Hallucination Detection in LLMs Using Spectral Features of Attention Maps

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Abstract

Large Language Models (LLMs) have demonstrated remarkable performance across various tasks but remain prone to hallucinations. Detecting hallucinations is essential for safetycritical applications, and recent methods leverage attention map properties to this end, though their effectiveness remains limited. In this work, we investigate the spectral features of attention maps by interpreting them as adjacency matrices of graph structures. We propose the LapEigvals method, which utilises the topk eigenvalues of the Laplacian matrix derived from the attention maps as an input to hallucination detection probes. Empirical evaluations demonstrate that our approach achieves stateof-the-art hallucination detection performance among attention-based methods. Extensive ablation studies further highlight the robustness and generalisation of LapEigvals, paving the way for future advancements in the hallucination detection domain.

1 Introduction

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The recent surge of interest in Large Language Models (LLMs), driven by their impressive performance across various tasks, has led to significant advancements in their training, fine-tuning, and application to real-world problems. Despite progress, many challenges remain unresolved, particularly in safety-critical applications where the cost of errors is high. A significant issue is that LLMs are prone to hallucinations, i.e. generating "content that is nonsensical or unfaithful to the provided source content" (Farquhar et al., 2024; Huang et al., 2023). Since eliminating hallucinations is impossible (Lee, 2023; Xu et al., 2024), there is a pressing need for methods to detect when a model produces hallucinations. In addition, uncovering internal behaviour while studying hallucinations of LLMs might reveal significant progress in understanding their characteristics, fostering further development in the field. Recent studies have shown

that hallucinations can be detected using internal states of the model, e.g., hidden states (Chen et al., 2024) or attention maps (Chuang et al., 2024a), and that LLMs can internally "know when they do not know" (Azaria and Mitchell, 2023; Orgad et al., 2025). We provide new insights showing that spectral features of attention maps coincide with hallucinations, and based on that observation, we introduce a novel method for detecting hallucinations. 043

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As highlighted by (Barbero et al., 2024), attention maps can be viewed as weighted adjacency matrices of graphs. Building on this perspective, we performed statistical analysis and demonstrated that the eigenvalues of a Laplacian matrix derived from attention maps serve as good predictors of hallucinations. We propose the LapEigvals method, which utilises the top-k eigenvalues of the Laplacian as input features of a probing model to detect hallucinations. We share full implementation in a public repository: https://anonymous.4open. science/r/lapeig-acl-2025.

We summarise our contributions as follows:

- We perform statistical analysis of the Laplacian matrix derived from attention maps and show that it could serve as a better predictor of hallucinations compared to the previous method relying on the log-determinant of the maps.
- (2) Building on that analysis and advancements in the graph-processing domain, we propose leveraging the top-k eigenvalues of the Laplacian matrix as features for hallucination detection probes and empirically show that it achieves state-of-the-art performance among attention-based approaches.
- (3) Through extensive ablation studies, we demonstrate properties, robustness and generalisation of LapEigvals and suggest promising directions for further development.

2 Motivation

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Considering the attention matrix as an adjacency matrix representing a set of Markov Chains, each corresponding to one layer of an LLM (Barbero et al., 2024) (Figure 2), we can leverage its spectral properties, as was done in many successful graphbased methods (Mohar, 1997; von Luxburg, 2007; Bruna et al., 2013; Topping et al., 2022). In particular, it was shown that graph Laplacian might help to describe several graph properties, like the presence of bottlenecks (Topping et al., 2022; Black et al., 2023). We hypothesise that hallucinations may be related to disturbance of information flow caused by some form of bottleneck.

To assess whether our hypothesis holds, we measured if graph spectral features provide a stronger coincidence with hallucinations than the previous attention-based method - AttentionScore (Sriramanan et al., 2024). We prompted an LLM with questions from the TriviaQA dataset (Joshi et al., 2017) and extracted attention maps, differentiating by layers and heads. We then computed the spectral features, i.e., the 10 largest eigenvalues of the Laplacian matrix from each head and layer. Further, we conducted a two-sided Mann-Whitney U test to compare whether Laplacian eigenvalues and the values of AttentionScore are different between hallucinated and non-hallucinated examples. Figure 1 shows *p*-values for all layers and heads, indicating that AttentionScore often results in higher *p*-values compared to Laplacian eigenvalues. Overall, we studied 6 datasets and 5 LLMs and found similar results, and present all results in Appendix A. Based on these findings, we propose leveraging top-k Laplacian eigenvalues as features for a hallucination probe.

