# C.A. Meredith, A.N. Prior, and Possible Worlds

*Zuzana Rybaříková*
*Univeristy of West Bohemia, the Czech Republic*
*zuzka.rybarikova@gmail.com*

*Per Hasle*
*University of Copenhagen, Denmark*
*per.hasle@hum.ku.dk*

**Abstract**

Arthur Prior and Carew Meredith cooperated on the formulation of several systems of logic. The cooperation was so close that on the basis of their joint work, they are both considered to be precursors of possible worlds semantics. However, their concept of possible worlds, their understanding of the relevant formal representations and indeed their general approach to modal logic considerably differed. These differences should be pointed out in order to more precisely appreciate the contribution of each of these authors. To neglect the differences could cause the misinterpretation of Meredith's and Prior's work. On the one hand, it might cause corruption of Meredith's system of logic and lead to paradoxes, as Prior pointed out in 'Modal Logic with Functorial Variables and a Contingent Constant'. On the other hand, considering Prior as a mere follower of Meredith could cause an underestimation of Prior's originality and contribution to this field.

**Keywords:** C. A. Meredith, A. N. Prior, Possible worlds, Possible worlds semantics, Modal logic, Many-valued logic

# 1 Introduction

Although a proof of consistency is a highly desirable result for a system of logic, such a proof is not always uncontroversial. Arthur Prior (1967, 77 [35]) pointed out this issue in '*Logic of Successive World-States*', chapter V of *Past, Present and Future*.[1] Namely, he stressed that Smiley's proof did show a consistency of most systems of tense and modal logic, but modal operators appeared in the light of this proof trivialised, and modal calculi are insufficiently characterised by them. Therefore, the proof raised the need for a de-trivialising of systems of modal logic. There are several solutions to the problem. Ivo Thomas and Jan Łukasiewicz (1970b, 353 [9]) favoured the reversed turnstile to indicate that what follows is not a thesis of the system. Another solution is propositional quantification in systems of modal logic which was represented for instance by Saul Kripke (1959 [7]). Lastly, Prior (1967, 77–78 [35]) also presented his solution which consists in introducing a propositional variable '$a$' with certain decisive properties.[2] Prior pointed out that the idea could be found in Meredith's system of modal logic in which it figured, albeit as a constant called '$n$'.

Prior did not deal with the variable '$a$' for the first time in chapter V of *Past, Present and Future*, the '*Logic of Successive World-States*'. It was already introduced in Prior's and Meredith's joint paper '*Interpretations of Different Modal Logics in the "Property Calculus"*', published in 1996 [40] but originally written in 1956 and at the time distributed in mimeographed form. Their joint introduction of the propositional variable '$a$' certainly suggests a relation between this '$a$' and Meredith's '$n$'. Additionally, and not least on account of this paper, Meredith and Prior are considered to be precursors of possible worlds semantics, as was extensively argued in (Copeland, 2006 [3]). Copeland's argument was to some extent dependent on the relation between Prior's '$a$' and Meredith's '$n$'. In the paper '*Interpretations of Different Modal Logics in the "Property Calculus"*', written three years before the publication of Kripke's '*A*

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[2]Even though Prior described '$a$' as a constant in V. chapter of *Past, Present and Future*, he later identified it with a variable (see Prior, 2003b, 183 [38]). It also acts as a variable in his systems of logic (see, e.g., Prior 1967, 90 [35]). Therefore, Prior's '$a$' will be labelled as a variable elsewhere in this paper.

*Completeness Theorem in Modal Logic'*, Prior and Meredith introduced a system of logic later named $U$-calculus where variables '$a$', '$b$', and '$c$' are bound by a binary operator '$U$'. Neither the operator nor the variables are, however, interpreted in the paper but as already noted Prior later claimed the similarity between the variables and Meredith's constant '$n$', namely in chapter V of *Past, Present and Future*, as was mentioned in a previous paragraph. Moreover, as demonstrated in (Copeland 2006), the '$U$' arguably anticipates Kripke's accessibility relation. The interpretation of '$U$' will be discussed more later.

However, the translation from '$a$' to '$n$' is not as straightforward as it appears to be from previous claims. Namely, it is not certain that Prior and Meredith shared a similar approach to modal logic. There are at least two different approaches to systems of logic which deal with future contingents. They are firstly systems of many-valued logic which follow Łukasiewicz's (1970a, 125–127 [8]) rejection of the rule of bivalence and which contain three or more truth-values. Łukasiewicz's solution thus is extensional and plainly truth-functional, but of course at the cost of the complications of defining the truth-functions which many-valued logic introduces. Differing decisively from Łukasiewicz's solution, the second approach retains bivalence and 'instead' consists in the enlargement of semantics through the introduction of intensional systems of logics. The latter approach is also linked with possible worlds semantics. While Prior in his later works undoubtedly discussed possible worlds and his systems of logic were intensional, Meredith's approach is more unclear since Meredith did very little to explain it in his papers. There are certain indications that it might differ from Prior's view. The first Meredith's system of modal logic was many-valued and in general, Meredith was deeply influenced by Łukasiewicz.

