

---

# Diffusion-based Reinforcement Learning via Q-weighted Variational Policy Optimization

---

Shutong Ding<sup>1,3</sup> Ke Hu<sup>1</sup> Zhenhao Zhang<sup>1</sup> Kan Ren<sup>1,3</sup> Weinan Zhang<sup>2</sup>  
Jingyi Yu<sup>1,3</sup> Jingya Wang<sup>1,3</sup> Ye Shi<sup>1,3</sup> \*

<sup>1</sup>ShanghaiTech University    <sup>2</sup>Shanghai Jiao Tong University

<sup>3</sup>MoE Key Laboratory of Intelligent Perception and Human Machine Collaboration

{dingsht, v-huke, v-zhangzh1, renkan}@shanghaitech.edu.cn  
{yujingyi, wangjingya, shiye}@shanghaitech.edu.cn  
wnzhang@sjtu.edu.cn

## Abstract

Diffusion models have garnered widespread attention in Reinforcement Learning (RL) for their powerful expressiveness and multimodality. It has been verified that utilizing diffusion policies can significantly improve the performance of RL algorithms in continuous control tasks by overcoming the limitations of unimodal policies, such as Gaussian policies. Furthermore, the multimodality of diffusion policies also shows the potential of providing the agent with enhanced exploration capabilities. However, existing works mainly focus on applying diffusion policies in offline RL, while their incorporation into online RL has been less investigated. The diffusion model’s training objective, known as the variational lower bound, cannot be applied directly in online RL due to the unavailability of ‘good’ samples (actions). To harmonize the diffusion model with online RL, we propose a novel model-free diffusion-based online RL algorithm named Q-weighted Variational Policy Optimization (QVPO). Specifically, we introduce the Q-weighted variational loss and its approximate implementation in practice. Notably, this loss is shown to be a tight lower bound of the policy objective. To further enhance the exploration capability of the diffusion policy, we design a special entropy regularization term. Unlike Gaussian policies, the log-likelihood in diffusion policies is inaccessible; thus this entropy term is nontrivial. Moreover, to reduce the large variance of diffusion policies, we also develop an efficient behavior policy through action selection. This can further improve its sample efficiency during online interaction. Consequently, the QVPO algorithm leverages the exploration capabilities and multimodality of diffusion policies, preventing the RL agent from converging to a sub-optimal policy. To verify the effectiveness of QVPO, we conduct comprehensive experiments on MuJoCo continuous control benchmarks. The final results demonstrate that QVPO achieves state-of-the-art performance in terms of both cumulative reward and sample efficiency. Our official implementation is released in <https://github.com/wadx2019/qvpo/>.

## 1 Introduction

Recent years have witnessed significant success in the application of diffusion policy in imitation learning [6, 22, 27, 32], and offline reinforcement learning [40, 1, 16, 4, 5, 11, 44, 26]. In imitation

---

\*Corresponding author.

learning, diffusion models [13, 35, 7] are often employed to capture the intricate distributions found within expert datasets. Furthermore, in offline RL, certain works replace unimodal policies with diffusion models to enrich the diversity in sampled actions or decision sequences. This success can be largely attributed to the robust expressiveness and multimodality inherent in diffusion models [13, 35]. Such methods have demonstrated their effectiveness in practical scenarios like robotic navigation [36, 43], robot arm manipulation [6, 17, 33, 23], dexterous hand and legged robot locomotion control [14].

While diffusion policies have been extensively explored, their application in online RL has received relatively less attention. In online RL, value estimation varies with policy changes [45], posing a significant challenge. While diffusion models can effectively capture data distribution, they cannot directly improve the policy due to their training objectives [42]. Consequently, integrating diffusion models into traditional online RL framework [34, 10, 9, 38] is challenging. Some works, such as DIPO [42], propose leveraging Q-function gradients to update actions for higher rewards, followed by employing diffusion models to fit the updated action distribution. However, relying on gradient updates constrains the algorithm’s exploration capability. QSM [31] directly aligns the score and the gradient of the Q-function under the perspective of score-matching. However, since the gradient of the Q-network is inaccurate, QSM has a doubled approximation error from the alignment process, which prevents the policy from converging to optimality.

To address these issues, we propose a novel model-free online RL algorithm called Q-weighted Variational Policy Optimization (QVPO). The core idea behind QVPO is straightforward yet effective. By revisiting the VLO objective of diffusion models and the policy objective of online RL, we discovered that the Variation Lower Bound (VLO) objective, with appropriate weights for state-action pairs, can form a tight lower bound of the policy objective under certain conditions. This new objective, termed Q-weighted VLO loss, is complemented by Q-weight transformation functions, allowing it to be applied to general RL tasks. Additionally, we add an entropy term to the diffusion policy loss to enhance exploration. Calculating the exact entropy is intractable due to the inaccessibility of the probability density function (PDF) of diffusion policies. Therefore, we design an entropy regularization term from another perspective. Moreover, while diffusion policies offer advantages like multimodality, they also introduce large policy variance, leading to inefficient interaction with environments. To address this, we develop an efficient behavior policy through action selection to improve sample efficiency. With QVPO, diffusion policies can fully leverage their exploration capability and multimodality in the online RL paradigm. QVPO achieves state-of-the-art performance in terms of both cumulative reward and sample efficiency compared to traditional online RL methods and existing diffusion-based RL methods in MuJoCo locomotion tasks [39].

Our contributions are summarized as follows:

- 1) **Q-weighted VLO Loss.** By revisiting the VLO loss of diffusion models and policy loss in online RL, we propose the Q-weighted VLO loss, which is the core component of the proposed QVPO method. We further extend its application to general RL tasks via equivalent Q-weight transformation functions. Additionally, we theoretically prove that the Q-weighted VLO loss is a tight lower bound of the policy objective in online RL.
- 2) **Diffusion Entropy Regularization & Efficient Behavior Policy.** We find that the exploration ability of diffusion policies declines with limited diffusion steps. To address this, we apply a special entropy regularization term to the policy loss, which is nontrivial for diffusion policies. Additionally, to cope with the large policy variance of the diffusion model, we develop an efficient behavior policy through action selection to improve sample efficiency.
- 3) **State-of-the-art Performance.** We verified the effectiveness of QVPO on MuJoCo locomotion benchmarks. Experimental results indicate that QVPO achieves state-of-the-art performance in terms of sample efficiency and episodic reward compared to both previous traditional and diffusion-based online RL algorithms.

## 2 Related Works

In this section, we will review the existing works that utilize diffusion models in decision-making tasks and generally divide them into four categories according to different applications.

**Diffusion Policies for Imitation Learning.** Imitation learning is a framework for learning a behavior policy from expert datasets without explicit reward. Diffusion models can effectively fit the distribution of given datasets due to their powerful expressiveness. Some research works such as Diffusion

Policy [6], Crossway Diffusion [22], AVDC [18] and [27] exemplify this by generating robot action sequences via diffusion based policy conditioned on visuomotor observation. Considering the reward is sparse or inaccessible in the expert dataset, BESO [32] leverages the diffusion model in the domain of goal-conditioned imitation learning to learn a goal-specified policy.

**Diffusion Planners.** Planning in RL involves using the dynamic function to inform decision-making over a long horizon. Diffusion planners reduce compounding errors by directly constructing the complete trajectory rather than relying on one-step transitions. Diffuser [15] arranges state-action sequences into a structured two-dimensional array and samples the trajectories based on reward guidance. In subsequent work, Decision Diffuser [2] generates state sequences with classifier-free guidance and then uses inverse dynamic function to derive actions to alleviate the distribution shift problem. Latent Diffuser [20] generates sequences in a learned latent space and then reconstructs the original trajectories. A bunch of works such as [21, 3, 8] leverage hierarchical structures to enhance learning efficiency and decision-making capabilities by decomposing complex tasks into a hierarchy of sub-tasks.

**Diffusion Policies for Offline RL.** Unlike most imitation learning problems without reward and assuming an optimal expert to provide the data, offline RL faces the challenge of learning optimal policy from suboptimal offline datasets [19]. Diffusion-QL [40] integrates the diffusion model with the conventional Q-learning framework. However, the training of Diffusion-QL is unstable in the out-of-distribution (OOD) state region, SRDP [1] reconstructs the state to alleviate the distribution shift caused by OOD. EDP [16] reduces the computation cost during diffusion inference by applying Tweedie’s formula to predict actions. CEP [24] draws samples by diffusion sampling with exact guidance defined by the energy function based on contrastive learning. CPQL [5] uses a consistency model to represent the policy. Some works [12, 4, 11] utilize diffusion models to construct the optimal policy by weighted regression [29, 28]

**Diffusion Policies for Online RL.** Online RL faces a more challenging problem in that it requires algorithms to learn the policy and make decisions in real-time interaction with the environment. DIPO [42] is the first to employ diffusion policies in online RL and proposes a novel policy improvement method. During diffusion policy updates, each action in the reply buffer is updated via action gradient to receive higher rewards. However, the action of gradient updating may deviate from the behavior policy, and the paradigm of gradient updating limits the algorithm’s ability of global exploration and raises computational costs. QSM [31] introduces a new update role for diffusion policy by aligning the score of the diffusion model with the gradient of the Q-function to increase the Q-value of state-action pairs. However, it overlooks the impact of inaccuracies in the value function gradient and biases introduced during the alignment process when updating the policy. This can lead to the policy being influenced by suboptimal data within the buffer. Compared with these existing diffusion-based online RL algorithms, QVPO performs policy optimization with the Q-weighted VLO loss which can be theoretically proven to be the tight lower bound of the policy objective of online RL. This means QVPO does not incur additional errors in policy optimization. Besides, diffusion policy entropy regularization and efficient behavior policy via action selection techniques are provided to further enhance the performance of the diffusion policy in online RL.

### 3 Preliminaries

#### 3.1 Reinforcement Learning

For reinforcement learning (RL), an MDP is defined as  $(\mathcal{S}, \mathcal{A}, p, r, \rho_0, \gamma)$ , where  $\mathcal{S}$  is the state space,  $\mathcal{A}$  is the action space,  $p : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, \infty)$  is the transition probability function of the next state  $s_{t+1}$  given the current state  $s_t$  and the action  $a_t$ ,  $r : \mathcal{S} \times \mathcal{A} \rightarrow [r_{\min}, r_{\max}]$  is the bounded reward function,  $\rho_0 : \mathcal{S} \rightarrow [0, \infty)$  is the distribution of the initial state  $s_0$  and  $\gamma \in [0, 1]$  is the discount factor for value estimation. In an MDP, the process starts from an initial state  $s_0$  sampled from  $\rho_0$  and then samples actions  $a_t$  from the policy  $\pi(a|s) : \mathcal{S} \times \mathcal{A} \rightarrow [0, \infty)$  given the state  $s_t$ . Here  $\tau = (s_0, a_0, s_1, a_1 \dots)$  denotes this kind of trajectory and  $\tau \sim \pi$  denotes the distribution of trajectory  $\tau$  given the policy  $\pi(a|s)$ . The state-value function  $V_\pi(s) = \mathbb{E}_{\tau \sim \pi}[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s]$  represents the expected reward that the agent can obtain under the policy  $\pi$  in the state  $s$ . Compared to the state-value function, the action-state function is  $Q_\pi(s, a) = \mathbb{E}_{\tau \sim \pi}[\sum_{t=0}^{\infty} \gamma^t r_t | a_0 = a, s_0 = s]$  where action  $a_0$  is provided as input. Further, the advantage function refers to the extra benefit of taking a specific action relative to taking the average action at a given state which is defined as the following:  $A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s)$ . The goal of RL is to learn the policy that maximizes the discounted expected

cumulative reward, defined as  $J(\pi) = \mathbb{E}_{\tau \sim \pi} [\sum_{t=0}^{\infty} \gamma^t r_t]$ , which can be optimized via performing multiple steps of policy improvement  $\pi_{k+1} = \arg \max_{\pi} \mathbb{E}_{\tau \sim \pi_k} [Q_{\pi_k}(s, a) \log(\pi(a | s))]$ .

