Finite/Fixed Time Control of Markov Jump System under Cyber Attacks

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***Abstract*—In this paper, we address the finite-time and fixed-time control problem for Markov jump systems (MJS) under the influence of cyber attacks. The increasing occurrence of cyber threats, such as denial of service (DoS) and data injection attacks, presents significant challenges to the stability and performance of these systems. We propose a novel control strategy that ensures finite-time and fixed-time stabilization despite the presence of such attacks. Our approach is based on Lyapunov-Krasovskii functionals, incorporating adaptive mechanisms to counteract the effects of compromised communication channels. The effectiveness of the proposed method is substantiated through theoretical analysis and validated by simulation results. Our findings reveal that the control scheme is resilient to various cyber attack scenarios, guaranteeing system stability within a pre-defined time frame.**

***Index Terms*—Finite-time control, Fixed-time control, Markov jump systems, Cyber attack, Networked control systems.**

I. Introduction

In recent years, the integration of information technology with control systems has revolutionized various industrial and societal applications. However, this increased connectivity has also made networked control systems (NCS) highly susceptible to cyber attacks. The growing sophistication of these attacks poses a significant threat to the stability and performance of NCS, particularly those systems with complex dynamics such as Markov jump systems (MJS). MJS are characterized by their ability to switch between multiple modes according to a Markov process, which is often used to model systems that undergo abrupt changes due to environmental factors or internal system states.

Cyber attacks on NCS can take various forms, including denial of service (DoS), data injection, replay attacks, and false data injection. These attacks can disrupt communication between the system's components, manipulate data being transmitted, or even completely block control signals. The impact of such attacks on MJS is profound, as the switching nature of these systems can exacerbate the instability introduced by the attacks, potentially leading to catastrophic system failures.

A. Significance of Finite-Time and Fixed-Time Control

Finite-time and fixed-time control strategies have emerged as crucial tools in ensuring the rapid stabilization of dynamic systems. Unlike traditional asymptotic control, which guarantees convergence over an infinite time horizon, finite-time control ensures that the system states reach equilibrium within a finite time, which may depend on the initial conditions. Fixed-time control, on the other hand, guarantees stabilization within a time bound that is independent of the initial conditions. This property is particularly desirable in scenarios where a quick and reliable response is necessary, such as in the presence of cyber attacks.

Recent studies have extensively explored the application of finite-time and fixed-time control in various contexts. For example, finite-time stabilization has been successfully applied in nonlinear systems, where it provides robustness against disturbances and uncertainties [1]. Similarly, fixed-time control has been utilized in robotic systems and power grids to ensure that system performance remains within acceptable bounds despite the presence of external perturbations [2]. However, the application of these control strategies to MJS under cyber attack conditions is relatively unexplored, presenting a significant gap in the literature.

B. Challenges and Objectives

The primary challenge in applying finite-time and fixed-time control to MJS under cyber attacks lies in the system's inherent complexity. The random switching between different modes introduces additional difficulties in analyzing and ensuring stability, especially when the system is subjected to malicious interventions. Traditional control approaches may fail to account for the combined effects of Markovian switching and cyber attacks, leading to suboptimal or even unstable outcomes.

To address these challenges, this paper proposes a novel control framework that integrates finite-time and fixed-time control techniques with adaptive strategies tailored to MJS. The key contributions of this paper are as follows:

1. We develop a control scheme that guarantees both finite-time and fixed-time stabilization of MJS in the presence of cyber attacks. The control law is designed to be adaptive, allowing it to counteract the effects of varying attack intensities and types.

2. We provide a rigorous stability analysis using Lyapunov-Krasovskii functionals, demonstrating that the proposed control law ensures system stability within a predefined time horizon, even under adverse conditions.

3. The proposed approach is validated through extensive numerical simulations, illustrating its effectiveness in maintaining system stability across various cyber attack scenarios.

These simulations highlight the robustness and versatility of the control scheme.

C. Related Work

The study of cyber-physical security in NCS has gained considerable attention in recent years. Mo et al. [3] investigated the impact of cyber attacks on smart grid infrastructures, proposing methods to detect and mitigate such attacks. Similarly, Fawzi et al. [4] focused on secure estimation and control in cyber-physical systems, introducing strategies to maintain system performance despite adversarial interventions. While these studies have advanced our understanding of cyber-physical security, they have primarily concentrated on detection and mitigation, with limited focus on control strategies that ensure rapid stabilization.