The aim of this paper is to point out certain differences in their systems of logic and even question whether Prior's '$a$' and Meredith '$n$' are translatable into one-another. In order to introduce the issue, Meredith's systems of logic, his constant '$n$', and his overall approach to logic will be presented. Secondly, Prior's '$a$' will be discussed. The chapter will also touch on his critique of Meredith's system of modal logic. The final chapter includes arguments that support the view that there are several differences between Meredith's '$n$' and Prior's '$a$' and reasons why these differences are important.

## 2 Meredith's Systems of Modal Logic

Meredith was originally a mathematician. As he switched from mathematics to logic, he demonstrated his excellence in the formal logic. David Meredith (1977, 514–516 [22]), C.A. Meredith's cousin, reported that a lot of Meredith's papers, which Prior published, arose as Meredith's response to logical queries of Meredith's colleagues (cf. appendix). In contrast to the formalism, philosophical implications were nearly not discussed in Meredith's published works. Meredith had taken up formal logic even before Łukasiewicz's arrival to Ireland shortly after World War II, but his work was henceforth deeply influenced by this Polish logician, who spent his last years until his death in 1956 in Dublin where Meredith lived. It is worth mentioning this influence in general, since not only Meredith's systems of modal logic – which are in fact only a minor part of his work – carry vestiges of Łukasiewicz's influence. Indeed this influence spread through almost all (or maybe all) of Meredith's published work. Meredith is acknowledged primarily as an author of condensed detachment, which helped him to abbreviate proofs (see Kalman 1983, 443 [6]). The detachment operations were for the first time introduced to Meredith by Łukasiewicz (D. Meredith 1977, 514 [22]). The rule of detachment was used for the shortening of axioms, which was a central endeavour among Łukasiewicz and his students (see Skolimowski 1967, 61 [42]). Meredith's application of Łukasiewicz's approach was so successful that he was able to find shorter axioms than Łukasiewicz himself. (see Church 1951, 229 [2]).

There are two systems of modal logic in Meredith's work.[3] The system ($\supset$, $\Box$, 0, $n$, $\delta$, $p$) was introduced by Meredith in 1953 and published in a joint paper with Prior in 1965 (Meredith and Prior 1965 [20]).[4] The constant '$n$' appeared for the first time in this system of logic. It represents 'the world' and also stands for 'contingently true', i.e., true in this world but false in another world.[5] However, it also takes on a sec-

[3]Despite several differences, both systems are based on Meredith's work on the calculus of properties (Meredith and Prior 1965, 102 [20]; Prior and Meredith 1996, 133–134 [40]). The calculus of properties appeared for the first time in the paper '*Ein erweiterter Klassenkalkül*', which was written by Mordechaj Wajsberg, Łukasiewicz's student in Warsaw. Łukasiewicz recommended this paper to Meredith (see Copeland 2006, 379).

[4]It is worth mentioning that under the title '*Note on my modal system*' in Prior's archive [13] could be found Meredith's paper which deals with this system of logic.

[5]False in every other world, if there are more than two – see the remarks below on having three or more possible worlds. Meredith investigates various scenarios, the

ond meaning, wherein '$n$' acts as a truth-value – as in the truth table below (Table 1). Moreover, '$n$' has a counterpart in '$\dot{n}$', which means false in this world but true in another world, i.e. contingently false. The constants '$n$' and '$\dot{n}$' were originally introduced as constants for a many-valued system of modal logic. The axioms of the system are:

1. $\Box[\delta[(p{\supset}0){\supset}(q{\supset}r)]{\supset}\delta[(r{\supset}p){\supset}(q{\supset}p)]]$[6]
2. $\Box p{\supset}[\delta(p{\supset}q){\supset}\delta q]$
3. $\delta(0){\supset}[\delta(0{\supset}0){\supset}\delta(\Box p)]$
4. $n$
5. $p{\supset}\Box(n{\supset}p)$
6. $\Box n{\supset}p$ (Meredith and Prior 1965, 103 [20])

Despite being constant, '$n$' is also an axiom in this system of modal logic. In addition, two other axioms which contain '$n$' characterise its function in the system. As is pointed out in *Computations and Speculations* (Meredith and Prior 1962, 118 [18]; see Appendix), the axiom $p{\supset}\Box(n{\supset}p)$ claims that any proposition which is true in the system is necessarily implied by '$n$' and the axiom $\Box n{\supset}p$ means that if '$n$' is necessary it could imply any proposition. Nonetheless, '$\Box n$' is not a theorem of the system.

