The Role of Language Imbalance in Cross-lingual Generalisation: Insights from Cloned Language Experiments

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Abstract

Multilinguality is crucial for extending recent advancements in language modelling to diverse linguistic communities. To maintain high performance while representing multiple languages, multilingual models ideally align representations, allowing what is learned in one language to generalise to others. Prior research has emphasised the importance of parallel data and shared vocabulary elements as key factors for such alignment. In this study, we investigate an unintuitive novel driver of cross-lingual generalisation: language imbalance. In controlled experiments on perfectly equivalent cloned languages, we observe that the existence of a predominant language during training boosts the performance of less frequent languages and leads to stronger alignment of model repre-017 sentations across languages. Furthermore, we find that this trend is amplified with scale: with large enough models or long enough training, we observe that bilingual training data with a 021 ⁹⁰/₁₀ language split yields better performance 022 on both languages than a balanced 5% split. Building on these insights, we design training schemes that can improve performance in all cloned languages, even without altering the training data. As we extend our analysis to real languages, we find that infrequent languages still benefit from frequent ones, yet whether language imbalance causes cross-lingual generalisation there is not conclusive.

1 Introduction

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In recent years, autoregressive language models (LMs) pretrained on massive text corpora have advanced the state of the art in NLP tasks across the board (Brown et al., 2020; Touvron et al., 2023a,b; Köpf et al., 2023). While most of the leading models are trained on English texts, multilingual capabilities are crucial to make these advances accessible to a broader user base with diverse linguistic backgrounds. Ideally, data in one language should improve these multilingual models' performance in others. Such multilingual models should thus display **cross-lingual generalisation**: by reusing circuits (Cammarata et al., 2020; Elhage et al., 2021) and aligning their internal representations across languages, they may generalise concepts learned in a language to another.¹ 043

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How can such cross-lingual generalisation be achieved? This has been a focus of much prior work. One previously identified driver of cross-lingual generalisation is parallel training data; empirical evidence shows that training the model on either parallel sentence pairs (Lample and Conneau, 2019) or on corpora which are comparable across languages (Dufter and Schütze, 2020) improves generalisation. Another driver of cross-lingual generalisation is the availability of anchor points, i.e., vocabulary elements that are shared between languages; these can be naturally occurring subwords (e.g., computer in English and computador in Portuguese may share the subword *comp*; Pires et al., 2019; Wu and Dredze, 2019), shared special tokens (e.g., mask or bos symbols; Dufter and Schütze, 2020), or even artificially inserted "code-switching" augmentations (Conneau et al., 2020b; Reid and Artetxe, 2022; K et al., 2020; Feng et al., 2022). Beyond these two drivers, limited model capacity has been found to improve generalisation by Dufter and Schütze (2020), but to constrain multilingual capabilities by Chang et al. (2023).

In this work, we identify a surprising new factor that can boost cross-lingual generalisation abilities: **language imbalance**. We first conduct

¹A circuit is typically defined as a subgraph of a neural network which performs a specific computation. E.g., a circuit could be responsible for computing "greater than" comparisons between numbers in English sentences (Hanna et al., 2024). If representations are aligned across languages (in terms of how they encode, e.g., numbers) and circuits are reused, a model should be able to apply what it learns in one language (e.g., "greater than" comparisons in English) to perform similar computations in another language (e.g., French).

experiments in a synthetic setting with perfectly 076 equivalent cloned languages; this allows us to in-077 vestigate LMs' generalisation abilities in isolation 078 from the effects of languages' dissimilarities, giving us a rough upper bound on the generalisation we should expect to see between real language pairs. In this cloned language setting, we find that having a dominant main language improves generalisation, significantly boosting the performance of less frequent languages. Furthermore, we find that this effect becomes stronger when we either increase our model's size or when we train it for longer. Based on these insights, we design training curricula that improve performance in all cloned languages without any modifications to the training data. In the second part of our paper, we investigate to what extent our insights transfer to real language pairs. While we find that lower resource languages typically do benefit from higher resource ones, 094 the impact of language imbalance on cross-lingual generalisation is much less clear in this more realistic setting. Overall, our results suggest an interesting attribute of model training dynamics: in some settings, having a main language can lead model components to be shared across languages. 100

2 Cross-lingual Generalisation

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While natural languages differ widely in their typological properties, any pair of languages will share at least a few grammatical and syntactic patterns. Further, as their semantics reflect the underlying processes of our world, language pairs should also have similarities in the types of messages their users typically convey. Intuitively, this suggests that what is learned about a language L_A should be useful to model another language L_B , and vice versa. The extent of such generalisation depends not only on how similar the two languages are, but also on the employed learning algorithm. We analyse such generalisation here, with a focus on how language imbalance influences multilingual LMs.

Intuitively, if a model generalises well across 116 languages, it should achieve better performance 117 in each language (in terms of, e.g., perplexity) 118 than a monolingual model trained on the same 119 data. Concretely, a model trained on a multi-120 lingual dataset $\mathcal{D}_{multi} = \mathcal{D}_{A} \cup \mathcal{D}_{B}$ containing 121 languages L_A and L_B should perform better than 122 monolingual models trained only on \mathcal{D}_{A} or \mathcal{D}_{B} . 123 This becomes clear when using definitions from 124 information theory: \mathcal{D}_{multi} contains at least as 125 much information about L_A as \mathcal{D}_A . However, such 126

a multilingual model could also perform worse. This could happen, for instance, if the data from different languages interfere with each other during optimisation through conflicting gradient update directions (Wang et al., 2020). It could also happen if the model has limited capacity: the multilingual model has to represent many languages, which intuitively requires more capacity than a single one, even if some parameters are shared across them (Conneau et al., 2020a; Pfeiffer et al., 2022).

