Leveraging Dynamic Modeling of Cart-like Nonholonomic Systems to Improve Contact Point's Location and Control

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***Abstract*— There is a wide range of cart-like systems over different environments such as hospitals, hotels, supermarkets, and warehouses, between many others. These cart-like systems are passive-wheeled objects with nonholonomic constraints with varying inertial parameters. To effectively plan and control for these systems we need to understand the ground-cart-robot interactions and leverage the existence of multiple contact points to minimize the energy used by the robot and improve the control of the system. We derive the dynamic equations of the cart-like system using a constrained Euler-Lagrange formulation and propose a Linear Quadratic Regulator controller to move the cart along a desired trajectory using external forces (applied by the robot). We discuss the selection of one or multiple contact points which can be optimize to improve the performance of the system. We present a brief description of the control architecture used for the Mobile Manipulator (MM). We validate our approach experimentally, using a MM to push a shopping cart and track desired trajectories. These experiments show the accuracy of the control architecture to track the desired trajectories for carts with different inertial parameters and improve the controllability of the system by changing the contact point on the cart.**

I. INTRODUCTION

A wide range of passive-wheeled objects need to be pushed around in different environments. In hospitals, we have hospital beds and wheelchairs with similar wheels configuration. Hotels have luggage carts that we would like robots to load and move around. People usually interact with shopping carts and strollers, and in construction sites, people push toolboxes and wheelbarrows between many other cart-like systems. All these can be described as passive nonholonomic wheeled objects with varying inertial parameters. They are passive because they have no motors and need an external force to be applied to have them move. They have two sets of wheels, with fixed and caster wheels, which introduce nonholonomic constraints as they cannot move laterally. During interaction with the object, its inertial parameters can widely vary (e.g., loading an object into a shopping cart while moving) or between interactions (e.g., fetching an empty wheelchair and then pushing it with a patient). The goal of the MM is to have the object follow a desired trajectory or reach a desired goal. The reaction of the cart to external forces depends on the cart's mass and Center of Mass (CoM), which can vary, and the location of the Contact Point (CP) at which the force is applied. We want to create the capability for MMs to control these cart-like systems along a desired trajectory with consideration of their dynamics, as shown in Fig. 1.

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Fig. 1. Example of commonly used cart-like systems and our mobile manipulator.

Previous research on control of wheeled carts, like [12], [10], [11], and [6] focuses on path planning. Assuming that the MM can change the cart's orientation as needed. This assumption does not hold for heavy carts with nonholonomic constraints. Likewise, research on humanoid robots pushing heavy objects [8] and [14] focuses mainly on the humanoid's posture and computing zero momentum points for the interaction between the robot and the object. [4] presents a similar robot to ours, which is used to push/pull a pallet-jack. Using a force control for the manipulator, this system can navigate the pallet-jack to a desired configuration. Even though the pallet-jack system has a similar wheel configuration with nonholonomic constraints, by rotating the handle, the robot can set the orientation of the front wheel, thus setting the direction of motion of the pallet-jack. This allows the robot to plan the trajectory of the pallet-jack kinematically. Control using MM with velocity control in the mobile base and torque control on the upper torso is discussed in [5], which describes how to decouple the base motion to the arm through compensation. Work has been done in pushing wheelchairs. For example, a MM using a modified arm with two grippers to push a wheelchair is introduced in [7]. Their approach uses an adaptive control formulation to control the wheelchair and estimate its mass. Similarly, [13] uses a MM to push a 4-caster wheels cart and performs mass estimation. In both cases, the object's CoM is considered known and constant, significantly simplifying the cart dynamics and estimating the cart's inertial parameters. All these previous works do not consider the location of the manipulators on the cart-like system and do not change the

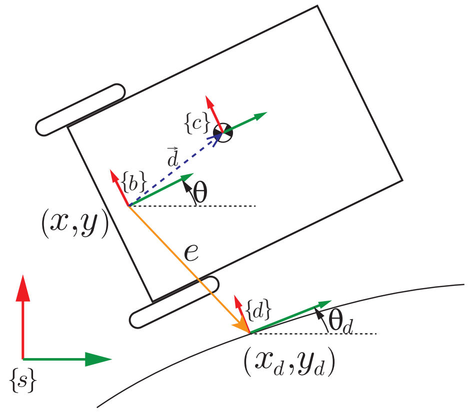

Fig. 2. Simplified cart-like system's diagram error trajectory tracking problem with a desired trajectory.

contact point with the object.