The smallest matrix satisfying the axioms could be defined as the table (Table 1) for the implication and '$\Box$'.

Since '$n$' and '$\dot{n}$' are truth-values, this Meredith system of modal logic is a many-valued system. The semantics of the system is based on truth-values. Although '$n$' was described by Meredith as 'the world' or 'the possible world', it is firstly and mostly the truth-value. In the previously defined matrix, it could be described as consisting of two truth-values '1' for this world and '0' for another world, e.g., $(1, 0)$. Consequently, it is argued in *Computations and Speculations* (Meredith and Prior 1962 [18]) that the minimal number of truth values of the system is

details of which go beyond the purpose of this paper.

[6] 'δ', 'ε' and 'ζ' are one-placed propositional functors (see Simons 2017 [41]) from formulas into truth-values. Their introduction allowed the shortening of axioms, which was discussed in a previous paragraph, and they are not vital to the understanding of this paper (even though the shortening of axioms was the most crucial of all endeavours to Meredith). Observe that the axioms can be read 'disregarding' the 'δ', i.e. considering the trivial case where a formula is substituted with itself – i.e. you may 'throw away' 'δ' simply by replacing its argument with itself. The constants '0', '1', '$n$', '$\dot{n}$' stand for the defined truth-values.

Table 1: The smallest matrix satisfying the axioms of Meredith's system $(\supset, \Box, 0, n, \delta, p)$ from Prior's *Past, Present and Future* (see Prior 1967, p. 78 [35]).

| $\supset$ | 1 | $n$ | $\dot{n}$ | 0 | $\Box$ |
|---|---|---|---|---|---|
| 1 | 1 | $n$ | $\dot{n}$ | 0 | 1 |
| $n$ | 1 | 1 | $\dot{n}$ | $\dot{n}$ | 0 |
| $\dot{n}$ | 1 | $n$ | 1 | $n$ | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |

four, i.e. 1, 0, $n$, and $\dot{n}$. If '$n$' is intended to be clearly distinguished from other contingent propositions, there should be at least three possible worlds and eight truth values (see Meredith and Prior 1962, 119 [18]). In this way '$n$' could be differentiated from other contingent propositions which are true in more than the actual world. There could be more possible worlds and more truth values. In these cases, '$n$' appears as a sequence of truth values, namely true in the actual world and false in others i.e. '$1, 0, 0, \ldots, 0$' sequence of truth values. For this reason, Meredith argues: "'$n$', though true, is next to absolute falsum, '$n \supset 0$', though false, is next to absolute verum." (Meredith and Prior 1965, 108 [20]).

The second system of modal logic to which Meredith contributed was $U$-calculus, which was introduced in 1956 in a joint paper with Prior '*Interpretations of Different Modal Logics in the "Property Calculus"*'. The system contains variables '$a$', '$b$', and '$c$', which were not interpreted in the paper, but Prior identified them later with possible worlds (see Prior 1962, 36 [32]) and time instants (see Prior 1967, 88 [35]). Since this system of logic was primarily discussed by Prior, it will be introduced in the next chapter.

## 3 Prior's Concept of Possible Worlds

While Meredith's system of modal logic is many-valued, Prior was a keen proponent of intensional logic. Therefore, he discussed intensively possible worlds in his works and had a certain concept of them. This was, however, not quite the case at the earlier period of his career during which he started developing his interest in tempo-modal logic (the early Fifties). His view on modal logic developed during his lifetime. In the early Fifties, Prior appreciated also Łukasiewicz's many-valued approach to modal logic and future contingents,[7] but he turned to intensional logic later, especially when he began his development of modern temporal logic (see Prior 1955a [28]).

As was mentioned in a previous chapter, Prior's '$a$' was introduced in the joint paper with Meredith in 1956. Prior interpreted this variable and the entire system of logic to which it belongs in 1962 in his papers '*Possible Worlds*' and '*Tense-Logic and the Continuity of Time*'. All three of the papers in question deal with $U$-calculus,[8] the system of logic based on the introduction of the operator '$U$'. This operator is in '*Possible Worlds*' and '*Tense-Logic and the Continuity of Time*' described as an operator which states accessibility between possible worlds. '$Uab$' means the move (or the jump as was suggested by Geach to Prior) from the possible world '$a$' to the possible world '$b$'. It could be also interpreted as asserting that the possible world '$b$' could be reached from the possible world '$a$'. (Prior 1962b, 36 [32]; Prior 1962c, 140 [33]).[9] In '*Tense-Logic and the Continuity of Time*', Prior (1962c, 140 [33]) suggested a temporal interpretation of $U$-calculus, when he interpreted '$Uab$' as '$b$' being the future outcome of '$a$'. This interpretation is close to the temporal interpretation of $U$-calculus, where '$Uab$' means 'The instant $a$ is earlier than the instant $b$' (see Prior 2003a, 118 [37]).

