L}_1 = -\frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \sum_{i \in \mathcal{Q}_u^S} \sigma(r_{ui}^T, \mathcal{Q}_u^S) \log \sigma(r_{ui}^S, \mathcal{Q}_u^S). \tag{7}$$

Note that $\mathcal{Q}_u^S$ satisfies Assumption 4.3. Therefore, $\mathcal{L}_1$ can make the student benefit from the teacher's ranking ability by exactly bounding $\text{NDCG}_{\mathcal{Q}_u^S}$. Since $(\mathcal{Q}_u^T)_1$ is a subset of $\mathcal{Q}_u^S$, it encourages the student to learn the rankings among $(\mathcal{Q}_u^T)_1$.

5.3 Loss for $(\mathcal{Q}_u^T)_2$

For the second subset, we propose to approximately maximize $\text{NDCG}_{(\mathcal{Q}_u^T)_2}$ by computing CE loss on the union of $(\mathcal{Q}_u^T)_2$ and a set of randomly sampled items. The probability of each item being sampled is defined as follows: For each item i in $(\mathcal{Q}_u^T)_2$, we raise the scores of all items ranked higher than i in the student's predicted ranking by 1. After iterating the entire $(\mathcal{Q}_u^T)_2$, let z_j denote the score of item j. Then, the probability of item j to be sampled is given by $p_j \propto e^{z_j/\tau}, \forall j \in \mathcal{I} \backslash \mathcal{Q}_u^T$, where τ is a hyperparameter and is fixed to 10 in our experiments.

Note that the sampling strategy is adaptive due to: 1) When the student assigns low rankings to all items in $(\mathcal{Q}_u^T)_2$, we sample nearly uniformly from the entire item set $\mathcal{I}$, allowing us to cover more items in multiple training epochs. 2) In contrast, we sample from highly ranked items when the student can already assign higher rankings to items in $(\mathcal{Q}_u^T)_2$. According to Theorem 4.4, these highly ranked items play a greater role in maximizing the partial NDCG on $(\mathcal{Q}_u^T)_2$ and enable us to efficiently approximate the fulfillment of Assumption 4.3.

Using the above sampling strategy, we sample L items and combine them with $(\mathcal{Q}_u^T)_2$ to form the set $\mathcal{A}^u$ (note that we resample in each epoch). Then, CE loss is computed on $\mathcal{A}^u$ as follows:

$$\mathcal{L}_2 = -\frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \sum_{i \in \mathcal{A}^u} \sigma(r_{ui}^T, \mathcal{A}^u) \log \sigma(r_{ui}^S, \mathcal{A}^u). \tag{8}$$

5.4 Adaptive Loss Fusion

Note that $\mathcal{L}_1$ and $\mathcal{L}_2$ play different roles. $\mathcal{L}_1$ focuses on $(\mathcal{Q}_u^T)_1$, consisting of top items considered by both the student and teacher. The goal on these items is to distill their fine-grained rankings, which is done by exactly maximize the partial NDCG. On the contrary, $\mathcal{L}_2$ focuses on items not well-mastered by the student. The goal for these items is to improve their rankings, which is done by making the student imitate the teacher's ranking on $(\mathcal{Q}_u^T)_2$ and randomly sampled items.

To combine the two losses, we propose an adaptive weighting scheme. Specifically, the final loss is

$$\mathcal{L}_{RCE-KD} = (1-\gamma) \cdot \mathcal{L}_1 + \gamma \cdot \mathcal{L}_2, \tag{9}$$

where γ is updated at the beginning of each epoch by the following equation:

$$\gamma = \exp\left(-\beta \cdot \frac{|(\mathcal{Q}_u^T)_1|}{|\mathcal{Q}_u^T|}\right), \tag{10}$$

where $|\cdot|$ denotes the cardinality of the set and β is a hyperparameter. When $|(\mathcal{Q}_u^T)_1|$ is small, we make the student overlap more with the teacher's top items by increasing the weight of $\mathcal{L}_2$. Otherwise, we assign a large weight to $\mathcal{L}_1$ because it is more useful when we want to distill fine-grained rankings.

Table 1: Recommendation performance. The best results are in boldface, and the best baselines are underlined. *Improv.b* denotes the relative improvement of RCE-KD over the best baseline. LGCN stands for LightGCN. A paired t-test is performed over 5 independent runs for evaluating p-value (≤ 0.05 indicates statistical significance).

