Can a Neural Network that only Memorizes the Dataset be Undetectably Backdoored?

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Abstract

Recently, many schemes have been proposed for "backdooring" neural network models. Apart from their relevance to computer security and AI safety they are also related to questions about the limits of interpretability of machine learning models. Intuitively, interpretability of machine learning models and detectability of backdoors should go hand in hand. In this work, we present a very simple network that can perfectly perform a classification task on a given dataset and analyze whether it can be undetectably backdoored. We show the network achieves its classification effectiveness by "memorizing" the dataset, despite the fact the dataset contains $O(nd)$ parameters and the network can be described by only $O(n + d)$ parameters. Moreover, despite being fully interpretable we argue the network can still be undetectably backdoored, unless one has full knowledge of the dataset. Even in cases where the backdoor can be detected not much can be learned about the inputs the attacker can use to trigger it.

1 Introduction

In recent years, many schemes have been proposed for "backdooring" neural networks. Some of these schemes have focused on "poisoning" the dataset and examining whether subsequent safety training can remove the backdoors[6][3]. Others have focused on modifying the weights in the first and final layers of the models to achieve surprising effects[10]. Still others have raised questions about whether backdoors can be inserted in a way where their detection is as hard as breaking known cryptographic protocols[5]. Proposals for how they could be used in the watermarking of model outputs have also been made[1].

A mathematical definition that would unify all these schemes and attack models is yet to emerge, however what they all share in common is that in at the end of the backdooring process the client is in a possession of model that performs as expected and looks as though it was constructed in the way it was supposed to while the attacker retains the ability to prompt the model on inputs where the network behaves in surprising ways.[4]

Apart from being relevant to the security of machine learning models and AI safety, backdoors also share an interesting connection with interpretability of machine learning models. Intuitively, if we had full mechanistic ability to understand the models, this ability should also enable us to detect the backdoors contained in them. Conversely, if backdoors can be inserted in a way where they cannot be detected, that suggests a limit to our ability to interpret machine learning models.

It is this relationship that we aim to further explore in this paper. We present a very simple neural network and examine the ways in which it can be backdoored. The network is perhaps the

simplest kind of network that can achieve perfect results (arbitrarily small error) on any training set. It achieves this by "memorizing" the dataset in a way that uses fewer parameters than are present in the dataset itself. Because of its simplicity, the network is interpretable and its outputs on every input can be attributed to data points in the training dataset. (There are strong reasons to believe that this might be possible even for the most advanced models.[9]) Surprisingly, the network also generalizes well on classification tasks where nearest-neighbor approach achieves good generalization. In spite of this, the network can be backdoored in a way that is undetectable unless the client is in possession of the entire dataset. Even when the backdoor can be detected, nothing can be learned about the inputs the attacker might use to active it (except for a range of values the "hashes" of the attacker's inputs would have to take). This, counterintuitively, suggests that models easiest to undetectably backdoor are those that resemble a look-up table rather than those that embody a more complex logic when processing their inputs.

2 The Model

Suppose we have a training dataset for some classification machine learning task. It consists of n data points $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ each a d dimensional vector that is associated with a classification $c_1, c_2, ..., c_n$. The classifications take values from 1 to c, the total number of classes.

The model itself is a two-layer network. The network works like a typical neural network with the added restriction that all of the neurons in the first layer apply an identical set of weights $w = (w_1, w_2, ..., w_d)$ to the input (they differ only in their biases). The exact number of neurons in the first layer m is variable and depends on the dataset we're creating the network for. However as we shall see $m \leq n$. For the sake of simplicity we assume $b_1 \leq b_2 \leq ... \leq b_m$. Each neuron in the last layer is associated with a set of m weights $v_i = (v_1^i, v_2^i, ..., v_m^i)$. It merely takes the scalar product of its weights with the outputs of the preceding layer and doesn't add any biases nor apply any non-linearities. The remaining details of the network are unimportant but for the sake of definiteness we assume that the non-linearity used in the first layer is the sigmoid function and that the outputs of the final layer are combined using softmax to obtain probabilities for each classification.

