ILENS: Iterative Logical Enhancement via Neurosymbolic Computation and Common Sense

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Abstract

Trained on internet-scale datasets, large language models (LLMs) excel in tasks relying on surface patterns and exhibit strong common sense knowledge. However, their performance decreases on tasks requiring deeper reasoning steps. Recent techniques aim to combine the strengths of both reasoning programs and LLMs by converting natural language problems into formal logic specifications, thereby enhancing reasoning task performance. Despite these advancements, LLMs 011 often struggle with ambiguities and complex cases, leading to reasoning errors in the formal method step. In this paper, based on the observation that LLMs can provide the implicit common sense facts when asked explicitly, we 017 propose ILENS (Iterative Logical Enhancement 018 via Neurosymbolic Computation and Common 019 Sense), a new iterative neurosymbolic system for logical inferences which integrates the two systems in an iterative manner. Initially, we translate the problem specifications into AMR graphs, and then convert them into first-order logic (FOL) expressions to minimize inaccurate interpretations from natural language to FOL. Subsequently, we use formal theorem provers (Prover9, Mace4) to deduce the conclu-027 sion. Within this process, we ask the theorem prover to generate counterexamples based on the given premises when the theorem prover fails to provide a definite answer, then prompting the LLM to identify any implicit common sense facts. These facts are then incorporated back into the theorem to attempt proof completion. Through the iterative steps and leveraging the GPT-4 API in conjunction with Prover9 and Mace4, our new proposed ILENS system significantly reduces uncertain and error cases and achieves 80.22% accuracy on the challenging FOLIO dataset, setting a new state of the art.

1 Introduction

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Recent advancements in large language models (LLMs), such as ChatGPT/InstructGPT (Ouyang

et al., 2022), GPT-3 (Brown et al., 2020), GPT-4 (Achiam et al., 2023), LLAMA (Touvron et al., 2023), and PALM (Chowdhery et al., 2023), have demonstrated significant success across a variety of tasks including text generation, classification, coding, and problem-solving. LLMs are transformer-based models that operate on statistical principles. Despite their considerable success, the generation of outputs in these models relies on probabilistic token prediction (Naveed et al., 2023). However, real-world natural language (NL) is often complex and ambiguous (Nadkarni et al., 2011). Therefore, tasks that require long sequences of logical reasoning, comprehension of implicit natural language statements, or reasoning out of domain remain challenging for LLMs (Liang et al., 2022; Saparov et al., 2024; Anil et al., 2022). Although techniques such as chain of thought (CoT) (Nye et al., 2021; Wei et al., 2022; Wang et al., 2022; Huang and Chang, 2022; Kojima et al., 2022) and in-context learning (ICL) (Min et al., 2021; Dong et al., 2022; Min et al., 2022; Schick et al., 2024) have been proposed to address some of these difficulties, recent studies suggest that the inherent architecture of transformer-based language models still lacks optimal efficiency in deeper proofs logical reasoning (Dziri et al., 2024; Olausson et al., 2023).

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Additionally, logical reasoning is crucial for AI-based tasks such as theorem proving, solving mathematical problems with step-by-step solutions, efficient code generation, algorithm design, and answering complex queries. These type of tasks can be challenging because they require long chain of logical reasoning or step by step problem solving. Hence, enhancing the logical capabilities of LLMs and their ability to apply common sense knowledge can significantly improve their performance in mathematics and science-based applications (Li et al., 2021; Jain et al., 2023). The use of logical reasoning can lead to more accurate



Figure 1: Iterative workflow of ILENS.

and reliable results by reducing hallucinations across a wide range of applications compared to probabilistic token prediction (Xu et al., 2024; Olausson et al., 2023; Zhang et al., 2024). Recent research has combined powerful LLMs with formal theorem provers, leveraging the strengths of both approaches to create more robust and capable systems. This integration aims to enhance the logical reasoning capabilities of LLMs, enabling them to perform tasks that require rigorous logical reasoning alongside natural language understanding (Olausson et al., 2023; Pan et al., 2023).