### 3.2 Denoising Diffusion Probabilistic Models

Denoising diffusion probabilistic models (DDPM) [13] are powerful latent variable generative models that transform any data distribution into a simple Gaussian distribution by adding noise (forward process) and then denoise it using neural networks (reverse process). Given a dataset  $\{\mathbf{x}_0^i\}_{i=1}^N$  for  $\mathbf{x}_0^i \sim q(\mathbf{x}_0)$ , the forward process of DDPM transforms the distribution  $q(\mathbf{x}_0)$  into a tractable prior Gaussian distribution by incorporating Gaussian noise in  $T$  steps with the transitions:

$$q(\mathbf{x}_t | \mathbf{x}_{t-1}) := \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t} \mathbf{x}_{t-1}, \beta_t \mathbf{I}), \quad (1)$$

where  $\beta_t$  is the variance schedule. Using the Markov chain property, we can obtain the distribution of  $\mathbf{x}_t$  conditioned on  $\mathbf{x}_0$ :

$$q(\mathbf{x}_t | \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t} \mathbf{x}_0, (1 - \bar{\alpha}_t) \mathbf{I}), \quad (2)$$

where  $\alpha_t = 1 - \beta_t$  and  $\bar{\alpha}_t = \prod_{s=0}^t \alpha_s$ . Eventually, when  $T \rightarrow \infty$ ,  $\mathbf{x}_T$  converges to an isotropic Gaussian distribution. In the reverse process, DDPM first generates noise from the prior distribution  $p(\mathbf{x}_T) = \mathcal{N}(\mathbf{x}_T; 0, \mathbf{I})$  and then gradually denoises it by learning parameterized transitions  $p_{\theta}(\mathbf{x}_{t-1} | \mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_{\theta}(\mathbf{x}_t, t), \Sigma_{\theta}(\mathbf{x}_t, t))$  to fit  $q(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{x}_0)$ , where  $\theta$  denotes the learnable parameters. The training objective of DDPM is to maximize the Variational Lower Bound (VLO) defined as

$$L_{\text{VLO}} = \mathbb{E}_{q(\mathbf{x}_{0:T})} \left[ \log \frac{p_{\theta}(\mathbf{x}_{0:T})}{q(\mathbf{x}_{1:T} | \mathbf{x}_0)} \right]. \text{ Finally, with } \epsilon \sim \mathcal{N}(0, \mathbf{I}), \text{ the loss in DDPM takes the form of:} \quad (3)$$

$$\mathbb{E}_{t \sim [1, T], \mathbf{x}_0, \epsilon_t} [ \|\epsilon_t - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon_t, t)\|^2 ].$$

## 4 Q-weighted Variational Policy Optimization

In this section, we will first revisit the VLO loss [13] of the diffusion model and the policy objective of online RL [37]. Then, we derive the Q-weighted VLO loss, which is the principal component of the proposed Q-weighted Variational Policy Optimization (QVPO). However, the Q-weighted VLO loss cannot be directly applied in general RL tasks. To address this issue, we also provide equivalent Q-weight transformation functions. Since the derived Q-weighted VLO loss is the tight lower bound of the policy loss in online RL, QVPO can effectively avoid the drawbacks of existing online RL methods for diffusion policies [42, 31] mentioned in Section 2.

QVPO also involves practical techniques for the underlying issues of optimizing diffusion policies. Firstly, to further enhance the exploration ability of diffusion policy, we develop a special entropy regularization. This is not trivial for diffusion policy since the log probability of the state-action pair is not available. Besides, we devise an efficient behavior policy via action selection for the diffusion model to avoid the large policy variance of diffusion, which results in sample inefficiency. With these practical techniques, diffusion policy can achieve better performance with fewer online interactions with the environments. Finally, the training procedure of QVPO is shown in Figure 1.

### 4.1 Q-weighted Variational Objective for Diffusion Policy

As mentioned in DIPO [42], optimizing diffusion policies in the online RL paradigm is nontrivial, which principally results from two reasons. In one aspect, if we directly apply the deterministic policy gradient to diffusion policies like Diffusion-QL [40], the backpropagation chain through the denoising procedure of the diffusion model becomes quite long. This leads to high computational costs and instability during training, which severely restricts the performance of diffusion policies in online RL. In another aspect, samples from the optimal policy are required when directly using the variational bound objective to train diffusion policies. However, these optimal samples are typically unavailable in online RL.

In this context, we revisited the VLO objective of diffusion models and the RL policy objective. To our surprise, we discovered that by adding the appropriate weights to the VLO objective, it becomes a tight lower bound of the RL policy objective under certain conditions.

**Theorem 1. (Lower Bound of RL Policy Objective)** *If  $Q(s, a) \geq 0$  for any state-action pair  $(s, a)$ , the Q-weighted variational bound objective of diffusion policy*

$$\mathbb{E}_{s, a \sim p(s' | s, a), \pi_k(a | s)} \left[ Q(s, a_0) \cdot \mathbb{E}_{a_{1:T} \sim q(a_{1:T} | s, a_0)} \left[ \log \frac{\pi_{\theta}(a_{0:T} | s)}{q(a_{1:T} | s, a_0)} \right] \right]$$

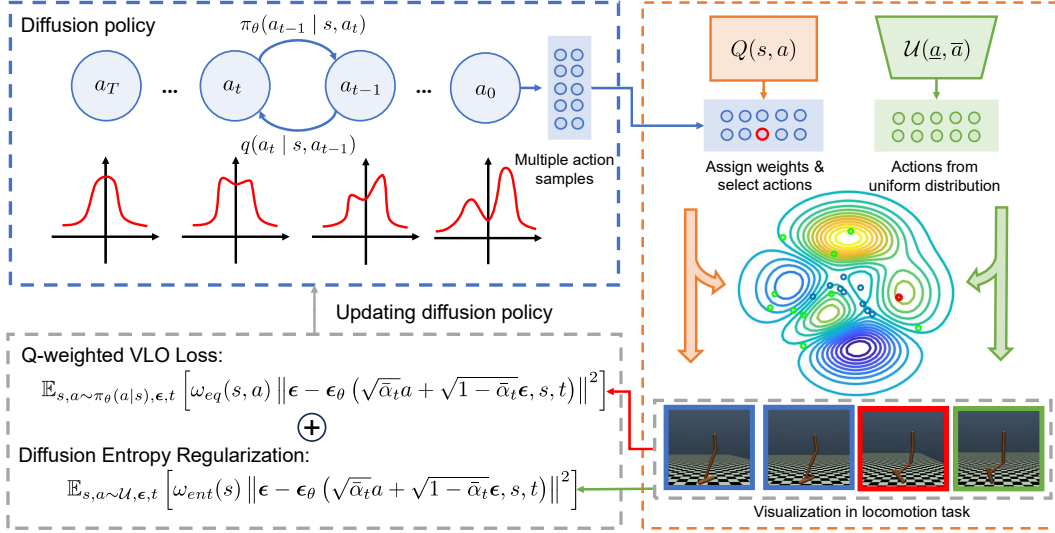


Figure 1: The training pipeline of QVPO. In each training epoch, QVPO first utilizes the diffusion policy to generate multiple action samples for every state. Then, these action samples will be selected and endowed with different weights according to the Q value given by the value network. Besides, action samples from uniform distribution are also created for the diffusion entropy regularization term. With these action samples and weights, we can finally optimize the diffusion policy via the combined objective of Q-weighted VLO loss and diffusion entropy regularization term.

is the tight lower bound of the objective of RL policy

$$\mathbb{E}_{s, a \sim p(s' | s, a), \pi_k(a | s)} [Q(s, a) \log(\pi_\theta(a | s))],$$

and the equality holds when the policy converges.

The proof can be referred to in the supplementary materials. According to this theorem, we can derive the Q-weighted variational bound loss as (4), which can be applied to diffusion policy optimization.

$$\mathcal{L}(\theta) \triangleq \mathbb{E}_{s, a \sim \pi_k(a | s), \epsilon, t} \left[ \frac{\beta_t^2}{2\sigma_t^2 \alpha_t (1 - \bar{\alpha}_t)} Q(s, a) \cdot \|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}a + \sqrt{1 - \bar{\alpha}_t}\epsilon, s, t)\|^2 \right]. \quad (4)$$

According to [13], removing the coefficient  $\frac{\beta_t^2}{2\sigma_t^2 \alpha_t (1 - \bar{\alpha}_t)}$  does not affect the training of diffusion. Hence, the final Q-weighted VLO loss is defined as

$$\mathcal{L}(\theta) \triangleq \mathbb{E}_{s, a \sim \pi_k(a | s), \epsilon, t} \left[ Q(s, a) \cdot \|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}a + \sqrt{1 - \bar{\alpha}_t}\epsilon, s, t)\|^2 \right]. \quad (5)$$

While we can now use (5) to optimize the diffusion policy, two issues remain to be resolved.

1) **Negative Q value.** In real-world decision-making tasks, it is difficult to ensure that the returned reward is always non-negative, which means the Q value may be negative for some state-action pairs. Therefore, to apply QVPO in practical tasks, we must address situations where the  $Q(s, a)$  value is negative for certain states and actions.

2) **High-quality Training Samples.** According to (5), achieving significant policy improvement requires obtaining certain rare state-action samples with high Q values. However, this is a significant challenge with limited interaction with the environment. This issue is also common in other online RL methods based on weighted policy training, such as RWR [30], and it hinders their application to real-world tasks.