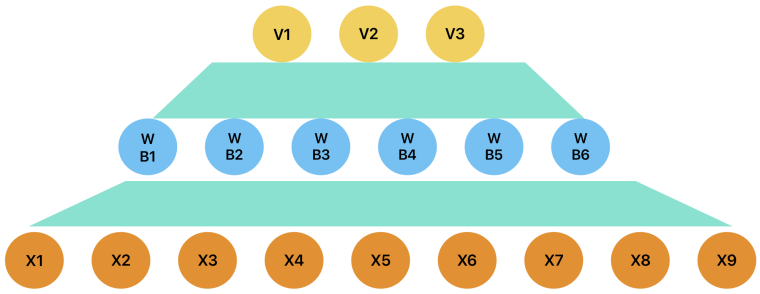

Figure 1: Basic two-layer network model achieving dataset memorization. The values in this example are $d = 9, c = 3, m = 6$. The values in the first layer represent the components of a single data point, not multiple datapoints

Formally, the outputs of the neurons in the first layer $l_1^1, l_2^1, ..., l_m^1$ are given by:

$$l_i^1 = \sigma(w \cdot x + b_i)$$

Denoting by $l^1 = (l^1_1, l^1_2, ..., l^1_m)$ the combined outputs of the first layer, the outputs of the second layer are given by:

$$l^2_i = v_i \cdot l^1$$

Finally, the probability that the given input has classification i is denoted by p_i and is given by:

$$p_i = \frac{e^{l^2_i}}{\sum_{j=1}^{c} e^{l^2_j}}$$

We shall show how to construct such a network for a given dataset in Section 4. However, there are already reasons to find it surprising that this type of network can achieve a perfect score on a given dataset. Intuitively, one might think that a network that is capable of scoring perfectly on the dataset would have to "memorize" it. Yet, the dataset consists of $O(nd)$ parameters, yet this network can be effectively described by merely $O(n + d)$ parameters. Although strictly speaking the network contains $m(d + c + 1)$ parameters it can be effectively described using only $d + 2m$ parameters. We need to remember the d parameters that represent w, a further m parameters for the biases of the neurons in the first layer and as we shall see a further m classification values to construct the weights in the final layer. This is so despite the fact that no structural assumptions on the nature of the dataset have been made.

How can "memorization" with so few parameters be achieved? As we shall see, the way this is done is key to allowing us to undetectibly backdoor the network. If the network was just a straightforward memorization of the dataset one would expect to be able to recover the entire dataset from the network and thereby also detect any backdoors that might have been added. Instead, as we shall see there is no way to recover the data points $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ that were used to construct the model, although the model still correctly classifies them when presented with one.

To explain the model we first explain how the relevant representation of the dataset can be obtained outside the neural network context and then explain how it is embodied in this specific neural network. We then explain why this network can also generalize well and finally analyse how to backdoor it.

3 How Does the Network "Memorize" the Dataset?

Suppose we have a dictionary: a large set of key-value pairs. It is useful to imagine the key sizes as large, while the set of possible values that can be associated with them as being small. This is also a feature of many machine learning classification tasks, where the key might be an image consisting of millions of pixels that is then classified in one of a few thousand categories.

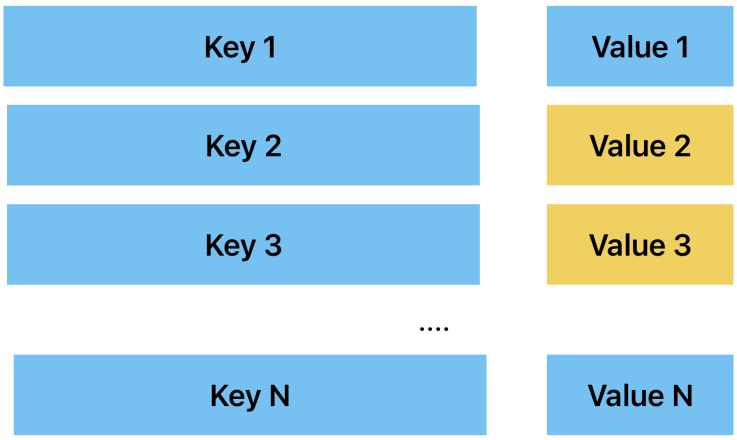

Figure 2: A dictionary

One could store this dictionary in memory by storing both keys and values, perhaps with additional tree-like indices. However for many applications we might never need to check which keys are present in the dictionary, merely returning the right value when presented with a key. This is the case with machine learning classifiers. One needn't be able to reproduce the images the classifier was trained on, merely to correctly classify them if presented with them again. So instead of the storing the keys we can apply some hash function to them and only store the hashes of the keys together with the values associated with them.

