

TRANSDUCTIVE VISUAL PROGRAMMING: EVOLVING TOOL LIBRARIES FROM EXPERIENCE FOR SPATIAL REASONING

000
001
002
003
004
005
006
007
008
009
010
011
012
013
014
015
016
017
018
019
020
021
022
023
024
025
026
027
028
029
030
031
032
033
034
035
036
037
038
039
040
041
042
043
044
045
046
047
048
049
050
051
052
053
Anonymous authors
Paper under double-blind review
ABSTRACT
The composition of specialized tools offers a powerful approach for complex visual reasoning, particularly for tasks involving 3D spatial understanding. However, existing visual programming methods are often constrained by fixed toolsets or offline tool induction, which leads to suboptimal solutions and poor tool reuse. We introduce Transductive Visual Programming (TVP), a novel framework that dynamically evolves a library of reusable tools by learning from its problem-solving experience. TVP abstracts recurring solution patterns into new, higher-level tools, which are then used to construct simpler and more effective programs for new tasks. On the challenging Omni3D-Bench, TVP establishes a new state of the art, outperforming both specialized vision-language models and prior visual programming systems. The evolved tools also exhibit strong generalization to out-of-domain queries on 3DSRBench, SpatialSense, and VGBench. Our work demonstrates that transductive tool evolution is a powerful and generalizable paradigm for building robust visual reasoning systems.

1 INTRODUCTION

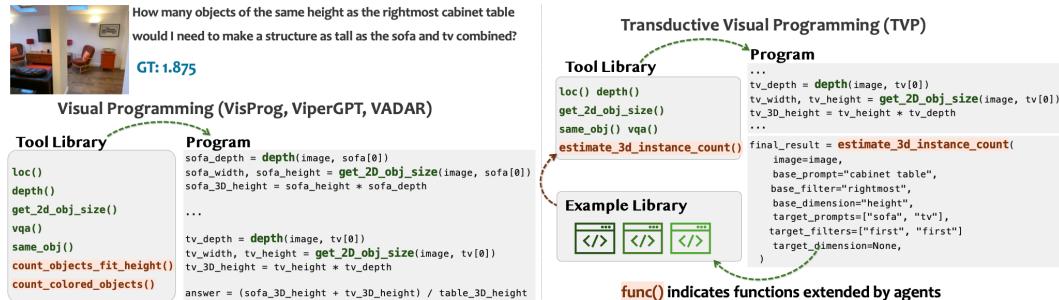


Figure 1: (Left) Prior methods operate in an open-loop manner—tools are created without experience from actual problem-solving. (Right) TVP maintains both an Example Library of successful program solutions and a Tool Library of abstracted functions. Through this closed-loop system, TVP abstracts tools from patterns in proven solutions.

Reasoning on complex visual scenes, including localization, understanding spatial relations, and counting objects, is not a single visual skill. While pre-trained Vision-Language Model (VLM) have made progress in processing images, spatial reasoning remains challenging for frontier models such as GPT-4o (Lee et al., 2025; Marsili et al., 2025). This limitation motivates a compositional approach: decomposing complex visual problems into discrete computational steps executable by specialized tools — a paradigm known as visual programming.

An effective programming system should emulate how human programmers learn: first by solving concrete problems, and only then by abstracting recurring patterns into reusable functions. Current visual programming paradigms do the opposite. They either rely on fixed, predefined tool sets that cannot learn (Gupta & Kembhavi, 2023; Surís et al., 2023), or they speculatively synthesize new

054 tools inductively before their utility is proven. For example, VADAR (Marsili et al., 2025), a prior
 055 system that induces tools from questions upfront without grounding to any solutions, still relies
 056 94.2% of the time on the basic predefined APIs despite the presence of its proposed tools.
 057

058 We propose **Transductive Visual Programming** (TVP), a
 059 framework that generates program solutions while continually
 060 evolving a dynamic and highly-reusable tool library built from
 061 past experience. Analogous to human programmers who iden-
 062 tify recurring patterns and abstract them into reusable func-
 063 tions only after solving concrete problems, TVP learns through
 064 *transductive* abstraction. This approach *directly leverages suc-
 065 cessful program solutions from specific examples to form new,
 066 higher-level tools*. Rather than “inducing” potentially useful
 067 functions before solving problems, TVP recognizes repeated
 068 patterns after implementing multiple concrete solutions, en-
 069 suring that learned abstractions are useful and grounded in ex-
 070 perience.

071 As illustrated in Fig. 1, TVP’s architecture is centered around
 072 a dual-library system designed to facilitate this transductive
 073 learning loop. The **Example Library** serves as a memory,
 074 accumulating a growing corpus of successful program sol-
 075 utions. In parallel, the **Tool Library** maintains a dynamic set
 076 of higher-level functions that are continuously evolved by ab-
 077 stracting common patterns from the Example Library. When faced with a new query, TVP first
 078 retrieves relevant demonstrations from its experience, then leverages its evolved, higher-level tools
 079 to construct simpler and more efficient programs.

080 On the challenging Omni3D-Bench for 3D spatial reasoning, TVP achieves a new state-of-the-art
 081 overall accuracy of 33.3%, outperforming both generic VLMs like GPT-4o (+22.4%) or SpaceMan-
 082 tis (Jiang et al., 2024; Chen et al., 2024) built with spatial-specific finetuning, and previous visual
 083 programming systems including VADAR (+11.3%). The superior performance of TVP is a direct
 084 result of its effective tool discovery and reuse. Our analysis shows that tools learned through TVP are
 085 used far more frequently than those of baseline methods; 36.3% of our learned tools are employed
 086 as the only functions in final program solutions, compared to just 5.8% for VADAR’s speculatively
 087 induced APIs.

088 Moreover, the tool set evolved by TVP generalizes to unseen tasks with strong performance. After
 089 the TVP agent evolves its dual libraries on Omni3D-Bench §3.1, it is directly applied to handle
 090 novel spatial reasoning queries sampled from SpatialScore-Hard collection §3.2 (including 3DSR-
 091 Bench (Ma et al., 2024), SpatialSense (Yang et al., 2019), and VG-Bench (Wu et al., 2025)), where it
 092 also delivers superior performance with zero-shot generation across task categories, demonstrating
 093 that its learned skills are highly transferable. Taken together, this paper demonstrates that program-
 094 ming with transductive tool creation from experience is a powerful paradigm for tackling complex
 095 spatial reasoning tasks.

096 2 TRANSDUCTIVE VISUAL PROGRAMMING

097 Transductive Visual Programming (TVP) is a framework that learns to create and refine a library
 098 of reusable tools from its own problem-solving experience. It operates via a closed-loop process
 099 centered on a dual-library architecture (Fig. 3): an **Example Library** \mathcal{E} accumulates successful
 100 program solutions, while a **Tool Library** \mathcal{T} maintains an evolving set of functions abstracted from
 101 the experience. This design allows TVP to emulate human learning: first solve concrete problems,
 102 then generalize successful patterns into reusable skills. The entire workflow is formalized in Alg. 1.

103 2.1 PROGRAM GENERATION AND EXECUTION

104 **Initialization.** TVP begins with an empty Example Library $\mathcal{E} \leftarrow \emptyset$ and a Tool Library \mathcal{T} ini-
 105 tialized with predefined basic vision tools. These predefined tools, inherited from Marsili et al.
 106 (2025), include: object localization (`loc`) and bounding box detection (`get_2d_object_size`)

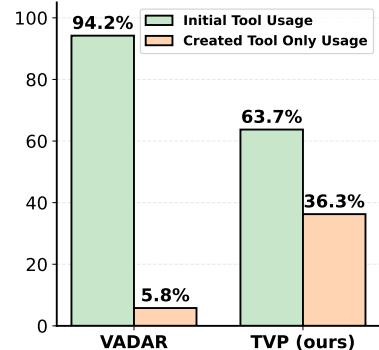


Figure 2: Tool usage distribution: transductive (TVP) vs. inductive abstraction (VADAR)

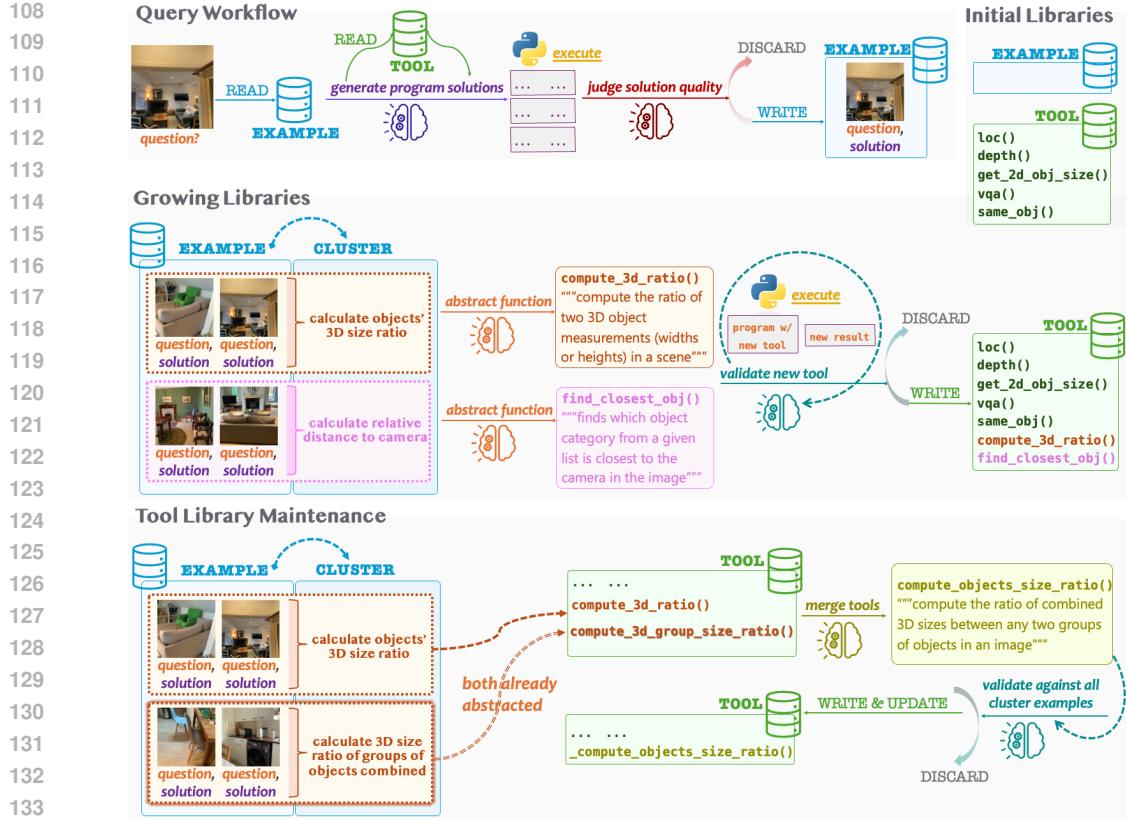


Figure 3: **TVP’s dual-library architecture and pipeline.** (Top) Query workflow: For each visual reasoning question, TVP retrieves similar examples from the Example Library, generates candidate programs using available tools and high-quality solutions join the Example Library. (Middle) Tool abstraction: As examples accumulate, TVP clusters similar queries and abstracts common solution patterns into parameterized tools. (Bottom) Tool maintenance: When functionally similar tools emerge from different clusters, TVP identifies and merges them into unified tools that generalize both functionalities.

with GroundingDINO (Liu et al., 2024), depth estimation (depth) via UniDepth (Piccinelli et al., 2024), object property queries with GPT-4o (vqa), and overlapping bounding box verification with same_object. These basic spatial computations serve as building blocks for more complex reasoning.

Example retrieval. Given a question q_i , TVP first retrieves k most similar queries from the current Example Library \mathcal{E} . Retrieval is based on the embedding similarity of question texts. Since question texts are straightforward, similar embeddings indicate similar query semantics.

Program generation and execution. We provide the question q_i , the retrieved k examples with their solutions as in-context demonstrations, and the current Tool Library \mathcal{T} (in the form of tool signatures and docstrings) to the Program Generator, which is an LLM that explores m different candidate program solutions for q_i , as visual reasoning problems often admit multiple valid approaches. TVP executes each program candidate with access to the full Tool Library \mathcal{T} implementations, producing execution traces as namespaces and a final calculated answer to q_i .

2.2 EXAMPLE LIBRARY: ADDING HIGH-QUALITY PROGRAMS

Quality judge. TVP employs a VLM judge to assess each candidate program’s quality. The judge has access to the program implementation, full execution trace and the produced answer, evaluating

162 them against the question and image as visual evidence. The best candidate represents the final
 163 solution to the question. [More details on the judge criteria are provided in §C.3.](#)
 164

165 **Joining Example Library.** We maintain a quality threshold τ_q that gates entry to the Example
 166 Library. This ensures \mathcal{E} contains only high-quality program solutions, forming the foundation for
 167 both effective in-context examples (§2.1) and high-quality tool abstractions (§2.3).

168
 169
 170 **2.3 TOOL LIBRARY: TOOL ABSTRACTION FROM EXPERIENCE**
 171

172 At intervals of n_a processed questions, TVP mines its Example Library for recurring patterns that
 173 merit abstraction into reusable tools for the Tool Library (see Alg. 2 2 for the complete abstraction
 174 process).

