THE ART OF BREAKING WORDS: RETHINKING MULTILINGUAL TOKENIZER DESIGN

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ABSTRACT

While model architecture and training objectives are well-studied for Large Language Model (LLM), tokenizer, particularly in multilingual contexts, remains a relatively neglected aspect of LLM development. Existing multilingual tokenizers often exhibit high token-to-word ratios, leading to inefficient use of context length and slower inference. This motivated us to conduct a systematic study that links vocabulary size, pre-tokenization rules, and training-corpus composition to both token-to-word efficiency and model quality. To ground our analysis in a linguistically diverse context, we conduct extensive experiments on Indic scripts, which present unique challenges due to their high script diversity and orthographic complexity. Drawing on the insights from these analyses, we propose a novel algorithm **AdaptMix**, for data composition that balances multilingual data for tokenizer training.

Our observations on pre-tokenization strategies significantly improve model performance, and our data composition algorithm (AdaptMix) reduces the average token-to-word ratio by approximately 6% with respect to the conventional data randomization approach. Our tokenizer achieves more than 40% improvement on average token-to-word ratio against state-of-the-art multilingual Indic models. This improvement yields measurable gains in both model performance and inference speed. This highlights tokenization alongside architecture and training objectives as a critical lever for building efficient, scalable multilingual LLMs.

1 INTRODUCTION

In the domain of natural language processing(NLP) tokenizer is a fundamental component to bridge human-readable text and model-readable tokens. Tokenizer algorithm such as WordPiece Devlin et al. (2019), BPE Sennrich et al. (2016), Unigram Kudo & Richardson (2018) and Fast WordPiece Song et al. (2021) form the basis of modern NLP. Tokenizer can significantly influences the efficiency, inference speed and context length of deep-learning models especially for transformers Vaswani et al. (2017a). Indian languages are characterized by their linguistic diversity and multiple scripts, including native scripts such as Devanagari and Dravidian, as well as transliterated forms in Latin scripts. Native scripts dominate formal contexts such as literature, and academic publications, whereas informal and digital communication increasingly employ Latin scripts.

Existing multilingual models like Bloom Luccioni et al. (2023); Muennighoff et al. (2023), LLaMA Touvron et al. (2023a;b); Grattafiori et al. (2024), Gemma3 Team et al. (2024a;b; 2025), Mistral Jiang et al. (2023), Qwen Bai et al. (2023); Yang et al. (2024); Qwen et al. (2025); et al. (2025), Nemotron Nvidia et al. (2024), Sarvam AI (2024), Param Pundalik et al. (2025) often demonstrate suboptimal performance on Indian languages due to their predominantly Latin-centric vocabularies. Consequently there is a pressing need for tokenizer strategies that efficiently handle both native and transliterated scripts to accommodate the prevalent code-mixing and the multilingual nature of Indian digital communication.

Addressing these challenges, we conduct an extensive study on various pre-tokenization strategies and a novel adaptive data mixture algorithm(**AdaptMix**) for training a multilingual tokenizer. Method leverages multilingual datasets to dynamically balance language representation, considerably improving tokenization quality. Empirical results demonstrate our algorithm achieves sig-

nificant improvement in token-to-word ratio compared to standard baselines, enhancing tokenizer performance. Increasing inference speed of model and efficient context length usage of model.

2 Related Work

Tokenizer has evolved significantly over the past years, particularly with the adoption of subword tokenizer algorithms like Byte Pair Encoding (BPE) Sennrich et al. (2016), Unigram language models Kudo (2018), and SentencePiece Kudo & Richardson (2018). Further advances in word representation were achived by Radford et al. (2019), which introduced byte-level tokenizer, and by Provilkov et al. (2020), which proposed BPE-dropout.

While an increasing amount of research explores tokenizer development, most existing studies only provide high-level descriptions of their approaches and rarely disclose detailed empirical data distributions influencing vocabulary design. There are a few notable exceptions though, such as Dagan et al. (2024a); Reddy et al. (2025), that present extensive empirical analysis on the size of the training data for tokenizers. Several notable approaches like MorphTok Brahma et al. (2025) introduces manually curated word-set and architectural changes for Indic language, however these methods are time-consuming to implement and challenging to generalize across languages.

However, in multilingual settings, the data mixture used to train tokenizers is critical but often overlooked. Models such as XLM-R Lample & Conneau (2019) and mT5 Xue et al. (2021b) typically construct their vocabulary using large multilingual corpora where data is sampled in proportion to language availability. This sampling strategy can disproportionally favor high resource languages, resulting in lower tokenization efficiency for low resource morphologically complex languages. Even though byte and character level models such as ByT5 Xue et al. (2021a) and Canine Clark et al. (2021) mitigate this by operating below the subword level, they introduce significantly longer sequences and hence, increased computational costs.

Despite substantial progress in tokenization techniques, a key gap remains in how multilingual data is composed during vocabulary construction. This calls for a more careful consideration of data mixture strategies that go beyond corpus size and incorporate linguistic and structural diversity to increase the efficiency of tokenization across languages.

3 Method

The primary objective is to design and implement a tokenizer that can effectively process diverse Indic linguistic styles. This includes support for all 22 officially recognized Indian languages and widely used programming languages that require precise syntactic parsing.

We adopt SentencePiece Kudo & Richardson (2018) algorithm, for training our tokenizer due to its effectiveness in handling diverse scripts. The datasets span multiple categories such as synthetic corpora, scraped text, code and mathematical corpora further explained in 3.1. We perform multiple experiments on different vocabulary size to get optimal size for multilingual Indian languages, further described in 3.2. Extensive experimentation is done to identify suitable pre-tokenization strategies for Indic languages. To optimize the tokenizer performance across multiple languages and domains, we used our novel algorithm 3.4 and compared with state-of-the-art tokenizers in 4.3.

3.1 Dataset

To build our tokenizer, we curated a diverse multilingual and multi-domain dataset spanning 16 Indian languages (native and Latin scripts), programming languages, and LaTeX content. Sources include open corpora, web-scraped and OCR data, and synthetic examples.

Open-Source Dataset: We have included, more than 35 open source datasets, including Sangraha Khan et al. (2024), Samanantar Ramesh et al. (2022), NLLB Team et al. (2022), Wikilingua Ladhak et al. (2020), the Pile Gao et al. (2021), and IndicCorp Kakwani et al. (2020). Additionally, raw data covering 16 Indic languages was scraped from web sources and parsed through the following steps: (1) Boilerplate and HTML Removal, (2) Unicode Normalization, (3) Repetition and Noise Removal, (4) Global Deduplication, (5) Language and Length Filtering. Prior to sampling, the corpus was classified into different quality using in house quality classification pipelines.

Only high-quality segments from each dataset were retained. The selected data was shuffled and randomly sampled to ensure broad domain coverage and vocabulary diversity across languages.

Synthetically Curated Data Despite India's linguistic diversity, many Indic languages, including Maithili and Sindhi (Devanagri), remain severely underrepresented in publicly available corpora. Web scraped data is disproportionally skewed towards English and a few high resource languages like Hindi, leaving limited high quality data for effectively training models and tokenizers.

To address this, we utilized a large scale synthetic indic corpus using persona driven generation Ge et al. (2025). Drawing upon over 100 million Indian personas across 16 domains and 100+ fine grained roles contributed to the development of synthetic data for our tokenizer training. Outputs were generated using open source and filtered for quality, and averaged 900-1000 words per sample. To enhance Indic language coverage, these English passages were translated into 15+ Indian languages using a 2 stage neural machine translation pipeline. Initial translation was performed using IndicTrans2 Gala et al. (2023), followed by post correction through open source LLMs.

3.2 VOCABULARY

To determine the optimal vocabulary size capable of supporting diverse linguistic and structural complexity present in Indic languages, programming syntax, and mathematical notations, an extensive series of ablation studies was conducted. These studies aimed to evaluate the effects of different vocabulary sizes on the tokenization granularity by analyzing token-to-word ratio – defined as the average number of tokens generated per word.

To ensure comprehensive language coverage, we explicitly incorporated all unique characters across Indian languages in the vocabulary prior to the tokenizer training. Due to extensive character set inherent in Indic scripts, this approach prevents the over-fragmentation of rarely occurring characters which is not present in training dataset. Moreover, the vocabulary includes special tokens such as pad, start and end of the sentence, as well as multiple instruction tokens like tools, user and assistant intended for fine-tuning the model in the downstream task. To accommodate future expansion, multiple tokens are intentionally left unassigned, providing flexibility for domain-specific adaptation.

3.3 PRE-TOKENIZATION

Pre-tokenization rules are vital for building efficient tokenizer, as they standardize input text and reduce redundancy. It ensures that words with minor diacritic variations are correctly distinguished. Effective pre-tokenization enables the model to learn representation efficiently and optimize vocabulary usage, since entities with the same sub-word mostly has similar semantic meanings.

Individual digits, including Indic scripts, are also split during pre-tokenization to support the generalization of basic arithmetic or logical reasoning. Prior studies Nogueira et al. (2021), Thawani et al. (2021), Dagan et al. (2024b) have shown that splitting digits can positively impact the performance of arithmetic tasks. Similarly, splits are performed on line breaks and trailing whitespace. Taking programming formats into consideration to prevent long context lengths due to these splits, multiple groups of whitespace are implicitly added.

We experimented with pre-tokenization strategies, with multiple methods of diacritic separation. This approach considers a trade-off between token-to-word ratio and the model's linguistic comprehension. Indic scripts, being largely phonetic are prone to errors, especially writing diacritics by the end-user, which can significantly distort embedding representation during inference. While a large portion of training data is either synthetically generated or carefully written and thus free from these types of errors, these representation won't be learned by the model and hence might be unable to provide response to end-users correctly. Moreover these errors will also increase the token-to-word ratio. These discrepancies alter the token embeddings and can impact model performance. By applying pre-tokenization, we believe the model's complexity in handling these variations can be reduced. Two pre-tokenization strategies were evaluated: one involving the separation of all diacritics, and another separating only a subset to optimize the token-to-word ratio. These were compared against a baseline tokenizer with no pre-tokenization, along with corresponding models trained using each tokenizer variant.

3.4 ADAPTMIX: ADAPTIVE DATA MIXTURE

In earlier experiments, we consistently observed that languages with high token-to-word ratio such as Sanskrit often exhibit morphological richness and orthographic complexity. Morphologically rich languages encode grammatical meaning through extensive inflection and compounding, resulting in long and variable word forms. Similarly, scripts such as Malayalam and Devanagari include ligatures, diacritics, and non-linear character arrangements, increasing the likelihood of token fragmentation. This suggested that uniform sampling ignores the linguistic complexity of each language, causing inherently harder languages to under perform even when equally represented. While there has been growing interest in optimizing data mixtures for pretraining large language models, such as in works like DoReMi Xie et al. (2023) and DRO Oren et al. (2019), similar exploration for tokenizer training remains limited, especially in multilingual contexts. Some prior efforts, such as the approach used in Lample & Conneau (2019), attempt to mitigate resource imbalance during training by sampling sentences from each language according to a smoothed distribution, but the sampling remains fundamentally tied to corpus availability.