## 4.2 Equivalent Transformation for Q-weighted VLO Loss

To resolve these two problems in training diffusion policies with the Q-weighted VLO loss, we propose equivalent Q-weight transformation functions to convert the Q value into an equivalent positive Q weight and leverage the powerful data-synthesizing ability of the diffusion model to generate high-quality samples for training. The new equivalent Q-weighted VLO loss is defined in (6). We will introduce the detailed implementation of these solutions in the following contexts.

$$\mathcal{L}(\theta) \triangleq \mathbb{E}_{s, a \sim \pi_k(a | s), \epsilon, t} \left[ \omega_{eq}(s, a) \|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}a + \sqrt{1 - \bar{\alpha}_t}\epsilon, s, t)\|^2 \right]. \quad (6)$$

**Theorem 2. (Optimal Solution in One Policy Improvement Step)** Considering the optimization problem in policy improvement step  $\pi_{k+1} = \arg \max_{\pi} \mathbb{E}_{\tau \sim \pi_k} [Q_{\pi_k}(s, a) \log(\pi(a | s))]$ , the optimal policy in a given state  $s$  is

$$p(a | s) = \begin{cases} \frac{\pi(a|s)Q(s,a)}{\int \mathbb{I}_{Q(s,a)>0}(a)\pi(a|s)Q(s,a)da}, & Q(s, a) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

when there exists a such that  $Q(s, a) > 0$ , and

$$p(a | s) = \frac{1}{N} \sum_{i=1}^N \delta(x - a_i), a_i \in \left\{ a \mid Q(s, a) = \max_a Q(s, a), \pi(a | s) > 0 \right\}, \quad (8)$$

when  $Q(s, a) \leq 0$  for any  $a \in \mathcal{A}$ ,  $\delta(x - a)$  is the Dirac function,  $N$  is the cardinality of set  $\{a \mid Q(s, a) = \max_a Q(s, a), \pi(a | s) > 0\}$ ,  $\pi(a | s)$  is the behavior policy,  $p(a | s)$  is the policy to be optimized and  $\mathbb{I}_{Q(s,a)>0}$  is the indicator function that judges whether  $Q(s, a) > 0$ .

The proof can be referred to in the supplementary materials. Hence, with Theorem 2, we can obtain the probability density function (PDF) of the optimal policy in one policy improvement step and further use  $\frac{p(a|s)}{\pi(a|s)}$  as the weight to achieve an equivalent optimized diffusion policy according to (6).

However, the difficulty of judging whether any  $Q(s, a) \leq 0$  in a certain state  $s$  is still a problem. Therefore, we first come up with an approximate solution "qcut" weight transformation function, which returns  $Q(s, a)$  when  $Q(s, a) \geq 0$  and a small value  $\epsilon > 0$  when  $Q(s, a) < 0$ . Furthermore, the "qcut" weight transformation function just endows the best action with the weight mentioned above and discards the left action samples via setting them as zero.

In short, qcut weight transformation function takes the formulation of the above two situations into account and "cuts" them via a small value  $\epsilon > 0$ . Notably, we remove the denominator  $\int \mathbb{I}_{Q(s,a)>0}\pi(a | s)Q(s, a)da$  in qcut weight transformation function. This is because the denominator  $\int \mathbb{I}_{Q(s,a)>0}(a)\pi(a | s)Q(s, a)da$  is somewhat like the  $V(s)$ , which denotes the "importance" of the state  $s$ . Therefore, removing this item can allow important states to obtain larger weights during training, which is confirmed in practice.

However, we find that the qcut weight transformation function does not work very well when  $Q$  values in most state-action pairs are less than zero. To address this issue, we propose another "qadv" weight transformation function, which is much simpler but performs better in our experiments.

$$\omega_{eq}(s, a) \triangleq \omega_{qadv}(s, a) = \begin{cases} A(s, a), & A(s, a) \geq 0 \\ 0, & A(s, a) < 0 \end{cases} \quad (9)$$

where  $A(s, a) = Q(s, a) - V(s)$  is the advantage function. qadv weight transformation function replaces the  $Q(s, a)$  with the advantage  $A(s, a)$  to avoid the second situation. Moreover, applying  $Q$  value and advantage in policy optimization has been proved equivalent and can also reduce the training variance in previous works like [25].

It is worth noting that, for qadv weight transformation function, we do not require an extra network to approximate  $V(s)$  to calculate  $A(s, a)$ . We can always sample a certain number of actions for a state  $s$  with diffusion policy and estimate  $V(s)$  using the average of their  $Q(s, a)$ . Besides, for qadv function, we also recommend just selecting the best action sample with the largest  $A$  value for each state in policy optimization, which can further improve the quality of training samples and reduce the training cost.

**Remark.** Although the weighted VLO loss of QVPO looks similar to AWR [28] and EDP [16], they are different intrinsically. With the Q-weighted VLO loss, QVPO trains the diffusion policy with the weighted sample from the current policy, while AWR or EDP trains the diffusion policy with the weighted sample from the policy in the offline dataset (replay buffer). These "on-policy" samples truly benefit the online training. Moreover, AWR and EDP utilize the exp weight function; but this function is too conservative for online RL.

### 4.3 Enhancing Policy Exploration via Diffusion Entropy Regularization

While diffusion policies can achieve impressive performance with the Q-weighted VLO loss in online RL, their powerful exploration capabilities have yet to be fully harnessed. As noted in Diffusion-QL

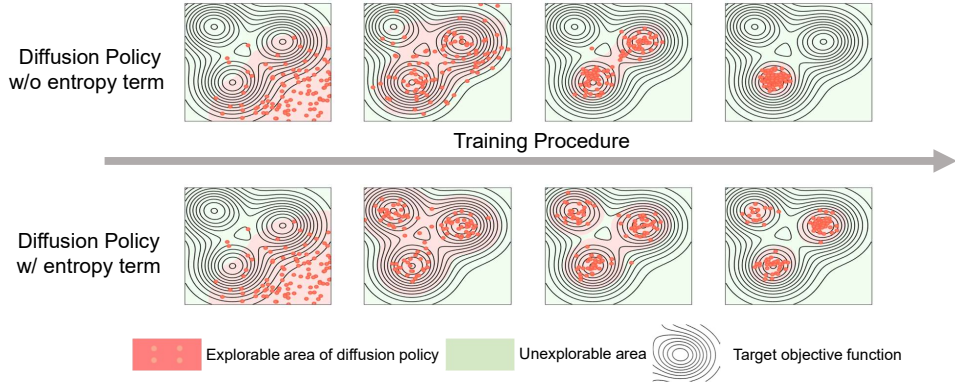


Figure 2: A toy example on continuous bandit to show the effect of diffusion entropy regularization term via the changes of the explorable area for diffusion policy with the training procedure. The contour lines indicate the reward function of continuous bandit, which is an arbitrarily selected function with 3 peaks.

[40], the policy expressiveness of the diffusion model decreases with fewer diffusion steps. In our experiments, we find that not only does policy expressiveness decline, but the exploration capability of the diffusion model also diminishes when the number of diffusion steps is reduced. However, it is essential to limit the number of diffusion steps to avoid excessive training and evaluation costs in real-world applications. Therefore, studying how to enhance the exploration ability of the diffusion model with a limited number of diffusion steps is necessary.

Adding an extra entropy regularization term to the policy loss appears to be a good solution, as it has been validated for categorical policies in discrete action spaces and Gaussian policies in continuous action spaces. However, estimating the entropy for a diffusion policy is nontrivial due to the inaccessibility of the log-likelihood of action samples. Moreover, maximizing the entropy of a policy can be viewed as narrowing the distance between the policy and a maximum entropy distribution (i.e., uniform distribution) in some sense. Thus, by recalling the VLO loss of the diffusion model, the entropy of a diffusion model can be increased with training samples from the uniform distribution. Based on this idea, we propose an entropy regularization term for diffusion policies as follows,

$$\mathcal{L}_{ent}(\theta) \triangleq \mathbb{E}_{s,a \sim \mathcal{U}, \epsilon, t} \left[ \omega_{ent}(s) \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t}a + \sqrt{1 - \bar{\alpha}_t}\epsilon, s, t) \right\|^2 \right], \quad (10)$$

where  $\omega_{ent}(s) = \omega_{ent} \sum_{i=1}^N \frac{\omega_{eq}(s, a_i)}{N}$  is the coefficient related to state  $s$  for balancing the trade-off between exploitation and exploration. Here,  $N$  represents the number of selected training samples from the diffusion policy for state  $s$ . When  $\omega_{ent}$  is large, the exploration ability of the diffusion policy is improved, whereas the exploitation ability is reduced. Figure 2 is the experiment results on a continuous bandit toy example that clearly illustrates the effect of this entropy regularization term on the diffusion policy, and the concrete reward function of this bandit problem is  $R(x) = \sum_{i=1}^3 w_i \frac{1}{2\pi\sigma_i^2} \exp\left(-\frac{1}{2\sigma_i^2}(x - \mu_i)^T(x - \mu_i)\right)$ , where  $w_i = 1.5$ ,  $\sigma_i = 0.1$ , and  $\mu_i = [-1.35, 0.65]^T$ ,  $[-0.65, 1.35]^T$  and  $[-1.61, 1.61]^T$  respectively.

#### 4.4 Reducing Diffusion Policy Variance via Action Selection

Despite the diffusion model that allows the online RL agent to seek better policies, it also introduces a large policy variance. This results in inefficiency for online interactions of the behavior policy with the environment. To address this issue, we propose an efficient behavior policy via action selection for diffusion model  $\pi_{\theta}^K(a | s)$ , which improves the sample efficiency. Specifically, it is motivated by the idea that the efficiency of action samples from behavior policy depends on their Q values. In other words, action samples from behavior policy with high Q values indicate that the following trajectory also has a large underlying reward and is of great significance for training. Specifically, the efficient behavior policy  $\pi_{\theta}^K(a | s)$  is defined as:

$$\pi_{\theta}^K(a | s) \triangleq \underset{a \in \{a_1, \dots, a_K \sim \pi_{\theta}(a|s)\}}{\operatorname{argmax}} Q(s, a).$$

Furthermore, while using the efficient behavior policy  $\pi_{\theta}^K(a | s)$  as a behavior policy can improve sample efficiency, it is not recommended to apply the same action selection number  $K$  to the target

policy for calculating the TD target. This is due to the large policy variance of the diffusion policy, which can result in a serious overestimation of the target Q value. Hence, if we choose the action selection number  $K_b$  for the behavior policy, a smaller action selection number  $K_t < K_b$  for the target policy is often a good choice in practice.

Table 1: Comparison of QVPO and 6 other online RL algorithms in evaluation results. (N/A indicates the algorithm does not work)

Environments	PPO	SPO	TD3	SAC	DIPO	QSM	QVPO(*)
Hopper-v3	3154.3(426.2)	2212.8(988.4)	3267.5(8.5)	2996.6(111.9)	3295.4(7.0)	2154.7(998.2)	<b>3728.5(13.8)</b>
Walker2d-v3	3751.5(609.1)	3321.8(1328.9)	3513.9(40.7)	4888.1(80.0)	4681.7(25.7)	3613.4(1443.5)	<b>5191.8(60.2)</b>
Ant-v3	2781.9(74.1)	2100.2(302.4)	4583.8(69.5)	5030.9(1000.3)	5665.9(54.7)	N/A	<b>6425.1(67.6)</b>
HalfCheetah-v3	4773.5(53.4)	4008.2(246.8)	10388.6(80.4)	10616.9(72.8)	9590.5(67.5)	3888.2(632.6)	<b>11385.6(164.5)</b>
Humanoid-v3	713.7(85.9)	797.4(262.1)	<b>5353.5(53.7)</b>	5159.7(475.3)	4945.5(898.6)	4793.1(229.5)	<b>5306.6(14.5)</b>

## 5 Experiments

In this section, we first describe a practical implementation of QVPO, as shown in Algorithm 1, which is implemented in an off-policy fashion. Then, we evaluate the proposed algorithm on different decision-making tasks and with various hyper-parameters. With these experimental results, we aim to answer three questions:

- How does QVPO compare to previous popular online RL algorithms and existing diffusion-based online RL algorithms?
- How does the entropy regularization term affect the performance of QVPO?
- Can the  $K$ -efficient behavior policy significantly improve the sample efficiency of QVPO?

