EFFICIENT PRECISION AND RECALL METRICS FOR ASSESSING GENERATIVE MODELS USING HUBNESS-AWARE SAMPLING

Anonymous authors
Paper under double-blind review

ABSTRACT

Despite impressive results, deep generative models require massive datasets for training. As dataset size increases, effective evaluation metrics like precision and recall (P&R) become computationally infeasible on commodity hardware. In this paper, we address this challenge by proposing efficient P&R (eP&R) metrics that give almost identical results as the original P&R but with much lower computational costs. Specifically, we identify two redundancies in the original P&R: i) redundancy in ratio computation and ii) redundancy in manifold inside/outside identification. We find both can be effectively removed via hubness-aware sampling, which extracts representative elements from synthetic/real image samples based on their hubness values, *i.e.*, the number of times a sample becomes a k-nearest neighbor to others in the feature space. Thanks to the insensitivity of hubness-aware sampling to exact k-nearest neighbor (k-NN) results, we further improve the efficiency of our eP&R metrics by using approximate k-NN methods. Extensive experiments show that our eP&R matches the original P&R but is far more efficient in time and space.

1 INTRODUCTION

Deep generative models have achieved great success by combining deep learning with generative modeling. However, they have also inherited the data-hungry nature of deep learning, requiring massive datasets for training. For instance, the FFHQ dataset used to train StyleGAN contains 70 thousand images (Karras et al., 2019), while the Latent Diffusion model leveraged LAION-400M's 400 million text-image pairs (Rombach et al., 2022a). Stable Diffusion[1] pushed this even further, training its models on LAION-5B's 5 billion pairs (Schuhmann et al., 2022). Despite their impressive results, the massive scale of datasets used to train modern deep generative models presents challenges for evaluation. As dataset size increases, some of the most effective evaluation metrics (Salimans et al., 2016; Heusel et al., 2017; Sajjadi et al., 2018; Simon et al., 2019; Kynkäänniemi et al., 2019), which compare generated and real image distributions, may become computationally infeasible for commodity GPUs and ordinary research institutions. Going forward, developing more efficient evaluation metrics becomes critical to continue advancing the state of the art (SOTA).

Among the most effective evaluation metrics, Fréchet Inception Distance (FID) (Heusel et al., 2017) and Inception Score (IS) (Salimans et al., 2016) are relatively computationally efficient. Let n be the number of samples, they have linear time and space complexity of $O(n)$, as they rely on simple statistics of extracted features. Specifically, the feature extraction takes $O(n)$ time and space while the statistics (*e.g.*, mean) computation also takes linear time $O(n)$ for a given feature extractor like Inception v3 (Szegedy et al., 2015) with fixed feature dimensions. However, both FID and IS are one-dimensional scores that cannot distinguish between different failure modes. Addressing this issue, the precision and recall (P&R) metrics (Sajjadi et al., 2018; Simon et al., 2019; Kynkäänniemi et al., 2019) were employed. Intuitively, precision measures the *quality* of synthesized images while recall measures their *diversity*. Although effective, the SOTA version of P&R (Kynkäänniemi et al., 2019) requires costly pairwise distance calculations (*e.g.*, in k-nearest neighbor algorithm) between

[1] https://stability.ai/stable-diffusion

extracted features of samples and sorting, consuming $O(n^2 \log n)$ time and space, and thus becomes computationally infeasible when evaluating deep generative models trained on large-scale datasets.

In this work, we address the high computational costs of precision and recall (P&R) metrics with a novel solution based on *hubness-aware sampling*. Specifically, we have identified two important types of *redundancies* in the computation of P&R: i) redundancy in the P&R ratio computation and ii) redundancy in identifying whether a sample is within or outside of a manifold (*e.g.*, synthetic or real image manifold). Interestingly, we find that both these redundancies can be effectively removed by hubness-aware sampling. In a nutshell, hubness-aware sampling extracts a small number of m representative elements from the real/synthetic samples based on their *hubness values*, defined as the number of times a sample becomes a k-nearest neighbor (k-NN) to others in the feature space (Radovanovic et al., 2010; Liang et al., 2022). We denote such representative elements as "hubs", which have higher hubness values than their peers. We conjecture that the validity of our approach comes from the fact that hubness values are effective importance identifiers for samples with respect to the k-NN results on which P&R (Kynkäänniemi et al., 2019) relies. In addition, utilizing the fact that the identification of hubness points relies on their relatively higher hubness values rather than exact k-NN results, we further improve the efficiency of our eP&R metrics using approximate k-NN methods. Extensive experimental results demonstrate that the P&R calculated using such representative elements is almost identical to the original P&R, but consumes much less time and space. Our contributions include:

- We propose the *efficient precision and recall* (eP&R) metrics for assessing generative models, which give almost identical results as the original P&R metrics (Kynkäänniemi et al., 2019) but consume much less time and space. Theoretically, our eP&R metrics run in $O(mn \log n)$ time and consume $O(mn)$ space (m is the number of of hubs samples and $m \ll n$), which are much more efficient than the original P&R metrics that run in $O(n^2 \log n)$ time and consumes $O(n^2)$ space.
- We identify two important types of redundancies in the original P&R metrics and uncover that both of them can be effectively removed by hubness-aware sampling (Radovanovic et al., 2010; Liang et al., 2022). In addition, the insensitivity of hubness-aware sampling to exact k-nearest neighbor (k-NN) results allows for further efficiency improvement by using approximate k-NN methods.
- Extensive experimental results demonstrate the effectiveness of our eP&R metrics.

2 Related Work

2.1 Deep Generative Models

Deep generative models have achieved impressive results in image-generation tasks. Major models include Variational Autoencoders (VAEs), Generative Adversarial Networks (GANs), and diffusion models, which will be reviewed below respectively.

Variational Autoencoders (VAEs). VAEs use variational inference to approximate posterior inference, training an encoder network to map inputs to a latent space and a decoder network to reconstruct the inputs from the latents (Kingma & Welling, 2013). Despite their elegant theory, images generated by early VAEs are usually blurry, which was improved by incorporating latent quantization to produce models like VQ-VAE (Van Den Oord et al., 2017) and VQ-VAE2 (Razavi et al., 2019) that can synthesize sharp and high-resolution images.

Generative Adversarial Networks (GANs). GANs (Goodfellow et al., 2014) train two neural networks concurrently: a generator network that produces synthetic outputs, and a discriminator network that distinguishes real from synthetic data. The two networks are pitted against each other in a minimax adversarial game, where the generator tries to fool the discriminator and the discriminator tries to identify fakes. This creates a constant evolutionary pressure that enables GANs to produce increasingly realistic outputs. A core innovation of GANs is using the discriminator not just for evaluation, but directly in the training loop to guide the generator. GANs can produce sharp and photorealistic images, but are notoriously difficult to train due to mode collapse, optimization instability, and other challenges. This has led to significant efforts to stabilize GAN training (Arjovsky et al., 2017; Gulrajani et al., 2017; Miyato et al., 2018; Zhou et al., 2019; Qin et al., 2020).

Along with these efforts, researchers have extended the synthesis capabilities of GANs to a variety of image generation tasks, including unconditional image synthesis (Karras et al., 2017; 2019; 2020; 2021; Sauer et al., 2022), conditional image synthesis (Mirza & Osindero, 2014; Miyato et al., 2018; Brock et al., 2019), image-to-image translation (Isola et al., 2017; Park et al., 2019; Zhu et al., 2020), image editing (Abdal et al., 2019; 2020), etc.

Diffusion Models. Diffusion models (Sohl-Dickstein et al., 2015; Ho et al., 2020; Dhariwal & Nichol, 2021) train a neural network to reverse a stochastic diffusion process. They start with a data sample x and apply a diffusion process that gradually adds Gaussian noise over multiple timesteps to arrive at a noisy sample $\hat{x}$. A neural network is then trained to take $\hat{x}$ and predict the noise that was added at each step, allowing the original x to be reconstructed. By training the model to denoise the diffused samples, it learns to generate high-quality samples. Diffusion models avoid problematic generator-discriminator training and provide exact log-likelihoods. However, sampling requires running the full diffusion process in reverse, which is computationally expensive. Extensions like DDIM (Song et al., 2020) have made diffusion models more efficient. Thanks to their training stability, diffusion models have been widely used in text-to-image synthesis and editing tasks, including the Latent Diffusion model (Rombach et al., 2022a) that inspired Stable Diffusion, DALLE-2 (Ramesh et al., 2022), Imagen (Saharia et al., 2022), DreamBooth (Ruiz et al., 2022), MUSE (Chang et al., 2023).

Nevertheless, despite their success, modern deep generative models require massive datasets for training. This makes their evaluation challenging as the most effective evaluation metrics compare the full distributions of generated and real images, which may become computationally infeasible for commodity GPUs and non-prestigious institutions.