T→S	Method	CiteULike				Gowalla				Yelp			
		R@10	N@10	R@20	N@20	R@10	N@10	R@20	N@20	R@10	N@10	R@20	N@20
MF→MF	Teacher	0.0283	0.0155	0.0442	0.0198	0.1088	0.0907	0.1544	0.1053	0.0394	0.0253	0.0660	0.0339
	Student	0.0177	0.0098	0.0284	0.0128	0.0946	0.0820	0.1329	0.0939	0.0348	0.0222	0.0586	0.0299
	CD	0.0239	0.0131	0.0347	0.0158	0.0979	0.0855	0.1389	0.0977	0.0370	0.0236	0.0608	0.0310
	RRD	0.0251	0.0135	0.0362	0.0169	0.0977	0.0861	0.1395	0.0987	0.0362	0.0230	0.0626	0.0319
	DCD	0.0254	0.0136	0.0375	0.0173	0.1007	0.0871	0.1413	0.0999	0.0377	0.0240	0.0639	0.0330
	HetComp	0.0255	0.0135	0.0391	0.0177	0.1028	0.0874	0.1427	0.1010	0.0383	0.0245	0.0644	0.0331
	RCE-KD	**0.0278**	**0.0152**	**0.0431**	**0.0194**	**0.1082**	**0.0905**	**0.1525**	**0.1047**	**0.0400**	**0.0259**	**0.0667**	**0.0345**
	Improv.b	9.02%	11.76%	10.23%	9.60%	5.25%	3.55%	6.87%	3.66%	4.44%	5.71%	3.57%	4.23%
	p-value	1.71e-4	1.98e-5	5.47e-4	7.12e-5	4.22e-5	2.79e-4	8.22e-4	3.37e-3	6.32e-4	1.22e-4	1.72e-3	3.17e-5
LGCN→LGCN	Teacher	0.0296	0.0160	0.0461	0.0205	0.1236	0.1035	0.1730	0.1190	0.0432	0.0276	0.0716	0.0367
	Student	0.0215	0.0113	0.0344	0.0148	0.1098	0.0928	0.1550	0.1069	0.0363	0.0235	0.0621	0.0308
	CD	0.0234	0.0125	0.0354	0.0161	0.1132	0.0951	0.1592	0.1100	0.0385	0.0247	0.0669	0.0342
	RRD	0.0247	0.0125	0.0359	0.0158	0.1142	0.0969	0.1627	0.1109	0.0391	0.0245	0.0671	0.0338
	DCD	0.0243	0.0124	0.0360	0.0155	0.1149	0.0971	0.1631	0.1108	0.0403	0.0247	0.0676	0.0340
	HetComp	0.0248	0.0127	0.0362	0.0160	0.1150	0.0981	0.1636	0.1120	0.0405	0.0256	0.0691	0.0350
	RCE-KD	**0.0262**	**0.0133**	**0.0377**	**0.0171**	**0.1196**	**0.1011**	**0.1681**	**0.1163**	**0.0431**	**0.0277**	**0.0716**	**0.0369**
	Improv.b	5.65%	4.73%	4.12%	6.21%	4.00%	3.06%	2.75%	3.84%	6.42%	8.20%	3.62%	5.43%
	p-value	4.72e-4	7.33e-4	9.87e-4	3.73e-3	1.31e-3	4.52e-4	2.01e-3	7.38e-4	3.77e-4	5.92e-5	3.01e-3	9.00e-5
HSTU→HSTU	Teacher	0.0463	0.0291	0.0613	0.0333	0.1124	0.0901	0.1625	0.1063	0.0482	0.0314	0.0800	0.0417
	Student	0.0262	0.0159	0.0371	0.0189	0.0974	0.0781	0.1416	0.0923	0.0391	0.0254	0.0664	0.0342
	CD	0.0428	0.0264	0.0565	0.0285	0.1029	0.0817	0.1527	0.0997	0.0415	0.0271	0.0701	0.0365
	RRD	0.0433	0.0263	0.0562	0.0297	0.1048	0.0835	0.1538	0.1002	0.0433	0.0274	0.0712	0.0373
	DCD	0.0461	0.0292	0.0599	0.0319	0.1060	0.0857	0.1541	0.1021	0.0443	0.0290	0.0728	0.0382
	HetComp	0.0470	0.0299	0.0609	0.0331	0.1049	0.0840	0.1532	0.1018	0.0440	0.0285	0.0719	0.0377
	RCE-KD	**0.0524**	**0.0325**	**0.0670**	**0.0366**	**0.1106**	**0.0902**	**0.1594**	**0.1058**	**0.0459**	**0.0305**	**0.0754**	**0.0400**
	Improv.b	11.49%	8.70%	10.02%	10.57%	4.34%	5.25%	3.44%	3.62%	3.61%	5.17%	3.57%	4.71%
	p-value	1.73e-4	4.22e-4	8.92e-5	3.77e-4	3.52e-4	9.92e-4	4.57e-3	8.20e-3	7.32e-4	3.71e-5	2.23e-3	2.38e-4
LGCN→MF	Teacher	0.0296	0.0160	0.0461	0.0205	0.1236	0.1035	0.1730	0.1190	0.0432	0.0276	0.0716	0.0367
	Student	0.0177	0.0098	0.0284	0.0128	0.0946	0.0820	0.1329	0.0939	0.0348	0.0222	0.0586	0.0299
	CD	0.0240	0.0133	0.0365	0.0170	0.1097	0.0917	0.1549	0.1072	0.0368	0.0247	0.0610	0.0342
	RRD	0.0247	0.0137	0.0367	0.0169	0.1098	0.0932	0.1577	0.1070	0.0377	0.0249	0.0622	0.0340
	DCD	0.0260	0.0139	0.0387	0.0177	0.1123	0.0966	0.1600	0.1098	0.0392	0.0258	0.0641	0.0347
	HetComp	0.0263	0.0142	0.0402	0.0180	0.1110	0.0943	0.1604	0.1103	0.0399	0.0260	0.0657	0.0344
	RCE-KD	**0.0285**	**0.0156**	**0.0437**	**0.0197**	**0.1200**	**0.1013**	**0.1677**	**0.1163**	**0.0419**	**0.0271**	**0.0692**	**0.0360**
	Improv.b	8.37%	9.86%	8.71%	9.44%	6.86%	4.87%	4.55%	5.44%	5.01%	4.23%	5.33%	3.75%
	p-value	1.77e-5	1.92e-4	4.29e-4	6.99e-5	4.33e-5	3.52e-4	1.21e-3	3.23e-4	7.38e-4	3.52e-3	4.77e-4	5.83e-3
HSTU→MF	Teacher	0.0463	0.0291	0.0613	0.0333	0.1124	0.0901	0.1625	0.1063	0.0482	0.0314	0.0800	0.0417
	Student	0.0177	0.0098	0.0284	0.0128	0.0946	0.0820	0.1329	0.0939	0.0348	0.0222	0.0586	0.0299
	CD	0.0361	0.0209	0.0502	0.0251	0.1047	0.0834	0.1520	0.1021	0.0433	0.0276	0.0743	0.0370
	RRD	0.0379	0.0224	0.0520	0.0270	0.1054	0.0831	0.1511	0.1018	0.0430	0.0271	0.0734	0.0359
	DCD	0.0411	0.0253	0.0533	0.0295	0.1078	0.0854	0.1552	0.1029	0.0449	0.0297	0.0759	0.0392
	HetComp	0.0401	0.0239	0.0524	0.0278	0.1066	0.0840	0.1531	0.1031	0.0453	0.0290	0.0765	0.0382
	RCE-KD	**0.0457**	**0.0285**	**0.0595**	**0.0324**	**0.1128**	**0.0905**	**0.1624**	**0.1065**	**0.0485**	**0.0316**	**0.0805**	**0.0419**
	Improv.b	11.19%	12.65%	11.63%	9.83%	4.64%	5.97%	4.64%	3.30%	7.06%	6.40%	5.23%	6.89%
	p-value	1.27e-5	5.22e-5	4.73e-4	2.20e-3	1.37e-3	2.52e-4	3.31e-4	2.83e-3	1.07e-4	3.88e-4	1.17e-3	4.92e-4

Finally, the total loss for training the student is given by

$$\mathcal{L} = \mathcal{L}_{Base} + \lambda \cdot \mathcal{L}_{RCE-KD}\,, \tag{11}$$

where $\mathcal{L}_{Base}$ is the loss of the base recommendation model, such as BPR loss. λ is a hyperparameter.

