

000 001 002 003 004 005 TUNE: FREQUENCY-GUIDED TOKEN GATING FOR 006 ROBUST CONTINUAL LEARNING IN LLMs 007 008

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011 ABSTRACT

012 Continual learning (CL) in large language models (LLMs) remains a critical chal-
013 lenge, as sequential training often results in catastrophic forgetting of previously
014 learned knowledge. To our knowledge, no prior work has approached CL in LLMs
015 from a frequency perspective, despite strong evidence that spectral properties
016 directly govern model robustness and vulnerability to forgetting. Recent meth-
017 ods based on Low-Rank Adaptation (LoRA) have shown promise for parameter-
018 efficient CL, but remain preliminary, relying on task-specific subspace expansion
019 with additional regularization. We propose TUNE (Token Update via Noise-robust
020 Frequency Encoding), a frequency-guided token modulation mechanism that sta-
021 bilizes LoRA residual updates. TUNE employs a stationary wavelet transform
022 (SWT) to decompose token embeddings into multi-resolution subbands, where
023 token saliency is derived from high-frequency activations and reliability is assessed
024 through cross-scale agreement. These signals are fused into token-wise scaling that
025 amplifies reliable updates while suppressing noisy fluctuations. Without introducing
026 additional trainable parameters beyond LoRA expansion, TUNE achieves signifi-
027 cant improvements over the SOTA baselines, establishing frequency-aware token
028 adaptation as a promising direction for CL in LLMs.

029 1 INTRODUCTION

030 **Continual Learning in LLMs** Continual learning (CL) aims to adapt models to new tasks while
031 retaining prior knowledge (Silver et al., 2013; De Lange et al., 2021). This is especially critical for
032 Large language models (LLMs), which operate in evolving domains at scale. Yet, when fine-tuned or
033 continually trained on new data, LLMs often suffer from catastrophic forgetting—the tendency to
034 overwrite previously acquired knowledge (Luo et al., 2023; Shi et al., 2024; Li et al., 2024). This
035 phenomenon limits the adaptability and reliability of LLMs, making it a critical area of research.
036 Existing CL paradigms include (i) rehearsal Huang et al. (2024); Pillai (2025), (ii) regularization such
037 as EWC/SI Kirkpatrick et al. (2017); Zenke et al. (2017), and and (iii) architecture-based subspace
038 isolation or expansion strategies Mallya & Lazebnik (2018); Wang et al. (2023b). While effective,
039 these methods are often computationally overload for LLMs due to their massive scale.

040 Recent advances in parameter-efficient fine-tuning (PEFT) have shifted attention toward lightweight
041 adaptation methods. In particular, Low-Rank Adaptation (LoRA) Hu et al. (2021) inserts small
042 trainable low-rank matrices while freezing the original weights, enabling efficient continual updates.
043 Extensions such as O-LoRA Wang et al. (2023a) and N-LoRA Yang et al. (2024) introduce explicit
044 regularization on inter-task orthogonality or parameter collision to mitigate forgetting while preserving
045 efficiency. These approaches highlight the promise of PEFT-based CL, where the cost of adaptation
046 is significantly reduced.

047 **High-Frequency Drift as the Substrate of Forgetting** Prior work has shown that model robustness
048 is closely tied to its frequency response (Yin et al., 2020; Ilyas et al., 2019; Sun et al., 2024). Spectral
049 analyses consistently reveal that low-frequency components encode the core global context, serving
050 as the primary carrier of semantics that discriminates between tasks or domains. In contrast, high-
051 frequency components act largely as detail enhancers—capturing local, instance-specific refinements
052 rather than general task-level structure (Cooley et al., 1969; Rahaman et al., 2019; Zhi-Qin John Xu
053 et al., 2020; Pan et al., 2023). Since they emphasize sample-wise variations, high-frequency features
054 are typically noisier and less stable, making them unreliable anchors for continual task adaptation.
055 Such abrupt feature drift in high-frequency band causes unstable representation shifts across tasks
056 and makes it a primary substrate of catastrophic forgetting.

To our knowledge, no prior work has addressed continual learning in LLMs from a frequency perspective. In spectral view, low-frequency components provide the stable semantic backbone for each task, while high-frequency components capture instance-wise local variations that are useful for capturing details but also prone to drift. This contrast is evident in Figure 1: low-frequency coefficients (A_{10}) form well-separated clusters between tasks, reflecting their role as stable semantic carriers, whereas high-frequency coefficients (D_1) collapse into overlapping clusters scattered broadly, showing their noisy property.

Motivated by the high-frequency-induced challenges in continual learning, we propose **TUNE** (Token Update via Noise-robust Frequency Encoding), a spectral token gating mechanism that integrates with LoRA residual adaptation to mitigate noisy high-frequency drift while amplifying reliable, salient signals. TUNE implements this by applying a stationary wavelet transform (SWT) along each token embedding sequence, decomposing it into aligned multi-resolution frequency subbands while strictly preserving original token structure. This decomposition enables multi-scale inspection of token activity and identifies whether the given token features are stable or noise-prone.

Specifically, TUNE introduces two complementary measures. First, a high-frequency saliency score quantifies the strength of token-level fine-grained variation, highlighting where meaningful novelty arises. Second, a parent-guided reliability score measures the consistency of high-frequency activations across scales, i.e., the agreement between a child detail coefficient and its reliable parent in the SWT hierarchy. This agreement-based reliability suppresses unstable spikes that fail to persist across scales while emphasizing coefficients with strong cross-scale support. Combining these two measures yields a token-dependent gating factor that rescales the LoRA residual updates, assigning larger adaptation capacity to reliable, salient tokens while damping noisy ones. In doing so, TUNE curbs catastrophic forgetting mainly caused by high-frequency drifts, while directing token plasticity toward trustworthy novel signals. We summarize our main contributions as follows:

- To our knowledge, this is the first work to tackle continual learning in LLMs explicitly from a frequency perspective, identifying high-frequency drift as a primary substrate of catastrophic forgetting.
- We propose TUNE, a spectral token gating mechanism that integrates SWT with LoRA residual adaptation. TUNE combines high-frequency saliency with parent-guided reliability to produce token-dependent gating factors that regulate adaptation.
- TUNE introduces no additional trainable parameters beyond standard continual LoRA paradigm with minimal computational overhead.
- Extensive experiments on both small- and large-scale CL benchmarks demonstrate that TUNE consistently improves over strong state-of-the-art baselines.

2 RELATED WORKS

2.1 LORA IN CONTINUAL LEARNING

Low-Rank Adaptation (LoRA) (Hu et al., 2021) constrains task adaptation to a low-rank update $\Delta W = AB$, enabling parameter-efficient finetuning by freezing the pretrained weights. This approach has recently been extended to continual learning settings. In particular, Orthogonal LoRA (O-LoRA) (Wang et al., 2023a) allocates a new LoRA subspace for each task while freezing those from previous tasks, and further imposes an orthogonality constraint so that the update directions of new tasks do not overlap with earlier ones. LB-CL (Qiao & Mahdavi, 2024) further develops this line by balancing LoRA consolidation across tasks to improve stability. Non-collision LoRA

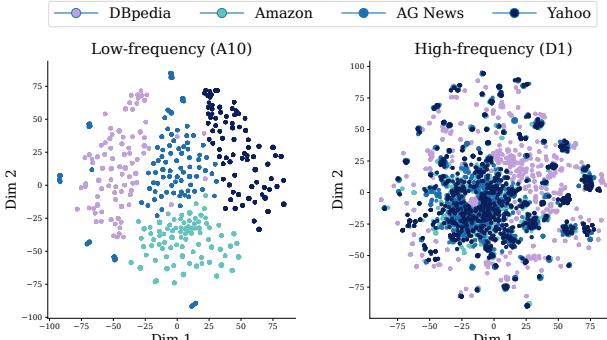


Figure 1: **t-SNE visualization of SWT-decomposed token features across 4 different tasks in standard CL Benchmark.** Left: level-10 low-frequency coefficients (A_{10}) exhibit clear separation by task. Right: level-1 high-frequency coefficients (D_1) show overlapping clusters.

(N-LoRA) (Yang et al., 2024) instead focuses on preventing parameter collisions by enforcing sparsity on task-specific LoRA parameters, ensuring that updates from different tasks occupy distinct, non-overlapping coordinates. These approaches establish LoRA as a representative backbone for continual learning in LLMs.

2.2 FREQUENCY PERSPECTIVE ON CATASTROPHIC FORGETTING

Representation drift caused by noisy features has been identified as a major driver of catastrophic forgetting. When models are updated on new tasks, parameters often overfit to unstable or spurious patterns in the incoming data, producing abrupt representational change that overwrites consolidated knowledge from earlier tasks. Prior work has shown that catastrophic forgetting is tightly linked to such drift. Ramasesh et al. (2020) dissected layer-wise representation dynamics and found that forgetting coincides with sharp shifts in internal activations. Caccia et al. (2022) further demonstrated that sudden representational change across tasks is predictive of performance collapse. Toneva et al. (2019) revealed that examples most frequently forgotten are those aligned with noisy or unstable decision boundaries. Together, these studies suggest that catastrophic forgetting emerges from updates driven by noisy, non-robust features that destabilize the feature space across tasks.