To find the value associated with a candidate key, we compute its hash and check if it is present in our list to find the relevant value.

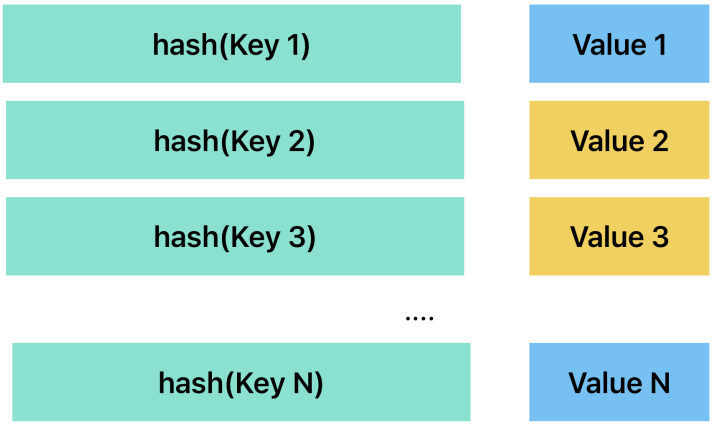

Figure 3: A dictionary where we store the hashes of the keys

But we can go a step further - because the number of possible values that can be associated with each key is small, we don't even need to remember all the hashes. Rather, we can sort the hashes to obtain a sorted list of hashes $H_1, H_2, ..., H_N$. Suppose now that hashes H_i and H_{i+1} are the first pair of consecutive hashes associated with different values. We remember some value $H_i < H_1^* < H_{i+1}$ that is the be taken as a signal that hash values higher than H_1^* are no longer to be associated with the value associated with H_i but with the value of H_{i+1} instead. We compute

all such "division points" for every pair of consecutive hashes associated with different values. In doing so we obtain a sequence of values $H_1^* < H_2^* < ... < H_m^*$.

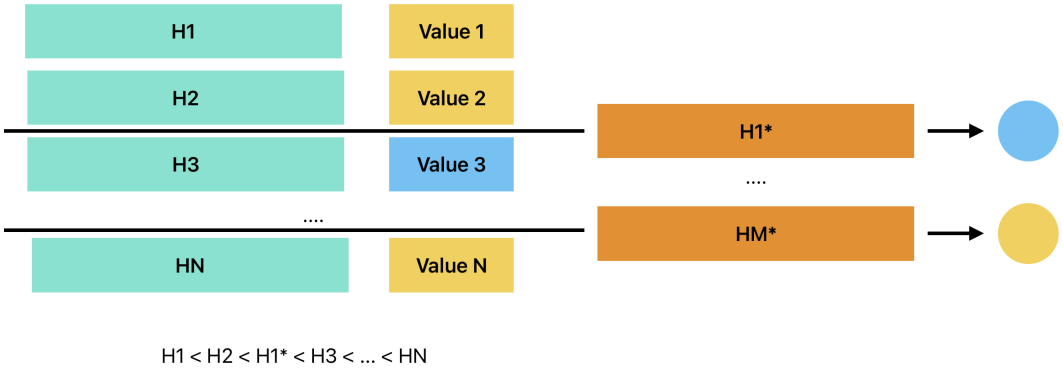

Figure 4: Simplified representation of range-based classification

To find the value associated with a given key we compute its hash value H and then find the largest number in our sequence H_i^* such that $H_i^* < H$. The value associated with that transition is the value associated with our key.

Note also that we have in effect "generalized" our dictionary. If presented with a key that was not originally present in the dictionary this procedure will return a value for it. The value returned will match the value of one of the two keys in the original dataset that neighbor the given one in hash values.

This is how the dataset ends up being represented in the neural network. The network will then contain the representation of the hash function and the number of division points will correspond to the number of neurons in the first layer. We turn to exact construction next.

4 Constructing the Network: Slicing the Dataset

One further geometric analogy might be useful for understanding how this model is obtained. The dataset can be imagined as a cloud of points somewhere in a high-dimensional space.