---

### Algorithm 1 Q-weighted Variational Policy Optimization

---

**Input:** Diffusion policy  $\pi_\theta(a | s)$ , value network  $Q_\omega(s, a)$ , replay buffer  $\mathcal{D}$ ,  $K_b$ -efficient diffusion policy for behavior policy,  $K_t$ -efficient diffusion policy for target policy, number of training samples  $N_d$  from diffusion policy, number of training samples  $N_e$  from uniform distribution  $\mathcal{U}(\underline{a}, \bar{a})$ .

- 1: **for**  $t$  **in**  $1, 2, \dots, T$  **do**
  - 2:   Sample the action using the diffusion policy  $\pi_\theta^{K_b}(a | s_t)$ .
  - 3:   Take the action  $a_t$  in the environment and store the returned transition in  $\mathcal{D}$ .
  - 4:   Sample a mini-batch  $\mathcal{B}$  of transitions in  $\mathcal{D}$ .
  - 5:   Generate  $N_d$  samples from  $\pi_\theta(a | s)$ , and  $N_e$  samples from  $\mathcal{U}(\underline{a}, \bar{a})$  for each state  $s$  in  $\mathcal{B}$ .
  - 6:   Endow the  $N_d$  samples with weights (9).
  - 7:   Select an action sample  $a_{max}$  with maximum weight among  $N_d$  samples for training.
  - 8:   Endow the  $N_e$  samples with the weight  $\omega_{ent}(s) = \omega_{ent} \cdot \omega_{eq}(s, a_{max})$ .
  - 9:   Update the parameters of the diffusion policy using the summation of (6) and (10).
  - 10:   Construct TD target as  $y_t = r_t + \gamma Q_\omega(s_{t+1}, \pi_\theta^{K_t}(a | s_{t+1}))$  for each  $(s_t, a_t, r_t, s_{t+1})$  in  $\mathcal{B}$ .
  - 11:   Update the parameters of the value network using MSE loss.
  - 12: **end for**
- 

### 5.1 Comparative Evaluation

To demonstrate the effectiveness of our method, we compared QVPO with six other online model-free RL algorithms: two off-policy algorithms (TD3 [9] and SAC [10]), two on-policy algorithms (PPO [34] and SPO [3]), and one advanced diffusion-based online RL algorithm (DIPO [42], QSM [31]). These comparisons were conducted on five MuJoCo locomotion tasks [39]. We plotted the learning curves of QVPO and the six other algorithms over five random runs, as shown in Figure 3. The solid curve represents the average return, and the transparent shaded region represents the standard deviation. Each experiment was conducted over  $1e6$  training epochs. According to the learning curves, QVPO achieves state-of-the-art performance compared to the other six algorithms, and converges much faster than the other algorithms, further demonstrating its sample efficiency. Additionally, the practical implementation of QVPO follows SAC in critic part, which utilizes doubled Q networks and only use the minimum for policy and critic update.

Table 1 presents the evaluation results of QVPO and six other algorithms on the MuJoCo locomotion tasks after  $1e6$  iterations, further confirming the superiority of QVPO. Notably, during the evaluation



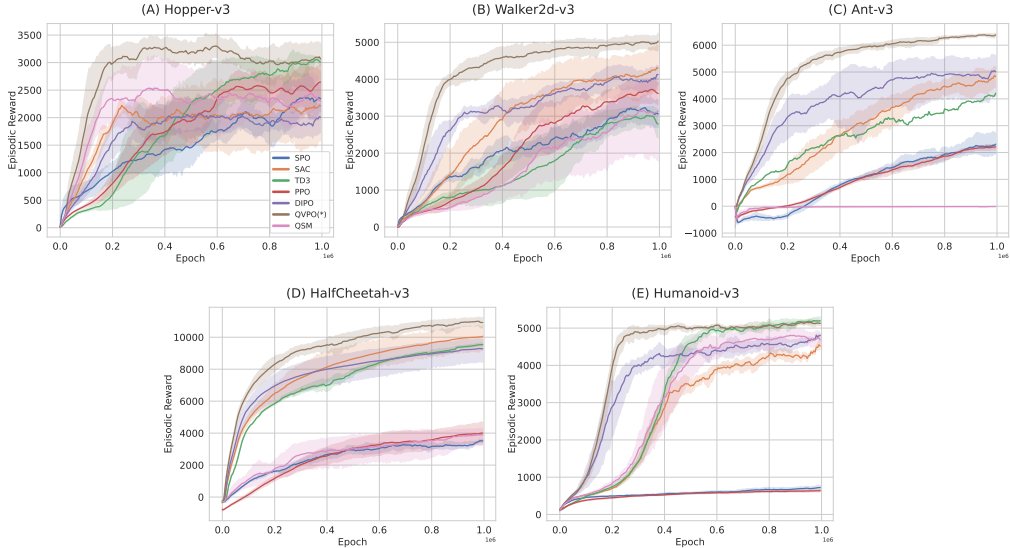


Figure 3: Learning Curves of different algorithms on 5 Mujoco locomotion benchmarks across 5 runs. The x-axis is the number of training epochs. The y-axis is the episodic reward. the plots smoothed with a window of 5000.

stage, we apply the efficient behavior policy with a large action selection number  $K = 32$  instead of the deterministic denoising procedure used in DIPO [42]. As illustrated in [41], the high-probability region in each diffusion step is not the origin but a Gaussian sphere. Hence, using the deterministic denoising procedure in the evaluation stage is not a good choice. In that case, using the proposed efficient behavior policy during evaluation is more reasonable. More details related to the experiments can be found in the supplementary materials.

## 5.2 Ablation Study and Parameter Analysis

To further examine the significance of each component in QVPO, we also conducted the ablation study and parameter analysis on the effects of the entropy regularization and the  $K$ -efficient behavior policy. Here We use the experiment on Ant-v3 as an example since the final results are similar across different locomotion tasks.

**Effect of Diffusion Entropy Regularization.** As shown in Figure 4, without the diffusion entropy regularization loss, QVPO with 100 diffusion steps achieves much better performance than QVPO with 20 diffusion steps. However, when the entropy regularization loss is applied, QVPO with 20 diffusion steps achieves close performance to QVPO with 100 diffusion steps, which verifies the effectiveness of the entropy regularization term.

**Action Selection number  $K$  for Efficient Behavior Policy.** Figure 5 presents the performance of QVPO with different action selection numbers for behavior policy  $K_b$  and for target policy  $K_t$ . It can be observed that QVPO with action selection numbers  $K_b = 4, K_t = 1$  is superior to QVPO with action selection numbers  $K_b = 1, K_t = 1$ , while the training stability of QVPO with  $K_b = 4, K_t = 2$  and  $K_b = 4, K_t = 1$  is better than that of QVPO with  $K_b = 4, K_t = 4$ . This indicates that efficient behavior policy via action selection can improve sample efficiency and setting the action selection numbers  $K_t < K_b$  can help reduce the overestimation error of the target Q value. Besides, the experiment with  $K_b = 20, K_t = 2$  shows that a

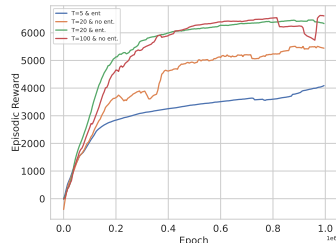


Figure 4: Comparison between QVPO with and without the diffusion entropy regularization.

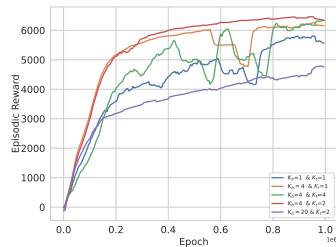


Figure 5: Comparison of QVPO with different action selection numbers for behavior policy  $K_b$  and for target policy  $K_t$ .

too-high action selection number will lead to a limited exploration. Therefore, setting  $K_b = 4$  can balance exploration and exploitation well.

## 6 Conclusion

In this paper, we proposed a novel diffusion-based online RL algorithm called QVPO that sufficiently exploits the expressiveness and multimodality of diffusion policy. QVPO leverages the Q-weighted VLO loss, a core component that serves as a tight lower bound of the policy objective in online RL under certain conditions, facilitated by Q-weight transformation functions. Additionally, we designed a special entropy regularization term to enhance the exploration capabilities of diffusion policies and the efficient behavior policy to improve sample efficiency by reducing the behavior policy variance. Our comprehensive experiments on MuJoCo continuous control benchmarks demonstrate that QVPO achieves state-of-the-art performance in terms of both cumulative reward and sample efficiency, surpassing both traditional and existing diffusion-based online RL methods.

However, there are still several underlying challenges in applying diffusion policies to online RL that require further exploration. For instance, although we developed a special entropy regularization loss to approximate the effect of maximizing entropy, it lacks the ability to adaptively adjust the entropy term and cannot incorporate the entropy of the diffusion policy into the TD target for soft policy iteration, as seen in SAC. Future work will focus on developing adaptive entropy adjustment mechanisms and integrating entropy into the TD target to enable soft policy iteration, which we believe will further enhance the performance of diffusion policies in online RL.

## Acknowledgement

This work was supported by NSFC (No.62303319), Shanghai Sailing Program (22YF1428800), Shanghai Local College Capacity Building Program (23010503100), ShanghaiTech AI4S Initiative SHTAI4S202404, Shanghai Frontiers Science Center of Human-centered Artificial Intelligence (ShangHAI), HPC Platform of ShanghaiTech University, MoE Key Laboratory of Intelligent Perception and Human-Machine Collaboration (ShanghaiTech University) and Shanghai Engineering Research Center of Intelligent Vision and Imaging. We also thank Ming Zhou, Shanghai AI Laboratory, for his discussion and suggestion in our proof of Theorem 4.1.