Building on this view, recent analyses suggest that the noisy features most responsible for drift often reside in high-frequency components of the representation. While low-frequency bands encode stable, task-relevant structure, high-frequency bands tend to amplify local variations and spurious correlations that carry weak or noisy semantics (Cooley et al., 1969; Ilyas et al., 2019; Yin et al., 2020; Pan et al., 2023; Sun et al., 2024). There is converging evidence that high-frequency components are the primary channel for model brittleness. Ablation studies conducted in Abello et al. (2021) shows that the largest error sensitivity is placed in mid/high bands and Wang et al. (2020) demonstrates that small-norm adversarial attacks concentrate their perturbations at mid/high frequencies while robust models tend to shift attribution toward low-frequency cues. These findings support treating high-frequency activations as unstable carriers of non-robust features that are prone to triggering abrupt shifts when emphasized during training. In continual learning setting, this implies that high-frequency activations can serve as a marker of noisy or fragile examples that destabilize previously learned representations if not treated carefully.

2.3 STATIONARY WAVELET TRANSFORM (SWT)

Discrete Wavelet Transform (DWT) The Fourier transform decomposes a sequence into sinusoidal bases, fully describing its frequency content but without temporal localization, since each coefficient reflects contributions from the entire signal. This limitation makes it less suitable for non-stationary data such as language, where preserving the temporal structure is essential. The wavelet transform overcomes this by analyzing a signal $x(t)$ with functions localized in both time and frequency. These functions are generated by shifting and scaling a prototype function, the mother wavelet ψ . This is particularly effective for signals whose statistical properties vary over time, such as language sequences where both global and local dependencies must be captured.

Since our focus is on language data, which naturally appears as discrete token sequences, we restrict our attention to the discrete formulation of the transform. For discrete signals, this expansion is realized through the discrete wavelet transform (DWT), implemented via a pair of analysis filters for low-pass (h) and high-pass (g) decomposition, together with corresponding synthesis filters \tilde{h}, \tilde{g} for reconstruction. Writing convolution as $(f * k)[n] = \sum_m f[m] k[n-m]$, the level- j DWT coefficients and its reconstruction are computed as

$$A_j[n] = (A_{j-1} * h)[2n], \quad D_j[n] = (A_{j-1} * g)[2n], \quad A_0 \equiv x, \\ A_{j-1}[n] = (\uparrow 2 A_j * \tilde{h})[n] + (\uparrow 2 D_j * \tilde{g})[n],$$

where $\uparrow 2$ denotes upsampling operator by two. The recursive filtering and decimation imply that each scale isolates a specific portion of the frequency spectrum. In the dyadic DWT with orthonormal, ideal half-band filters, the frequency support of the coefficients is exactly partitioned as

$$\Omega(A_j) = \left\{ \omega : |\omega| \leq \frac{\pi}{2^j} \right\}, \quad \Omega(D_j) = \left\{ \omega : \frac{\pi}{2^j} < |\omega| \leq \frac{\pi}{2^{j-1}} \right\}, \quad \omega \in [-\pi, \pi).$$

For practical wavelets such as Symlets and Daubechies, the separation between bands is only approximate. Yet A_j consistently captures progressively coarser low-frequency trends, while D_j emphasizes higher-frequency, localized variations.

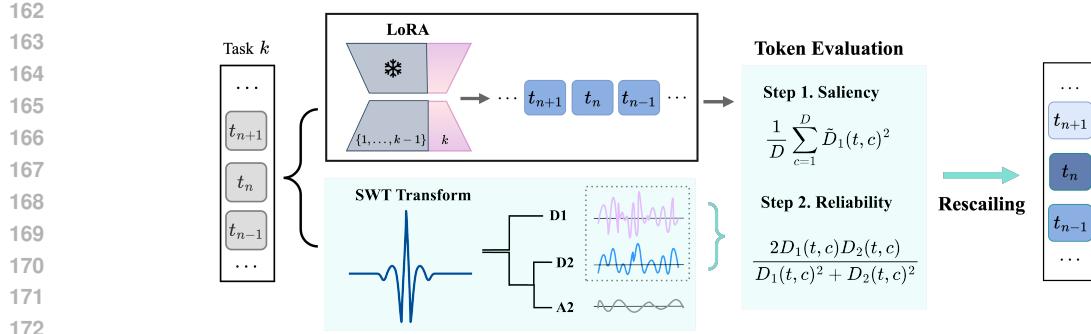


Figure 2: **Overview of TUNE.** Sequence of token embeddings are decomposed by SWT into three subbands (A_2 : low-frequency approximation, D_2 : mid-frequency detail, D_1 : high-frequency detail). Then evaluate for saliency and reliability based on D_1 and D_2 coefficients, producing token-dependent scaling factors that modulate LoRA residual updates for current task k .

Stationary Wavelet Transform (SWT) While the DWT provides an efficient multi-resolution decomposition, its use of decimation introduces shift variance, a small translation of the input sequence can cause large changes in the wavelet coefficients. This sensitivity is undesirable for language data, where precise token ordering is important to capture semantics. The stationary wavelet transform (SWT), also called the undecimated or shift-invariant wavelet transform, removes the downsampling step and instead upsamples the filters at each level. As a result, all subbands retain the same length as the original signal, and coefficient alignment across scales is preserved.

Formally, let $h^{(j)}$ and $g^{(j)}$ denote the level- j analysis filters obtained by inserting $2^{j-1} - 1$ zeros between each of the filter coefficients of h and g . The recursive SWT decomposition and reconstruction are computed as

$$A_j[n] = (A_{j-1} * h^{(j)})[n], \quad D_j[n] = (A_{j-1} * g^{(j)})[n], \quad A_0 \equiv x,$$

$$A_{j-1}[n] = (A_j * \tilde{h}^{(j)})[n] + (D_j * \tilde{g}^{(j)})[n].$$

After j levels of decomposition, the SWT produces a set of coefficient components

$$\{A_j, D_j, D_{j-1}, \dots, D_1\},$$

where A_j represents the coarsest approximation (lowest-frequency trend) and $\{D_\ell\}_{\ell=1}^j$ capture progressively finer detail bands ordered from low to high frequency. Unlike the DWT, no decimation is applied, so each A_j and D_j has the same length as x . While this redundancy makes SWT less efficient, it ensures shift invariance and exact token-level alignment across scales. As a result, SWT offers a decomposition that is both semantically stable and structurally faithful to discrete token sequences.

Interscale Correlation in SWT A distinctive property of wavelet coefficients is their persistence across scales: significant coefficients tend to propagate from child $D_j[n]$ to parent $D_{j+1}[n]$, while noise coefficients do not (Xu et al., 1994; Luisier et al., 2006; He et al., 2015). This phenomenon, often referred to as the parent-child relationship, reflects strong interscale correlation, where large (small) child coefficients are likely to correspond to large (small) parent coefficients. Xu et al. (1994) proposed to quantify this propagation by a normalized correlation index measuring the raw coefficient correlation between child $W_{j,n}$ and parent $W_{j+1,n}$ at location n . Luisier et al. (2006) further extend this property to develop inter-scale-dependent thresholding functions where weak parent-child correlation enforces stronger attenuation to suppress noise whereas high inter-scale consistency preserves reliable signal structure. By exploiting this inter-scale correlation as a key indicator of feature reliability in SWT, one can identify coefficients that carry robust linguistic information while suppressing unstable, noise-driven activations.