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In this work, we implement ILENS (Iterative Logical Enhancement via Neurosymbolic Computation and Common Sense), a system that combines 100 LLMs with a theorem prover in an iterative fashion, 101 thereby enhancing the logical capabilities of LLMs. 102 ILENS is an iterative neurosymbolic system where 103 the language model converts the natural language 104 statements first to abstract meaning representation 105 (AMR) (Banarescu et al., 2013; Knight et al., 2021) using a parser, then translates the AMR 107 graphs to first-order logic (FOL) expressions (Enderton, 2001; Barker-Plummer et al., 2011). 109 The translated FOL expressions are fed to the 110 theorem provers (Prover9, Mace4) (McCune, 111 2005-2010) to determine the truth value of the 112 inference. In the cases of indefinite responses or 113 syntax errors generated by the prover, our system 114 improves them. If the theorem prover fails to 115

provide a definite answer, we ask it to generate a counter-example. This example is then used as a reference for the language model to find any missing links or facts in the given natural language premises (NL) to enrich the NL premises. The improved NL statements are then passed through the parsers and theorem prover to get updated inference output. If there is a syntax error in the FOL expressions, the language model is prompted to fix the error and the improved FOL is passed through the prover again. This iteration continues until the theorem prover can find a definite answer or reach a specified threshold.

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ILENS leverages the ability of LLM to follow 129 instructions and its strength of common sense 130 knowledge for FOL translation while offloading 131 logical reasoning deduction to a formal theorem 132 prover. Hence, the success of ILENS lies in two 133 novel ideas. The first key idea is translating AMR 134 to FOL rather than directly translating FOL from 135 NL. Abstract Meaning Representation (AMR) 136 tends to be more structured and semantically clearer, making it potentially easier to translate into 138 FOL without errors whereas translating natural 139 language statements to first-order logic (FOL) 140 can be more complex due to the ambiguity and 141 nuances of human language (as shown in Figure 142 2). Our baseline system performs well with this 143 added step in FOL translation with improved FOL 144 expressions in secenarios where the NL statements 145 have implicit information. The second key idea 146 is to **iterate logical reasoning** with theorem prover which updates the NL premises with any missing facts provided by LLM based on the counter-example from the theorem prover. By adding this iterative step, our system improves accuracy significantly from the baseline system by 29%. We can summarize our contributions:

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- We introduce ILENS, a new iterative neurosymbolic system for logical inferences. Our system successfully improves the accuracy on task that requires logic reasoning with LLMs and external theorem prover setting a new state of the art. On FOLIO dataset (Han et al., 2022) the accuracy achieved is 80.22% which is about 7.7% higher than the previous benchmark model (Olausson et al., 2023).
 - Our experiments demonstrates that combining common sense knowledge of LLM and iterative logic inference by formal theorem provers can improve the task of logic inference deduction.
 - We do a thorough comparison of ILENS to baseline systems and also perform error analysis.



Figure 2: Comparing conversion of first-order logic (FOL) from abstract meaning representation (AMR) and natural language (NL).

2 Related work

Semantic parsing with language models is the process of converting natural language into a structured, machine-readable representation which has seen significant advancements with the advent of LLMs. Semantic parsing traditionally involves mapping natural language utterances to formal representations like AMR, lambda calculus, or SQL queries (Ge and Mooney, 2005; Kamath and Das, 2018). The goal is to capture the underlying meaning of the input text in a way that facilitates further processing by downstream applications (Wang et al., 2015; Berant and Liang, 2014). Works by (Zhang et al., 2019; Bevilacqua et al., 2021) use a transformer-based architecture to achieve stateof-the-art results in AMR parsing tasks. Recent developments have leveraged the power of LMs, such as GPT-3, BERT, and T5, to enhance semantic parsing capabilities (Raffel et al., 2020; Shin and Van Durme, 2021; Hahn et al., 2022; Wong et al., 2023). LogicLLAMA (Yang et al., 2023) can directly translate natural language into FOL rules along with correct predictions made by GPT-3.5, which achieves performance comparable to GPT-4 with a fraction of the cost.

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Common sense reasoning in language models allows language models to interpret implicit information, disambiguate meanings, and make logical inferences that are intuitive for humans. COMET (Bosselut et al., 2019) uses the transformer model to generate inferential knowledge, augmenting the language model's ability to reason about everyday scenarios. Models like BERT and GPT-3 have been fine-tuned on datasets specifically designed to improve the capability of common sense reasoning (Talmor et al., 2018; Rajani et al., 2019; Sap et al., 2020; Liu et al., 2021; Bian et al., 2023).