2.2 Evaluation Metrics for Assessing Deep Generative Models

Fréchet Inception Distance (FID). The FID metric, introduced by Heusel et al. (2017), computes the Fréchet distance between features of the real and generated images extracted by an Inception-V3 feature extractor. A lower FID score indicates a higher similarity between the distributions of real and generated images, implying better image quality and diversity in the generated samples. Thus, the computation of FID consumes $O(n)$ time and space as the feature extraction takes $O(n)$ time and space while the Fréchet distance computation also takes linear time $O(n)$ when using a fixed Inception-V3 network.

Inception Score (IS). The IS metric, proposed by Salimans et al. (2016), uses a pre-trained Inception-v3 classification model to compute the conditional label distribution $p(y|x)$ for each generated image x. IS measures two main aspects: i) the diversity of generated images, indicated by the entropy of $p(y|x)$, and the precision of generated images, indicated by the KL divergence between the marginal distribution $p(y)$ and the conditional distribution $p(y|x)$ for each x. A higher IS generally indicates the model can generate more realistic and diverse images. Similar to that of FID, the computation of IS consumes $O(n)$ time and space as the feature extraction takes $O(n)$ time and space while the computation of IS metric takes linear time $O(n)$.

The FID and IS metrics were improved by Chong & Forsyth (2020) to FID_∞ and IS_∞, which apply Quasi-Monte Carlo integration to reduce bias and improve reliability of them for finite samples.

Precision and Recall (P&R). Despite their effectiveness, FID and IS metrics are one-dimensional scores and thus cannot differentiate between specific failure modes, *e.g.*, mode dropping or collapsing (Lin et al., 2018), or provide insights into the underlying causes of poor performance. The P&R metrics were employed to address this issue (Sajjadi et al., 2018; Simon et al., 2019; Kynkäänniemi et al., 2019). In short, precision measures the percentage of generated samples that are considered high-quality and indistinguishable from real data, indicating the *quality* of generated samples; recall measures the percentage of all potential high-quality samples that the generator was able to produce, indicating the *diversity* of generated samples. Specifically, Sajjadi et al. (2018) formulated P&R through relative probabilistic densities between the distributions of real and generated images, which are non-trivial to estimate. Addressing this issue, they proposed a practical algorithm based on the maximal achievable values of an alternative definition of P&R. Their method was generalized by Simon et al. (2019) to accommodate arbitrary distributions and link P&R to type I and type II errors of likelihood ratio classifiers. Observing that the P&R implementation proposed by Sajjadi et al. (2018) relies on relative densities and thus cannot correctly identify mode collapse/truncation, Kynkäänniemi et al. (2019) propose to model the real and generated image manifolds directly using the k-nearest neighbors of samples, which is the state-of-the-art (SOTA) version of P&R for assessing generative models. Although more accurate, their method is computationally expensive as k-NN consumes $O(n^2)$ time and space, making their metrics infeasible to compute using commodity hardware on the large datasets used by modern deep generative models.

Addressing this issue, we propose the efficient precision and recall (eP&R) metrics based on a novel strategy named *hubness*-aware sampling, which give almost identical results to (Kynkäänniemi et al., 2019) but consume much less time and space.

2.3 Hubness Phenomenon

Hubness is a widely recognized phenomenon of nearest neighbors search in high-dimensional spaces that arises from the well-known "curse of dimensionality" (Radovanovic et al., 2010). It pertains to the inherent characteristics of data distributions in high-dimensional spaces and reveals a counter-intuitive fact: even with uniform distributions, high dimensionality gives rise to the emergence of "popular" nearest neighbors (Newman et al., 1983; Newman & Rinott, 1985; Radovanovic et al., 2010), *i.e.*, points that are significantly more likely to be among the k-nearest neighbors of other points within a given sample set, denoted as *hubs*.

To mitigate the effects of the hubness phenomenon, researchers have developed various solutions in different domains, including gene expression classification (Buza, 2016a;b), time-series classification (Tomašev et al., 2015), and electroencephalograph classification (Buza & Koller, 2016). Furthermore, hubness-aware k-nearest neighbor (k-NN) methods have been developed, including hubness-weighted k-NN (Radovanovic et al., 2010), hubness-fuzzy k-NN (Tomašev et al., 2014), hubness-information k-NN (Tomasev & Mladenic, 2011), Naive Hubness-Bayesian k-NN (Tomašev et al., 2011), and Augmented Naive Hubness-Bayesian k-NN (Tomašev & Mladenić, 2013). By incorporating measures and adjustments specifically designed for the hubness phenomenon, these methods have improved the accuracy and reliability of k-NN algorithms.

However, instead of mitigation, recent works have demonstrated that depending on the task, the hubness phenomenon can be very useful. For example, Liang et al. (2022) showed that the hubness phenomenon can be used as a prior to identify high-quality latents in GAN latent spaces. Following the same philosophy, this work introduces a new method to improve the computational efficiency of P&R metrics by incorporating hubness-aware sampling.

3 Preliminaries

As proposed by Kynkäänniemi et al. (2019), the precision and recall (P&R) metrics for assessing generative models are defined as:

$$\text{precision}(\mathbf{\Phi}_r, \mathbf{\Phi}_g) = \frac{1}{|\mathbf{\Phi}_g|} \sum_{\phi_g \in \mathbf{\Phi}_g} f(\phi_g, \mathbf{\Phi}_r), \quad \text{recall}(\mathbf{\Phi}_r, \mathbf{\Phi}_g) = \frac{1}{|\mathbf{\Phi}_r|} \sum_{\phi_r \in \mathbf{\Phi}_r} f(\phi_r, \mathbf{\Phi}_g) \tag{1}$$

where $\mathbf{\Phi_g}$ and $\mathbf{\Phi_r}$ are the sets of feature vectors corresponding to the generated and real image samples, respectively; $|\mathbf{\Phi}|$ denotes the number of samples in set $\mathbf{\Phi}$ and $|\mathbf{\Phi}_g| = |\mathbf{\Phi}_r|$; $f(\phi, \mathbf{\Phi})$ is a binary function determining whether a sample ϕ lies on a manifold represented by $\mathbf{\Phi}$ that:

$$f(\phi, \mathbf{\Phi}) = \begin{cases} 1, \text{ if } \|\phi - \phi'\|_2 \le \|\phi' - \text{NN}_k(\phi', \mathbf{\Phi})\|_2 \text{ for } \textbf{at least one } \phi' \in \mathbf{\Phi} \\ 0, \text{ otherwise,} \end{cases} \tag{2}$$

where $\text{NN}_k(\phi', \mathbf{\Phi})$ denotes the kth nearest neighbour of ϕ' in $\mathbf{\Phi}$. Intuitively, their precision and recall metrics estimate the generative and real image manifolds with a collection of hyperspheres, respectively, with each feature vector sample as the center and the distance between it and its kth nearest neighbor as the radius. A sample ϕ is determined to lie on a manifold if it lies within the hyperspheres of that manifold and vice versa.

4 Efficient Precision and Recall

Although effective, Eqs. 1 and 2 are computationally expensive due to the calculation of pairwise distances between samples and the sorting required by k-NN, which grows quasi-quadratically with the number of samples. This prevents them from being computed on large datasets with commodity GPUs and hampers the progress of the field. To improve the computational efficiency of precision and recall (P&R) metrics, we identify two important types of *redundancies* in Eqs. 1 and 2 (Sec. 4.1) and propose to address them using *hubness-aware sampling*, whose insensitivity to exact k-NN results allows for further efficiency improvement (Sec. 4.2). We also conduct a computational complexity analysis (Sec. 4.3) to demonstrate the high computational efficiency of our method.