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In an attempt to make models better across many languages, many multilingual models these days are trained on somewhat balanced data (Scao et al., 2023; Faysse et al., 2024). In some of these cases, low-resource languages are upsampled to improve their performance under the model. As mentioned above, however, while balancing languages' appearance in a model's training set should intuitively improve performance, this is not necessarily true. In fact, (and perhaps surprisingly) some recent large language models trained in mostly English-focused settings perform reasonably well in a large sample of languages (Ahia et al., 2023; Blevins and Zettlemoyer, 2022; Briakou et al., 2023). These models' training data is typically highly imbalanced, with only a small fraction being composed of "non-English" languages. It is thus unclear whether multilingual models indeed benefit from training on datasets with balanced languages (Ye et al., 2023).

In smaller training scales, the benefits of multilingual training are better understood. In general, it has been found that low-resource languages tend to benefit from data in higher-resource languages, but high-resource languages benefit much less from each other (Conneau et al., 2020a; Chang et al., 2023). It is, however, unclear what causes crosslingual generalisation in this case. Is the model in fact able to generalise better in the imbalanced setting? Or does the model generalise equally well in the balanced case, but its capacity bottlenecks performance in higher-resource languages, stopping us from observing performance gains?

We investigate the role of language imbalance in cross-lingual generalisation here. Notably, Wendler et al. (2024) recently showed that LMs seem to perform internal computations in an abstract "concept space" which is closest to their main language (English in this case); representations are then mapped back into the input language only in the models' final layers. Alabi et al. (2024) observe a similar trend when using language adapters.

3 Experimental Setup

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In this section, we provide a brief overview over models, data, and metrics used; for more details, see App. A. Our code will be made available on GitHub. All of our experiments use GPT-2style decoder-only transformers (Radford et al., 2019). We base our implementation on the Languini Kitchen codebase (Stanić et al., 2023), and unless otherwise noted, we use the gpt-small configuration with 85M non-embedding parameters, training on 1.2B tokens of English or French books. We use separate tokenisers for English and French. For some of our experiments, we treat their vocabularies as **disjoint** and do not merge them. If we merge subwords that occur in both vocabularies, we make this clear with the label **anchored**.

As our main evaluation metric, we report our models' perplexity (PPL) on the test set. Further, we define three metrics that allow for easy comparison of monolingual and multilingual models. Let t_A and t_B be the number of tokens a multilingual model is trained on in languages L_A and $L_{\rm B}$, respectively. We define monolingual token equivalence (MLTE) as the number of tokens that would be required by a monolingual model, trained only in either language $L_{\rm A}$ or $L_{\rm B}$, to achieve the same perplexity as the multilingual model does in that language. To determine MLTE, we fit a simple scaling law to predict perplexity from the number of training tokens (e.g., t_A) using the results from our trained monolingual models (see App. B for details). Analogously, we define monolingual PPL equivalence (MLPE) as the perplexity a monolingual model would reach when trained on the same number of L_A tokens (i.e., t_A) as a given multilingual model. Finally, we define token efficiency (**TEff**) as the fraction between MLTE and the number of tokens used for multilingual training, e.g., $\text{TEff}_{A} = \frac{\text{MLTE}_{A}}{t_{A}}$. Intuitively, if TEff > 1, performance improves due to multilinguality, while if TEff < 1, multilinguality hurts performance.

4 Cloned Languages

In this section, we examine the model's capability to generalise across perfectly equivalent **cloned languages**. We create a cloned language by duplicating the language model's vocabulary; this allows us to encode each sequence in either the original language (using the original vocabulary) or in the cloned language (using the cloned vocabulary). This experimental paradigm was originally proposed by K et al. (2020) and Dufter and Schütze (2020).² Formally, let L_{orig} be an "original" language with a vocabulary of subword units Σ ; we denote each subword $w \in \Sigma$. This language can be described by a probability distribution $p(\boldsymbol{w}_{\text{orig}})$, where $\boldsymbol{w}_{\text{orig}} \in \Sigma^*$. We clone language L_{orig} by creating multiple instantiations of it: L_1, L_2 , $\dots L_N$. These languages have vocabularies Σ_n , each of which has symbols that are equivalent to the original ones.³ Furthermore, these languages define probability distributions which are isometric to the original language. If we denote equivalence as $\boldsymbol{w}_n \stackrel{\circ}{=} \boldsymbol{w}_{\text{orig}}$ for $\boldsymbol{w}_n \in \Sigma_n^*$ and $\boldsymbol{w}_{\text{orig}} \in \Sigma^*$, we have $\boldsymbol{w}_n \stackrel{\circ}{=} \boldsymbol{w}_{\text{orig}} \implies p(\boldsymbol{w}_n) = p(\boldsymbol{w}_{\text{orig}})$.

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Given dataset $\mathcal{D}_{orig} = \{\boldsymbol{w}_{orig}^{(k)}\}_{k=1}^{K}$ with $\boldsymbol{w}_{orig}^{(k)} \sim p(\boldsymbol{w}_{orig})$, we can now create a multilingual dataset \mathcal{D}_{multi} by independently mapping each sequence to one of the cloned languages: For each $\boldsymbol{w}^{(k)}$, we first sample a language $L^{(k)} \sim p(L)$ from a categorical distribution over languages, then we map the sequence to $L^{(k)}$ by encoding it using the corresponding vocabulary. We can write $\mathcal{D}_{multi} = \bigcup_{n=1}^{N} \mathcal{D}_n$ where

 $\mathcal{D}_n = \left\{ \boldsymbol{w}_n^{(k)} \mid \boldsymbol{w}_n^{(k)} \stackrel{\circ}{=} \boldsymbol{w}_{ ext{orig}}^{(k)} ext{ and } L^{(k)} = L_n
ight\}$

denotes the subset in language L_n .

Importantly, cloned languages are perfectly equivalent, having the same syntax, semantics, and distribution. They differ only in the symbols used to encode their vocabularies. Any generalisation we observe in this setting should thus serve as an upper bound on the potential to generalise across non-identical natural languages.⁴ In other words, if our model cannot generalise across cloned languages, we would have strong reason to believe it shouldn't generalise across distinct languages. If we observe that a model can generalise across cloned languages, however, we may or may not observe the same to happen across non-cloned languages. We'll investigate the latter in Section 5.

