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A Token-Level Decoding Algorithm for Ensembling Models Across Vocabulary Sizes

Anonymous ACL submission

Abstract

Model ensembling is a technique to combine the predicted distributions of two or more models, often leading to improved robustness and performance. For ensembling in text generation, the next token's probability distribution is derived from a weighted sum of the distributions of each individual model. This requires the underlying models to share the same subword vocabulary, limiting the applicability of ensembling as many open-sourced models have distinct vocabularies. In research settings, experimentation or upgrades to vocabularies may introduce multiple vocabulary sizes. This paper proposes an inference-time only algorithm that allows for ensembling models with different vocabularies, without the need to learn additional parameters or alter the underlying models. Instead, the algorithm ensures that tokens generated by the ensembled models agree in their surface form. We apply this technique to combinations of traditional encoder-decoder models and decoder-only LLMs and evaluate on machine translation. In addition to expanding to model pairs that were previously incapable of token-level ensembling, our algorithm frequently improves translation performance over either model individually.

1 Introduction

Text generation takes place as a sequence of token predictions. At each time steps, the model, conditioned on some input, produces a probability distribution over the vocabulary. From this distribution, the next token is selected to extend the hypothesis—the text generated thus far.

Individual models may be sensitive to noise or lack coverage in certain domains. Model ensembling is a method to combine outputs from multiple models, which often provides for more robust outputs and increases in performance. The traditional model ensembling approach assumes a shared vocabulary and computes a new distribution

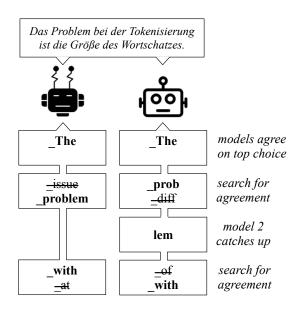


Figure 1: Agreement-Based Ensembling (ABE) enables ensembling among models with different vocabularies. Token generation for each beam item is constrained to tokens with agreeing detokenized forms.

as a weighted sum of its component vocabularies:

$$p(x_t) = \sum_{i} \lambda_i p_{m_i}(x_t \mid x_{1..t-1})$$
 (1)

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where all interpolation weights, λ_i , sum to 1. The new ensembled distribution functions the same as if it originated from a single model and the next token prediction proceeds as usual.

In practice, most models do *not* share vocabularies. When the vocabularies differ, the resulting probability distributions are no longer comparable. Then, it is no longer straightforward to ensemble these outputs. To address this, we introduce Agreement-Based Ensembling (ABE), an inference-time ensembling algorithm that requires no new parameters or model adaptation, but instead works by coordinating token selection across models under the notion of *agreement* (§ 3.1). At each decoding timestep, each model produces its distribution over the next token; our method efficiently searches over their cross-product for tokens

that are contextually compatible with the currently generated surface string (§ 3.2). When the tokens are different (but agreeing), the longer token constrains future search (§ 3.3). This is caricatured in Figure 1). Our approach easily extends to other inference algorithms such as beam search (§ 3.4).

Our contributions are as follows. We

- introduce an inference-time algorithm for ensembling models with different vocabularies¹,
- demonstrate the ability to ensemble across varying architectures (encoder-decoder, LLMs, or both), and
- show improved results in machine translation across a range of models.

Our code is implemented in Python using the Huggingface transformers library (Wolf et al., 2019) and is open-source.²

2 Related Work

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Ensembling is a generally reliable technique for increasing the quality of model outputs that goes back at least as far as Hansen and Salamon (1990). Although it is more expensive, and therefore often prohibitive in production inference settings, it is useful for example in competitions or for production training scenarios, such as for distillation. In such settings, the user typically has complete control over model training; ensembled models can be taken from different checkpoints (Sennrich et al., 2016) or from completely different training runs initialized from different random checkpoints, and therefore all have the same vocabularies. Hoang et al. (2024) move a step beyond this by ensembling models with divergent architectures (an MT system and an LLM) and across contexts longer than are supported by all models, but the models still share the same vocabulary.

The situation becomes more difficult when the vocabularies are not shared. One way to address this is to work at the sequence level instead of the token level. One such approach is that of Jiang et al. (2023), who propose LLM-Blender. It comprises a ranking function that computes pairwise comparisons of complete model outputs and then selects from among them; this approach completely

avoids the need to do any kind of token-level ensembling. Farinhas et al. (2023) generate multiple translation hypotheses and then explore selecting from among them using voting, minimum Bayes risk, and LLM-based selection.

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Sequence-level ensembling has limitations, and the reality of disjoint vocabularies has motivated prior work in token-level ensembling even across different vocabularies. Existing work, however, requires extra model training. Xu et al. (2024) learned mappings across vocabularies that map token representations into a joint space, and employ a variety of filtering methods for efficiency. Shen et al. (2024) present a "collaborative decoding" framework between a lead and assistant model where a classifier dynamically selects which of them will produce the next token at each step of generation; their approach also appears to require a shared vocabulary.

Our work is distinct in that it requires no further training or parameters. Our approach manages token-level ensembling across different vocabularies by ensuring that all models in the ensemble agree on the *string* being generated, and interleaves model steps for models that fall behind.