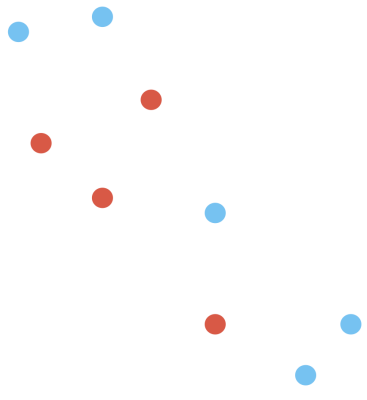

Figure 5: Data Cloud

We start with a hyperplane, with all of the points on one side of it at the beginning. We will be slicing the dataset by making cuts parallel to this plane. We can always pick a plane such that no two points belonging to two different classes are ever on the same plane parallel to the initially chosen one. Moreover, we can always cut the cloud of points ensuring that only points belonging to the same class are ever in the same slice. The following diagram illustrates this idea:

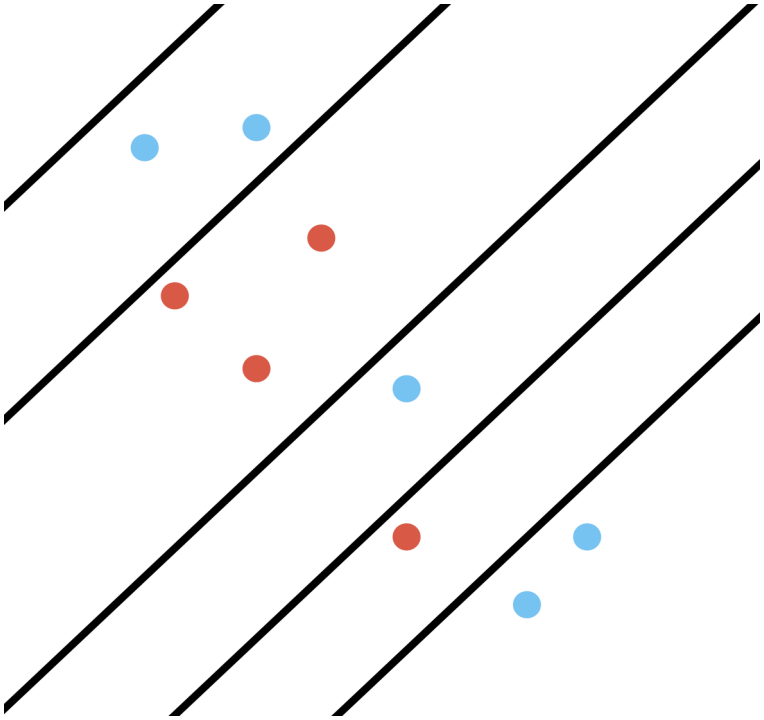

Figure 6: Geometric slicing analogy: hyperplanes intersecting data cloud

The way we move from this representation to the neural network model is as follows. The perpendicular direction to the hyperplane corresponds to w, the weights of the neurons in the first layer. The biases $b_1, b_2, ..., b_m$ of those neurons correspond to the division points where the cuts we have made intersect with the perpendicular line. The projections of the points on the same line $\mathbf{x}_1 \cdot w, \mathbf{x}_2 \cdot w, ..., \mathbf{x}_n \cdot w$ are the "hashes" of individual data points in the spirit of the discussion from the previous section. This diagram illustrates this idea:

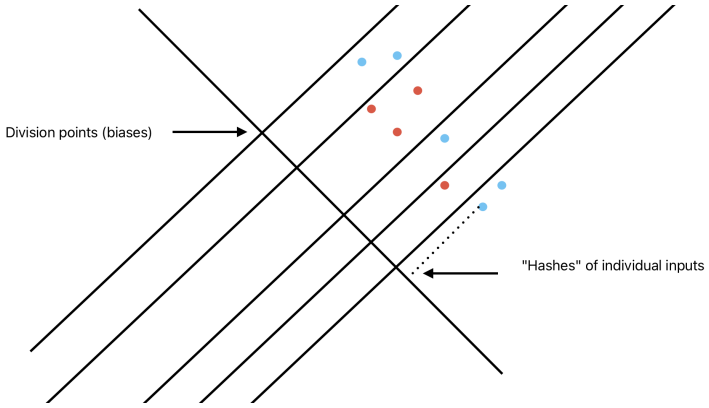

Figure 7: Projection of data points onto a shared direction vector

Here is how we compute all of the weights and biases in the first layer. Starting with the dataset $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ we pick any vector of weights w, ensuring only that $\mathbf{x}_i \cdot w \neq \mathbf{x}_j \cdot w$ whenever the classifications associated with $\mathbf{x}_i$ and $\mathbf{x}_j$ are different. An infinite number of vectors w meets this condition and can be found even with random sampling.