3 Method

In our method, we train a hallucination probe using only attention maps, which we extracted during LLM inference, as illustrated in Figure 2. The attention map is a matrix containing attention scores for all tokens processed during inference, while the hallucination probe is a logistic regression model that uses features derived from attention maps as input. This work's core contribution is using the top-k eigenvalues of the Laplacian matrix as input features, which we detail below.

Denote $\mathbf{A}^{(l,h)} \in \mathbb{R}^{T \times T}$ as the attention map matrix for layer $l \in \{1 \dots L\}$ and attention head $h \in \{1 \dots H\}$, where T is the total number of tokens generated by an LLM (including input tokens), L the number of layers (transformer blocks), and H the number of attention heads. The attention matrix is row-stochastic, meaning each row sums to $1 (\sum_{j=0}^{T} \mathbf{A}_{:,j}^{(l,h)} = \mathbf{1})$. It is also lower triangular $(a_{ij}^{(l,h)} = 0 \text{ for all } j > i)$ and nonnegative $(a_{ij}^{(l,h)} \ge 0 \text{ for all } i, j)$. We can view $\mathbf{A}^{(l,h)}$ as a weighted adjacency matrix of a directed graph, where each node represents processed token, and each directed edge from token i to token j is weighted by the attention score, as depicted in Figure 2. 133

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Then, we define the Laplacian of a layer l and attention head h as:

$$\mathbf{L}^{(l,h)} = \mathbf{D}^{(l,h)} - \mathbf{A}^{(l,h)}, \qquad (1)$$

where $\mathbf{D}^{(l,h)}$ is a diagonal degree matrix. Since the attention map defines a directed graph, we distinguish between the *in-degree* and *out-degree* matrices. The *in-degree* is computed as the sum of attention scores from preceding tokens, and due to the softmax normalization, it is uniformly 1. Therefore, we define $\mathbf{D}^{(l,h)}$ as the *out-degree* matrix, which quantifies the total attention a token receives from tokens that follow it. To ensure these values remain independent of the sequence length, we normalize them by the number of subsequent tokens (i.e., the number of outgoing edges).

$$d_{ii}^{(l,h)} = \frac{\sum_{u} a_{ui}^{(l,h)}}{T-i},$$
(2)

where $i, u \in \{0...(T-1)\}$ denote token indices. Such defined Laplacian is bounded, i.e. $\mathbf{L}_{ij}^{(l,h)} \in [-1,1]$ (see Appendix B). Intuitively, the resulting Laplacian for each processed token represents the average attention score to previous tokens reduced by the attention score to itself. As eigenvalues of the Laplacian can encode information about information flow in graph (von Luxburg, 2007; Topping et al., 2022), we take eigenvalues of $\mathbf{L}^{(l,h)}$, which are diagonal entries, due to the lower triangularity of the Laplacian matrix, and sort them:

$$\tilde{z}^{(l,h)} = \operatorname{sort}\left(\operatorname{diag}\left(\mathbf{L}^{(l,h)}\right)\right)$$
 (3)

Recently, (Zhu et al., 2024) found features from the entire token sequence, rather than a single token, improving hallucination detection. Similarly, (Kim et al., 2024) demonstrated that information from all



Figure 1: Visualisation of *p*-values from the two-sided Mann-Whitney U test for all layers and heads of Llama-3.1-8B across two feature types: AttentionScore and the k = 10 Laplacian eigenvalues. These features were derived from attention maps collected when the LLM answered questions from the TriviaQA dataset. Higher *p*-values indicate no significant difference in feature values between hallucinated and non-hallucinated examples. For AttentionScore, 80% of heads have p < 0.05, while for Laplacian eigenvalues, this percentage is 91%. Therefore, Laplacian eigenvalues may be better predictors of hallucinations, as feature values across more heads exhibit statistically significant differences between hallucinated and non-hallucinated examples.



Figure 2: The autoregressive inference process in an LLM is depicted as a graph for a single attention head h (as introduced by (Vaswani, 2017)) and three generated tokens $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$. Here, $\mathbf{h}_i^{(l)}$ represents the hidden state at layer l for the input token i, while $a_{i,j}^{(l,h)}$ denotes the scalar attention score between tokens i and j at layer l and attention head h. Arrows direction refers to information flow during inference.