6 Experiments

Section 6.1 first introduces the experimental settings. The **implementation details** are shown in Appendix C.1. Then, the overall performance comparison is shown in Section 6.2. Consequently, we investigate the training efficiency of all compared KD methods in Section 6.3. The ablation study is conducted in Section 6.4. To verify our sampling strategy's **efficiency for approximating Assumption 4.3**, we conduct experiments in Appendix C.2. We present **hyperparameter analysis** in Appendix C.4. Finally, in Appendix C.5, we demonstrate the effectiveness of **applying our method to sequential recommendation** to showcase its generalization capability for recommendation tasks.

6.1 Experimental Settings

Datasets. We conduct experiments on three public datasets, including **CiteULike** Wang et al. (2013); Kang et al. (2022; 2021), **Gowalla** Cho et al. (2011); Tang & Wang (2018); Lee et al. (2019), and **Yelp2018** Lee et al. (2019); Kweon et al. (2021). Detailed statistics and methods of constructing training and test sets are given in Appendix C.1.

Evaluation Protocols. Per the custom, we adopt the full-ranking evaluation to achieve an unbiased evaluation. We employ Recall (Recall@N) and normalized discounted cumulative gain (NDCG@N) and report the results for $N \in \{10, 20\}$. We conduct five independent runs for each configuration and report the average results.

Baselines. We compare our method with five response-based KD methods: CD Lee et al. (2019), RRD Kang et al. (2020), DCD Lee & Kim (2021), and HetComp Kang et al. (2023). The introduction of these methods is in Appendix C.1.

Backbones. We refer to previous works Chen et al. (2023); Kang et al. (2020; 2021), and use MF Rendle et al. (2012) and LightGCN He et al. (2020). We also add HSTU Zhai et al. (2024) as a new backbone, which is a popular generative recommendation model.

Teacher/Student. For each backbone, we create two instances, one large and one small. We use the large instance as the teacher and the small one as the student. Details are provided in Appendix C.1.

6.2 Performance Comparison

The performance of all methods is provided in Table 1. From the results, we observe that:
• Different KD methods perform differently. We find that CD performs poorly compared to other methods. We attribute this to CD using a pair-wise loss to align teachers' and students' predictions. In training recommendation models, pair-wise loss is usually less effective than list-wise losses, such as RRD loss and CE loss.
• Our method significantly outperforms all other methods in all cases, suggesting it effectively aligns the teacher's and student's predictions and utilizes the teacher's predictions to enhance the student. This also demonstrates that utilizing CE loss for KD and using teacher and student predictions to collaboratively decide on sampling strategies are effective.
• In all scenarios, students can perform similarly to teachers. This suggests that with the proper knowledge distillation approach, we can significantly reduce the model size and improve the model's inference efficiency with little to no degradation of the model's recommendation accuracy.

Table 2: The comparison of the training time (seconds) per epoch.

Method	CiteULike			Gowalla			Yelp		
	MF	LightGCN	HSTU	MF	LightGCN	HSTU	MF	LightGCN	HSTU
Student	4.25	5.47	8.12	27.33	50.11	58.72	26.93	36.53	49.29
CD	14.37	20.80	29.03	82.77	137.63	201.70	77.72	126.38	210.69
RRD	19.67	23.81	39.09	132.37	167.90	231.12	119.49	152.27	271.86
DCD	21.37	26.82	38.63	145.87	158.60	241.35	108.37	162.92	292.00
HetComp	16.32	24.87	40.03	137.62	144.53	239.07	121.36	156.04	281.91
RCE-KD	15.23	21.79	33.62	99.30	141.72	221.65	81.17	129.74	233.38

Table 3: The comparison of GPU Memory (GB) required by our method and comparison methods.

Method	CiteULike			Gowalla			Yelp		
	MF	LightGCN	HSTU	MF	LightGCN	HSTU	MF	LightGCN	HSTU
Student	0.39	0.60	3.11	0.45	1.08	1.10	0.45	0.81	1.36
CD	1.07	2.52	6.27	6.27	8.81	19.37	4.99	7.02	17.62
RRD	0.92	2.42	6.81	5.23	8.64	19.93	4.85	6.30	19.01
DCD	1.41	2.93	7.22	7.89	9.37	21.52	6.03	7.22	20.89
HetComp	1.09	2.69	6.97	6.78	9.02	19.99	5.87	7.01	19.97
RCE-KD	1.05	2.47	6.52	6.37	8.90	19.98	5.41	6.90	18.74

6.3 Training Efficiency

In this section, we report the training efficiency of our method and comparison methods. All results are obtained by testing with PyTorch on a GeForce RTX 3090 GPU.

In RCE-KD, we only need to add the cost of time and space required for random sampling on top of CE loss. Therefore, it has very high training efficiency. To empirically validate the training efficiency of our method, we report the training time and storage cost of our method and comparison methods. The results are presented in Table 2 and Table 3. The method *Student* denotes that we train the student model without KD. Note that since we save the teacher's predictions before KD and simply load the predictions without rerunning the teacher during KD, the architecture of the teachers does not affect the training inference. Therefore, we only report the results with different students.

From the results, we find that:

- All KD methods inevitably increase training costs. In most cases, all KD methods have similar training costs. We believe this is attributed to the fact that they all follow a similar pattern of sampling a subset of items before computing the loss functions.
- Among all baseline methods, we find that CD and RRD have smaller training costs than others. We believe this is because CD and RRD are simpler and require fewer intermediate computational processes. However, they do not perform as well as the more complex methods. This forces baselines to face a trade-off between training cost and recommendation accuracy.
- Our method has similar training efficiency as CD and RRD. This can be attributed to the simplicity of our method. Moreover, we empirically find that the number of items that need to be sampled by our method is often smaller than that of other methods, significantly reducing the cost required in the sampling phase. Together, these two make our method highly efficient in training.