²K et al. (2020) perform duplication on the character IDs, i.e., before tokenisation, while Dufter and Schütze (2020) adopt an approach equivalent to ours. Both of these works term L_2 a "fake" language. Since there is no distinction between L_1 and L_2 , however, we call them cloned languages instead. Other related studies have investigated the effect of infinitely many cloned languages on LMs' performance (Huang et al., 2023; Chen et al., 2023), or employed duplicated vocabularies at the token level to study their impact on LMs' memorisation or performance (Kharitonov et al., 2021; Schäfer et al., 2024).

³Unless otherwise noted, these vocabularies are defined as disjoint sets in our experiments, meaning that no anchor points exist across languages.

⁴As for most of our experiments we consider cloned languages' alphabets to be disjoint, in practice our results only upper bound the cross-lingual generalisation of models with no anchor points (i.e., with disjoint vocabularies).



Figure 1: LM performance by imbalance ratio. (top) LM perplexity. (bottom) LM accuracy on GLUE; models were fine-tuned in EN_1 and evaluated on either EN_1 and EN_2 .

4.1 Generalisation

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Due to the equivalence of cloned languages, one may expect language models to easily generalise across them. In that case, training a multilingual model on datasets \mathcal{D}_1 and \mathcal{D}_2 would lead to similar performance to training a monolingual model on the original dataset $\mathcal{D}_{\text{orig}}$ (note that $|\mathcal{D}_{\text{orig}}| = |\mathcal{D}_1| +$ $|\mathcal{D}_2|$). We perform this experiment here, training either monolingual models on English (EN), or multilingual models on cloned English (EN_1 and EN_2), setting $p(EN_1) = p(EN_2) = 0.5$. Perhaps surprisingly, when training in this balanced multilingual setting, language modelling performance is significantly worse than in the monolingual setting (see Table 1, rows 2 & 4). In fact, one would obtain better performance training two monolingual models for half as many steps than training on this combined data. Training data in one language seems to hurt performance in the other language instead of boosting it. This indicates that the model is not able to generalise well across languages in this setting.

Takeaway 1. *The model does not generalise well across cloned languages given a ^{50/50} data split.*

4.2 Language Imbalance

How does the balance of the languages' data affect generalisation performance? Will the multilingual model still underperform its monolingual equivalents when trained on an uneven language distribution? When varying the ratio of EN_1 to EN_2 data shown during training (while keeping the total number of training steps constant), we observe that the rarer "lower resource" language, here always EN_2 , benefits from the presence of a dominant "main language". Fig. 1 (left) shows that, under higher imbalance, the model's performance on EN_2 becomes much better than that of a monolingual model trained on the same amount of EN_2 data. For example, when training in the ⁹/₁₀ regime, we obtain a $TEff_{EN_2}$ of over 2 (see Table 1, row 5). Do these improvements translate to better crosslingual generalisation on downstream tasks as well? We test this by fine-tuning models on the GLUE benchmark (Wang et al., 2019) in EN_1 only, and evaluating them on EN_1 and EN_2 . We observe that models trained under higher language imbalance indeed have significantly better EN_2 zero-shot performance (see Fig. 1 right). Together, these results suggest that cross-lingual generalisation is occurring. 301

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Is this generalisation attained due to the model's internal computations being shared across languages? To answer this question, we analyse how language imbalance affects the cross-lingual alignment of our models' representations. Looking at the cosine similarity of equivalent subwords $w_1 \stackrel{\circ}{=} w_2$ in EN₁ and EN₂, we find that similarity steadily increases with higher imbalance: in the ⁵⁰/₅₀ setting, embeddings are not aligned (exhibiting an average cosine similarity of 0.02), while, e.g., in the ⁹⁰/₁₀ setting, equivalent subwords are much more aligned, showing a similarity of 0.28 (details in App. C). Comparing the cosine similarity of hidden states when the LM is given equivalent sequences $w_1 \stackrel{\circ}{=} w_2$, we also observe stronger alignment for a model trained in the imbalanced ^{90/10} regime, compared to the ⁵⁰/₅₀ counterpart (see App. F). Interestingly, the cosine similarity between gradients is also higher in the imbalanced setting: when processing equivalent sequences, the gradients with respect to w_1 or w_2 have an average cosine similarity of 0.53 for the model trained in the ^{90/10} setting, compared to 0.07 in the 50/50 setting (see full plots of similarities per model component in App. G). This suggests that the gradient updates with respect to one language may benefit the optimisation process of that language's cloned counterpart more when training under higher imbalance.

Takeaway 2. Language imbalance improves generalisation and leads to representations which are more aligned across cloned languages.

4.3 Many Languages

How does this trend transfer to settings with more than two languages? In such cases, sharing circuits across languages might be even more crucial due to the model's limited capacity. Instead of cloning

			Т	Pl	PL	TEff			
Run Type	Row	# Tokens	$p(EN_1)$	$p(EN_2)$	$p(\mathrm{EN}_3),\ldots,p(\mathrm{EN}_{10})$	EN_1	EN_2	EN_1	EN_2
	1	1.2B	100%	0%	0%	21.9	-	1	-
Monolingual	2	0.5 imes 1.2B	100%	0%	0%	25.3	-	1	-
	3	0.1 imes 1.2B	100%	0%	0%	48.4	-	1	-
2.1	4	1.2B	50%	50%	0%	26.1	26.1	0.89	0.89
2 languages	5	1.2B	90%	10%	0%	22.5	32.8	1.00	2.08
10.1	6	1.2B	10%	10%	10%,,10%	35.5	35.7	1.69	1.67
10 languages	7	1.2B	50%	$\frac{1}{18}$	$\frac{1}{18}, \dots, \frac{1}{18}$	24.6	33.4	1.15	3.56
Sabadula	8	1.2B	$100\% \downarrow 0\%$	0% ↑ 100%	0%	>1B	31.4	-	0.47
Schedule	9	1.2B	90% ५ 10%	10% 🖻 90%	0%	26.5	24.4	0.83	1.18
Dr. data	10	$2 \times 1.2B$	50%	50%	0%	23.3	23.3	0.73	0.73
2x data	11	$2 \times 1.2 B$	90% ५ 10%	10% ሾ 90%	0%	22.8	20.4	0.83	1.60
2m data	12	$3 \times 1.2B$	50%	50%	0%	22.2	22.2	0.64	0.64
ox data	13	3 imes 1.2B	90% ५ 10%	10% ↑ 90%	0%	21.5	19.3	0.77	1.63

Table 1: Performance of language models trained on different compositions of EN_1 and EN_2 . a% \lor b% indicates an immediate decrease from a% down to b% halfway during training. Analogously, a% r b% indicates an immediate increase.

the language only once, we now clone it nine times, obtaining in total 10 languages. In Table 1 (rows 6 & 7), we report the performance when sampling the languages in a balanced way and when having a much stronger main language.