178layers, instead of a single one in isolation, yields179better results on this task. Motivated by these find-180ings, our method uses features from all tokens and181all layers as input to the probe. Therefore, we take

the top-k largest values from each head and layer, and concatenate them into a single feature vector z, where k is a hyperparameter of our method:

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$$z = \left\| \left[\tilde{z}_T^{(l,h)}, \tilde{z}_{T-1}^{(l,h)}, \dots, \tilde{z}_{T-k}^{(l,h)} \right] \right\|$$
(4)

Since LLMs contain dozens of layers and heads, the probe input vector $z \in \mathbb{R}^{T*L*H*k}$ would suffer from large dimensionality. Thus, we project it to lower dimensionality using the PCA (Jolliffe and Cadima, 2016). We call our approach LapEigvals.

4 Experimental setup

4.1 Dataset construction

We use annotated QA datasets to construct the hallucination detection datasets and label incorrect LLM answers as hallucinations. To determine whether the generated answers were correct, we adopted the *llm-as-judge* approach (Zheng et al., 2023), as in previous studies (Orgad et al., 2025). Specifically, we prompted a large LLM to classify each response as either *hallucination*, *nonhallucination*, or *rejected*, where *rejected* indicates that it was unclear whether the answer was correct, e.g., the model refused to answer due to insufficient knowledge. Based on the manual qualitative inspection of several LLMs, we employed gpt-4o-mini (OpenAI et al., 2024) as the judge model since it



Figure 3: Overview of the methodology used in this work. Solid lines indicate the test-time pipeline, while dashed lines represent additional pipeline steps for generating labels for training the hallucination probe (logistic regression). The primary contribution of this work is leveraging the top-k eigenvalues of the Laplacian as features for the hallucination probe, highlighted with a bold box on the diagram.

provides the best trade-off between accuracy and cost.

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For experiments, we selected 6 QA datasets previously utilised in the context of hallucination detection (Chen et al., 2024; Kossen et al., 2024; Chuang et al., 2024b; Mitra et al., 2024). Specifically, we used the validation set of NQOpen (Kwiatkowski et al., 2019), comprising 3610 question-answer pairs, and the validation set of TriviaQA (Joshi et al., 2017), containing 7983 pairs. To evaluate our method on longer inputs, we employed the development set of CoQA (Reddy et al., 2019) and the rc.nocontext portion of the SQuADv2 (Rajpurkar et al., 2018) datasets, with 5928 and 9960 examples, respectively. Additionally, we incorporated the QA part of the HaluEval (Li et al., 2023) dataset, containing 10000 examples, and the generation part of the TruthfulQA (Lin et al., 2022) benchmark with 817 examples. For TriviaQA, CoQA, and SQuADv2, we followed the same preprocessing procedure as (Chen et al., 2024).

We generate answers using 5 open-source LLMs: Llama-3.1-8B¹ and Llama-3.2-3B² (Grattafiori et al., 2024), Phi-3.5³ (Abdin et al., 2024), Mistral-Nemo⁴ (Mistral AI Team and NVIDIA, 2024), Mistral-Small-24B⁵ (Mistral AI Team, 2025). We use two softmax temperatures for each LLM when decoding ($temp \in \{0.1, 1.0\}$) and one prompt (showed on Listing 3). Overall, we evaluated hallucination detection probes on 10 LLM configurations and 6 QA datasets. We present the frequency of classes for answers from each configuration in Figure 9 (Appendix E).

4.2 Hallucination Probe

As a hallucination probe, we take a logistic regression model, using the implementation from scikit-learn (Pedregosa et al., 2011) with all parameters default, except for $max_iter = 2000$ and $class_weight = "balanced"$. For top-k eigenvalues, we tested 5 values of $k \in \{5, 10, 20, 50, 100\}^6$ and selected the result with the highest efficacy. All eigenvalues are projected with PCA onto 512 dimensions, except in *per-layer* experiments where there may be fewer than 512 features. In these cases, we apply PCA projection to match the input feature dimensionality, i.e., decorrelating them. As an evaluation metric, we use AUROC on the test split.