Our current work builds on our research [2] on modeling and online inertial parameter estimation of cart-like systems and the control aspects are being presented on IROS 2023 [1]. In this work, we discuss a derivation of a simplify dymamic model and the selection of contact points to improve control. We present experimental results for a MM pushing a shopping cart with different inertial parameters for linear and curved trajectories.

II. Dynamic model and control of cart-like systems

To control the cart-like system with a MM, we will start by understanding the object being pushed. We will describe the cart-like system as a passive nonholonomic system and study the control inputs required to follow a desired trajectory.

A. Dynamic model

As mentioned, cart-like systems are passive and can have a wide range of inertial parameters. The cart-like object may have a considerable mass, e.g., a wheelchair with a patient or a full shopping cart. Since the objects are passive (no motors), we want to control them by applying an external wrench at a given set of contact points using a MM. To predict the cart's behavior, we model its dynamics with consideration of its inertial parameters (mass, center of mass, and inertia) and input wrench to control the cart.

We propose a simplified dynamic model for these cart-like systems using Euler-Lagrange dynamics. First, we consider the motion of the system in $SE(2)$ and define the following coordinate frames: spatial/world frame $\{s\}$, a fixed frame on the object $\{b\}$, and the CoM frame $\{c\}$. Since the CoM location is unknown, we will develop the Euler-Lagrange equations about frame $\{b\}$. We consider that the fixed wheels introduce nonholonomic constraints due to no lateral slippage of the wheels. While the caster wheels also introduce constraints, due to the off-set of these wheels, they eventually align with the direction of motion, thus we will disregard them an consider them as noise. We define the frame $\{b\}$ at the middle point between the fixed wheels and define the generalized coordinates of the cart as $q = \begin{bmatrix} x & y & \theta \end{bmatrix}$. Seen from the fixed frame of the cart, frame $\{b\}$, the nonholonomic constraint have the form

$$\Lambda(q_b)\dot{q}_b = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix} \dot{q}_b = 0 \qquad (1)$$

where Λ is the constraint matrix. We consider the Lagrangian $\mathscr{L} = T - V$, where T is the kinetic energy and V is the potential energy. Since we are working in the plane, we assume that the potential energy equals zero. For the kinetic energy of the cart system, we should consider the velocity of the main body, with mass m_{cart}, and the velocity of each of the wheels rotating, with mass m_i. In general, carts either have small, light wheels that rotate fast (e.g., shopping cart) or large, heavier wheels that rotate slowly (wheelchair). Regardless, the kinematic energy of the wheels is negligible compared to the energy of the cart itself. Thus, we will look at the energy of the cart as a whole, with mass $m = m_{cart} + \sum m_i$, and neglect the kinetic energy contribution of the wheel's angular velocity about their axis of rotation. The kinematic energy of the cart's CoM in the $\{b\}$ frame is given by

$$\begin{aligned} T =\ & \frac{1}{2} m \left(\dot{x}_b^2 + \dot{y}_b^2\right) - m d_x \dot{\theta}_b \left[\dot{x}_b s_{\theta_b} - \dot{y}_b c_{\theta_b}\right] \\ & - m d_y \dot{\theta}_b \left[\dot{x}_b c_{\theta_b} + \dot{y}_b s_{\theta_b}\right] + \frac{1}{2} m \dot{\theta}_b^2 \left(d_x^2 + d_y^2\right) + \frac{1}{2} I \dot{\theta}_b^2 \end{aligned} \qquad (2)$$