3 Agreement-Based Ensembling

Autoregressive models produce distributions over their vocabularies at each decoding time step. This process generally continues until the end-ofsequence token is produced or some maximum length reached. Greedy decoding, beam search, and sampling are all search algorithms that change how the next token is selected.

The traditional model ensembling approach (also called here *interpolation-based ensembling*) fits nicely within any of these frameworks, but requires the models to share the same vocabulary. This approach simply alters the probability distribution to be a weighted sum of the distributions from each model. Any search algorithm proceeds as before, selecting a token from this new distribution.

When the vocabularies differ, the distributions do not match and we cannot so nicely factor the probability computation from the algorithm. In Agreement-Based Ensembling, each model produces its distribution over its own target vocabulary as usual, but algorithmic changes are required to coordinate on the selection of the next token to ensure they agree on the detokenized surface string.

In this section, we will describe these changes.

¹Our sole requirement is that models are open-vocabulary so that they can generate any string the other model can.

²Outputs and code available anonymously at https://anonymous.4open.science/r/anon-abe-073B. It will be released as Apache 2.0.

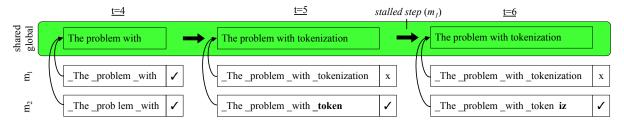


Figure 2: A global state maintains the shared detokenized string, which is determined by the local hypotheses. Associated with each model is a flag denoting whether the model is stalled (\times) or able to generate (\checkmark) . In stalled steps (§ 3.3), only the trailing model(s) generate(s) a token, catching up with the shared string. The stalled model is prevented from generating additional content.

At a high level, this requires maintaining a shared global agreement state (§ 3.1), efficiently searching the cross-product of the models' vocabularies (§ 3.2), and handling the varying token lengths of the models' differing vocabularies (§ 3.3). For ease of presentation, we will describe the algorithm using two models in a greedy decoding setting; this allows us to focus on these new ideas, without the complexity of beam search. However, the algorithm works with any number of models, so long as they all have open-vocabularies, and the extensions to beam search (which we used for all our experiments) are straightforward.

3.1 Agreement

The fundamental difficulty when ensembling models with different vocabularies is to ensure that they reach consensus on the shared output string, despite the fact that the string will have been generated via different tokenizations. In Agreement-Based Ensembling, we maintain a shared string—the global hypothesis—which is updated at each time step by the predicted tokens. It is important to store and compare against this string in *detokenized* form³ for precise comparison. Each model separately maintains its own local hypothesis under its own tokenization, which is a substring of this global hypothesis. This is visualized in Figure 2.

We define the notion of agreement. Consider a set of strings S. The global hypothesis, g, of this set is defined by (1) the shortest terminated string (ends with end-of-sequence token) or (2) the longest unterminated sequence—whichever is satisfied first. A set of strings S is in agreement if and only if all $s_i \in S$ are substrings of g. Note that agreement does not mean the models have produced the exact same string, only that their strings do not disagree. The algorithm provides a core inductive guarantee

that the detokenized string for every model will always agree with the shared global hypothesis.

3.2 Efficient Search

At each decoding timestep, each model takes its forward step from its current state and produces a distribution over its vocabulary. We need to efficiently search the intersection of their vocabularies for extensions to the current shared hypothesis that are in agreement. This space has dimensions $V_1 \times V_2$ and is too large to search completely.

We therefore apply a variant of cube pruning (Chiang, 2007; Huang and Chiang, 2007) with an "agreement filter" to search this space efficiently. The distributions from each model are sorted, per usual, and arranged into a two-dimensional grid. This is depicted visually in Figure 3. Each box in the grid denotes the selection of a token from each vocabulary, each of which is associated with a score, computed as the weighted sum of the length-normalized model scores for each local hypothesis. Normalization is important as model hypotheses are not guaranteed to be the same length.

To enumerate these items, we maintain a heap, which stores tuple items (i, j, s), where i and j index the candidate vocabulary items, and s records their weighted score. The heap is seeded with the tuple (1, 1, s) denoting the top left corner of this grid, representing the most probable token extension from each model. We now iterate as follows:

```
    while True do
    Pop item from heap
    Compute strings s<sub>1</sub> and s<sub>2</sub>
    if agrees(s<sub>1</sub>, s<sub>2</sub>) then
    return item
    end if
    Add neighbors of item to heap
    end while
```

³We store byte-strings so byte fall-back tokenization and non-Latin scripts to work.

⁴In all experiments, models are evenly weighted.

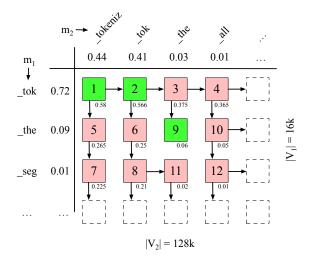


Figure 3: The first 12 candidates in ABE search space for unstalled m_1, m_2 . Each model's vocabulary is sorted by score. The top left corner is pushed onto a heap with its score, 0.58. The loop then pops from the heap, checks for agreement, and adds unvisited neighbors onto the heap. Numbers denote visitation order.