3 METHOD

We now describe Token Update via Noise-robust Frequency Encoding (TUNE), our frequency-aware mechanism for regulating LoRA residual adaptation. The key idea is to decompose token embeddings

216 into multi-resolution subbands and construct a token-dependent gating factor that amplifies reliable
 217 novelty while suppressing noisy, unstable fluctuations. This section details how TUNE leverages
 218 stationary wavelet transform (SWT) to extract frequency-specific signals and integrates them into the
 219 LoRA update path to mitigate catastrophic forgetting. See Figure 2 for full diagram.
 220

221 **3.1 SETTING AND NOTATION**
 222

223 Let $x \in \mathbb{R}^{L \times D}$ denote a token sequence with length L and hidden size D . Applying a two-level
 224 stationary wavelet transform (SWT) with analysis filters (h, g) produces three subbands
 225

$$226 \quad A_2, D_2, D_1 \in \mathbb{R}^{L \times D},$$

227 where A_2 is the level-2 low-pass approximation, D_2 the level-2 coarser detail, and D_1 the level-1
 228 finest detail. For practical finite impulse response (FIR) filters such as Daubechies and Symlets, the
 229 frequency separation across bands is approximate, but the dominant ordering, high: $D_1 > \text{mid}: D_2 >$
 230 low: A_2 , consistently holds.
 231

232 Following the inter-scale correlation framework (Xu et al.,
 233 1994; Luisier et al., 2006; He
 234 et al., 2015), we treat D_1 as the
 235 child and D_2 as its parent in
 236 the hierarchy, while A_2 provides
 237 the low-frequency semantic back-
 238 bone. Because SWT is undecimated,
 239 all subbands retain length
 240 L , ensuring exact positional align-
 241 ment between coefficients across
 242 scales. We index tokens by $t \in$
 243 $\{1, \dots, L\}$ and channels by $c \in$
 244 $\{1, \dots, D\}$.
 245

246 Figure 3 illustrates this fre-
 247 quency hierarchy on 5 example
 248 token sequences where A_2 varies
 249 smoothly and forms a stable con-
 250 text, whereas D_2 and especially D_1 display
 251 stronger oscillations. Figure 3 (c) quantifies oscillation
 252 strength : for each sequence, we compute the first-order difference of coefficients over time and
 253 then evaluate the standard deviation within a ω -width sliding window (practically, we used $\omega = 7$).
 254 A larger value means the signal fluctuates more strongly within that local region, reflecting higher
 255 oscillation strength. Together, the plots indicate that A_2 provides stable global structure, D_2 carries
 256 mid-scale details, and D_1 concentrates the most volatile fluctuations, which motivates the need to
 257 assess high-frequency (D_1) reliability before using it for adaptation.
 258

259 **3.2 HIGH-FREQUENCY SALIENCY**
 260

261 High-frequency coefficients D_1 capture local fine-grained variation that holds the details, but also
 262 include noise. To quantify their strength in a robust manner, we apply a soft shrinkage relative to a
 263 per-channel noise scale σ_c , a MAD-derived statistic computed along the token axis. This produces
 264 denoised details
 265

$$266 \quad \tilde{D}_1(t, c) = \text{sign}(D_1(t, c)) \text{ReLU}(|D_1(t, c)| - \sigma_c).$$

267 We then compute normalized token-wise energy and map it into $[0, 1]$ with a sigmoid,
 268

$$269 \quad E(t) = \sigma \left(\frac{1}{D} \sum_{c=1}^D \frac{\tilde{D}_1(t, c)^2}{\sigma_c^2 + \varepsilon} \right), \quad \sigma(u) = \frac{1}{1+e^{-u}}.$$

270 This yields a high-frequency saliency score such that tokens with strong, non-trivial energy across
 271 feature channels receive higher $E(t)$.
 272

270 3.3 PARENT-GUIDED RELIABILITY
271

272 Saliency alone may respond to unstable spikes, which can add noisy perturbation to semantically
273 meaningful high-frequency representations. To capture stability, we exploit the persistence of
274 coefficients across scales. Using level-2 details D_2 as reference coefficients, we compute the inter-
275 scale agreement as

$$276 \quad r(t, c) = \frac{2D_1(t, c)D_2(t, c)}{D_1(t, c)^2 + D_2(t, c)^2 + \varepsilon\sigma_c^2}, \quad g(t, c) = \tanh(\gamma r(t, c)),$$

$$277$$

278 with sharpness control hyperparameter $\gamma > 0$. The agreement score $g(t, c)$ takes values in $[-1, 1]$
279 and reflects both the sign and relative magnitude of D_1 and D_2 . The score becomes stronger when
280 child (D_1) and parent (D_2) coefficients are aligned in both sign and magnitude and weaker when
281 they are misaligned. However, values of g near zero lie in the steep, unsaturated region of the tanh
282 nonlinearity, where small changes in r cause large fluctuations in g , making such scores unreliable.

283 To account for the instability of g around zero, we further introduce a light-weight computable
284 confidence term $\mathcal{C}(t, c)$ derived from the local sensitivity of g w.r.t. r . Since

$$285 \quad \frac{\partial g}{\partial r}(t, c) = \gamma \operatorname{sech}^2(\gamma r(t, c)),$$

$$286$$

287 large derivatives occur when g is close to 0, indicating unstable response to small changes in r .
288 Conversely, when g saturates near ± 1 , the derivative vanishes, indicating strong confidence in the
289 agreement. We therefore define the confidence score as

$$291 \quad \mathcal{C}(t, c) = 1 - \frac{1}{\gamma} \left(\frac{\partial g}{\partial r}(t, c) \right) = 1 - \operatorname{sech}^2(\gamma r(t, c)) = \tanh^2(\gamma r(t, c)),$$

$$292$$

293 which maps sensitivity into $[0, 1]$. The final parent-guided reliability for each token t is then given by

$$295 \quad R(t) = \frac{1}{D} \sum_{c=1}^D \underbrace{\sigma(g(t, c))}_{\text{agreement}} \cdot \underbrace{\mathcal{C}(t, c)}_{\text{confidence}},$$

$$296$$

$$297$$

298 This reliability is high when child coefficients are consistent with their parent and the gate is confident
299 (saturated), selectively emphasizing cross-scale supported features.

300 Figure 4 illustrates whether the saliency and reliability measures behave in accordance with their
301 intended design. For this, we define per-channel token scores:

$$302 \quad E(t, c) = \sigma\left(\frac{\tilde{D}_1(t, c)^2}{\sigma_c^2 + \varepsilon}\right), \quad R(t, c) = \sigma(g(t, c)) \cdot \mathcal{C}(t, c).$$

$$303$$

$$304$$

305 For visualization purposes, $E(t, c)$ and $R(t, c)$ are further normalized to the range $[0, 1]$. In panel (a),
306 each scatter point corresponds to a coefficient pair $(D_1(t, c), D_2(t, c))$, colored by its saliency $E(t, c)$.
307 Points with larger $D_1(t, c)$ map to higher saliency, confirming that $E(t, c)$ captures the intensity
308 of fine-scale variations. In panel (b), reliability $R(t, c)$ is maximized when $D_1(t, c)$ and $D_2(t, c)$
309 are aligned in both sign and magnitude, and weakest off-diagonal when they disagree. Together,
310 these plots show that $E(t, c)$ highlights token-level detail, while $R(t, c)$ refines it by emphasizing
311 cross-scale consistent activations and suppressing unstable ones.

312 3.4 TOKEN-WISE SCORE AND LORA INTEGRATION
313

314 As a final token-wise gating score, we combine saliency and reliability through a geometric mean,

$$316 \quad S(t) = (E(t))^{1-\lambda} (R(t))^\lambda, \quad \lambda \in [0, 1],$$

$$317$$

318 and map this token-level score to a scaling factor $s(t) = \sigma(S(t))$ that modulates the LoRA residual
319 path. As our baseline LoRA structure, we adopt orthogonal LoRA (O-LoRA) (Wang et al., 2023a),
320 which assigns each task a separate LoRA subspace while freezing those from previous tasks, and
321 imposes cross-task orthogonality constraints to reduce subspace overlap across tasks. This provides a
322 strong and widely used starting point for LoRA-based continual learning.

323 On top of this baseline, TUNE introduces token-dependent rescaling of the LoRA residual updates:

$$y_{\text{out}}(t) = Wx(t) + s(t) \Delta W_{\text{LoRA}}(x(t); \theta_k),$$

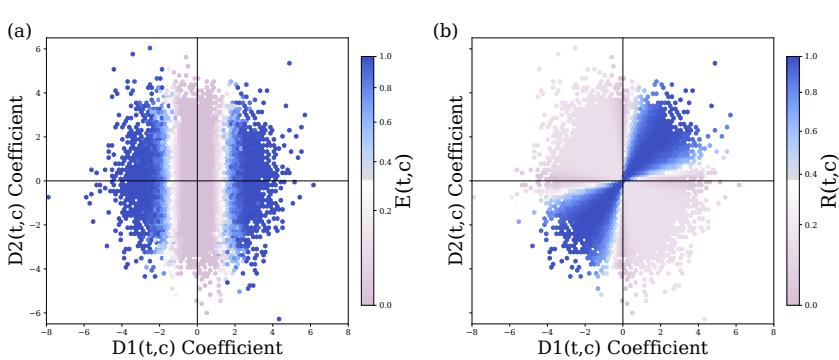


Figure 4: **Scatter plots of normalized saliency $E(t, c)$ and reliability $R(t, c)$.** Each point corresponds to a coefficient pair $(D_1(t, c), D_2(t, c))$, colored by its normalized score in $[0, 1]$. (a) Larger $D_1(t, c)$ values generally correspond to higher $E(t, c)$. (b) $R(t, c)$ is strongest along the diagonal where $D_1(t, c)$ and $D_2(t, c)$ align in both sign and magnitude, and weak off-diagonal.

where θ_k denotes the learnable set of LoRA parameters for the current task k . This design provides two complementary benefits. During the forward pass, the scaling factor $s(t)$ adjusts the contribution of each token, effectively regulating its influence in the residual representations and thereby controlling token-wise attention to salient versus noisy signals. During the backward pass, treating $s(t)$ as constant with respect to θ_k yields

$$\frac{\partial \mathcal{L}}{\partial \theta_k} = \sum_{t=1}^L \underbrace{s(t)}_{\text{token-dependent LR}} \frac{\partial \mathcal{L}}{\partial y_{\text{out}}(t)} \frac{\partial \Delta W_{\text{LoRA}}(x(t); \theta_k)}{\partial \theta_k}.$$

Here $s(t)$ acts as a token-dependent learning rate (LR), scaling the gradient update contributed by each token according to its saliency and reliability. Tokens with high $s(t)$ exert stronger influence on parameter updates, promoting plasticity for novel yet reliable signals. In contrast, tokens with low $s(t)$ contribute little, helping to preserve stability against noisy or unstable activations. In this way, the scale factor regulates not only the forward significance of each token in residual adaptation but also its backward learning step size, effectively controlling how spectral cues shape the stability–plasticity trade-off during optimization.