4.3 Baselines

Our method is a supervised approach to detect hallucinations solely from attention maps. For a fair comparison, we modify unsupervised AttentionScore (Sriramanan et al., 2024) to take log-determinants for each head as features instead of summing them. We also add original AttentionScore with the summation over heads for a reference. To evaluate the effectiveness of our proposed Laplacian eigenvalues, we also compare it to using raw attention maps and call it AttnEigvals. Additionally, in Appendix F.1, we provide results for each approach but *per-layer*, and in Appendix F.3 we showcase comparison with method relying on hidden states. We provide implementation and hardware details in Appendix C. 241

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¹hf.co/meta-llama/Llama-3.1-8B-Instruct

²hf.co/meta-llama/Llama-3.2-3B-Instruct

³hf.co/microsoft/Phi-3.5-mini-instruct

⁴hf.co/mistralai/Mistral-Nemo-Instruct-2407

⁵hf.co/mistralai/Mistral-Small-24B-Instruct-2501

Table 1: Test AUROC for LapEigvals and several baseline methods. AUROC values were obtained in the single run of logistic regression training on features from a dataset generated with temp = 1.0. We marked results for AttentionScore in gray as it is unsupervised approach, not directly comparable to the other ones. In **bold**, we highlight the best performance individually for each dataset and LLM. See Appendix F for extended results.

LLM	Feature			Test A	UROC (†)		
		CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Llama3.1-8B	AttentionScore	0.493	0.589	0.556	0.538	0.532	0.541
Llama3.1-8B	AttnLogDet	0.769	0.827	0.793	0.748	0.842	0.814
Llama3.1-8B	AttnEigvals	0.782	0.819	0.790	0.768	0.843	0.833
Llama3.1-8B	LapEigvals	0.830	0.874	0.827	0.791	0.889	0.829
Llama3.2-3B	AttentionScore	0.509	0.588	0.546	0.530	0.515	0.581
Llama3.2-3B	AttnLogDet	0.700	0.801	0.690	0.734	0.789	0.795
Llama3.2-3B	AttnEigvals	0.724	0.819	0.694	0.749	0.804	0.723
Llama3.2-3B	LapEigvals	0.812	0.828	0.693	0.757	0.832	0.787
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Mistral-Nemo	AttentionScore	0.493	0.531	0.529	0.510	0.532	0.494
Mistral-Nemo	AttnLogDet	0.728	0.798	0.769	0.772	0.812	0.852
Mistral-Nemo	AttnEigvals	0.778	0.781	0.761	0.758	0.821	0.802
Mistral-Nemo	LapEigvals	0.835	0.833	0.795	0.812	0.865	0.828
Mistral-Small-24B	AttentionScore	0.516	0.504	0.462	0.455	0.463	0.451
Mistral-Small-24B	AttnLogDet	0.766	0.842	0.747	0.753	0.833	0.735
Mistral-Small-24B	AttnEigvals	0.805	0.848	0.751	0.760	0.844	0.765
Mistral-Small-24B	LapEigvals	0.861	0.882	0.791	0.820	0.876	0.748

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5 Results

Table 1 presents the results of our method compared to the baselines. LapEigvals achieved the best performance among all tested methods on 5 out of 6 datasets. Moreover, our method consistently performs well across all 5 LLM architectures ranging from 3 up to 24 billion parameters. TruthfulQA was the only exception where LapEigvals was the second-best approach, yet it might stem from the small size of the dataset or severe class imbalance (depicted in Figure 9). In contrast, using eigenvalues of vanilla attention maps in AttnEigvals leads to worse performance, which suggests that transformation to Laplacian is the crucial step to uncover latent features of an LLM corresponding to hallucinations. In Appendix F, we show that LapEigvals consistently demonstrates a smaller generalisation gap, i.e., the difference between training and test performance is smaller for our method. While the AttentionScore method performed poorly, it is fully unsupervised and should not be directly compared to other approaches. However, its supervised counterpart - AttnLogDet - remains inferior to methods based on spectral features, namely LapEigvals and AttnEigvals. In Table 4 in Appendix F.1, we present extended results, including *per-layer* and *all-layers* breakdowns, two temperatures used during answer generation, and a comparison between training and test AUROC. Moreover, compared to probes based on hidden states, our method performs best in most of the tested settings, as shown in Appendix F.3.

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6 Ablation studies

To better understand the behaviour of our method under different conditions, we conduct a comprehensive ablation study. This analysis provides valuable insights into the factors driving the LapEigvals performance and highlights the robustness of our approach across various scenarios. In order to ensure reliable results, we perform all studies on the TriviaQA dataset, which has a reasonable input size and number of examples.