Although we need only one valid item for our greedy search example, Figure 3 depicts the first twelve loop iterations for illustrative purposes. At each step, the current item is popped from the heap and checked for agreement. This item is checked to determine whether the set of proposed local hypotheses are in agreement. Arrows denote "neighbor" items (the next vocabulary extension in each dimension), which are used to create updated tuples that are then added to the heap.

The algorithm can be extended to an arbitrary number of ensembled models by making use of an n-dimensional hypercube,⁵ and extending the tuples to include n vocabulary position indices.

3.3 Stalled steps

Models with larger vocabularies are likely to generate longer subwords at each timestep. This means that one model may be ahead of the rest and need to be *stalled*. We define *stalling*. Consider a set of models, M. The set of local hypotheses generated by M is S, where s_i was generated by m_i . Recall that the global hypothesis is represented by g. A model, m_i , is stalled when $s_i = g$ and at least one other model is not stalled: $\exists (m_j, s_j) \ni s_j \neq g$. An example of when a model becomes stalled is illustrated in time steps t = 5 and t = 6 in Figure 2.

Stalled steps aim to restore this imbalance by al-

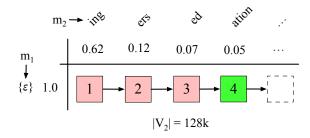


Figure 4: Search space when m_1 is stalled. m_1 has generated *tokenization* while m_2 has only generated *token iz*.

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lowing the unstalled models to generate without the stalled models in order to catch up. Conceptually, stalling a model is simple We prevent the model from being able to generate a token by replacing V (its vocabulary) with $\{\epsilon\}$ —an empty transition. We illustrate the reduction in search space in Figure 4. Note that for each stalled model, the dimensionality of the search space is effectively reduced by one.

3.4 Beam Search

Greedy decoding is a special case of beam search where the beam size is 1. It is simple to extend ABE to handle larger beams. The main conceptual difference is that the search space includes an additional dimension, the beam index. For a beam size of k, the search space is $k \times V_1 \times V_2$. Similar to the extension beyond two models (end of Section 3.2), we add an additional index to denote which beam item each vocabulary pair comes from. Then, instead of terminating after the first valid item, we iterate until we have encountered k of them. For instance, three models with a beam would have a 4-dimensional search space of $\{k \times V_1 \times V_2 \times V_3\}$. The k items become the beam at the next time step. Note that *neighbors* of a given candidate must come from the same beam item; beam number 2 cannot have neighbors in beam number 3.

Beam lengths may be ragged due to stalling, but this is handled with padding, normalization, and selecting hidden states based on hypothesis length.

4 Experiments

Agreement-Based Ensembling constrains the output of each model by the output of all models. We therefore choose to evaluate against machine translation (MT) due to its constrained nature. We primarily evaluate on the WMT24 test set (Kocmi et al., 2024) en-de but extend to several other out-of-English directions (cs, es, uk) from the

⁵For simplicity, we use the term hypercube, though not all dimensions are equal.

same test set. For evaluation, we consider both COMET (Rei et al., 2022) and BLEU (Papineni et al., 2002). We computed COMET scores with with pymarian⁶ (Gowda et al., 2024), and BLEU scores with sacrebleu⁷ (Post, 2018).

We examine ensembling within and between different classes of models:

- Custom MT. We train our own encoderdecoder models on the same pool of data with different vocabulary sizes.
- **Public MT**. Large-scale, multilingual, publicly-available MT models.
- LLMs. Decoder-only LLMs with demonstrated capabilities in MT.

4.1 Models

For preliminary experiments, we start by ensembling models that we trained. This allows us to have control over the vocabulary while also guaranteeing the models are reasonably similar and will frequently agree during generation. We then extend to off-the-shelf models, covering both encoder-decoder and decoder-only architectures.

Custom MT We train transformer base models using Marian (Junczys-Dowmunt et al., 2018) on approximately 600m lines of filtered English—German data downloaded using mtdata (Gowda et al., 2021) (details in Appendix A). We perform standard data filtering to include deduplication, language identifiation, length ratios, and margin-scoring. We train four unigram-based sentencepiece tokenization models (Kudo, 2018; Kudo and Richardson, 2018) with sizes of 8k, 16k, 32k, and 64k. Using these four tokenizers, we train four associated machine translation models.

Each model is a standard transformer base model (Vaswani et al., 2023) with 6/6 layers, embeddings size 1024, and hidden sizes of 8192. The entire configuration can be found in Table 6 in the Appendix. The data is randomly shuffled for infinite streaming via sotastream (Post et al., 2023), so we use logical epochs (1b tokens) rather than exact passes over the training set. We train for 25 logical epochs on one 24GB Titan RTX. In our experiments, we use various checkpoints of these models.⁸

Public MT In addition to custom models that only support English and German, we also consider two widely used multilingual MT models, M2M (Fan et al., 2020) and NLLB (Team et al., 2022) in multiple size and distillation variants. The former covers 100 languages with a 128k multilingual vocabulary, while the latter covers 202 languages with a 256k multilingual vocabulary. The hugging-face repository ids for all off-the-shelf models are listed in Table 7 in the Appendix.