Reasoning through neurosymbolic approaches in LLMs combines neural networks with symbolic reasoning systems to integrate structured knowledge and logical inference capabilities (Hitzler et al., 2022). The integration of structured knowledge bases with neural models can enhance the reasoning abilities of LLMs (Zhang et al., 2023b). Recent studies have shown how LLMs have a significant gap in logical reasoning when compared to human judgment (Press et al., 2022; Wang et al., 2024; Gu et al., 2024).

Given these extensive backgrounds, several works have been done regarding the optimal methods for integrating LLMs with symbolic components to enhance logical reasoning capabilities. Arabshahi et al. (2021) show how combining a neurosymbolic system with common sense through conversation can complete its reasoning chains. Similarly, Manhaeve et al. (2021) develops systems that combine neural and symbolic components to perform complex reasoning tasks. In DSR-LM (Zhang et al., 2023a), pre-trained LMs govern the perception of factual knowledge, and a symbolic module performs deductive reasoning. Logic-LM proposed by



Figure 3: Brief description of the models used to compare the performance of ILENS.

(Pan et al., 2023) integrates LLMs with symbolic solvers to improve logical problem-solving. Additionally, they introduce a self-refinement module that learns to modify inaccurate logical formulations using error messages from the symbolic the-237 orem prover as feedback. However, their idea of self-refinement only focuses on syntax correction, which is significantly different from our approach 240 241 and contributions. SATLM (Ye et al., 2023) uses an LLM to generate a declarative task specification 242 rather than an imperative program and leverage an 243 off-the-shelf automated theorem prover to derive the final answer. LINC (Olausson et al., 2023) uses 245 246 LLMs as the semantic parser and offloads the logical reasoning task to an external theorem prover. 247 Our work is inspired by LINC, where our methodology enhances the natural language understanding of LLMs by converting nature language premises to AMR, thereby reducing ambiguity. Moreover, iterative logical reasoning can incrementally add more facts or rules to the theorem prover, resulting in significant performance improvements in our 254 system.

256Tool usage for application task augments LMs257with external tools such as mathematical and sci-258entific computation tools, code interpreters, knowl-259edge base retrieval systems, and translation ser-260vices. This approach leverages the strengths of261both the LMs and specialized tools, resulting in a262more powerful and versatile system. Tool usage263can be done in two ways. First, External Tool In-

tegration Without Direct LM Awareness where the language model is not directly aware of the external tool or the procedures it uses. The integration occurs at a higher level, where the outputs from the LM are processed by external systems to perform specific tasks, such as external code interpreters (Gao et al., 2023; Drori et al., 2022; Azerbayev et al., 2022). For theorem proving, existing works (Wu et al., 2022; Jiang et al., 2022) rely on external theorem provers to get inferences. We follow this approach in our work by invoking external theorem provers (Prover9, Mace4) for logical reasoning. Second, Direct Tool Invocation by the LM where the LM is responsible for invoking external tools through API calls (Schick et al., 2024; Thoppilan et al., 2022).

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3 ILENS

ILENS (Iterative Logical Enhancement via Neurosymbolic Computation and Common Sense) is an iterative neurosymbolic system augmented with external theorem provers (Prover9, Mace4) for end-to-end logical reasoning. Our framework (Figure 1) consists of six stages.

• Step 1: We use a semantic parser (Goodman, 2020) with LM to translate NL premises and conclusion pair to AMR graphs. The given premises and conclusion pair is converted to AMR graphs using an LM pre-trained with AMR dataset. More details on the LM used

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can be found in Section 4.3.

- Step 2: The AMR graphs are translated to FOL expressions using GPT-4 API through prompts with ICL. More details are provided in Section 4.2 and Appendix B
- Step 3: The translated FOL is then fed to the formal theorem prover to determine the truth value of the inference. We use Prover9, an automated theorem-proving system for firstorder and equational logic extensively utilized within the logic research community (Mc-Cune, 2005) for inference deduction. For the response of Prover9, we closely follow the work done in LINC (Olausson et al., 2023), therefore the prover either returns a value from the set {True, False, Uncertain} or raises an exception due to incorrect FOL syntax (e.g., if the formulae have unbalanced parentheses or any unknown symbols or operators are not recognized by the prover).