6.1 How does the number of eigenvalues influence performance?

First, we verify how the number of eigenvalues317influences the performance of the hallucination318probe and present results for Mistral-Small-24B319in Figure 4 (results for all models are showcased in320

 $^{^6 {\}rm For}$ datasets with examples having less than 100 tokens, we stop at k=50

Figure 10 in Appendix G). Generally, using more 321 eigenvalues improves performance, but there is less variation in performance among different values of k for LapEigvals. Moreover, LapEigvals achieves significantly better performance with smaller input sizes, as AttnEigvals with the largest k = 100326 fails to surpass LapEigvals's performance at k =327 5. These results confirm that spectral features derived from the Laplacian carry a robust signal indicating the presence of hallucinations and highlight 330 the strength of our method. 331



Figure 4: Probe performance across different top-k eigenvalues: $k \in \{5, 10, 25, 50, 100\}$ for TriviQA dataset with temp = 1.0 and Mistral-Small-24B LLM.

6.2 Does using all layers at once improve performance?

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Second, we demonstrate that using all layers of an LLM instead of a single one improves performance. In Figure 5, we compare *per-layer* to *all-layer* efficacy for Mistral-Small-24B (results for all models are showcased in Figure 11 in Appendix G). For the *per-layer* approach, better performance is generally achieved in later LLM layers. Notably, peak performance varies across LLMs, requiring an additional search for each new LLM. In contrast, the all-layer probes consistently outperform the best per-layer probes across all LLMs. This finding suggests that information indicating hallucinations is spread across many layers of LLM, and considering them in isolation limits detection accuracy. Further, Table 4 in Appendix F summarises outcomes for the two variants on all datasets and LLM configurations examined in this work.

6.3 Does sampling temperature influence results?

Here, we compare LapEigvals to baselines on hallucination datasets produced with several temperatures used during decoding. Higher temperatures



Figure 5: Analysis of model performance across different layers for Mistral-Small-24B and TriviaQA dataset with temp = 1.0 and k = 100 top eigenvalues (results for models operating on all layers provided for reference).

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typically produce more hallucinated examples (Lee, 2023; Renze, 2024), leading to dataset imbalance. Thus, to mitigate the effect of data imbalance, we sample a subset of 1000 hallucinated and 1000 non-hallucinated examples 10 times for each temperature and train hallucination probes. Interestingly, in Figure 6, we observe that all models improve their performance at higher temperatures, but LapEigvals consistently achieves the best accuracy on all considered temperature values. The correlation of efficacy with temperature may be attributed to differences in the characteristics of hallucinations at higher temperatures compared to lower ones (Renze, 2024). Also, hallucination detection might be facilitated at higher temperatures due to underlying properties of softmax function (Veličković et al., 2024), and further exploration of this direction is left for future work.

6.4 How does LapEigvals generalizes?

To check whether our method generalises across datasets, we trained the hallucination probe on features from the training split of one QA dataset and evaluated it on the features from the test split of a different QA dataset. Due to space limitations, we present results for selected datasets and provide extended results and absolute efficacy values in Appendix H. Figure 7 showcases the percentage drop in Test AUROC when using a different training dataset compared to training and testing on the same QA dataset. We can observe that LapEigvals provides a performance drop comparable to other baselines, and in several cases, it generalises best. Interestingly, all methods exhibit poor generalisation on TruthfulQA, possibly due to dataset size 390



Figure 6: Test AUROC for different sampling *temp* values during answer decoding on the TriviaQA dataset, using k = 100 eigenvalues for LapEigvals and AttnEigvals with the Llama-3.1-8B LLM. Error bars indicate the standard deviation over 10 balanced samples containing N = 1000 examples per class.

or imbalance. Additionally, in Appendix H, we show that LapEigvals achieves the highest test performance in all scenarios (except for TruthfulQA).

6.5 How does performance vary across prompts?

Lastly, to assess the stability of our method across different prompts used for answer generation, we compared the results of the hallucination probes trained on features from four distinct prompts, the content of which is included in Appendix I. As shown in Table 2, LapEigvals consistently outperforms all baselines across all four prompts. While we can observe variations in performance across prompts, LapEigvals demonstrates the lowest standard deviation (0.05) compared to AttnLogDet (0.016) and AttnEigvals (0.07), indicating its greater robustness.

Table 2: Test AUROC across four different prompts for answers on the TriviaQA dataset using Llama-3.1-8B with temp = 1.0 and k = 50 (some prompts have led to fewer than 100 tokens). Prompt p_3 was the main one used to compare our method to baselines, as presented in Tables 1.

Feature		Test AU	ROC (†)	
	p_1	p_2	p_3	p_4
AttnLogDet	0.847	0.855	0.842	0.860
AttnEigvals	0.840	0.870	0.842	0.875
LapEigvals	0.882	0.890	0.888	0.895

7 Related Work

Hallucinations in LLMs were proved to be inevitable (Xu et al., 2024), and to detect them, one can leverage either *black-box* or *white-box* approaches. The former approach uses only the outputs from an LLM, while the latter uses hidden states, attention maps, or logits corresponding to generated tokens.