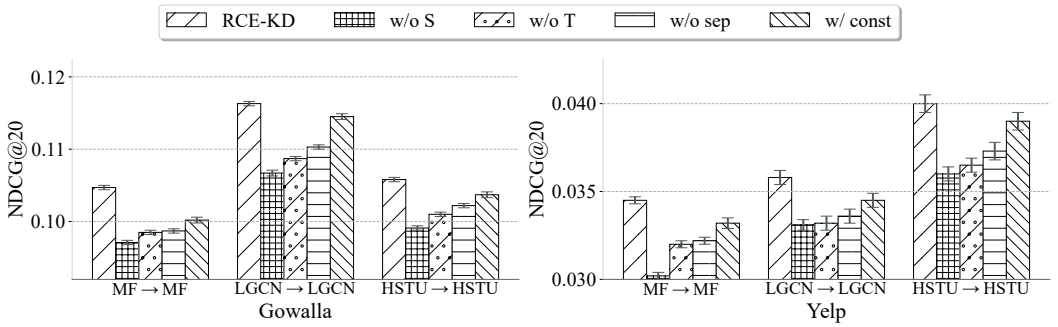

Figure 3: Ablation study on Gowalla and Yelp, including the results in three homogeneous Teacher → Student settings.

6.4 Ablation Study

RCE-KD consists of three key components: 1) It divides the teacher's top items into two subsets; 2) It computes CE loss on items selected from both the teacher's and the student's top items; 3) An adaptive mechanism is proposed to combine the losses on these subsets. To validate the effectiveness of these key components, we design four variants: 1) **RCE-KD w/o sep** does not compute losses for $(\mathcal{Q}_u^T)_1$ and $(\mathcal{Q}_u^T)_1$, separately. It only computes CE loss on $\mathcal{A}^u \cup \mathcal{Q}_u^S$; 2) **RCE-KD w/o S** only aligns the predictions on the teacher's predicted top items, i.e., $\mathcal{Q}_u^T$; 3) Similarly, **RCE-KD w/o T** only aligns the predictions on the student's predicted top items, i.e., $\mathcal{Q}_u^S$; 4) **RCE-KD w/ const** replaces the adaptive weight derived from Eq.(10) with a constant hyperparameter γ.

Figure 3 shows the results of these four variants on Gowalla and Yelp, and three Teacher/Student settings. The results of the remaining settings are provided in Appendix C.3. We find that all variants are inferior to the original RCE-KD, which demonstrates the effectiveness of all key components. Moreover, **RCE-KD w/o S** usually performs worse than **RCE-KD w/o T**. We believe that the reason is that the top items given by the student can exactly satisfy Assumption 4.3, while the top items given by the teacher do not. The superiority of **RCE-KD w/ const** over **RCE-KD w/o T** demonstrates the necessity of involving top items from both the student and the teacher. Finally, the superiority of RCE-KD over **RCE-KD w/ const** and **RCE-KD w/o sep** validates the effectiveness of our adaptive weighting scheme and the necessity of splitting out the two subsets and treating them separately.

7 Conclusion

This paper analyzes CE loss in the real KD scenario for recommender systems, where loss is computed using a subset of items. We prove that CE loss bounds NDCG. It makes CE loss suitable for recommender systems, where rankings are essential. We also theoretically provide a critical assumption about the item subset, on which CE loss is computed, for the conclusion to hold. Based on the above analysis, we propose RCE-KD to fully unleash the potential of CE loss by approximately satisfying the assumption through teacher-student collaboration. Extensive experiments on both homogeneous and heterogeneous settings demonstrate the effectiveness of our method.

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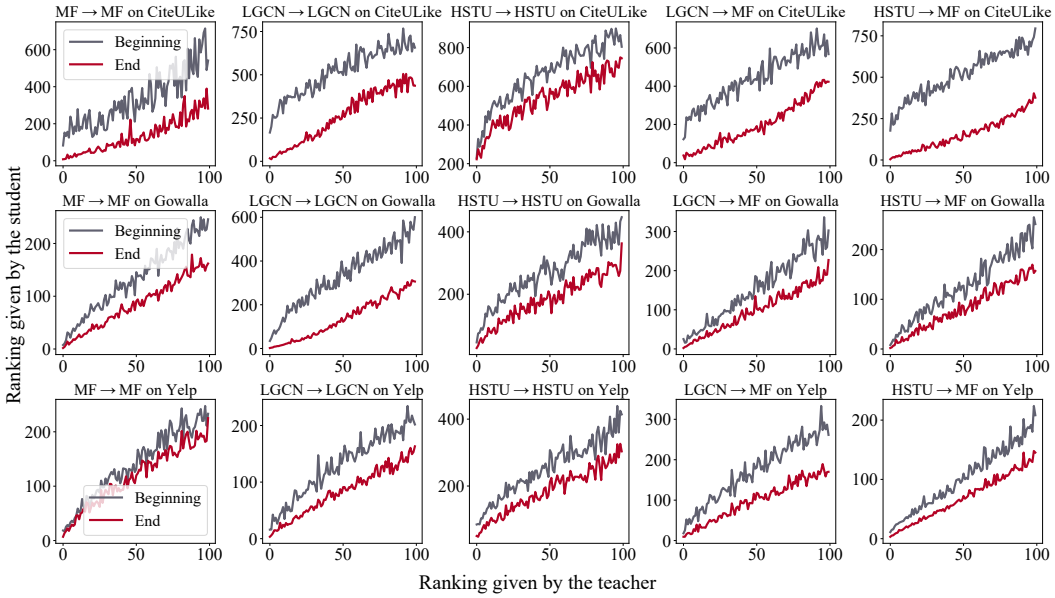


Figure 4: Relationship between rankings given by the teacher (shown in x-axis) and the student (shown in y-axis) on all datasets. Items are sorted in decreasing order according to the teacher's rankings.

A Relationship between the Rankings Given by the Student and the Teacher

To investigate whether we can satisfy Assumption 4.3 without explicitly considering the student's top items, we report the rankings given by the student and the teacher. The student is trained with vanilla CE loss, which is computed using the teacher's top items. The items are sorted in decreasing order according to the rankings given by the teacher. The results on all three datasets are provided in Figure 4. In each subfigure, we give two lines. The grey one represents the results at the beginning of training (after about $0.2\times$ total epoch number of rounds of training). The red one represents the results after training is complete.

The results show similar trends in all cases. Concretely, we observe that: 1) There is a significant positive correlation between the rankings given by the teacher and the student. This suggests that through knowledge distillation, students do learn some of the teacher's ranking results. 2) For top items given by the teacher (ranked higher than 100), students often give lower rankings (lower than 100 and even 200 on CiteULike). 3) The phenomenon is particularly acute at the beginning of training.