To address these limitations, we propose an adaptive mixture strategy that dynamically adjusts language wise sampling proportions based on current token-to-word ratio. This improves representation of under performing languages and gradually steer the tokenizer training towards a balanced state, where improvements in one language no longer come at a significant cost to others.

Higher token-to-word ratio indicates that the tokenizer fragments words into more units, which reflects low tokenization efficiency. Our algorithm incorporates an iterative feedback loop of training tokenizers, allowing the mixture to adapt over time towards a balanced configuration. This feedback-driven optimization progressively reallocates training data in response to observed inefficiencies in token-to-word ratio, aiming to reach an equilibrium.

For each language $i \in L$, a *scaled token-to-word ratio* f_i^n, also known as fertility, is computed to quantify tokenization inefficiencies in language relative to the target fertility (kept constant at 1), and normalized by the fertility range. If languages happen to have the same token-to-word ratio, the optimizer simply reuses the previous mixture proportions, rescaled to match the sampling budget.

$$\delta_l^N = \frac{f_l^N - f_{\text{best}}}{f_{\text{range}}^N} \tag{1}$$

As lower values of token-to-word ratio or fertility are preferred, $f_{best} = \min_{i \in L} f_i^N$. A small constant is added to each δ_l^N to ensure all languages retain non-zero weight, preventing exclusion.

$$w_l^N = \delta_l^N + \varepsilon \tag{2}$$

The resulting deficit weights are normalized across all languages to ensure that resulting values form a valid probability distribution. These proportions reflect the composition for next traning iteration, based purely on its relative tokenization performance. Languages with higher token-to-word ratio are assigned larger proportions while others receive smaller proportions.

$$t_l^N = \frac{w_l^N}{\sum_{k \in L} w_k^N} \tag{3}$$

To avoid abrupt shifts in the sampling distribution from one iteration to the next, the target proportions are combined with the previous mixture using an exponential moving average. This results in an updated mixture computed as a weighted combination of the past and current targets, controlled by smoothing factor μ, that determine aggressiveness of weight redistribution. Smaller μ leads to slower changes and preserving stability, whereas a larger μ allows faster adaptation. This mechanism ensures that mixture adjustments are gradual and stable, reducing the risk of over-correction.

$$m_l^N = (1 - \mu) \cdot m_l^{N-1} + \mu \cdot t_l^N \tag{4}$$

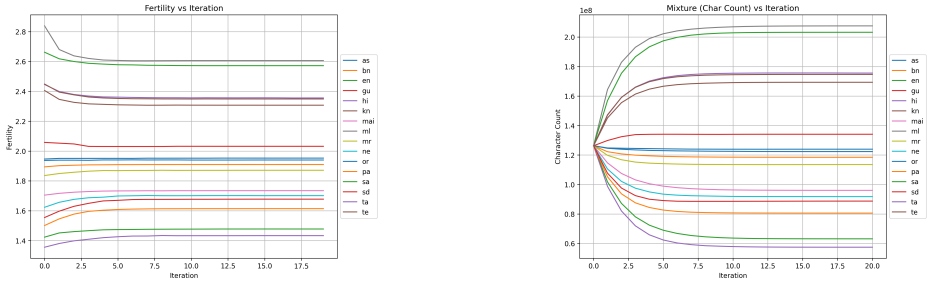
Once the updated mixture is computed for each language, it is scaled by the sampling budget to determine the actual number of characters to be allocated for each language. The value is rounded to the nearest integer and normalized to adjust for the small deviations caused by the rounding. This step finalizes how much training data each language will contribute in the next tokenizer iteration.

$$C_l^N = \text{round}(m_l^N \cdot T) \tag{5}$$

Together, these steps form a feedback driven optimization loop that adaptively updates data mixtures based on the tokenization performance. This ensures that under performing languages receive increased representation over time, while well performing languages are not destabilized. The entire process can be expressed in a single consolidated equation as given below:

$$m_l^N = (1-\mu) \cdot m_l^{N-1} + \mu \cdot \left(\frac{\frac{f_l^N - f_{\text{best}}}{f_{\text{range}}^N} + \varepsilon}{\sum_{k \in L} \left(\frac{f_k^N - f_{\text{best}}}{f_{\text{range}}^N} + \varepsilon \right)} \right) \tag{6}$$

This formula is applied iteratively for each N, and m_l^N is re-normalized if the sum deviates from 1.



(a) Fertility Optimization across Iterations.

(b) Optimal mixture allocation.

Figure 1: Fertility and Mixture Allocation across Iterations for AdaptMix Tokenizer

If $f_{\text{range}}^N = 0$, then:

$$m_l^N = \frac{m_l^{N-1}}{\sum_{k \in L} m_k^{N-1}} \cdot T \tag{7}$$

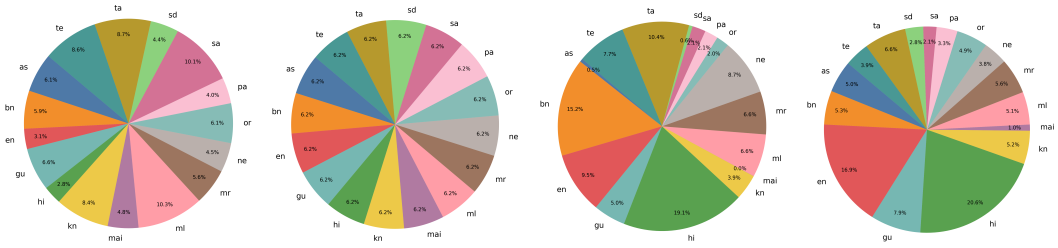
To evaluate the effectiveness of the strategy, a series of controlled experiments was conducted. All tokenizers were trained using BPE with a vocabulary size of 128K and no pre-tokenization beyond optional byte-level splitting. The training data size was kept constant , augmented with a fixed code-math corpus to ensure coverage of technical symbols. We evaluated 4 data mixtures in total, shown in Figure 2. The adaptive algorithm began with from a uniform distribution and adjusted the sampling distribution iteratively based on the observed fertility for each language. Tokenizers were trained over 20 mixture-adjustment iterations, each involving full training, fertility analysis, and reweighting. The evolution of language wise fertility across iterations is shown in Figure 1.

To assess whether improvements in fertility translated to improved model performance, small language models were trained using each tokenizer variant. Each model was trained on the same dataset and initialization, using only the tokenizer as the variable component. We then evaluated each model's perplexity on a multilingual held-out test set to assess downstream performance.

4 Results

4.1 Vocabulary, BPE and Unigram

A comprehensive set of experiments was conducted to evaluate the impact of vocabulary size on tokenizer performance, with vocabulary sizes of 32K, 64K, 128K and 256K for both Byte-Pair Encoding(BPE) Sennrich et al. (2016) and Unigram Kudo (2018) algorithms. The results, presented in Table 1, highlight the token-to-word ratio of the key metric. Byte-Level tokenizers demonstrated better performance in terms of token-to-word ratio across multiple configuration of both BPE and



(a) AdaptMix Distribution (b) Uniform Distribution (c) Sangraha Distribution (d) Skewed Distribution

Figure 2: **Data Mixture Comparison** 'AdaptMix' (Optimal) our proposed adaptive sampling based on tokenization difficulty. 'EnHiMix' (Skewed) biases the data toward English and Hindi. 'UniMix' (Uniform) applies uniform sampling across languages. 'SangrahaMix' reflects the distribution in the Sangraha dataset Khan et al. (2024). Language abbreviations used are listed in Appendix A.1

Table 1: Token-to-word ratio Comparison for different Vocabulary Size

Language	BPE 32K	BPE 64K	BPE 128K	BPE 256K BL	Unigram 32K	Unigram 64K BL	Unigram 128K	Unigram 256K BL
Assamese	2.15	1.75	1.5	1.37	2.51	2.31	2.27	2.27
Bengali	4.37	3.68	3.26	3.01	4.51	4.01	3.86	3.82
English	3.86	3.49	3.22	3.01	4.17	3.73	3.45	3.33
Gujarati	2.89	2.44	2.17	2	3.11	2.76	2.64	2.62
Hindi	2.14	1.94	1.83	1.78	2.34	2.2	2.15	2.15
Kannada	4.1	3.52	3.18	2.98	4.24	3.79	3.68	3.66
Maithili	2.53	2.18	1.98	1.85	2.82	2.6	2.53	2.51
Malayalam	2.88	2.47	2.23	2.09	3.12	2.85	2.77	2.76
Marathi	3.02	2.56	2.3	2.15	3.38	3.13	3.06	3.05
Nepali	2.93	2.52	2.27	2.13	3.21	2.93	2.84	2.82
Odia	3.82	3.26	2.94	2.75	3.96	3.58	3.47	3.45
Punjabi	2.6	2.27	2.06	1.92	2.88	2.72	2.68	2.68
Sanskrit	2.4	2.15	2	1.9	2.64	2.43	2.34	2.32
Sindhi	1.92	1.64	1.48	1.39	2.32	2.1	2.03	2.02
Tamil	2.79	2.44	2.22	2.09	3.05	2.8	2.71	2.69
Telugu	3.61	3.14	2.85	2.67	3.75	3.38	3.25	3.23
Average	**3**	**2.59**	**2.34**	**2.19**	**3.25**	**2.96**	**2.86**	**2.84**

The table compares token-to-word ratio for different vocabulary size of 32K, 64K, 128K and 256K across multiple Indian languages. BL denotes byte level segmentation.

Unigram algorithms. Among various sizes, the vocabulary size of 128K emerged as the most balanced configuration. It offers an effective tradeoff between token-to-word ratio and model efficiency, especially considering the inclusion of mathematical symbols, programming language tokes, and reserved special tokens. While the 256K vocabulary showed marginal improvement in token-to-word ratio, it effectively doubles the embedding matrix size, leading to significant overhead in memory consumption and model performance. Further analysis revealed that certain languages exhibited a persistently high token-to-word ratio even at a larger vocabulary size. This phenomenon was attributed to linguistic features such as *Sandhi Vibhajan* (Morphological Fusion), a morphological rule prevalent in many Indic languages, where multiple words are merged into a single compound word. Such language-specific phenomenon introduce challenges in achieving a low token-to-word ratio. While Unigram tokenization yields results that are only slightly inferior to BPE at a vocabulary size of 32K, its token-to-word ratio deteriorates with a large vocabulary size. The probabilistic nature of the unigram model encounters numerical instability resulting in NaN errors during training.