## References

- [1] Suzan Ece Ada, Erhan Oztop, and Emre Ugur. Diffusion policies for out-of-distribution generalization in offline reinforcement learning. *IEEE Robotics and Automation Letters*, 9(4):3116–3123, April 2024.
- [2] Anurag Ajay, Yilun Du, Abhi Gupta, Joshua Tenenbaum, Tommi Jaakkola, and Pulkit Agrawal. Is conditional generative modeling all you need for decision-making? *arXiv preprint arXiv:2211.15657*, 2022.
- [3] Chang Chen, Fei Deng, Kenji Kawaguchi, Caglar Gulcehre, and Sungjin Ahn. Simple hierarchical planning with diffusion. *arXiv preprint arXiv:2401.02644*, 2024.
- [4] Huayu Chen, Cheng Lu, Chengyang Ying, Hang Su, and Jun Zhu. Offline reinforcement learning via high-fidelity generative behavior modeling. *arXiv preprint arXiv:2209.14548*, 2022.
- [5] Yuhui Chen, Haoran Li, and Dongbin Zhao. Boosting continuous control with consistency policy. *arXiv preprint arXiv:2310.06343*, 2023.
- [6] Cheng Chi, Siyuan Feng, Yilun Du, Zhenjia Xu, Eric Cousineau, Benjamin Burchfiel, and Shuran Song. Diffusion policy: Visuomotor policy learning via action diffusion. *arXiv preprint arXiv:2303.04137*, 2023.
- [7] Prafulla Dhariwal and Alexander Nichol. Diffusion models beat gans on image synthesis. *Advances in neural information processing systems*, 34:8780–8794, 2021.
- [8] Zibin Dong, Jianye Hao, Yifu Yuan, Fei Ni, Yitian Wang, Pengyi Li, and Yan Zheng. Diffuser-Lite: Towards real-time diffusion planning. *arXiv preprint arXiv:2401.15443*, 2024.

- [9] Scott Fujimoto, Herke Hoof, and David Meger. Addressing function approximation error in actor-critic methods. In *International conference on machine learning*, pages 1587–1596. PMLR, 2018.
- [10] Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, and Sergey Levine. Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor. In *International conference on machine learning*, pages 1861–1870. PMLR, 2018.
- [11] Philippe Hansen-Estruch, Ilya Kostrikov, Michael Janner, Jakub Grudzien Kuba, and Sergey Levine. Idql: Implicit q-learning as an actor-critic method with diffusion policies. *arXiv preprint arXiv:2304.10573*, 2023.
- [12] Longxiang He, Linrui Zhang, Junbo Tan, and Xueqian Wang. Diffcps: Diffusion model based constrained policy search for offline reinforcement learning. *arXiv preprint arXiv:2310.05333*, 2023.
- [13] Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. *Advances in neural information processing systems*, 33:6840–6851, 2020.
- [14] Xiaoyu Huang, Yufeng Chi, Ruofeng Wang, Zhongyu Li, Xue Bin Peng, Sophia Shao, Borivoje Nikolic, and Koushil Sreenath. Diffuseloco: Real-time legged locomotion control with diffusion from offline datasets, 2024.
- [15] Michael Janner, Yilun Du, Joshua B Tenenbaum, and Sergey Levine. Planning with diffusion for flexible behavior synthesis. *arXiv preprint arXiv:2205.09991*, 2022.
- [16] Bingyi Kang, Xiao Ma, Chao Du, Tianyu Pang, and Shuicheng Yan. Efficient diffusion policies for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 36, 2024.
- [17] Tsung-Wei Ke, Nikolaos Gkanatsios, and Katerina Fragkiadaki. 3d diffuser actor: Policy diffusion with 3d scene representations, 2024.
- [18] Po-Chen Ko, Jiayuan Mao, Yilun Du, Shao-Hua Sun, and Joshua B. Tenenbaum. Learning to act from actionless videos through dense correspondences, 2023.
- [19] Sergey Levine, Aviral Kumar, George Tucker, and Justin Fu. Offline reinforcement learning: Tutorial, review, and perspectives on open problems. *arXiv preprint arXiv:2005.01643*, 2020.
- [20] Wenhao Li. Efficient planning with latent diffusion. *arXiv preprint arXiv:2310.00311*, 2023.
- [21] Wenhao Li, Xiangfeng Wang, Bo Jin, and Hongyuan Zha. Hierarchical diffusion for offline decision making. In *International Conference on Machine Learning*, pages 20035–20064. PMLR, 2023.
- [22] Xiang Li, Varun Belagali, Jinghuan Shang, and Michael S Ryoo. Crossway Diffusion: Improving diffusion-based visuomotor policy via self-supervised learning. *arXiv preprint arXiv:2307.01849*, 2023.
- [23] Toru Lin, Yu Zhang, Qiyang Li, Haozhi Qi, Brent Yi, Sergey Levine, and Jitendra Malik. Learning visuotactile skills with two multifingered hands, 2024.
- [24] Cheng Lu, Huayu Chen, Jianfei Chen, Hang Su, Chongxuan Li, and Jun Zhu. Contrastive energy prediction for exact energy-guided diffusion sampling in offline reinforcement learning. In *International Conference on Machine Learning*, pages 22825–22855. PMLR, 2023.
- [25] Volodymyr Mnih, Adria Puigdomenech Badia, Mehdi Mirza, Alex Graves, Timothy Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *International conference on machine learning*, pages 1928–1937. PMLR, 2016.
- [26] Fei Ni, Jianye Hao, Yao Mu, Yifu Yuan, Yan Zheng, Bin Wang, and Zhixuan Liang. Metadiffuser: Diffusion model as conditional planner for offline meta-rl. In *International Conference on Machine Learning*, pages 26087–26105. PMLR, 2023.
- [27] Tim Pearce, Tabish Rashid, Anssi Kanervisto, Dave Bignell, Mingfei Sun, Raluca Georgescu, Sergio Valcarcel Macua, Shan Zheng Tan, Ida Momennejad, Katja Hofmann, and Sam Devlin. Imitating human behaviour with diffusion models, 2023.
- [28] Xue Bin Peng, Aviral Kumar, Grace Zhang, and Sergey Levine. Advantage-weighted regression: Simple and scalable off-policy reinforcement learning. *arXiv preprint arXiv:1910.00177*, 2019.

- [29] Jan Peters, Katharina Mulling, and Yasemin Altun. Relative entropy policy search. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 24, pages 1607–1612, 2010.
- [30] Jan Peters and Stefan Schaal. Reinforcement learning by reward-weighted regression for operational space control. In *Proceedings of the 24th international conference on Machine learning*, pages 745–750, 2007.
- [31] Michael Psenka, Alejandro Escontrela, Pieter Abbeel, and Yi Ma. Learning a diffusion model policy from rewards via q-score matching. *arXiv preprint arXiv:2312.11752*, 2023.
- [32] Moritz Reuss, Maximilian Li, Xiaogang Jia, and Rudolf Lioutikov. Goal-conditioned imitation learning using score-based diffusion policies. *arXiv preprint arXiv:2304.02532*, 2023.
- [33] Paul Maria Scheickl, Nicolas Schreiber, Christoph Haas, Niklas Freymuth, Gerhard Neumann, Rudolf Lioutikov, and Franziska Mathis-Ullrich. Movement primitive diffusion: Learning gentle robotic manipulation of deformable objects, 2023.
- [34] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*, 2017.
- [35] Yang Song, Jascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. Score-based generative modeling through stochastic differential equations. *arXiv preprint arXiv:2011.13456*, 2020.
- [36] Ajay Sridhar, Dhruv Shah, Catherine Glossop, and Sergey Levine. Nomad: Goal masked diffusion policies for navigation and exploration, 2023.
- [37] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.
- [38] Richard S Sutton, David McAllester, Satinder Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. *Advances in neural information processing systems*, 12, 1999.
- [39] Emanuel Todorov, Tom Erez, and Yuval Tassa. Mujoco: A physics engine for model-based control. In *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 5026–5033. IEEE, 2012.
- [40] Zhendong Wang, Jonathan J Hunt, and Mingyuan Zhou. Diffusion policies as an expressive policy class for offline reinforcement learning. In *The Eleventh International Conference on Learning Representations*, 2022.
- [41] Lingxiao Yang, Shutong Ding, Yifan Cai, Jingyi Yu, Jingya Wang, and Ye Shi. Guidance with spherical gaussian constraint for conditional diffusion. *arXiv preprint arXiv:2402.03201*, 2024.
- [42] Long Yang, Zhixiong Huang, Fenghao Lei, Yucun Zhong, Yiming Yang, Cong Fang, Shiting Wen, Binbin Zhou, and Zhouchen Lin. Policy representation via diffusion probability model for reinforcement learning. *arXiv preprint arXiv:2305.13122*, 2023.
- [43] Gengyu Zhang, Hao Tang, and Yan Yan. Versatile navigation under partial observability via value-guided diffusion policy, 2024.
- [44] Zhengbang Zhu, Minghuan Liu, Liyuan Mao, Bingyi Kang, Minkai Xu, Yong Yu, Stefano Ermon, and Weinan Zhang. Madiff: Offline multi-agent learning with diffusion models, 2023.
- [45] Zhengbang Zhu, Hanye Zhao, Haoran He, Yichao Zhong, Shenyu Zhang, Yong Yu, and Weinan Zhang. Diffusion models for reinforcement learning: A survey. *arXiv preprint arXiv:2311.01223*, 2023.

## A Proofs

### A.1 Proof of Theorem 1

**Theorem 1. (Lower Bound of RL Policy Objective)** If  $Q(s, a) \geq 0$  for any state-action pair  $(s, a)$ , the  $Q$ -weighted variational bound objective of diffusion policy

$$\mathbb{E}_{s, a \sim p(s'|s, a), \pi_k(a|s)} \left[ Q(s, a_0) \cdot \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{\pi_\theta(a_{0:T} | s)}{q(a_{1:T} | s, a_0)} \right] \right]$$

is the tight lower bound of the objective of RL policy

$$\mathbb{E}_{s, a \sim p(s'|s, a), \pi_k(a|s)} [Q(s, a) \log(\pi_\theta(a|s))],$$

and equality holds when the policy converges.

*Proof.* The policy objective of online RL is

$$\max_{\theta} \mathbb{E}_{s, a \sim p(s'|s, a), \pi_k(a|s)} [Q(s, a) \log(\pi_\theta(a|s))],$$

where  $\pi_k(a | s)$  indicates the behavior policy in state  $s$ ,  $\pi_\theta(a | s)$  is the policy to be optimized, and  $Q(s, a)$  represents the Q value of the state-action pair  $(s, a)$ . Then, if  $\pi_\theta(a | s)$  is a diffusion policy, the inner-term  $Q(s, a) \log(\pi_\theta(a|s))$  equals to

$$Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{\pi_\theta(a_{0:T} | s)}{q(a_{1:T} | s, a_0)} \right] + Q(s, a_0) D_{KL}(q(a_{1:T} | s, a_0) \parallel \pi_\theta(a_{1:T} | s, a_0)).$$

According to the Jensen's Inequality (convexity of  $-\log$ ) and  $Q(s, a) \geq 0$ , the second KL divergence term

$$\begin{aligned} & Q(s, a_0) D_{KL}(q(a_{1:T} | s, a_0) \parallel \pi_\theta(a_{1:T} | s, a_0)) \\ &= Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{q(a_{1:T} | s, a_0)}{\pi_\theta(a_{1:T} | s, a_0)} \right] \\ &= Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \frac{\pi_\theta(a_{1:T} | s, a_0)}{q(a_{1:T} | s, a_0)} \right] \\ &\geq -Q(s, a_0) \log \left( \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \frac{\pi_\theta(a_{1:T} | s, a_0)}{q(a_{1:T} | s, a_0)} \right] \right) \\ &= -Q(s, a_0) \log(1) \\ &= 0. \end{aligned}$$