Where s_θ and c_θ are the sine and cosine functions of θ, d_x and d_y are the position of the CoM in the cart frame, and I is the inertia of the system. The nonholonomic constraint appears in this kinematic energy, so we can set it to zero. For the external wrenches, we consider three main elements, the external wrench applied by the MM, Γ_{MM}; non-conservative forces, $N(q,\dot{q})$ (e.g., viscous force as the wheels rotate); and the reaction forces introduced by the nonholonomic constraints of the wheels due to no lateral slippage. The reactive forces due to the nonholonomic constriant oppose any force that would create a motion along the nonholonomic constraint. Then, we can write the dynamic equations of the system as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \Gamma_{MM} - N(q,\dot{q}) - \Lambda^T(q)\lambda \qquad (3)$$

Where λ is a Lagrange multiplier vector (one for each linearly independent constraint) that solves for the nonholonomic constraints. Then, our dynamic system will be given by

$$M = \begin{bmatrix} m & 0 & -m d_y c_\theta \\ 0 & m & -m d_y s_\theta \\ -m d_y c_\theta & -m d_y s_\theta & I + m(d_x^2 + d_y^2) \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & m\dot{\theta} d_y s_\theta \\ 0 & 0 & -m\dot{\theta} d_y c_\theta \\ 0 & 0 & 0 \end{bmatrix} \quad \Gamma_{MM} = \begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} \quad N = \sigma \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\Lambda(q) = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix} \quad \lambda = \begin{bmatrix} \lambda_1 \end{bmatrix}$$

Where σ is a positive definite, viscous coefficient that opposes the system's motion. The Lagrange multipliers add additional unknowns to the system, which can be solved by the forward dynamics of the system and then by solving for the Lagrange multipliers such that the acceleration in the directions of the constraints is zero. Similarly, we can compute

$$P_\Lambda(q) = \left[I - \Lambda^T \left(\Lambda M^{-1} \Lambda^T\right)^{-1} \Lambda M^{-1}\right] \tag{4}$$

where $P_\Lambda(q) \in \mathbb{R}^{n\times n}$ is a projection matrix with rank $n-k$, and k is the number of constraints. $P_\Lambda(q)$ maps the generalized forces Γ_{MM} to $P_\Lambda \Gamma_{MM}$, which is an input force that only applies forces in the allowed directions of motion. If we look at the control problem from the origin of frame $\{b\}$ the projection matrix will nullify any generalize force in the lateral direction (along y^b-axis) due to the wheel no-lateral slippage constraint.

B. Control of the Cart-like System

Given a desired trajectory as $q_d \in SE(2)$, we will look at the control problem from the perspective of the cart-like system in frame $\{b\}$. A diagram of the trajectory tracking problem is shown in fig. 2.

If we control the cart-like system by applying a wrench at the origin of frame $\{b\}$ the applied wrench can be computed using the projection matrix where $\theta_b = 0$, such that the dynamic system becomes

$$\Gamma_{MM} = \begin{bmatrix} f_x \\ 0 \\ \tau_z \end{bmatrix} = \begin{bmatrix} m & 0 & -md_y \\ 0 & 0 & 0 \\ -md_y & 0 & I' \end{bmatrix} \ddot{q}_b + \begin{bmatrix} \sigma_x \dot{x}_b \\ 0 \\ \sigma_\theta \dot{\theta}_b \end{bmatrix} \tag{5}$$

It can be seen that we cannot apply lateral forces into the system since they will be counteracted by the nonholonomic constraints (or tilt and drop the cart if the force is too large). Regardless, we can formulate the subsystem that has a control on the linear acceleration x and angular acceleration θ and propose a control architecture similar to the control of Differential Drive Robots. By incorporating the lateral error into our desired orientation, we pivot the cart in a direction that approaches the desired trajectory. Once the cart is on the desired trajectory, the goal orientation is the same as the desired orientation. We define a goal trajectory based on the desired trajectory for the cart as

$$\begin{bmatrix} x^{des}(t) \\ \theta^{des}(t) \end{bmatrix} = \begin{bmatrix} x_d(t) \\ \theta_d + atan2(y_d - y, x_d - x) \end{bmatrix} \tag{6}$$