B Proofs

B.1 Proof of Theorem 4.1

Proof. By inserting $\boldsymbol{y} = \log_2(\sigma(\boldsymbol{r}_u^T) + 1)$ into the definition of DCG in Eq.(3), we have

$$\mathrm{DCG}(\boldsymbol{\pi}, \boldsymbol{y}) = \sum_{i \in \mathcal{I}} \frac{\sigma(\boldsymbol{r}_u^T)_i}{\log_2(1 + \pi^{-1}(i))}. \tag{12}$$

Then, similar to the proof for Theorem 3 in Bruch et al. (2019), we have

$$\text{DCG}(\boldsymbol{\pi}, \boldsymbol{y}) = \sum_{i \in \mathcal{I}} \frac{\sigma(\boldsymbol{r}_u^T)_i}{\log_2(1 + \pi^{-1}(i))} \tag{13}$$

$$\geq \sum_{i \in \mathcal{I}} \frac{\sigma(\boldsymbol{r}_u^T)_i}{\pi^{-1}(i)} \tag{14}$$

$$\geq \sum_{i \in \mathcal{I}} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{\exp(r_{ui}^S)}{\sum_{k \in \mathcal{I}} \exp(r_{uk}^S)}, \tag{15}$$

where $\boldsymbol{r}_u^S$ is the student's predictive score vector that derives the permutation $\boldsymbol{\pi}$.

For the ideal DCG, we have

$$\text{DCG}(\widetilde{\boldsymbol{\pi}}, \boldsymbol{y}) = \sum_{i \in \mathcal{I}} \frac{\sigma(\boldsymbol{r}_u^T)_i}{\log_2(1 + \widetilde{\pi}^{-1}(i))} \tag{16}$$

$$\leq \sum_{i \in \mathcal{I}} \sigma(\boldsymbol{r}_u^T)_i \tag{17}$$

$$= 1 \tag{18}$$

Finally,

$$\log \text{NDCG}(\boldsymbol{\pi}, \boldsymbol{y}) = \log \left(\frac{\text{DCG}(\boldsymbol{\pi}, \boldsymbol{y})}{\text{DCG}(\widetilde{\boldsymbol{\pi}}, \boldsymbol{y})} \right) \tag{19}$$

$$\geq \log \left(\sum_{i \in \mathcal{I}} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{\exp(r_{ui}^S)}{\sum_{k \in \mathcal{I}} \exp(r_{uk}^S)} \right) \tag{20}$$

$$\geq \sum_{i \in \mathcal{I}} \sigma(\boldsymbol{r}_u^T)_i \log \left(\frac{\exp(r_{ui}^S)}{\sum_{k \in \mathcal{I}} \exp(r_{uk}^S)} \right), \tag{21}$$

where the final inequality holds because of Jensen's inequality. We complete the proof by noting that the right-hand side of the final inequality is the negative of CE loss. □

B.2 Proof of Theorem 4.4

Proof.

$$\text{NDCG}_{\mathcal{J}^u}(\boldsymbol{\pi}, \boldsymbol{y}) = \frac{\text{DCG}(\boldsymbol{\pi}, \boldsymbol{y}_{\mathcal{J}^u})}{\text{DCG}(\widetilde{\boldsymbol{\pi}}_{\mathcal{J}^u}, \boldsymbol{y}_{\mathcal{J}^u})} \tag{22}$$

$$\geq \text{DCG}(\boldsymbol{\pi}, \boldsymbol{y}_{\mathcal{J}^u}) \qquad \text{(Due to DCG}(\widetilde{\boldsymbol{\pi}}_{\mathcal{J}^u}, \boldsymbol{y}_{\mathcal{J}^u}) \leq 1.\text{)}$$

$$= \sum_{i \in \mathcal{J}^u} \frac{\sigma(\boldsymbol{r}_u^T)_i}{\log_2(1 + \pi^{-1}(i))} \tag{23}$$

$$\geq \sum_{i \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{1}{\pi^{-1}(i)} \tag{24}$$

$$\geq \sum_{i \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{1}{\sum_{\pi^{-1}(j) \leq \pi^{-1}(i)} \exp(r_{uj}^S - r_{ui}^S)} \tag{25}$$

$$= \sum_{i \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{\exp(r_{ui}^S)}{\sum_{\pi^{-1}(j) \leq \pi^{-1}(i)} \exp(r_{uj}^S)} \tag{26}$$

$$\geq \sum_{i \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)}. \tag{27}$$

Table 4: Statistics of the preprocessed datasets.

Dataset	#Users	#Items	#Interactions	#Sparsity
CiteULike	5,219	25,181	125,580	99.89%
Gowalla	29,858	40,981	1,027,370	99.92%
Yelp2018	41,801	26,512	1,022,604	99.91%

Table 5: Dimensions of teachers and students for MF and LightGCN.

Model	CiteULike		Gowalla		Yelp	
	MF	LightGCN	MF	LightGCN	MF	LightGCN
Teacher	400	2000	300	2000	300	1000
Student	20	20	20	20	20	20

Therefore,

$$\log \text{NDCG}_{\mathcal{J}^u}(\boldsymbol{\pi}, \boldsymbol{y}) \geq \log \sum_{i \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_i \cdot \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)} \tag{28}$$

$$= \log \sum_{i \in \mathcal{J}^u} \frac{\exp(r_{ui}^T)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^T)} \cdot \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)} + \log \sum_{j \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_j \tag{29}$$

$$\geq \sum_{i \in \mathcal{J}^u} \frac{\exp(r_{ui}^T)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^T)} \log \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)} + \log \sum_{j \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_j \tag{30}$$

$$= \sum_{i \in \mathcal{J}^u} \frac{\exp(r_{ui}^T)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^T)} \log \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)} + \log C_{\mathcal{J}^u}, \tag{31}$$

where $C_{\mathcal{J}^u} \triangleq \sum_{j \in \mathcal{J}^u} \sigma(\boldsymbol{r}_u^T)_j$ is a constant, given $\mathcal{J}^u$.