Obviously, the equality holds when  $\pi_\theta(a_{1:T} | s, a_0) = q(a_{1:T} | s, a_0)$ . Finally, the  $Q$ -weighted variational bound objective

$$\mathbb{E}_{s, a_0 \sim p(s'|s, a_0), \pi_k(a|s)} \left[ Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{\pi_\theta(a_{0:T} | s)}{q(a_{1:T} | s, a_0)} \right] \right]$$

is the tight lower bound of the RL policy objective

$$\mathbb{E}_{s, a \sim p(s'|s, a), \pi_k(a|s)} [Q(s, a) \log(\pi_\theta(a|s))]$$

□

## A.2 Derivation of Q-weighted VLO Loss

For each state  $s$ , we have

$$\begin{aligned}
& \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{q(a_{1:T} | a_0)}{\pi_\theta(a_{0:T})} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{\prod_{t=1}^T q(a_t | a_{t-1})}{\pi_\theta(a_T) \prod_{t=1}^T \pi_\theta(a_{t-1} | a_t)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \log \frac{\prod_{t=1}^T q(a_t | a_{t-1})}{\prod_{t=1}^T \pi_\theta(a_{t-1} | a_t)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=1}^T \log \frac{q(a_t | a_{t-1})}{\pi_\theta(a_{t-1} | a_t)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \log \frac{q(a_t | a_{t-1})}{\pi_\theta(a_{t-1} | a_t)} + \log \frac{q(a_1 | a_0)}{\pi_\theta(a_0 | a_1)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \log \frac{q(a_{t-1} | a_t, a_0) q(a_t | a_0)}{\pi_\theta(a_{t-1} | a_0) q(a_{t-1} | a_0)} + \log \frac{q(a_1 | a_0)}{\pi_\theta(a_0 | a_1)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \log \left( \frac{q(a_{t-1} | a_t, a_0)}{\pi_\theta(a_{t-1} | a_t)} \cdot \frac{q(a_t | a_0)}{q(a_{t-1} | a_0)} \right) + \log \frac{q(a_1 | a_0)}{\pi_\theta(a_0 | a_1)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \left( \log \frac{q(a_{t-1} | a_t, a_0)}{\pi_\theta(a_{t-1} | a_t)} + \log \frac{q(a_t | a_0)}{q(a_{t-1} | a_0)} \right) + \log \frac{q(a_1 | a_0)}{\pi_\theta(a_0 | a_1)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \log \frac{q(a_{t-1} | a_t, a_0)}{\pi_\theta(a_{t-1} | a_t)} + \log \frac{q(a_T | a_0)}{q(a_1 | a_0)} + \log \frac{q(a_1 | a_0)}{\pi_\theta(a_0 | a_1)} \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ -\log \pi_\theta(a_T) + \sum_{t=2}^T \log \frac{q(a_{t-1} | a_t, a_0)}{\pi_\theta(a_{t-1} | a_t)} + \log q(a_T | a_0) \right. \\
&\quad \left. - \log q(a_1 | a_0) + \log q(a_1 | a_0) - \log \pi_\theta(a_0 | a_1) \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{q(a_T | a_0)}{\pi_\theta(a_T)} + \sum_{t=2}^T \log \frac{q(a_{t-1} | a_t, a_0)}{\pi_\theta(a_{t-1} | a_t)} - \log \pi_\theta(a_0 | a_1) \right] \\
&= \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ D_{KL}(q(a_T | a_0) \| \pi_\theta(a_T)) + \sum_{t=2}^T D_{KL}(q(a_{t-1} | a_t, a_0) \| \pi_\theta(a_{t-1} | a_t)) - \log \pi_\theta(a_0 | a_1) \right] \\
&= \mathbb{E}_{a_0, \epsilon} \left[ \frac{\beta_t^2}{2\sigma_t^2 \alpha_t (1 - \bar{\alpha}_t)} \left\| \epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} a_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, s, t) \right\|^2 \right].
\end{aligned}$$

Thus, the Q-weighted VLO loss can be derived as

$$\begin{aligned}
& \max_{\theta} \mathbb{E}_{s, a_0 \sim \pi_k(a|s)} \left[ Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{\pi_\theta(a_{0:T} | s)}{q(a_{1:T} | s, a_0)} \right] \right] \\
& \triangleq \min_{\theta} \mathbb{E}_{s, a_0 \sim \pi_k(a|s)} \left[ Q(s, a_0) \mathbb{E}_{a_{1:T} \sim q(a_{1:T}|s, a_0)} \left[ \log \frac{q(a_{1:T} | s, a_0)}{\pi_\theta(a_{0:T} | s)} \right] \right] \\
& \triangleq \min_{\theta} \mathbb{E}_{s, a \sim \pi_k(a|s), \epsilon, t} \left[ \frac{\beta_t^2}{2\sigma_t^2 \alpha_t (1 - \bar{\alpha}_t)} Q(s, a) \cdot \left\| \epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} a + \sqrt{1 - \bar{\alpha}_t} \epsilon, s, t) \right\|^2 \right].
\end{aligned}$$

## A.3 Proof of Theorem 2

**Theorem 2. (Optimal Solution in One Policy Improvement Step)** Considering the optimization problem in policy improvement step  $\pi_{k+1} = \arg \max_{\pi} \mathbb{E}_{\tau \sim \pi_k} [Q_{\pi_k}(s, a) \log(\pi)]$ , the optimal policy

in a given state  $s$  is

$$p(a | s) = \begin{cases} \frac{\pi(a|s)Q(s,a)}{\int \mathbb{1}_{Q(s,a)>0}(a)\pi(a|s)Q(s,a)da}, & Q(s,a) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

when there exists a such that  $Q(s,a) > 0$ , and

$$p(a | s) = \frac{1}{N} \sum_{i=1}^N \delta(x - a_i), a_i \in \left\{ a \mid Q(s,a) = \max_a Q(s,a) \right\}, \quad (12)$$

when  $Q(s,a) \leq 0$  for any  $a \in \mathcal{A}$ ,  $\delta(x - a)$  is the Dirac function,  $N$  is the cardinality of set  $\{a \mid Q(s,a) = \max_a Q(s,a)\}$ ,  $\pi(a | s)$  is the behavior policy,  $p(a | s)$  is the policy to be optimized and  $\mathbb{1}_{Q(s,a)>0}$  is the indicator function that judges whether  $Q(s,a) > 0$ .

*Proof.* Consider the optimization problem of the policy improvement step in a certain state  $s$ :

$$\begin{aligned} & \min_{p(a|s)} - \int_{\mathcal{A}} \pi(a | s) Q(s, a) \log p(a | s) da \\ & \text{subject to } \int p(a | s) da = 1 \\ & p(a | s) \geq 0, a \in \mathcal{A} \end{aligned}$$

where  $p(a | s)$  is a probability density function defined on  $\mathcal{A}$  that represents the policy to be optimized in the state  $s$ .

Here, we can derive the solution to this optimization problem according to KKT conditions by classifying it into three cases.

- $Q(s,a) > 0$  for any action  $a \in \mathcal{A}$ . In this case, the optimization problem is convex, and we can use KKT conditions directly to obtain its optimal solution as below:

$$\begin{aligned} \frac{\pi(a | s)Q(s,a)}{p(a | s)} - \lambda + \nu(a | s) &= 0 \\ \int p(a | s) da &= 1 \\ \nu(a | s) &\geq 0, a \in \mathcal{A} \\ \nu(a | s)p(a | s) &= 0, a \in \mathcal{A} \end{aligned} \quad (13)$$

Then, consider

$$p(a|s) = \frac{\pi(a | s)Q(s,a)}{\lambda - \nu(a|s)}.$$

Since  $Q(s,a) > 0$  and  $p(a|s) \geq 0$ , the denominator

$$\lambda - \nu(a | s) \geq 0.$$

Hence,

$$p(a | s) > 0, \quad \nu(a | s) = 0,$$

which means

$$p(a | s) = \frac{\pi(a | s)Q(s,a)}{\int \pi(a | s)Q(s,a)da}.$$

- $Q(s,a) \leq 0$  for any action  $a \in \mathcal{A}$ . When  $Q(s,a) \leq 0$  for any action  $a \in \mathcal{A}$ , it can be found that the objective will be  $-\infty$  if there exists an action  $a$  such that  $p(a|s) = 0$  and  $\pi(a|s)Q(a | s) < 0$ . In that case, only considering the original optimization problem is not meaningful. Hence, we can further consider the optimality of  $\int Q(s,a)p(a|s)da$ . Finally, we can drive the optimal policy pdf is the summation of the Dirac function

$$p(a | s) = \frac{1}{N} \sum_{i=1}^N \delta(x - a_i), a_i \in \left\{ a \mid Q(s,a) = \max_a Q(s,a), \pi(a | s) > 0 \right\}$$

where  $N$  is the cardinality of set  $\{a \mid Q(s,a) = \max_a Q(s,a), \pi(a | s) > 0\}$ .

- $Q(s, a) > 0$  for some actions  $a$  and  $Q(s, a) \leq 0$  for the left actions  $a$ . Combining the above two situations, the optimal policy pdf is

$$p(a | s) = \begin{cases} \frac{\pi(a|s)Q(s,a)}{\int \mathbb{I}_{Q(s,a)>0} \pi(a|s)Q(s,a)da} & Q(s, a) > 0 \\ 0 & otherwise \end{cases}.$$

Finally, we can summarize the optimal policy in a certain state  $s$  as

$$p(a | s) = \begin{cases} \frac{\pi(a|s)Q(s,a)}{\int \mathbb{I}_{Q(s,a)>0} \pi(a|s)Q(s,a)da}, & Q(s, a) > 0 \\ 0, & otherwise \end{cases} \quad (14)$$

when there exists  $a$  such that  $Q(s, a) > 0$ , and

$$p(a | s) = \frac{1}{N} \sum_{i=1}^N \delta(x - a_i), a_i \in \left\{ a \mid Q(s, a) = \max_a Q(s, a), \pi(a | s) > 0 \right\}, \quad (15)$$

when  $Q(s, a) \leq 0$  for any  $a \in \mathcal{A}$ .  $\square$

#### A.4 Proof for Convergence of QVPO

Assume the new diffusion policy after one QVPO iteration can be approximately expressed as

$$\pi_{new}(a | s) \approx (1 - p_{data}(s)A_{\pi_{old}}(s, a^*)\eta)\pi_{old}(a | s) + p_{data}(s)A_{\pi_{old}}(s, a^*)\eta \frac{\mathbb{I}_{a \in \mathcal{N}(a^*|s, \epsilon)}(a)}{S_{\mathcal{N}(a^*|s, \epsilon)}}$$

where  $a^* | s$  is the action that can maximize  $Q_{\pi_{old}}(s, a)$  in the state  $s$ ,  $\mathcal{N}(a^* | s, \epsilon)$  is the neighborhood of  $a^* | s$  with a small radius  $\epsilon$ ,  $S_{\mathcal{N}(a^*|s, \epsilon)}$  is the area of this neighborhood, and  $p_{data}(s)$  is the sampling distribution of the state. This assumption is straightforward: the training sample's generation probability in the diffusion model will be improved and the improved probability is proportional to its weight.