175 **Example clustering.** First, all queries in \mathcal{E} are clustered by embedding similarity of question texts
 176 (similar to example retrieval). This creates initial groups of related questions with high potential for
 177 similar program solutions. For clusters with similarity surpassing threshold τ_{sim} and size exceeding
 178 τ_{cluster} , we query an LLM for abstraction analysis to evaluate the cluster’s abstraction potential.
 179 High-potential patterns (score $\geq \tau_{\text{potential}}$) trigger tool abstraction from these examples’ solutions.
 180 [More details on the criteria for abstraction potential are provided in §C.3.](#)
 181

182 **Tool abstraction.** We provide the Tool Abstractor with all cluster examples’ questions, program
 183 solutions, execution results, and the current Tool Library. The Tool Abstractor creates a parameter-
 184 ized function capturing the cluster’s shared program logic. For instance, as shown in Fig. 3, clusters
 185 calculating 3D size ratios yield `compute_3d_ratio`, while clusters finding nearest objects pro-
 186 duce `find_closest_obj`. The new function replaces step-by-step programming in the cluster
 187 examples.

188 **Validating new tool.** Each new abstract function is rigorously vetted (Alg. 3). TVP rewrites each
 189 cluster example using the new tool; with no ground truth, a rewrite is accepted if it yields identical
 190 results or—common for floating-point cases—is judged by a VLM to be equally valid or better given
 191 the visual evidence. This safeguards or improves visual-program quality while enabling generaliza-
 192 tion. Unlike prior speculative induction, TVP is transductive: it lifts concrete solutions built from
 193 low-level tools directly into abstractions, ensuring every new tool is grounded in experience from
 194 the Example Library.

195 **Example status update.** When cluster examples are abstracted into a common function, we rewrite
 196 these examples in the Example Library to use the new tool. This better guides future similar ques-
 197 tions to employ abstract tools in their solutions. Once successfully abstracted, we mark these cluster
 198 examples as closed for future clustering and abstractions.

199
 200
 201 **2.4 DUAL-LIBRARY MAINTENANCE**
 202

203 As more datapoints are processed and both libraries grow, TVP performs periodic maintenance for
 204 both libraries.

205 **Periodic Tool Library update with tool merging.** Tool abstractions performed with accumulative
 206 cluster examples may lead to functionally similar abstract tools created at different intervals. We
 207 periodically merge functionally similar tools in \mathcal{T} (see Alg. 4). As illustrated in Fig. 3, tools like
 208 `compute_3d_ratio` and `compute_3d_group_size_ratio` serving similar purposes merge
 209 into a more general `compute_objects_size_ratio`. Merged tools undergo the same rigorous
 210 validation against all examples using the original tools. This periodic merging reduces redundancy
 211 while preserving all abstracted functionalities.

212 **Example Library update across iterations.** TVP runs through multiple iterations T over the entire
 213 dataset \mathcal{D} (as shown in Alg. 1). Examples in the Example Library can be updated across iterations by
 214 comparing quality ratings—better program solutions replace existing ones, as later iterations with
 215 access to more abstract tools produce more efficient and cleaner programs.

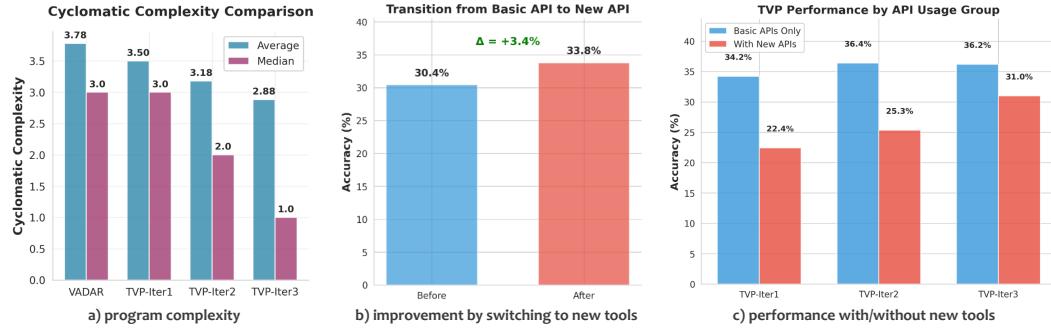


Figure 4: Tool abstraction improves both program efficiency and accuracy. (a) Program cyclomatic complexity steadily decreases across TVP iterations. (b) Programs gain $+3.4\%$ accuracy when switching from basic to abstracted tools. (c) Performance with new APIs improves 38% across iterations as TVP gradually masters its learned abstractions

3 EXPERIMENTS

3.1 3D SPATIAL REASONING PERFORMANCE

TVP is a general visual programming system applicable to various tasks given appropriate initialization (particularly the initial toolkit, §2.1). 3D spatial reasoning is a representative challenge where mere visual perception in the form of VQA fails short of the precise geometric calculations required by the 3D spatial inference.

Setup. We evaluate our approach on **Omni3D-Bench** (Marsili et al., 2025), a challenging test set containing 501 non-templated spatial queries on diverse real-world scenes, requiring 3D object grounding and reasoning about distances and dimensions. Among the four question types, we evaluate: (1-3) Yes/No, Multiple Choice, and Counting via exact-match accuracy (with possible fuzzy string normalization); (4) Float-point calculations via Mean Relative Accuracy (Yang et al., 2025): $\mathcal{MRA} = \frac{1}{|C|} \sum_{\theta \in C} \mathbb{1} \left(\frac{|\hat{y} - y|}{y} < 1 - \theta \right)$ with $C = \{0.5, 0.55, \dots, 0.95\}$, and $\text{Float}(\pm 10\%)$ accuracy for predictions within 10% error tolerance. We run TVP for $T = 3$ iterations with quality threshold $\tau_q = 8.5$ for example library entry, similarity threshold $\tau_{\text{sim}} = 0.8$, and minimum cluster size $\tau_{\text{cluster}} = 4$ for clustering. Tool abstraction and deduplication occur at every step ($n_a = n_d = 1$). We use GPT-4o for program generation and image-based quality judge, and 4o-mini for abstraction and auxiliary tasks.

For baselines, we compare against generic VLMs (GPT-4o, LLaVA-OneVision-7B-Chat Li et al., 2024, Qwen2-VL-7B-Inst Wang et al., 2024a, Molmbo-7B-D Deitke et al., 2025), spatial-finetuned VLMs (SpaceMantis Jiang et al., 2024¹ and SpatialBot-3B Cai et al., 2025), and prior visual programming systems (VisProg Gupta & Kembhavi, 2023, ViperGPT Suris et al., 2023, VADAR Marsili et al., 2025). Refer to §C for additional implementation details.

TVP achieves state-of-the-art through experience-grounded tool creation. As shown in Tab. 1, TVP achieves new state-of-the-art performance on Omni3D-Bench with 33.3% overall accuracy, outperforming all baselines including the previous best visual programming system VADAR (29.9%) by 11.4% relative improvement. The advantage is most pronounced on precise floating-point spatial calculations: TVP reaches 19.3% accuracy within $\pm 10\%$ tolerance, vs. VADAR’s 15.9% and GPT-4o’s 8.2%. Monolithic VLMs handle perception-heavy yes/no and multiple-choice tasks reasonably well, but falter on exact 3D measurements—where compositional programming excels. Even spatial-finetuned models (*e.g.*, SpaceMantis) show no meaningful gains over generic VLMs on these calculations, again underscoring the necessity for stronger compositional programming approach like TVP.

TVP enables effective program compression. As TVP creates more higher-level tools, repetitive low-level operations are gradually eliminated. Fig. 4 (panel a) illustrates how our TVP steadily re-

¹Finetuned following SpatialVLM (Chen et al., 2024)

270 Table 1: Performance comparison on Omni3D-Bench. **Best scores bolded**, second best underlined.
271 *Results from VADAR paper.

273 Method	274 Accuracy by Question Type (%)					275 Overall (%)
	276 Yes/No	277 Multiple Choice	278 Counting	279 Float MRA	280 Float ($\pm 10\%$)	
<i>276 Generic VLMs</i>						
GPT-4o	65.3	60.5	18.6	26.7	8.2	27.2
Qwen2-VL-7B-Inst	58.7	33.7	12.9	21.5	10.0	21.8
LLaVA-OV-7B-Chat	<u>60.0</u>	27.9	<u>22.9</u>	26.8	11.1	23.0
Molmo-7B-D	46.7	41.9	18.6	28.4	8.9	21.6
<i>281 Spatial-Finetuned VLMs</i>						
SpaceMantis	53.3	30.2	4.3	21.4	8.2	18.2
SpatialBot-3B	<u>60.0</u>	30.2	0.0	17.7	8.5	18.8
<i>284 Visual Programming</i>						
VisProg*	54.7	25.9	2.9	0.9	—	—
ViperGPT*	56.0	42.4	20.0	15.4	—	—
VADAR	56.0*	57.6*	21.7*	<u>35.5*</u>	15.9	29.9
TVP (ours)-iter1	50.7	62.8	21.4	34.7	<u>18.5</u>	31.3
TVP (ours)-iter2	<u>60.0</u>	59.3	24.3	34.7	17.4	<u>31.9</u>
TVP (ours)-iter3	<u>60.0</u>	<u>61.6</u>	24.3	36.5	19.3	33.3
w/o Tool Lib	<u>60.0</u>	<u>61.6</u>	21.4	35.5	17.0	31.7

duce program complexity across iterations. The average [McCabe’s Cyclomatic Complexity Number \(CCN\) \(McCabe, 1976\)](#) (§C.2) decreases from 3.5 to 2.88 (-17.7%), with the median CCN dropping from 3.0 to 1.0. The program compression leads to two key benefits: (1) improved efficiency and reduced potential errors in reimplementation of branching logic; (2) improved interpretability of generated programs, as a single function call replace otherwise whole paragraphs of code (see qualitative examples in Fig. 10).

Tool abstraction facilitates progressive self-improvement, especially on hard problems. To isolate the contribution of our tool abstraction from in-context learning with examples, we run TVP with only the Example Library active (w/o Tool Lib in Tab. 1). This configuration still benefits from retrieved similar examples as few-shot demonstrations but relies solely on basic initial tools. Notably, the Example-Library-Only variant **already achieves a competitive 31.5% accuracy overall, outperforming all prior baselines**. This strong performance demonstrates the quality of our accumulated experience, enabled by our example library admission design (§2.2).

While the Example Library provides strong foundation, our full TVP system with active tool creation shows more **significant improvement across iterations** (31.3% \rightarrow 31.9% \rightarrow 33.3%), while the Example-Library-Only variant maintains static (31.7% \rightarrow 31.5% \rightarrow 31.5%). This progressive improvement stems from the **closed-loop design** as illustrated in Fig. 1: abstracted tools encapsulate past experience and enable better future programs, which become better examples, from which better tools can be abstracted. Fig. 4 (panel b) also indicates this effect: **when programs switch from basic tools to newly created abstractions, they achieve +3.4% accuracy improvement**. Without tool creation in the loop, this self-improving cycle is weakened.

Furthermore, we find that the tool abstraction contributes most value on hard problems. We use GPT-5 with high reasoning effort to rate the difficulty of the spatial reasoning questions on a 1.0–10.0 scale (details in §C.4), then divide questions into three groups: Easy (1–3), Medium (4–6), and Hard (7–10). **Fig. 5 shows accuracy across methods for different complexity levels. TVP (Full) delivers the best performance on both Easy and Hard batches.** For easy questions, thoroughly validated created tools avoid potential reimplementation errors, leading to more stable performance. For harder questions, created tools provide simpler solution steps that eliminate complicated logic, thus easing the program reasoning. **Fig. 6 reveals the evolution of the benefits brought by our active tool abstraction across iterations**, as we compare the the performance delta between TVP (Full) and TVP (Example-Lib-Only) for each complexity level. **On the hardest batch, TVP (Full) shows the most significant improvement trajectory**, starting at -4.5% relative to Example-Lib-

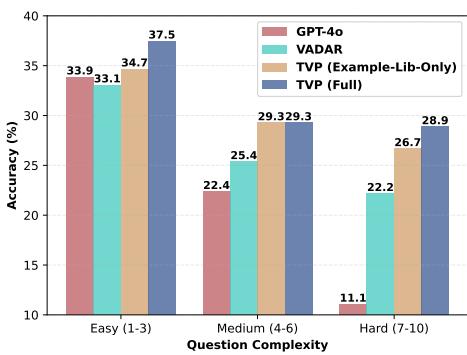


Figure 5: Performance comparison across question complexity levels on Omni3D-Bench. The complexity scores are rated with criteria defined in §C.4

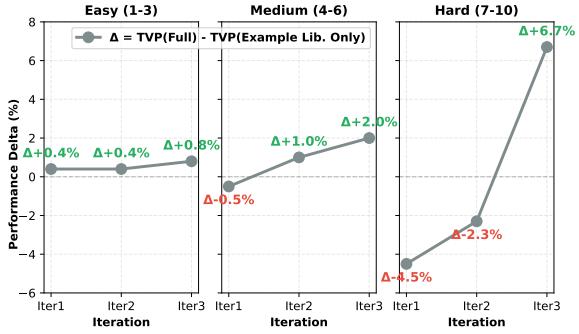


Figure 6: Performance delta between TVP (Full) and TVP (Example-Lib-Only) across iterations for each question complexity level. TVP full system (with active tool creation) shows most significant gains on the hardest batch of questions.

Only in iteration 1, but ultimately surpassing it by +6.7% in iteration 3. This demonstrates that our created tools – beyond just in-context examples – effectively reduce the reasoning workload for most challenging questions, as they encapsulate past experience of complicated code logic into simple function calls. An example is illustrated in Fig. 10(panel b), where a newly created tool serves as a convenient step in solving a spatial reasoning problem.