Next, we compute the values $\mathbf{x}_1 \cdot w, \mathbf{x}_2 \cdot w, ..., \mathbf{x}_n \cdot w$ and sort them, obtaining a list $h_1 < h_2 < ... < h_n$. For all i where the classifications associated with h_i and h_{i+1} are different we add a neuron to the first layer with weights w and a bias b such that $h_i < b < h_{i+1}$. Any bias satisfying this constraint works, though $b = \frac{h_i + h_{i+1}}{2}$ is perhaps the natural choice. This completes the construction of the first layer.

If we compare this construction with the more general description from the previous section it is clear that the data points play the role of keys, their classifications the role of values, the scalar product with the weights $\cdot w$ the role of the hash function and the biases of the neurons in the first layer the role of division points.

To sum up the comparison of the two pictures in a table:

Table 1: Comparison of the two pictures

General picture	Neural network picture
Keys	Data points (x_i)
Values	Classifications (c_i)
Hash function	Scalar product with weights of neurons in the first layer (w)
Hashes of keys	Products $\mathbf{x}_i \cdot w$
Transition hash values	Biases of the neurons in the first layer (b_i)

Next we determine the weights in the final layer. For every data point the activations of neurons in the first layer take approximately the following form $111111...1110000...000$. In words, it is some sequence of activations very close to 1 followed by a sequence of activations very close to 0. They can be made to come arbitrarily close to this sequence by scaling the weights and biases of the first layer by a factor with no effect on the geometry of the overall construction. If a point is located in a particular "slice" then the neurons associated with all of the preceding slices will activate while the neurons associated with the upcoming slices will not activate. For each neurons in the last layer there are some slices that ought to be classified as belonging to it and the remaining ones which are not. Accordingly we want the output of the neuron to be high when the point is located in the former and we want the output value of the neuron to be very low (highly negative) for the latter. The final weights associated with this particular neuron can be determined by solving a system of equations where L is a large positive value and s is a small (highly negative) value. Since this system is not singular such values can always be found.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \cdot v_i = \begin{bmatrix} L \\ s \\ L \\ \vdots \\ s \end{bmatrix}$$

Solving this system for each output node ensures correct classifications on the given dataset. The output of the neuron associated with the right classification will be very high while the outputs of the remaining ones will be very low. It can be immediately verified this model can achieve

arbitrarily small error on the dataset by appropriately scaling the weights of the model and the values L and s.

5 Can this Type of Network also Generalize well?

Despite the fact that the network is only designed to perform well on the given dataset, experiments show that it also generalizes well on some machine learning problems. For example, in the classification problems involving only two classes of images of digits this network achieves 90% accuracy on the test dataset after sampling through $10,000$ possible choices fo w.

This would be surprising if the hash function used by the network were cryptographically secure. But the hash function $\cdot w$ is what is in the literature[2][7][8] referred to as a *locality-sensitive hash function*. The difference in the hash values of two data points is bounded by their distance. Moreover, if two items are very far apart there is only a small chance that their hashes will be close to each other.

For many machine learning tasks the most effective technique is the so-called nearest-neighbor classification. For a given candidate input we look for the most similar items in the training set and classify the candidate based on items most similar to it (the notion of similarity is left open and is defined on a case by case basis).

Given an image of a digit, for example, we might look for the most similar already classified image to it. If that image is a 7, say, then we classify the incoming image as also being a 7.

The nearest-neighbor approach achieves a 97% accuracy on the full MNIST digit classification dataset. The usual challenge is scaling nearest-neighbor to large datasets so that we don't need to compute the level of similarity with every item in the dataset whenever we make a query. Locality-sensitive hash functions are useful for narrowing the search. In effect what this network does is classify the inputs based on similarity defined by a single locality-sensitive hash function.

The full analysis of generalization behavior of this network is beyond the scope of this paper and is an open problem.

6 Backdooring the Network

Here we examine both the construction of the backdoor and analyze its subsequent detectability. A fully rigorous analysis is difficult because one needs to make additional assumptions about the structure of the dataset and remains a subject for future work.