Now consider the improvement of the RL objective:

$$\begin{aligned} \mathcal{J}(\pi_{new}) - \mathcal{J}(\pi_{old}) &= \mathbb{E}_{s \sim \rho_0} [V_{\pi_{new}}(s) - V_{\pi_{old}}(s)] \\ &= \mathbb{E}_{s \sim \rho_0} [\mathbb{E}_{a \sim \pi_{new}(a|s)} [Q_{\pi_{new}}(s, a)] - V_{\pi_{old}}(s)] \\ &= \mathbb{E}_{s \sim \rho_0} [\mathbb{E}_{a \sim \pi_{new}(a|s)} [Q_{\pi_{new}}(s, a) - Q_{\pi_{old}}(s, a)]] \\ &\quad + \mathbb{E}_{s \sim \rho_0} [\mathbb{E}_{a \sim \pi_{new}(a|s)} [Q_{\pi_{old}}(s, a)] - V_{\pi_{old}}(s)] \\ &= \mathbb{E}_{s \sim \rho_0} [\mathbb{E}_{a \sim \pi_{new}(a|s)} [Q_{\pi_{new}}(s, a) - Q_{\pi_{old}}(s, a)]] + \mathbb{E}_{s, a \sim \rho_0, \pi_{new}(a|s)} [A_{\pi_{old}}(s, a)] \end{aligned}$$

The first term here can be further expanded according to the Bellman equation:

$$\mathbb{E}_{s \sim \rho_0} [\mathbb{E}_{a \sim \pi_{new}(a|s)} [Q_{\pi_{new}}(s, a) - Q_{\pi_{old}}(s, a)]] = \gamma \mathbb{E}_{s \sim d_{\pi_{new}}^1} [V_{\pi_{new}}(s) - V_{\pi_{old}}(s)]$$

where  $d_{\pi_{new}}^1$  denotes the probability distribution of state in time step  $t = 1$  with policy  $\pi_{new}$ . Repeating the above operation, we will obtain:

$$\begin{aligned} \mathcal{J}(\pi_{new}) - \mathcal{J}(\pi_{old}) &= \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{s, a \sim d_{\pi_{new}, \pi_{new}}^t} [A_{\pi_{old}}(s, a)] \\ &= \frac{1}{1 - \gamma} \mathbb{E}_{s \sim d_{\pi_{new}}} [\mathbb{E}_{a \sim \pi_{new}(\cdot|s)} [A_{\pi_{old}}(s, a)]] \\ &\approx \frac{1}{1 - \gamma} \mathbb{E}_{s \sim d_{\pi_{new}}} \left[ (1 - p_{data}(s)A_{\pi_{old}}(s, a^*)\eta) \mathbb{E}_{a \sim \pi_{old}(\cdot|s)} [A_{\pi_{old}}(s, a)] \right. \\ &\quad \left. + p_{data}(s)A_{\pi_{old}}^2(s, a^*)\eta \frac{\mathbb{I}_{a \in \mathcal{N}(a^*|s, \epsilon)}(a)}{S_{\mathcal{N}(a^*|s, \epsilon)}} \right] \\ &= \frac{1}{1 - \gamma} \mathbb{E}_{s \sim d_{\pi_{new}}} \left[ p_{data}(s)A_{\pi_{old}}^2(s, a^*)\eta \frac{\mathbb{I}_{a \in \mathcal{N}(a^*|s, \epsilon)}(a)}{S_{\mathcal{N}(a^*|s, \epsilon)}} \right] \geq 0 \end{aligned}$$



## B Hyperparameters

All of our experiments are implemented on a GPU of NVIDIA GeForce RTX 4090 with 24GB and a CPU of Intel Xeon w5-3435X. The implementation of SAC, TD3, PPO, SPO, and DIPO is based on <https://github.com/toshikwa/soft-actor-critic.pytorch>, <https://github.com/sfujim/TD3>, <https://github.com/ikostrikov/pytorch-a2c-ppo-acktr-gail>, <https://github.com/MyRepositories-hub/Simple-Policy-Optimization>, and <https://github.com/BellmanTimeHut/DIPO> respectively, which are official code library. Table 2 and 3 present the hyper-parameters used in our experiments. Notably, we find that the performance of DIPO with 20 diffusion steps is worse. Thus, different from 20 diffusion steps in QVPO, 100 diffusion steps are set for DIPO, which are also recommended in the original paper [42].

Table 2: Hyper-parameters used in the experiments.

Hyperparameters	QVPO	DIPO	SAC	TD3	SPO	PPO
No. of hidden layers	2	2	2	2	2	2
No. of hidden nodes	256	256	256	256	64	256
Activation	mish	mish	relu	relu	tanh	tanh
Batch size	256	256	256	256	256	256
Discount for reward $\gamma$	0.99	0.99	0.99	0.99	0.99	0.99
Target smoothing coefficient $\tau$	0.005	0.005	0.005	0.005	0.0	0.005
Learning rate for actor	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$7 \times 10^{-4}$
Learning rate for critic	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$3 \times 10^{-4}$	$7 \times 10^{-4}$
Actor Critic grad norm	N/A	2	N/A	N/A	0.5	0.5
Memory size	$1 \times 10^6$	$1 \times 10^6$	$1 \times 10^6$	$1 \times 10^6$	$1 \times 10^6$	$1 \times 10^6$
Entropy coefficient	N/A	N/A	0.2	N/A	N/A	0.01
Value loss coefficient	N/A	N/A	N/A	N/A	N/A	0.5
Exploration noise	N/A	N/A	N/A	$\mathcal{N}(0, 0.1)$	N/A	N/A
Policy noise	N/A	N/A	N/A	$\mathcal{N}(0, 0.2)$	N/A	N/A
Noise clip	N/A	N/A	N/A	0.5	N/A	N/A
Use gae	N/A	N/A	N/A	N/A	True	True
Diffusion steps	20	100	N/A	N/A	N/A	N/A

Table 3: Hyper-parameters used in QVPO.

Hyperparameters	Ant-v3	HalfCheetah-v3	Hopper-v3	Humanoid-v3	Waller2d-v3
Number of samples from diffusion policy $N_d$	64	64	64	64	64
Number of samples from $\mathcal{U}(\underline{a}, \bar{a})$ $N_e$	10	10	10	10	10
Action selection number for behavior policy $K_b$	4	4	4	4	4
Action selection number for target policy $K_t$	2	4	1	2	2
Entropy weight $\omega_{ent}$	0.01	0.01	0.01	0.01	0.01
Q-weight transformation function	<i>qadv</i>	<i>qadv</i>	<i>qadv</i>	<i>qadv</i>	<i>qadv</i>

## C Training and Inference Time

The comparison of training and inference time is shown in the following tables. Notably, since the official implementation of QSM is based on the jax and other algorithms are based on pytorch, it is not a fair comparison. In practice, the algorithm realized with jax is 6-10 times faster than that realized with pytorch. Besides, to fairly compare the diffusion-based RL methods in training and inference time, we set the same number of diffusion steps (T=20) for all of them (i.e., QVPO, QSM, and DIPO). The results in Table 4 imply that QVPO can sufficiently use the parallel computing ability of GPU and the multiple-sampling and selecting procedure does not take much time compared with gradient-based optimization like DIPO.

Table 4: The training and inference time comparison on Ant-v3 Benchmarks.

Method	QVPO	DIPO	TD3	SAC	PPO	SPO	QSM (jax)
Training Time (h)	6.8	10.5	0.5	2.5	0.3	0.3	1.0
Inference Time (ms)	6.2	5.7	0.2	0.3	0.2	0.3	0.9

Moreover, although QVPO is much slower than classical RL methods like SAC in inference, the inference time (6ms) still is acceptable. To our knowledge, most existing real robots only require 50-Hz control policy (i.e. output action per 20 ms). Besides, just like QSM, the inference time of QVPO can be further improved with jax framework if it is necessary. Hence, the inference time is not a bottleneck to applying QVPO to real applications.

## NeurIPS Paper Checklist

### 1. Claims

Question: Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope?

Answer: [Yes]

Justification: As stated in the abstract and introduction, this paper proposes a novel diffusion-based model-free online RL method, accurately reflecting the key contribution and scope.

Guidelines:

- The answer NA means that the abstract and introduction do not include the claims made in the paper.
- The abstract and/or introduction should clearly state the claims made, including the contributions made in the paper and important assumptions and limitations. A No or NA answer to this question will not be perceived well by the reviewers.
- The claims made should match theoretical and experimental results, and reflect how much the results can be expected to generalize to other settings.
- It is fine to include aspirational goals as motivation as long as it is clear that these goals are not attained by the paper.

### 2. Limitations

Question: Does the paper discuss the limitations of the work performed by the authors?

Answer: [Yes]

Justification: The analysis of limitations is provided in Section 6.

Guidelines:

- The answer NA means that the paper has no limitation while the answer No means that the paper has limitations, but those are not discussed in the paper.
- The authors are encouraged to create a separate "Limitations" section in their paper.
- The paper should point out any strong assumptions and how robust the results are to violations of these assumptions (e.g., independence assumptions, noiseless settings, model well-specification, asymptotic approximations only holding locally). The authors should reflect on how these assumptions might be violated in practice and what the implications would be.
- The authors should reflect on the scope of the claims made, e.g., if the approach was only tested on a few datasets or with a few runs. In general, empirical results often depend on implicit assumptions, which should be articulated.
- The authors should reflect on the factors that influence the performance of the approach. For example, a facial recognition algorithm may perform poorly when image resolution is low or images are taken in low lighting. Or a speech-to-text system might not be used reliably to provide closed captions for online lectures because it fails to handle technical jargon.
- The authors should discuss the computational efficiency of the proposed algorithms and how they scale with dataset size.
- If applicable, the authors should discuss possible limitations of their approach to address problems of privacy and fairness.
- While the authors might fear that complete honesty about limitations might be used by reviewers as grounds for rejection, a worse outcome might be that reviewers discover limitations that aren't acknowledged in the paper. The authors should use their best judgment and recognize that individual actions in favor of transparency play an important role in developing norms that preserve the integrity of the community. Reviewers will be specifically instructed to not penalize honesty concerning limitations.

### 3. Theory Assumptions and Proofs

Question: For each theoretical result, does the paper provide the full set of assumptions and a complete (and correct) proof?

Answer: [Yes]

Justification: The proofs are provided in Appendix A.

Guidelines:

- The answer NA means that the paper does not include theoretical results.
- All the theorems, formulas, and proofs in the paper should be numbered and cross-referenced.
- All assumptions should be clearly stated or referenced in the statement of any theorems.
- The proofs can either appear in the main paper or the supplemental material, but if they appear in the supplemental material, the authors are encouraged to provide a short proof sketch to provide intuition.
- Inversely, any informal proof provided in the core of the paper should be complemented by formal proofs provided in appendix or supplemental material.
- Theorems and Lemmas that the proof relies upon should be properly referenced.

#### 4. Experimental Result Reproducibility

Question: Does the paper fully disclose all the information needed to reproduce the main experimental results of the paper to the extent that it affects the main claims and/or conclusions of the paper (regardless of whether the code and data are provided or not)?

Answer: [Yes]

Justification: These details are provided in Appendix B.