Black-box approaches focus on the text generated by LLMs. For instance, (Li et al., 2024) verified the truthfulness of factual statements using external knowledge sources, though this approach relies on the availability of additional resources. Alternatively, *SelfCheckGPT* (Manakul et al., 2023) generates multiple responses to the same prompt and evaluates their consistency, with low consistency indicating potential hallucination.

White-box methods have emerged as a promising approach for detecting hallucinations (Farquhar et al., 2024; Azaria and Mitchell, 2023; Arteaga et al., 2024; Orgad et al., 2025). These methods are universal across all LLMs and do not require additional domain adaptation compared to black-box ones (Farquhar et al., 2024). They draw inspiration from seminal works on analysing the internal states of simple neural networks (Alain and Bengio, 2016), which introduced *linear classifier probes* – models operating on the internal states of neural networks. Linear probes have been widely applied to the internal states of LLMs, e.g., for detecting hallucinations.

One of the first such probes was SAPLMA (Azaria and Mitchell, 2023), which demonstrated that one could predict the correctness of generated text straight from LLM's hidden states. Further, the INSIDE method (Chen et al., 2024) tackled hallucination detection by sampling multiple responses from an LLM and evaluating consistency between their hidden states using a normalised sum of the eigenvalues from their covariance matrix. Also, (Farquhar et al., 2024) proposed a complementary probabilistic approach, employing entropy to quantify the model's intrinsic uncertainty. Their method involves generating multiple responses, clustering them by semantic similarity, and calculating Semantic Entropy using an appropriate estimator. To address concerns regarding the validity of LLM probes, (Marks and Tegmark, 2024) introduced a high-quality QA dataset with simple true/false answers and causally demonstrated that the truthfulness of such statements is linearly represented in

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Figure 7: Generalisation across datasets measured as a per cent performance drop in Test AUROC (less is better) when trained on one dataset and tested on the other. Training datasets are indicated in the plot titles, while test datasets are shown on the x-axis. Results computed on Llama-3.1-8B with k = 100 top eigenvalues and temp = 1.0. Results for all datasets are presented in Appendix H.

LLMs, which supports the use of probes for short texts.

Self-consistency methods (Liang et al., 2024), like INSIDE or Semantic Entropy, require multiple runs of an LLM for each input example, which substantially lowers their applicability. Motivated by this limitation, (Kossen et al., 2024) proposed to use Semantic Entropy Probe, which is a small model trained to predict expensive Semantic Entropy (Farquhar et al., 2024) from LLM's hidden states. Notably, (Orgad et al., 2025) explored how LLMs encode information about truthfulness and hallucinations. First, they revealed that truthfulness is concentrated in specific tokens. Second, they found that probing classifiers on LLM representations do not generalise well across datasets, especially across datasets requiring different skills. Lastly, they showed that the probes could select the correct answer from multiple generated answers with reasonable accuracy, which they concluded with the LLM making mistakes at the decoding stage besides knowing the correct answer.

Recent studies have started to explore hallucination detection exclusively from attention maps. (Chuang et al., 2024a) introduced the lookback ratio, which measures how much attention LLMs allocate to relevant input parts when answering questions based on the provided context. The work most closely related to ours is (Sriramanan et al., 2024), which introduces the AttentionScore method. Although the process is unsupervised and computationally efficient, the authors note that its performance can depend highly on the specific layer from which the score is extracted. Compared to AttentionScore, our method is fully supervised and grounded in graph theory, as we interpret inference in LLM as a graph. While AttentionScore aggregates only the attention diagonal to compute

its log-determinant, we instead derive features from the graph Laplacian, which captures all attention scores (see Eq. (1) and (2)). Additionally, we utilize all layers for detecting hallucination rather than a single one, demonstrating effectiveness of this approach. We also demonstrate that it performs poorly on the datasets we evaluated. Nonetheless, we drew inspiration from their approach, particularly using the lower triangular structure of matrices when constructing features for the hallucination probe. 497

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8 Conclusions

In this work, we demonstrated that the spectral features of LLMs' attention maps, specifically the eigenvalues of the Laplacian matrix, carry a signal capable of detecting hallucinations. Specifically, we proposed the LapEigvals method, which employs the top-k eigenvalues of the Laplacian as input to the hallucination detection probe. Through extensive evaluations, we empirically showed that our method consistently achieves state-of-the-art performance among all tested approaches. Furthermore, multiple ablation studies demonstrated that our method remains stable across varying numbers of eigenvalues, diverse prompts, and generation temperatures while offering reasonable generalisation.