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Interestingly, when sampling uniformly, we obtain TEff \approx 1.7; performance is thus better than with a monolingual model trained on an equivalent amount of monolingual data (compare rows 6 & 3). This differs from our observations for the bilingual setting, where uniform language sampling performed worse than the equivalent monolingual models. Presumably, modelling this many languages effectively with limited model capacity may lead the model to share its circuits, improving cross-lingual generalisation (Dufter and Schütze, 2020). The limit of infinite languages (in which a model never observes the same language more than once) was analysed by Huang et al. (2023); interestingly, LMs still seem to learn the language, to some extent, even in that setting.

In the imbalanced setting where we sample a stronger "main language" 50% of the time, we observe even stronger performance on all languages. Despite the model seeing only roughly 67M tokens in each of the rarer languages (1/18 of all steps), it achieves **better** performance in these languages than in the uniform setting with 120M tokens (1/10 of all steps) per language. In fact, on the rarer languages, the model achieves TEff \approx 3.6, matching the performance of a monolingual model trained on 240M tokens.

Takeaway 3. When training on many cloned languages, sampling a main language disproportionately improves generalisation.



Figure 2: TEff as we train LMs with (left) more data, or (right) larger architectures. mini, small and medium denote GPT sizes in Languini (Stanić et al., 2023), with 11M, 85M, and 303M non-embedding parameters.

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4.4 Effect of scaling

Model and data size are crucial factors for the performance of LMs. Here, we investigate how the previously identified trends are affected by scaling the model architecture or training data. Fig. 2 (left) shows that the effect of imbalance on cross-lingual generalisation appears to increase when we train on twice as much data (2.4B tokens instead of 1.2B), reaching TEff > 3; this corresponds to a "chinchilla-optimal" setup for our GPT small model (Hoffmann et al., 2022). At the same time, the TEff of the 5% setting seems to be decreasing under prolonged training. This might be caused by the heightened importance of model capacity under longer training, which may have a stronger impact on performance when representations are less aligned across languages. Overall, the disparity in effectiveness between the imbalanced and balanced settings grows with longer training. Remarkably, when training for 4.8B tokens, the ⁹⁰/₁₀ setting yields better performance in both languages, compared to the 50/50 setting.

When decreasing the model size, we also

observe higher performance benefits in the 410 imbalanced setting (see Fig. 2 right), potentially 411 due to the capacity argument described above. 412 Interestingly, however, the effect of imbalance 413 appears to be significantly stronger for larger 414 models as well. When training a larger model 415 with around 300M parameters (GPT medium in 416 Languini; Stanić et al., 2023), in the ⁹⁰/₁₀ setting, we 417 achieve better performance on both languages than 418 under the 5% split. This might be because larger 419 models generally exhibit better generalisation 420 ability than smaller ones (Brown et al., 2020). 421

> **Takeaway 4.** Longer training and larger models lead to stronger performance benefits due to language imbalance.

4.5 Language Sampling Schedule

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Knowing that language imbalance boosts generalisation, how can we use this insight to train better models? Is there a way to leverage our insights in order to improve performance on two languages, even with the same training data? In Table 1 (rows 8, 9, 11, and 13), we report results when training with a language sampling schedule that ensures a language imbalance throughout all of training, but which still leads to an overall $5\%_{50}$ split between EN_1 and EN_2 data seen by the model. We sample EN_1 with a higher probability during the first half of training. Then, we sample EN_2 more often to achieve a marginal split of $5\%_{50}$.

When showing exclusively EN_1 at first, and then showing only EN_2 ($^{100 \ l \ 0/0 \ r \ 100}$; row 8), we observe bad overall performance. By the end of training, perplexity on EN_1 is very high, presumably due to catastrophic forgetting (McCloskey and Cohen, 1989; French, 1999). Further, EN_2 does not seem to benefit from the EN_1 data, achieving very low performance, which might be due to the lower learning rate in the second half of training. ⁵

On the other hand, if we avoid catastrophic forgetting, making sure that the model encounters at least some samples of both languages at every point in training, via a ^{90 \L 10}/10 r⁹⁰ split (first sampling languages with ratio ⁹⁰/10, and then switching to ¹⁰/90 after half of training), we can mitigate these issues. On our standard training set (1.2B tokens, row 9), we observe almost equivalent performance to uniform language sampling on EN_1 , but significantly improved performance on EN_2 . Under longer training, these benefits become more pronounced: this language schedule improves performance on both languages compared to the simple ⁵⁰/₅₀ setting (compare row 10 vs 11 and row 12 vs 13).

Takeaway 5. Compared to uniform language sampling, an imbalanced ratio throughout training can lead to better results on all languages, even if the overall language split remains balanced.

5 Real Languages

To verify whether the insights from our clonedlanguage experiments hold in a more natural setting, we now run experiments with multilingual models on English (EN) and French (FR).