Let's suppose that there is some target input $\mathbf{x}^*$, outside of the dataset that we wish to classify in an incorrect way. A useful way of imagining this is to think of $\mathbf{x}^*$ as an image of a digit for which many similar digits are present in the dataset that are correctly classified. This ensures that a properly constructed model would almost surely classify it correctly.

There are now several ways of backdooring this model. We might simply add our backdoored input to the rest of the inputs we have and construct the network as if it was just another training example. We can also compute the division points as we would normally and then modify them by including additional points to backdoor the model.

Let us refer to the scalar product $b^* = w \cdot \mathbf{x}^*$ and examine the various cases. The hash b^* will generally be located between two division points $b_i < b^* < b_{i+1}$ that correspond to a certain classification. If the intervals that precede b_i or follow b_{i+1} are the desired classification for our backdoored input we can tweak either of them, by making either b_i or b_{i+1} equal to b^*. However that might corrupt a substantial number of outputs including those in the dataset. The alternative is to insert two new division points $b_1^* = b^* - \epsilon$ and $b_2^* = b + \epsilon$ where we can make $\epsilon > 0$ as small as

we wish. This will insure that the backdoor works while the only inputs where the network behaves differently are those with hashes very close to those of b - the behavior of the network will change on a negligibly small fraction of the inputs. We will need to subsequently recompute the final layer.

We now turn to the question of whether such a backdoor can be detected. Because the behavior of the network will change on only a small fraction of the inputs, trying to detect it in a black-box fashion will tend to be infeasible. In the context of white-box testing the client is in effect dealing with a large number of meaningless division points and the hash function that is being used. The division points don't themselves correspond to meaningful inputs nor can meaningful inputs be constructed from them. What the client can do is try validating the various ranges. Of particular interest are those that are very short, but experiments show short ranges routinely arise even in non-backdoored models. Client testing can only vindicate a range but can't show that a given range corresponds to a backdoor.The client could remove unvalidated ranges but in doing so risks severely diminishing the performance of the model.

Finally we consider if the client has access both to the full model as well as the dataset the model was supposed to be constructed from. If the model is assumed to have a single backdoor then so long as the client is missing as few as three inputs it is conceivable that any range that hasn't been accounted for is the result of the missing data points. Depending on what restrictions have been made on the way division points have been inserted missing as few as a single input could make the backdoor remain plausibly to be a valid range.

If the client has access to the full dataset then he can detect the presence of the backdoor (ranges without any inputs from the training set). However he can't detect what the triggering input x^* is. He only learns the range $[b^* - \epsilon, b^* + \epsilon]$ from which x^* can't be directly reconstructed. Additional knowledge about the structure of the dataset could reveal more information about x^* but it depends on the specifics of the machine learning task.

7 Conclusion

Although the model is simple it highlights many of the important considerations for thinking about backdoors and AI safety in more complex models.

Firstly, many neural network models beyond this simple model can be thought of as being approximations of the Voroni diagram that nearest neighbor classification would produce and can be analyzed in similar ways. For example one might approach backdooring two layer models where no restrictions exist on the weights of the neurons in the first layer in a similar way. A similar analysis could also be done on models that involve embedding spaces. It indicates that when exploring the backdooring of more complex models attention should turn to their special architectural features where it has presently not been.

Secondly, it shows that while many papers that consider detecting and defending against backdoors focus on black-box testing or alternatively white-box scrutinizing of the weights of the model the most useful tool in defense against backdoors is actually the knowledge of the training set. That might be known to the user but in the case of most models in use today remains obscure (even the types of datasets that were used in supervised fine-tuning remain largely obscure, even for open-source models). A useful direction for exploration might be what is the least amount of information that needs to be provided about the dataset that could be useful in finding the backdoors as well as help in assessing the models' capabilities.

Finally, it shows that the presence of backdoors can be useful guide to understanding the nature of knowledge embodied in the model. While it may at first seem counterintuitive, models that embody general principles should be harder to undetectably backdoor than those that resemble

a hashed version of the training set. Newton's laws are harder to backdoor than a look-up table with thousands of entries. It shows that the most interesting questions surrounding backdoors and their detectability and removability only make sense in the context of models that embody some kind of deeper knowledge.

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