Note that by minimizing CE loss on $\mathcal{J}^u$, which is defined as follows:

$$- \sum_{i \in \mathcal{J}^u} \frac{\exp(r_{ui}^T)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^T)} \log \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)}, \tag{32}$$

we also maximize

$$\sum_{i \in \mathcal{J}^u} \frac{\exp(r_{ui}^T)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^T)} \log \frac{\exp(r_{ui}^S)}{\sum_{j \in \mathcal{J}^u} \exp(r_{uj}^S)} + \log C_{\mathcal{J}^u}, \tag{33}$$

because $C_{\mathcal{J}^u}$ is a constant when $\mathcal{J}^u$ is fixed. □

C More Experimental Results

C.1 Experimental Settings

Datasets. We conduct experiments on three public datasets, including **CiteULike**[1] Wang et al. (2013); Kang et al. (2022; 2021), **Gowalla**[2] Cho et al. (2011); Tang & Wang (2018); Lee et al. (2019), and **Yelp2018**[3] Lee et al. (2019); Kweon et al. (2021).

Following the previous method Xu et al. (2023), we filter out users and items with less than 10 interactions and then split the rest chronologically into training, validation, and test sets in an 8:1:1 ratio. The statistics of the preprocessed datasets are summarized in Table 4.

[1] https://github.com/changun/CollMetric/tree/master/citeulike-t

[2] http://dawenl.github.io/data/gowallapro.zip

[3] https://github.com/hexiangnan/sigir16-eals

Table 6: The Number of transformer blocks (#Block) and number of heads (#Head) for HSTU.

Model	CiteULike		Gowalla		Yelp	
	#Block	#Head	#Block	#Head	#Block	#Head
Teacher	8	4	8	4	8	8
Student	1	2	1	1	1	2

Table 7: Overlap rate at the beginning (2% of total training epochs), midpoint (20% of total training epochs), and end (100% of total training epochs) of training. Denoted as OV@2, OV@20, and OV@100 respectively.

T→S	CiteULike			Gowalla			Yelp		
	OR@2	OR@20	OR@100	OR@2	OR@20	OR@100	OR@2	OR@20	OR@100
MF→MF	0.57	0.89	0.98	0.69	0.93	0.98	0.64	0.94	0.95
LGCN→LGCN	0.67	0.94	0.97	0.60	0.90	0.96	0.71	0.95	0.97
HSTU→HSTU	0.69	0.96	0.98	0.67	0.92	0.97	0.73	0.93	0.98
LGCN→MF	0.52	0.92	0.95	0.56	0.90	0.96	0.62	0.93	0.95
HSTU→MF	0.54	0.88	0.97	0.61	0.92	0.95	0.67	0.95	0.96

Teacher/Student. For each backbone, we create two instances, one large and one small. We use the large instance as the teacher and the small one as the student. For the large instance, we increase the model size until the recommendation performance no longer improves and adopt the model with the best performance. For the small instance, we select the hyperparameters to enlarge the performance gap between the student and the teacher.

Concretely, for MF and LightGCN, we choose different embedding dimensions for the teacher and the student while keeping other hyperparameters the same. The detailed embedding dimensions are provided in Table 5. As for HSTU, we decrease the number of transformer blocks and the number of heads to obtain the student model. The final number of blocks and heads for HSTU is given in Table 6.

In addition to homogeneous settings, we consider two heterogeneous settings where teachers and students have different architectures: 1) LightGCN as the teacher and MF as the student, and 2) HSTU as the teacher and MF as the student.

Implementation Details. We implement all the methods with PyTorch and use Adam as the optimizer. Before distillation, we save the teacher's predictions and load them during KD instead of rerunning the teacher. In the case of using HSTU as the student, we fix the batch size to 128. In other cases, we fix it to 2048. For our method, the weight decay is selected from {1e-3, 1e-5, 1e-7}. The search space of the learning rate is {1e-3, 1e-4}. β is selected from {0.5,1,3,5,7,9}. λ is selected from {0.5,1,5,10,50,100,500,5000,10000}. K and L are both selected from {10,50,100,500,1000}. We conduct early stopping according to the NDCG@20 on the validation set and stop training when the NDCG@20 does not increase for 30 consecutive epochs. All hyperparameters of the compared baselines are tuned to ensure optimal performance.

Baselines. We compare our method with the following knowledge distillation methods:
• CD Lee et al. (2019) samples unobserved items with a ranking-related distribution and uses a point-wise KD loss.
• RRD Kang et al. (2020) adopts a list-wise loss to maximize the likelihood of the teacher's recommendation list.
• DCD Lee & Kim (2021) corrects what the student has failed to predict with a dual correction loss accurately.
• HetComp Kang et al. (2023) guides the student model by transferring easy-to-hard knwoledge sequences generated from the teacher's trajectories.

C.2 Approximate Efficiency of Assumption 4.3

In Theorem 4.4, we demonstrate that the relationship between CE loss and NDCG can only be established when Assumption 4.3 holds. To address the practical limitation of precisely satisfying Assumption 4.3 in real-world scenarios, we devise a novel sampling strategy for $(\mathcal{Q}_u^T)_2$ in Section 5.3.