B. Jack Copeland (2006, 378–380 [3]) emphasises that the papers '*Possible Worlds*' and '*Tense-Logic and the Continuity of Time*' are unique in

[7] This fact is evident from Prior's appraisal of Łukasiewicz's system of modal logic in papers, which were published in those years (see Prior, 1952a; Prior 1952b; Prior 1953a; Prior 1953b [24, 25, 26]).

[8] The origins of $U$-calculus are unclear. Copeland (2006, 377–378 [3]) argues that it was based on $l$-calculus, which Prior introduced previously. There is also a '$U$' operator which was introduced by Jerzy Łoś in the paper '*Foundations of the Methodological Analysis of Mill's Canons*'. Prior was acquainted with this paper by Henry Hiż's review [5] and discussed this operator in his *Time and Modality*. The formula '$Ut_1p_1$' means '$p_1$ is satisfied in the $t_1$', where '$p_1$' stands for a proposition and '$t_1$' can be understood as a time instant. Both variables belong to the semantical category of propositions (see Hiż 1951, 58–59 [5]; Prior 1957, 19–28 [30]). However, there is no direct reference to Łoś in Prior's discussion of $U$-calculus. Prior (1967, 42 [35]) argued that $U$-calculus was formalised by Meredith.

[9] The closeness to Kripke's accessibility relation in Kripke 1959 [7] is obvious. In so far as these understandings were also present, albeit implicitly, in Meredith and Prior's 1956 paper (*Interpretations of Different Modal Logics in the 'Property Calculus'*, published 1996 [40]) it is fair to say that they anticipated Kripke semantics by several years, as Copeland argues in (Copeland 2006 [3]).

their presentation of possible worlds semantics. Published soon after Kripke's famous paper '*A Completeness Theorem in Modal Logic*' (Kripke 1959 [7]), they made no reference to this paper, even though Prior as a reviewer of it was obviously aware of its publication as well as its content. This apparent omission might be caused by the fact that the two papers in question are based on a previously written paper '*Interpretations of Different Modal Logics in the "Property Calculus"*' which Prior wrote in cooperation with Meredith in 1956.[10]

The papers discussed so far were not the only papers dealing with modal logic which Prior and Meredith wrote together. In 1965, the paper '*Modal Logic with Functorial Variables and a Contingent Constant*' was published, in which Meredith's systems of modal logic and Prior's discussion of them appear. Prior acknowledged that he was influenced by Meredith's system of logic in his previous papers, namely by Meredith's constant '$n$' which in various contexts is interpreted to stand for 'possibility', 'the world', or the Wittgensteinian 'the world is the case', or plainly a truth-value (but then in line with the previously mentioned understandings). Nonetheless, in the discussion of Meredith's system in this paper Prior (1965, 100 [20]) argued: 'Formally, the system is elegant and ingenious; philosophically, it may well give rise to misgivings.' Prior demonstrated that the identification of the constant '$n$' with a possible world could lead to a problem with propositional identity, which is so serious that Prior claimed that there could be no such proposition as Meredith's '$n$' (Meredith and Prior 1965, 100–101 [20]). Prior (1967, 77–82 [35]) further refined and enlarged his criticism in the previously mentioned '*Logic of Successive World-States*' (chapter V in *Past, Present, and Future*).

Instead of a constant '$n$', Prior introduced world-propositions, which are formed by two operators '$W$' and '$Q$'. While '$Wp$' means '$p$ comprehends all truths' (Meredith and Prior 1965, 101 [20]), '$Qp$' stands for '$p$ is the totality of truth at some time' (Prior 1967, 80 [35]). The operators are defined as:

$$
\begin{aligned}
Wp &\stackrel{\text{def}}{=} p \land \forall q \, [(q \supset \Box(p \supset q)] \\
Qp &\stackrel{\text{def}}{=} \Diamond p \land \forall q \, [\Box(p \supset q) \lor \Box(p \supset \neg q)]
\end{aligned}
$$

Prior (1967, 79 [35]) claimed that these functors were able to prevent triv-

[10]The paper was, however, not published during their lifetime. It was discovered by Copeland and published by him in 1996 [40].