Note that TUNE adds no additional learnable parameters beyond the continual expansion of LoRA modules already inherent to the O-LoRA framework. The scaling factors $s(t)$ are computed directly from spectral cues given by SWT decomposition of token embeddings without any parameterization. A detailed analysis of the resulting computational overhead is provided in the next section.

4 EXPERIMENTS

4.1 DATASETS

We evaluate TUNE under the widely used continual learning benchmarks for LLMs. The Standard CL benchmark consists of four text classification tasks (AG News, Amazon, Yelp, DBPedia, Yahoo) arranged in three different orders. For a greater challenge on longer sequence of continual tasks, we also adopt a Large-scale CL benchmark comprising fifteen different tasks: the five standard CL benchmark tasks, four GLUE tasks (MNLI, QQP, RTE, SST-2), five SuperGLUE tasks (WiC, CB, COPA, MultiRC, BoolQ), and IMDB reviews, also in three different orders. All task instructions follow the unified instruction-tuning schema from prior works (Qin & Joty, 2022; Wang et al., 2023a; Qiao & Mahdavi, 2024; Yang et al., 2024).

4.2 MODELS AND TRAINING

Experiments are conducted on two representative LLMs: the encoder–decoder T5-large (Raffel et al., 2023) and the decoder-only LLaMA-7B (Touvron et al., 2023). LoRA modules are inserted into the query and value projection matrices, while pretrained weights are frozen. For continual learning, we adopt O-LoRA (Wang et al., 2023a) as the baseline framework. TUNE is integrated into this baseline by introducing token-dependent rescaling of the LoRA residual updates. Unless otherwise noted, we

378 Table 1: Testing performance on two standard CL benchmarks with T5-large.¹ from Wang et al.
 379 (2023a),² from Qiao & Mahdavi (2024),³ reproduced by us.

Method	Standard CL Benchmark				Large Number of Tasks			
	Order-1	Order-2	Order-3	avg	Order-4	Order-5	Order-6	avg
ProgPrompt ¹	75.2	75.0	75.1	75.1	78.0	77.7	77.9	77.9
PerTaskFT ¹	70.0	70.0	70.0	70.0	78.1	78.1	78.1	78.1
MTL ¹	80.0	80.0	80.0	80.0	76.5	76.5	76.5	76.5
SeqFT ¹	18.9	24.9	41.7	28.5	7.4	7.4	7.5	7.4
IncLoRA ¹	66.0	64.9	68.3	66.4	63.3	58.5	61.7	61.2
Replay ¹	55.2	56.9	61.3	57.8	55.0	54.6	53.1	54.2
EWC ¹	48.7	47.7	54.5	50.3	45.3	44.5	45.6	45.1
L2P ¹	60.3	61.7	61.1	60.7	57.5	53.8	56.9	56.1
LFPT5 ¹	67.6	72.6	77.9	72.7	70.4	68.2	69.1	69.2
O-LoRA ¹	75.4	75.7	76.3	75.8	72.3	64.8	71.6	69.6
LB-CL ²	76.9	76.5	76.8	76.7	68.4	67.3	71.8	69.2
N-LoRA ³	76.0	78.1	77.6	77.2	74.3	69.4	64.8	69.5
TUNE	78.5	78.4	78.0	78.3	75.4	73.0	75.9	74.8

395 follow the same experimental configurations of O-LoRA with 1-epoch training for every task except
 396 the learning rate and TUNE-specific hyperparameters. All experiments were conducted on NVIDIA
 397 A6000 GPUs, using the DeepSpeed framework for implementation. For evaluation, performance is
 398 measured by Average Accuracy (AA) after training the final task, i.e., $AA = \frac{1}{T} \sum_{i=1}^T a_{i,T}$.

4.3 BASELINES

401 We compare TUNE against a broad set of baselines commonly used in continual learning with LLMs.
 402 Independent training methods include **PerTaskFT**, which trains a separate model for each task, and
 403 **ProgPrompt** (Razdaibiedina et al., 2023), which learns independent prompts. As an optimistic upper
 404 bound, Multi-Task Learning (**MTL**) jointly trains on all tasks simultaneously. Non-continual fine-
 405 tuning methods include **SeqFT** (de Masson d’Autume et al., 2019), which updates all parameters on
 406 a sequence of tasks, and **IncLoRA**, an incremental learning of new LoRA parameters on a sequence
 407 of tasks without any regularization or replay. Regularization and replay-based approaches are also
 408 considered: **EWC** (Kirkpatrick et al., 2017) fine-tunes entire model with a regularization loss based
 409 on Fisher information and **Replay** maintains a buffer of prior samples for rehearsal. Prompt-based
 410 methods such as **L2P** (Wang et al., 2022) and **LFPT5** (Qin & Joty, 2022) dynamically select or
 411 generate prompts to adapt to new tasks. Finally, we compare against recent LoRA-based continual
 412 learning methods, including **O-LoRA** (Wang et al., 2023a), which enforces orthogonality between
 413 task-specific subspaces, **LB-CL** (Qiao & Mahdavi, 2024), which balances LoRA consolidation across
 414 tasks and **N-LoRA** (Yang et al., 2024), which reduces parameter collisions via sparsity.

415 Note that most of the baselines are taken from O-LoRA, as we adopt same experimental setting and
 416 codebase provided in their repository. For LB-CL, we report the results from the original paper due
 417 to the absence of publicly available code. For N-LoRA, we observe that they increase the number of
 418 training epochs per task from 1 (as used in O-LoRA and other baselines) to 10, which significantly
 419 inflates computational cost while yielding only marginal performance gains. To ensure consistency
 420 and fairness, we reproduce N-LoRA using the official codebase but adjust the training epoch to 1.

4.4 RESULTS

423 Tables 1 and 2 summarize the re-
 424 sults on the standard and large-scale
 425 CL benchmarks with T5-large and
 426 LLaMA-7B. On the standard 4-task
 427 benchmark, TUNE achieves the high-
 428 est average accuracy of 78.3, surpass-
 429 ing all other baselines. The improve-
 430 ment is more pronounced in the large-
 431 scale 15-task benchmark, where TUNE reaches 74.8 as average accuracy, significantly higher than
 432 O-LoRA (69.6) and N-LoRA (69.5). These gains indicate that token-wise frequency-guided modula-
 433 tion provides better stability under long sequences of tasks where forgetting is most severe.

422 Table 2: Testing performance on two standard CL benchmarks with LLaMA-7B.¹ reproduced by us.

CL Benchmark	Order-1	Order-2	Order-3	avg
O-LoRA ¹	74.8	73.7	78.3	75.6
N-LoRA ¹	71.2	75.1	76.2	74.2
TUNE	77.8	78.1	79.8	78.6

On LLaMA-7B in Table 2, where model scale makes forgetting even more challenging, TUNE again achieves the best performance. It improves the average accuracy on the standard benchmark to 78.6, compared to 75.6 for O-LoRA and 74.2 for N-LoRA. This demonstrates that the effectiveness of TUNE generalizes across architectures, showing consistent benefits in CL. Taken together, these results highlight that TUNE not only strengthens the stability–plasticity trade-off but also scales effectively to larger models and longer task sequences. By leveraging frequency cues to guide token-level adaptation, TUNE delivers SOTA performance among LoRA-based continual learning methods. We further analyze catastrophic forgetting using the Backward Transfer (BWT) metric, as detailed in Appendix H. The results show TUNE consistently reduces negative BWT compared to O-LoRA and N-LoRA, confirming its ability to better preserve prior knowledge across tasks.

Computational Analysis. TUNE introduces no additional trainable parameters beyond O-LoRA, as shown in Table 3, and incurs only negligible runtime overhead. The extra cost comes from the SWT decomposition and token-level scoring, both of which scale linearly with BLD . This cost is minor compared to the dominant Transformer operations such as quadratic attention $\mathcal{O}(BL^2D)$ and feed-forward layers $\mathcal{O}(BLD^2)$. A more detailed complexity analysis is provided in Appendix G.