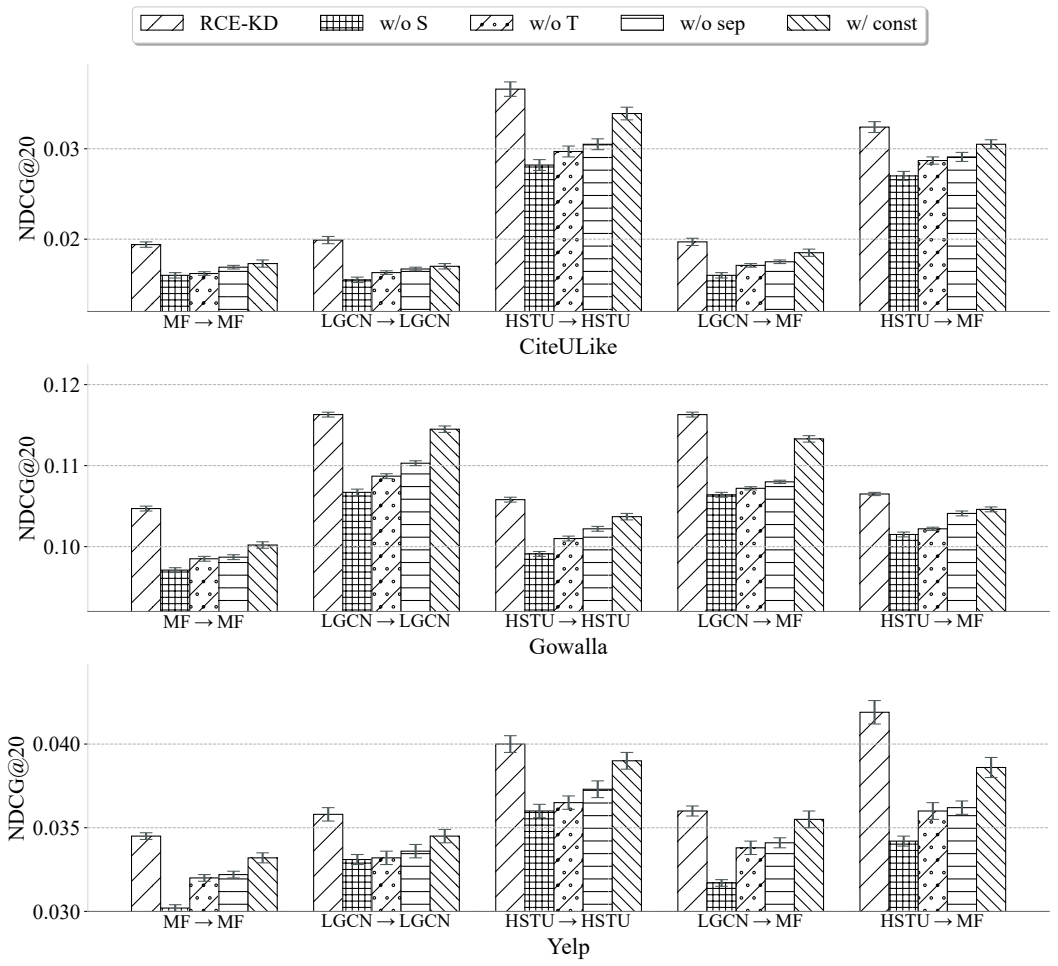


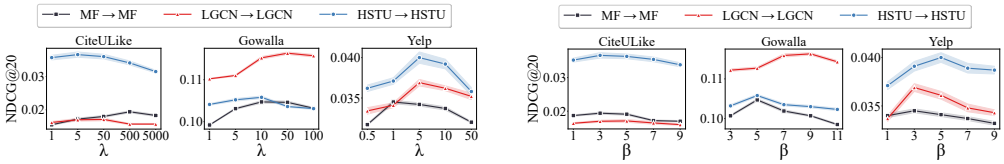
Figure 5: Ablation study on all datasets and all Teacher → Student settings. The average NDCG@20 and standard deviation over 5 independent runs are provided.

This strategy enables the extended set $\mathcal{A}^u$ to closely approximate Assumption 4.3. In this section, we design experiments to validate the efficiency of this approximation. Specifically, we compute the degree of overlap between the set we constructed (i.e., $\mathcal{A}^u$) and the ideal set as training progressed. Formally, we take the top-$|\mathcal{A}^u|$ items given by the student as the ideal set (denoted as $Idea^u$) because it strictly satisfies the closure assumption. Then, in Table 7, we show the overlap rate between $\mathcal{A}^u$ and the ideal set $Idea^u$ at the beginning, midpoint, and end of the training. The overlap rate is computed as $OV = |\mathcal{A}^u \cap Idea^u|/|\mathcal{A}^u \cup Idea^u|$.

From the results in Table 7, we observed that during the early stages of training (approximately 2% of total training epochs), a high overlap rate (exceeding 60%) is typically achieved. As training progresses, the overlap rate increases rapidly, reaching approximately 95% by the mid-training phase (around 20% of total training epochs). By the end of training, the overlap rate reached approximately 98%.

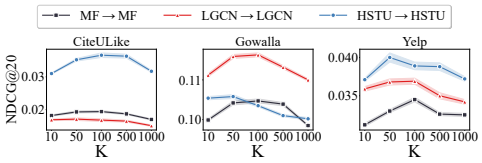
C.3 Ablation Study

This section presents additional results of the ablation study. In Figure 5, we give the results on all KD settings and all datasets. The results suggest similar trends to Figure 3. Specifically, we find that all variants are inferior to the original RCE-KD, demonstrating the effectiveness of all key components. Moreover, **RCE-KD w/o S** usually performs worse than **RCE-KD w/o T**. We believe that the reason is that the top items given by the student can exactly satisfy Assumption 4.3, while

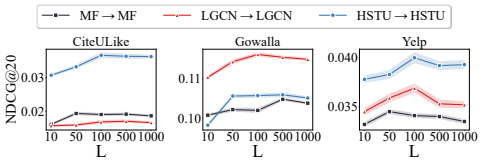


(a) Effect of λ.

(b) Effect of β.

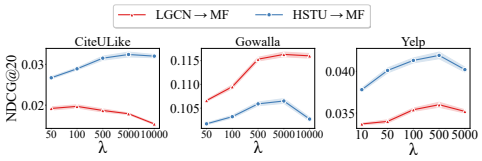


(c) Effect of K.

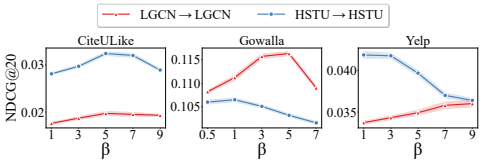


(d) Effect of L.

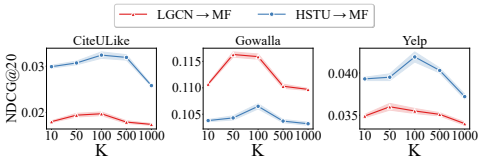
Figure 6: Hyperparameter study on three datasets. We report the results on three homogenous Teacher → Student settings. The average NDCG@20 and standard deviation over 5 independent runs are provided.



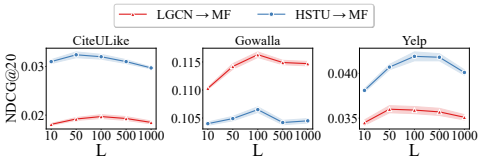
(a) Effect of λ.



(b) Effect of β.



(c) Effect of K.



(d) Effect of L.

Figure 7: Hyperparameter study on three datasets. We report the results on two heterogeneous Teacher → Student settings. The average NDCG@20 and standard deviation over 5 independent runs are provided.

the top items given by the teacher do not. On the other hand, the superiority of **RCE-KD w/ const** over **RCE-KD w/o T** demonstrates the necessity of involving top items from both the student and the teacher. Finally, the superiority of RCE-KD over **RCE-KD w/ const** and **RCE-KD w/o sep** validates the effectiveness of our adaptive weighting scheme and the necessity of splitting out the two subsets and treating them separately.