4.1 Redundancies in Precision and Recall Calculations

As mentioned above, we have identified two important types of *redundancies* in P&R calculations: i) redundancy in the P&R ratio computation and ii) redundancy in identifying whether a sample is within or outside of a manifold (*e.g.*, synthetic or real image manifold) as follows:

Observation 4.1 (Redundancy in Ratio Estimation) *As Eq. 1 shows, the P&R metrics are essentially ratios of the number of samples in a set* $\mathbf{\Phi}$ *that lie on a given manifold to the number of all samples in* $\mathbf{\Phi}$*. Thus, we can obtain similar P&R ratios by using representative samples of* $\mathbf{\Phi}$ *with the rest as redundant.*

Observation 4.2 (Redundancy in Inside/Outside Manifold Identification) *As shown in Eq. 2,* $f(\phi, \mathbf{\Phi})$ *is* 1 *as long as* ϕ *is within the* k*-NN hypersphere of at least one sample* $\phi' \in \mathbf{\Phi}$*. This means that we only need to find one valid* ϕ' *for each* ϕ *and all the other* ϕ'*s are redundant.*

4.2 Redundancy Reduction using Hubness-aware Sampling

Interestingly, we find hubness-aware sampling to be an effective solution for both redundancies. Specifically, for Observation 4.1, we find that samples with similar hubness values are effective representative samples of set $\mathbf{\Phi}$ in terms of P&R ratios as they share similar ratios of samples identified as 1 *vs.* 0 by Eq. 2 (Fig. 1), indicating that we can use a small number of hubs samples to approximate P&R; for Observation 4.2, we find that most ϕ with $f(\phi, \mathbf{\Phi}) = 1$ (Eq. 2) are included in the k-NN hypersphere of at least one ϕ' with high hubness values, *i.e.*, hubs samples (Fig. 2), indicating that we can obtain similar outputs of Eq. 2 using a small number of hubs samples. Thus, our efficient P&R metrics can be defined as:

$$\text{precision}^{hub}(\mathbf{\Phi}_r, \mathbf{\Phi}_g) = \frac{1}{|\mathbf{\Phi}_g^{hub}|} \sum_{\phi_g^{hub} \in \mathbf{\Phi}_g^{hub}} f(\phi_g^{hub}, \mathbf{\Phi}_r^{hub}) \tag{3}$$

$$\text{recall}^{hub}(\mathbf{\Phi}_r, \mathbf{\Phi}_g) = \frac{1}{|\mathbf{\Phi}_r^{hub}|} \sum_{\phi_r^{hub} \in \mathbf{\Phi}_r^{hub}} f(\phi_r^{hub}, \mathbf{\Phi}_g^{hub}) \tag{4}$$

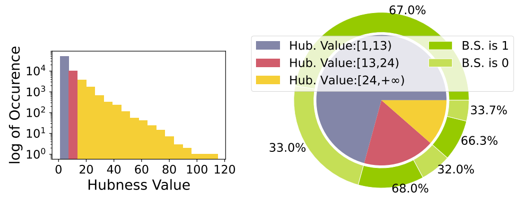
where $\mathbf{\Phi}_g^{hub}$ and $\mathbf{\Phi}_r^{hub}$ are the sets of feature vectors with hubness values $m > t$ corresponding to the generated and real image samples, respectively; t is a threshold hyper-parameter.

Efficient Hubs Sample Identification. Despite their effectiveness, the identification of hub samples is also based on the $O(n^2)$ k-NN algorithm which is expensive in both time and space. Fortunately, such identification is insensitive to exact k-NN results as it only relies on a rough threshold t of the hubness values. Thus, we can use an approximate k-NN algorithm for the identification of hub samples that further improves the efficiency of our metrics.

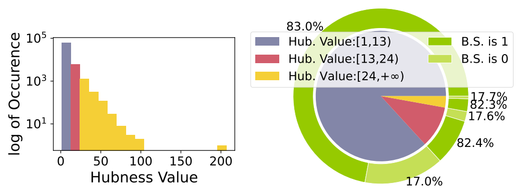
4.3 Computational Complexity Analysis

To provide a clear demonstration of the computational efficiency of our metrics, we conduct a computational complexity analysis as follows. Given two sets $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$ ($|\mathbf{\Phi}_r| = |\mathbf{\Phi}_g| = n$), the calculation of the original P&R metrics (Kynkäänniemi et al., 2019) can be divided into five stages:

1. **[Distance Matrices of $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$]** Calculating pairwise distances for samples in $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$ respectively, which consumes $O(n^2)$ time and space for each set.
2. **[Sorting]** Sorting the distance matrices as required by the k-NN algorithm, which consumes $O(n^2 \log n)$ time and no extra space.

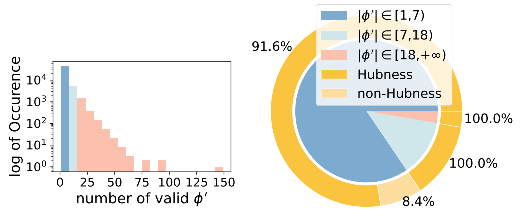


(a) All 70k images in the FFHQ dataset

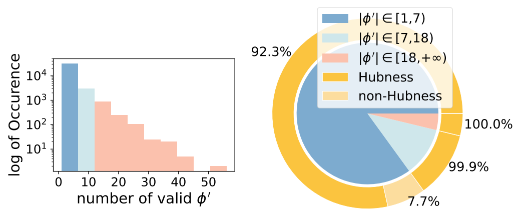


(b) 70k images generated by StyleGAN2

Figure 1: Samples with similar hubness values are effective representative samples in terms of P&R ratios calculation. (a) Left: Histogram of sample occurrences (log scale) *vs.* hubness value (FFHQ). The samples are grouped into different colors based on similar hubness values. Right: Pie chart showing that all three groups share similar ratios of samples identified as 1 vs. 0 (green *vs.* light green) using Eq. 2 for recall calculation. (b) The same experiment as (a) but on StyleGAN-generated samples for precision calculation. Following (Kynkäänniemi et al., 2019), we use VGG-16 as a feature extractor and StyleGAN2 trained on the FFHQ dataset as the generative model to be assessed. Please see Sec. A.4 for more details. Hub.: Hubness; B.S: binary score.



(a) All 70k images in the FFHQ dataset



(b) 70k images generated by StyleGAN2

Figure 2: Most samples ϕ with $f(\phi, \mathbf{\Phi}) = 1$ (Eq. 2) are included in the k-NN hypersphere of at least one hubs sample ($t = 3$) of the other distribution. (a) Left: Histogram of sample occurrences (log scale) *vs.* the times a sample is included in the k-NN hypersphere of a sample of the other distribution, *i.e.*, valid ϕ' (FFHQ). The samples are grouped into different colors based on similar numbers of valid ϕ'. Right: Pie chart showing the ratio of samples within the k-NN hypersphere of *hubness* vs. *non-hubness* samples from the other distribution, to the total number of samples ϕ with $f(\phi, \mathbf{\Phi}) = 1$ in each group. Hubness: points with hub values above a threshold $t \geq 3$; Non-hubness: $t < 3$. (b) The same experiment as (a) but on StyleGAN-generated samples. Following (Kynkäänniemi et al., 2019), we use VGG-16 as a feature extractor and StyleGAN2 trained on the FFHQ dataset as the generative model to be assessed. Please see Sec. A.4 for more details.

3. **[Radii]** Recording the distance from each sample to its kth nearest neighbour as the radius of its hypersphere, taking $O(n)$ time and space.
4. **[Distance Matrix between $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$]** Calculating pairwise distances between samples of $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$, which consumes $O(n^2)$ time and space.
5. **[P&R]** Calculating P&R ratios, taking $O(n^2)$ time and no extra space for each metric.

In contrast, our efficient P&R can be divided into seven stages:

1. **[Subspace Construction for $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$]** Constructing subspaces of samples for $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$ as required by the approximate k-NN algorithm HNSW (Malkov & Yashunin, 2018), taking $O(\log n)$ time and $O(n)$ space for each set.
2. **[Approx. Hubs Identification for $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$]** Computing the approximate hubness value for each sample in $\mathbf{\Phi}_r$ and $\mathbf{\Phi}_g$ using the approximate k-NN algorithm and extracting hubs set $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_g^{hub}$ with m_r and m_g ($m_r < n, m_g < n$) hubs samples respectively using a user-specified threshold t, taking $O(m_r)$, $O(m_g)$ time and space for each set, respectively.
3. **[Efficient Distance Matrices]** Calculating pairwise distances for samples between $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_r$, and $\mathbf{\Phi}_g^{hub}$ and $\mathbf{\Phi}_g$, which consumes $O(m_r n)$ and $O(m_g n)$ time and space, respectively.
4. **[Efficient Sorting]** Sorting the distance matrices as required by the k-NN algorithm, which consumes $O(m_r n \log n)$ and $O(m_g n \log n)$ time respectively and no extra space.

5. **[Radii]** Recording the distance from each sample to its kth nearest neighbour as the radius of its hypersphere, taking $O(m_r)$ and $O(m_g)$ time and space, respectively.
6. **[Efficient Distance Matrix between Φ_r^{hub} and Φ_g^{hub}]** Calculating pairwise distances between samples of Φ_r^{hub} and Φ_g^{hub}, which consumes $O(m_r m_g)$ time and space.
7. **[Efficient P&R]** Calculating P&R ratios, taking $O(m_g^2)$ and $O(m_r^2)$ time and no extra space for each metric.