LLMs We consider TOWER (Alves et al., 2024) and LLaMa 3.x (Grattafiori et al., 2024). TOWER is an LLM specifically fine-tuned for the task of translation whereas LLaMa is general purpose. LLaMa models use a vocab of 128k while TOWER uses 32k. TOWER was finetuned with the following prompt:

Translate the following text from English into German. \n English: {source sentence} \n German:

For LLaMa models, we use both 0-shot prompts and 3-shot prompts derived from the WMT24 baseline evaluation scripts. Exact verbiage of prompts can be found in Table 8 in the Appendix. LLMs differ in architecture from the previous settings as they lack an encoder. This further illustrates that ABE is architecture-agnostic.

We note that ensembling two large LLMs with 3-shot prompts requires an additional memory footprint. These experiments were run on a single 80GB A100 though can be managed with approximately 48GB.

4.2 Baselines

To compare the results of our ensembling, we have two baseline generation algorithms. The first is vanilla translation: using the model as intended. For the MT models, this is only passing the source input (with some language id tags for the multilingual models) to the huggingface generate function. For TOWER and LLaMa, we use the huggingface pipeline function with the aforementioned prompts (explicitly listed in Table 8).

We additionally consider linear interpolation as an ensembling baseline. In this traditional setting, two models' output distributions can only be interpolated when they are over the same event space (i.e., have the same vocabulary). We therefore only run this baseline over our custom MT models, making use of different checkpoints along the training trajectories of the different models.

⁶Version v1.12.31, wmt22-comet-da model

⁷Version 2.5.1, standard params.

⁸Namely epochs {1, 5, 10, 15, 20, 25}

⁹https://github.com/wmt-conference/ wmt-collect-translations

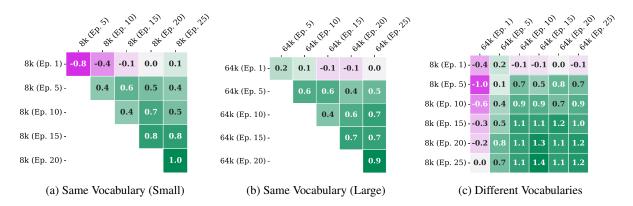


Figure 5: Δ COMET results on our custom English–German models using Agreement-Based Ensembling. Δ COMET is the improvement of ensembling two models via ABE over the best individual model. Labeling indicates vocab size followed by epoch checkpoint. All results on en-de WMT24.

For both baselines and Agreement-Based Ensembling, we use a beam size of 5 for all models. We generate with a maximum length of 256 tokens. ¹⁰

5 Results

We demonstrate the effectiveness of our ensembling algorithm by comparing the sequences generated by ABE over the performance of the best individual model. Given two models m_i , m_j , the translations produced by either model alone are T_i and T_j , respectively. The translations produced by ensembling these two models with ABE are denoted as $ABE_{i,j}$. We define the delta as:

$$\Delta S = S(ABE_{i,j}) - \max(S(T_i), S(T_j))$$
 (2)

where S may refer to BLEU or COMET scores.

5.1 Custom MT Models

In Figure 5, we display the $\Delta COMET$ scores across various combinations of custom MT models. We provide the $\Delta BLEU$ for all custom models in Appendix Figure A. We see consistent positive improvements across many checkpoints. In Figure 5(a) and Figure 5(b), we ensemble the smallest and largest custom MT models with vocabulary sizes of 8k and 64k, respectively, across various checkpoints. Further, we successfully do token-level ensembling of models with differing vocabularies (Figure 5(c))—a previously impossible task.

A persistent trend we find is that under-fitted models (e.g., Ep. 1) do not ensemble well. This is evidenced by negative $\Delta COMET$ scores across the

first row. In all other combinations, we see improvement, thus demonstrating the power of ensembling via ABE over using individual models. We also seek to demonstrate that these ensembling results are at least as good as a naive interpolation-based ensembling baseline. In order to do this, we compare the relative improvement using interpolation-based ensembling to the improvement gained from ABE. Note that this restricts the setting in which we can ensemble as the vocabularies must match. In Table 1, we display the relative Δ BLEU improvements and see that ABE is often a bigger improvement in these models.

BLEU	Δ Interpolation	ΔABE
27.7	0.16	1.07

Table 1: All scores are averages across all experiments. We report the average BLEU across models. For all model pairs we report the average improvement in BLEU over the score of m_1 or m_2 individually when using Interpolation or ABE.

5.2 Public MT Models

Our custom models are well-suited for ABE, since they were trained on the same data and potentially have related vocabulary distributions even when their vocabularies differ. We next consider models over which we have less control. As large multilingual models, M2M and NLLB are quite different from our custom ones. In Figure 6, we display Δ BLEU. ABE creates positive improvements though not across all combinations as seen in Custom MT.

We see an improvement when ensembling our largest custom model (64k) with larger multilin-

¹⁰If a model is stalled at this length, there is no agreed hypothesis and we return an empty string.



Figure 6: Δ BLEU of ensembling different encoderdecoder model pairs using ABE. This includes our largest custom model (bilingual) and publicly available multilingual models.

gual MT models. We suspect the smaller multilingual model (M2M 418M) performs less well than a bilingual model or a larger multilingual model, and these negative trends may be further examples of underfit models. The Δ COMET scores (displayed in the Figure 10) with ABE are more negative than their BLEU equivalents. This may indicate that ABE may do better at surfacing particular n-grams but may affect other aspects such as fluency that COMET or other neural metrics may penalize.