We can compute the optimal controller u^* for this system, and the final required wrench for the cart-like system, at the origin of frame $\{b\}$ will be given by

$$\Gamma_{MM} = M\ddot{q}^{des} + C\dot{q}^{des} + N - u^* \tag{7}$$

C. Contact Point Location

The mobile manipulator will not always be able to grasp the object about the origin of frame $\{b\}$ or want to grasp from that point, but to choose from different contact points to minimize the required torque by the manipulator. Thus, we can define the contact frame at the location of the contact point of the MM, in the object frame, as $p^b_{cp} = \begin{bmatrix} x^b_{cp} & y^b_{cp} & 0 \end{bmatrix}$, with the same orientation as the cart. We define a transformation matrix g^i_j as the transformation pair (p^i_j, R^i_j), which takes a vector on the j frame to the i frame. The transformation from the CP to frame $\{b\}$ will be given by

$$g^b_{cp} = \begin{bmatrix} I_{3\times 3} & p^b_{cp} \\ 0_{3\times 1} & 1 \end{bmatrix} \tag{8}$$

Then, we can transform the required force at the origin of frame $\{b\}$ into an equivalent wrench applied at the origin of the contact frame using the adjoint transformation

$$\Gamma_{cp} = Ad^T_{g^b_{cp}} \Gamma_b = \begin{bmatrix} R^b_{cp} & \hat{p}^b_{cp} R^b_{cp} \\ 0 & R^b_{cp} \end{bmatrix}^T \Gamma_b \tag{9}$$

where $\hat{p}^b_{cp}$ is the skew-symmetric matrix of the vector p^b_{cp}.

As an example, lets consider that we want the system to accelerate with some linear acceleration $\ddot{x}^{des}_b$ and no angular acceleration $\ddot{\theta}^{des}_b = 0$. According to eq. 5, the required wrench will be equal to

$$\begin{aligned} f_x &= m\ddot{x}^{des}_b \\ \tau_z &= -md_y \ddot{x}^{des}_b = -d_y f_x \end{aligned} \tag{10}$$

Which means that we need to apply the required linear force and counteract any torque due to the displacement of the CoM. If we consider the possible contact points on the shopping cart as a set that can move laterally along the handle of the shopping cart, the adjoint transformation will give us the following relationship as

$$\begin{aligned} f^{cp}_x &= m\ddot{x}^{des}_b \\ \tau^{cp}_x &= y_{cp} f^b_x - d_y f^b_x \end{aligned}$$

We can see that we need to apply the same linear force to accomplish the desired acceleration, but if we pick the contact point to be equal to the lateral displacement of the CoM ($y_{cp} = d_y$), then no torque need to be applied to counteract the torque due to the CoM displacement.

To compute the desired wrench to be applied onto the cart, we need to know the inertial parameters of the system, specifically m and d_y. In [2] we present a online estimation method to improve parameter estimation during pushing maneuvers.

III. Parameter Estimation and Control of the MM

To have a accurate control input and the location of the CoM to select the best CP location for control, we need to have an accurate estimation of the inertial parameters of the system. in [2] we show good results on the mass estimation under basic trajectories using an augmented state to include the parameters m and d_y using an extended Kalman filter for state estimation on these parameters. Here we show an improvement on the estimation of the lateral displacement.

For control, we compute the desired torque required to follow the desired trajectory using the estimated parameters and then have the MM apply the required torque onto the cart. The control architecture is presented in [1], where we

Fig. 3. Picture of the MM pushing the heavy shopping cart during a curved trajectory tracking experiment.