Guidelines:

- The answer NA means that the paper does not include experiments.
- If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
- If the contribution is a dataset and/or model, the authors should describe the steps taken to make their results reproducible or verifiable.
- Depending on the contribution, reproducibility can be accomplished in various ways. For example, if the contribution is a novel architecture, describing the architecture fully might suffice, or if the contribution is a specific model and empirical evaluation, it may be necessary to either make it possible for others to replicate the model with the same dataset, or provide access to the model. In general, releasing code and data is often one good way to accomplish this, but reproducibility can also be provided via detailed instructions for how to replicate the results, access to a hosted model (e.g., in the case of a large language model), releasing of a model checkpoint, or other means that are appropriate to the research performed.
- While NeurIPS does not require releasing code, the conference does require all submissions to provide some reasonable avenue for reproducibility, which may depend on the nature of the contribution. For example
  - (a) If the contribution is primarily a new algorithm, the paper should make it clear how to reproduce that algorithm.
  - (b) If the contribution is primarily a new model architecture, the paper should describe the architecture clearly and fully.
  - (c) If the contribution is a new model (e.g., a large language model), then there should either be a way to access this model for reproducing the results or a way to reproduce the model (e.g., with an open-source dataset or instructions for how to construct the dataset).
  - (d) We recognize that reproducibility may be tricky in some cases, in which case authors are welcome to describe the particular way they provide for reproducibility. In the case of closed-source models, it may be that access to the model is limited in some way (e.g., to registered users), but it should be possible for other researchers to have some path to reproducing or verifying the results.

#### 5. Open access to data and code

Question: Does the paper provide open access to the data and code, with sufficient instructions to faithfully reproduce the main experimental results, as described in supplemental material?

Answer: [Yes]

Justification: We will release the code once the paper is accepted.

Guidelines:

- The answer NA means that paper does not include experiments requiring code.
- Please see the NeurIPS code and data submission guidelines (<https://nips.cc/public/guides/CodeSubmissionPolicy>) for more details.
- While we encourage the release of code and data, we understand that this might not be possible, so “No” is an acceptable answer. Papers cannot be rejected simply for not including code, unless this is central to the contribution (e.g., for a new open-source benchmark).
- The instructions should contain the exact command and environment needed to run to reproduce the results. See the NeurIPS code and data submission guidelines (<https://nips.cc/public/guides/CodeSubmissionPolicy>) for more details.
- The authors should provide instructions on data access and preparation, including how to access the raw data, preprocessed data, intermediate data, and generated data, etc.
- The authors should provide scripts to reproduce all experimental results for the new proposed method and baselines. If only a subset of experiments are reproducible, they should state which ones are omitted from the script and why.
- At submission time, to preserve anonymity, the authors should release anonymized versions (if applicable).
- Providing as much information as possible in supplemental material (appended to the paper) is recommended, but including URLs to data and code is permitted.

## 6. Experimental Setting/Details

Question: Does the paper specify all the training and test details (e.g., data splits, hyper-parameters, how they were chosen, type of optimizer, etc.) necessary to understand the results?

Answer: [Yes]

Justification: These details are provided in Section 5 and Appendix B.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The experimental setting should be presented in the core of the paper to a level of detail that is necessary to appreciate the results and make sense of them.
- The full details can be provided either with the code, in appendix, or as supplemental material.

## 7. Experiment Statistical Significance

Question: Does the paper report error bars suitably and correctly defined or other appropriate information about the statistical significance of the experiments?

Answer: [Yes]

Justification: The standard deviation in our experiment results (Figure 3 and Table 1) shows the statistical significance.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The authors should answer "Yes" if the results are accompanied by error bars, confidence intervals, or statistical significance tests, at least for the experiments that support the main claims of the paper.
- The factors of variability that the error bars are capturing should be clearly stated (for example, train/test split, initialization, random drawing of some parameter, or overall run with given experimental conditions).
- The method for calculating the error bars should be explained (closed form formula, call to a library function, bootstrap, etc.)
- The assumptions made should be given (e.g., Normally distributed errors).

- It should be clear whether the error bar is the standard deviation or the standard error of the mean.
- It is OK to report 1-sigma error bars, but one should state it. The authors should preferably report a 2-sigma error bar than state that they have a 96% CI, if the hypothesis of Normality of errors is not verified.
- For asymmetric distributions, the authors should be careful not to show in tables or figures symmetric error bars that would yield results that are out of range (e.g. negative error rates).
- If error bars are reported in tables or plots, The authors should explain in the text how they were calculated and reference the corresponding figures or tables in the text.

## 8. Experiments Compute Resources

Question: For each experiment, does the paper provide sufficient information on the computer resources (type of compute workers, memory, time of execution) needed to reproduce the experiments?

Answer: [Yes]

Justification: The information on the computer resources is provided in Appendix B.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The paper should indicate the type of compute workers CPU or GPU, internal cluster, or cloud provider, including relevant memory and storage.
- The paper should provide the amount of compute required for each of the individual experimental runs as well as estimate the total compute.
- The paper should disclose whether the full research project required more compute than the experiments reported in the paper (e.g., preliminary or failed experiments that didn't make it into the paper).

## 9. Code Of Ethics

Question: Does the research conducted in the paper conform, in every respect, with the NeurIPS Code of Ethics <https://neurips.cc/public/EthicsGuidelines>?

Answer: [Yes]

Justification: Our research conformed with the NeurIPS Code of Ethics.

Guidelines:

- The answer NA means that the authors have not reviewed the NeurIPS Code of Ethics.
- If the authors answer No, they should explain the special circumstances that require a deviation from the Code of Ethics.
- The authors should make sure to preserve anonymity (e.g., if there is a special consideration due to laws or regulations in their jurisdiction).

## 10. Broader Impacts

Question: Does the paper discuss both potential positive societal impacts and negative societal impacts of the work performed?

Answer: [NA]

Justification: Our paper primarily focuses on theoretical research in how to employ the diffusion model in online reinforcement learning, with no consideration of societal impacts.

Guidelines:

- The answer NA means that there is no societal impact of the work performed.
- If the authors answer NA or No, they should explain why their work has no societal impact or why the paper does not address societal impact.
- Examples of negative societal impacts include potential malicious or unintended uses (e.g., disinformation, generating fake profiles, surveillance), fairness considerations (e.g., deployment of technologies that could make decisions that unfairly impact specific groups), privacy considerations, and security considerations.

- The conference expects that many papers will be foundational research and not tied to particular applications, let alone deployments. However, if there is a direct path to any negative applications, the authors should point it out. For example, it is legitimate to point out that an improvement in the quality of generative models could be used to generate deepfakes for disinformation. On the other hand, it is not needed to point out that a generic algorithm for optimizing neural networks could enable people to train models that generate Deepfakes faster.
- The authors should consider possible harms that could arise when the technology is being used as intended and functioning correctly, harms that could arise when the technology is being used as intended but gives incorrect results, and harms following from (intentional or unintentional) misuse of the technology.
- If there are negative societal impacts, the authors could also discuss possible mitigation strategies (e.g., gated release of models, providing defenses in addition to attacks, mechanisms for monitoring misuse, mechanisms to monitor how a system learns from feedback over time, improving the efficiency and accessibility of ML).

## 11. Safeguards

Question: Does the paper describe safeguards that have been put in place for responsible release of data or models that have a high risk for misuse (e.g., pretrained language models, image generators, or scraped datasets)?

Answer: [NA]

Justification: Our experiment of this paper was conducted on five MuJoCo locomotion tasks as Hopper-v3, Walker2d-v3, Ant-v3, HalfCheetah-v3, and Humanoid-v3, which poses no such risks.

Guidelines:

- The answer NA means that the paper poses no such risks.
- Released models that have a high risk for misuse or dual-use should be released with necessary safeguards to allow for controlled use of the model, for example by requiring that users adhere to usage guidelines or restrictions to access the model or implementing safety filters.
- Datasets that have been scraped from the Internet could pose safety risks. The authors should describe how they avoided releasing unsafe images.
- We recognize that providing effective safeguards is challenging, and many papers do not require this, but we encourage authors to take this into account and make a best faith effort.

## 12. Licenses for existing assets

Question: Are the creators or original owners of assets (e.g., code, data, models), used in the paper, properly credited and are the license and terms of use explicitly mentioned and properly respected?

Answer: [Yes]

Justification: The implementation of SAC, TD3, PPO, SPO, and DIPO are cited properly in Appendix B. Other assets are not applied here.

Guidelines:

- The answer NA means that the paper does not use existing assets.
- The authors should cite the original paper that produced the code package or dataset.
- The authors should state which version of the asset is used and, if possible, include a URL.
- The name of the license (e.g., CC-BY 4.0) should be included for each asset.
- For scraped data from a particular source (e.g., website), the copyright and terms of service of that source should be provided.
- If assets are released, the license, copyright information, and terms of use in the package should be provided. For popular datasets, [paperswithcode.com/datasets](https://paperswithcode.com/datasets) has curated licenses for some datasets. Their licensing guide can help determine the license of a dataset.

- For existing datasets that are re-packaged, both the original license and the license of the derived asset (if it has changed) should be provided.
- If this information is not available online, the authors are encouraged to reach out to the asset’s creators.

### 13. **New Assets**

Question: Are new assets introduced in the paper well documented and is the documentation provided alongside the assets?

Answer: [NA]

Justification: The paper does not introduce new assets.

Guidelines:

- The answer NA means that the paper does not release new assets.
- Researchers should communicate the details of the dataset/code/model as part of their submissions via structured templates. This includes details about training, license, limitations, etc.
- The paper should discuss whether and how consent was obtained from people whose asset is used.
- At submission time, remember to anonymize your assets (if applicable). You can either create an anonymized URL or include an anonymized zip file.

### 14. **Crowdsourcing and Research with Human Subjects**

Question: For crowdsourcing experiments and research with human subjects, does the paper include the full text of instructions given to participants and screenshots, if applicable, as well as details about compensation (if any)?

Answer: [NA]

Justification: The paper does not involve crowdsourcing nor research with human subjects.

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Including this information in the supplemental material is fine, but if the main contribution of the paper involves human subjects, then as much detail as possible should be included in the main paper.
- According to the NeurIPS Code of Ethics, workers involved in data collection, curation, or other labor should be paid at least the minimum wage in the country of the data collector.

### 15. **Institutional Review Board (IRB) Approvals or Equivalent for Research with Human Subjects**

Question: Does the paper describe potential risks incurred by study participants, whether such risks were disclosed to the subjects, and whether Institutional Review Board (IRB) approvals (or an equivalent approval/review based on the requirements of your country or institution) were obtained?

Answer: [NA]

Justification: The paper does not involve crowdsourcing nor research with human subjects.

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Depending on the country in which research is conducted, IRB approval (or equivalent) may be required for any human subjects research. If you obtained IRB approval, you should clearly state this in the paper.
- We recognize that the procedures for this may vary significantly between institutions and locations, and we expect authors to adhere to the NeurIPS Code of Ethics and the guidelines for their institution.
- For initial submissions, do not include any information that would break anonymity (if applicable), such as the institution conducting the review.