ialisation of modal logic and therefore they could replace the variable '$a$' or Meredith's constant '$n$'. Further in '*Logic of Successive World-States*', Prior (1967, 88–92 [35]) suggested the translation between this calculus and $U$-calculus. He followed this up in 1968 in the chapter '*Tense Logic and the Logic of Earlier and Later*' in *Papers on Time and Tense*, where the possibilities of translation are fully explored as '*Four Grades of Tense Logical Involvement*' (Prior 2003a [37]). The operators '$W$' and '$Q$' led to the postulation of Prior's world-propositions and instant-propositions. World-propositions (and instant-propositions) are according to Prior the maximal conjunct of propositions. It means that if any proposition which is not implied by this conjunct is added to it by conjunction a contradiction appears. World-propositions represent Prior's concept of possible worlds respectively time instants and at the same time they are ingeniously suited to Prior's nominalism. They allowed him to claim that there are no possible worlds as real existent entities but only propositions bound by the propositional quantifiers (see Meredith and Prior 1965, 99–102 [20]) (and this, in turn, strengthened one of his most crucial tense-logical points, the idea that instants do not exist in their own right but are to be understood as "logical constructions").[11]

## 4 '$a$' and '$n$' as Possible Worlds

There are certain similarities between Prior's '$a$' and Meredith's '$n$'. Prior also seemed to derive the inspiration for his variable '$a$' from Meredith's constant '$n$'. However, the aim of this chapter is to point out several reasons why a translation between Prior's '$a$' and Meredith's '$n$' is not straightforward.

Firstly, it is not certain to which of two traditions of modal logic Meredith belonged, as was mentioned previously. On the one hand, Meredith was deeply influenced by Łukasiewicz after the latter's appointment to Ireland, and Meredith took part in the development of Łukasiewicz's systems of logic. On the other hand, his important works dealing with modal logic were written in cooperation with Prior, who

[11] Prior said this more or less directly in many places. For example, in chapter V of *PPF* he argues that time instants consist of propositions, and in (2003a [37]) he strongly and thoroughly lays out the same idea. In what may well be the last note he ever wrote (Prior 1969 [36]), in a hotel in Ånsdalsnes in Norway shortly before he arrived in Trondheim where he died, he wrote most succinctly: "What is time? Time is a logical construction."

represented the intensional approach to modal logic. In addition, there is this certain concept of possible worlds in Meredith's works. The decision as to where Meredith really stood is crucial since the traditions differ radically in their approach to possible worlds. There are in our opinion certain features which indicate that Meredith approach to modal logic belongs more to Łukasiewicz's tradition than to Prior's.

As already mentioned there are features which suggest a closeness between Prior's '$a$' and Meredith's '$n$', not least the simple fact that the variable '$a$' and $U$-calculus were introduced in Prior and Meredith's joint paper '*Interpretations of Different Modal Logics in the "Property Calculus"*'. The question is, however, to what extent $U$-calculus was principally Meredith's system, or to what extent he contributed to the development of it. Namely, there could be a close similarity between '$a$' and '$n$', if Meredith's considered $U$-calculus as a system of modal logic and variables '$a$', '$b$', '$c$' possible worlds. It was already mentioned in this paper that Prior's interpretation of this system was linked with possible worlds semantics. In *Past, Present and Future* he however also reports:

> In some notes made in 1956, C. A. Meredith related modal logic to what he called the 'property calculus' in the following way: Suppose we use $a$, $b$, $c$, etc., as name-variables, and $U$ as a constant 2-place predicate. What the sentence-form $Uab$ means does not matter. (Prior 1967, 42).

It seems that Prior acknowledged Meredith to be the originator of $U$-calculus – indeed in all his remarks regarding $U$-calculus, Prior claimed that it was Meredith's formalisation. However, Meredith's interpretation of the system, if there was any, is unclear. Although Prior identified the variables '$a$', '$b$' and '$c$' with possible worlds, the quotation seems to imply that '$a$', '$b$' and '$c$' were not interpreted as possible worlds by Meredith.[12] Moreover, in spite of the fact that Prior attributed the invention of $U$-calculus to Meredith's formalisation, he did not acknowledge him as an author of possible worlds semantics – and Prior was always prepared to acknowledge any contribution which a colleague made to his own work (e.g., Øhrstrøm and Hasle 1995, 171 [23]). – But as ob-

[12]The interpretation appears to be closer to the operators in Leśniewski's system of logic. Nevertheless, there is no evidence that he has this system of logic in mind here and Wajsberg's original paper also referred to a different source of inspiration, namely the calculus of David Hilbert and Wilhelm Ackermann (Wajsberg 1933, 113 [43]).

served, when Prior discussed Meredith's possible worlds it is always linked to the constant '$n$'.