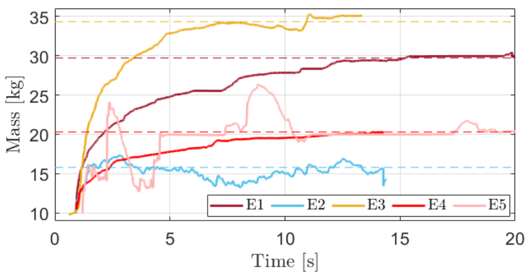

Fig. 4. Estimation of the Mass with the cart with different load.

are controlling the base of the robot to keep a relative distance with the cart and have the manipulator apply the required wrench using a compliant controller.

IV. EXPERIMENTS

The setup for the hardware experiments has the mobile manipulator, composed of a "Ridgeback" mobile base from Clearpath Robotics, an omnidirectional base that has a velocity input in $\mathbb{R}^3$; a Kuka IIWA 14 manipulator, which has 7 Degrees of Freedom and a maximum payload of $14kg$; a "Robotiq FTS-300" Force/Torque sensor and "Robotiq 2F-85" two-finger gripper. The cart is a medium shopping cart with a weight of $15kg$. To keep track of the position and velocity of the MM and the cart, we are using a Vicon camera system for motion capture. We have markers on both MM and object, and we get the pose of the bodies with a $0.4mm$ mean error. To move the MM-cart system, we have a $5\times 6m^2$ space.

We present the result for the mass and lateral displacement estimation and then the control improvement enable by changing the CP during control

A. parameter estimation

For parameter estimation we can see that we can do online learning while pushing on a straight line with a accurate mass estimation in about 10 seconds as shown in Fig. 4, while the estimation of the lateral displacement of the CoM takes about 5 to 10 seconds as shown in Fig. 5.

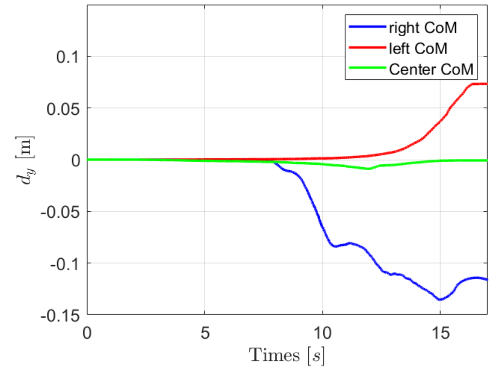

Fig. 5. Estimation of the CoM with the cart with a heavy load on the right, the center and the left

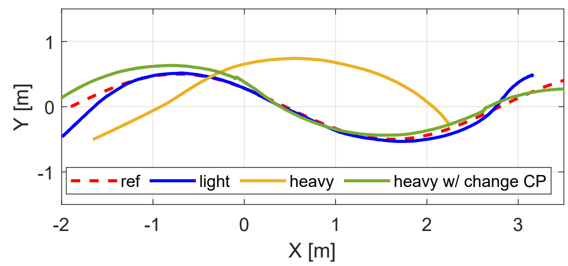

Fig. 6. Trajectory tracking experiments for a curved trajectory.

B. Curve Trajectory Tracking and Contact Points

We present the result of the trajectory tracking experiment of a curve for 3 attempts, one with the light curve and two for the heavy cart. The following trajectories are shown in Fig. 6. For the light cart, the manipulator has enough torque to rotate the cart fast enough to keep the cart close to the trajectory. When we double the weight of the cart and do not allow the change of contact point, the MM does not have enough torque to change the heading of the cart fast enough, thus failing the control goal. Finally, we allow the MM to change contact point discretely when there is a change in the inflection of the desired trajectory, which allow us to once more have the cart follow the desired trajectory with minimum error.

V. CONCLUSIONS

Using the dynamic model of the cart-like system and understanding the location of the contact point between the object and the MM, allow us to improve the capabilities of the MM and minimize the energy required by the MM to push these systems. We are able to learn the parameters during control using an extended Kalman filter that allow us to subsequently improve the selection of the contact point and the required wrench needed to follow a trajectory. In the future, we want to leverage the learning of this parameter to improve path-planning by taking consideration of the inertial parameters of the cart-like system, the selection of contact points and the maximum torques that the MM can apply.

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