Theoretically, the proposed eP&R metrics run in $\max(O(m_r n \log n), O(m_g n \log n))$ time and consumes $\max(O(m_r n), O(m_g n))$ space while the original P&R metrics run in $O(n^2 \log n)$ time and consumes $O(n^2)$ space. Since $m_r < n$, $m_g < n$, the proposed eP&R metrics are far more efficient than the original P&R metrics.

5 Experiments

5.1 Experimental Setup

Hardware. We use a PC with an Intel(R) Core(TM) i7-10875H CPU, an NVIDIA RTX 4090 24GB GPU for small datasets and a GPU node with 2 NVIDIA V100 32GB GPUs for large datasets.

Datasets. We use the FFHQ (Kazemi & Sullivan, 2014) dataset containing 70k portrait images, the CelebA-HQ (Karras et al., 2017) dataset containing 30k portrait images and the LSUN (Church, Cat, and Horse) dataset (Yu et al., 2015) containing 120k, 1.5m and 1.5m images of corresponding categories respectively in our experiments.

Generative Models. Following (Kynkäänniemi et al., 2019), we test our eP&R metrics with StyleGAN2 (Karras et al., 2020) trained on the FFHQ and LSUN-Cat, LSUN-Church and LSUN-Horse datasets mentioned above. To demonstrate the generalizability of our metrics, we further test them with the other members of the StyleGAN family, including StyleGAN1 (Karras et al., 2019), StyleGAN3 (Karras et al., 2021), VQ-VAE-2 (Razavi et al., 2019) and the Latent Diffusion model (Rombach et al., 2022b) trained on the FFHQ dataset.

Hyper-parameters. Unless specified, we follow Kynkäänniemi et al. (2019) and use $k = 3$ in (approximate) k-NN algorithms for all P&R, eP&R calculations and hubness-aware sampling, and $t = 3$ as the threshold to extract hubs samples.

Unless specified, we use the FFHQ dataset and a StyleGAN2 model trained on it in our experiments.

5.2 Efficient vs. Original Precision and Recall

Approximation Error. Our eP&R is an approximation of the original P&R (Kynkäänniemi et al., 2019), which inevitably introduces errors. To demonstrate the validity of our approximation, we record the relative errors $\epsilon = \frac{|x-\hat{x}|}{|x|}$ in Table 1, where x is the original P&R result and $\hat{x}$ is our approximation. It can be observed that our eP&R metrics share almost identical results to the original P&R with very small relative errors around 1%. Please see Appendix A.1 for a comparison with reduced sampling of the original P&R, which further justifies the effectiveness of our metrics.

Time and Memory Consumption. We profile the running time and memory consumption to compare the computational efficiency of our eP&R and the original P&R metrics. As Table 2 shows, our eP&R metrics run significantly faster and consume much less memory than the baseline, which justifies our complexity analysis in Sec. 4.3.

5.3 Ablation Study

As mentioned in Secs. 4.1 and 4.2, the proposed eP&R metrics consist of three components addressing Observation 4.1, Observation 4.2, and "Efficient Hubs Sample Identification" (approx. k-NN) respectively. To show their effectiveness, we conduct an ablation study as shown in Table 3. It can be observed that each of the proposed components contributes to the success of our eP&R metrics.

[2] https://github.com/pythonprofilers/memory_profiler/tree/master

Table 1: Approximation errors compared to the original Precision and Recall (P&R) metrics. B.L.: the original P&R metrics as the baseline (Kynkäänniemi et al., 2019). eP&R: our efficient P&R metrics. Error(%): relative error $\epsilon = \frac{|x-\hat{x}|}{|x|}$, where x is the B.L. result and $\hat{x}$ is our eP&R result.

(a) Approximation errors of eP&R computed using StyleGAN2 trained on different datasets.

	FFHQ		CelebA-HQ		LSUN-Church		LSUN-Cat		LSUN-Horse	
	Precision	Recall	Precision	Recall	Precision	Recall	Precision	Recall	Precision	Recall
eP&R	0.719±0.002	0.501±0.002	0.709±0.001	0.401±0.002	0.608±0.002	0.362±0.003	0.758±0.001	0.408±0.003	0.782±0.001	0.414±0.003
B.L.	0.716±0.001	0.493±0.001	0.713±0.001	0.394±0.001	0.614±0.001	0.359±0.002	0.766±0.001	0.401±0.002	0.778±0.001	0.409±0.002
Error(%)	0.4%	0.2%	0.6%	1.7%	0.9%	0.8%	1.0%	1.7%	0.5%	1.2%

(b) Approximation errors of eP&R calculated using different generative models and the FFHQ dataset ($t = 4$).

	StyleGAN3		StyleGAN1		VQ-VAE-2		Latent Diffusion	
	Precision	Recall	Precision	Recall	Precision	Recall	Precision	Recall
eP&R	0.684±0.002	0.559±0.001	0.724±0.001	0.389±0.001	0.719±0.001	0.163±0.004	0.749±0.001	0.262±0.002
B.L.	0.680±0.001	0.553±0.001	0.719±0.001	0.392 ±0.001	0.716±0.001	0.162±0.002	0.743±0.001	0.269±0.001
Error(%)	0.6%	1.0%	0.7%	0.7%	0.4%	0.6%	0.8%	2.6%

Table 2: Time and space consumption of our eP&R metrics compared to the original P&R metrics using StyleGAN2 trained on the FFHQ and LSUN-Church datasets respectively. B.L.: the original P&R metrics as the baseline (Kynkäänniemi et al., 2019). eP&R: our efficient P&R metrics. Time (S): serial implementation. Time (P): parallel implementation using CUDA. The profiling items are in one-to-one correspondence with the lists in Sec. 4.3 using Python time() and memory-profiler[2].

(a) FFHQ (70k images).

Profiling	B.L.			Profiling	eP&R		
	Time (S)	Time (P)	Memory		Time (S)	Time (P)	Memory
DMs ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	160s	66s	15.84 GB	Subspace ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	4s	3s	3.01 GB
				A. hubs ($\mathbf{\Phi}_r^{hub}$, $\mathbf{\Phi}_g^{hub}$)	2s	1.2s	–
				eDMs	72s	32s	11.23 GB
Sorting	104s	22s	–	eSorting	50s	12s	–
Radii	2.2s	2.2s	0.58 GB	Radii	1.7s	1.7s	0.30 GB
DM ($\mathbf{\Phi}_r \leftrightarrow \mathbf{\Phi}_g$)	85s	34s	19.24 GB	eDM ($\mathbf{\Phi}_r^{hub} \leftrightarrow \mathbf{\Phi}_g^{hub}$)	18s	9s	8.74 GB
P&R	48s	28s	–	eP&R	11s	6s	–
Total/Peak	399s	144s	19.90 GB	Total/Peak	165s	**75s**	**14.24 GB**

(b) LSUN-Church (120K images).

Profiling	B.L.			Profiling	eP&R		
	Time (S)	Time (P)	Memory		Time (S)	Time (P)	Memory
DMs ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	211s	110s	35.24 GB	Subspace ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	9s	7s	5.40 GB
				A. hubs ($\mathbf{\Phi}_r^{hub}$, $\mathbf{\Phi}_g^{hub}$)	3.3s	3.3s	–
				eDMs	107s	48s	18.83 GB
Sorting	164s	42s	–	eSorting	95s	28s	–
Radii	5s	5s	0.81 GB	Radii	4s	4s	0.51 GB
DM ($\mathbf{\Phi}_r \leftrightarrow \mathbf{\Phi}_g$)	143s	61s	37.04 GB	eDM ($\mathbf{\Phi}_r^{hub} \leftrightarrow \mathbf{\Phi}_g^{hub}$)	36s	15s	9.54 GB
P&R	90s	40s	–	eP&R	17s	8s	–
Total/Peak	613s	238s	37.24 GB	Total/Peak	269s	**113s**	**25.01 GB**

5.4 Choice of Hyperparameters

The proposed eP&R metrics have two hyperparameters: i) k used by the (approximate) k-NN algorithm; and ii) threshold t used to identify hubs samples.

Choice of number of nearest neighours k. As Table 4 shows, it can be observed that improvements of our eP&R metrics are stable under different choices of k. Therefore, without loss of generality, we use $k = 3$ following Kynkäänniemi et al. (2019).

Choice of threshold t. As Table 5 shows, our eP&R metrics introduce a trade-off between error and efficiency with t: the higher t, the more efficient our metrics but at the cost of higher errors. Thus, in our experiments, we strike a balance by using $t = 3$ for (FFHQ, StyleGAN2) combination.