Both libraries evolve steadily through transductive learning. Fig. 9 visualizes TVP’s evolution through three iterations on Omni3D-Bench. The system exhibits steady growth in both libraries: the Example Library accumulates from 0 to 304 high-quality solutions while the Tool Library expands from 5 initial tools to 11 active abstractions (after creating 61 total and merging redundant ones). This controlled growth—creating many abstract functions but keeping only the most general through periodic merging (§2.4)—ensures the tool library remains manageable while capturing diverse functionalities. The impact of this evolution is evident in Fig. 4(panel c). Programs using only basic APIs maintain stable performance across iterations, confirming that TVP preserves its ability to leverage fundamental tools. Meanwhile, programs utilizing new APIs show dramatic accuracy improvement—from 22.4% in iteration 1 to 31.0% in iteration 3—as the system masters applying its learned abstractions. This +38% relative improvement demonstrates that TVP doesn’t just create tools but learns to use them more effectively over time.

TVP is robust to backbone LLM choice, showing a clear scaling trend with model sizes. We further investigate TVP’s robustness to open-source smaller LLMs, represented by the Qwen2.5-Coder-Instruct (Hui et al., 2024) family as the backbone program generator, spanning from 7B to 32B parameters. Specific configurations are given in §C.2. Fig. 7 presents the scaling behavior, where TVP exhibits a clear performance improvement with increasing model capacity. Notably, using an open-source 32B model, TVP achieves performance close to our GPT-4o-backed variant (30.7% vs. 31.3%), and surpasses the previous best baseline VADAR (29.9%) despite its more capable GPT-4o backbone. This result underscores that our TVP does not rely on proprietary-specific optimal LLMs but can achieve strong performance with more accessible open-source alternatives. The consistent scaling trend also validates TVP’s architecture as model-agnostic, and suggests significant future potential of our transductive tool creation, as foundation models with enhanced capabilities become available.

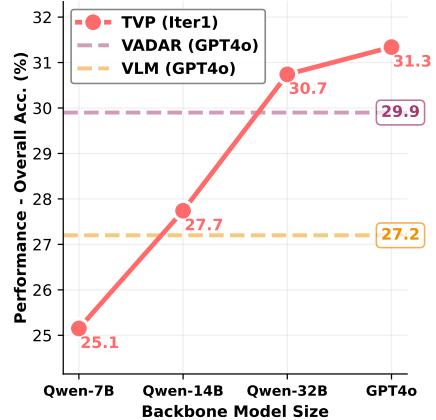


Figure 7: TVP performance scales consistently on Omni3D-Bench with backbone model capacity (Qwen2.5-Coder-7/14/32B & GPT-4o).

378
379

3.2 GENERALIZING ACROSS UNSEEN SPATIAL REASONING QUERIES

380
381
382

As TVP builds its dual libraries through one test set (Omni3D-Bench), we evaluate whether the learned capabilities transfer to unseen spatial reasoning queries in the wild. We take the agent built from §3.1 and directly apply it to new benchmarks without any additional example or tool creation.

383
384
385
386
387
388
389
390
391
392
393

Setup. We sample from the SpatialScore-*Hard* collection (Wu et al., 2025) that contains the most challenging spatial understanding queries. In total, we collected 256 test datapoints from 3DSR-Bench (Ma et al., 2024), SpatialSense (Yang et al., 2019), and VG-Bench (Wu et al., 2025) respectively, keeping single-image, visual-bounding-box-free samples (aligning with the structural setup on Omni3D-Bench). These encompass 4 categories of spatial reasoning capabilities (Fig. 8), covering Yes/No, Multiple-Choice, and Open-Ended (numeric calculation) questions types. Same as in §3.1, we evaluate the former two question types with accuracy, and the last type with $\text{Float}(\pm 10\%)$ accuracy within 10% error range tolerance. We compare our TVP agent’s zero-shot transfer performance against various VLMs and VADAR as representative visual programming system. Notably, we run VADAR on these test sets, meaning that it has new tools created specifically for these test sets while our TVP directly applies its Omni3D-Bench libraries without any modification.

394

395
396
397
398

Table 2: Results on benchmarks from sampled SpatialScore-*Hard* collection; TVP (built on Omni3D-Bench) evaluated zero-shot. **Best scores bolded**, second underlined.

399

Method	3DSR-B.	SpatialSense	VG-B.	Overall
<i>Generic VLMs</i>				
GPT-4o	<u>52.1</u>	46.5	20.3	<u>42.6</u>
LLaVA-OV-7B-Chat	12.4	9.9	9.4	<u>10.9</u>
Qwen2-VL-Inst	49.6	32.4	7.8	34.4
Molmo-7B-D	41.3	54.9	12.5	37.9
<i>Spatial-Finetuned VLMs</i>				
SpaceMantis	37.2	19.7	7.8	25.0
SpatialBot-3B	20.7	62.0	6.2	28.5
<i>Visual Programming</i>				
VADAR	24.8	40.8	<u>39.1</u>	32.8
TVP-Generalizing	52.9	<u>59.2</u>	43.8	52.3

413

414

Transductively learned tools generalize with superior performance. As shown in Tab. 2, TVP achieves 52.3% overall accuracy on SpatialScore-*Hard*, outperforming even VADAR (32.8%) which inductively creates tools specifically for these test sets. The performance breakdown in Fig. 8 shows TVP excels particularly on challenging spatial reasoning categories. For 3D positional relations, TVP achieves 59.2% vs VADAR’s 40.8% and GPT-4o’s 46.5%. On depth and distance estimation (from VG-Bench), TVP reaches 43.8% while VADAR manages 39.1% and GPT-4o only 20.3%. The consistent superior performance across spatial tasks validates that our transductive learning builds up genuinely reusable libraries rather than overfitted solutions. Fig. 10 (panel b) provides qualitative evidence of this generalization. Tools like `find_largest_by_3d_metric`, originally abstracted from Omni3D-Bench solutions, now handles the a new 3D comparison queries from 3DSR-Bench without any modification, demonstrating that transductive abstraction captures fundamental reasoning patterns rather than dataset-specific tricks.

426
427
428

3.3 QUALITATIVE ANALYSIS ON TOOL UTILIZATION AND EVOLUTION

429
430
431

Transductively abstracted tools are easily applicable to diverse tasks. Fig. 10 illustrates how TVP’s tool creation achieves high utilization. Panel (a) shows `estimate_3d_instance_count` handling diverse ratio calculations within Omni3D-Bench—from computing how many cabinet tables would match a sofa-TV height to determining television stacking requirements. Panel

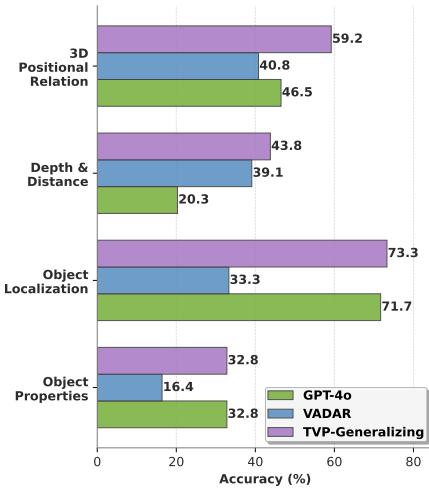


Figure 8: TVP’s libraries transfer well on SpatialScore-*Hard*, particularly for 3D spatial and depth/distance reasoning.

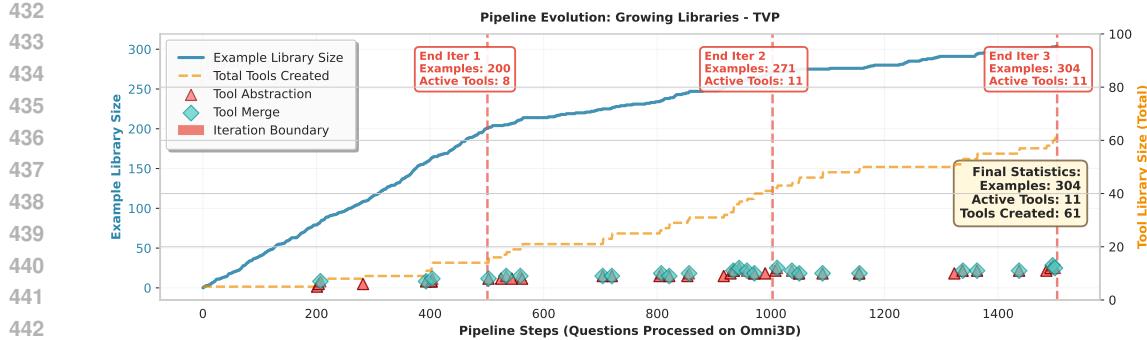


Figure 9: Evolution of TVP’s dual libraries through iterations on Omni3D-Bench. The Example Library grows steadily to 304 solutions while the Tool Library expands strategically—creating 61 tools total but maintaining 11 active abstractions with periodic merging.

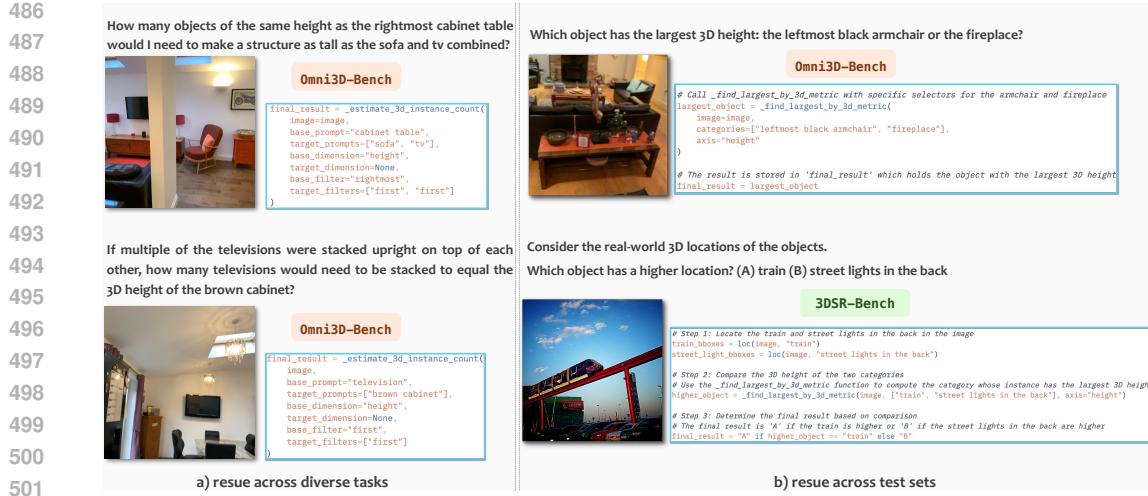
(b) reveals even stronger evidence: `find_largest_by_3d_metric` seamlessly transfers from Omni3D-Bench (comparing armchair and fireplace heights) to completely unseen 3DSR-Bench queries (comparing train and street light elevations). These tools succeed because they encode proven solution patterns validated through actual problem-solving, not hypothetical utilities.

Tool hierarchies emerge naturally through iterative refinement. Fig. 11 traces the evolution of a representative tool hierarchy. Starting from basic step-by-step solutions using primitive operations (Level-0), TVP first abstracts `estimate_3d_height_from_reference` to handle height estimation tasks (Level-1). As more examples accumulate, the tool maintenance mechanism identifies similarity with a width-estimation function, merging them into the more general `estimate_object_dimension_by_reference` (Level-2). Later iterations add filtering capabilities, creating an even more powerful abstraction. This hierarchical evolution mirrors how human programmers refactor code—starting with specific solutions, recognizing patterns, and progressively generalizing. The Tool Library maintenance mechanism (Fig. 3) ensures this evolution produces increasingly powerful tools while avoiding redundancy. Through periodic merging, functionally similar tools like `compute_3d_ratio` and `compute_3d_group_size_ratio` combine into unified abstractions that handle broader use cases. The quantitative impact is clear: programs using these evolved tools achieve both higher accuracy (+3.4% when switching to new tools, Fig. 4-panel b) and lower complexity (median cyclomatic complexity drops 66% from 3.0 to 1.0, Fig. 4-panel a). This demonstrates that transductive visual programming doesn’t just solve problems—it continuously improves its problem-solving capabilities through experience.

4 RELATED WORK

4.1 SPATIAL REASONING

Spatial reasoning requires understanding of precise real-world spatial relationships between 3D objects beyond the pixel space of images, which is challenging for monolithic VLMs (Kamath et al., 2023; Majumdar et al., 2024; Fu et al., 2024; Tong et al., 2024; Cai et al., 2025; Zhang et al., 2024). Even for recent VLMs built with specialized spatial finetuning (e.g., SpatialVLM Chen et al., 2024, SpatialRGPT Cheng et al., 2024, SpatialBot Cai et al., 2025), these models still struggle with more diverse 3D reasoning queries (Lee et al., 2025; Marsili et al., 2025) – similar to our findings revealed from experimenting on SpatialVLM and SpatialBot (§3.1). These limitations motivated visual programming approaches that decompose complex visual tasks into executable steps, where each step leverages specialized vision tools. VisProg (Gupta & Kembhavi, 2023) introduced a domain-specific language (DSL) for composing vision specialists. ViperGPT (Surís et al., 2023) directly generates Python code for calling APIs. Both systems rely on predefined static APIs, limiting their ability to handle queries beyond their initial toolkit. VADAR (Marsili et al., 2025) attempts a dynamic tool set by **proposing** potentially useful functions as new APIs, which are created through pure “**induction**”—based solely on question texts before solving any problems. This speculative approach leads to low utilization of the generated APIs in actual solutions (see Fig. 2). Our TVP belongs to the



502 Figure 10: Tool reuse across diverse tasks and benchmarks. (a) Transductively learned tools handle
 503 diverse problems within Omni3D-Bench. (b) The tool learned from Omni3D-Bench transfers to
 504 unseen benchmarks.