5.3 Off-the-Shelf LLMs

We demonstrate the flexible nature of the algorithm by extending our ensembling results to distinct architectures—encoder-decoder with decoder-only LLMs. In Figure 7, we display Δ BLEU improvements. In this section, we only display 3-shot experiments with LLaMa but a more comprehensive results table is available in Figure 9 in the Appendix.

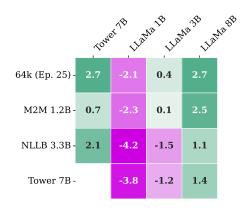


Figure 7: $\Delta BLEU$ of ensembling various encoder-decoder models with LLMs using ABE.

We still see consistent positive gains from ensembling models—particularly when ensembling

the bilingual models with the larger multilingual models. One crucial trend we notice is that poorer performing models, such as the smaller instances of M2M or LLaMa get consistent *negative* results. This indicates that poorer performing models will only deteriorate the performance of the better model which is also typical of other ensembling approaches. However, we see improvements when ensembling a small bilingual model with Tower or LLaMa. We further see improvements when ensembling two LLMs (+1.4 with Tower and LLaMa8b). As before, we observe more negative results when using COMET (Figure 10 in Appendix).

5.4 Additional languages

We additionally study the ensembling of these models with ABE by comparing the performance in other languages (cs, es, uk). We compare NLLB, Tower, and LLaMa and display the results in Table 2. Similar to before, we notice mixed performance across model pairs and target languages. We suspect this is due to underlying model differences.

		m_1	m_2	ABE	$ \Delta$
	cs	26.8	14.1	24.0	-2.8
NLLB + Tower	es	43.2	41.0	44.4	+1.2
	uk	26.3	6.1	24.1	-2.2
	cs	26.8	19.6	25.3	-1.5
NLLB + LLaMa	es	43.2	37.1	42.7	-0.5
	uk	26.3	20.3	26.2	-0.1
	cs	14.1	19.6	21.7	+2.1
Tower + LLaMa	es	41.0	37.1	42.0	+1.0
	uk	6.1	20.3	22.4	+2.1

Table 2: BLEU scores for different ensembling pairs and their individual models. m_1 and m_2 denote the individual model score while ABE denotes the ensembled score. Δ is the difference between ABE and the higher of m_1 and m_2 . The model versions are M2M 1.2B, NLLB 3.3B, Tower v0.2 7B, LLaMa 3.1 8B 3-shot.

Tower and LLaMa, which have been a consistently successful ensembling pair, see improvements in all three of these languages. We further note that according to their respective documentation, neither model supports cs or uk, 11 but we see improvements in both using ABE.

6 Analysis

We seek to answer why our ensembling is successful in some scenarios, though not all. We provide

¹¹There was likely substantial amounts of these languages in the pretraining data.

both a quantitative and qualitative study.

6.1 Model Preferences

One effect we wish to disentangle is whether this algorithm is an improvement on the search space or on the modeling. As previously mentioned, interpolation-based ensembling only affects the intermediate token probabilities (a modeling change) and makes no changes to the search procedure. ABE does a bit of both by severely altering the search and mildly altering the modeling (scoring by the weighted sum of two models instead of one).

To answer this question, we seek to quantify the preferences of each translation under each model. Given m_1 , m_2 and the associated translations T_1 , T_2 . We can ensemble these models with ABE to generate ABE_{m_1,m_2} . We then determine the ranking of these three translations under each modeling scheme— m_1 and m_2 —by comparing the models' likelihoods of each translation. In Table 3, we see that models that ensemble well together (top, custom models) also consistently rank the ABE output as the most likely. They also agree on the most likely output 86% of the time. Conversely, we see more mixed preference with M2M and NLLB (Δ BLEU=-0.2) suggesting that ABE cannot overcome underlying modeling disagreements. This indicates our method is more effectively exploring the search space when models agree.

		T_1	\mathtt{T}_2	ABE	Same %
8k+64k	$m_1 \\ m_2$	102 198	106 223	2207 2028	86.0
M2M+NLLB	m_1 m_2	1002 840	1012 809	1092 1096	54.5

Table 3: Preference. Top: m_1 and m_2 are our bilingual 8k and 64k models (+ Δ under ABE). Bottom: m_1 and m_2 are M2M1.2B and NLLB3.3B (- Δ with ABE). T_i shows counts when outputs of m_i were ranked highest (or tied). ABE shows counts when the outputs of the ensemble were ranked highest. "Same %" designates when models had the same ranking.

6.2 Constraining Hallucinations

Standard (same-vocabulary) ensembling can have a normalizing effect on models, for example helping increase their robustness to noise. Upon examining outputs, we found a recuring trend that ABE also helps prevent models that have begun to hallucinate. An example is shown in Table 4. Here, noisy inputs that are included by design in the WMT24 test sets occasionally trip up individual models, including

Llama-3.2 (3B-Instruct-3-SHOT). Using ABE on all pairs of these models yields the correct output.

source	lfg \$sqqq
16k 64k	lfg \$sqqq {m} {m} {m} {m} {m}
Llama	lfg \$qqqq\$qqqqqqqqqqqqq Es scheint, dass das ursprüngliche
Ziania	Textstück fehlt oder nicht verfügbar
	ist. Die gegebene Zeichenkombina-
	tion "lfg \$sqqq" ist nicht
ABE	lfg \$sqqq

Table 4: (Truncated) examples of individual models hallucinating or becoming overly verbose on noisy input, but in different ways. Any ABE pairing of these models produces the correct output.