5.1 Generalisation

In the cloned setting, we observed no significant generalisation when training on a balanced language mix (i.e., TEff < 1, representations were unaligned, and zero-shot GLUE accuracy on EN_2 was bad). Similarly, when sampling EN and FR data uniformly, we also obtain TEff < 1. A multilingual model's performance is thus worse than a monolingual model trained only in the same EN or FR data (see Table 2, row 7). Notably, prior work has identified anchors (shared vocabulary items across languages) help generalisation (Dufter and Schütze, 2020; Pires et al., 2019; Wu and Dredze, 2019). We thus experiment with similarly merging vocabulary items shared between EN and FR, and confirm this helps performance (compare Table 2, row 7 vs 11). We run more experiments analysing the impact of anchor points in both cloned and real languages, see App. D. Note that, with an anchored vocabulary, generalisation across EN and FR is not necessarily upper bounded by our results on disjoint cloned languages. In fact, in the 50/50 setting, we observe a marginally higher TEff for EN-FR models with an anchored vocabulary than for EN_1 -EN2 models where we used disjoint vocabularies (compare Table 1 row 4 and Table 2 row 11).

5.2 Language Imbalance

Analogous to the cloned setting, we observe that an imbalanced EN/FR ratio leads to improved performance (TEff > 1), on the rarer language (see Table 2, rows 7-9 & 11-13). This is the case for both, a $^{9}/_{10}$ and a $^{10}/_{90} EN/FR$ ratio. Fig. 3 shows PPL and TEff in EN and FR as a function of the language imbalance. We observe that large 480

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⁵Chen et al. (2023) find that an equivalent setting can still be beneficial when using many more languages: they periodically reinitialise the learned embeddings (which is equivalent to switching to a new cloned language) and obtain models that are better adaptable to new languages.

			Training Data	Pl	PL	TEff		
Run Type	Row	# Tokens	p(EN)	p(FR)	EN	\mathbf{FR}	EN	\mathbf{FR}
	1	1.2B	100%	0%	21.9	-	1	-
	2	0.5 imes 1.2B	100%	0%	25.3	-	1	-
Monolingual	3	$0.1 \times 1.2B$	100%	0%	48.4	-	1	-
Wollollingual	4	1.2B	0%	100%	-	16.0	-	1
	5	$0.5 \times 1.2B$	0%	100%	-	18.4	-	1
	6	$0.1 \times 1.2B$	0%	100%	-	34.1	-	1
	7	1.2B	50%	50%	26.4	19.4	0.85	0.82
Multilingual	8	1.2B	90%	10%	22.5	31.9	1.00	1.05
disjoint vocabs	9	1.2B	10%	90%	43.5	16.4	1.10	0.97
	10	1.2B	90% ५ 10%	10% ሾ 90%	29.1	20.5	0.60	0.66
	11	1.2B	50%	50%	26.0	19.0	0.91	0.88
	12	1.2B	90%	10%	22.5	29.0	1.00	1.27
Multilingual	13	1.2B	10%	90%	39.5	16.5	1.33	0.96
anchored vocabs	14	1.2B	90% ५ 10%	10% ሾ 90%	28.9	19.3	0.61	0.83
	15	1.2B	90%५10% ≓50% →50%	10%	26.4	18.5	0.85	1.00
	16	1.2B	$95\% {\downarrow} 35\% \rightarrow \!\! 35\% \rightarrow \!\! 35\%$	$5\%^{\scriptscriptstyle 0}65\%\rightarrow 65\%\rightarrow 65\%$	26.3	18.7	0.86	0.95
2. data	17	$2 \times 1.2B$	50%	50%	23.0	16.9	0.79	0.76
2x data	18	$2 \times 1.2B$	90% ५ 10%	10% ሾ 90%	26.1	17.1	0.44	0.70
2m data	19	$3 \times 1.2B$	50%	50%	21.8	16.0	0.70	0.67
ox data	20	$3 \times 1.2B$	90% \ 10%	10% ሾ 90%	25.1	16.2	0.35	0.63

Table 2: Performance of language models trained on different compositions of EN and FR. $a\% \rightarrow b\% \rightarrow c\% \rightarrow d\%$ indicates a four stage language schedule, switching immediately between, e.g., c% and d% after 75% of training.

imbalances generally seem to yield TEff > 1; the worst TEff is reached with a balanced EN/FR ratio. These trends are in line with our findings in the cloned setting. However, especially with disjoint vocabularies, the observed performance benefits due to generalisation are less significant. Presumably, this is due to EN and FR not being equivalent and thus generally allowing less generalisation.

Does imbalance again improve generalisation due to a better alignment of the model's representations in the two languages? As in the cloned language setting, we investigate the cosine similarity between the models' hidden states when processing "equivalent" sequences in the two languages. For real languages, however, we do not have access to perfectly equivalent sequences. Instead, we mimick this scenario using parallel translated sequences in the two languages, which should contain roughly similar properties. Differently from the cloned language setting, we do not observe higher hidden state similarities for models trained on imbalanced data (see App. F). Further, we find that gradient similarities barely differ across balanced and imbalanced settings when using disjoint vocabularies. For the anchored vocabulary they are even marginally higher in the balanced setting (see App. G). We thus do not find evidence that the improved TEff in the imbalanced setting is caused by a stronger alignment of model updates across languages in this setting. A possible reason for this discrepancy could be that, at the scales of our experiments, LMs tend to rely on



Figure 3: LM performance on EN and FR by imbalance ratio.

language specific surface-level features (which are shared by cloned languages, but not by distinct real languages) and show less understanding of complex semantics which might be more generalisable. Future research might thus consider investigating these trends at larger scales.

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Takeaway 6. Imbalanced multilinguality boosts the performance of real low-resource languages. However, this effect is weaker here than for cloned languages. Further, for real languages, we do not find evidence of language imbalance leading to representations which are more cross-lingually aligned.

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5.3 Effect of Scaling

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In the cloned setting, we observed that prolonging training significantly decreased TEff in the $5\%_{50}$ setting. We hypothesised that this might be caused by a stronger influence of the limited model capacity with longer training, and poor sharing of representations between languages. As EN and FR are distinct languages that require at least some language specific representations, we might expect this trend to be even more pronounced for these languages. However, compared to the cloned setting, prolonging training leads to a smaller decline in TEff in the $5\%_{50}$ setting here. Presumably, the anchored vocabulary allows for better generalisation compared to the cloned setting, despite the languages being distinct.