4.5 ABLATION STUDIES

We conduct ablation studies to examine the impact of different design choices in TUNE. Detailed analyses on wavelet properties, scaling distributions, and additional variants are provided in Appendix D, E, and F. The main results are highlighted in Figure 5.

Wavelet function We adopt Symlets-8 (sym8) as the default wavelet for TUNE, as also reported in Table 1. Sym8 achieves the strongest performance due to its near-symmetry, which reduces phase distortion and improves cross-scale alignment, together with a moderate filter length that balances frequency resolution and temporal localization. Shorter filters such as Daubechies-4 (db4) are computationally cheaper but suffer from stronger phase misalignment, resulting in a modest drop in accuracy. These findings show that wavelet choice matters, with sym8 offering the most consistent gains, while TUNE remains competitive across various filters and continues to outperform non-frequency baselines.

No reliability regulation We next ablate the inter-scale reliability term $R(t)$. Using saliency alone for gating tokens, i.e., $s(t) = E(t)$, causes a clear degradation in accuracy, demonstrating that reliability plays a critical role in filtering out unstable high-frequency activations.

Random scaling Finally, we replace TUNE’s spectral scaling with random scaling factors drawn uniformly from the same empirical range of $s(t)$ (0.55–0.65) driven by TUNE with sym8 (see Appendix E). Despite matching the overall scale distribution, this variant performs substantially worse, indicating that the gains of TUNE cannot be attributed to random scaling of token activations. Instead, the improvements stem from the structured use of spectral cues—saliency and reliability—that adaptively modulate tokens in a task-sensitive manner.

5 CONCLUSION

We proposed TUNE, a frequency-guided token modulation method for continual learning in LLMs. By integrating wavelet-based saliency and reliability into token-wise LoRA updates, TUNE mitigates forgetting while enabling stable transfer. Across standard and large-scale benchmarks, it achieves consistent gains over strong baselines with no extra parameters and negligible overhead, establishing frequency-aware modulation as an effective principle for stable continual learning.

Table 3: Comparison of training computation cost.

Method	GPU Memory	Num of training params
O-LoRA	23.68 GB	$r(m + n)$
LB-CL	28.28 GB	$r(m + n) + r$
N-LoRA	23.69 GB	$r(m + n)$
TUNE	24.02 GB	$r(m + n)$

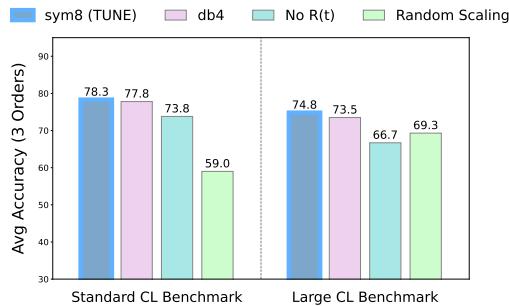


Figure 5: **Ablation results on TUNE.** Effect of wavelet function choice, removal of inter-scale reliability $R(t)$, and replacement with random scaling.

We next ablate the inter-scale reliability term $R(t)$. Using saliency alone for gating tokens, i.e., $s(t) = E(t)$, causes a clear degradation in accuracy, demonstrating that reliability plays a critical role in filtering out unstable high-frequency activations.

Finally, we replace TUNE’s spectral scaling with random scaling factors drawn uniformly from the same empirical range of $s(t)$ (0.55–0.65) driven by TUNE with sym8 (see Appendix E). Despite matching the overall scale distribution, this variant performs substantially worse, indicating that the gains of TUNE cannot be attributed to random scaling of token activations. Instead, the improvements stem from the structured use of spectral cues—saliency and reliability—that adaptively modulate tokens in a task-sensitive manner.

486 ETHICS STATEMENT.
487488 This work does not involve human subjects, personal data, or sensitive attributes. The proposed
489 method builds upon publicly available benchmark datasets (CL benchmark, GLUE, SuperGLUE,
490 IMDB), all of which are widely used in the research community. Our contributions focus on
491 algorithmic improvements for continual learning in large language models, with no foreseeable risks
492 of ethical harm beyond those already present in standard LLM training.
493494 REPRODUCIBILITY STATEMENT.
495496 We have made all implementation details explicit, including hyperparameters, model configurations,
497 datasets, and task sequences. Detailed training settings for both T5-large and LLaMA-7B are provided
498 in the appendix (Tables 7, 8). All benchmark datasets are publicly accessible, and the exact task
499 orders used for continual learning are documented in Appendix I.2. We will release the source code
500 and scripts to fully reproduce the reported results upon publication.
501502 THE USAGE OF LLMs.
503504 In the preparation of this paper, we used large language models (LLMs) in a limited and supporting
505 capacity. Specifically:506 • **Writing aid and polishing:** LLMs were employed to improve the clarity, readability, and
507 grammar of the manuscript. Their role was restricted to stylistic suggestions and refinement of
508 phrasing, without altering the scientific content, claims, or conclusions.
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648 **A CODE AVAILABILITY**
649650 The source code for TUNE is available at <https://anonymous.4open.science/r/TUNE-C52F>
651
652653 **B FULL ALGORITHMIC DETAILS FOR TUNE**
654655 We present the full step-by-step procedure of TUNE. The algorithm outlines how token embeddings
656 are decomposed by a two-level SWT, how saliency and reliability scores are computed, and how
657 their fusion produces a token-wise scaling factor that modulates the LoRA residual updates. All
658 input–output shapes are specified for clarity.
659660 **Algorithm 1** TUNE for LoRA-residual scaling

661 1: **Input:** dataset of N token sequences; stream of mini-batches $\{x^{(b)}\}_{b=1}^{N/B}$ where each $x^{(b)} \in \mathbb{R}^{B \times L \times D}$;
662 two-level SWT filters (h, g) ; hyperparameters γ, κ, λ ; numeric $\varepsilon > 0$.

663 2: **for** each mini-batch $x^{(b)}$ **do**

664 3: **2-Level SWT decomposition.**

665 4: $(A_2, D_2, D_1) \leftarrow \text{SWT}(x^{(b)}; h, g, \text{level} = 2)$ $(A_2, D_2, D_1 \in \mathbb{R}^{B \times L \times D})$

666 5: **High-frequency saliency $E(t)$.**

667 6: $\tilde{D}_1 \leftarrow \text{sign}(D_1) \cdot \text{ReLU}(|D_1| - \sigma_c)$

668 7: where $\sigma_c \in \mathbb{R}^{B \times D}$ is a per-channel noise scale (MAD statistic along tokens).

669 8: $E(t) \leftarrow \sigma\left(\frac{1}{D} \sum_{c=1}^D \frac{\tilde{D}_1^2}{\sigma_c^2 + \varepsilon}\right)$ $(E \in [0, 1]^{B \times L \times 1})$

670 9: **Parent-guided reliability $R(t)$.**

671 10: $r(t, c) \leftarrow \frac{2 D_1 \odot D_2}{D_1^2 + D_2^2 + \varepsilon \sigma_c^2}$ $(r \in \mathbb{R}^{B \times L \times D})$

672 11: $g(t, c) \leftarrow \tanh(\gamma r)$; $\mathcal{C}(t, c) \leftarrow \tanh^2(\gamma r)$ $(g \in (-1, 1)^{B \times L \times D}, \mathcal{C} \in (0, 1)^{B \times L \times D})$

673 12: $R(t) \leftarrow \frac{1}{D} \sum_{c=1}^D \sigma(\kappa g(t, c)) \odot \mathcal{C}(t, c)$ $(R \in [0, 1]^{B \times L \times 1})$

674 13: **Token score and scaling.**

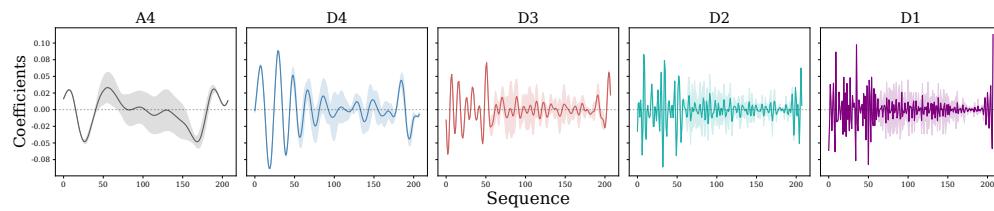
675 14: $S(t) \leftarrow E(t)^{1-\lambda} \odot R(t)^\lambda$

676 15: $s(t) \leftarrow \sigma(S)$ $(s \in (0, 1)^{B \times L \times 1})$

677 16: **LoRA integration.**

678 17: $y_{\text{out}}(t) \leftarrow W x^{(b)}(t) + s(t) \Delta_{\text{LoRA}}(x^{(b)}(t); \theta)$ $(y_{\text{out}} \in \mathbb{R}^{B \times L \times d_{\text{out}}})$

679 18: **end for**

687 **C FREQUENCY-BAND ANALYSIS OF TOKEN REPRESENTATIONS**
688698 **Figure 6: Example level 4 SWT traces on DBPedia with sym8 filter.** The embedding sequence
699 is decomposed into approximation (A_4) and detail components (D_4 – D_1). Each subplot shows the
700 mean coefficient trajectory across tokens with 95% confidence intervals, illustrating how different
701 frequency bands emphasize smooth trends (A_4) versus progressively higher-frequency fluctuations
(D_4 – D_1).
702

To better understand how token representations are distributed across frequency bands, we decompose the embedding sequence using a four-level stationary wavelet transform (SWT). This yields one approximation component (A_4) and four detail components (D_4 – D_1), which progressively capture lower- to higher-frequency variations. For each token position, the coefficients are averaged across embedding dimensions and aggregated over multiple samples to produce mean traces with confidence intervals. Figure 6 shows an SWT-decomposed example trace across scales from the DBpedia dataset using the Symlet-8 (sym8) wavelet filter, illustrating how different frequency bands emphasize distinct patterns of variation along the sequence.