• Step 4: If Prover9 is able to determine a definitive response, the program proceeds with the next example. However, the process involves iterative methods when more complex scenarios arise. Specifically, if Prover9 returns an {Uncertain} response which indicates the inability to find a definitive solution, the FOL premises are forwarded to Mace4, a software tool designed to find finite models for firstorder logic statements and is often used in conjunction with theorem provers such as Prover9. In this case, Mace4 attempts to identify a counterexample to the premises and advances the process to Step 5. On the other hand, if an error occurs during inference, the error message is passed to the LLM along with the FOL expressions. The LLM then attempts to correct the errors in the FOL statements before going back to Step 3.

- Step 5: The counterexample and the NL statements are then sent to the LLM (GPT-4 API) to find any missing fact or value. Using its common sense ability, the LLM updates the NL statements with the newly found missing fact.
- Step 6: The updated NL premise and conclusion pair goes back to the FOL parser in Step 2 and continues through the process till the prover is able to find a definite inference.

The success of ILENS hinges on accurately trans-342 lating NL statements into FOL expressions and 343 augmenting both Prover9 and Mace4 to perform 344 logical inference and identify any missing links in the given premises. Given the complexity of hu-346 man language, our framework prioritizes formal 347 provers over LLMs to capture every nuance of fac-348 tual information. This approach ensures there are 349 no hallucinations or incorrect representations in the updated NL statements in Step 4 and Step 5. 351 However, a notable drawback is the potential for 352 semantic and syntax errors in the FOL expressions 353 produced by the LLM. To mitigate this, we first 354 convert NL statements into AMR graphs and then 355 transform these AMR graphs into FOL expressions. 356 The key advantage of AMR lies in its structured 357 representation of entities and relationships in nat-358 ural language, thereby reducing the ambiguity of 359 NL statements (as shown in Figure: 2). 360

4 **Experimental Setup**

In this section, we introduce our experimental setup, including the dataset and models used, as well as the baselines against which ILENS is compared. We provide the source code $link^1$ to our experiments.

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4.1 Dataset

For our experiments, we use the FOLIO dataset (Han et al., 2022), which is a collection of annotated natural language statements converted into first-order logic (FOL) expressions, designed for evaluating the performance of logical inference systems. We have considered the validation set of FOLIO for evaluation. Additionally, the dataset requires pre-processing in order to have the right syntax compatible with Prover9, which can reference LINC's pre-processing step (Olausson et al., 2023). The original FOLIO validation set contains 204 examples. However, after pre-processing the dataset, 22 examples contained syntax errors, leaving 182 examples for evaluation. The pre-processing is conducted to ensure that the FOL expressions in FOLIO are in the correct syntactical format for Prover9 and Mace4. We run the 182 examples on our systems ILENS⁻ and ILENS. (System details are included in Section 4.3 and 4.4). More information on pre-processing is in Appendix A

¹Code: Project Code



Figure 4: Performance of ILENS⁻ (baseline) and ILENS(Mode-2).

4.2 In-context learning

We have chosen six diverse examples from the FO-LIO training set for in-context learning (ICL). For our baseline experiment, we use only four out of the six examples. Here, the NL statements in the examples are converted to AMR graphs. Along with the FOL expressions and AMR graphs, the updated examples from the training set are provided to the GPT-4 API through prompts. For ILENS, we run the experiment with two different modes. Mode-1 uses two out of six examples for ICL and iteration happens only twice. Mode-2 uses six examples for ICL and iteration happens four times. More details on ICL can be found under Appendices A and B

4.3 Models

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In our experiments, we use different language models for different tasks. For converting NL statements to AMR graphs, we use parse_xfm_bart_large (base model:facebook/bartlarge, version: 0.1.0 date: 2022-02-16, size: 1.4GB, smatch score: 83.7 SMATCH) for sentence to graph conversion. It is trained and scored on AMR-3 (LDC2020T02) (Knight et al., 2021) using num_beams=4. For more information, please refer to the model on GitHub.² We use GPT-4 (OpenAI, 2023) API ^{3 4} for AMR to FOL conversion as well as for updating the NL statements with missing links found from Mace4. We consider temperature

³https://openai.com/index/gpt-4/

T = 0 for our baseline system and temperature T = 0.2 for the main system. We use the NLTK ⁵ extension of Prover9 and Mace4 which is a python extension for the provers.