Table 5: Choice of threshold t.

t	Precision	Recall	Time (P)	Memory
1	0.717±0.001	0.500±0.002	116s	16.34 GB
2	0.718±0.001	0.501±0.002	95s	15.27 GB
3	0.719±0.002	0.501±0.001	75s	14.21 GB
4	0.726±0.002	0.507±0.001	68s	13.10 GB
5	0.730±0.001	0.515±0.002	63s	12.28 GB
B.L.	0.716±0.001	0.493±0.001	144s	19.90GB

Table 3: Ablation study. Alg. Variations: variants of our metrics. Time (P): parallel implementation using CUDA. P&R: original P&R metrics (Kynkäänniemi et al., 2019). Ob. 4.1: we replace the $\mathbf{\Phi}_g$ in precision calculation and $\mathbf{\Phi}_r$ in recall calculation with their hubs versions $\mathbf{\Phi}_g^{hub}$ and $\mathbf{\Phi}_r^{hub}$ respectively (Eq. 1). Ob. 4.2: we replace the $\mathbf{\Phi}_r$ in precision calculation and $\mathbf{\Phi}_g$ in recall calculation with their hubs versions $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_g^{hub}$ respectively (Eq. 1). eP&R: our efficient P&R metrics, which uses Ob. 4.1, 4.2 and "Efficient Hubs Sample Identification" (approximate k-NN) together. *: when used alone, (2)(3) cannot save time and space as they still require the full distance matrices.

Alg. Variations	Precision	Recall	Time (P)	Memory
(1) P&R (Original)	0.716±0.001	0.493±0.001	144s	19.90 GB
(2) P&R + Ob. 4.1	0.715±0.005	0.497±0.004	138s*	19.32* GB
(3) P&R + Ob. 4.2	0.708±0.005	0.501±0.005	140s*	19.31* GB
(4) P&R + Ob. 4.1, 4.2	0.719±0.002	0.494±0.001	104s	15.84 GB
(5) eP&R (Ours)	0.719±0.002	0.501±0.001	**75s**	**14.21 GB**

Table 4: Choice of k for the k-NN algorithm used in our eP&R metric. B.L.: the original P&R metrics as the baseline (Kynkäänniemi et al., 2019). eP&R: our efficient P&R metrics. Time (S): serial implementation. Time (P): parallel implementation using CUDA.

k	B.L.				eP&R			
	Precision	Recall	Time (P)	Memory	Precision	Recall	Time (P)	Memory
3	0.716±0.001	0.493±0.001	144s	19.90 GB	0.719±0.001	0.494±0.001	75s	14.21 GB
5	0.814±0.001	0.614±0.001	150s	19.90 GB	0.813±0.002	0.615±0.002	73s	13.71 GB
7	0.865±0.001	0.683±0.001	144s	19.91 GB	0.868±0.004	0.689±0.002	73s	13.51 GB
9	0.893±0.001	0.730±0.001	147s	19.91 GB	0.899±0.006	0.737±0.002	72s	13.39 GB
10	0.907±0.001	0.758±0.001	147s	19.91 GB	0.915±0.006	0.767±0.002	71s	13.39 GB

5.5 Robustness against the Truncation Trick

The truncation trick is a widely used technique that improves GAN sample quality by truncating the latent vector z fed into the generator (Brock et al., 2019; Karras et al., 2019; 2020). As Table 6 shows, our eP&R metrics are robust against the truncation trick with $\phi = 0.5, 0.7$, where $\phi = 0.7$ is the recommended value.

Table 6: Robustness against the truncation trick (Karras et al., 2019). We calculate the metrics using StyleGAN2 trained on the FFHQ dataset and $t = 4$. Please note that $\phi = 0.7$ is the recommended value Karras et al. (2019; 2020) for the truncation trick and $\phi = 1.0$ means no truncation is applied at all. B.L.: the original P&R metrics as the baseline (Kynkäänniemi et al., 2019). eP&R: our efficient P&R metrics. Error(%): relative error $\epsilon = \frac{|x-\hat{x}|}{|x|}$, where x is the B.L. result and $\hat{x}$ is our eP&R result. We did not include Time and Memory costs are the truncation trick does not affect the number of samples, hence consuming the same amount of time and memory.

	$\phi = 0.5$		$\phi = 0.7$		$\phi = 1.0$	
	Precision	Recall	Precision	Recall	Precision	Recall
eP&R	0.932±0.002	0.089±0.002	0.890±0.002	0.297±0.002	0.714±0.002	0.493±0.001
B.L.	0.935±0.001	0.101±0.001	0.885±0.001	0.308±0.001	0.716±0.001	0.493±0.001

6 Conclusion

In conclusion, we have proposed efficient precision and recall (eP&R) metrics that provide almost identical results as the original P&R metrics but with much lower computational costs. By identifying and removing redundancies in P&R computation through hubness-aware sampling and approximate k-NN methods, we have developed a highly efficient yet accurate approach to evaluating generative models. Extensive experiments demonstrate the effectiveness of our eP&R metrics. Going forward, eP&R provides an important step towards feasible and insightful assessment of state-of-the-art generative models trained on massive datasets. We believe eP&R can enable more rapid progress in this exciting field.

REFERENCES

Rameen Abdal, Yipeng Qin, and Peter Wonka. Image2stylegan: How to embed images into the stylegan latent space? In *Proceedings of the IEEE/CVF international conference on computer vision*, pp. 4432–4441, 2019. 3

Rameen Abdal, Yipeng Qin, and Peter Wonka. Image2stylegan++: How to edit the embedded images? In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 8296–8305, 2020. 3

Martin Arjovsky, Soumith Chintala, and Léon Bottou. Wasserstein generative adversarial networks. In *International conference on machine learning*, pp. 214–223. PMLR, 2017. 2

Martin Aumüller, Erik Bernhardsson, and Alexander Faithfull. Ann-benchmarks: A benchmarking tool for approximate nearest neighbor algorithms. In *International conference on similarity search and applications*, pp. 34–49. Springer, 2017. 18

Andrew Brock, Jeff Donahue, and Karen Simonyan. Large scale GAN training for high fidelity natural image synthesis. In *International Conference on Learning Representations*, 2019. URL `https://openreview.net/forum?id=B1xsqj09Fm`. 3, 9

Krisztian Buza. Classification of gene expression data: a hubness-aware semi-supervised approach. *Computer methods and programs in biomedicine*, 127:105–113, 2016a. 4

Krisztian Buza. Semi-supervised naive hubness Bayesian k-nearest neighbor for gene expression data. In *Proceedings of the 9th International Conference on Computer Recognition Systems CORES 2015*, pp. 101–110. Springer, 2016b. 4

Krisztián Antal Buza and Júlia Koller. Classification of electroencephalograph data: A hubness-aware approach. *Acta Polytechnica Hungarica*, 13(2):27–46, 2016. 4

Huiwen Chang, Han Zhang, Jarred Barber, AJ Maschinot, Jose Lezama, Lu Jiang, Ming-Hsuan Yang, Kevin Murphy, William T Freeman, Michael Rubinstein, et al. Muse: Text-to-image generation via masked generative transformers. *arXiv preprint arXiv:2301.00704*, 2023. 3

Min Jin Chong and David Forsyth. Effectively unbiased fid and inception score and where to find them. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2020. 3

Prafulla Dhariwal and Alexander Nichol. Diffusion models beat gans on image synthesis. *Advances in neural information processing systems*, 34:8780–8794, 2021. 3

Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, and K.Q. Weinberger (eds.), *Advances in Neural Information Processing Systems*, volume 27. Curran Associates, Inc., 2014. 2

Ishaan Gulrajani, Faruk Ahmed, Martin Arjovsky, Vincent Dumoulin, and Aaron C Courville. Improved training of wasserstein gans. *Advances in neural information processing systems*, 30, 2017. 2

Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, and Sepp Hochreiter. Gans trained by a two time-scale update rule converge to a local nash equilibrium. *Advances in neural information processing systems*, 30, 2017. 1, 3

Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. *Advances in neural information processing systems*, 33:6840–6851, 2020. 3

Phillip Isola, Jun-Yan Zhu, Tinghui Zhou, and Alexei A Efros. Image-to-image translation with conditional adversarial networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1125–1134, 2017. 3

Tero Karras, Timo Aila, Samuli Laine, and Jaakko Lehtinen. Progressive growing of gans for improved quality, stability, and variation. *arXiv preprint arXiv:1710.10196*, 2017. 3, 7

Tero Karras, Samuli Laine, and Timo Aila. A style-based generator architecture for generative adversarial networks. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 4401–4410, 2019. 1, 3, 7, 9

Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Analyzing and improving the image quality of StyleGAN. In *Proc. CVPR*, 2020. 3, 7, 9

Tero Karras, Miika Aittala, Samuli Laine, Erik Härkönen, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Alias-free generative adversarial networks. In *Proc. NeurIPS*, 2021. 3, 7

Vahid Kazemi and Josephine Sullivan. One millisecond face alignment with an ensemble of regression trees. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1867–1874, 2014. 7