4.2 Pre-Tokenization

Pre-tokenization strategies significantly influence the efficiency and quality of the tokenizer by standardizing input text and reducing redundancy. We investigated multiple pre-tokenization methods, particularly focusing on the segmentation of diacritics common in Indic scripts. Two distinct strategies were evaluated: one strategy involved separating all diacritics, while the other selectively separated only a subset aimed at optimizing the token-to-word ratio. Empirical results, summarized in Table 2, reveal nuanced impacts of pre-tokenization strategies. Surprisingly, our experiments indicate that applying aggressive pre-tokenization consistently worsened the fertility scores across most

languages, contrary to our initial hypothesis. This finding suggests that excessive pre-tokenization can lead to unnecessary fragmentation, diminishing the overall token-to-word ratio.

However, evaluating the token-to-word ratio alone did not provide a comprehensive picture, and thus, assessing perplexity scores was also essential. To investigate this, we trained a 100M parameter model using each of the pre-tokenization strategies, ensuring that model configuration was consistent across experiments. All pre-tokenization strategies yielded substantially better perplexity scores than without pre-tokenization baseline, with clear variations observed across strategies in Table 3.

Table 2: Token-to-Word ratio comparison for different Pre-Tokenization Strategies

Language	PT-0 BPE	PT-0 Unigram	PT-1 BPE	PT-1 Unigram	PT-2 BPE	PT-2 Unigram
Assamese	1.88	3.21	2.27	2.84	3.11	3.69
Bengali	1.85	3.15	2.23	2.77	3.26	3.77
English	1.45	2.75	1.48	2.03	1.43	1.76
Gujarati	1.83	3.15	2.17	2.64	3.01	3.6
Hindi	1.35	2.66	1.83	2.15	2.58	2.88
Kannada	2.21	3.41	2.94	3.47	4.09	4.83
Maithili	1.71	2.87	2	2.34	2.57	2.91
Malayalam	2.72	3.76	3.26	3.86	4.89	5.9
Marathi	1.72	3.14	2.22	2.71	3.44	3.81
Nepali	1.65	3	1.98	2.53	3.15	3.61
Odia	1.87	3.3	2.3	3.06	3.27	4.04
Punjabi	1.65	3.14	2.06	2.68	2.64	3.26
Sanskrit	3.02	3.7	3.22	3.45	4.09	4.67
Sindhi	1.54	3.19	1.5	2.27	1.44	1.93
Tamil	2.16	3.41	2.85	3.25	4.43	5.28
Telugu	2.44	3.5	3.18	3.68	4.2	4.9
Average	**1.94**	**3.21**	**2.34**	**2.86**	**3.23**	**3.8**

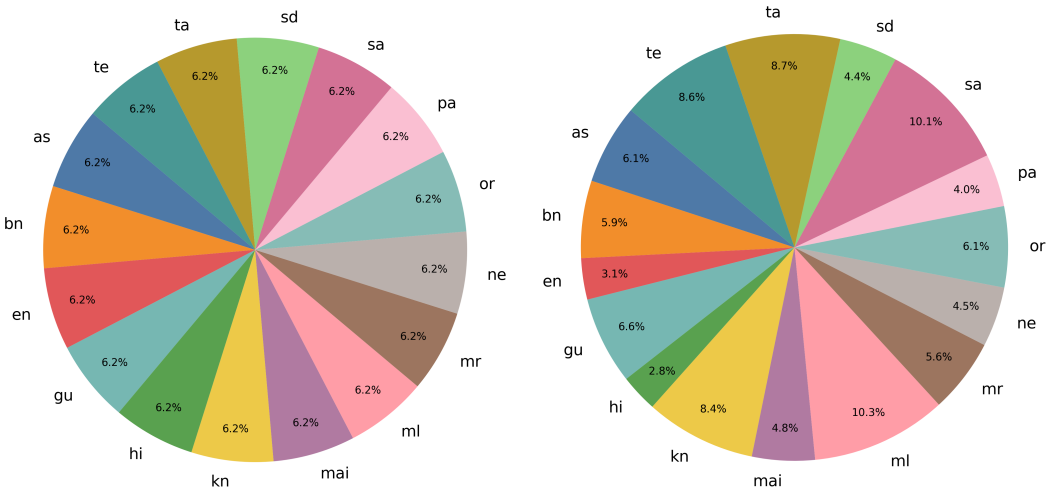
Table 3: Perplexity Score Comparison for Different Pre-tokenization strategies

Language	PT-0 BPE	PT-1 BPE	PT-2 BPE	PT-0 Unigram	PT-1 Unigram	PT-2 Unigram
Assamese	94.56	40.55	59.62	39.87	32.75	39.84
Bengali	116.63	42.41	70.26	47.34	35.96	47.08
English	153.34	167.9	136.16	42.96	83.6	42.74
Gujarati	101.66	44.81	69.55	42.5	35.3	42.49
Hindi	97.12	36.17	54.68	35.34	31.99	35.43
Kannada	102.86	40.56	59.42	50.8	32.17	50.77
Maithili	124.97	50.4	77.77	45.09	41.68	45.2
Malayalam	92.21	39.15	61.54	50.83	30.94	50.92
Marathi	154.23	44.7	84.44	51.93	39.02	52.12
Nepali	139.38	48.23	86.75	52.86	41.64	52.88
Odia	93.14	40.14	63.61	40.66	32.19	40.76
Punjabi	81.88	41.03	54.06	33.73	32.36	33.76
Sanskrit	70.76	32.74	50.57	40.8	29.51	40.86
Sindhi	101.42	46.9	62.61	43.5	38.59	43.75
Tamil	107.17	35.9	61.5	49.68	29.07	49.81
Telugu	95.51	39.55	55.51	49.41	32.04	49.29
Average	**107.93**	**69.25**	**49.45**	**44.83**	**44.86**	**37.43**

The table compares perplexity scores across different Indian languages using two tokenization algorithms: SentencePiece Byte Pair Encoding (BPE) and Unigram, under three distinct pre-tokenization strategies: PT-0 (Baseline, without pre-tokenization), PT-1 (Pre-tokenization of certain diacritics), and PT-2 (Pre-tokenization of all diacritics). Lower perplexity scores indicate better tokenization performance.

Table 4: Token-to-word ratio Comparison Across Mixtures

Language	**AdaptMix**	EnHiMix	UniMix	SangrahaMix
Assamese	1.93	2.47	1.93	2.35
Bengali	1.90	2.22	1.89	1.74
English	1.47	1.27	1.42	1.39
Gujarati	2.03	8.60	2.05	2.20
Hindi	1.43	1.18	1.35	1.30
Kannada	2.30	2.92	2.40	2.56
Maithili	1.73	1.71	1.70	1.88
Malayalam	2.60	3.45	2.83	2.77
Marathi	1.87	1.86	1.83	1.78
Nepali	1.70	1.92	1.62	1.58
Odia	1.95	2.64	1.94	2.33
Punjabi	1.61	2.05	1.50	1.80
Sanskrit	2.57	2.97	2.66	2.91
Sindhi	1.67	1.70	1.55	1.73
Tamil	2.35	2.84	2.44	2.22
Telugu	2.34	2.73	2.44	2.39
Average	**1.97**	**2.66**	**1.97**	**2.06**



(a) Data distribution before applying AdaptMix

(b) Data distribution after applying AdaptMix

Figure 3: **Data Mixture Transition** *Left data distribution:* Initial Uniform distribution across languages. *Right data distribution:* After applying 'AdaptMix' algorithm to uniform distribution. Language abbreviations used are listed in Appendix A.1

4.3 AdaptMix: Adaptive Data Mixture

Results demonstrated consistent and interpretable trends; languages such as Sanskrit, Tamil, Malayalam, etc. initially exhibited higher fertility, but showed steady reductions across iterations until the optimal mixture was reached. At the same time, languages that started with low fertility, such as English, Maithili and Punjabi, retained stable performance, indicating that the optimization algorithm retained its efficiency.

As per Figure 3 it is evident that substantial cross-linguistic variation necessitates assigning appropriate weights to each language so that a balanced tokenizer attains an optimal token-to-word ratio across languages. Sanskrit and Dravidian languages (Kannada, Malayalam, Telugu, Tamil) require greater weights. We found their agglutination and longer orthographic word forms yield higher token rates than in other languages. Similar insight is shared by Saluja et al. (2019) for these languages.

To assess generalization, we experimented with varying vocabulary sizes between 16K and 128K. Despite the variation, the results showed consistent allocation patterns with deviations within 1–2%, suggesting robustness to vocabulary size.

Tokenizer trained using AdaptMix algorithm, with the optimal data mixture shown in 2a, achieved the lowest fertility score among all evaluated tokenizers across all 16 languages. To validate the effectiveness of the optimal mixture, Table 4 shows a comparison between 4 different data sampling strategies, while keeping tokenizer configuration same.

To assess the downstream impact of tokenization strategies, identical models were trained using each tokenizer variant and evaluated on a held out test set. All models shared the same configuration and training data, ensuring that the tokenizer was the only differing factor. The optimal mixture tokenizer achieved the lowest overall perplexity, with improvements in morphologically rich languages like Bengali, Malayalam, Oriya, and Tamil, while maintaining strong performance on high resource languages like English and Hindi. Notably, the English/Hindi heavy tokenizer excels on its focus languages but performs poorly on others. Random and uniform mixtures show inconsistent results due to a lack of adaptive balancing. These findings reinforce the earlier analysis on fertility and parity, demonstrating that improvements in tokenization quality translate to downstream performance benefits.