505
 506 visual programming family but takes a fundamentally different approach: rather than inductively
 507 proposing tools, it learns them “**transductively**” through experience—solving problems with basic
 508 tools first, then abstracting recurring solution patterns into new functions.

510 4.2 TOOL USE AND ABSTRACTION

511 Agentic systems calling various specialist tools have shown superior performance across a wide
 512 range of domains, including web navigation (Zheng et al., 2025; Wang et al., 2025), robotic controls
 513 (Liang et al., 2022), graphics generation (Hu et al., 2024; Sun et al., 2025) and game exploration
 514 (Wang et al., 2023). In 3D visual tasks, Yuan et al. (2024) proposes view-dependent and
 515 -independent modules in visual programs for zero-shot open-vocabulary grounding. Mi et al. (2025)
 516 introduces “code as spatial relation encoders” optimized through test suites before deployment. Be-
 517 yond applying specialized tools, recent works have studied the automatic creation of new tools that
 518 enable self-evolving agents with a dynamic toolbox (Cai et al., 2023; Wang et al., 2024b; Yuan
 519 et al., 2023; Qian et al., 2023). LILO (Grand et al., 2023) compresses programs into symbolic
 520 λ -expressions for abstracting tools. Alita (Qiu et al., 2025) produces specialized model context pro-
 521 tocols (MCP) connecting web-search with tool generation and execution. Skillweaver (Zheng et al.,
 522 2025) identifies novel skills from web tasks following a simple-to-complex curriculum. ASI (Wang
 523 et al., 2025) shares our insight that tool abstractions come from experience in concrete solutions, and
 524 proposes novel functions from concrete action trajectories in web environments. Our TVP evolves its
 525 toolbox through a unique dual-library architecture that simultaneously gathers experience and cre-
 526 ates tools. By grounding tool abstraction in actual problem-solving experience, TVP demonstrates
 527 how transductive library learning produces effective self-evolving visual programming agent. §B
 528 provides an extended comparison between TVP and prior work on tool discovery.

529 5 CONCLUSION

530 We introduce Transductive Visual Programming (TVP), a novel paradigm where agents learn to
 531 build tools from problem-solving experience, moving beyond static or speculatively-created tool
 532 sets. By abstracting reusable functions from successful solutions, TVP mirrors human tool learning
 533 process. Our approach sets a new state-of-the-art on challenging 3D spatial reasoning benchmark
 534 Omni3D-Bench. The learned tools also demonstrate strong zero-shot generalization to unseen spatial
 535 queries from other test sets (3DSRBench, SpatialSense, and VGBench), proving that TVP builds
 536 robust, reusable knowledge. Our work opens exciting directions: TVP’s dual-library architecture is
 537 task-agnostic, and demonstrates a viable path toward continually self-learning agents that build hi-
 538 erarchical, compositional skills.

540 REFERENCES
541

542 Tianle Cai, Xuezhi Wang, Tengyu Ma, Xinyun Chen, and Denny Zhou. Large language models as
543 tool makers. *arXiv preprint arXiv:2305.17126*, 2023.

544 Wenxiao Cai, Iaroslav Ponomarenko, Jianhao Yuan, Xiaoqi Li, Wankou Yang, Hao Dong, and
545 Bo Zhao. Spatialbot: Precise spatial understanding with vision language models. In *2025 IEEE*
546 *International Conference on Robotics and Automation (ICRA)*, pp. 9490–9498. IEEE, 2025.

547 Boyuan Chen, Zhuo Xu, Sean Kirmani, Brain Ichter, Dorsa Sadigh, Leonidas Guibas, and Fei Xia.
548 Spatialvlm: Endowing vision-language models with spatial reasoning capabilities. In *Proceedings*
549 *of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 14455–14465,
550 2024.

551 An-Chieh Cheng, Hongxu Yin, Yang Fu, Qiushan Guo, Ruihan Yang, Jan Kautz, Xiaolong Wang,
552 and Sifei Liu. Spatialrgpt: Grounded spatial reasoning in vision-language models. *Advances in*
553 *Neural Information Processing Systems*, 37:135062–135093, 2024.

554 Matt Deitke, Christopher Clark, Sangho Lee, Rohun Tripathi, Yue Yang, Jae Sung Park, Moham-
555 madreza Salehi, Niklas Muennighoff, Kyle Lo, Luca Soldaini, et al. Molmo and pixmo: Open
556 weights and open data for state-of-the-art vision-language models. In *Proceedings of the Com-*
557 *puter Vision and Pattern Recognition Conference*, pp. 91–104, 2025.

558 Xingyu Fu, Yushi Hu, Bangzheng Li, Yu Feng, Haoyu Wang, Xudong Lin, Dan Roth, Noah A
559 Smith, Wei-Chiu Ma, and Ranjay Krishna. Blink: Multimodal large language models can see but
560 not perceive. In *European Conference on Computer Vision*, pp. 148–166. Springer, 2024.

561 Gabriel Grand, Lionel Wong, Maddy Bowers, Theo X Olausson, Muxin Liu, Joshua B Tenenbaum,
562 and Jacob Andreas. Lilo: Learning interpretable libraries by compressing and documenting code.
563 *arXiv preprint arXiv:2310.19791*, 2023.

564 Tanmay Gupta and Aniruddha Kembhavi. Visual programming: Compositional visual reasoning
565 without training. In *Proceedings of the IEEE/CVF conference on computer vision and pattern*
566 *recognition*, pp. 14953–14962, 2023.

567 Ziniu Hu, Ahmet Iscen, Aashi Jain, Thomas Kipf, Yisong Yue, David A Ross, Cordelia Schmid, and
568 Alireza Fathi. Scenecraft: An llm agent for synthesizing 3d scenes as blender code. In *Forty-first*
569 *International Conference on Machine Learning*, 2024.

570 Binyuan Hui, Jian Yang, Zeyu Cui, Jiaxi Yang, Dayiheng Liu, Lei Zhang, Tianyu Liu, Jiajun Zhang,
571 Bowen Yu, Kai Dang, et al. Qwen2. 5-coder technical report. *arXiv preprint arXiv:2409.12186*,
572 2024.

573 Dongfu Jiang, Xuan He, Huaye Zeng, Con Wei, Max Ku, Qian Liu, and Wenhui Chen. Mantis:
574 Interleaved multi-image instruction tuning. *arXiv preprint arXiv:2405.01483*, 2024.

575 Danial Kamali and Parisa Kordjamshidi. Neptune: A neuro-pythonic framework for tunable com-
576 positional reasoning on vision-language. *arXiv preprint arXiv:2509.25757*, 2025.

577 Amita Kamath, Jack Hessel, and Kai-Wei Chang. What’s “up” with vision-language models? inves-
578 tigating their struggle with spatial reasoning. In *Proceedings of the 2023 Conference on Empirical*
579 *Methods in Natural Language Processing*, pp. 9161–9175, 2023.

580 Phillip Y Lee, Jihyeon Je, Chanho Park, Mikaela Angelina Uy, Leonidas Guibas, and Minhyuk
581 Sung. Perspective-aware reasoning in vision-language models via mental imagery simulation.
582 *arXiv preprint arXiv:2504.17207*, 2025.

583 Bo Li, Yuanhan Zhang, Dong Guo, Renrui Zhang, Feng Li, Hao Zhang, Kaichen Zhang, Yanwei Li,
584 Ziwei Liu, and Chunyuan Li. Llava-onevision: Easy visual task transfer, 2024. URL <https://arxiv.org/abs/2408.03326>.

585 Jacky Liang, Wenlong Huang, Fei Xia, Peng Xu, Karol Hausman, Brian Ichter, Pete Florence, and
586 Andy Zeng. Code as policies: Language model programs for embodied control. *arXiv preprint*
587 *arXiv:2209.07753*, 2022.

594 Shilong Liu, Zhaoyang Zeng, Tianhe Ren, Feng Li, Hao Zhang, Jie Yang, Qing Jiang, Chunyuan
 595 Li, Jianwei Yang, Hang Su, et al. Grounding dino: Marrying dino with grounded pre-training
 596 for open-set object detection. In *European conference on computer vision*, pp. 38–55. Springer,
 597 2024.

598 Wufei Ma, Haoyu Chen, Guofeng Zhang, Yu-Cheng Chou, Celso M de Melo, and Alan Yuille.
 599 3dsrbench: A comprehensive 3d spatial reasoning benchmark. *arXiv preprint arXiv:2412.07825*,
 600 2024.

602 Arjun Majumdar, Anurag Ajay, Xiaohan Zhang, Pranav Putta, Sriram Yenamandra, Mikael Henaff,
 603 Sneha Silwal, Paul Mcvay, Oleksandr Maksymets, Sergio Arnaud, Karmesh Yadav, Qiyang Li,
 604 Ben Newman, Mohit Sharma, Vincent Berges, Shiqi Zhang, Pulkit Agrawal, Yonatan Bisk, Dhruv
 605 Batra, Mrinal Kalakrishnan, Franziska Meier, Chris Paxton, Sasha Sax, and Aravind Rajeswaran.
 606 Openeqa: Embodied question answering in the era of foundation models. In *Conference on
 607 Computer Vision and Pattern Recognition (CVPR)*, 2024.

608 Damiano Marsili, Rohun Agrawal, Yisong Yue, and Georgia Gkioxari. Visual agentic ai for spatial
 609 reasoning with a dynamic api. In *Proceedings of the Computer Vision and Pattern Recognition
 610 Conference*, pp. 19446–19455, 2025.

612 T.J. McCabe. A complexity measure. *IEEE Transactions on Software Engineering*, SE-2(4):308–
 613 320, 1976. doi: 10.1109/TSE.1976.233837.

614 Boyu Mi, Hanqing Wang, Tai Wang, Yilun Chen, and Jiangmiao Pang. Evolving symbolic 3d visual
 615 grounder with weakly supervised reflection. *arXiv preprint arXiv:2502.01401*, 2025.

617 Luigi Piccinelli, Yung-Hsu Yang, Christos Sakaridis, Mattia Segu, Siyuan Li, Luc Van Gool, and
 618 Fisher Yu. Unidepth: Universal monocular metric depth estimation. In *Proceedings of the
 619 IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 10106–10116, 2024.

620 Cheng Qian, Chi Han, Yi R Fung, Yujia Qin, Zhiyuan Liu, and Heng Ji. Creator: Tool cre-
 621 ation for disentangling abstract and concrete reasoning of large language models. *arXiv preprint
 622 arXiv:2305.14318*, 2023.

624 Jiahao Qiu, Xuan Qi, Tongcheng Zhang, Xinzhe Juan, Jiacheng Guo, Yifu Lu, Yimin Wang, Zixin
 625 Yao, Qihan Ren, Xun Jiang, et al. Alita: Generalist agent enabling scalable agentic reasoning
 626 with minimal predefinition and maximal self-evolution. *arXiv preprint arXiv:2505.20286*, 2025.

627 Fan-Yun Sun, Shengguang Wu, Christian Jacobsen, Thomas Yim, Haoming Zou, Alex Zook,
 628 Shangru Li, Yu-Hsin Chou, Ethem Can, Xunlei Wu, et al. 3d-generalist: Self-improving vision-
 629 language-action models for crafting 3d worlds. *arXiv preprint arXiv:2507.06484*, 2025.

631 Dídac Surís, Sachit Menon, and Carl Vondrick. Viperpt: Visual inference via python execution
 632 for reasoning. In *Proceedings of the IEEE/CVF international conference on computer vision*, pp.
 633 11888–11898, 2023.

634 Peter Tong, Ellis Brown, Penghao Wu, Sanghyun Woo, Adithya Jairam Vedagiri IYER, Sai Charitha
 635 Akula, Shusheng Yang, Jihan Yang, Manoj Middepogu, Ziteng Wang, et al. Cambrian-1: A fully
 636 open, vision-centric exploration of multimodal llms. *Advances in Neural Information Processing
 637 Systems*, 37:87310–87356, 2024.

639 Guanzhi Wang, Yuqi Xie, Yunfan Jiang, Ajay Mandlekar, Chaowei Xiao, Yuke Zhu, Linxi Fan,
 640 and Anima Anandkumar. Voyager: An open-ended embodied agent with large language models.
 641 *arXiv preprint arXiv:2305.16291*, 2023.

642 Peng Wang, Shuai Bai, Sinan Tan, Shijie Wang, Zhihao Fan, Jinze Bai, Keqin Chen, Xuejing Liu,
 643 Jialin Wang, Wenbin Ge, Yang Fan, Kai Dang, Mengfei Du, Xuancheng Ren, Rui Men, Dayiheng
 644 Liu, Chang Zhou, Jingren Zhou, and Junyang Lin. Qwen2-vl: Enhancing vision-language model’s
 645 perception of the world at any resolution. *arXiv preprint arXiv:2409.12191*, 2024a.

646 Zhiruo Wang, Daniel Fried, and Graham Neubig. Trove: Inducing verifiable and efficient toolboxes
 647 for solving programmatic tasks. *arXiv preprint arXiv:2401.12869*, 2024b.