A variable '$a$' is mentioned in *Computations and Speculations*, in which is written:

> … n is represented by the property of being identical with a selected object a, formulae which express properties of a as well as formulae which express properties of all objects being taken as theorems. This is analogous to the use of matrices in which value n or 'true in n only', is designated as well as the value 'true in all worlds'.
>
> (Meredith and Prior 1962, 121 [18])

Nonetheless, there is no suggestion that this '$a$' is identified with possible worlds. It seems from this description that the variables '$a$', '$b$', and '$c$' stand for objects according to Meredith.

In addition, it is not exactly clear if Meredith had any elaborate philosophical concept of possible worlds as metaphysical entities. Meredith may have had certain ontological views on possible worlds, but they never occurred in his papers or even in his correspondence to Prior. However Prior reported certain views on this subject which attribute some metaphysical considerations or even views to Meredith. Namely:

> The system $(C, \Gamma, 0, n, \delta, p)$ introduces the more original feature of a constant $n$ to represent "the world" in the Wittgensteinian sense of "everything that is the case."
>
> (Meredith and Prior 1965, 99 [20]).

> For all his virtuosity in these formal manipulations, and his training being mathematical, Meredith likes to do philosophical jobs with his logic too. He has a modal system with a contingent constant n for 'the world' in Wittgenstein's (Tractatus) sense of 'everything that is the case' – the logical product of all true propositions. Developing Wittgenstein's other Tractatus statement that 'The world is the totality of facts, not of things' in the light of his insistence (e.g. in the Notebook 1914–16, p.93) that 'facts cannot be named' we may say that 'the world' is not the biggest nameable object, but the maximum that can be truly said, and so must be expressed

> by a proposition. Meredith's n, put to this use, has such laws as CpLCnp [$p \supset \Box(n \supset p)$] – any true proposition is strictly implied by n, since it is a conjunct of it. And a possible world is a proposition which, though possibly true, says so much that if any proposition be conjoined with it the result will be either an impossibility or strictly equivalent to the original. In a metaphysical mood Meredith once remarked that 'worlds' are the only real individuals; it is certainly true that his own interests have seldom taken in the ordinary calculus of names and predicates. (Prior 1962a, 9–10)

> I remember, too, C. A. Meredith remarking in 1956 that he thought the only genuine individuals were 'worlds', i.e. propositions expressing total world-states, as the opening of Wittgenstein's *Tractatus* ('The world is everything that is the case'). (Prior 2003c, 219)

This could illustrate what Meredith meant when he mentioned possible worlds, especially that he had in mind the view of the early (*Tractatus*) Wittgenstein. Nonetheless, does it imply that Meredith was committed to, or even seriously interested in possible worlds semantics? There are certain facts which might contradict this.

Prior's philosophical interpretation might not have been important for Meredith. This does not stand in contradiction to the fact that Meredith himself occasionally offered a philosophical remark, as reported by Prior. But such considerations may have been merely tentative and appear extraneous to where Meredith's real motivations and interests lay. This could be illustrated by the fact that Meredith never replied to Prior's philosophical objections to his system of logic. This by no means indicates that Meredith was indifferent to objections in general. On the contrary, in his correspondence with Prior Meredith was eager to respond to observations and queries regarding the formalism, and corrected mistakes if any appeared in his formal system.[13] But he did not discuss philosophical implications of his system nor the metaphysical queries Prior pointed out in the correspondence and in published works

[13] For instance, in his letter to Prior from 10th October 1956 [14], C. A. Meredith explained to Prior that he fixed objections of David Meredith regarding the problems with the rule of Modus Ponens in a certain system of logic.

as well. It could also be to some extent indicative that Meredith's possible ontological views are reported only by Prior, who was interested in ontological implications of his systems of logic, but not by Meredith the mathematician himself.

There is, however, a deeper difference between Meredith's system of modal logic and the joint work on the $U$-calculus. Meredith's system of modal logic was based on the many-valued matrix. The semantics for this system relies on truth-values only, even though certain truth-values are labelled as 'the world'. Though being titled as 'the world', '$n$' and '$\dot{n}$' are quite distant from what is meant by this term in modern modal logic. There is also a small remark on Meredith's part in '*Modal Logic with Functorial Variables and Contingent Constant*' which could indicate that he favoured an approach to modal logic as a many-valued system of logic:

> I do not know if there are any philosophical applications of this system. I can only suggest that these philosophers who think that logic must be two-valued are confusing $Hp$ and $p$.[14]. (Meredith and Prior 1965, 108)

While Prior's '$a$' is a part of $U$-calculus as an intensional system of logic, Meredith's '$n$' belongs to extensional systems of many-valued logic, in which it also played a role of a truth-value. This seems to open a considerable gap between Meredith and Prior. Meredith's contribution was, after all, not a contribution to (intensional) possible world semantics, hardly even a contribution to the very notion of possible worlds.