In addition, we hypothesise that self-supervised learning (Balestriero et al., 2023) could yield a more robust and generalisable approach while uncovering non-trivial intrinsic features of attention maps. Notably, results such as those in Section 6.3 suggest intriguing connections to recent advancements in LLM research (Veličković et al., 2024; Barbero et al., 2024), highlighting promising directions for future investigation.

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Limitations

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Supervised method In our approach, one must provide labelled hallucinated and non-hallucinated ex-535 amples to train the hallucination probe. While this 536 can be handled by the *llm-as-judge*, it might in-537 troduce some noise or pose a risk of overfitting. Limited generalisation across LLM architectures 539 The method is incompatible with LLMs having dif-540 ferent head and layer configurations. Developing 541 architecture-agnostic hallucination probes is left for future work. Minimum length requirement Computing top-k Laplacian eigenvalues demands attention maps of at least k tokens (e.g., k = 100require 100 tokens). Open LLMs Our method requires access to the internal states of LLM thus it cannot be applied to closed LLMs. Risks Please 548 note that the proposed method was tested on se-549 lected LLMs and English data, so applying it to 550 untested domains and tasks carries a considerable risk without additional validation. 552

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A Details of motivational study

We present a detailed description of the procedure used to obtain the results presented in Section 2 along with additional results for other datasets and LLM.

To test whether there is a statistically significant difference in the values of AttnEigvals and Laplacian eigenvalues, we first took QA datasets and ran inference with three LLMs, namely Llama-3.1-8B Llama-3.2-3B and Phi-3.5. Then, we extracted attention maps and computed AttentionScore (Sriramanan et al., 2024), i.e., log-determinant of attention maps. Unlike original work, we did not sum the scores over heads as we performed analysis at a single-head level of granularity. Also, we computed the Laplacian according to the definition presented in Section 3, took the 10 largest eigenvalues for each head, and treated each eigenvalue as a separate example. Finally, we ran the Mann–Whitney U test, leveraging SciPy implementation (Virtanen et al., 2020), and gathered *p*-values presented in Figure 1.

Table 3 presents the percentage of heads having a statistically significant difference in feature values between hallucinated and non-hallucinated examples, as indicated by p < 0.05 from the Mann-Whitney U test. These results show that the Laplacian eigenvalues better distinguish between the two classes for all considered LLMs and datasets.

B Bounds of the Laplacian

In the following section, we prove that the Laplacian defined in 3 is bounded and has at least one zero eigenvalue. Here, we denote eigenvalues as λ_i , and provide derivation for a single layer and head, which holds also after stacking them together into a single graph (set of per-layer graphs). For clarity we omit superscript (l, h) denoting layer and head.

Lemma 1. The Laplacian eigenvalues are bounded: $-1 \le \lambda_i \le 1$.

1178*Proof.* Due to the lower-triangular structure of the1179Laplacian, its eigenvalues lie on the diagonal and1180are given by:

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$$\lambda_i = \mathbf{L}_{ii} = d_{ii} - a_{ii}$$

1182 The out-degree are defined as:

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$$d_{ii} = \frac{\sum_{u} a_{ui}}{T - i}$$

Table 3: Percentage of heads having a statistically significant difference in feature values between hallucinated and non-hallucinated examples, as indicated by p < 0.05 from the Mann-Whitney U test. Results were obtained for AttentionScore and the 10 largest Laplacian eigenvalues on 6 datasets and 5 LLMs.

LLM	Dataset	% (of $p < 0.05$
		AttnScore	Laplacian eigvals
Llama3.1-8B	CoQA	40	87
Llama3.1-8B	HaluevalQA	91	93
Llama3.1-8B	NQOpen	78	83
Llama3.1-8B	SQuADv2	70	81
Llama3.1-8B	TriviaQA	80	91
Llama3.1-8B	TruthfulQA	40	60
Llama3.2-3B	CoQA	50	79
Llama3.2-3B	HaluevalQA	91	93
Llama3.2-3B	NQOpen	81	84
Llama3.2-3B	SQuADv2	69	74
Llama3.2-3B	TriviaQA	81	8
Llama3.2-3B	TruthfulQA	40	62
Phi3.5	CoQA	45	8
Phi3.5	HaluevalQA	80	80
Phi3.5	NQOpen	73	80
Phi3.5	SQuADv2	81	82
Phi3.5	TriviaQA	86	92
Phi3.5	TruthfulQA	41	53
Mistral-Nemo	CoQA	35	78
Mistral-Nemo	HaluevalQA	78	82
Mistral-Nemo	NQOpen	64	51
Mistral-Nemo	SQuADv2	54	50
Mistral-Nemo	TriviaQA	71	74
Mistral-Nemo	TruthfulQA	40	50
Mistral-Small-24B	CoQA	28	78
Mistral-Small-24B	HaluevalQA	68	70
Mistral-Small-24B	NQOpen	45	51
Mistral-Small-24B	SQuADv2	75	82
Mistral-Small-24B	TriviaQA	65	70
Mistral-Small-24B	TruthfulQA	43	52