Diederik P Kingma and Max Welling. Auto-encoding variational bayes. *arXiv preprint arXiv:1312.6114*, 2013. 2

Tuomas Kynkäänniemi, Tero Karras, Samuli Laine, Jaakko Lehtinen, and Timo Aila. Improved precision and recall metric for assessing generative models. *Advances in Neural Information Processing Systems*, 32, 2019. 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15

Wen Li, Ying Zhang, Yifang Sun, Wei Wang, Mingjie Li, Wenjie Zhang, and Xuemin Lin. Approximate nearest neighbor search on high dimensional data—experiments, analyses, and improvement. *IEEE Transactions on Knowledge and Data Engineering*, 32(8):1475–1488, 2019. 18

Yuanbang Liang, Jing Wu, Yu-Kun Lai, and Yipeng Qin. Exploring and exploiting hubness priors for high-quality GAN latent sampling. In *Proceedings of the 39th International Conference on Machine Learning*, volume 162 of *Proceedings of Machine Learning Research*, pp. 13271–13284. PMLR, 17–23 Jul 2022. URL `https://proceedings.mlr.press/v162/liang22b.html`. 2, 4

Zinan Lin, Ashish Khetan, Giulia Fanti, and Sewoong Oh. Pacgan: The power of two samples in generative adversarial networks. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett (eds.), *Advances in Neural Information Processing Systems*, volume 31. Curran Associates, Inc., 2018. 3

Yu A Malkov and Dmitry A Yashunin. Efficient and robust approximate nearest neighbor search using hierarchical navigable small world graphs. *IEEE transactions on pattern analysis and machine intelligence*, 42(4):824–836, 2018. 6

Mehdi Mirza and Simon Osindero. Conditional generative adversarial nets. *arXiv preprint arXiv:1411.1784*, 2014. 3

Takeru Miyato, Toshiki Kataoka, Masanori Koyama, and Yuichi Yoshida. Spectral normalization for generative adversarial networks. *arXiv preprint arXiv:1802.05957*, 2018. 2, 3

Charles M Newman and Yosef Rinott. Nearest neighbors and Voronoi volumes in high-dimensional point processes with various distance functions. *Advances in Applied Probability*, 17(4):794–809, 1985. 4

Charles M Newman, Yosef Rinott, and Amos Tversky. Nearest neighbors and Voronoi regions in certain point processes. *Advances in Applied Probability*, 15(4):726–751, 1983. 4

Taesung Park, Ming-Yu Liu, Ting-Chun Wang, and Jun-Yan Zhu. Semantic image synthesis with spatially-adaptive normalization. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 2337–2346, 2019. 3

Yipeng Qin, Niloy Mitra, and Peter Wonka. How does lipschitz regularization influence gan training? In *Computer Vision–ECCV 2020: 16th European Conference, Glasgow, UK, August 23–28, 2020, Proceedings, Part XVI 16*, pp. 310–326. Springer, 2020. 2

Milos Radovanovic, Alexandros Nanopoulos, and Mirjana Ivanovic. Hubs in space: Popular nearest neighbors in high-dimensional data. *Journal of Machine Learning Research*, 11(sept):2487–2531, 2010. 2, 4

Aditya Ramesh, Prafulla Dhariwal, Alex Nichol, Casey Chu, and Mark Chen. Hierarchical text-conditional image generation with clip latents. *arXiv preprint arXiv:2204.06125*, 2022. 3

Ali Razavi, Aaron van den Oord, and Oriol Vinyals. Generating diverse high-fidelity images with vq-vae-2. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alché-Buc, E. Fox, and R. Garnett (eds.), *Advances in Neural Information Processing Systems*, volume 32. Curran Associates, Inc., 2019. URL https://proceedings.neurips.cc/paper/2019/file/5f8e2fa1718d1bbcadf1cd9c7a54fb8c-Paper.pdf. 2, 7

Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, and Björn Ommer. High-resolution image synthesis with latent diffusion models. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 10684–10695, 2022a. 1, 3

Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, and BjÃ¶rn Ommer. High-resolution image synthesis with latent diffusion models. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2022b. 7

Nataniel Ruiz, Yuanzhen Li, Varun Jampani, Yael Pritch, Michael Rubinstein, and Kfir Aberman. Dreambooth: Fine tuning text-to-image diffusion models for subject-driven generation. 2022. 3

Chitwan Saharia, William Chan, Saurabh Saxena, Lala Li, Jay Whang, Emily Denton, Seyed Kamyar Seyed Ghasemipour, Burcu Karagol Ayan, S Sara Mahdavi, Rapha Gontijo Lopes, et al. Photorealistic text-to-image diffusion models with deep language understanding. *arXiv preprint arXiv:2205.11487*, 2022. 3

Mehdi SM Sajjadi, Olivier Bachem, Mario Lucic, Olivier Bousquet, and Sylvain Gelly. Assessing generative models via precision and recall. *Advances in neural information processing systems*, 31, 2018. 1, 3

Tim Salimans, Ian Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen. Improved techniques for training gans. *Advances in neural information processing systems*, 29, 2016. 1, 3

Axel Sauer, Katja Schwarz, and Andreas Geiger. Stylegan-xl: Scaling stylegan to large diverse datasets. volume abs/2201.00273, 2022. URL https://arxiv.org/abs/2201.00273. 3

Christoph Schuhmann, Romain Beaumont, Richard Vencu, Cade Gordon, Ross Wightman, Mehdi Cherti, Theo Coombes, Aarush Katta, Clayton Mullis, Mitchell Wortsman, et al. Laion-5b: An open large-scale dataset for training next generation image-text models. *Advances in Neural Information Processing Systems*, 35:25278–25294, 2022. 1

Larissa C Shimomura, Rafael Seidi Oyamada, Marcos R Vieira, and Daniel S Kaster. A survey on graph-based methods for similarity searches in metric spaces. *Information Systems*, 95:101507, 2021. 18

Loïc Simon, Ryan Webster, and Julien Rabin. Revisiting precision and recall definition for generative model evaluation. *arXiv preprint arXiv:1905.05441*, 2019. 1, 3

Jascha Sohl-Dickstein, Eric Weiss, Niru Maheswaranathan, and Surya Ganguli. Deep unsupervised learning using nonequilibrium thermodynamics. In *International Conference on Machine Learning*, pp. 2256–2265. PMLR, 2015. 3

Jiaming Song, Chenlin Meng, and Stefano Ermon. Denoising diffusion implicit models. In *International Conference on Learning Representations*, 2020. 3

Christian Szegedy, Wei Liu, Yangqing Jia, Pierre Sermanet, Scott Reed, Dragomir Anguelov, Dumitru Erhan, Vincent Vanhoucke, and Andrew Rabinovich. Going deeper with convolutions. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1–9, 2015. 1

Nenad Tomasev and Dunja Mladenic. Nearest neighbor voting in high-dimensional data: Learning from past occurrences. In *2011 IEEE 11th International Conference on Data Mining Workshops*, pp. 1215–1218. IEEE, 2011. 4

Nenad Tomašev and Dunja Mladenić. Hub co-occurrence modeling for robust high-dimensional kNN classification. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, pp. 643–659. Springer, 2013. 4

Nenad Tomašev, Miloš Radovanovic, Dunja Mladenic, and Mirjana Ivanovic. A probabilistic approach to nearest-neighbor classification: Naive hubness Bayesian kNN. In *Proc. 20th ACM Int. Conf. on Information and Knowledge Management (CIKM)*, pp. 2173–2176, 2011. 4

Nenad Tomašev, Miloš Radovanović, Dunja Mladenić, and Mirjana Ivanović. Hubness-based fuzzy measures for high-dimensional k-nearest neighbor classification. *International Journal of Machine Learning and Cybernetics*, 5(3):445–458, 2014. 4

Nenad Tomašev, Krisztian Buza, Kristóf Marussy, and Piroska B Kis. Hubness-aware classification, instance selection and feature construction: Survey and extensions to time-series. In *Feature selection for data and pattern recognition*, pp. 231–262. Springer, 2015. 4

Aaron Van Den Oord, Oriol Vinyals, et al. Neural discrete representation learning. *Advances in neural information processing systems*, 30, 2017. 2

Mengzhao Wang, Xiaoliang Xu, Qiang Yue, and Yuxiang Wang. A comprehensive survey and experimental comparison of graph-based approximate nearest neighbor search. *arXiv preprint arXiv:2101.12631*, 2021. 18

Fisher Yu, Ari Seff, Yinda Zhang, Shuran Song, Thomas Funkhouser, and Jianxiong Xiao. Lsun: Construction of a large-scale image dataset using deep learning with humans in the loop. *arXiv preprint arXiv:1506.03365*, 2015. 7

Zhiming Zhou, Jiadong Liang, Yuxuan Song, Lantao Yu, Hongwei Wang, Weinan Zhang, Yong Yu, and Zhihua Zhang. Lipschitz generative adversarial nets. In *International Conference on Machine Learning*, pp. 7584–7593. PMLR, 2019. 2

Peihao Zhu, Rameen Abdal, Yipeng Qin, and Peter Wonka. Sean: Image synthesis with semantic region-adaptive normalization. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 5104–5113, 2020. 3

A Additional Experiments

A.1 Comparison with Reduced Sampling

To further demonstrate the superiority of our eP&R metrics, we compare them with another baseline of reduced sampling, *i.e.*, instead of using the full dataset, we randomly sample a subset from it and use a reduced number of generated samples to calculate P&R accordingly. As Table 7 shows, our method provides much more accurate P&R results given the same number of samples, demonstrating the superiority of our metrics.