Further, unlike in the cloned setting, longer training significantly decreases the TEff of the lower-resource language in the imbalanced setting here (see Fig. 4). In fact, the ⁹/₁₀ TEff even falls below 1, approaching the TEff of the ⁵/₅₀ setting. This suggests that language imbalance might not improve generalisation across distinct real languages. Still, when scaling up the model, we observe an increase of almost 2x in the TEff of the lower-resource language (see Fig. 4). This is in line with our cloned languages observations, although the effect is weaker.

Takeaway 7. Performance benefits for real lowresource languages tend to decrease or vanish with longer training. Larger models, however, appear to yield higher performance benefits in both the balanced and imbalanced setting.

5.4 Language Sampling Schedule

For equivalent cloned languages, we found that an imbalanced language sampling schedule can lead to improvements upon simple uniform sampling. If this held for real languages as well, it could have important practical implications for future multilingual LM training. However, whereas a ^{90 \, 10}/_{10 \, f 90} schedule yielded strong performance on cloned languages, matching or outperforming the ⁵/₅₀ setting, this is not the case for EN and FR (see Table 2, row 10 vs 7 and row 14 vs 11). Furthermore, in line with the observations above, longer training does not make this schedule more effective, but instead increases its gap to the performance of the ⁵/₅₀ setting (see rows 17-20).

The discrepancy between these results and the ones in cloned languages might be explained by



Figure 4: TEff of models on EN and FR with anchored vocab as we train them with (left) more data, or (right) larger architectures.

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the reduced effect of imbalance on the generalisation and representation alignment in real languages. The schedules may be enough to force LMs to share circuits across cloned languages, but not across real ones. To investigate if this negative result was a particular property of our chosen schedule, we explore other more complex scheduling options. ⁶ In general, none of the tested schedules appears to outperform the ⁵%₅₀ setting (see rows 15, 16) on both languages. However, more complex 4-stage schedules, can obtain better performance on one language while incurring a slight performance drop in the other. Intriguingly, this allows trading off the performance of different languages without altering the training data.

Takeaway 8. For real languages, we do not find improvements on all languages due to the tested language schedules. However, they allow for trading off performance in different languages.

6 Conclusion

We ran experiments to measure cross-lingual generalisation in both a controlled setting with cloned English languages, as well as with English and French. In both settings, we find that, without vocabulary overlap, our models do not show strong cross-lingual generalisation when trained on a balanced language set. However, when training on an imbalanced mix of languages, we observe increased performance compared to monolingual settings. For cloned languages, we find that this can be explained by a higher alignment of the model's representations across languages, which indicates circuit reuse and improved cross-lingual generalisation. Yet, at the scales of our experiments, such a correlation is less evident in real languages. While our findings allow us to design an imbalanced language schedule that yields improved performance in the cloned setting, further research is required to extend these improvements to real-world settings.

⁶Future research might design these more carefully, also analysing the interplay of language- and learning rate schedule

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There are several limitations of our work, many of which present opportunities for future research.

Limitations

642Data and model size.While we conduct exper-643iments with varying data (up to 4.8B tokens) and644model size (up to 336M parameters), it is uncertain645whether the identified trends also apply at the scale646of modern large language models. Additionally, for647more capable models, cross-lingual generalisation648might be relevant in different aspects, with, e.g.,649semantics playing a larger role. As the semantic650content communicated in different languages might651be easily transferable, this might impact generali-652sation dynamics.

Languages. We only run experiments on English and French, two Indo-European languages. Further work could consider more languages and investigate the impact of language similarity in results more broadly.

658Model architecture.We run most of our exper-659iments on a Transformer decoder (we also mea-660sure embedding alignment for simpler Word2Vec661models). Future research could analyse the effects662of architecture in more depth to better understand663the drivers of representation alignment. Conneau664et al. (2020b), e.g., find that shared parameters in665the top layers lead to better cross-lingual transfer.666In our Word2Vec experiments, we do not observe667improvements in representation alignment due to668language imbalance (see Fig. 6), presumably due669to no parameters being shared between the two lan-670guages. Would this change when adding a shared671layer to the Word2Vec model?

Downstream performance. In our evaluation we mainly rely on perplexity as a metric, with a single experiment on GLUE accuracy. It might be insightful to analyze effects on downstream task performance more broadly.

677Quantifying generalisation. In this work, we678mainly measure cross-lingual generalisation by679comparing the performance of multilingual mod-680els with that of monolingual models trained on681the same amount of data in the given language.682If a multilingual model on languages L_A and L_B 683requires fewer L_A tokens to reach a given perplex-684ity on L_A than a monolingual model, we speak685of cross-lingual generalisation, knowing that per-686formance on L_A must have been boosted by data

in language $L_{\rm B}$. Future work could formalise this measure and aim to model/quantify the relationship between the number of training tokens seen in a language $L_{\rm B}$ and performance in another language $L_{\rm A}$, depending on model size, language imbalance, language similarity, anchor points, and other factors. An accurate model of these relationships could be of substantial practical value. 687

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1109 A Experimental Setup

1110Model. We use a GPT-2-style decoder-only transformer architecture in our experiments (Radford et al.,11112019). Unless otherwise noted, we instantiate our model with 12 layers and a hidden size of 768, which1112results in 85M non-embedding parameters; this corresponds to Languini's gpt-small configuration. We1113follow previous work and train our models with sequence length 512, batch size 128, the Adam optimiser1114(Kingma and Ba, 2015), and a cosine learning rate schedule from 6e-4 to 6e-6 with 500 warmup steps.