Table 5: Token-to-word ratio Comparison Across state-of-the-art Tokenizers

Language	**AdaptMix**	Qwen	LLaMA	Nemotron Mistral	Nemotron Mini	Sarvam-M	DeepSeekv3	Gemma 27B	Airavata
Assamese	**1.93**	7.18	8.06	4.24	4.58	4.24	3.59	2.68	8.94
Bengali	1.90	6.92	7.85	2.93	2.65	2.93	2.89	**1.74**	8.20
English	1.47	1.36	1.35	1.37	1.35	1.37	1.33	**1.35**	1.58
Gujarati	**2.03**	8.53	9.54	3.59	15.17	3.59	4.84	2.39	14.14
Hindi	1.43	4.66	2.65	1.97	1.77	1.97	2.92	**1.38**	1.80
Kannada	**2.30**	11.08	13.81	3.82	4.02	3.82	5.83	3.15	19.35
Maithili	**1.73**	4.67	2.85	2.53	2.28	2.53	3.28	1.90	2.45
Malayalam	**2.60**	13.30	16.00	4.88	4.71	4.88	7.83	3.39	11.38
Marathi	**1.87**	6.46	3.86	3.14	2.62	3.14	4.15	1.94	3.31
Nepali	**1.70**	6.28	3.61	3.04	2.32	3.04	4.07	2.06	3.05
Oriya	1.95	12.92	15.91	**1.71**	17.24	17.23	7.26	4.60	17.29
Punjabi	**1.61**	7.39	7.88	3.12	12.70	3.12	4.51	2.74	10.84
Sanskrit	**2.57**	8.00	4.75	4.26	4.32	4.26	4.95	3.36	4.66
Sindhi	**1.67**	3.09	2.99	2.65	2.83	2.65	2.98	2.14	5.04
Tamil	**2.35**	9.75	11.89	3.71	3.57	3.71	4.88	2.42	10.50
Telugu	**2.34**	11.45	13.30	3.90	3.77	3.90	5.99	2.93	19.51
Average	**1.97**	**7.69**	**7.89**	**3.18**	**5.37**	**4.15**	**4.46**	**2.51**	**8.88**

Table 5 shows AdaptMix performs better across the state-of-the-art tokenizers in terms of the token-to-word ratio properly balancing the ratio of different languages.

5 Conclusion

We presented a comprehensive analysis of multilingual tokenizer strategies and demonstrated that any optimal vocabulary size of 128K effectively balances tokenization efficiency and computational constraints, outperforming both smaller or larger vocabularies. Furthermore, various pre-tokenization methods improves models performance, despite a slight increase in the token-to-word ratio. In our experiment we varied the mixture weights across Indic languages to study their impact on tokenizer and model performance. Our proposed **AdaptMix** algorithm dynamically optimized multilingual training data composition for all languages, significantly reducing disparity of token-word-ratio across languages. Collectively these contribution underline tokenizer as a fundamental component on par with model architecture and training objectives in building scalable and efficient multilingual language models.

References

Sarvam AI. sarvamai/sarvam-m. `https://huggingface.co/sarvamai/sarvam-m`, 2024.

Jinze Bai, Shuai Bai, Yunfei Chu, Zeyu Cui, Kai Dang, Xiaodong Deng, Yang Fan, Wenbin Ge, Yu Han, Fei Huang, Binyuan Hui, Luo Ji, Mei Li, Junyang Lin, Runji Lin, Dayiheng Liu, Gao Liu, Chengqiang Lu, Keming Lu, Jianxin Ma, Rui Men, Xingzhang Ren, Xuancheng Ren, Chuanqi Tan, Sinan Tan, Jianhong Tu, Peng Wang, Shijie Wang, Wei Wang, Shengguang Wu, Benfeng Xu, Jin Xu, An Yang, Hao Yang, Jian Yang, Shusheng Yang, Yang Yao, Bowen Yu, Hongyi Yuan, Zheng Yuan, Jianwei Zhang, Xingxuan Zhang, Yichang Zhang, Zhenru Zhang, Chang Zhou, Jingren Zhou, Xiaohuan Zhou, and Tianhang Zhu. Qwen technical report, 2023. URL https://arxiv.org/abs/2309.16609.

Maharaj Brahma, N J Karthika, Atul Singh, Devaraj Adiga, Smruti Bhate, Ganesh Ramakrishnan, Rohit Saluja, and Maunendra Sankar Desarkar. Morphtok: Morphologically grounded tokenization for indian languages, 2025. URL https://arxiv.org/abs/2504.10335.

Jonathan H. Clark, Dan Garrette, Iulia Turc, and John Wieting. CANINE: pre-training an efficient tokenization-free encoder for language representation. *CoRR*, abs/2103.06874, 2021. URL https://arxiv.org/abs/2103.06874.

Gautier Dagan, Gabriel Synnaeve, and Baptiste Rozière. Getting the most out of your tokenizer for pre-training and domain adaptation. In *Proceedings of the 41st International Conference on Machine Learning*, ICML'24. JMLR.org, 2024a.

Gautier Dagan, Gabriel Synnaeve, and Baptiste Rozière. Getting the most out of your tokenizer for pre-training and domain adaptation, 2024b. URL https://arxiv.org/abs/2402.01035.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. Bert: Pre-training of deep bidirectional transformers for language understanding. In *North American Chapter of the Association for Computational Linguistics*, 2019. URL https://api.semanticscholar.org/CorpusID:52967399.

An Yang et al. Qwen3 technical report, 2025. URL https://arxiv.org/abs/2505.09388.

Jay Gala, Pranjal A. Chitale, Raghavan AK, Varun Gumma, Sumanth Doddapaneni, Aswanth Kumar, Janki Nawale, Anupama Sujatha, Ratish Puduppully, Vivek Raghavan, Pratyush Kumar, Mitesh M. Khapra, Raj Dabre, and Anoop Kunchukuttan. Indictrans2: Towards high-quality and accessible machine translation models for all 22 scheduled indian languages, 2023. URL https://arxiv.org/abs/2305.16307.

Leo Gao, Stella Biderman, Sid Black, Laurence Golding, Travis Hoppe, Charles Foster, Jason Phang, Horace He, Anish Thite, Noa Nabeshima, Shawn Presser, and Connor Leahy. The pile: An 800gb dataset of diverse text for language modeling. *CoRR*, abs/2101.00027, 2021. URL https://arxiv.org/abs/2101.00027.

Tao Ge, Xin Chan, Xiaoyang Wang, Dian Yu, Haitao Mi, and Dong Yu. Scaling synthetic data creation with 1,000,000,000 personas, 2025. URL https://arxiv.org/abs/2406.20094.

Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Alex Vaughan, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, Aurelien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Roziere, Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong, Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Allonsius, Daniel Song, Danielle Pintz, Danny Livshits, Danny Wyatt, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino, Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael Smith, Filip Radenovic, Francisco Guzmán, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Anderson, Govind Thattai, Graeme Nail, Gregoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Korevaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel Kloumann, Ishan Misra, Ivan Evtimov, Jack Zhang, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason Park, Jay Mahadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee, Jeremy Fu,

Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe Spisak, Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden Alwala, Karthik Prasad, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, Khalid El-Arini, Krithika Iyer, Kshitiz Malik, Kuenley Chiu, Kunal Bhalla, Kushal Lakhotia, Lauren Rantala-Yeary, Laurens van der Maaten, Lawrence Chen, Liang Tan, Liz Jenkins, Louis Martin, Lovish Madaan, Lubo Malo, Lukas Blecher, Lukas Landzaat, Luke de Oliveira, Madeline Muzzi, Mahesh Pasupuleti, Mannat Singh, Manohar Paluri, Marcin Kardas, Maria Tsimpoukelli, Mathew Oldham, Mathieu Rita, Maya Pavlova, Melanie Kambadur, Mike Lewis, Min Si, Mitesh Kumar Singh, Mona Hassan, Naman Goyal, Narjes Torabi, Nikolay Bashlykov, Nikolay Bogoychev, Niladri Chatterji, Ning Zhang, Olivier Duchenne, Onur Çelebi, Patrick Alrassy, Pengchuan Zhang, Pengwei Li, Petar Vasic, Peter Weng, Prajjwal Bhargava, Pratik Dubal, Praveen Krishnan, Punit Singh Koura, Puxin Xu, Qing He, Qingxiao Dong, Ragavan Srinivasan, Raj Ganapathy, Ramon Calderer, Ricardo Silveira Cabral, Robert Stojnic, Roberta Raileanu, Rohan Maheswari, Rohit Girdhar, Rohit Patel, Romain Sauvestre, Ronnie Polidoro, Roshan Sumbaly, Ross Taylor, Ruan Silva, Rui Hou, Rui Wang, Saghar Hosseini, Sahana Chennabasappa, Sanjay Singh, Sean Bell, Seohyun Sonia Kim, Sergey Edunov, Shaoliang Nie, Sharan Narang, Sharath Raparthy, Sheng Shen, Shengye Wan, Shruti Bhosale, Shun Zhang, Simon Vandenhende, Soumya Batra, Spencer Whitman, Sten Sootla, Stephane Collot, Suchin Gururangan, Sydney Borodinsky, Tamar Herman, Tara Fowler, Tarek Sheasha, Thomas Georgiou, Thomas Scialom, Tobias Speckbacher, Todor Mihaylov, Tong Xiao, Ujjwal Karn, Vedanuj Goswami, Vibhor Gupta, Vignesh Ramanathan, Viktor Kerkez, Vincent Gonguet, Virginie Do, Vish Vogeti, Vítor Albiero, Vladan Petrovic, Weiwei Chu, Wenhan Xiong, Wenyin Fu, Whitney Meers, Xavier Martinet, Xiaodong Wang, Xiaofang Wang, Xiaoqing Ellen Tan, Xide Xia, Xinfeng Xie, Xuchao Jia, Xuewei Wang, Yaelle Goldschlag, Yashesh Gaur, Yasmine Babaei, Yi Wen, Yiwen Song, Yuchen Zhang, Yue Li, Yuning Mao, Zacharie Delpierre Coudert, Zheng Yan, Zhengxing Chen, Zoe Papakipos, Aaditya Singh, Aayushi Srivastava, Abha Jain, Adam Kelsey, Adam Shajnfeld, Adithya Gangidi, Adolfo Victoria, Ahuva Goldstand, Ajay Menon, Ajay Sharma, Alex Boesenberg, Alexei Baevski, Allie Feinstein, Amanda Kallet, Amit Sangani, Amos Teo, Anam Yunus, Andrei Lupu, Andres Alvarado, Andrew Caples, Andrew Gu, Andrew Ho, Andrew Poulton, Andrew Ryan, Ankit Ramchandani, Annie Dong, Annie Franco, Anuj Goyal, Aparajita Saraf, Arkabandhu Chowdhury, Ashley Gabriel, Ashwin Bharambe, Assaf Eisenman, Azadeh Yazdan, Beau James, Ben Maurer, Benjamin Leonhardi, Bernie Huang, Beth Loyd, Beto De Paola, Bhargavi Paranjape, Bing Liu, Bo Wu, Boyu Ni, Braden Hancock, Bram Wasti, Brandon Spence, Brani Stojkovic, Brian Gamido, Britt Montalvo, Carl Parker, Carly Burton, Catalina Mejia, Ce Liu, Changhan Wang, Changkyu Kim, Chao Zhou, Chester Hu, Ching-Hsiang Chu, Chris Cai, Chris Tindal, Christoph Feichtenhofer, Cynthia Gao, Damon Civin, Dana Beaty, Daniel Kreymer, Daniel Li, David Adkins, David Xu, Davide Testuggine, Delia David, Devi Parikh, Diana Liskovich, Didem Foss, Dingkang Wang, Duc Le, Dustin Holland, Edward Dowling, Eissa Jamil, Elaine Montgomery, Eleonora Presani, Emily Hahn, Emily Wood, Eric-Tuan Le, Erik Brinkman, Esteban Arcaute, Evan Dunbar, Evan Smothers, Fei Sun, Felix Kreuk, Feng Tian, Filippos Kokkinos, Firat Ozgenel, Francesco Caggioni, Frank Kanayet, Frank Seide, Gabriela Medina Florez, Gabriella Schwarz, Gada Badeer, Georgia Swee, Gil Halpern, Grant Herman, Grigory Sizov, Guangyi, Zhang, Guna Lakshminarayanan, Hakan Inan, Hamid Shojanazeri, Han Zou, Hannah Wang, Hanwen Zha, Haroun Habeeb, Harrison Rudolph, Helen Suk, Henry Aspegren, Hunter Goldman, Hongyuan Zhan, Ibrahim Damlaj, Igor Molybog, Igor Tufanov, Ilias Leontiadis, Irina-Elena Veliche, Itai Gat, Jake Weissman, James Geboski, James Kohli, Janice Lam, Japhet Asher, Jean-Baptiste Gaya, Jeff Marcus, Jeff Tang, Jennifer Chan, Jenny Zhen, Jeremy Reizenstein, Jeremy Teboul, Jessica Zhong, Jian Jin, Jingyi Yang, Joe Cummings, Jon Carvill, Jon Shepard, Jonathan McPhie, Jonathan Torres, Josh Ginsburg, Junjie Wang, Kai Wu, Kam Hou U, Karan Saxena, Kartikay Khandelwal, Katayoun Zand, Kathy Matosich, Kaushik Veeraraghavan, Kelly Michelena, Keqian Li, Kiran Jagadeesh, Kun Huang, Kunal Chawla, Kyle Huang, Lailin Chen, Lakshya Garg, Lavender A, Leandro Silva, Lee Bell, Lei Zhang, Liangpeng Guo, Licheng Yu, Liron Moshkovich, Luca Wehrstedt, Madian Khabsa, Manav Avalani, Manish Bhatt, Martynas Mankus, Matan Hasson, Matthew Lennie, Matthias Reso, Maxim Groshev, Maxim Naumov, Maya Lathi, Meghan Keneally, Miao Liu, Michael L. Seltzer, Michal Valko, Michelle Restrepo, Mihir Patel, Mik Vyatskov, Mikayel Samvelyan, Mike Clark, Mike Macey, Mike Wang, Miquel Jubert Hermoso, Mo Metanat, Mohammad Rastegari, Munish Bansal, Nandhini Santhanam, Natascha Parks, Natasha White, Navyata Bawa, Nayan Singhal, Nick Egebo, Nicolas Usunier, Nikhil Mehta, Nikolay Pavlovich Laptev, Ning Dong, Norman Cheng, Oleg Chernoguz, Olivia Hart, Omkar Salpekar, Ozlem Kalinli, Parkin Kent,