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4.4 Baselines

We compare ILENS to three baselines namely ILENS⁻, LINC (Olausson et al., 2023), and LogicLLAMA*. For ILENS⁻, we consider our framework without the iteration process (Step 4, Step 5 and Step 6). From LINC, we consider their Naïve, Scratchpad, CoT and original models for GPT-3.5 and GPT-4. In Naïve, the model is given the NL premises and is asked to directly generate the label. In Scratchpad, the model is asked to first generate FOL expressions corresponding to the premises, and then generate the label. In CoT, standard technique of CoT prompting is used to deduce the truth value. In LINC, the LLM is used as a semantic parser and logical inference is done by Prover9. LogicLLAMA* is a joint system with LogicLLAMA (Yang et al., 2023) for FOL translation and Prover9 for logic inference. For better understanding of the baseline models used to compare ILENS, we show a schematic representation in Figure 3.

5 Results & Analysis

In this section we provide details of our experimental results, comparisons of our systems with the other baselines, and an analysis of different syntax and semantic errors.

²The parse xfm model

⁴The exact number of parameters in GPT-4 has not been officially disclosed by OpenAI. However, reports and credible sources suggest that GPT-4 is significantly larger than its predecessor, GPT-3, which has 175 billion parameters.

⁵NLTK python extension

5.1 Experimental results

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Figure 4 shows the performance of our baseline 447 system ILENS⁻ and iterative system ILENS. We 448 can observe from ILENS^{-'s} performance that even 449 though it is able to deduce truth values correctly 450 in 95 out of 182 cases, there are still 26 syntax 451 errors and 110 indefinite logic inferences. This 452 signifies that AMR to FOL conversion can help 453 during the translation step. However, ILENS⁻ does 454 not include the iteration process, therefore there is 455 no improved logical reasoning. We discuss error 456 analysis in detail under Section 5.3. Our iterated 457 system ILENS has been configured into two modes 458 based on Section 4. Mode-1 runs with two itera-459 tions and two-shot learning and Mode-2 runs with 460 four iterations and six examples for few-shot learn-461 ing. In figure 4 (b) we provide the performance 462 of ILENS Mode-2. As we can see with increased 463 iterations and examples for few-shot learning, the performance improves significantly. The syntax er-465 rors as well as the indefinite logic inferences have 466 been reduced by increasing the prediction accuracy 467 to 36%, 35%, and 13% for "True", "False" and 468 "Uncertain" labels respectively. We provide more 469 information on ILENS Mode-1 under Appendix 470 C. Due to resource limitation, we were unable to 471 experiment beyond four iterations. With more itera-472 tions, the system may be able to resolve the syntax 473 errors however the possibility of hallucination by 474 LLM needs to be further explored. 475

5.2 Comparison with other baseline models



Figure 5: Results of our systems ILENS⁻, ILENS with the baseline models.

We compare our baseline and iterative systems (Mode-1 and Mode-2) Figure 5 with the other baseline models described in Section 4.4. From this figure, we can see that ILENS(Mode-2) surpasses the performance of the previous models by achieving 80.22% accuracy which is about 4.92% higher than CoT-GPT4 and 7.72% higher than LINC-GPT4 (Olausson et al., 2023). Additionally, we notice a significant improvement in performance with added iterations between ILENS⁻ and ILENS(Mode-1 and Mode-2). This clearly shows how iterative logic inference with external theorem provers like Prover9 and Mace4 can help improve logical reasoning. Further, we notice that by increasing the iteration from two to four, the Mode-2 system improves its performance by 13%. Additionally, we investigate individual predictions of different labels and show the result in Figure 6.

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Figure 6: Comparison of accuracy in different categories across different models and systems. The categories considered here are "True", "False" and "Uncertain".

5.3 Error analysis

We perform a thorough error analysis on the models' performances. We notice that FOL expressions generated from GPT-4 API has some types of errors.