C.1 HIGH-FREQUENCY (D_1) TRACE ADJUSTED WITH RELIABILITY ($R(t)$)

To highlight the effect of $R(t)$, the reliability term, we plot the z-scored amplitude of the raw high-frequency coefficients $D_1(t)$ and their reliability-adjusted trace $D_1^{\text{adj}}(t) = R(t) \cdot D_1(t)$. Although $R(t)$ is not directly used to alter high-frequency components of token embeddings in TUNE, this visualization illustrates how it effectively suppresses spiky, unstable variations in D_1 .

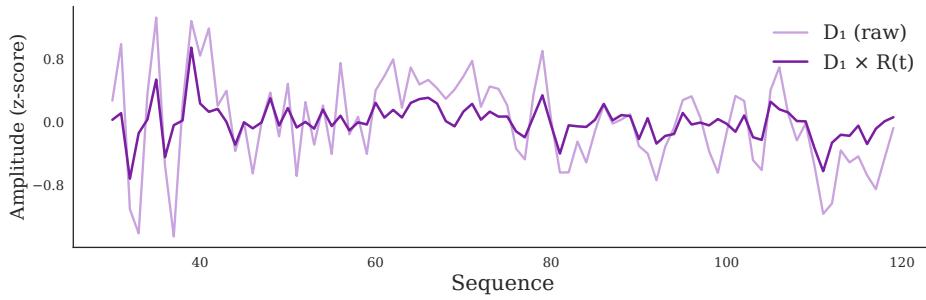


Figure 7: **High-frequency coefficients $D_1(t)$ with reliability adjustment.** Applying $R(t)$ to $D_1(t)$ suppresses erratic spikes in the raw $D_1(t)$ trace, yielding a smoother high-frequency profile.

C.2 TASK DISCRIMINABILITY OF FREQUENCY BANDS

To examine how different frequency bands contribute to task separation, we probe the task discriminability of low- (A_{10}), mid- (D_8), and high-frequency (D_1) SWT coefficients on four tasks from the standard CL benchmark. For each band, we train a logistic regression classifier with 5-fold cross-validation to obtain a confusion matrix, and compute inter-task distances using Maximum Mean Discrepancy (MMD). This analysis reveals how reliably each band preserves task-specific information versus collapsing into overlapping noise.

D CHOICE OF WAVELET FUNCTION

Selecting an appropriate wavelet is crucial for effectively decomposing token embeddings in our setup. In practice, the choice is guided by several factors: (i) orthogonality, which ensures non-redundant coefficient representations and stable reconstruction; (ii) symmetry, which mitigates phase distortions and preserves temporal alignment of token features across scales; and (iii) filter length, which balances frequency resolution against temporal localization. Longer filters (e.g., Coiflets) capture finer spectral structure at the cost of reduced localization, while shorter filters (e.g., Daubechies-4) emphasize compact support but often introduce greater asymmetry.

Figure 9 visualizes the filters used in our experiments. **Symlets-8 (sym8)** are nearly symmetric with a moderate filter length ($K = 16$), ensuring that low-pass and high-pass filters remain phase-aligned. This property reduces distortions in cross-scale comparisons and makes reliability estimation more stable, a particularly desirable feature for language data where both local token ordering and global semantic consistency must be preserved. **Coiflets-5 (coif5)** have longer filters ($K = 30$) with vanishing moments for both scaling and wavelet functions, providing stronger frequency selectivity but weaker temporal localization. **Daubechies-8 (db8)** extend the Daubechies family to a longer filter length ($K = 16$), gaining smoothness at the cost of greater asymmetry. Finally, **Daubechies-4 (db4)** are short filters ($K = 8$) that are computationally efficient but exhibit stronger phase distortion

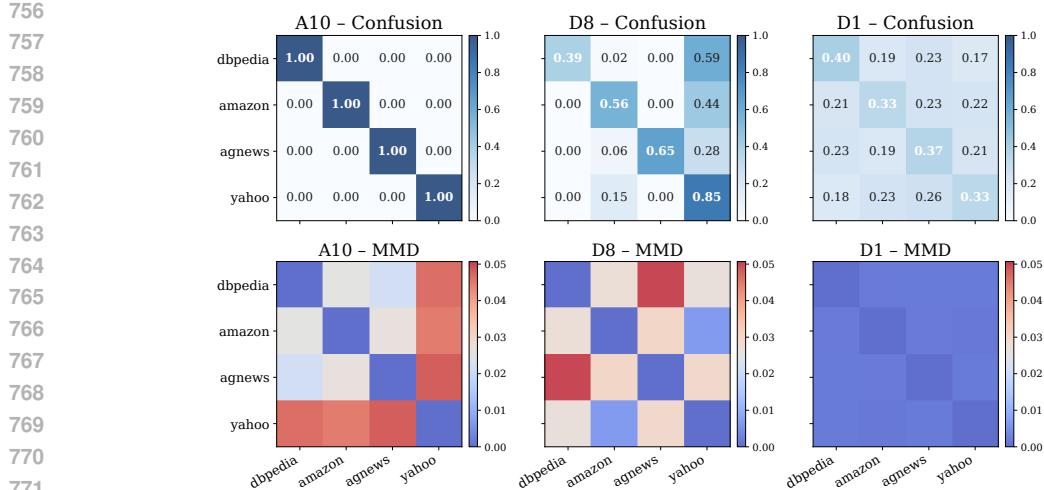


Figure 8: **Task discriminability across frequency bands.** Left: confusion matrices from a linear probe. Right: pairwise Maximum Mean Discrepancy (MMD) between tasks. A_{10} exhibits nearly perfect separation, confirming that low-frequency components encode stable, task-specific semantics. D_8 shows partial separation, reflecting mid-frequency structure. In contrast, D_1 collapses into overlapping clusters with weak inter-task separation, highlighting its noisy and unstable nature.