Parth Parekh, Paul Saab, Pavan Balaji, Pedro Rittner, Philip Bontrager, Pierre Roux, Piotr Dollar, Polina Zvyagina, Prashant Ratanchandani, Pritish Yuvraj, Qian Liang, Rachad Alao, Rachel Rodriguez, Rafi Ayub, Raghotham Murthy, Raghu Nayani, Rahul Mitra, Rangaprabhu Parthasarathy, Raymond Li, Rebekkah Hogan, Robin Battey, Rocky Wang, Russ Howes, Ruty Rinott, Sachin Mehta, Sachin Siby, Sai Jayesh Bondu, Samyak Datta, Sara Chugh, Sara Hunt, Sargun Dhillon, Sasha Sidorov, Satadru Pan, Saurabh Mahajan, Saurabh Verma, Seiji Yamamoto, Sharadh Ramaswamy, Shaun Lindsay, Shaun Lindsay, Sheng Feng, Shenghao Lin, Shengxin Cindy Zha, Shishir Patil, Shiva Shankar, Shuqiang Zhang, Shuqiang Zhang, Sinong Wang, Sneha Agarwal, Soji Sajuyigbe, Soumith Chintala, Stephanie Max, Stephen Chen, Steve Kehoe, Steve Satterfield, Sudarshan Govindaprasad, Sumit Gupta, Summer Deng, Sungmin Cho, Sunny Virk, Suraj Subramanian, Sy Choudhury, Sydney Goldman, Tal Remez, Tamar Glaser, Tamara Best, Thilo Koehler, Thomas Robinson, Tianhe Li, Tianjun Zhang, Tim Matthews, Timothy Chou, Tzook Shaked, Varun Vontimitta, Victoria Ajayi, Victoria Montanez, Vijai Mohan, Vinay Satish Kumar, Vishal Mangla, Vlad Ionescu, Vlad Poenaru, Vlad Tiberiu Mihailescu, Vladimir Ivanov, Wei Li, Wenchen Wang, Wenwen Jiang, Wes Bouaziz, Will Constable, Xiaocheng Tang, Xiaojian Wu, Xiaolan Wang, Xilun Wu, Xinbo Gao, Yaniv Kleinman, Yanjun Chen, Ye Hu, Ye Jia, Ye Qi, Yenda Li, Yilin Zhang, Ying Zhang, Yossi Adi, Youngjin Nam, Yu, Wang, Yu Zhao, Yuchen Hao, Yundi Qian, Yunlu Li, Yuzi He, Zach Rait, Zachary DeVito, Zef Rosnbrick, Zhaoduo Wen, Zhenyu Yang, Zhiwei Zhao, and Zhiyu Ma. The llama 3 herd of models, 2024. URL https://arxiv.org/abs/2407.21783.

Albert Q. Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier, Lélio Renard Lavaud, Marie-Anne Lachaux, Pierre Stock, Teven Le Scao, Thibaut Lavril, Thomas Wang, Timothée Lacroix, and William El Sayed. Mistral 7b, 2023. URL https://arxiv.org/abs/2310.06825.

Divyanshu Kakwani, Anoop Kunchukuttan, Satish Golla, Gokul N.C., Avik Bhattacharyya, Mitesh M. Khapra, and Pratyush Kumar. IndicNLPSuite: Monolingual corpora, evaluation benchmarks and pre-trained multilingual language models for Indian languages. In Trevor Cohn, Yulan He, and Yang Liu (eds.), *Findings of the Association for Computational Linguistics: EMNLP 2020*, pp. 4948–4961, Online, November 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.findings-emnlp.445. URL https://aclanthology.org/2020.findings-emnlp.445/.

Mohammed Khan, Priyam Mehta, Ananth Sankar, Umashankar Kumaravelan, Sumanth Doddapaneni, Suriyaprasaad B, Varun G, Sparsh Jain, Anoop Kunchukuttan, Pratyush Kumar, Raj Dabre, and Mitesh Khapra. Indicllmsuite: A blueprint for creating pre-training and fine-tuning datasets for indian languages. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 15831–15879. Association for Computational Linguistics, 2024. doi: 10.18653/v1/2024.acl-long.843. URL http://dx.doi.org/10.18653/v1/2024.acl-long.843.

Taku Kudo. Subword regularization: Improving neural network translation models with multiple subword candidates. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 66–75. Association for Computational Linguistics, 2018. URL https://aclanthology.org/P18-1007.

Taku Kudo and John Richardson. SentencePiece: A simple and language independent subword tokenizer and detokenizer for neural text processing. In Eduardo Blanco and Wei Lu (eds.), *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing: System Demonstrations*, pp. 66–71, Brussels, Belgium, November 2018. Association for Computational Linguistics. doi: 10.18653/v1/D18-2012. URL https://aclanthology.org/D18-2012/.

Faisal Ladhak, Esin Durmus, Claire Cardie, and Kathleen McKeown. WikiLingua: A new benchmark dataset for cross-lingual abstractive summarization. In Trevor Cohn, Yulan He, and Yang Liu (eds.), *Findings of the Association for Computational Linguistics: EMNLP 2020*, pp. 4034–4048, Online, November 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.findings-emnlp.360. URL https://aclanthology.org/2020.findings-emnlp.360/.

Guillaume Lample and Alexis Conneau. Cross-lingual language model pretraining. *CoRR*, abs/1901.07291, 2019. URL http://arxiv.org/abs/1901.07291.

Ilya Loshchilov and Frank Hutter. SGDR: stochastic gradient descent with restarts. *CoRR*, abs/1608.03983, 2016. URL http://arxiv.org/abs/1608.03983.

Ilya Loshchilov and Frank Hutter. Fixing weight decay regularization in adam. *CoRR*, abs/1711.05101, 2017. URL http://arxiv.org/abs/1711.05101.

Alexandra Sasha Luccioni, Sylvain Viguier, and Anne-Laure Ligozat. Estimating the carbon footprint of bloom, a 176b parameter language model. *J. Mach. Learn. Res.*, 24(1), January 2023. ISSN 1532-4435.

Niklas Muennighoff, Thomas Wang, Lintang Sutawika, Adam Roberts, Stella Biderman, Teven Le Scao, M Saiful Bari, Sheng Shen, Zheng Xin Yong, Hailey Schoelkopf, Xiangru Tang, Dragomir Radev, Alham Fikri Aji, Khalid Almubarak, Samuel Albanie, Zaid Alyafeai, Albert Webson, Edward Raff, and Colin Raffel. Crosslingual generalization through multitask finetuning. In Anna Rogers, Jordan Boyd-Graber, and Naoaki Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 15991–16111, Toronto, Canada, July 2023. Association for Computational Linguistics. doi: 10.18653/v1/2023.acl-long.891. URL https://aclanthology.org/2023.acl-long.891/.

Rodrigo Nogueira, Zhiying Jiang, and Jimmy Lin. Investigating the limitations of transformers with simple arithmetic tasks, 2021. URL https://arxiv.org/abs/2102.13019.