FOL generated by LLM with syntax errors

Figure 7 shows the details of different syntax errors generated by the API in FOL expressions.

* Arity issues: FOL expressions sometimes contain multiple arities or symbols/arities are used as both relation and function. One such example is: *NL Premise:* "If Greyhound is not an airline, then there are no Greyhound planes."

NL Conclusion: "A Greyhound is not a Boeing 707."

FOL Premise: "-Airline(Greyhound) -> -exists x. (Plane(x) & Greyhound(x))"

FOL Conclusion:"-Boeing707(Greyhound)"

Here, "Greyhound" is used both as a predicate and a constant value which is conflicting to Prover9.

* **Term error:** When the FOL statement has unnecessary tokens which is not readable by the

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(b) Syntax errors of ILENS(Mode-2).

Figure 7: Performance of ILENS⁻ (baseline) and ILENS(Mode-2).

517Prover9, it causes this type of error. For ex-518ample: FOL Premise: "all x. (design_style(x,519Zaha_Hadid) -> Timeless(x))" will generate an er-520ror like "sread_term error"

* Unexpected token: This type of error arises when the format of the FOL does not match the Prover9 format, for instance, when there are unbalanced parentheses or any unexpected symbols like *FOL Premise:* "all x, y, z. ((LocatedAt(x, y) & LocatedAt(y, z)) -> LocatedAt(x, z))

527 Unable to assume facts when given information528 is incomplete

We observe that ILENS is not able to deduce logic
inference correctly when there is incomplete information. Incomplete information is different from
missing links, missing rules, or implicit information hidden in the Natural Language. Here is one
such example:

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NL Premise: ["All rabbits are cute. ",
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- 536 "Some turtles exist. ",
- "An animal is either a rabbit or a squirrel.",
- "If something is skittish, then it is not still.",
- "All squirrels are skittish.",
- 540 "Rock is still."]

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NL Conclusion: "Rock is a turtle or cute."	54
Actual label: True	54
Predicted label: Uncertain	54
Unless the LLM assumes some information about	54
"Rock", it will not be able to get a definite answer	54
for the inference.	54
The error analysis shows us the scenarios where our	54
system fails to successfully deduce logic inference.	548
The scenarios include FOL expressions generated	549
by GPT-4 with syntax errors and NL statements	55
with incomplete information.	55

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6 Conclusion and Future Work

We present ILENS, an iterative neurosymbolic system augmented with external theorem provers. ILENS is built on two novel ideas: using abstract meaning representation (AMR) to convert text into first-order logic (FOL) expressions, and iterating logic inference using counterexamples generated by Mace4 to improve logical reasoning. Our system significantly outperforms baseline models using similar evaluation techniques. We successfully demonstrate that increasing the number of iterations can enhance the performance of logic inference. This work supports the hypothesis that augmenting an external theorem prover with a large language model (LLM) can improve truth value inference deduction. Thus, the success of ILENS paves the way for future research in neurosymbolic computation for reasoning from natural languages. Future work could explore integrating other forms of symbolic reasoning, expanding the range of natural language inputs, and enhancing the scalability of such systems.

Limitations

Due to limited resources, we were able to run our experiments on one dataset (FOLIO validation (Han et al., 2022)) and with iteration up to four. However, the workflow of our system is not specific to any dataset. Therefore, with some simple data pre-processing it can be used on multiple datasets and with different provers.

Ethics Statement

No ethical violations were anticipated or encountered during the course of this research. As such, no ethical approval was required for this work.

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A FOLIO Dataset

A.1 Pre-processing

In order to pre-process the data, we follow the same technique done in LINC (Olausson et al., 2023). We first reformat the dataset with correct symbols accepted by Prover9 and Mace4. For one of our baselines LogicLLAMA*, we preprocess the dataset generated through LogicLLAMA model (Yang et al., 2023) the same way.

A.2 Few-shot examples

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For in-context learning (ICL) we consider the training set of the FOLIO dataset. We pick six diverse examples whose labels were "True", "False" and "Uncertain". We consider the following examples from the Yale-LILY/FOLIO website with the following 24, 61, 149, 262, 264, 685. We do not provide the labels of the examples during few-shot learning. For our baseline and ILENS(Mode-1) we randomly pick four and two out of these six examples respectively.