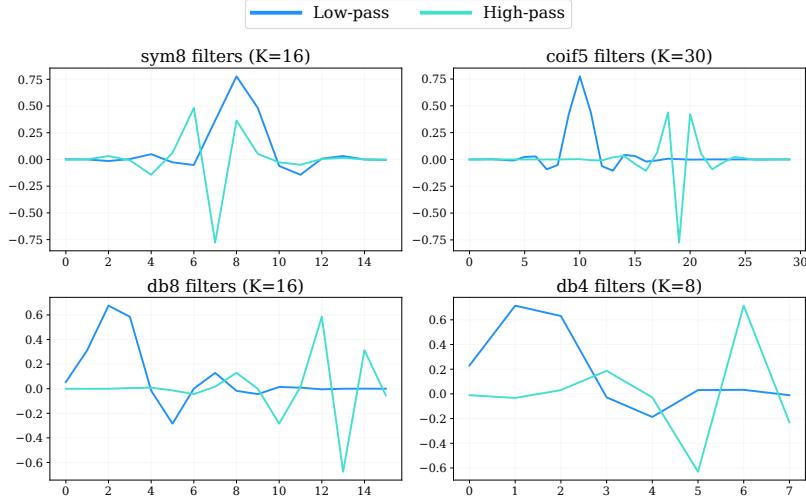


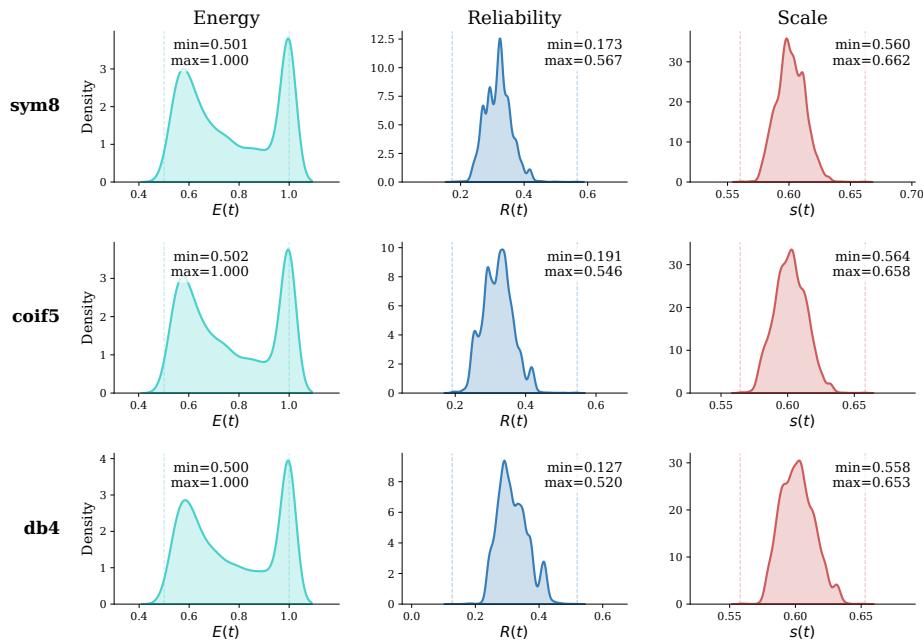
Figure 9: **Wavelet filters used in our experiments.** Low-pass (blue) and high-pass (green) decomposition filters for Symlets-8, Coiflets-5, Daubechies-8, and Daubechies-4. Sym8 is used as the default in the main results, while the others are considered in ablation studies.

compared to their longer counterparts. All wavelets considered (Symlets, Daubechies, Coiflets) are orthogonal families, ensuring energy-preserving, non-redundant filters.

We adopt sym8 as the default wavelet throughout the main experiments, as it provides the most effective balance between symmetry, frequency resolution, and temporal localization. Among these factors, symmetry is especially critical in our setup, since it minimizes phase distortions between low- and high-pass filters, ensuring stable cross-scale comparisons that underpin reliability estimation. This property is particularly important for language data, where local token structure and global semantic stability must be preserved simultaneously. Ablation studies with coif5, db8, and db4 (see Section F) further suggest that while performance can vary with filter choice, the overall method remains reasonably robust across different wavelet families.

810 E DISTRIBUTIONS OF $E(t)$, $R(t)$, AND $s(t)$

812 We visualize the pooled token-level distributions of the three measures used in our SWT analysis—energy/saliency $E(t)$, reliability $R(t)$ (the agreement score), and the combined scale $s(t)$ —for
 813 the DBpedia task in standard CL benchmark. Values are aggregated across all batches and samples;
 814 each row corresponds to one wavelet family (sym8, coif5, db4), and each column to a measure.
 815 Across filters, $E(t)$ is right-skewed with most mass near 1.0 and a light mode around 0.6 (bimodal
 816 shape), indicating many tokens receive high saliency while a smaller group is moderate. This re-
 817 flects the spiky and oscillatory behavior of high-frequency band. $R(t)$ concentrates in a mid range
 818 (around 0.13–0.56), presenting moderate but not extreme agreement. The scale $s(t)$ is narrow and
 819 stable (around 0.56–0.66) with minor shifts across filters, suggesting the combined scaling behaves
 820 consistently regardless of the wavelet choice. Vertical dashed lines mark the min/max of each pooled
 821 distribution.



844 Figure 10: Density estimates of pooled token-level measures on DBpedia. Rows: wavelet filters
 845 (sym8, coif5, db4). Columns: saliency $E(t)$, reliability $R(t)$, and combined token scale $s(t)$. Vertical
 846 dashed lines denote the observed min/max for each distribution; y-axis is density.

848 F ABLATION STUDIES

849 Table 4: Testing performance on two standard CL benchmarks with T5-large.¹:

851 Method	852 Standard CL Benchmark				853 Large Number of Tasks			
	854 Order-1	855 Order-2	856 Order-3	857 avg	858 Order-4	859 Order-5	860 Order-6	861 avg
TUNE + sym8 (Default)	78.5	78.4	78.0	78.3	75.4	73.0	75.9	74.8
TUNE + sym4	78.9	78.8	76.8	78.2	73.9	71.8	72.5	72.7
TUNE + db8	79.0	79.1	78.0	78.7	73.0	71.3	73.1	72.5
TUNE + db4	78.4	78.5	76.4	77.8	73.6	71.9	74.9	73.5
TUNE + coif5	78.5	78.3	77.2	78.0	71.5	71.1	75.0	72.5
TUNE w/o R(t)	72.9	77.2	71.3	73.8	68.8	65.2	66.0	66.7
Random Scaling	51.5	49.8	75.8	59.0	72.2	65.2	72.0	69.3

862 We conduct ablation studies to examine the influence of various components of TUNE such as wavelet
 863 choice and the role of inter-scale reliability regulation. Results are summarized in Table 4.

864 For wavelet selection, we test four widely used families: Symlets-8 (sym8), Symlets-4 (sym4),
 865 Daubechies-8 (db8), Daubechies-4 (db4), and Coiflets-5 (coif5). Among them, sym8 achieves
 866 the highest overall performance, confirming that its balanced design—moderate filter length with
 867 near-symmetry—offers a particularly suitable trade-off for decomposing language embeddings.
 868 Nevertheless, other filters also yield strong results, showing that while the exact choice of wavelet
 869 impacts final accuracy, the frequency-guided token scaling mechanism remains consistently effective
 870 across families and outperforms non-frequency baselines.

871 We also ablate the reliability regulation term $R(t)$. Removing $R(t)$ and using saliency $E(t)$ alone
 872 for scaling produces a clear drop in accuracy, indicating that inter-scale reliability is crucial for
 873 suppressing unstable high-frequency activations. Finally, we replace TUNE-driven scaling with
 874 random scaling values drawn uniformly from $[0.55, 0.65]$, matching the empirical range of scales
 875 driven by TUNE (Figure 10). This variant performs far worse, highlighting that the gains of TUNE
 876 stem from principled spectral cues rather than arbitrary rescaling.

877 Together, these ablations confirm that frequency-guided scaling is a robust and indispensable mecha-
 878 nism, with sym8 serving as the most effective default wavelet for language continual learning.
 879

880 G COMPUTATION ANALYSIS

881 We provide a detailed analysis of the additional computational cost introduced by TUNE. Let B
 882 denote the batch size, L the sequence length, D the hidden size, k the wavelet filter length, and ℓ the
 883 number of SWT decomposition levels.
 884

885 **SWT Decomposition.** At each level, the stationary wavelet transform (SWT) applies both a low-
 886 pass and a high-pass finite impulse response (FIR) filter. Thus, ℓ levels require 2ℓ FIR convolutions
 887 in total. Each convolution costs $\mathcal{O}(BkLD)$, giving

$$888 T_{\text{SWT}} = \mathcal{O}(2\ell BkLD).$$

889 Since k is small (e.g., $k \in [4, 16]$), this cost is linear in BLD and negligible compared to quadratic
 890 attention or large matrix multiplications.
 891

892 **Token Scoring.** The computation of token saliency $E(t)$ and reliability $R(t)$ consists primarily
 893 of elementwise operations. For saliency, squaring the high-frequency coefficients, normalizing by
 894 per-channel variance, and applying a sigmoid costs $\mathcal{O}(BLD)$. Reliability involves computing the
 895 agreement between child-parent coefficients (D_1, D_2), a sigmoid, and a lightweight confidence term
 896 $\mathcal{C}(t, c)$, also costing $\mathcal{O}(BLD)$. Finally, combining saliency and reliability into the token scaling
 897 factor $s(t)$ requires only a few elementwise operations, adding $\mathcal{O}(BL)$.

898 Summing the above contributions, the added complexity of TUNE is $T_{\text{TUNE}} = \mathcal{O}(2\ell BkLD) +$
 899 $\mathcal{O}(BLD)$, both terms scaling linearly with BLD and involving only small constants. In contrast, the
 900 dominant Transformer costs are

$$901 T_{\text{attn}} = \mathcal{O}(BL^2D), \quad T_{\text{FFN}} = \mathcal{O}(BLD^2),$$

902 which grow quadratically with L or D . In contrast, the additional terms introduced by TUNE only
 903 scale linearly with BLD and involve small constants tied to the filter length k and decomposition
 904 depth ℓ . Since $k \ll D$ and ℓ is typically small in practice, it follows that

$$905 T_{\text{TUNE}} \ll T_{\text{attn}}, T_{\text{FFN}},$$

906 indicating that the extra cost of TUNE is asymptotically insignificant compared to the dominant
 907 Transformer operations. This conclusion is further supported by the empirical memory usage
 908 comparison in Table 3, which confirms that TUNE adds negligible overhead while preserving the
 909 efficiency of the O-LoRA framework.