Nvidia, :, Bo Adler, Niket Agarwal, Ashwath Aithal, Dong H. Anh, Pallab Bhattacharya, Annika Brundyn, Jared Casper, Bryan Catanzaro, Sharon Clay, Jonathan Cohen, Sirshak Das, Ayush Dattagupta, Olivier Delalleau, Leon Derczynski, Yi Dong, Daniel Egert, Ellie Evans, Aleksander Ficek, Denys Fridman, Shaona Ghosh, Boris Ginsburg, Igor Gitman, Tomasz Grzegorzek, Robert Hero, Jining Huang, Vibhu Jawa, Joseph Jennings, Aastha Jhunjhunwala, John Kamalu, Sadaf Khan, Oleksii Kuchaiev, Patrick LeGresley, Hui Li, Jiwei Liu, Zihan Liu, Eileen Long, Ameya Sunil Mahabaleshwarkar, Somshubra Majumdar, James Maki, Miguel Martinez, Maer Rodrigues de Melo, Ivan Moshkov, Deepak Narayanan, Sean Narenthiran, Jesus Navarro, Phong Nguyen, Osvald Nitski, Vahid Noroozi, Guruprasad Nutheti, Christopher Parisien, Jupinder Parmar, Mostofa Patwary, Krzysztof Pawelec, Wei Ping, Shrimai Prabhumoye, Rajarshi Roy, Trisha Saar, Vasanth Rao Naik Sabavat, Sanjeev Satheesh, Jane Polak Scowcroft, Jason Sewall, Pavel Shamis, Gerald Shen, Mohammad Shoeybi, Dave Sizer, Misha Smelyanskiy, Felipe Soares, Makesh Narsimhan Sreedhar, Dan Su, Sandeep Subramanian, Shengyang Sun, Shubham Toshniwal, Hao Wang, Zhilin Wang, Jiaxuan You, Jiaqi Zeng, Jimmy Zhang, Jing Zhang, Vivienne Zhang, Yian Zhang, and Chen Zhu. Nemotron-4 340b technical report, 2024. URL https://arxiv.org/abs/2406.11704.

Yonatan Oren, Shiori Sagawa, Tatsunori B. Hashimoto, and Percy Liang. Distributionally robust language modeling. *CoRR*, abs/1909.02060, 2019. URL http://arxiv.org/abs/1909.02060.

Aleksandar Petrov, Emanuele La Malfa, Philip H. S. Torr, and Adel Bibi. Language model tokenizers introduce unfairness between languages, 2023. URL https://arxiv.org/abs/2305.15425.

Ivan Provilkov, Dmitrii Emelianenko, and Elena Voita. Bpe-dropout: Simple and effective subword regularization, 2020. URL https://arxiv.org/abs/1910.13267.

Kundeshwar Pundalik, Piyush Sawarkar, Nihar Sahoo, Abhishek Shinde, Prateek Chanda, Vedant Goswami, Ajay Nagpal, Atul Singh, Viraj Thakur, Vijay Dewane, Aamod Thakur, Bhargav Patel, Smita Gautam, Bhagwan Panditi, Shyam Pawar, Madhav Kotcha, Suraj Racha, Saral Sureka, Pankaj Singh, Rishi Bal, Rohit Saluja, and Ganesh Ramakrishnan. Param-1 bharatgen 2.9b model, 2025. URL https://arxiv.org/abs/2507.13390.

Qwen, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang,

Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu, Keqin Bao, Kexin Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji Lin, Tianhao Li, Tianyi Tang, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yichang Zhang, Yu Wan, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zihan Qiu. Qwen2.5 technical report, 2025. URL https://arxiv.org/abs/2412.15115.

Alec Radford, Jeff Wu, Rewon Child, David Luan, Dario Amodei, and Ilya Sutskever. Language models are unsupervised multitask learners. 2019. URL https://api.semanticscholar.org/CorpusID:160025533.

Gowtham Ramesh, Sumanth Doddapaneni, Aravinth Bheemaraj, Mayank Jobanputra, Raghavan AK, Ajitesh Sharma, Sujit Sahoo, Harshita Diddee, Mahalakshmi J, Divyanshu Kakwani, Navneet Kumar, Aswin Pradeep, Srihari Nagaraj, Kumar Deepak, Vivek Raghavan, Anoop Kunchukuttan, Pratyush Kumar, and Mitesh Shantadevi Khapra. Samanantar: The largest publicly available parallel corpora collection for 11 Indic languages. *Transactions of the Association for Computational Linguistics*, 10:145–162, 2022. doi: 10.1162/tacl_a_00452. URL https://aclanthology.org/2022.tacl-1.9/.

Varshini Reddy, Craig W. Schmidt, Yuval Pinter, and Chris Tanner. How much is enough? the diminishing returns of tokenization training data, 2025. URL https://arxiv.org/abs/2502.20273.

Rohit Saluja, Mayur Punjabi, Mark Carman, Ganesh Ramakrishnan, and Parag Chaudhuri. Sub-Word Embeddings for OCR Corrections in Highly Fusional Indic Languages . In *2019 International Conference on Document Analysis and Recognition (ICDAR)*, pp. 160–165, Los Alamitos, CA, USA, September 2019. IEEE Computer Society. doi: 10.1109/ICDAR.2019.00034. URL https://doi.ieeecomputersociety.org/10.1109/ICDAR.2019.00034.

Rico Sennrich, Barry Haddow, and Alexandra Birch. Neural machine translation of rare words with subword units. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 1715–1725. Association for Computational Linguistics, 2016. URL https://aclanthology.org/P16-1162.

Xinying Song, Alex Salcianu, Yang Song, Dave Dopson, and Denny Zhou. Fast WordPiece tokenization. In Marie-Francine Moens, Xuanjing Huang, Lucia Specia, and Scott Wen-tau Yih (eds.), *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pp. 2089–2103, Online and Punta Cana, Dominican Republic, November 2021. Association for Computational Linguistics. doi: 10.18653/v1/2021.emnlp-main.160. URL https://aclanthology.org/2021.emnlp-main.160/.

Gemma Team, Thomas Mesnard, Cassidy Hardin, Robert Dadashi, Surya Bhupatiraju, Shreya Pathak, Laurent Sifre, Morgane Rivière, Mihir Sanjay Kale, Juliette Love, Pouya Tafti, Léonard Hussenot, Pier Giuseppe Sessa, Aakanksha Chowdhery, Adam Roberts, Aditya Barua, Alex Botev, Alex Castro-Ros, Ambrose Slone, Amélie Héliou, Andrea Tacchetti, Anna Bulanova, Antonia Paterson, Beth Tsai, Bobak Shahriari, Charline Le Lan, Christopher A. Choquette-Choo, Clément Crepy, Daniel Cer, Daphne Ippolito, David Reid, Elena Buchatskaya, Eric Ni, Eric Noland, Geng Yan, George Tucker, George-Christian Muraru, Grigory Rozhdestvenskiy, Henryk Michalewski, Ian Tenney, Ivan Grishchenko, Jacob Austin, James Keeling, Jane Labanowski, Jean-Baptiste Lespiau, Jeff Stanway, Jenny Brennan, Jeremy Chen, Johan Ferret, Justin Chiu, Justin Mao-Jones, Katherine Lee, Kathy Yu, Katie Millican, Lars Lowe Sjoesund, Lisa Lee, Lucas Dixon, Machel Reid, Maciej Mikuła, Mateo Wirth, Michael Sharman, Nikolai Chinaev, Nithum Thain, Olivier Bachem, Oscar Chang, Oscar Wahltinez, Paige Bailey, Paul Michel, Petko Yotov, Rahma Chaabouni, Ramona Comanescu, Reena Jana, Rohan Anil, Ross McIlroy, Ruibo Liu, Ryan Mullins, Samuel L Smith, Sebastian Borgeaud, Sertan Girgin, Sholto Douglas, Shree Pandya, Siamak Shakeri, Soham De, Ted Klimenko, Tom Hennigan, Vlad Feinberg, Wojciech Stokowiec, Yu hui Chen, Zafarali Ahmed, Zhitao Gong, Tris Warkentin, Ludovic Peran, Minh Giang, Clément Farabet, Oriol Vinyals, Jeff Dean, Koray Kavukcuoglu, Demis Hassabis, Zoubin Ghahramani, Douglas Eck, Joelle Barral, Fernando Pereira, Eli Collins, Armand Joulin, Noah Fiedel, Evan Senter, Alek Andreev, and Kathleen Kenealy. Gemma: Open models based on gemini research and technology, 2024a. URL https://arxiv.org/abs/2403.08295.

Gemma Team, Morgane Riviere, Shreya Pathak, Pier Giuseppe Sessa, Cassidy Hardin, Surya Bhupatiraju, Léonard Hussenot, Thomas Mesnard, Bobak Shahriari, Alexandre Ramé, Johan Ferret, Peter Liu, Pouya Tafti, Abe Friesen, Michelle Casbon, Sabela Ramos, Ravin Kumar, Charline Le Lan, Sammy Jerome, Anton Tsitsulin, Nino Vieillard, Piotr Stanczyk, Sertan Girgin, Nikola Momchev, Matt Hoffman, Shantanu Thakoor, Jean-Bastien Grill, Behnam Neyshabur, Olivier Bachem, Alanna Walton, Aliaksei Severyn, Alicia Parrish, Aliya Ahmad, Allen Hutchison, Alvin Abdagic, Amanda Carl, Amy Shen, Andy Brock, Andy Coenen, Anthony Laforge, Antonia Paterson, Ben Bastian, Bilal Piot, Bo Wu, Brandon Royal, Charlie Chen, Chintu Kumar, Chris Perry, Chris Welty, Christopher A. Choquette-Choo, Danila Sinopalnikov, David Weinberger, Dimple Vijaykumar, Dominika Rogozińska, Dustin Herbison, Elisa Bandy, Emma Wang, Eric Noland, Erica Moreira, Evan Senter, Evgenii Eltyshev, Francesco Visin, Gabriel Rasskin, Gary Wei, Glenn Cameron, Gus Martins, Hadi Hashemi, Hanna Klimczak-Plucińska, Harleen Batra, Harsh Dhand, Ivan Nardini, Jacinda Mein, Jack Zhou, James Svensson, Jeff Stanway, Jetha Chan, Jin Peng Zhou, Joana Carrasqueira, Joana Iljazi, Jocelyn Becker, Joe Fernandez, Joost van Amersfoort, Josh Gordon, Josh Lipschultz, Josh Newlan, Ju yeong Ji, Kareem Mohamed, Kartikeya Badola, Kat Black, Katie Millican, Keelin McDonell, Kelvin Nguyen, Kiranbir Sodhia, Kish Greene, Lars Lowe Sjoesund, Lauren Usui, Laurent Sifre, Lena Heuermann, Leticia Lago, Lilly McNealus, Livio Baldini Soares, Logan Kilpatrick, Lucas Dixon, Luciano Martins, Machel Reid, Manvinder Singh, Mark Iverson, Martin Görner, Mat Velloso, Mateo Wirth, Matt Davidow, Matt Miller, Matthew Rahtz, Matthew Watson, Meg Risdal, Mehran Kazemi, Michael Moynihan, Ming Zhang, Minsuk Kahng, Minwoo Park, Mofi Rahman, Mohit Khatwani, Natalie Dao, Nenshad Bardoliwalla, Nesh Devanathan, Neta Dumai, Nilay Chauhan, Oscar Wahltinez, Pankil Botarda, Parker Barnes, Paul Barham, Paul Michel, Pengchong Jin, Petko Georgiev, Phil Culliton, Pradeep Kuppala, Ramona Comanescu, Ramona Merhej, Reena Jana, Reza Ardeshir Rokni, Rishabh Agarwal, Ryan Mullins, Samaneh Saadat, Sara Mc Carthy, Sarah Cogan, Sarah Perrin, Sébastien M. R. Arnold, Sebastian Krause, Shengyang Dai, Shruti Garg, Shruti Sheth, Sue Ronstrom, Susan Chan, Timothy Jordan, Ting Yu, Tom Eccles, Tom Hennigan, Tomas Kocisky, Tulsee Doshi, Vihan Jain, Vikas Yadav, Vilobh Meshram, Vishal Dharmadhikari, Warren Barkley, Wei Wei, Wenming Ye, Woohyun Han, Woosuk Kwon, Xiang Xu, Zhe Shen, Zhitao Gong, Zichuan Wei, Victor Cotruta, Phoebe Kirk, Anand Rao, Minh Giang, Ludovic Peran, Tris Warkentin, Eli Collins, Joelle Barral, Zoubin Ghahramani, Raia Hadsell, D. Sculley, Jeanine Banks, Anca Dragan, Slav Petrov, Oriol Vinyals, Jeff Dean, Demis Hassabis, Koray Kavukcuoglu, Clement Farabet, Elena Buchatskaya, Sebastian Borgeaud, Noah Fiedel, Armand Joulin, Kathleen Kenealy, Robert Dadashi, and Alek Andreev. Gemma 2: Improving open language models at a practical size, 2024b. URL https://arxiv.org/abs/2408.00118.

