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Quantifying Influence of Network Structure on Opinion Dynamics

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Extended Abstract

The rise of online social networks and recommender systems is changing how people share information and form opinions. Individual interactions strongly influence opinion dynamics, and interaction patterns can create echo chambers, reinforcing existing biases and reducing openness to new ideas. These processes can result in social polarization and widespread disagreement, with significant societal consequences. It is crucial to develop a quantitative framework for understanding large-scale social dynamics.

Most existing studies on opinion dynamics [1–3] have focused on qualitative characterization of opinion distributions, such as consensus and polarization. But how to quantify the relationship between the opinion evolution and network structure is still unclear. In this work, we develop probabilistic inequalities to approximate opinion distributions for a class of opinion dynamics, providing insights into how macroscopic behaviors emerge from microscopic agent updates and enabling quantitative predictions for opinion evolution.

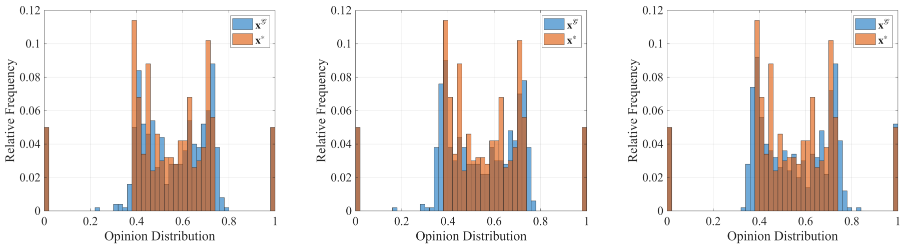
To capture the network structure, we introduce a random graph model with independent edges: Let the agent set be $\mathscr{V} := \{1, \dots, n\}$, and add an edge $\{i, j\}$ to the edge set $\mathscr{E}$ with probability $p_{ij} \in [0, 1]$, where $i \neq j$. We call the resulting graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ a realization. The model includes commonly-used graph models such as the Erdős–Rényi graph, the stochastic block model (SBM), and the Chung–Lu configuration model [4].

Given a realization graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, we define the following discrete-time randomized gossip opinion dynamics over $\mathscr{G}$. Each agent holds a real-valued opinion $X_i(t) \in \mathbb{R}$ at time $t \in \{0, 1, 2, \dots\}$. At each time step t, an edge $\{i, j\}$ of $\mathscr{G}$ is selected uniformly and independently of previous selections, and the two endpoints i and j interact. There are two types of agents, stubborn and regular. Stubborn agents (e.g., agent j) do not change their opinions: $X_j(t) \equiv X_j(0)$ for all t. Regular agents (e.g., agent i) update their opinions by averaging their own opinions with that of the other agent: $X_i(t+1) = (X_i(t) + X_j(t))/2$. Note that $\{X(t)\}$ is a Markov chain, and admits a unique stationary distribution if for every regular agent there exists a path linking it to a stubborn agent [5]. Denote the mean of the stationary distribution by $\mathbf{x}^{\mathscr{G}} := \lim_{t\to\infty} \mathbb{E}_{\mathscr{G}}\{X(t)\}$, where $\mathbb{E}_{\mathscr{G}}$ is the conditional expectation with respect to the randomness of $\mathscr{G}$. The final opinion vector $\mathbf{x}^{\mathscr{G}}$ is a random vector dependent of the graph $\mathscr{G}$.

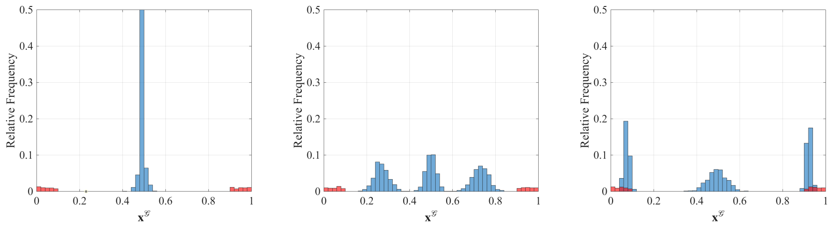
We aim to characterize the properties of $\mathbf{x}^{\mathscr{G}}$ (and, more generally, $X(t)$) by using deterministic quantities derived from the probability matrix $P = [p_{ij}]$ of the random graph model. To achieve this, we consider a reference system over the expected graph $\bar{\mathscr{G}} = (\mathscr{V}, \bar{\mathscr{E}}, \bar{A})$ with the weighted adjacency matrix $\bar{A} = P$. In a similar way we can define gossip dynamics $X^*(t)$ over the expected graph $\bar{\mathscr{G}}$, and the dynamics has the final opinion $\mathbf{x}^* := \lim_{t\to\infty} \mathbb{E}\{X^*(t)\}$. Note $\bar{\mathscr{G}}$ is deterministic, so is $\mathbf{x}^*$. Our major result can be stated as follows, which provides a probabilistic bound for the distance between $\mathbf{x}^{\mathscr{G}}$ and $\mathbf{x}^*$.

Theorem 1 (Informal)**.** *Under certain degree conditions on the random graph model,* $\mathbf{x}^{\mathscr{G}}$ *and* $\mathbf{x}^*$ *are close with high probability, i.e.,* $\mathbb{P}\{\|\mathbf{x}^{\mathscr{G}} - \mathbf{x}^*\| = o(\sqrt{n})\} = 1 - o(1)$.

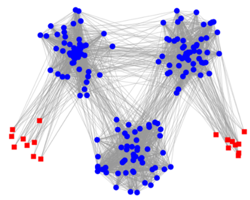
The theorem enables an entry-wise approximation of $\mathbf{x}^{\mathscr{G}}$ by $\mathbf{x}^*$. That is, most of the entries of $\mathbf{x}^{\mathscr{G}}$ are close to those of $\mathbf{x}^*$ except for $o(n)$ agents (See Figure 1(a)). Another important consequence is that analyzing $\mathbf{x}^*$ provides a simpler way to characterize the behavior of $\mathbf{x}^{\mathscr{G}}$ (See Figure 1(b)). We emphasize that similar results hold for the time average $S(t) := \frac{1}{t}\sum_{i=0}^{t-1} X(i)$ and for transient opinions $X(t)$, and can also be extended to certain nonlinear models. The proposed framework enables quantitative prediction of opinion evolution models, and provides insights into simulating and optimizing large-scale dynamics while handling the randomness of the network.



(a) Histograms of the final opinions $\mathbf{x}^{\mathscr{G}}$ (blue) for three different realizations $\mathscr{G}$, compared with the final opinions $\mathbf{x}^*$ (brown) of the expected graph $\bar{\mathscr{G}}$. Note that $\mathbf{x}^*$ is the same in all three subfigures, with stubborn agents holding opinions 0 or 1. The final opinions $\mathbf{x}^{\mathscr{G}}$ differ across the subfigures, but are well approximated by $\mathbf{x}^*$.



(b) Emergent behaviors of $\mathbf{x}^{\mathscr{G}}$ resulting from the SBM structure in (c). Regular agents (blue) are close to consensus (left), when they have a few edges to stubborn ones. They form three clusters (middle), as the influence of stubborn agents increases. Their opinions polarize (right) when they are strongly influenced by two sets of stubborn agents holding different opinions.



(c) An SBM with three regular agent communities (blue) and two stubborn agent communities (red).

Figure 1: Illustration of Theorem 1 and emergent behaviors of $\mathbf{x}^{\mathscr{G}}$.

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