648 Zora Zhiruo Wang, Apurva Gandhi, Graham Neubig, and Daniel Fried. Inducing programmatic
 649 skills for agentic tasks. *arXiv preprint arXiv:2504.06821*, 2025.
 650

651 Haoning Wu, Xiao Huang, Yaohui Chen, Ya Zhang, Yanfeng Wang, and Weidi Xie. Spati-
 652 alscore: Towards unified evaluation for multimodal spatial understanding. *arXiv preprint*
 653 *arXiv:2505.17012*, 2025.

654 Shitao Xiao, Zheng Liu, Peitian Zhang, and Niklas Muennighoff. C-pack: Packaged resources to
 655 advance general chinese embedding, 2023.
 656

657 Jihan Yang, Shusheng Yang, Anjali W Gupta, Rilyn Han, Li Fei-Fei, and Saining Xie. Thinking in
 658 space: How multimodal large language models see, remember, and recall spaces. In *Proceedings*
 659 *of the Computer Vision and Pattern Recognition Conference*, pp. 10632–10643, 2025.

660 Kaiyu Yang, Olga Russakovsky, and Jia Deng. Spatialsense: An adversarially crowdsourced bench-
 661 mark for spatial relation recognition. In *Proceedings of the IEEE/CVF International Conference*
 662 *on Computer Vision*, pp. 2051–2060, 2019.

663 Lifan Yuan, Yangyi Chen, Xingyao Wang, Yi R Fung, Hao Peng, and Heng Ji. Craft: Customizing
 664 llms by creating and retrieving from specialized toolsets. *arXiv preprint arXiv:2309.17428*, 2023.
 665

666 Zhihao Yuan, Jinke Ren, Chun-Mei Feng, Hengshuang Zhao, Shuguang Cui, and Zhen Li. Vi-
 667 sual programming for zero-shot open-vocabulary 3d visual grounding. In *Proceedings of the*
 668 *IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 20623–20633, 2024.

669

670 Zheyuan Zhang, Fengyuan Hu, Jayjun Lee, Freda Shi, Parisa Kordjamshidi, Joyce Chai, and Ziqiao
 671 Ma. Do vision-language models represent space and how? evaluating spatial frame of reference
 672 under ambiguities. *arXiv preprint arXiv:2410.17385*, 2024.

673

674 Boyuan Zheng, Michael Y Fatemi, Xiaolong Jin, Zora Zhiruo Wang, Apurva Gandhi, Yueqi Song,
 675 Yu Gu, Jayanth Srinivasa, Gaowen Liu, Graham Neubig, et al. Skillweaver: Web agents can
 676 self-improve by discovering and honing skills. *arXiv preprint arXiv:2504.07079*, 2025.

677 Shuyan Zhou, Frank F Xu, Hao Zhu, Xuhui Zhou, Robert Lo, Abishek Sridhar, Xianyi Cheng,
 678 Tianyue Ou, Yonatan Bisk, Daniel Fried, et al. Webarena: A realistic web environment for build-
 679 ing autonomous agents. *arXiv preprint arXiv:2307.13854*, 2023.

680

681 A THE USE OF LARGE LANGUAGE MODELS (LLMs)

682 In this paper, we used LLMs only for grammar checking and light editing of the manuscript to polish
 683 the writing.

684 B FURTHER COMPARISON WITH RELATED TOOL-USE WORK

685 Complementing §4.2, here we discuss in more depth how TVP’s transductive approach fundamen-
 686 tally differs from representative methods in tool use and discovery.

687 **TVP vs. Skillweaver.** Skillweaver (Zheng et al., 2025) follows a **purely inductive** approach similar
 688 to VADAR: it speculatively proposes potentially useful functions before attempting any problem-
 689 solving experience, then synthesizes artificial test cases to validate these speculated functions. In
 690 contrast, TVP’s tool creation is grounded in actual problem-solving experience, as TVP first accu-
 691 mulates experience solving problems, then parameterizes this experience into new tools, guaran-
 692 teeing usefulness because each new tool encapsulates concrete, tested program solution patterns.
 693 Moreover, while Skillweaver **relies on a human-defined curriculum** with a predefined task order
 694 (procedural → navigational → information-seeking) tailored to the WebArena (Zhou et al., 2023)
 695 evaluation environment, TVP is **prior-free** and allows random ordering of datapoints, making it
 696 more generally applicable (see §D.1 for analysis of TVP’s resilience to randomness).

TVP vs. ASI. While ASI (Wang et al., 2025) shares our key insight of creating tools from concrete program solutions, TVP differs in three critical aspects. First, ASI abstracts every episode individually, so each new tool simply represents one single episode. TVP abstracts a cluster of similar queries’ solutions together, ensuring **each new tool generalizes over multiple example solutions** for better reusability. Second, ASI lacks library maintenance and cannot handle overlapping skills, leading to redundancy. TVP includes **explicit library maintenance** that merges similar skills, keeping the tool library clean and concise. Third, ASI extracts multiple tools from a single action trajectory, proposing multiple useful functions from one solution rather than abstracting the whole solution itself. TVP directly lifts an entire cluster of program solutions into a single abstract tool through transductive parameterization.

TVP and NePTune. NePTune (Kamali & Kordjamshidi, 2025) focuses on combining programmatic control flow with **symbolic logic operators** to enhance program expressiveness and execution. TVP’s contribution is orthogonal, focusing on **novel tool creation** to enable more expressive programs through abstraction.

C IMPLEMENTATION DETAILS

C.1 VADAR REPRODUCING CONFIGURATIONS

We directly utilized VADAR’s official code-base and adhered to the official hyperparameter settings throughout, including: random batches of 15 questions for API proposal, GroundingDINO-SwinT-OGC (Liu et al., 2024) for object detection and UniDepth-v2-ViT14 (Piccinelli et al., 2024) for depth estimation – the exact same tools used in our TVP implementation.

C.2 TVP CONFIGURATIONS

We run the TVP pipeline on the Omni3D-Bench (§3.1) with the following configurations:

For the main pipeline (Alg. 1), we process all $N = 501$ questions from the entire dataset over $T = 3$ iterations. During each iteration, we generate $m = 4$ program candidates per question and retrieve $k = 3$ similar examples from the example library \mathcal{E} using BGE-Large-En-v1.5 embeddings (Xiao et al., 2023) with a embedding similarity threshold $\tau_{\text{sim}} = 0.8$. Programs are accepted into the example library only if their quality score exceeds $\tau_q = 8.5$ on a 10-point scale. The tool abstraction process (Alg. 2) is triggered continuously after every $n_a = 1$ step (effectively at every step). We cluster examples using a similarity threshold of $\tau_{\text{sim}} = 0.8$ and require a minimum cluster size of $\tau_{\text{cluster}} = 4$ examples. Clusters with an abstraction potential score above $\tau_{\text{potential}} = 9.0$ are considered for tool creation. The validation process (Alg. 3) requires a minimum execution success rate of 100% and a correctness rate of at least 85% for divergent results. Both abstraction and program rewriting allow up to $R_{\text{max}} = 2$ and $R_{\text{rewrite}} = 2$ retry attempts respectively. Tool deduplication (Alg. 4) is also performed after every $n_d = 1$ step. Tools are considered duplicates when their similarity exceeds 0.95. The merge process allows up to $R_{\text{merge}} = 2$ retry attempts to create a unified tool that passes validation.

Throughout our experiments, we maintain a **uniform random seed** of 42 across all pipeline components, governing aspects such as datapoint order (discussed more in §D.1).

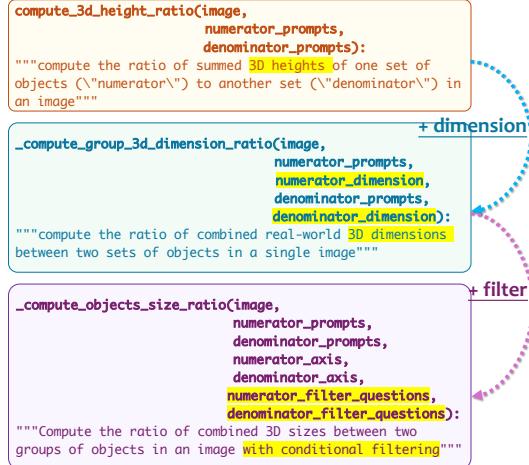


Figure 11: Hierarchical evolution of tool abstractions through transductive learning. From concrete step-by-step solutions (Level-0), TVP progressively abstracts more general and powerful tools through clustering (Level-1) and merging (Level-2), mirroring how human programmers build reusable function

In our main experiments (Tab. 1), we employ GPT-4o as the backbone program generator (LLM_{prog}), and as the VLM-based quality judge (LLM_{judge}) & correctness validator ($LLM_{correct}$). We use the reasoning model o4-mini for clustering ($LLM_{cluster}$), abstraction ($LLM_{abstract}$), deduplication (LLM_{dedup}), merging (LLM_{merge}), and program rewriting tasks. **In our ablations on the scaling behavior (Fig. 7), we switch to Qwen2.5-Coder-Instruct-7/14/32B (Hui et al., 2024) for the backbone program generation, and run TVP for $T = 1$ iteration. Results discussed in §3.1 demonstrate TVP’s robustness to the backbone LLM, as well as the clear scaling trend with model capacity.**

Unless required by specific reasoning models like o4-mini (temperature = 1.0), **LLM temperatures** are set to 0.0 for deterministic tasks (quality judgment and correctness validation), ensuring rigorous assessment; and 1.0 for more creative tasks (program generation, abstraction, and rewriting), increasing the likelihood of finding better solutions.

In Fig. 4 (panel a), we use McCabe’s Cyclomatic Complexity Number (CCN) (McCabe, 1976) as the program complexity measure, computed via the Lizard library ², following the practice in Yuan et al. (2023).

C.3 CRITERIA IN TVP’S JUDGE COMPONENTS

The program quality judge (§2.2) gates the admission to TVP’s Example Library through evaluation across five comprehensive dimensions (as shown in Prompt 1): (1) 3D spatial understanding, following Marsili et al. (2025)’s official implementation for 3D concepts and definitions; (2) answer correctness with visual verification against the provided image; (3) appropriate program tool usage; (4) code quality including readability and efficiency; and (5) robustness to edge cases. These dimensions align with the critical requirements of both spatial reasoning and programming. The reliability of our quality judge is empirically validated in Tab. 1, where **enabling only the Example-Library in TVP already outperforms all baselines**. This demonstrates accurate admission of high-quality solutions in our Example Library that provide strong in-context examples.

The criteria for **tool abstraction potential** can be found in Prompt 2, which analyzes a group of program solutions clustered via question embeddings (embedding similarity is the first step of clustering, refer to §2.3). The abstraction potential focuses on general code abstraction requirements: (1) common computational patterns; (2) logical flow; (3) generalization capability; and (4) parameterization potential. We allow this flexibility in tool abstraction to **enable more diverse exploration of higher-level tools**, while still ensuring new tools’ quality through the rigorous validation against all examples in the cluster before Tool Library admission (refer to §2.3).

C.4 COMPLEXITY RATING OF 3D SPATIAL REASONING QUESTIONS

In both our complexity-grouped evaluation illustrated in Figs. 5 and 6), and the curriculum-ordered TVP run discussed in §D.1, we use the **question complexity scores** rated via GPT-5 (high reasoning effort) with the prompt given in Prompt 3. The complexity rating evaluates questions along five axes, considering e.g., 3D understanding; single-/multi-object relationships and multi-step reasoning; cognitive and computational load. Based on these scores on the scale of 1.0–10.0, we partition questions into three complexity buckets: Easy (1–3), Medium (4–6), and Hard (7–10).

D ADDITIONAL EMPIRICAL ANALYSES AND DISCUSSION

D.1 TVP’S RESILIENCE TO RANDOM DATAPPOINT ORDERING

TVP is designed to operate on the fly without any dataset-specific priors, unlike previous methods such as Skillweaver (Zheng et al., 2025) that depends on human-defined curriculum for structured progression (see also discussion in §B). To validate our prior-free design choice, we compare the

²<https://github.com/terryyin/lizard>

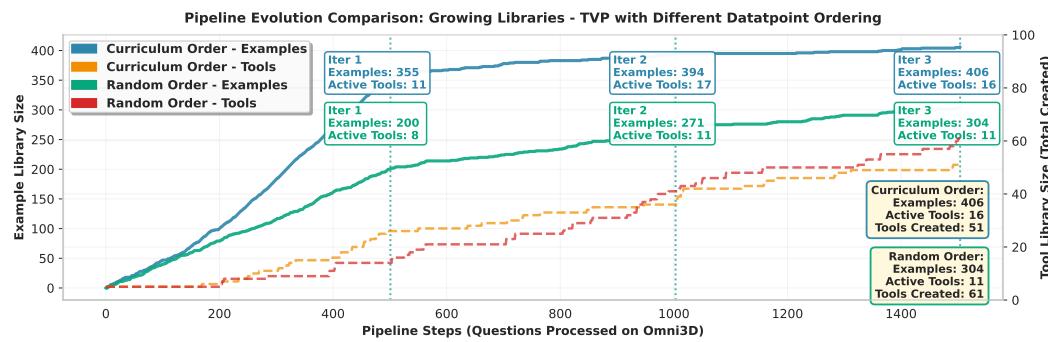


Figure 12: Pipeline evolution comparison between curriculum-ordered (as defined in §D.1) and random-ordered (as in our main experiments §3.1) datapoint processing. While curriculum ordering enables faster initial accumulation of examples and tools, random ordering ultimately creates more diverse tools through broader exploration of the problem space.

original TVP with **random ordering** (as used in our main experiments, §3.1) to **curriculum ordering** based on easy-to-hard progression through question complexity scores (details in §C.4).