7 Conclusion

We have presented an algorithm that enables tokenlevel ensembling of models with distinct vocabularies. In contrast to prior relevant work, our approach requires no learned mappings of token representations (Xu et al., 2024) or other model fine-tuning. Instead, we run models in parallel, using a classical approach from parsing and statistical machine translation to efficiently select tokens whose surface representation all models agree on.

We believe the algorithm itself is an interesting contribution to the literature, since it enables (and makes easy) a task that was previously impossible. Traditional ensembling is a technique that introduces improvements in some, but not all, settings. It is therefore interesting that our approach also (a) produces gains in a variety of machine translation settings and (b) also often improves over standard ensembling. Our analysis shows how this variant of ensembling seems to help address search errors in the underlying models, since those models often prefer (as measured by likelihood) the ensembled results to their own selections.

Machine translation was a natural task for this approach. For one, ensembling is often used to produce higher-quality distilled results. Second, the translation task helps constraint the generative output to a subset of tokens that meaningful capture the source semantics. Our agreement-based approach might falter in less constrained tasks. The implementation is conceptually simple and factored and allows for easy experimentation with different methods for agreement-based search. We therefore view this as a fruitful topic for future research.

Limitations and Ethics

We note a few limitations with our work. The first is our focus on one task, machine translation. Machine translation is heavily conditioned on the input, and the accepted translation set is relatively small compared to other tasks. Though this approach works on Large Language Models, it may not easily extend to other more diverse tasks such as summarization.

We also acknowledge that machine translation is still a generation task, and is prone to the typical generation pitfalls of hallucinations, or erroneous translations—particularly when using LLMs. Overly relying on error-prone automated translation without a human review can have unintended consequences when used as a means of distributing information.

The authors also acknowledge the assistance of LLMs in the work in this paper—in particular using AI agents like CoPilot and ChatGPT to write code and edit plots.

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A Appendix

Below we describe each step of our filtering pipeline:

- 1. Remove items when equal to source and target pair in our validation set.
- 2. Remove lines without both source and target.
- 3. Remove lines where langid (Lui and Baldwin, 2012) on source is < 0.5 for English and on target is < 0.5 for German.
- 4. Remove lines when more than half of the line is punctuation.
- 5. Remove lines that have too many characters with frequencies outside of the expected language set (Fan et al., 2020). 12
- 6. LASER based Margin-scoring (Artetxe and Schwenk, 2019) (done in 2.5M line chunks for computation).
- 7. Deduplicate all training data.

¹²https://github.com/facebookresearch/
fairseq/blob/main/examples/m2m_100/
README.md

Data Name	Filtered Size	Paper (if applicable)
ELRC	6.5M	
ELRA	66k	
EU (dcep, eac, ecdc)	1.8M	
Wikimatrix	5.6M	Schwenk et al. (2021a)
WikiTitles	2.9M	
TedTalks	166k	
Bible	35k	
OPUS Books	43k	Tiedemann (2012)
CC-Aligned	12M	El-Kishky et al. (2020)
CC-Matrix	244M	Schwenk et al. (2021b)
DGT	4M	
European Central Book (ECB)	83k	
ELITR	232k	
EMEA	233k	
EU Bookshop	5.1M	
EU Const.	4k	
Europarl (v3,7,8,10)	6.3M	Koehn (2005)
EuroPat (v1-3)	47M	Heafield et al. (2022)
Global Voices	174k	Nguyen and Daumé III (2019)
JRC	457k	Steinberger et al. (2006)
KDE/GNome	110k	Hätty et al. (2017)
MultiUN	118k	Chen and Eisele (2012)
MultiCCAligned	60M	
MultiParaCrawl	70M	
News Commentary (v9,14,16)	937k	
OPUS Train	580k	Tiedemann (2012)
ParaCrawl (v9)	242M	Esplà et al. (2019)
PHP	7k	
QED	400k	
Tanzil	476k	
Tatoeba	1.8M	Tiedemann (2020)
TED (2013)	403k	Cettolo et al. (2013)
XLEnt	1.4M	El-Kishky et al. (2021)
Tilde	4.8M	Rozis and Skadiņš (2017)
StatMT 13 (CommonCrawl)	1.8M	
Deduplicated	618M	

Table 5: We aggregate most English–German bitext listed on mtdata (available at https://github.com/thammegowda/mtdata). The above is the filtered text sizes.

