Event-Triggered Prescribed Performance Fuzzy Optimal Tracking Control for Strict-Feedback Nonlinear Systems

Zongsheng Huang
School of Automation Engineering
University of Electronic Science and Technology of China
Chengdu 611731, China
zs__Huang@163.com

***Abstract*—The prescribed performance event-triggered optimal tracking control problem is considered for a class of uncertain strict-feedback nonlinear systems with external disturbances. The disturbance observers are employed to estimate external disturbances. The fuzzy logic systems are employed to approximate unknown nonlinearities and cost function. The controller contains an adaptive fuzzy controller and an optimal compensation term. Firstly, an adaptive fuzzy controller is established by the dynamic surface control (DSC) technique, which is addressed to handle "computation complexity" issue occurred in conventional backstepping approach. An event-triggered mechanism is established to reduce communication load. Subsequently, based on adaptive dynamic programming (ADP) method, an optimal compensation term is developed by minimizing the cost function. In addition, the tracking error can be restricted in the prescribed region with the aid of the prescribed performance control. Thus, the whole control scheme can not only ensure that the tracking error converges to a boundary with prescribed performance but also minimize the cost function, reduce the communication burden and avoid "computation complexit" issue. The boundedness of all signals in the closed-loop is proved. Finally, the simulation example illustrates the validity of the designed control scheme.**

***Index Terms*—Fuzzy control, adaptive dynamic programming, event-triggered control, prescribed performance, strict-feedback nonlinear systems.**

I. Introduction

As a matter of fact, strict-feedback nonlinear form can be adopted to characterize many classical physical systems, such as the hypersonic flight vehicle [1], spacecrafts [2], robot manipulators [3] and so on. Thus, strict-feedback nonlinear systems have received extensive attention and lots of remarkable control approaches have been discussed. In particular, adaptive backstepping method has been regarded as an effective approach in the control field of strict-feedback nonlinear systems with uncertainties [4]. It is worth noting that there exists a widespread class of uncertainties named unstructured uncertainties, which cannot be modeled or repeated and bring a great difficulty for controller design. To handle this challenge, adaptive control approaches with approximators were explored to handle unstructured uncertainties using neural network (NN) or fuzzy logic systems (FLSs) approximators, for example, [5]. Nevertheless, these control schemes mentioned above suffered from a major limitation of "computation complexity" problem. The "computation complexity" issue is generated by the repeated derivation of the virtual controllers in every step within the conventional backstepping method. To cope with this weakness, the dynamic surface control (DSC) technique was firstly presented in [6], in which the filtering variable is generated via a first-order filter and the "computation complexity" can be eliminated. The DSC technique was expanded to adaptive control area for strict-feedback nonlinear systems with parametric uncertainties in [7]. Later, the NN-based adaptive control for the SISO systems with arbitrary uncertainty was proposed in [8]. Thereafter, in [9], combined with "minimal learning parameter" technique, the adaptive NN DSC control method proposed in [8] was improved to settle the " explosion of learning parameters" issue caused by introducing NN as approximator for the investigated systems. Recently, the DSC-based adaptive NN control scheme in [8] was further extended for pure-feedback stochastic nonlinear systems [10], nonstrict-feedback stochastic nonlinear systems [11] and so on.

