On the Extension and Sampling Theorems for the Coupled Fractional Fourier Transform

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In 1980 a fractional version of the Fourier transform was introduced by V. Namias, but did not received much attention until the early 1990s when it was found to have numerous applications in optics and time-frequency representation.

The fractional Fourier transform, denoted by F_θ, depends on a parameter $0 \leq \theta \leq \pi/2$, so that when $\theta = 0$, F_0 is the identity transformation and when $\theta = \pi/2$, $F_{\pi/2}$ is the standard Fourier transform. The transform has been extended to higher dimensions by taking tensor products of one-dimensional transforms.

In 2018 the author of this article introduced a novel generalization of the fractional Fourier transform to two dimensions, which is called the coupled fractional Fourier transform and is denoted by $F_{\alpha,\beta}$. This transform depends on two independent angles α and β, with $0 \leq \alpha, \beta \leq \pi/2$, so that $F_{0,0}$ is the identity transformation and $F_{\pi/2,\pi/2}$, is the two-dimensional Fourier transform. For other values of α and β, we obtain other interesting configurations of the transform. One immediate application of this transform is in time-frequency representation because of its close relationship to the Wigner distribution function.

In this talk we will discuss sampling theorems and extensions of this transform to spaces of generalized functions.