Algorithm 1 Agreement-Based Decoding Using Beam Search (One Time-Step)

```
scores is a BEAM_SIZE x MODEL_NUMBER x VOCABULARY_SIZE list.
2
   Section 3.2: scores are sorted so we can enumerate as shown in Figure 2
   Section 3.3: if models are stalled, we only consider epsilon transitions
   scores = [[model.step(j) for model in models] for j in range(beam_size)]
6
   scores = [torch.sort(beam_score) for beam_score in scores]
   scores = mask_stalled_beams(scores)
   class State:
10
     def __init__(self, beam_index, grid_indices, token_id, score):
11
       # set values ...
12
     def find_neighbors(self):
       # enumerate neighbors ...
14
15
     def score(self):
        # score is the weighted sum of model's beam scores
16
17
18
   # now we search the cross-product of vocabulary items
   next_beam = []
19
20
   heap = heap()
   for j in range(beam_size):
21
22
     We seed (0 index for all model) our heap to search our grid (Figure 2).
23
24
     For stalled models, this is the epsilon transition
     The token_ids is the list of tokens belonging to each model's vocabulary
25
     The token_scores is the associated score of these tokens
26
27
     token_ids = [scores[j][i].idx[0] for i in range(len(models))]
28
     token_scores = [scores[j][i].value[0] for i in range(len(models))]
29
     state = State(
30
31
                    beam_index = j,
                    grid_indices = [0 for _ in models],
32
33
                    token_ids = token_ids,
                    token_scores = token_scores)
34
35
     heap.push(state)
36
   # now we expand the search space until we find beam size agreeing extensions
37
   while len(next_beam) < beam_size:</pre>
38
     item = heap.pop()
39
40
      # Each model has a local hypothesis (specific to internal state)
41
42
     local_hypotheses = [model.extend_beam(item) for model in models]
43
      # global hypothesis will define agreement
44
45
     global_hypothesis = determine_global(local_hypothesis)
46
     if agreement(local_hypotheses, global_hypothesis):
47
       next_beam.append(item)
48
49
     for neighbor in find_neighbors(item):
50
       heap.push(neighbor)
51
52
   return next_beam
```

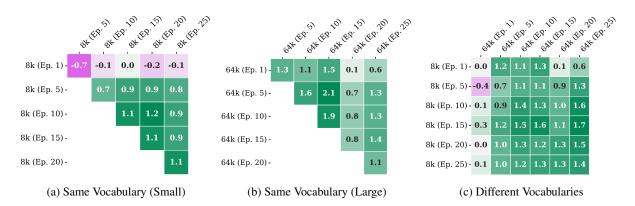


Figure 8: BLEU results on our custom English–German models using Agreement-Based Ensembling. These charts show the Δ BLEU improvement of ensembling two models via ABE over the best individual model. Labeling indicates vocab size followed by epoch checkpoint.

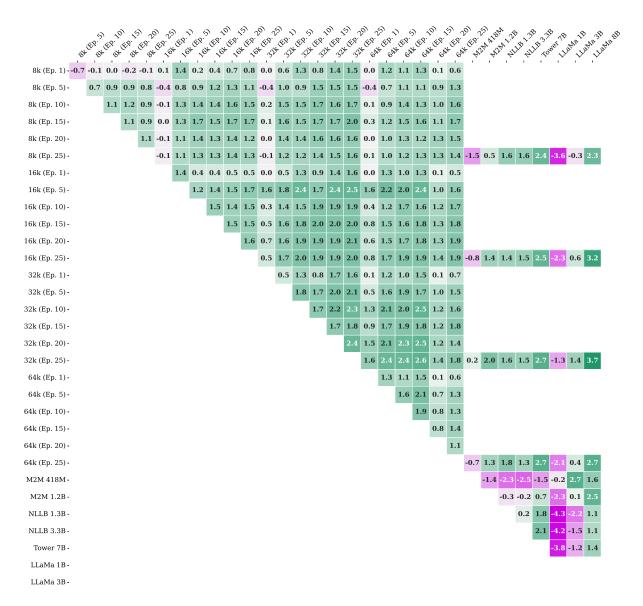


Figure 9: The Δ BLEU scores for all model pairs.

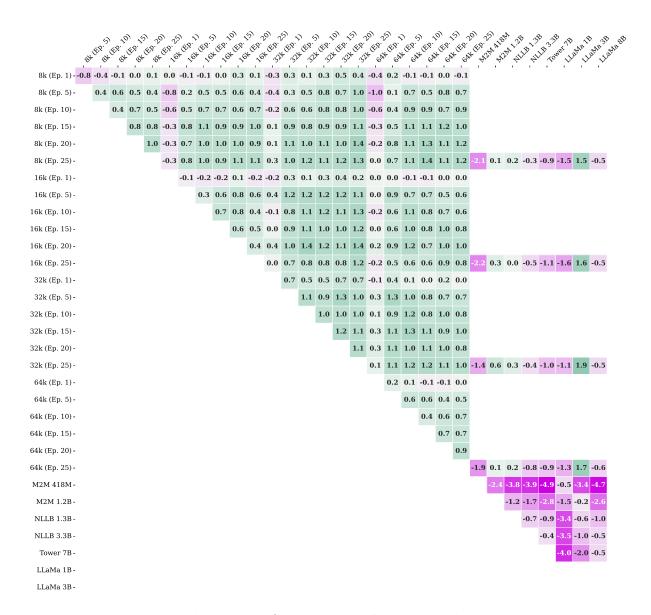


Figure 10: The Δ COMET scores for all model pairs.