As a popular control method, optimal control has attracted much attention, such as [12]. As we all know, for linear systems, with the aid of dynamic programming (DP) method, the optimal strategy is calculated associated with Ricatti equation. For nonlinear systems, the optimal solution can be produced related to Hamilton-Jacobian-Bellman (HJB) equation[13]. Nevertheless, the HJB equation can not be obtained via numerical methods. To break through this limitation, adaptive dynamic programming (ADP) method was firstly developed by Werbos [14]. The basic idea of ADP is to approximate the solution of HJB equation by using the function approximation structure according to the optimality principle. Murray *et al.* [15] firstly proposed the ADP-based value iteration algorithm for continuous systems and proved the convergence of algorithm, which is viewed as a major improvement in ADP area. On the basis of [15], in [16], the ADP-based policy iteration algorithm was designed to approximate optimal saturation controller for nonlinear systems with saturation constraints. With the development of ADP, some modified ADP algorithms have been derived. For instance, in [17], a new concept called "min-Hamiltonian" was defined, and an iterative ADP algorithm considering the approximation error was established. In [18], by applying the control input into the tracking error, the optimal strategy can be yielded without taking the reference signal into account. Based on ADP, many practical optimal control problems had been investigated, such as [19] and [20]. These results made a significant contribution on optimal control on the basis of ADP method.

Some of the aforementioned control schemes can allow that the error converges to a small residual set with unknown size. In other words, control accuracy cannot be specified a priori [21]. Nevertheless, in engineering fields, the designed control scheme often requires to meet the prescribed transient and steady-state performance. As an alternative, the prescribed performance control (PPC) was developed in [22] for the first time, which is an effective method to change the steady state and transient state tracking error performance into the performance constraints. With an error transformation, an expected tracking performance can be obtained accordingly. Subsequently, Bechlioulis and Rovithakis extended PPC to MIMO strict-feedback nonlinear systems in [23]. Combined with PPC, many researches have been carried out, such as neural control [24], sliding mode control[25], finite-time control [26] and so on. Currently, numerous improved PPC approaches have also been put forward to realize lower complexity. For example, in [27], an improved PPC with low complexity was provided to deal with the presence of discontinuous reference signal.

These aforementioned results belong to time-driven-based control methods. In the classical time-driven control framework, the actuator updates at any time instant, which results in wasting of resources and wearing of actuator. The event-triggered (ET) control strategy has several advantages over classical time-driven control strategy, such as releasing the communication load, saving the computation resources and reducing the abrasion of actuators [28]. Therefore, ET control approaches have been extensively adopted for varied systems, such as discrete-time systems [29], networked control systems [30], stochastic systems [31], linear systems [32], constrained nonlinear systems [33] and so on. The ET fuzzy controller was developed for the considered system in [34].

The main contributions are summarized as follows.

(1) Compared with [35], [36] that did not employ PPC technique, in this paper, to keep the transient state and steady state performance within the predefined residual set, a performance function is introduced in the procedure of controller design, thus the tracking performance has been improved.

(2) In this paper, the DSC method is combined with ADP for the investigated systems. The proposed controller is composed of an adaptive fuzzy controller and an optimal compensation term, so that the stability of the closed-loop system is guaranteed and the optimal tracking performance is achieved.

(3) In contrast to [37] without discussing saving unnecessary waste of communication, the relative-threshold-based ET mechanism is applied to reduce the communication load. Meanwhile, DSC technique is introduced to circumvent the "computation complexity" issue in this paper.

The structure is listed as follows. In Section II, the considered system and some concepts are given. In Section III, the main results for designing disturbance observer and controller are presented. In Section IV, the stability analysis is provided. Simulation results and conclusion are presented in Section V and Section VI, respectively.

Notations: Throughout this article, for any value or function ϱ, $\hat{\varrho}$ denotes the estimation value. $\varepsilon_{fi}(\cdot), i = 1, \cdots, n$ is the approximation error.

II. Problem Formulation and Preliminaries

A. System Model

Give the model as follows

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + x_{i+1} + d_i(t), 1 \le i \le n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + u + d_n(t), n \ge 2 \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, denotes the state vector, $u \in \mathbb{R}$ is the control input and $y \in \mathbb{R}$ is the output of the system. $f_i(\bar{x}_i) \in \mathbb{R}$, $i = 1, \ldots, n$ is the unknown smooth nonlinear function, and $d_i(t) \in \mathbb{R}$, $i = 1, \ldots, n$ is unknown external disturbance of the nonlinear system.

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