Finally, Prior was aware that there are also formal differences between the constant '$n$' from Meredith's system of modal logic and the variable '$a$' from $U$-calculus, even though both are entitled as 'possible worlds' in Prior's papers. In his paper '*Now*', Prior (2003b, 183 [38]) pointed out that '$n$' cannot be replaced by '$a$', since '$n$' is a constant and '$a$' a variable. Prior introduced an '$n$' as a constant in his $UT$-calculus. In his interpretation '$n$' is an instant-constant and also a propositional constant. It stood for a proposition which is true only in the moment of the utterance. For any utterance, that would obviously mean the "now" in which it was uttered. It was a part of his calculus for 'now'. '$a$', '$b$' and '$c$' were introduced as variables which stood for propositions similar to the proposition which was represented by '$n$', i.e. true only in one

[14] '$Hp$' is defined as '$\delta\Box(n\supset p)\supset\delta Hp$' by Meredith (Meredith and Prior 1965, 108 [20]).

current moment – Prior's famous instant propositions, which paved the way for hybrid logic (see Blackburn 2000 [1]) in the concomitant systems laid out as '*Four Grades of Tense-Logical Involvement*' (Prior 2003a [37]). This interpretation of Prior's without doubt bears some similarity to Meredith's '$n$', but it does not cover the function it has in Meredith's system of modal logic and it is also used in a different type of calculus.

# 5 Conclusion

Although Meredith and Prior wrote several joint papers, this paper attempts to stress certain differences in their concepts of possible worlds including the variable '$a$' and the constant '$n$' which are associated with them (especially in Prior's various remarks, as we have seen). It is important to have these differences in a mind during the evaluation of Meredith's and Prior's work since the underestimation of the differences could – paradoxically - lead to an underestimation of both Meredith's and Prior's contribution to modern logic, as it emerges from the papers discussed.

When disregarding the philosophical queries, which Prior pointed out, Meredith's system of modal logic loses its weaknesses. If formal logic and a formally correct system were Meredith's essential interests, as indicated through his correspondence with Prior as well as the description found in the Necrology by his cousin and fellow logician David Meredith (D. Meredith 1977 [22]), then his system of modal logic fulfilled his goals. Prior could have objections to philosophical implications of the system, but they might have been unimportant to Meredith.

Due to the formulation of the paper '*Interpretations of Different Modal Logics in the "Property Calculus"*' Meredith and Prior were considered to be precursors of possible worlds semantics, as discovered and argued by (Copeland 2006 [3]). However, it was only Prior who needed this type of semantics for his intensional systems of modal and temporal logic. Although Prior benefited from Meredith's formal introduction (and Geach's sci-fi suggestions) in his $U$-calculus, the final semantics and the relation of accessibility were primarily his work. It might be more precise to say that it was just Prior, who was a precursor of possible world semantics. The Prior-Meredith's correspondence indicates that Meredith had no intentions to reserve a part of this honour to himself.

# 6 Appendix on *Computations and Speculations*

Prior's Nachlass is kept in a number of boxes (1–22, see http://www.priorstudies.org) in the Bodleian Libraries, Oxford – specifically, in the Weston Library, Special Collections.[15] In Box 8 is found an incomplete book manuscript with this frontpage:

Computations and Speculations

By

C.A. Meredith.

Edited by

A.N. Prior

Professor of Philosophy, University of Manchester.

Many, though not all observations on Meredith's '$n$' are based indirectly, or sometimes directly, on this manuscript. The manuscript is typed, but incomplete. It originally consists of over 207 pages – cf. its Table of Contents, whose last section VIII.1. starts at p. 207. The manuscript as found in Prior's Papers Box 8 is somewhat complicated to overview. This is described in greater detail at (Hasle and Øhrstrøm 2014 [4]) http://www.priorstudies.org (Box 8, First Folder, Info). It should be noted that the manuscript is enclosed in a folder designated 'Miss P. Horne', who was the secretary in the department of Philosophy at the University of Manchester during Prior's years there. She on several occasions typed manuscripts for Prior.

According to a handwritten note by Mary Prior, found at the beginning of the folder, the manuscript was at one time submitted to OUP but was not accepted. According to a further note by Mary Prior, the missing parts formed the basis of five other publications, namely

1. C.A. Meredith & A.N. Prior. (1963). '*Notes on the axiomatics of the propositional calculus*', Notre Dame Journal of Formal Logic, vol. **4** (1963), pp. 171–187.