Table 7: Comparison with reduced sampling. # of Spls: number of samples. R.S.: reduced sampling, *i.e.*, instead of using the full dataset, we randomly sample a subset from it and use a reduced number of generated samples to calculate P&R accordingly. eP&R: our efficient P&R metrics. Time (S): serial implementation. Time (P): parallel implementation using CUDA. The last row shows the results of the original P&R as a reference.

# of Spls (%)	R.S.				eP&R			
	Precision	Recall	Time (P)	Memory	Precision	Recall	Time (P)	Memory
72.50	0.724±0.002	0.511±0.002	108s	13.12 GB	0.717±0.001	0.500±0.002	116s	16.34 GB
52.24	0.730±0.002	0.522±0.002	73s	12.23 GB	0.718±0.001	0.501±0.002	95s	15.27 GB
38.21	0.734±0.005	0.533±0.005	49s	8.47 GB	0.719±0.002	0.501±0.001	75s	14.21 GB
28.48	0.742±0.005	0.540±0.005	34s	6.22 GB	0.726±0.002	0.507±0.001	68s	13.10 GB
21.65	0.743±0.005	0.551±0.006	25s	5.01 GB	0.730±0.001	0.515±0.002	63s	12.28 GB
P&R (70k)	0.716±0.001	0.493±0.001	144s	19.90GB	-	-	-	-

A.2 Time and Space Consumption for large datasets

Due to hardware limitations, we have to perform matrix tiling when calculating P&R and eP&R on large datasets which splits a given matrix into tiles (submatrices) that can fit into GPU memory. However, this introduces additional overheads and is not desirable (Table 8), which further justifies our motivation to design efficient evaluation metrics for generative models. Nevertheless, we show the results of our eP&R metrics on the LSUN-Horse dataset containing 1.5m images in Table 9. It can be observed that our metrics still save a lot of time when matrix tiling is used.

Table 8: Time costs when matrix tiling is used. The experiments are conducted using the FFHQ dataset and a StyleGAN2 model trained on it.

# of Tiles	1 (no tiling)	2	5	10	50	100
Time (P)	144s	146s	148s	150s	174s	192s

Table 9: Time and space consumption of our eP&R metrics compared to the original P&R metrics using StyleGAN2 trained on the LSUN-Horse dataset. B.L.: the original P&R metrics as the baseline (Kynkäänniemi et al., 2019). eP&R: our efficient P&R metrics. Time (S): serial implementation. Time (P): parallel implementation using CUDA. The profiling items are in one-to-one correspondence with the stages listed in Sec. 4.3.

Profiling	B.L.		Profiling	eP&R	
	Time (S)	Time (P)		Time (S)	Time (P)
DMs ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	12h02min	54min	Subspace ($\mathbf{\Phi}_r$, $\mathbf{\Phi}_g$)	8min	3min
			A. hubs ($\mathbf{\Phi}_r^{hub}$, $\mathbf{\Phi}_g^{hub}$)	2min	1min
			eDMs	5h56min	26min
Sorting	2h56min	22min	eSorting	1h20min	11min
Radii	1min	1min	Radii	50s	50s
DM ($\mathbf{\Phi}_r \leftrightarrow \mathbf{\Phi}_g$)	6h30min	34min	eDM ($\mathbf{\Phi}_r^{hub} \leftrightarrow \mathbf{\Phi}_g^{hub}$)	1h27min	8min
P&R	2h12min	17min	eP&R	56min	4min
Total	23h45min	2h10min	Total	9h50min	**64min**

A.3 P&R Curves

We follow (Kynkäänniemi et al., 2019) and include the original P&R (baseline) and our eP&R curves against the parameter of the truncation trick in Fig. 3. The results show that our method approximates the original P&R curves well on both FFHQ and LSUN-Church datasets.

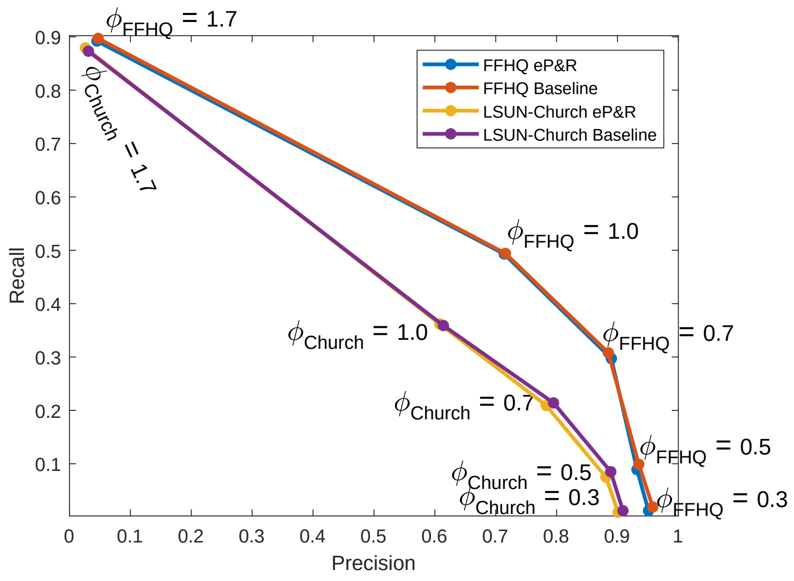


Figure 3: Original P&R (baseline) and our eP&R curves on the FFHQ and LSUN-Church datasets.

A.4 Insensitivity to Group Split Points

As shown in Table 10, the ratios of binary scores are similar for each hubness value on the FFHQ dataset, which validates the insensitivity of the choice of group split points for Observation 4.1 and Fig. 1 in the main paper.

Similarly, as shown in Table 11, the ratios of hubness samples increase quickly to 1 with the increase of $|\phi'|$ on the FFHQ dataset, which validates the insensitivity of the choice of group split points for Observation 4.2 and Fig. 2.

We show the same conclusions hold on the LSUN-Church dataset as well (Table 12 and Table 13).

A.5 Justification of pairwise distance calculation between $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_r$

As Table 14 shows, we calculate the pairwise distances between $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_r$ as it provides lower approximation errors than calculating pairwise distances for samples in $\mathbf{\Phi}_r^{hub}$. We conjecture the reason is that $\mathbf{\Phi}_r^{hub}$ is much sparser than $\mathbf{\Phi}_r$ and thus the pairwise distances for samples in it will be much larger than those of the original P&R, resulting in much larger k-NN hyperspheres that increase the approximation error. The same conclusion holds for $\mathbf{\Phi}_g^{hub}$ and $\mathbf{\Phi}_g$.

A.6 Additional experiments about the choice of t

As Table 5 shows, our eP&R metrics introduce a trade-off between error and efficiency with t: the higher t, the more efficient our metrics but at the cost of higher errors. In Table 15, we provide more experimental results on the choices of t.

B Limitations and Future Work

Although effective and efficient, the proposed eP&R metrics are not fully optimized. One area for improvement is in Stage 3 (Efficient Distance Matrices), which currently calculates pairwise

Table 10: Insensitivity to group split points for Observation 4.1 and Fig. 1 in the main paper (FFHQ). Hub. Value: hubness value, B.S.: binary score.