Gemma Team, Aishwarya Kamath, Johan Ferret, Shreya Pathak, Nino Vieillard, Ramona Merhej, Sarah Perrin, Tatiana Matejovicova, Alexandre Ramé, Morgane Rivière, Louis Rouillard, Thomas Mesnard, Geoffrey Cideron, Jean bastien Grill, Sabela Ramos, Edouard Yvinec, Michelle Casbon, Etienne Pot, Ivo Penchev, Gaël Liu, Francesco Visin, Kathleen Kenealy, Lucas Beyer, Xiaohai Zhai, Anton Tsitsulin, Robert Busa-Fekete, Alex Feng, Noveen Sachdeva, Benjamin Coleman, Yi Gao, Basil Mustafa, Iain Barr, Emilio Parisotto, David Tian, Matan Eyal, Colin Cherry, Jan-Thorsten Peter, Danila Sinopalnikov, Surya Bhupatiraju, Rishabh Agarwal, Mehran Kazemi, Dan Malkin, Ravin Kumar, David Vilar, Idan Brusilovsky, Jiaming Luo, Andreas Steiner, Abe Friesen, Abhanshu Sharma, Abheesht Sharma, Adi Mayrav Gilady, Adrian Goedeckemeyer, Alaa Saade, Alex Feng, Alexander Kolesnikov, Alexei Bendebury, Alvin Abdagic, Amit Vadi, András György, André Susano Pinto, Anil Das, Ankur Bapna, Antoine Miech, Antoine Yang, Antonia Paterson, Ashish Shenoy, Ayan Chakrabarti, Bilal Piot, Bo Wu, Bobak Shahriari, Bryce Petrini, Charlie Chen, Charline Le Lan, Christopher A. Choquette-Choo, CJ Carey, Cormac Brick, Daniel Deutsch, Danielle Eisenbud, Dee Cattle, Derek Cheng, Dimitris Paparas, Divyashree Shivakumar Sreepathihalli, Doug Reid, Dustin Tran, Dustin Zelle, Eric Noland, Erwin Huizenga, Eugene Kharitonov, Frederick Liu, Gagik Amirkhanyan, Glenn Cameron, Hadi Hashemi, Hanna Klimczak-Plucińska, Harman Singh, Harsh Mehta, Harshal Tushar Lehri, Hussein Hazimeh, Ian Ballantyne, Idan Szpektor, Ivan Nardini, Jean Pouget-Abadie, Jetha Chan, Joe Stanton, John Wieting, Jonathan Lai, Jordi Orbay, Joseph Fernandez, Josh Newlan, Ju yeong Ji, Jyotinder Singh, Kat Black, Kathy Yu, Kevin Hui, Kiran Vodrahalli, Klaus Greff, Linhai Qiu, Marcella Valentine, Marina Coelho, Marvin Ritter, Matt Hoffman, Matthew Watson, Mayank Chaturvedi, Michael Moynihan, Min Ma, Nabila Babar, Natasha Noy, Nathan Byrd, Nick Roy, Nikola Momchev, Nilay Chauhan, Noveen Sachdeva, Oskar Bunyan, Pankil Botarda, Paul Caron, Paul Kishan Ruben-

stein, Phil Culliton, Philipp Schmid, Pier Giuseppe Sessa, Pingmei Xu, Piotr Stanczyk, Pouya Tafti, Rakesh Shivanna, Renjie Wu, Renke Pan, Reza Rokni, Rob Willoughby, Rohith Vallu, Ryan Mullins, Sammy Jerome, Sara Smoot, Sertan Girgin, Shariq Iqbal, Shashir Reddy, Shruti Sheth, Siim Põder, Sijal Bhatnagar, Sindhu Raghuram Panyam, Sivan Eiger, Susan Zhang, Tianqi Liu, Trevor Yacovone, Tyler Liechty, Uday Kalra, Utku Evci, Vedant Misra, Vincent Roseberry, Vlad Feinberg, Vlad Kolesnikov, Woohyun Han, Woosuk Kwon, Xi Chen, Yinlam Chow, Yuvein Zhu, Zichuan Wei, Zoltan Egyed, Victor Cotruta, Minh Giang, Phoebe Kirk, Anand Rao, Kat Black, Nabila Babar, Jessica Lo, Erica Moreira, Luiz Gustavo Martins, Omar Sanseviero, Lucas Gonzalez, Zach Gleicher, Tris Warkentin, Vahab Mirrokni, Evan Senter, Eli Collins, Joelle Barral, Zoubin Ghahramani, Raia Hadsell, Yossi Matias, D. Sculley, Slav Petrov, Noah Fiedel, Noam Shazeer, Oriol Vinyals, Jeff Dean, Demis Hassabis, Koray Kavukcuoglu, Clement Farabet, Elena Buchatskaya, Jean-Baptiste Alayrac, Rohan Anil, Dmitry, Lepikhin, Sebastian Borgeaud, Olivier Bachem, Armand Joulin, Alek Andreev, Cassidy Hardin, Robert Dadashi, and Léonard Hussenot. Gemma 3 technical report, 2025. URL https://arxiv.org/abs/2503.19786.

NLLB Team, Marta R. Costa-jussà, James Cross, Onur Çelebi, Maha Elbayad, Kenneth Heafield, Kevin Heffernan, Elahe Kalbassi, Janice Lam, Daniel Licht, Jean Maillard, Anna Sun, Skyler Wang, Guillaume Wenzek, Al Youngblood, Bapi Akula, Loic Barrault, Gabriel Mejia Gonzalez, Prangthip Hansanti, John Hoffman, Semarley Jarrett, Kaushik Ram Sadagopan, Dirk Rowe, Shannon Spruit, Chau Tran, Pierre Andrews, Necip Fazil Ayan, Shruti Bhosale, Sergey Edunov, Angela Fan, Cynthia Gao, Vedanuj Goswami, Francisco Guzmán, Philipp Koehn, Alexandre Mourachko, Christophe Ropers, Safiyyah Saleem, Holger Schwenk, and Jeff Wang. No language left behind: Scaling human-centered machine translation, 2022. URL https://arxiv.org/abs/2207.04672.

Avijit Thawani, Jay Pujara, Filip Ilievski, and Pedro Szekely. Representing numbers in NLP: a survey and a vision. In Kristina Toutanova, Anna Rumshisky, Luke Zettlemoyer, Dilek Hakkani-Tur, Iz Beltagy, Steven Bethard, Ryan Cotterell, Tanmoy Chakraborty, and Yichao Zhou (eds.), *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pp. 644–656, Online, June 2021. Association for Computational Linguistics. doi: 10.18653/v1/2021.naacl-main.53. URL https://aclanthology.org/2021.naacl-main.53/.

Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, Aurelien Rodriguez, Armand Joulin, Edouard Grave, and Guillaume Lample. Llama: Open and efficient foundation language models, 2023a. URL https://arxiv.org/abs/2302.13971.

Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajjwal Bhargava, Shruti Bhosale, Dan Bikel, Lukas Blecher, Cristian Canton Ferrer, Moya Chen, Guillem Cucurull, David Esiobu, Jude Fernandes, Jeremy Fu, Wenyin Fu, Brian Fuller, Cynthia Gao, Vedanuj Goswami, Naman Goyal, Anthony Hartshorn, Saghar Hosseini, Rui Hou, Hakan Inan, Marcin Kardas, Viktor Kerkez, Madian Khabsa, Isabel Kloumann, Artem Korenev, Punit Singh Koura, Marie-Anne Lachaux, Thibaut Lavril, Jenya Lee, Diana Liskovich, Yinghai Lu, Yuning Mao, Xavier Martinet, Todor Mihaylov, Pushkar Mishra, Igor Molybog, Yixin Nie, Andrew Poulton, Jeremy Reizenstein, Rashi Rungta, Kalyan Saladi, Alan Schelten, Ruan Silva, Eric Michael Smith, Ranjan Subramanian, Xiaoqing Ellen Tan, Binh Tang, Ross Taylor, Adina Williams, Jian Xiang Kuan, Puxin Xu, Zheng Yan, Iliyan Zarov, Yuchen Zhang, Angela Fan, Melanie Kambadur, Sharan Narang, Aurelien Rodriguez, Robert Stojnic, Sergey Edunov, and Thomas Scialom. Llama 2: Open foundation and fine-tuned chat models, 2023b. URL https://arxiv.org/abs/2307.09288.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Ł ukasz Kaiser, and Illia Polosukhin. Attention is all you need. In I. Guyon, U. Von Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett (eds.), *Advances in Neural Information Processing Systems*, volume 30. Curran Associates, Inc., 2017a. URL https://proceedings.neurips.cc/paper_files/paper/2017/file/3f5ee243547dee91fbd053c1c4a845aa-Paper.pdf.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N. Gomez, Lukasz Kaiser, and Illia Polosukhin. Attention is all you need. *CoRR*, abs/1706.03762, 2017b. URL http://arxiv.org/abs/1706.03762.