918 **H CATASTROPHIC FORGETTING MEASURED BY BACKWARD TRANSFER**
919 **(BWT)**
920

921 Backward Transfer (BWT) quantifies catastrophic forgetting by measuring how much the accuracy
922 on earlier tasks degrades after training on subsequent tasks. Formally, it is defined as
923

924
$$BWT = \frac{1}{T-1} \sum_{i=1}^{T-1} (a_{i,T} - a_{i,i}),$$

925
926

927 where $a_{i,j}$ denotes the test accuracy on the i -th task after training up to the j -th task. A negative
928 BWT value indicates forgetting, with larger magnitudes corresponding to more severe degradation.
929
930 Table 5: Backward Transfer (BWT) across task orders on T5-Large.
931
932

Method	Orders 1–3				Orders 4–6			
	1	2	3	Avg	4	5	6	Avg
O-LoRA	-2.15	-12.55	-7.80	-7.50	-9.98	-8.02	-6.67	-8.22
N-LoRA	-2.68	1.00	-0.17	-0.62	-4.89	-15.09	-9.56	-9.85
TUNE	-0.08	-1.60	-1.04	-0.91	-5.24	-8.74	-3.89	-5.95

938 Table 6: Backward Transfer (BWT) for task orders 1–3 on LLaMA-7B.
939
940

Method	Orders 1–3			
	1	2	3	Avg
O-LoRA	-14.65	-8.70	-3.77	-9.04
N-LoRA	-8.69	-0.57	-0.34	-3.20
TUNE	-1.29	-3.56	-1.61	-2.15

941 Before delving into the details, we note that all reported BWT values are computed from the same
942 models whose performance is summarized in Table 1.
943

944 Table 5 reports BWT across different task orders on T5-Large. Compared to O-LoRA and N-LoRA,
945 TUNE consistently achieves less negative BWT, indicating reduced forgetting. On Orders 1–3,
946 TUNE reaches an average of -0.91 , close to stable retention, while O-LoRA suffers a much larger
947 drop of -7.50 . Although N-LoRA achieves a slightly lower average (-0.62) on Orders 1–3, it
948 deteriorates significantly in later tasks (Orders 4–6), where its BWT falls to -9.85 . In contrast,
949 TUNE maintains a substantially smaller degradation (-5.95), confirming its robustness as tasks
950 accumulate. These results highlight that frequency-aware scaling not only preserves early tasks but
951 also mitigates long-term forgetting across extended sequences.
952

953 Table 6 presents the same analysis on LLaMA-7B for Orders 1–3. Here the trend is even clearer:
954 O-LoRA exhibits severe forgetting (-9.04 on average), while N-LoRA performs better (-3.20) but
955 still suffers from unstable retention across tasks. TUNE consistently achieves the best balance with an
956 average of -2.15 , showing substantially reduced catastrophic forgetting compared to both baselines.
957 Together, these results across two architectures confirm that TUNE’s frequency-guided modulation
958 provides a principled mechanism for alleviating forgetting, improving both short-term stability and
959 long-term robustness.
960
961 **I EXPERIMENTAL SETUP**
962963 **I.1 HYPERPARAMETERS**
964
965 For all experiments, we follow the same training configurations as O-LoRA (Wang et al., 2023a),
966 ensuring a fair comparison. The only differences are the learning rate, which we adjust for stability,
967 and the TUNE-specific hyperparameters (γ, κ, λ). Importantly, these hyperparameters are kept
968

fixed across all tasks and orders without per-task tuning. This consistency indicates that TUNE is robust to hyperparameter selection: the chosen values provide a balanced integration of token saliency and reliability into the final scaling factor, without requiring task-specific adjustment. Such stability is advantageous in continual learning, where excessive hyperparameter tuning across tasks would undermine practicality. Tables 7 and 8 summarize the detailed settings used for T5-large and LLaMA-7B experiments, respectively.

Table 7: Detailed training hyper-parameters and configuration for continual learning on T5-Large.

Config	Order-1	Order-2	Order-3	Order-4	Order-5	Order-6
<i>TUNE Configs</i>						
γ	2.0	2.0	2.0	2.0	2.0	2.0
κ	2.0	2.0	2.0	2.0	2.0	2.0
λ	0.7	0.7	0.7	0.7	0.7	0.7
<i>O-LoRA Configs</i>						
lambda 1	0.5	0.5	0.5	0.5	0.5	0.5
lambda 2	0.0	0.0	0.0	0.0	0.0	0.0
lora rank per task	8	8	8	8	8	8
lora dropout	0.1	0.1	0.1	0.1	0.1	0.1
lora target modules	Query, Value projection					
<i>General Configs</i>						
Epoch	1	1	1	1	1	1
Learning rate	1e-3	5e-4	4e-4	1e-3	5e-4	4e-4
Gradient Accumulation Step	1	1	1	1	1	1
Train Batch size / GPU	8	8	8	8	8	8
Eval Batch size / GPU	128	128	128	128	128	128
GPU (A6000 48G)	1	1	1	4	4	4

Table 8: Detailed training hyper-parameters and configuration for continual learning on LLaMA-7B.

Config	Order-1	Order-2	Order-3
<i>TUNE Configs</i>			
γ	2.0	2.0	2.0
κ	2.0	2.0	2.0
λ	0.7	0.7	0.7
<i>O-LoRA Configs</i>			
lambda 1	0.5	0.5	0.5
lambda 2	0.0	0.0	0.0
lora rank per task	8	8	8
lora dropout	0.1	0.1	0.1
lora target modules	Query, Value projection		
<i>General Configs</i>			
Epoch	1	1	1
Learning rate	5e-4	5e-4	5e-4
Gradient Accumulation Step	8	8	8
Train Batch size / GPU	1	1	1
Eval Batch size / GPU	16	16	16
GPU (A6000 48G)	4	4	4

I.2 DATASETS AND CL TASK SEQUENCES

We evaluate on 15 datasets spanning diverse domains and tasks, including sentiment classification, topic classification, natural language inference, and question answering. These datasets are drawn from established benchmarks: the standard CL benchmark with five tasks (Yelp, Amazon, DBpedia, Yahoo, AG News), GLUE (MNLI, QQP, RTE, SST-2), SuperGLUE (WiC, CB, COPA, MultiRC, BoolQ), and IMDB reviews. Table 9 summarizes their categories, tasks, and domains.

1026 To assess both short- and long-horizon continual learning, we follow prior work (Wang et al., 2023a;
 1027 Qiao & Mahdavi, 2024; Yang et al., 2024) and construct two settings. The Standard CL benchmark
 1028 includes four tasks (AG News, Amazon, Yahoo, DBpedia) arranged in three different orders. The
 1029 Large-scale CL benchmark expands to fifteen tasks by adding four GLUE tasks, five SuperGLUE
 1030 tasks, and IMDB, also evaluated under three task sequences. Table 10 details the task orders used in
 1031 our experiments.

1032

1033 Table 9: Datasets and Their Tasks, Categories, Domains, and Metrics

1034

No.	Dataset name	Category	Task	Domain	Metric
1	Yelp	CL Benchmark	sentiment analysis	Yelp reviews	accuracy
2	Amazon	CL Benchmark	sentiment analysis	Amazon reviews	accuracy
3	DBpedia	CL Benchmark	topic classification	Wikipedia	accuracy
4	Yahoo	CL Benchmark	topic classification	Yahoo Q&A	accuracy
5	AG News	CL Benchmark	topic classification	news	accuracy
6	MNLI	GLUE	NLI	various	accuracy
7	QQP	GLUE	paragraph detection	Quora	accuracy
8	RTE	GLUE	NLI	news, Wikipedia	accuracy
9	SST-2	GLUE	sentiment analysis	movie reviews	accuracy
10	WiC	SuperGLUE	word sense disambig.	lexical databases	accuracy
11	CB	SuperGLUE	NLI	various	accuracy
12	COPA	SuperGLUE	QA	blogs, encyclopedia	accuracy
13	BoolQ	SuperGLUE	boolean QA	Wikipedia	accuracy
14	MultiRC	SuperGLUE	QA	various	accuracy
15	IMDB	SuperGLUE	sentiment analysis	movie reviews	accuracy

1049

1050 Table 10: Task Sequences for Continual Learning

1051

Order	Model	Task Sequence
1	T5, LLaMA	dbpedia → amazon → yahoo → ag
2	T5, LLaMA	dbpedia → amazon → ag → yahoo
3	T5, LLaMA	yahoo → amazon → ag → dbpedia
4	T5	mnli → cb → wic → copa → qqp → boolqa → rte → imdb → yelp → amazon → sst-2 → dbpedia → ag → multirc → yahoo
5	T5	multirc → boolqa → wic → mnli → cb → copa → qqp → rte → imdb → sst-2 → dbpedia → ag → yelp → amazon → yahoo
6	T5	yelp → amazon → mnli → cb → copa → qqp → rte → imdb → sst-2 → dbpedia → ag → yahoo → multirc → boolqa → wic

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