Despite the intuition that starting experience with simpler problems, then gradually attempting harder problems seems a natural fit, we show in Fig. 13 that the overall performance is mostly on-par (both outperforming baselines), with the randomly-ordered TVP gradually getting better than the curriculum-ordered variant.

To understand this result, Fig. 12 reveals the evolution dynamics under both ordering strategies. The curriculum prior introduces two notable early-stage effects. First, it enables **earlier example accumulation**: datapoints with similar complexity clustered together facilitate more frequent example retrieval, resulting in 355 accumulated examples versus 200 with random ordering at the end of iteration 1. Second, it promotes **earlier tool creation**, as similar and simpler examples form eligible clusters sooner, yielding 11 active tools compared to 8 with random ordering after the first iteration.

However, **random ordering proves more beneficial for sustained library growth**. By encountering datapoints of varying complexities and patterns throughout processing, TVP explores a more diverse solution space. Although initial accumulation may be slower, this diversity enables continued progression as both libraries capture richer patterns. By completion, random ordering generates 61 total tools compared to 51 with curriculum ordering, demonstrating the value of diverse exploration over structured progression.

The above analysis speaks for TVP’s design choice of resilience to random datapoint ordering. First, it enables **truly on-the-fly** operation without requiring any dataset-specific priors. Second, the diverse exploration inherent to random ordering fosters greater variety in accumulated experiences, leading to **more comprehensive tool creation** that better covers the problem space. This mirrors human learning that benefits from exposure to varied challenges rather than strictly structured curricula.

D.2 COMPUTATIONAL COST AND EFFICIENCY

We detail computational requirements for running TVP, as our transductive system should be highly accessible for research and deployment.

Cost structure and runtime. TVP operates in two distinct stages with different cost profiles:

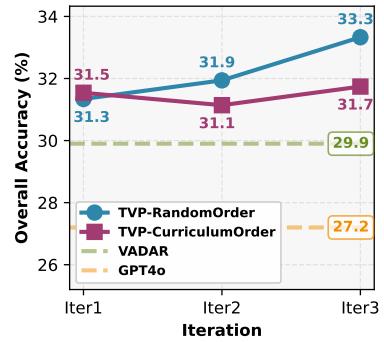


Figure 13: Performance comparison between TVP runs with random vs. curriculum-based datapoint ordering.

864 (1) *Building TVP’s dual libraries from scratch* involves processing the test set on the fly (Omni3D-
 865 Bench in §3.1), analogous to training a model. This stage requires approximately \$80 per iteration
 866 with our GPT-4o + GPT-4o-mini configuration, equivalent to about \$0.16 per question per iteration.
 867

868 (2) *Applying TVP’s built libraries to solve questions* (SpatialScore-Hard collection in §3.2) has
 869 minimal cost, usually equivalent to a single GPT-4o call per query.

870 GPUs are strictly optional in both stages. When used, the system requires under 4GB VRAM only
 871 to store the basic vision tools (GroundingDINO (Liu et al., 2024) and UniDepth (Piccinelli et al.,
 872 2024)), which can also run on CPUs.

873 The runtime for building TVP’s dual libraries (analogous to training) stands at approximately 7
 874 hours per iteration with our current implementation that executes programs sequentially.
 875

876 **Efficiency optimizations.** We implement several strategies to improve TVP’s cost-efficiency when
 877 building its dual libraries:

878 (1) *Early exit in tool validation*: Abstracted tools must achieve 100% execution success and 85%
 879 correctness on their validation cluster as per our current configurations (§C.2). For instance, for a
 880 cluster of 7 examples, validation exits early – thus **avoiding unnecessary computation** – when any
 881 one example fails execution (100% requirement not met); or when two fail the correctness check
 882 ($5/7 = 71\%$, drops below 85% pass rate)

883 (2) *Easy resumability*: We maintain comprehensive state checkpoints, supporting TVP’s pause and
 884 resume at any point.

885 (3) *Embedding bank*: Since question embeddings remain unchanged, we keep a persistent storage of
 886 embedding vectors that enables simple lookup when retrieving (§2.1) or clustering examples (§2.3).

888 (4) *Parallel program generation and quality judge*: We generate program candidates in parallel and
 889 batch the quality judging for all valid candidates to reduce run-time.

890 E PROMPT TEMPLATES

893 Prompt 1: Quality Judge

```

894 You are an expert judge evaluating the quality of a program that solves a 3D spatial reasoning
895 problem with tools (functions). Your task is to assess the program based on specific
896 criteria and assign a quality rating from 1.0 to 10.0.
897
898 ## TASK OVERVIEW
899 ### Question
900 question
901
902
903 ### Program to Evaluate
904 ````python
905 program_code
906 ````

907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
1014
1015
1016
1017
1018
1019
1020
1021
1022
1023
1024
1025
1026
1027
1028
1029
1030
1031
1032
1033
1034
1035
1036
1037
1038
1039
1040
1041
1042
1043
1044
1045
1046
1047
1048
1049
1050
1051
1052
1053
1054
1055
1056
1057
1058
1059
1060
1061
1062
1063
1064
1065
1066
1067
1068
1069
1070
1071
1072
1073
1074
1075
1076
1077
1078
1079
1080
1081
1082
1083
1084
1085
1086
1087
1088
1089
1090
1091
1092
1093
1094
1095
1096
1097
1098
1099
1100
1101
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112
1113
1114
1115
1116
1117
1118
1119
1120
1121
1122
1123
1124
1125
1126
1127
1128
1129
1130
1131
1132
1133
1134
1135
1136
1137
1138
1139
1140
1141
1142
1143
1144
1145
1146
1147
1148
1149
1150
1151
1152
1153
1154
1155
1156
1157
1158
1159
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
1171
1172
1173
1174
1175
1176
1177
1178
1179
1180
1181
1182
1183
1184
1185
1186
1187
1188
1189
1190
1191
1192
1193
1194
1195
1196
1197
1198
1199
1200
1201
1202
1203
1204
1205
1206
1207
1208
1209
1210
1211
1212
1213
1214
1215
1216
1217
1218
1219
1220
1221
1222
1223
1224
1225
1226
1227
1228
1229
1230
1231
1232
1233
1234
1235
1236
1237
1238
1239
1240
1241
1242
1243
1244
1245
1246
1247
1248
1249
1250
1251
1252
1253
1254
1255
1256
1257
1258
1259
1260
1261
1262
1263
1264
1265
1266
1267
1268
1269
1270
1271
1272
1273
1274
1275
1276
1277
1278
1279
1280
1281
1282
1283
1284
1285
1286
1287
1288
1289
1290
1291
1292
1293
1294
1295
1296
1297
1298
1299
1300
1301
1302
1303
1304
1305
1306
1307
1308
1309
1310
1311
1312
1313
1314
1315
1316
1317
1318
1319
1320
1321
1322
1323
1324
1325
1326
1327
1328
1329
1330
1331
1332
1333
1334
1335
1336
1337
1338
1339
1340
1341
1342
1343
1344
1345
1346
1347
1348
1349
1350
1351
1352
1353
1354
1355
1356
1357
1358
1359
1360
1361
1362
1363
1364
1365
1366
1367
1368
1369
1370
1371
1372
1373
1374
1375
1376
1377
1378
1379
1380
1381
1382
1383
1384
1385
1386
1387
1388
1389
1390
1391
1392
1393
1394
1395
1396
1397
1398
1399
1400
1401
1402
1403
1404
1405
1406
1407
1408
1409
1410
1411
1412
1413
1414
1415
1416
1417
1418
1419
1420
1421
1422
1423
1424
1425
1426
1427
1428
1429
1430
1431
1432
1433
1434
1435
1436
1437
1438
1439
1440
1441
1442
1443
1444
1445
1446
1447
1448
1449
1450
1451
1452
1453
1454
1455
1456
1457
1458
1459
1460
1461
1462
1463
1464
1465
1466
1467
1468
1469
1470
1471
1472
1473
1474
1475
1476
1477
1478
1479
1480
1481
1482
1483
1484
1485
1486
1487
1488
1489
1490
1491
1492
1493
1494
1495
1496
1497
1498
1499
1500
1501
1502
1503
1504
1505
1506
1507
1508
1509
1510
1511
1512
1513
1514
1515
1516
1517
1518
1519
1520
1521
1522
1523
1524
1525
1526
1527
1528
1529
1530
1531
1532
1533
1534
1535
1536
1537
1538
1539
1540
1541
1542
1543
1544
1545
1546
1547
1548
1549
1550
1551
1552
1553
1554
1555
1556
1557
1558
1559
1560
1561
1562
1563
1564
1565
1566
1567
1568
1569
1570
1571
1572
1573
1574
1575
1576
1577
1578
1579
1580
1581
1582
1583
1584
1585
1586
1587
1588
1589
1590
1591
1592
1593
1594
1595
1596
1597
1598
1599
1600
1601
1602
1603
1604
1605
1606
1607
1608
1609
1610
1611
1612
1613
1614
1615
1616
1617
1618
1619
1620
1621
1622
1623
1624
1625
1626
1627
1628
1629
1630
1631
1632
1633
1634
1635
1636
1637
1638
1639
1640
1641
1642
1643
1644
1645
1646
1647
1648
1649
1650
1651
1652
1653
1654
1655
1656
1657
1658
1659
1660
1661
1662
1663
1664
1665
1666
1667
1668
1669
1670
1671
1672
1673
1674
1675
1676
1677
1678
1679
1680
1681
1682
1683
1684
1685
1686
1687
1688
1689
1690
1691
1692
1693
1694
1695
1696
1697
1698
1699
1700
1701
1702
1703
1704
1705
1706
1707
1708
1709
1710
1711
1712
1713
1714
1715
1716
1717
1718
1719
1720
1721
1722
1723
1724
1725
1726
1727
1728
1729
1730
1731
1732
1733
1734
1735
1736
1737
1738
1739
1740
1741
1742
1743
1744
1745
1746
1747
1748
1749
1750
1751
1752
1753
1754
1755
1756
1757
1758
1759
1760
1761
1762
1763
1764
1765
1766
1767
1768
1769
1770
1771
1772
1773
1774
1775
1776
1777
1778
1779
1780
1781
1782
1783
1784
1785
1786
1787
1788
1789
1790
1791
1792
1793
1794
1795
1796
1797
1798
1799
1800
1801
1802
1803
1804
1805
1806
1807
1808
1809
1810
1811
1812
1813
1814
1815
1816
1817
1818
1819
1820
1821
1822
1823
1824
1825
1826
1827
1828
1829
1830
1831
1832
1833
1834
1835
1836
1837
1838
1839
1840
1841
1842
1843
1844
1845
1846
1847
1848
1849
1850
1851
1852
1853
1854
1855
1856
1857
1858
1859
1860
1861
1862
1863
1864
1865
1866
1867
1868
1869
1870
1871
1872
1873
1874
1875
1876
1877
1878
1879
1880
1881
1882
1883
1884
1885
1886
1887
1888
1889
1890
1891
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1910
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926
1927
1928
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
2033
2034
2035
2036
2037
2038
2039
2040
2041
2042
2043
2044
2045
2046
2047
2048
2049
2050
2051
2052
2053
2054
2055
2056
2057
2058
2059
2060
2061
2062
2063
2064
2065
2066
2067
2068
2069
2070
2071
2072
2073
2074
2075
2076
2077
2078
2079
2080
2081
2082
2083
2084
2085
2086
2087
2088
2089
2090
2091
2092
2093
2094
2095
2096
2097
2098
2099
2099
2100
2101
2102
2103
2104
2105
2106
2107
2108
2109
2109
2110
2111
2112
2113
2114
2115
2116
2117
2118
2119
2119
2120
2121
2122
2123
2124
2125
2126
2127
2128
2129
2129
2130
2131
2132
2133
2134
2135
2136
2137
2138
2139
2139
2140
2141
2142
2143
2144
2145
2146
2147
2148
2149
2149
2150
2151
2152
2153
2154
2155
2156
2157
2158
2159
2159
2160
2161
2162
2163
2164
2165
2166
2167
2168
2169
2169
2170
2171
2172
2173
2174
2175
2176
2177
2178
2178
2179
2180
2181
2182
2183
2184
2185
2186
2187
2187
2188
2189
2189
2190
2191
2192
2193
2194
2195
2196
2197
2197
2198
2199
2199
2200
2201
2202
2203
2204
2205
2205
2206
2207
2208
2209
2209
2210
2211
2212
2213
2214
2215
2216
2216
2217
2218
2219
2219
2220
2221
2222
2223
2224
2225
2225
2226
2227
2228
2229
2229
2230
2231
2232
2233
2234
2234
2235
2236
2237
2237
2238
2239
2239
2240
2241
2242
2243
2243
2244
2245
2245
2246
2247
2247
2248
2249
2249
2250
2251
2252
2252
2253
2254
2254
2255
2256
2256
2257
2258
2258
2259
2259
2260
2261
2261
2262
2263
2263
2264
2265
2265
2266
2267
2267
2268
2269
2269
2270
2271
2271
2272
2273
2273
2274
2275
2275
2276
2277
2277
2278
2279
2279
2280
2281
2281
2282
2283
2283
2284
2285
2285
2286
2287
2287
2288
2289
2289
2290
2291
2291
2292
2293
2293
2294
2295
2295
2296
2297
2297
2298
2299
2299
2300
2301
2301
2302
2303
2303
2304
2305
2305
2306
2307
2307
2308
2309
2309
2310
2311
2311
2312
2313
2313
2314
2315
2315
2316
2317
2317
2318
2319
2319
2320
2321
2321
2322
2323
2323
2324
2325
2325
2326
2327
2327
2328
2329
2329
2330
2331
2331
2332
2333
2333
2334
2335
2335
2336
2337
2337
2338
2339
2339
2340
2341
2341
2342
2343
2343
2344
2345
2345
2346
2347
2347
2348
2349
2349
2350
2351
2351
2352
2353
2353
2354
2355
2355
2356
2357
2357
2358
2359
2359
2360
2361
2361
2362
2363
2363
2364
2365
2365
2366
2367
2367
2368
2369
2369
2370
2371
2371
2372
2373
2373
2374
2375
2375
2376
2377
2377
2378
2379
2379
2380
2381
2381
2382
2383
2383
2384
2385
2385
2386
2387
2387
2388
2389
2389
2390
2391
2391
2392
2393
2393
2394
2395
2395
2396
2397
2397
2398
2399
2399
2400
2401
2401
2402
2403
2403
2404
2405
2405
2406
2407
2407
2408
2409
2409
2410
2411
2411
2412
2413
2413
2414
2415
2415
2416
2417
2417
2418
2419
2419
2420
2421
2421
2422
2423
2423
2424
2425
2425
2426
2427
2427
2428
2429
2429
2430
2431
2431
2432
2433
2433
2434
2435
2435
2436
2437
2437
2438
2439
2439
2440
2441
2441
2442
2443
2443
2444
2445
2445
2446
2447
2447
2448
2449
2449
2450
2451
2451
2452
2453
2453
2454
2455
2455
2456
2457
2457
2458
2459
2459
2460
2461
2461
2462
2463
2463
2464
2465
2465
2466
2467
2467
2468
2469
2469
2470
2471
2471
2472
2473
2473
2474
2475
2475
2476
2477
2477
2478
2479
2479
2480
2481
2481
2482
2483
2483
2484
2485
2485
2486
2487
2487
2488
2489
2489
2490
2491
2491
2492
2493
2493
2494
2495
2495
2496
2497
2497
2498
2499
2499
2500
2501
2501
2502
2503
2503
2504
2505
2505
2506
2507
2507
2508
2509
2509
2510
2511
2511
2512
2513
2513
2514
2515
2515
2516
2517
2517
2518
2519
2519
2520
2521
2521
2522
2523
2523
2524
2525
2525
2526
2527
2527
2528
2529
2529
2530
2531
2531
2532
2533
2533
2534
2535
2535
2536
2537
2537
2538
2539
2539
2540
2541
2541
2542
2543
2543
2544
2545
2545
2546
2547
2547
2548
2549
2549
2550
2551
2551
2552
2553
2553
2554
2555
2555
2556
2557
2557
2558
2559
2559
2560
2561
2561
2562
2563
2563
2564
2565
2565
2566
2567
2567
2568
2569
2569
2570
2571
2571
2572
2573
2573
2574
2575
2575
2576
2577
2577
2578
2579
2579
2580
2581
2581
2582
2583
2583
2584
2585
2585
2586
2587
2587
2588
2589
2589
2590
2591
2591
2592
2593
2593
2594
2595
2595
2596
2597
2597
2598
2599
2599
2600
2601
2601
2602
2603
2603
2604
2605
2605
2606
2607
2607
2608
2609
2609
2610
2611
2611
2612
2613
2613
2614
2615
2615
2616
2617
2617
2618
2619
2619
2620
2621
2621
2622
2623
2623
2624
2625
2625
2626
2627
2627
2628
2629
2629
2630
2631
2631
2632
2633
2633
2634
2635
2635
2636
2637
2637
2638
2639
2639
2640
2641
2641
2642
2643
2643
2644
2645
2645
2646
2647
2647
2648
2649
2649
2650
2651
2651
2652
2653
2653
2654
2655
2655
2656
2657
2657
2658
2659
2659
2660
2661
2661
2662
2663
2663
2664
2665
2665
2666
2667
2667
2668
2669
2669
2670
2671
2671
2672
2673
2673
2674
2675
2675
2676
2677
2677
2678
2679
2679
2680
2681
2681
2682
2683
2683
2684
2685
2685
2686
2687
2687
2688
2689
2689
2690
2691
2691
2692
2693
2693
2694
2695
2695
2696
2697
2697
2698
2699
2699
2700
2701
2701
2702
2703
2703
2704
2705
2705
2706
```

```

918
919     - Expected intermediate calculations
920     - Appropriate object filtering
921     - Correct depth-based 3D conversions
922     - Mismatched bounding boxes or coordinates
923     - Unexpected intermediate calculation results
924     - Objects that should have been filtered but weren't
925
926     #### 3D Spatial Concepts & Definitions
927     **Core Definitions:**
928     - **Coordinate system:** (width, height, length) = (x, y, z) axis
929     - **Depth:** Distance from camera (smaller depth = closer to camera)
930     - **2D measurements:** Size/distance in pixel space (image coordinates)
931     - **3D measurements:** Size/distance in real world
932     - **3D size formula:** '3D size = 2D size * depth'
933     - **2D distance formula:** Euclidean distance between object center coordinates:  $\sqrt{(x1-x2)^2 + (y1-y2)^2}$ 
934     - **3D distance formula:**  $3D\_distance = \sqrt{(2D\_distance)^2 + (depth1 - depth2)^2}$ 
935     - **Distance to camera:** Simply the object's depth value
936
937     **Key Considerations:**
938     - 2D sizes are in pixel space. To convert to 3D size, multiply by depth
939     - Objects with same 2D dimensions but different depths have different 3D sizes
940     - 3D distance requires the Pythagorean formula combining 2D distance and depth difference - as
941     - defined above
942     - Center coordinates should determine "leftmost"/"rightmost"
943     - The 'loc()' function should not handle compound descriptions - must locate base objects then
944     - filter for the desired condition
945     - All objects satisfying a condition must be checked, not just the first
946     - Multiple objects with same property values require proper tie-breaking
947     - Hypothetical object counts (e.g., "if a table has X legs") require counting actual objects
948     - in image
949
950     ## RATING CRITERIA (1.0 - 10.0 Scale)
951     ### 1. **3D Spatial Understanding**
952     - Properly converts between 2D and 3D measurements
953     - Correctly handles 3D size/distance calculation
954     - Correctly uses center coordinates for distance calculations and leftmost/rightmost
955     - determinations
956     - Interprets spatial relationships correctly (e.g., "largest" means 3D, not 2D)
957     - Answer is visually verifiable and reasonable
958
959     ### 2. **Correctness and Visual Verification**
960     - Solves the problem correctly based on the actual image
961     - Aligns with visual evidence from the image
962     - Intermediate results are consistent with visible scene
963     - Spatial relationships match visual reality
964
965     ### 3. **Tool Usage Efficiency**
966     - Uses appropriate tools for the task
967     - Leverages higher-level "learned" tools when suitable
968     - Avoids reimplementing existing functionality
969     - Note: Basic tools are acceptable when no higher-level tools fit
970
971     ### 4. **Code Quality**
972     - Well-structured with clear variable names
973     - Follows tool usage patterns correctly
974     - Efficient without unnecessary operations
975     - Includes helpful comments
976
977     ### 5. **Robustness and Edge Cases**
978     - Properly filters located objects for properties rather than using complex 'loc()' queries
979     - Handles multiple objects with same property (proper tie-breaking)
980     - Manages empty lists and None values appropriately
981     - Manages container relationships (e.g., "in", "on") properly
982     - Includes appropriate error checking
983
984     ## REQUIRED OUTPUT FORMAT
985     You MUST provide your response in exactly this format:
986
987     <rating>NUMBER</rating>
988     <reasoning>
989     [Detailed explanation covering:
990     - How visual evidence supports/contradicts the program's logic
991     - Specific strengths and weaknesses identified
992     - Analysis of 3D spatial reasoning approach
993     - Evaluation of intermediate execution results
994     - Missed opportunities to use available tools
995     - Overall assessment based on all criteria]
996     </reasoning>
997     Where NUMBER is a decimal between 1.0 and 10.0.