Data. For the English settings, we use Languini's default datasets to train and evaluate our models. 1115 These are English books from a filtered version of the books3 subset from the Pile (Gao et al., 2020). 1116 The train set consists of a total of 23.9B tokens, while the test set contains i.i.d. books, with a total of 1117 roughly 11M tokens. This data is tokenised into a vocabulary of size 16k, obtained using a BPE tokeniser 1118 trained with SentencePiece (Gage, 1994; Sennrich et al., 2016; Kudo and Richardson, 2018). For our 1119 experiments in French, we use the French-PD-Books dataset (PleIAs, 2024), to which we apply the 1120 preprocessing pipeline of the Languini Kitchen, but for French. We train a separate BPE tokeniser on 1121 this French dataset, using a 16k-sized vocabulary. Depending on the experiment, the French and English 1122 vocabularies are either kept separate (disjoint) or merged (anchored). Unless otherwise noted, we train 1123 our models for 18,265 steps—i.e., the first 1.2B tokens in our dataset; this corresponds to a GPT small 1124 model trained for 6h on an RTX 3090 GPU, the Languini GPT small 6h setting (Stanić et al., 2023). For 1125 experiments where we compare hidden representations or gradients on parallel French-English or cloned 1126 English sequences, we use data from the Europarl parallel corpus (Koehn, 2005). 1127

Evaluation. When evaluating PPL (from which we also compute MLPE, MLTE and TEff) on the held-out test set, we want to ensure sufficient context for all predictions. To this end, we use a sliding window with steps of 128: we fill in a 512 tokens context, ignore the model's outputs on the initial 384, and evaluate it only using the last 128 tokens.

B Fitted Scaling Laws

To predict the performance of monolingual models depending on the amount of tokens they are trained on, we fit a power law curve to predict the relationship between number of training tokens and perplexity for models of all three sizes and for both languages (see Fig. 5).



Figure 5: Fitted power laws curves predicting perplexity depending on the fraction of training tokens (compared to our standard 1.2B tokens) for different languages and model sizes.

1136 C Alignment of EN_1 and EN_2 Representations

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1137 While, under balanced language sampling, embeddings of corresponding subwords are not much more 1138 aligned than embeddings of random pairs, we observe an increase in cosine similarity with increasing 1139 language imbalance: from 0.02 for $5\%_0$ to 0.28 for $9\%_5$ (see Fig. 6). Fig. 7 shows that this alignment is 1140 higher for frequent subwords. This seems natural: at initialisation, subword embeddings are random and 1141 not aligned. Then, they become more and more aligned over the course of training.

Interestingly, the embeddings of a simple word2vec (Mikolov et al., 2013) model do not show stronger alignment under higher imbalance. This might be due to a lack of shared parameters between the languages (Conneau et al., 2020b).



Figure 6: Embedding cosine similarity of corresponding duplicate subwords from EN_1 and EN_2 and random pairs to control for anisotropy. Left: our GPT model. Right: Word2vec embeddings trained on the same data (computed with Gensim).



Figure 7: Embedding cosine similarity of corresponding cloned subwords $w_1 \stackrel{\circ}{=} w_2$ from EN₁ and EN₂, by frequency.

Anchor Points D

D.1 Anchors on Cloned Languages

As described earlier, previous works found that anchor points—i.e., lexical items which overlap between 1147 languages—can lead to better generalisation and alignment of representations (Dufter and Schütze, 2020; 1148 Pires et al., 2019; Wu and Dredze, 2019). In our cloned setting, we can investigate this in a controlled 1149 manner by varying the number of vocabulary elements we duplicate. While in the experiments described 1150 above we created EN_2 by duplicating the entire vocabulary, we now duplicate only a fraction. The 1151 remaining vocabulary is shared between EN_1 and EN_2 . In this experiment, we observe that a small 1152 number of anchor points already significantly boosts model performance (see Fig. 8), which indicates 1153 improved generalisation. 1154



Figure 8: Perplexity by percentage of anchor points, i.e., overlap between EN_1 and EN_2 vocabularies. Models trained on balanced EN_1/EN_2 split.

D.2 Anchors on Real Languages

English and French vocabularies naturally overlap, having common subwords. These shared elements 1156 potentially act as anchors, facilitating better cross-lingual generalisation. However, the effectiveness of such anchor points may be moderated by semantic differences; for instance, a shared subword might carry a different meaning or connotations in English and French, affecting its utility as an anchor. Despite these nuances, anchor points appear to boost generalisation between real languages: when we merge the EN 1160 and FR vocabularies, we obtain better performance on both languages (compare Table 2, row 7 vs 11) as 1161 well as higher alignment of gradients (see App. G). This aligns with our findings from the cloned language 1162 setting (see App. D.1). Given these benefits, it is natural to use an anchored (i.e., merged) vocabulary 1163 when possible.7 1164

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⁷In practice, this is usually achieved by training a tokeniser on multilingual data, instead of merging monolingually trained vocabularies.

E Larger Models and More Data

Fig. 9 and Fig. 10 contain results for the full array of model- and dataset size combinations we ran for cloned languages and for English and French, respectively.



Figure 9: Performance with balanced and imbalanced EN_1 and EN_2 data for different configurations of model- and dataset size

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Figure 10: Performance with balanced and imbalanced EN and FR data for different configurations of model- and dataset size. Using anchored vocabulary.

F Hidden State Similarity

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Here, we compare the hidden states of our model when processing parallel sequences, both in cloned languages (see Table 3) and in English and French (see Table 4). I.e., for a given trained model and parallel sequences w_a and w_b , we first feed w_a through the model, then w_b , and finally compute the cosine similarities for the hidden states of pairs of corresponding tokens from w_a and w_b (see App. H for details on how these pairs are determined). We use 500 parallel sequences obtained from the Europarl parallel corpus (Koehn, 2005). For cloned languages, we observe that hidden states of the model trained under higher language imbalance generally have higher cosine similarity than the those of the model trained in a balanced setting. For English and French such a trend is less clear. Interestingly, however, an anchored vocabulary seems to lead to slightly higher similarities of the hidden states.

Training Data		Layer											
$p(\mathrm{EN}_1)$	$p(\mathrm{EN}_2)$	1	2	3	4	5	6	7	8	9	10	11	12
50%	50%	0.55	0.79	0.83	0.88	0.85	0.83	0.78	0.66	0.56	0.46	0.25	-0.21
90%	10%	0.86	0.93	0.96	0.96	0.96	0.96	0.96	0.95	0.94	0.90	0.67	0.11
Δ		0.31	0.14	0.13	0.09	0.11	0.14	0.18	0.28	0.38	0.44	0.42	0.32

Table 3: Hidden states' cosine similarity when LM is fed equivalent inputs in cloned languages. Similarity is computed per token (i.e., comparing pairs of equivalent tokens).