Hub. Value	1	2	3	4	5	6	7	8	9	10	11	12
B.S. = 1	9038	7489	5795	4579	3525	2741	2188	1842	1474	1244	931	818
All Samples	13382	11292	8615	6815	5250	4158	3235	2664	2190	1791	1389	1231
Ratio	0.675	0.663	0.673	0.672	0.671	0.659	0.676	0.691	0.673	0.695	0.67	0.665
Hub. Value	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
B.S. = 1	679	593	508	425	361	305	272	246	200	177	153	160
All Samples	1046	883	776	640	543	469	409	361	291	264	232	230
Ratio	0.649	0.672	0.655	0.664	0.665	0.65	0.665	0.681	0.687	0.67	0.659	0.696

(a) All 70k images in the FFHQ dataset

Hub. Value	1	2	3	4	5	6	7	8	9	10	11	12	
B.S.e = 1	10030	9150	7375	5858	4555	3664	2952	2266	1848	1515	1358	1120	
All Samples	12083	10988	8833	7015	5452	4402	3561	2743	2237	1835	1624	1344	
Ratio	0.83	0.833	0.835	0.835	0.835	0.832	0.829	0.826	0.826	0.826	0.836	0.833	
Hub. Value	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
B.S.e = 1	1120	911	800	646	522	456	374	350	306	261	228	187	178
All Samples	1117	992	762	635	539	467	420	379	317	277	217	208	
Ratio	0.816	0.806	0.848	0.822	0.846	0.801	0.833	0.807	0.823	0.823	0.862	0.856	

(b) 70k images generated by StyleGAN2

Table 11: Insensitivity to group split points for Observation 4.2 and Fig. 2 in the main paper (FFHQ).

ϕ'	0	1	2	3	4	5	≥ 6
Hubness	11922	8625	6290	4540	3516	2741	10818
Non-hubness	13254	9089	6519	4598	3537	2749	10818
Ratio	0.9	0.949	0.965	0.987	0.994	0.997	1.000

(a) All 70k images in the FFHQ dataset

ϕ'	0	1	2	3	4	≥ 5
Hubness	11792	6596	4136	2745	1860	5531
Non-hubness	13482	7210	4328	2788	1874	5531
Ratio	0.875	0.915	0.956	0.985	0.992	1.000

(b) 70k images generated by StyleGAN2

distances between hub samples of one set and all samples of the other set to compute radii. A significant amount of time is spent on this step. We could optimize this by utilizing the subspace constructed by the approximate k-NN algorithms. Instead of comparing hubs to the full set, we would only need to calculate distances between hubs and samples within the relevant subspace of the other set. This would allow us to find radii much more quickly. While the current metrics are fast and accurate, optimizations like these could push the efficiency even higher without sacrificing effectiveness. We therefore see continued refinement of the eP&R metrics represents an exciting opportunity for future work.

C A Brief Introduction to Approximate k-NN algorithm

The k-nearest neighbors (k-NN) algorithm is a popular machine learning method for classification and regression. Given a new data point, it finds the k closest training examples based on a distance metric like Euclidean distance. A major limitation of k-NN is that it requires computing the distance between the new point and all points in the training set, which can be slow for large datasets.

Approximate k-NN algorithms are techniques that try to speed up neighbor search by sacrificing some accuracy. The key idea is to avoid exhaustively calculating distances to all points. Some common approaches include:

Table 12: Insensitivity to group split points for Observation 4.1 and Fig. 1 in the main paper (LSUN-Church). Hub. Value: hubness value, B.S.: binary score.

Hubness Value	1	2	3	4	5	6	7	8	9
Binary Score = 1	51045	17414	6653	2906	1413	658	355	202	108
All Samples	75650	25836	9866	4296	2074	1006	562	291	153
Ratio	0.675	0.674	0.674	0.676	0.681	0.654	0.631	0.694	0.708

(a) All 120k images in the LSUN-Church dataset

Hubness Value	1	2	3	4	5	6	7	8	9	10
Binary Score = 1	24459	4658	1081	405	161	71	35	19	9	6
All Samples	55401	10564	2385	957	371	154	81	38	21	12
Ratio	0.441	0.441	0.453	0.423	0.434	0.461	0.432	0.5	0.429	0.5

(b) 100k images generated by StyleGAN2

Table 13: Insensitivity to group split points for Observation 4.2 and Fig. 2 in the main paper (LSUN-Church).

ϕ'	0	1	2	3	4	5	$\geq$6
Hubness	29986	18482	12819	9106	6749	5037	20441
Non-hubness	32669	19615	13346	9213	6773	5040	20441
Ratio	0.918	0.942	0.961	0.988	0.996	0.999	1

(a) All 120k images in the LSUN-Church dataset

ϕ'	0	1	2	3	4	5	6	$\geq$7
Hubness	15731	7063	3984	2405	1505	963	728	2007
Non-hubness	17590	7821	4158	2443	1513	965	729	2007
Ratio	0.894	0.903	0.958	0.984	0.995	0.998	0.999	1

(b) 100k images generated by StyleGAN2

Table 14: Justification of pairwise distance calculation between $\mathbf{\Phi}_r^{hub}$ and $\mathbf{\Phi}_r$. P: Precision; R: Recall. Error(%): relative error $\epsilon = \frac{|x-\hat{x}|}{|x|}$, where x denotes the baseline precision 0.716 and recall 0.493 of the original P&R metric.

t	Pairwise distance between $\mathbf{\Phi}_r^{hub}$, $\mathbf{\Phi}_r^{hub}$				Pairwise distance between $\mathbf{\Phi}_r^{hub}$, $\mathbf{\Phi}_r$			
	P	R	P Error(%)	R Error(%)	P	R	P Error(%)	R Error(%)
1	0.713	0.484	0.4	1.8	0.717	0.500	0.3	0.0
2	0.723	0.506	1.0	2.6	0.718	0.501	0.3	0.0
3	0.746	0.534	4.2	8.3	0.719	0.501	0.4	0.2
4	0.768	0.562	7.3	14.0	0.726	0.507	1.6	0.6
5	0.787	0.588	9.9	19.3	0.730	0.515	1.9	0.6

Table 15: The eP&R scores with different threshold t. Error(%): relative error $\epsilon = \frac{|x-\hat{x}|}{|x|}$

	Hubness			Error(%)		
t	Percent(%)	Precision	Recall	Precision	Recall	Mean
1	72.50±0.01	0.718±0.001	0.494±0.002	0.3	0.0	0.1
2	52.24±0.04	0.718±0.001	0.494±0.002	0.3	0.0	0.1
3	38.21±0.04	0.719±0.002	0.494±0.001	0.4	0.2	0.3
4	28.48±0.04	0.726±0.002	0.496±0.001	1.6	0.6	1.1
5	21.65±0.04	0.730±0.001	0.496±0.002	1.9	0.6	1.3
6	16.64±0.02	0.732±0.002	0.497±0.003	2.2	0.6	1.4
7	12.99±0.02	0.739±0.002	0.498±0.003	3.2	2.6	2.8
8	10.22±0.01	0.747±0.002	0.509±0.003	4.3	4.3	4.3
9	8.15±0.03	0.747±0.003	0.509±0.004	5.5	8.1	6.8
10	6.55±0.01	0.748±0.004	0.517±0.003	9.9	8.4	8.4
B.L.	—	0.716±0.001	0.493±0.001	—	—	—

- Tree-based data structures like kd-trees that allow efficient searching of nearest points without checking all data.
- Hashing techniques that map similar points to the same buckets, narrowing the search.
- Dimensionality reduction methods like random projections that can compress data while preserving relative distances.
- Graph-based algorithms that connect neighboring points then traverse the graph instead of computing all distances.
- Sampling/filtering methods that find candidates in subsections of data.

The tradeoff is between accuracy and speed. Approximate methods may miss some true nearest neighbors, but can query large datasets much more efficiently. Performance gains allow k-NN to scale better to big data. Appropriate techniques depend on factors like data size, dimension, and desired accuracy. We refer interested audiences to (Aumüller et al., 2017; Li et al., 2019; Shimomura et al., 2021; Wang et al., 2021) for more details.

D Illustration figure and relevant discussions for valid ϕ'

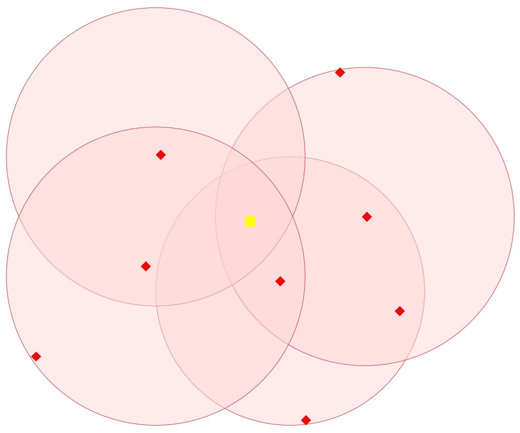


Figure 4: Illustration of valid ϕ'. ϕ is represented by a yellow cube and $\phi' \in \Phi$ set are represented by red rhombuses.

As Fig. 4 shows, by "the times a sample is included in the k-NN hypersphere of a sample of the other distribution, *i.e.*, valid ϕ'", we count the number of times ϕ (yellow cube) is within the k-NN hypersphere of $\phi' \in \Phi$ (red rhombuses).