B Prompts used for FOL generation

For our system, we use in-context learning, therefore using prompts to ask the GPT-4 API to generate FOL expressions or correct/update FOL expressions. We have provided the details of the prompts we have used in Table: 1

C Detailed result of ILENS (Mode-1) and LogicaLLAMA*

We provide below the performances of ILENS(Mode-1) and LogicLLAMA* in Figure 8. For ILENS(Mode-1), we can see a clear improvement in performance over ILENS⁻.
However, there still exists considerable amount of errors which is overcome by ILENS(Mode-2) indicating more iterations for logic inference can be helpful to draw the truth values.

LogicaLLAMA* on the other hand, performs very poorly on FOLIO dataset. After generating the validation dataset with LogicLLAMA (Yang et al., 2023) we preprocess the dataset (mentioned under Appendix A). Then we pass it through the Prover9 for logic inference. As we can see from Figure 8(b), it contains many errors in the conversion. The errors include the ones mentioned in Section 5.3 and several other syntax errors. Also, we know that the conversion done by LogicLLAMA* is incorrect semantically since most of the labels are incorrectly predicted "Uncertain".



Figure 8: Confusion Matrix of ILENS(Mode-1) and LogicLLAMA*.

	I will provide you with premises and conclusions in AMR, and you will convert them into
iLens [–]	First Order Logic (FOL) expressions. Follow the given examples for format and syntax.
	<examples:></examples:>
	Ensure that:
	- Symbols are consistently used as either predicates or functions
	- Quantifiers are correctly placed
	- No quotations are required for any proper noun
	The FOL expressions are valid and wall formed for use in theorem provers like Prover0
	- The FOL expressions are valid and well-formed for use in theorem provers like Provers.
	- Make sure the FOL expressions are consistent, syntactically correct, and have balanced
	parentheses.
	- Make sure the output is not like a chat response.
	Your output should be a dictionary with the keys "premise-fol" for
	premise_graphs_list with all FOL expressions /in a single list and "conclusion-fol" for
	conclusion_graphs_list with all FOL expressions in a single list.
	Your task is to read and understand the <i>counter_example</i> generated from Mace4 and
	use common sense knowledge to find any missing information or logic chain and generate
	First Order Logic (FOL) from the /provided natural language <i>premises</i> and <i>conclusion</i> .
	Follow the given example for format and syntax.
	<example:></example:>
	Ensure that
	- You do not use symbols/arities as both relation and function
ILENS	- The FOL expressions are valid and well-formed for use in theorem provers like Prover9
(Update	with consistent arities
with	The EQL expressions are consistent suntestically connect and have belenced normatheses
counter- example)	- The FOL expressions are consistent, syntactically correct, and have balanced parentneses.
	- You do not describe your answer like a chat You do not put quotations around any
	proper nouns or person's names.
	- You do not use decimal numbers - You respond only with the JSON dictionary and
	nothing else.
	- Your output includes both premise and conclusion expressions.
	Your output should be a dictionary with the keys "premises-FOL" for premises with
	all FOL expressions in a single list and "conclusion-FOL" for conclusion with FOL
	expression in a single list.
ILENS	Your task is to fix some errors in first order logic statements. I will provide you with
	the <i>error</i> , the <i>premise_fol</i> , and the <i>conclusion_fol</i> such that they do not contain that
	error. Follow the given examples for format and syntax:
	<examples:></examples:>
	Ensure that:
	- You use common sense and do not use symbols/arities as both relation and function.
	- The FOL expressions are valid and well-formed for use in theorem provers like Prover9
	with consistent arities.
(Fix er-	- The FOL expressions are consistent syntactically correct and have balanced parentheses
ror)	- You do not describe your answer like a chat - You do not use decimal numbers
	- You do not put quotations around any proper pouns or person's names
	- You respond only with the ISON dictionary and nothing also
	Vour output includes both promise and conclusion expressions
	- Your output metades both premise and conclusion expressions.
	indu output must be a JSON dictionary with the keys "premises-FOL" for premises (a
	single list of FOL expressions) and "conclusion-FOL" for the conclusion (a single list of
	FOL expressions).

Table 1: 2-6-shot prompts for ILENS⁻, ILENS(Mode-1 and Mode-2)