```

```

972
973 ---  

974 ## APPENDIX: Available Tools Reference  

975 The program had access to tool_counts tools total: num_basic_tools basic tools and  

976 num_created_tools learned tools.  

977 ### Basic Tools (Level 0)  

978 tool_signature, tool_docstring  

979 ### Learned Tools (Level 1+)  

980 tool_signature, tool_docstring
981
982
983
```

Prompt 2: Abstraction Potential Analysis

```

984 You are an expert at analyzing visual reasoning programs to identify common patterns that  

985 could be abstracted into reusable functions.  

986 ## Your Task  

987 Analyze num_cluster_examples visual reasoning examples to:  

988 1. Identify common computational patterns across examples  

989 2. Group them into clusters based on shared functionality  

990 3. Rate each cluster's abstraction potential (0-10 scale)  

991 ## Examples to Analyze  

992 examples (question, program solution)  

993 ## Clustering Criteria  

994 Identify clusters based on:  

995 1. **Common computational patterns** - e.g., finding largest/smallest, counting with  

   conditions  

996 2. **Similar operations sequence** - e.g., locate -> filter -> compute -> compare  

997 3. **Shared logic structure** - e.g., iteration patterns, comparison logic  

998 4. **Abstractable functionality** - can be parameterized into a reusable function  

999 ## Evaluation Requirements  

1000 ### For Each Cluster Provide:  

1001 - **Example IDs** that belong to it  

1002 - **Common pattern** explanation  

1003 - **Parameters** that vary between examples  

1004 - **Abstraction potential rating** (0-10) based on:  

   * How well the pattern generalizes  

   * Parameter variability coverage  

   * Clarity of the abstraction  

   * Reusability across similar tasks  

1005 - **Reasoning** for the rating  

1006 ## Critical Constraints  

1007 - **Each example must belong to exactly ONE cluster or be marked as unclustered**  

1008 - Focus on computational patterns, not surface similarities  

1009 - Only create clusters with strong shared patterns  

1010 ## Response Format  

1011 Provide your analysis using this exact format. Include as many cluster blocks as needed,  

   followed by an optional unclustered block:  

1012 ``  

1013 <cluster>  

1014 <example_ids>[comma-separated list of example IDs]</example_ids>  

1015 <pattern>[Description of the common computational pattern]</pattern>  

1016 <parameters>[List of parameters that vary between examples]</parameters>  

1017 <abstraction_potential>[Rating from 0-10]</abstraction_potential>  

1018 <reasoning>[Explanation for the rating and how the pattern could be abstracted]</reasoning>  

1019 </cluster>  

1020 [Additional <cluster> blocks as needed...]  

1021 <unclustered>  

1022 <example_ids>[comma-separated list of example IDs that don't fit clusters]</example_ids>  

1023 <reasoning>[Explanation of why these examples don't cluster well]</reasoning>  

1024 </unclustered>  

1025 ``  

1026 **Remember:** Every example ID must appear in exactly ONE cluster or in the unclustered group.

```

Prompt 3: Question Complexity Rating

```

1026
1027 You are an expert in evaluating the complexity of 3D spatial reasoning questions. Your task is
1028 to assign a complexity score (1.0 - 10.0 scale) to a single question based on its
1029 inherent spatial reasoning difficulty.
1030
1031 ## QUESTION TO EVALUATE
1032
1033 **Question:** question
1034
1035 **Answer Type:** answer type: float/integer/multiple-choice/etc.
1036
1037 ## EVALUATION FRAMEWORK
1038
1039 ### 1. **3D Spatial Reasoning Requirements**
1040 - Does the question require understanding of 2D (pixel/image space) vs 3D (real-world)
1041 measurements?
1042 - Does it involve depth understanding and distance-from-camera concepts?
1043 - Does it require 3D size calculations or understanding that same 2D size at different depths
1044 means different 3D sizes?
1045 - Does it involve 3D distance calculations (combining 2D distance and depth differences)?
1046 - Does it require converting between measurement spaces?
1047
1048 ### 2. **Spatial Relationship Complexity**
1049 - How many objects are involved in the spatial reasoning?
1050 - Types of relationships:
1051   - Simple property identification (color, material, count)
1052   - Spatial relationships (distance, size comparison, relative position)
1053   - Complex spatial relationships (e.g., "to the right of X and behind Y")
1054 - Does it require multi-step reasoning with intermediate conclusions?
1055 - Comparative judgments vs. absolute measurements
1056
1057 ### 3. **Cognitive Load and Constraints**
1058 - Number of constraints or conditions that must be simultaneously satisfied
1059 - Need to identify and distinguish between multiple candidate objects
1060 - Hypothetical or conditional reasoning ("if X is Y meters, then...")
1061 - Handling of multiple objects with potentially ambiguous descriptions
1062 - Container relationships (objects "in" or "on" other objects)
1063
1064 ### 4. **Calculation and Quantitative Complexity**
1065 - Simple identification or counting vs. numerical calculations
1066 - Ratio, proportion, or percentage calculations
1067 - Multiple measurement comparisons
1068 - Distance or size computations requiring formulas
1069 - Precision requirements
1070
1071 ### 5. **Answer Type Indicators**
1072 - **yes/no (binary):** Often simpler verification tasks but can be complex depending on what's
1073   being verified
1074 - **multiple choice (str with options):** Requires discrimination among bounded options
1075 - **numerical (float/int):** Often requires precise calculations and measurements
1076 - **open string:** May require identification and categorization
1077
1078 ## COMPLEXITY SCORING GUIDELINES
1079
1080 Consider the full spectrum of complexity:
1081
1082 **Lower end:** Simple, direct questions requiring minimal spatial reasoning
1083 - Single object property identification
1084 - Basic counting
1085 - Simple yes/no verification with clear criteria
1086
1087 **Middle range:** Moderate spatial reasoning and calculation
1088 - Size or distance comparisons between pairs of objects
1089 - Simple ratio calculations
1090 - Object identification with multiple constraints
1091 - Basic 2D-to-3D conversions
1092
1093 **Higher end:** Complex multi-step spatial reasoning
1094 - Multiple object comparisons with numerous constraints
1095 - Complex calculations involving combined measurements
1096 - Hypothetical reasoning with conditional calculations
1097 - Spatial relationships involving many objects with interdependencies
1098 - Ratios of combined or derived quantities
1099
1100 Assign a score on the scale of 1.0 - 10.0 based on the question's position in this complexity
1101 spectrum. Consider ALL evaluation dimensions together.
1102
1103 ## REQUIRED OUTPUT FORMAT
1104
1105 Provide your response in EXACTLY this format:

```

```

1080 <score>X.X</score>
1081 <reasoning>
1082 [Detailed explanation covering:
1083 - Which evaluation dimensions contribute most to complexity
1084 - Specific aspects that increase or decrease difficulty
1085 - Why this score is appropriate
1086 - Key spatial reasoning challenges in the question]
1087 </reasoning>
1088 The score should be a decimal number between 1.0 and 10.0. Use your judgment to place the
1089 question appropriately on the complexity spectrum.
1090
1091
1092
1093
1094
1095
1096
1097
1098
1099
1100
1101
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112
1113
1114
1115
1116
1117
1118
1119
1120
1121
1122
1123
1124
1125
1126
1127
1128
1129
1130
1131
1132
1133

```

F COMPLETE TVP ALGORITHM

1134
1135
1136
1137

Algorithm 1 Transductive Visual Programming (TVP) Pipeline

1138
1139
1140 **Input:** Dataset $\mathcal{D} = \{(I_i, q_i)\}_{i=1}^N$ (images, questions)
 1141 1: **Initialize:** Example Library $\mathcal{E} \leftarrow \emptyset$, Tool Library $\mathcal{T} \leftarrow \{\text{predefined tools}\}$
 1142 2: **Parameters:** quality threshold τ_q , abstraction interval n_a , deduplication interval n_d
 1143 3: **for** iteration $t = 1$ to T **do**
 1144 4: **for** each question $q_i \in \mathcal{D}$ **do**
 1145 5: # Step 1: Retrieve similar examples
 1146 6: $\mathcal{E}_{\text{sim}} \leftarrow \text{RetrieveSimilar}(\mathcal{E}, q_i, k = 3)$ ▷ Exclude q_i itself
 1147 7: # Step 2: Generate program candidates
 1148 8: $\mathcal{C} \leftarrow \emptyset$
 1149 9: **for** $j = 1$ to m **do** ▷ m candidates per question
 1150 10: $p_j \leftarrow \text{LLM}_{\text{prog}}(q_i, \mathcal{E}_{\text{sim}}, \mathcal{T})$ ▷ In-context learning
 1151 11: $\mathcal{C} \leftarrow \mathcal{C} \cup \{p_j\}$
 1152 12: **end for**
 1153 13: # Step 3: Execute and filter
 1154 14: $\mathcal{C}_{\text{succ}} \leftarrow \emptyset$
 1155 15: **for** each $p_j \in \mathcal{C}$ **do**
 1156 16: $\text{result}_j, \text{namespace}_j \leftarrow \text{Execute}(p_j, I_i, \mathcal{T})$
 1157 17: **if** $\text{result}_j \neq \text{None} \wedge \neg \text{error}$ **then**
 1158 18: $\mathcal{C}_{\text{succ}} \leftarrow \mathcal{C}_{\text{succ}} \cup \{(p_j, \text{result}_j, \text{namespace}_j)\}$
 1159 19: **end if**
 1160 20: **end for**
 1161 21: # Step 4: Judge quality with vision model
 1162 22: **for** each $(p_j, \text{result}_j, \text{namespace}_j) \in \mathcal{C}_{\text{succ}}$ **do** ▷ 1-10 scale
 1163 23: $\text{quality}_j \leftarrow \text{LLM}_{\text{judge}}(q_i, p_j, \text{namespace}_j, I_i)$
 1164 24: **end for**
 1165 25: # Step 5: Select best and update library
 1166 26: $p^*, \text{quality}^*, \text{namespace}^* \leftarrow \arg \max_j \text{quality}_j$
 1167 27: **if** $\text{quality}^* \geq \tau_q$ **then**
 1168 28: $e \leftarrow \text{Example}(q_i, p^*, \text{quality}^*, \text{namespace}^*)$
 1169 29: **if** $\exists e' \in \mathcal{E}$ with $e'.q = q_i$ **then** ▷ Update existing
 1170 30: **if** $\text{quality}^* > e'.\text{quality}$ or different tools used **then**
 1171 31: $\mathcal{E} \leftarrow (\mathcal{E} \setminus \{e'\}) \cup \{e\}$
 1172 32: **end if**
 1173 33: **else**
 1174 34: $\mathcal{E} \leftarrow \mathcal{E} \cup \{e\}$ ▷ Add new
 1175 35: **end if**
 1176 36: **end if**
 1177 37: # Step 6: Abstraction interval
 1178 38: **if** $|\mathcal{E}| \bmod n_a = 0$ **then**
 1179 39: $\mathcal{T} \leftarrow \text{AbstractTools}(\mathcal{E}, \mathcal{T})$ ▷ See Algorithm 2
 1180 40: **end if**
 1181 41: # Step 7: Deduplication interval
 1182 42: **if** $|\mathcal{E}| \bmod n_d = 0$ **then**
 1183 43: $\mathcal{T} \leftarrow \text{DeduplicateTools}(\mathcal{T})$ ▷ See Algorithm 4
 1184 44: **end if**
 1185 45: **end for**
 1186 46: **end for**
 1187 47: **return** \mathcal{E}, \mathcal{T}

1184
1185
1186
1187

1188
 1189
 1190
 1191
 1192
 1193
 1194
 1195
 1196
 1197

Algorithm 2 AbstractTools - Tool Abstraction from Example Clusters

Input: Example Library \mathcal{E} , Tool Library \mathcal{T}

1: **Parameters:** similarity threshold $\tau_{\text{sim}} = 0.8$, cluster size threshold $\tau_{\text{cluster}} = 4$, potential threshold $\tau_{\text{potential}} = 9.0$

2: # Step 1: Filter eligible examples

3: $\mathcal{E}_{\text{eligible}} \leftarrow \{e \in \mathcal{E} : e.\text{status} \neq \text{"abstracted"}\}$

4: # Step 2: Cluster by similarity

5: $\mathcal{G} \leftarrow \text{ClusterBySimilarity}(\mathcal{E}_{\text{eligible}}, \tau_{\text{sim}})$

6: **for** each cluster $G \in \mathcal{G}$ with $|G| \geq \tau_{\text{cluster}}$ **do**

7: # Step 3: Analyze cluster for patterns

8: pattern, potential $\leftarrow \text{LLM}_{\text{cluster}}(G)$

9: **if** potential $\geq \tau_{\text{potential}}$ **then** ▷ Abstraction potential threshold

10: # Step 4: Create tool with retry

11: **for** retry = 1 to R_{max} **do**

12: **if** retry = 1 **then**

13: $t \leftarrow \text{LLM}_{\text{abstract}}(G, \text{pattern}, \mathcal{T})$

14: **else**

15: $t \leftarrow \text{LLM}_{\text{abstract}}(G, \text{pattern}, \mathcal{T}, \text{feedback})$

16: **end if**

17: # Step 5: Validate tool

18: val $\leftarrow \text{ValidateTool}(t, G, \mathcal{T})$ ▷ See Algorithm 3

19: **if** val.passed **then**

20: $\mathcal{T} \leftarrow \mathcal{T} \cup \{t\}$

21: # Update examples with new tool

22: **for** each $e \in G$ with successful rewrite **do**

23: $e.\text{program} \leftarrow \text{val.rewritten}[e]$

24: $e.\text{status} \leftarrow \text{"abstracted"}$

25: $e.\text{tools_used} \leftarrow e.\text{tools_used} \cup \{t\}$

26: **end for**

27: **break**

28: **else**

29: feedback $\leftarrow \text{val.errors}$ ▷ For retry

30: **end if**

31: **end for**

32: **end if**

33: **end for**

34: **return** \mathcal{T}

1233
 1234
 1235
 1236
 1237
 1238
 1239
 1240
 1241

1242
 1243
 1244
 1245
 1246
 1247
 1248
 1249
 1250
 1251
 1252

Algorithm 3 ValidateTool - Two-Stage Tool Validation

1253

Input: Tool t , Examples G , Tool Library \mathcal{T}

Output: Validation result with rewritten programs

1254
 1255
 1256
 1257
 1258
 1259
 1260
 1261
 1262
 1263
 1264
 1265
 1266
 1267
 1268
 1269
 1270
 1271
 1272
 1273
 1274
 1275
 1276
 1277
 1278
 1279
 1280
 1281
 1282
 1283
 1284
 1285
 1286
 1287
 1288
 1289
 1290
 1291
 1292
 1293
 1294
 1295

1: # Stage 1: Execution validation
 2: successes $\leftarrow 0$, rewrites $\leftarrow \{\}$
 3: **for** each example $e \in G$ **do**
 4: **for** retry = 1 to R_{rewrite} **do**
 5: $p' \leftarrow \text{RewriteProgram}(e.\text{program}, t)$
 6: result', namespace' $\leftarrow \text{Execute}(p', e.\text{image}, \mathcal{T} \cup \{t\})$
 7: **if** result' $\neq \text{None} \wedge \neg \text{error}$ **then**
 8: successes \leftarrow successes + 1
 9: rewrites[e] $\leftarrow (p', \text{result}', \text{namespace}')$
 10: **break**
 11: **end if**
 12: **end for**
 13: **if** successes/| G | < 1.0 **then** ▷ Early exit
 14: **return** {passed : False, errors : execution_failures}
 15: **end if**
 16: **end for**
 17: # Stage 2: Correctness validation for divergent results
 18: correct $\leftarrow 0$, divergent $\leftarrow 0$
 19: **for** each $e \in G$ with successful rewrite **do**
 20: **if** rewrites[e].result $\neq e.\text{result}$ **then**
 21: divergent \leftarrow divergent + 1
 22: verdict $\leftarrow \text{LLM}_{\text{correct}}(e, \text{rewrites}[e], e.\text{image})$
 23: **if** verdict = "CORRECT" **then**
 24: correct \leftarrow correct + 1
 25: **end if**
 26: **end if**
 27: **end for**
 28: overall_correct $\leftarrow (|G| - \text{divergent} + \text{correct})/|G|$
 29: **if** overall_correct ≥ 0.85 **then**
 30: **return** {passed : True, rewrites : rewrites}
 31: **else**
 32: **return** {passed : False, errors : correctness_failures}
 33: **end if**

1296
 1297
 1298
 1299
 1300
 1301
 1302
 1303
 1304
 1305
 1306
 1307

Algorithm 4 DeduplicateTools - Merge Similar Tools

1308
Input: Tool Library \mathcal{T}
 1309 1: # Filter eligible tools
 1310 2: $\mathcal{T}_{\text{eligible}} \leftarrow \{t \in \mathcal{T} : t.\text{level} > 0 \wedge \neg t.\text{deprecated}\}$
 1311 3: # Find duplicate groups
 1312 4: $\mathcal{M} \leftarrow \text{LLM}_{\text{dedup}}(\mathcal{T}_{\text{eligible}})$ ▷ Groups with similarity ≥ 0.95
 1313 5: **for** each merge group $M \in \mathcal{M}$ **do**
 1314 6: # Get examples using these tools
 1315 7: $\mathcal{E}_M \leftarrow \{e \in \mathcal{E} : \exists t \in M, t \in e.\text{tools_used}\}$
 1316 8: **for** retry = 1 to R_{merge} **do**
 1317 9: **if** retry = 1 **then**
 1318 10: $t_{\text{merged}} \leftarrow \text{LLM}_{\text{merge}}(M, \text{strategy})$
 1319 11: **else**
 1320 12: $t_{\text{merged}} \leftarrow \text{LLM}_{\text{merge}}(M, \text{strategy}, \text{feedback})$
 1321 13: **end if**
 1322 14: val $\leftarrow \text{ValidateTool}(t_{\text{merged}}, \mathcal{E}_M, \mathcal{T})$
 1323 15: **if** val.passed **then**
 1324 16: $\mathcal{T} \leftarrow \mathcal{T} \cup \{t_{\text{merged}}\}$
 1325 17: **for** each $t \in M$ **do**
 1326 18: $t.\text{deprecated} \leftarrow \text{True}$
 1327 19: $t.\text{reason} \leftarrow \text{"Merged into } t_{\text{merged}}\text{"}$
 1328 20: **end for**
 1329 21: # Update examples
 1330 22: **for** each $e \in \mathcal{E}_M$ with successful rewrite **do**
 1331 23: Update e with merged tool
 1332 24: **end for**
 1333 25: **break**
 1334 26: **else**
 1335 27: feedback $\leftarrow \text{val.errors}$
 1336 28: **end if**
 1337 29: **end for**
 1338 30: **end for**
 1339 31: **return** \mathcal{T}

1340
 1341
 1342
 1343
 1344
 1345
 1346
 1347
 1348
 1349