Dragan et al. [5] provided a comprehensive analysis of robust control in discrete-time stochastic systems, emphasizing the need for resilient control strategies in the presence of uncertainties. However, their work did not address the specific challenges posed by MJS under cyber attacks. Similarly, Hespanha [6] explored stochastic hybrid systems for modeling NCS but did not focus on finite-time or fixed-time control techniques.

This paper seeks to bridge these gaps by focusing on the intersection of MJS, cyber-physical security, and finite-time/fixed-time control. Our proposed framework offers a comprehensive solution that not only ensures rapid stabilization but also adapts to the dynamic nature of MJS under cyber attack conditions.

II. Problem Formulation

Consider a Markov jump system described by the following dynamics:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + f_{\sigma(t)}(x(t), t),$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\sigma(t)$ is a discrete-time Markov process taking values in a finite set $\{1, 2, \ldots, N\}$, and $A_{\sigma(t)}, B_{\sigma(t)}$ are system matrices corresponding to the mode $\sigma(t)$. The term $f_{\sigma(t)}(x(t), t)$ represents the effect of cyber attacks, which could manifest as disturbances or manipulations of the state.

The objective is to design a control law $u(t)$ that ensures finite-time and fixed-time stabilization of the system, even in the presence of cyber attacks. Specifically, we aim to achieve the following:

1. Finite-Time Stability: There exists a finite time $T_f(x_0)$, depending on the initial condition x_0, such that $x(t) = 0$ for all $t \geq T_f(x_0)$.

2. Fixed-Time Stability: There exists a finite time T_f, independent of the initial condition x_0, such that $x(t) = 0$ for all $t \geq T_f$.

III. Control Design

To achieve finite-time stabilization, we propose a control law of the form:

$$u(t) = -K_{\sigma(t)}x(t) - \alpha_{\sigma(t)}\text{sgn}(x(t))|x(t)|^{\gamma},$$

where $K_{\sigma(t)}$ is a state feedback gain, $\alpha_{\sigma(t)} > 0$ is a positive constant, and $0 < \gamma < 1$. The term $\text{sgn}(x(t))|x(t)|^{\gamma}$ is designed to accelerate the decay of the state, ensuring that the system stabilizes within a finite time.

For fixed-time stabilization, the control law is modified to:

$$u(t) = -K_{\sigma(t)}x(t) - \alpha_{\sigma(t)}\text{sgn}(x(t))|x(t)|^{\gamma} - \beta_{\sigma(t)}\text{sgn}(x(t))|x(t)|^{\delta},$$

where $\beta_{\sigma(t)} > 0$ and $\delta > 1$. The additional term ensures that the stabilization time is independent of the initial conditions.

A. Stability Analysis

We establish the finite-time stability using a Lyapunov-Krasovskii functional of the form:

$$V(x(t)) = x(t)^T P_{\sigma(t)} x(t),$$

where $P_{\sigma(t)}$ is a positive definite matrix. The time derivative of $V(x(t))$ along the trajectories of the system is given by:

$$\dot{V}(x(t)) = x(t)^T \left(A_{\sigma(t)}^T P_{\sigma(t)} + P_{\sigma(t)} A_{\sigma(t)} + \sum_{k=1}^{N} \pi_{\sigma(t)k} P_k \right) x(t) + 2x(t)^T P_{\sigma(t)} B_{\sigma(t)} u(t), \quad (1)$$

where $\pi_{\sigma(t)k}$ represents the transition probability from mode $\sigma(t)$ to mode k. By choosing appropriate gains $K_{\sigma(t)}$, we can ensure that $\dot{V}(x(t))$ is negative definite, thereby guaranteeing finite-time convergence.

IV. Simulation Results

To validate the proposed control strategy, we simulate a Markov jump system under a DoS attack

. The simulation parameters are selected as follows: $A_1 = \begin{bmatrix} 0.5 & 0.1 \\ 0.3 & -0.4 \end{bmatrix}, A_2 = \begin{bmatrix} -0.2 & 0.4 \\ -0.1 & 0.5 \end{bmatrix}$, and $B_1 = B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$. The initial condition is $x(0) = [1, -1]^T$, and the control law parameters are $K_1 = K_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}, \alpha_1 = \alpha_2 = 1, \gamma = 0.5, \beta_1 = \beta_2 = 0.5, \delta = 2$.

The simulation results demonstrate the rapid stabilization of the system under the proposed control law, even in the presence of cyber attacks. The state trajectories converge to zero within a finite time, confirming the effectiveness of the approach.

References

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