Hyper-Parameter	Value
label smoothing	0.1
learning rate	0.0005
lr warmup	4000
lr decay inv sqrt	4000
mini batch warmup	4000
mini batch	1000
mini batch words	500000
max length	256
mini batch fit	true
early stopping	40
logical epoch	1Gt
shuffle	batches
fp16	false
tied embeddings	true
tied embeddings all	true
dim emb	1024
enc depth	6
dec depth	6
transformer dim ffn	8192
transformer decoder dim ffn	8192
transformer depth scaling	true
lemma dim emb	0
transformer ffn activation	relu
transformer-heads	8
transformer dropout	0.1
transformer dropout attention	0
transformer dropout ffn	0.1

Table 6: The above enumerate the Marian hyperparameters used for all of our custom models.

Model Type	Repo ID	m Size	V Size	Languages	License
	meta-llama/	8B	128k	de,es	LLaMa3
	Llama-3.1-8B-				
LLM	Instruct				
	meta-llama/	1B	128k	de, es	LLaMa3
	Llama-3.2-1B-				
	Instruct				
	meta-llama/	3B	128k	de, es	LLaMa3
	Llama-3.2-3B-				
	Instruct				
	Unbabel/	7B	32k	de, es	CC-BY-NC-4.0, LLaMa2
	TowerInstruct-7B-v0.2				
	Unbabel/	7B	32k	de, es	CC-BY-NC-4.0, LLaMa2
	TowerInstruct-				
	Mistral-7B-v0.2				
	facebook/m2m100_1.2B	1.2B	128k	de, es, cs, uk	MIT
	facebook/m2m100_418M	418M	128k	de, es, cs, uk	MIT
Public MT	facebook/	1.3B	256k	de, es, cs, uk	CC-BY-NC
r ublic IVI I	nllb-200-1.3B				
	facebook/	3.3B	256k	de, es, cs, uk	CC-BY-NC
	nllb-200-3.3B				
	facebook/nllb-200-	1.3B	256k	de, es, cs, uk	CC-BY-NC
	distilled-1.3B				
	Facebook/nllb-200-	600M	256k	de, es, cs, uk	CC-BY-NC
	distilled-600M				

Table 7: Huggingface Repo Ids for our publicly available models. LLaMa3 license refers to https://www.llama.com/llama3/license/. LLaMa2 refers to https://ai.meta.com/llama/license/. Tower also states the LLaMa license as it uses the LLaMa 2 pretraining weights. Language set only covers those addressed in this paper.

```
Shot | Prompt
0
     {"role": "system", "content": "Cutting Knowledge Date: December
     2023\nToday Date: 26 Jul 2024"}
     {"role": "user", "content": "Translate the following segment into XX.
     Do not add any additional content. Do not add parentheticals. Only
     provide the translation. The English segment:"}
 3
     The example translations are identical to the WMT24 evaluation scripts specific to the target language. The
     examples can be found at https://github.com/wmt-conference/
     wmt-collect-translations/tree/main/few_shots. Each example is put in the same format.
     Language names exchanged when necessary:
     {"role": "user", "content": "Translate the following text from
     English into German. The English Segment: example source}
     {"role": "assistant", "content": "{example translation}"}
     1
```

Table 8: LLaMa prompting messages.

B Sampling

One common use case with autoregressive models is sampling. As with other search procedures, standard ensembling works transparently with sampling. As a procedure, sampling is easy to implement with ABE. Instead of searching over the grid, we sample from each model consecutively (skipping over stalled models). The vocabulary which we sample from is renormalized to only allow for *agreeing* tokens.

We experimented with adding sampling to Agreement-Based Ensembling but found that it did not work well. We hypothesize the instability of sampling with this method stems in some part from the underlying idea that most tokenizers denote whitespace as *leading* (designating word beginnings) and not as *trailing* (designating word endings). This idea has been shown to have interesting effects on probability distributions (Oh and Schuler, 2024).

As an illustrative example, consider the following German indefinite articles: "ein" and "eine." The key difference being that "eine" is feminine. Both of these words are short and fundamental to the German vocabulary, so it is almost guaranteed that both words in their full form are in the model vocabulary. We further suspect that models with both of these words in their vocabulary have *never* seen "eine" tokenized as "_ein" + "e" in their training data.

Now consider our previously stated sampling procedure. Assume from m_1 , we sample "_Eine." When conditioned on this decision, we are likely to see both "_Ein" and "_Eine" holding most of the probability mass of m_2 . Let's assume we sample "_Ein" from m_2 . Since the local hypothesis of m_1 ("_Eine") and the local hypothesis of m_2 ("_Ein") are in agreement, this is a valid state to be in. However, when we next sample from m_2 to catch up to m_1 it is *not* going to have a high probability on "e" because it has never seen "Eine" tokenized that way during training.

We understand that m_1 has implicitly decided to generate the entire word "Ein", but it was unable to convey that it was *also* modeling the end of that word due to the tokenization scheme.

Now consider a word-ending tokenization scheme. Now, m_1 samples "Eine_" signifying that it is *done* with this word. When we constrain the output of m_2 on this hypothesis, "Ein_" is *not* going to be sampled because it does not agree. In

order to get into the same predicament, it would need to place high probability on "Ein", specifically *not* ending the word which is unlikely if both models wish to generate some version of the word "a."