[15]Weston Library, Broad Street, Oxford, OX1 3BG
Enquiries: specialcollections.enquiries@bodleian.ox.ac.uk
Bookings: specialcollections.bookings@bodleian.ox.ac.uk

2. C.A. Meredith & A.N. Prior. /1964). *'Investigations Into Implicational S5'*, Zeitschrift für mathematische Logik und Grundlagen der Mathematik, vol. **10** (1964), pp. 203–220.
3. C.A. Meredith & A.N. Prior. (1965). *'Modal logic with functorial variables and a contingent constant'*, Notre Dame Journal of Formal Logic, vol. **6** (1965), pp. 99–109.
4. C.A. Meredith & A.N. Prior. (1968). *'Equational logic'*, Notre Dame Journal of Formal Logic, vol. **9** (1968), pp. 212–226.
5. E.J. Lemmon, C.A. Meredith, D.Meredith, A.N. Prior, & I. Thomas. (1969). *'Calculi of pure strict implication'*, Philosophical Logic, ed. by J.W. Davis, D.J. Hockney, & W.K. Wilson, D. Reidel, Dordrecht, 1969, pp. 215–250. (Previously published in mimeograph form, University of Canterbury, 1957.)

## Dating

It is immediately obvious that the manuscript must have been produced during Prior's time as professor in Manchester, i.e., 1959–1965. Moreover the latest reference which can be found, in a footnote on p. 194, is from 1961 (namely, the second edition of Prior's *Formal Logic*). This makes it clear that the manuscript must have been produced in 1961 or later. Furthermore, since paper 1) above was published in 1963 this narrows down the possible dating to be 1963 at the very latest, and in all likelihood somewhat earlier. Finally it may be observed that even in *CaS* there is no clear identification of '$a$' and possible worlds. Since in papers published in 1962 there is such an interpretation, *CaS* must have been produced by 1962 at the very latest, and in all likelihood somewhat earlier. In conclusion, we end up with a dating most likely 1962.

## Authorship

As seen from the front page, C.A. Meredith is the author of the manuscript, and Prior figures as the editor. Nevertheless, it would appear that Prior did more than mere editing and indeed had an active hand in producing the running text. First of all, Meredith is mentioned in the third person throughout, e.g., *CaS* p. 111 and p. 112. This of course could have been a stylistic choice, even if somewhat unusual, but other passages seem to indicate quite strongly that Prior is the 'direct writer'. On page 138 we find this passage:

> Meredith first presented his work on D at a logical colloquium in Oxford in 1956; and it was there suggested at that colloquium by his cousin D. Meredith that there ought to be an axiomatization of that part of modal logic which employs no constant but strict implication. A number of us were provoked by his suggestion to begin work on this [which would later lead to the joint publication 1969f].[16] (*CaS* p. 138)

Moreover, in the five papers derived from *CaS*, Prior in all cases is presented not as editor but as the second author. David Meredith, cousin of C.A. Meredith and a fellow logician, stated this in his Necrology of C.A. Meredith:

> Many of the results that were prepared for publication by A. N. Prior in the sixties had been discovered as [CA] Meredith attempted to respond to queries from logical colleagues. (D. Meredith 1977, 514 [22]).

Here, 'published' certainly means more than simply accepting them for journal publication – Prior was not even on the editorial board of any of the journals in question. Overall, these features indicate that Prior had a more active role in the creation of *CaS* than merely editing, as otherwise suggested by the front page's 'Edited by A.N. Prior'. On the other hand, the original ideas, proofs etc. etc. without doubt stem from Meredith – completely or at least to a very high degree (it cannot entirely be ruled out that Prior could have amended a formula, a proof or a formulation here and there). Prior was a keen admirer of Meredith's work and hoped to motivate him to publish more, and his role in editing and working on this manuscript must be seen in that light. However, the cumulative evidence leads us to include *CaS* in our references with Prior as more than the editor, to wit, as second author.

Finally, it is worth mentioning that Section VI: 109–135 extensively deals with 'Meredith's $n$', which is alternately presented as a truth-value or a contingent constant which can be understood as a possible world.

[16]The colloquium mentioned was the logic colloquium organised by Prior et. al, cf. the *Logic Colloquium Programme: Oxford, 1956* in Prior's Papers in the Bodleian Library, Box 11, First Folder (see http://www.priorstudies.org – Boxes). This event was crucial to the development of significant logical work including Meredith and Prior's further cooperation. Several of the publications mentioned in this paper emanated, sooner or later, from this colloquium.

In a sense this has been an inspiration to Prior's later work on world propositions and instant propositions – important in its own right but even more important because this makes it belong to early part of the (pre)history of hybrid logic. Unfortunately, pages 123–135 are missing.

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