	Training Data							La	iyer					
	$p(\mathrm{EN})$	p(FR)	1	2	3	4	5	6	7	8	9	10	11	12
Disjoint	50%	50%	0.68	0.80	0.84	0.88	0.86	0.84	0.80	0.75	0.62	0.53	0.34	-0.15
	90%	10%	0.71	0.83	0.88	0.87	0.86	0.84	0.81	0.74	0.69	0.57	0.40	-0.17
	Δ		0.03	0.03	0.04	-0.01	0.00	0.00	0.01	0.00	0.07	0.04	0.06	-0.03
.ed	50%	50%	0.73	0.84	0.88	0.91	0.89	0.88	0.85	0.78	0.71	0.61	0.36	0.10
ichor	90%	10%	0.78	0.87	0.89	0.91	0.89	0.87	0.84	0.77	0.72	0.63	0.28	0.06
Aı	Ĺ	7	0.05	0.03	0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.02	-0.08	-0.04

Table 4: Hidden states' cosine similarity for parallel inputs in EN and FR for anchored and disjoint vocabularies. We first match which tokens correspond to each other in the two languages, and then compare their representations (see App. H).

G Gradient Similarity

Here, we compare the cosine similarity of trained models' gradients with respect to parallel sequences in two different (possibly cloned) languages. For cloned languages, the alignment between gradients is significantly higher for the model trained in the imbalanced ⁹⁰/₁₀ setting (see Fig. 11). For EN and FR data, this does not seem to be the case, whether the vocabulary is anchored (see Fig. 12) or disjoint (see Fig. 13). However, under the anchored vocabulary, the gradient similarities appear to be generally higher, suggesting better cross-lingual representation alignment.



Figure 11: Similarity of gradients with respect to parallel sequences in EN_1 and EN_2 for models trained in balanced and imbalanced settings. Macro average for ⁵⁰/₅₀: 0.07. Macro average for ⁹⁰/₁₀: 0.53.

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Figure 12: Similarity of gradients with respect to parallel sequences in EN and FR for models with anchored (i.e., merged) vocabulary, trained in balanced and imbalanced settings. Macro average for 5%: 0.17. Macro average for 9%: 0.14.



Figure 13: Similarity of gradients with respect to parallel sequences in EN and FR for models with disjoint vocabularies, trained in balanced and imbalanced settings. Macro average for $5\%_{50}$: 0.04. Macro average for $9\%_{10}$: 0.05.

H Matching Corresponding Tokens

In our experiments in §5.2, we employ parallel sequences in different languages and compare both their hidden states' and their gradients' similarity.

When comparing gradients (see App. G), we adopt a setup that is analogous to the training process as we aim to understand how one language might affect optimisation of the other: we compute gradients with respect to a full sequence in each language, and then compare these sequence-level aggregated gradients. Analogously, during training, gradient updates are also aggregated for entire sequences. (In fact, during training, these updates are also aggregated for an entire batch, but we use a batch size of 1 for this evaluation.)

However, when comparing hidden states, we compare the individual representations of corresponding tokens in the two sequences. We first compute the cosine similarity of each equivalent token pair, and only then average over the sequence dimension; this provides us with a more informative signal. For parallel sequences $\boldsymbol{w}_{\text{EN}_1} \stackrel{\circ}{=} \boldsymbol{w}_{\text{EN}_2}$ in cloned languages, it is clear which token corresponds to which: At each given position *t*, we know that $\boldsymbol{w}_{\text{EN}_1,t} \stackrel{\circ}{=} \boldsymbol{w}_{\text{EN}_2,t}$ so we can simply compare the hidden states position by position (see Table 3).

Yet, this might not be the case for real languages EN and FR, e.g., due to differing word order or tokenisation. To ensure that we still compare the hidden states of tokens that approximately correspond to each other in the respective languages, we match them based on their cosine similarity scores. Concretely, we create a bipartite graph where the nodes consist of the tokens of the two sequences. For every pair of tokens $w_{\text{EN},t}$ and $w_{\text{FR},t'}$ we add an edge which is weighed by the mean cosine similarity of their hidden states across all layers. We then compute a maximum weight full matching in this graph.⁸ Such a matching maximises the average similarity across all token pairs. Indeed, the resulting token pairs appear to approximately correspond to each other (see Fig. 14). We can then compare the hidden states of these pairs (see Table 4).

Notably, the cosine similarities of hidden states of corresponding EN and FR tokens $w_{\text{EN},t}$ and $w_{\text{FR},t'}$ computed in this way generally appear to be slightly higher than for corresponding tokens $w_{\text{EN},t} \stackrel{\circ}{=} w_{\text{EN},t}$ of cloned languages (compare Table 4 (disjoint) and Table 3). This might seem unexpected, given that $w_{\text{EN},t}$ and $w_{\text{EN},t}$ are perfectly equivalent but $w_{\text{EN},t}$ and $w_{\text{FR},t'}$ are generally not. Could this be an artifact of the employed matching strategy which always maximises the average similarity, potentially matching tokens that have very high similarity but are completely unrelated? If this is the case, we should also obtain higher similarity scores in the cloned setting when using the described matching strategy instead of comparing position by position. After running this experiment, we find that using the matching strategy the similarities under the ⁵⁰/₅₀ cloned language split are indeed marginally higher, although only in the last layers. Under the ⁹⁰/₁₀ split, however, we observe no notable changes. It thus seems that the proposed matching strategy does not artificially inflate similarity scores too strongly.



Figure 14: Computed matching for an example sentence using a model trained under ⁵⁰/₅₀ split with anchored vocabulary. Pointers to "…" denote a match with a token earlier or later in the sequence.

⁸We compute the matching using the NetworkX (Hagberg et al., 2008) implementation of the algorithm proposed by Karp (1978).