C.4 Hyperparameter Analysis

Effects of λ. We use λ to balance the impact of our KD loss and the base loss in Eq.(11). In Figure 6(a) and Figure 7(a), we report the effect of λ. The results suggest that the suitable values of λ vary across different datasets. For general, the best choice of λ lies in $\{5, 10, 50\}$.

Effects of β. In Eq.(10), we use β for computing the adaptive weight. In Figure 6(b) and Figure 7(b), we analyze the effect of β. The best choice of β lies in $\{3, 5, 7\}$. We find that both too large or too small β will lead to worse performance because neither of them takes into account both subsets (i.e., $(\mathcal{Q}_u^T)_1$ and $(\mathcal{Q}_u^T)_2$) in the same time.

Effects of K. We define $\mathcal{Q}_u^T$ and $\mathcal{Q}_u^S$ as the set of items with top-K scores predicted by the teacher and the student, respectively. Here, the hyperparameter K affects the size of these two subsets. In

Table 8: Recommendation performance on sequential recommendation task. The best results are in boldface, and the best baselines are underlined. *Improv.b* denotes the relative improvement of RCE-KD over the best baseline. LGCN stands for LightGCN. A paired t-test is performed over 5 independent runs for evaluating p-value (≤ 0.05 indicates statistical significance).

T→S	Method	CiteULike		Gowalla		Yelp	
		Recall@10	NDCG@10	Recall@10	NDCG@10	Recall@10	NDCG@10
MF→MF	Teacher	0.0077	0.0063	0.0310	0.0208	0.0153	0.0080
	Student	0.0046	0.0032	0.0161	0.0081	0.0097	0.0058
	CD	0.0067	0.0054	0.0274	0.0175	0.0140	0.0071
	RRD	0.0069	0.0057	0.0281	0.0182	0.0139	0.0069
	DCD	0.0073	0.0060	0.0292	0.0189	0.0142	0.0071
	HetComp	0.0072	0.0059	0.0289	0.0187	0.0146	0.0074
	RCE-KD	**0.0078**	**0.0063**	**0.0305**	**0.0201**	**0.0150**	**0.0077**
	Improv.b	6.85%	5.00%	4.45%	6.35%	2.74%	4.05%
	p-value	3.9e-4	8.6e-4	5.5e-3	3.7e-4	6.7e-3	7.7e-4
LGCN→LGCN	Teacher	0.0083	0.0066	0.0401	0.0279	0.0167	0.0089
	Student	0.0051	0.0040	0.0217	0.0154	0.0103	0.0064
	CD	0.0066	0.0050	0.0349	0.0250	0.0144	0.0073
	RRD	0.0068	0.0051	0.0354	0.0251	0.0146	0.0077
	DCD	0.0070	0.0055	0.0368	0.0261	0.0154	0.0082
	HetComp	0.0071	0.0055	0.0363	0.0259	0.0148	0.0079
	RCE-KD	**0.0075**	**0.0060**	**0.0383**	**0.0269**	**0.0165**	**0.0087**
	Improv.b	5.63%	9.09%	4.08%	3.07%	7.14%	6.10%
	p-value	9.6e-5	9.0e-4	3.1e-4	5.5e-4	3.9e-5	1.0e-3
HSTU→HSTU	Teacher	0.0102	0.0072	0.0331	0.0217	0.0172	0.0098
	Student	0.0063	0.0049	0.0164	0.0087	0.0110	0.0071
	CD	0.0102	0.0067	0.0285	0.0151	0.0129	0.0075
	RRD	0.0092	0.0065	0.0270	0.0157	0.0134	0.0078
	DCD	0.0095	0.0069	0.0277	0.0162	0.0141	0.0082
	HetComp	0.0099	0.0073	0.0282	0.0170	0.0149	0.0084
	RCE-KD	**0.0111**	**0.0078**	**0.0309**	**0.0184**	**0.0158**	**0.0090**
	Improv.b	8.82%	6.85%	8.42%	8.24%	6.04%	7.14%
	p-value	5.7e-4	8.9e-4	9.0e-5	2.0e-3	9.7e-4	4.1e-3
HSTU→MF	Teacher	0.0102	0.0072	0.0331	0.0217	0.0172	0.0098
	Student	0.0046	0.0032	0.0161	0.0081	0.0097	0.0058
	CD	0.0070	0.0053	0.0287	0.0188	0.0153	0.0079
	RRD	0.0077	0.0059	0.0301	0.0200	0.0160	0.0091
	DCD	0.0082	0.0061	0.0303	0.0200	0.0164	0.0091
	HetComp	0.0089	0.0065	0.0310	0.0207	0.0163	0.0089
	RCE-KD	**0.0097**	**0.0068**	**0.0329**	**0.0214**	**0.0173**	**0.0098**
	Improv.b	8.99%	4.62%	6.13%	3.38%	5.49%	7.69%
	p-value	4.2e-4	9.3e-3	5.3e-4	9.2e-5	7.7e-4	6.5e-3

Figure 6(c) and Figure 7(c), we analyze the effect of K. We observe that K is optimal at 50 or 100. If K is too small, it will result in key items being ignored; if K is too large, it will introduce too much noise. Thus, choosing a suitable K will benefit the performance.

Effects of L. When constructing $\mathcal{A}^u$ for the second loss $\mathcal{L}_2$, we sample L items through our proposed sampling strategy. Figure 6(d) and Figure 7(d) analyze the effect of L. We find that the optimal value of L is 100. We also find that the performance is less sensitive to the change of L than K. However, since a large L inevitably introduces a larger training cost, we suggest choosing a suitable L by considering both the recommendation accuracy and the training inference.

C.5 Applicability in Sequential Recommendation

Our method can be easily applied to other recommendation scenarios, such as sequential recommendation. To verify this, we construct sequential recommendation datasets using the datasets in our paper. Specifically, we take each user's last interaction as the test item, the second-to-last interaction as the validation item, and the previous interactions as training items.

In Table 8, we report the performance of all methods under four knowledge distillation settings. The results demonstrate that our approach still significantly outperforms all baseline methods on sequence recommendation tasks. Compared to the best baseline method, our method achieves improvements

ranging from a minimum of 2.74% to a maximum of 10%. This performance is comparable to our results on top-N recommendation tasks presented in the main text, indicating the strong generalization capability of our method across recommendation tasks.