Sang Michael Xie, Hieu Pham, Xuanyi Dong, Nan Du, Hanxiao Liu, Yifeng Lu, Percy Liang, Quoc V. Le, Tengyu Ma, and Adams Wei Yu. Doremi: Optimizing data mixtures speeds up language model pretraining, 2023. URL https://arxiv.org/abs/2305.10429.

Linting Xue, Aditya Barua, Noah Constant, Rami Al-Rfou, Sharan Narang, Mihir Kale, Adam Roberts, and Colin Raffel. Byt5: Towards a token-free future with pre-trained byte-to-byte models. *CoRR*, abs/2105.13626, 2021a. URL https://arxiv.org/abs/2105.13626.

Linting Xue, Noah Constant, Adam Roberts, Mihir Kale, Rami Al-Rfou, Aditya Siddhant, Aditya Barua, and Colin Raffel. mt5: A massively multilingual pre-trained text-to-text transformer, 2021b. URL https://arxiv.org/abs/2010.11934.

An Yang, Baosong Yang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Zhou, Chengpeng Li, Chengyuan Li, Dayiheng Liu, Fei Huang, Guanting Dong, Haoran Wei, Huan Lin, Jialong Tang, Jialin Wang, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Ma, Jianxin Yang, Jin Xu, Jingren Zhou, Jinze Bai, Jinzheng He, Junyang Lin, Kai Dang, Keming Lu, Keqin Chen, Kexin Yang, Mei Li, Mingfeng Xue, Na Ni, Pei Zhang, Peng Wang, Ru Peng, Rui Men, Ruize Gao, Runji Lin, Shijie Wang, Shuai Bai, Sinan Tan, Tianhang Zhu, Tianhao Li, Tianyu Liu, Wenbin Ge, Xiaodong Deng, Xiaohuan Zhou, Xingzhang Ren, Xinyu Zhang, Xipin Wei, Xuancheng Ren, Xuejing Liu, Yang Fan, Yang Yao, Yichang Zhang, Yu Wan, Yunfei Chu, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, Zhifang Guo, and Zhihao Fan. Qwen2 technical report, 2024. URL https://arxiv.org/abs/2407.10671.

Yanli Zhao, Andrew Gu, Rohan Varma, Liang Luo, Chien-Chin Huang, Min Xu, Less Wright, Hamid Shojanazeri, Myle Ott, Sam Shleifer, Alban Desmaison, Can Balioglu, Pritam Damania, Bernard Nguyen, Geeta Chauhan, Yuchen Hao, Ajit Mathews, and Shen Li. Pytorch fsdp: Experiences on scaling fully sharded data parallel, 2023. URL https://arxiv.org/abs/2304.11277.

A Appendix

A.1 Abbreviations

Abbreviation	Language
as	Assamese
bn	Bengali
en	English
gu	Gujarati
hi	Hindi
kn	Kannada
mai	Maithili
ml	Malayalam
mr	Marathi
ne	Nepali
or	Odia
pa	Punjabi
sd	Sindhi
sa	Sanskrit
ta	Tamil
te	Telugu

A.2 Token to Word Ratio

Token to Word Ratio is also commonly known as Fertility score. It is defined as,

$$\text{Token-to-Word Ratio} = \frac{\text{Total Number of Tokens}}{\text{Total Number of Words}} \tag{8}$$

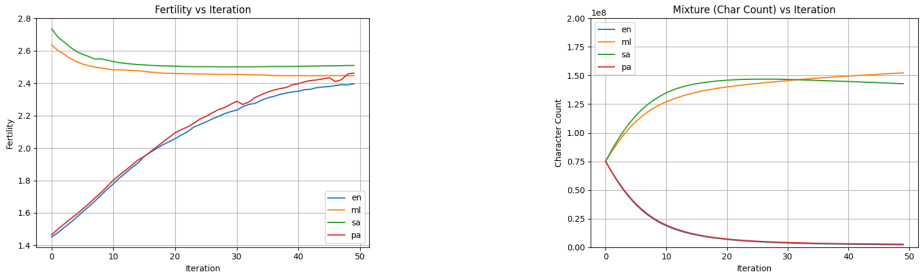
A.3 Model Training Details

All models were trained from scratch using causal decoder transformer Vaswani et al. (2017b) architecture with 16 layers and a hidden size of 512, resulting in approximately 100M parameters. Each model used a vocabulary size of 128k, based on the tokenizer being evaluated. The optimizer used was AdamW Loshchilov & Hutter (2017) with a learning rate of 3e-4, cosine learning rate decay Loshchilov & Hutter (2016) and the weight decay set to 0.1. Training was performed using BF16 precision on a single node with 2 GPUs, using Fully Sharded Data Parallelism (FSDP) Zhao et al. (2023) for efficient memory and compute scaling.

A.4 Problems with Weighted Data Mixture Optimization

Prior to scaling optimization algorithm to 16 languages, preliminary experiments were conducted on a subset of four languages: English, Punjabi, Malayalam and Sanskrit. The subset was chosen to reflect a range of token-to-word ratio behaviors as these languages have unique character sets and grammatical rules. English and Punjabi generally perform well over default mixtures, whereas Malayalam and Sanskrit are observed to exhibit higher token-to-word ratio. These small scale experiments helped to mold core algorithm without compute over-utilization.

However, during the early stage experiments, we observed unintended behavior in the way token-to-word ratio deficits were calculated. Initially, the best token-to-word ratio was defined dynamically as the lowest token-to-word ratio among all languages in each iteration. While this allowed the algorithm to adaptively update the mixture, it introduced a problematic patterns; instead of increasing the proportion of under-performing languages, the optimizer began decreasing the proportion of well performing ones, as observed in Figure 4. This occurred because Sanskrit and Malayalam could not realistically reach the same tokenization efficiency as English or Punjabi within the same vocabulary size. As a result, the algorithm minimized the overall deficit by degrading the performance of already efficient languages instead of improving under-performing ones.



(a) Weighted Fertility across Iterations.

(b) Weighted Mixture allocation.

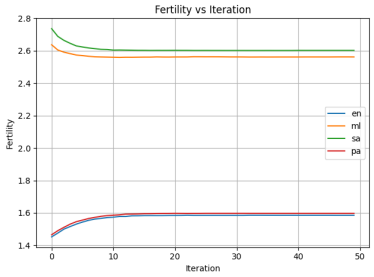
Figure 4: Fertility and Mixture Allocation across Iterations for Weighted Algorithm

This problem was resolved using our novel AdaptMix Algorithm on the same set of dataset and configuration as shown in 5.

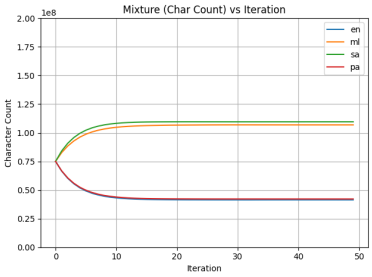
A.5 Parity Calculation

In addition to evaluating fertility, Table 6 shows tests conducted on Parity Petrov et al. (2023), which quantifies cross lingual fairness and bias in tokenization. Across the 16 Indian languages, the optimal data mixture consistently outperformed state of the art open source model tokenizers like Qwen, LLama, DeepSeek, and Gemma in achieving parity with English.

A.6 ApdatMix Pseudo Code



(a) AdaptMix Fertility across Iterations.



(b) AdaptMix Mixture allocation.

Figure 5: Fertility and Mixture Allocation across Iterations for AdaptMix Algorithm

Table 6: Parity Comparison Across state-of-art Tokenizers

Language	**AdaptMix**	Qwen	LLaMA	Nemotron Mistral	Nemotron Mini	Sarvam-M	DeepSeek v3	Gemma 27B
Assamese	1.31	5.27	5.97	3.09	3.39	3.09	2.69	1.98
Bengali	1.29	5.08	5.81	2.13	1.96	2.13	2.17	1.28
English	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Gujarati	1.37	6.27	7.06	2.62	11.23	2.62	3.63	1.77
Hindi	0.96	3.42	1.96	1.43	1.31	1.43	2.19	1.02
Kannada	1.56	8.14	10.22	2.78	2.97	2.78	4.38	2.33
Maithili	1.17	3.43	2.11	1.84	1.68	1.84	2.46	1.40
Malayalam	1.76	9.77	11.85	3.56	3.48	3.56	5.88	2.51
Marathi	1.26	4.75	2.85	2.29	1.94	2.29	3.12	1.43
Nepali	1.15	4.61	2.67	2.21	1.71	2.21	3.06	1.52
Oriya	1.32	9.50	11.78	12.57	12.77	12.57	5.45	3.40
Punjabi	1.09	5.43	5.83	2.27	9.40	2.27	3.39	2.02
Sanskrit	1.74	5.88	3.51	3.10	3.20	3.10	3.72	2.48
Sindhi	1.13	2.27	2.21	1.93	2.09	1.93	2.24	1.58
Tamil	1.59	7.16	8.80	2.70	2.64	2.70	3.66	1.79
Telugu	1.58	8.41	9.85	2.84	2.79	2.84	4.50	2.17

Algorithm 1 Calculate Mixture

Require: Current mixture M_t, token-to-word ratio/fertility scores F, learning rate λ, constant ϵ
Ensure: New mixture M_{t+1}

$total\_chars \leftarrow \sum_i current\_mixture[l_i]$

$best\_fertility \leftarrow \min(F.values())$
$worst\_fertility \leftarrow \max(F.values())$
$fertility\_range \leftarrow worst\_fertility - best\_fertility$
for each language l_i **do**
  if $fertility\_range > 0$ **then**
    $scaled\_deficit[l_i] \leftarrow (F[l_i] - best\_fertility)/fertility\_range$
  else
    $scaled\_deficit[l_i] \leftarrow 0$
  end if
  $deficit\_weight[l_i] \leftarrow scaled\_deficit[l_i] + \epsilon$
end for

$total\_deficit \leftarrow \sum_i deficit\_weight[l_i]$
for each language l_i **do**
  $deficit\_target[l_i] \leftarrow deficit\_weight[l_i]/total\_deficit$
  $new\_prop[l_i] \leftarrow current\_prop[l_i](1 - \lambda) + deficit\_target[l_i]\lambda$
  $current\_prop[l_i] \leftarrow current\_mixture[l_i]/total\_chars$
  $new\_mixture[l_i] \leftarrow \lfloor new\_prop[l_i] \times total\_chars \rfloor$
end for
return $new\_mixture$