Since $0 \le a_{ui} \le 1$, the sum in the numerator is upper bounded by T - i, therefore $d_{ii} \le 1$, and consequently $\lambda_i = \mathbf{L}_{ii} \le 1$, which concludes upper-bound part of the proof.

Recall, that eigenvalues lie on the main diagonal of the Laplacian, hence $\lambda_i = \frac{\sum_u a_{uj}}{T-i} - a_{ii}$. To find the lower bound of λ_i , we need to minimize $X = \frac{\sum_u a_{uj}}{T-i}$ and maximize $Y = a_{ii}$. First, we note that X's denominator is always positive T - i >0, since $i \in \{0 \dots (T-1)\}$ (as defined by Eq. (2)). For numerator, we recall that $0 \le a_{ui} \le 1$, therefore the sum has its minimum at 0, hence $X \ge 0$. Second, to maximize $Y = a_{ii}$, we can take maximum of $0 \le a_{ii} \le 1$ which is 1. Finally, X - Y = -1, consequently $\mathbf{L}_{ii} \ge -1$, which concludes the lower-bound part of the proof.

Lemma 2. For every \mathbf{L}_{ii} there exists at least one 1200 zero-eigenvalue and it corresponds to the last token 1201 T, i.e., $\lambda_T = 0$. 1202

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Proof. Recall, that eigenvalues lie on the main di-1203 agonal of the Laplacian, hence $\lambda_i = \frac{\sum_u a_{uj}}{T-i} - a_{ii}$. Consider last token, wherein the sum in the nu-1204 1205 merator reduces to $\sum_{u} a_{uj} = a_{TT}$, denomina-1206 tor becomes T - i = T - (T - 1) = 1, thus 1207 $\lambda_T = \frac{a_{TT}}{1} - a_{TT} = 0.$ 1208

Implementation details С

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In our experiments, we used HuggingFace Transformers (Wolf et al., 2020), PyTorch (Ansel et al., 2024), and scikit-learn (Pedregosa et al., 2011). We utilised Pandas (team, 2020) and Seaborn (Waskom, 2021) for visualisations and analysis. To version data, we employed DVC (Kuprieiev et al., 2025). We acquired attention maps using a single Nvidia A40 with 40GB VRAM, except for Mistral-Small-24B for which we used Nvidia H100 with 96GB VRAM. Training hallucination probe was done using CPU only. To compute labels using the *llm-as-judge* approach, we leveraged gpt-40-mini model available through OpenAI API. Detailed hyperparameter configurations and code to reproduce the experiments is available in the public Git repository.

Details of QA datasets D

In this work, we used 6 open and publicly available question answering datasets: NQ-Open (Kwiatkowski et al., 2019) (CC-BY-SA-3.0 license), SQuADv2 (Rajpurkar et al., 2018) (CC-BY-SA-4.0 license), TruthfulQA (Apache-2.0 license) (Lin et al., 2022), HALUEval (MIT license) (Li et al., 2023), CoQA (Reddy et al., 2019) (domain-dependent licensing, detailed on https: //stanfordnlp.github.io/coqa/), while TriviaQA (lacks clear licensing information, but was primarily shared as public benchmark). Research purposes fall into the intended use of these datasets. To preprocess and filter TriviaQA, CoQA, and SQuADv2 we utilized open-source code of (Chen et al., 2024)⁷, which also borrows from (Farquhar et al., 2024)⁸. In Figure 8, we provide histogram plots of the number of tokens for question and answer of each dataset computed with meta-llama/Llama-3.1-8B-Instruct tokenizer.

Ε Hallucination dataset sizes

In Figure 9, we show the number of examples for 1248 each label determined with the *llm-as-judge* heuris-1249 tic. It is worth noting that different generation 1250 configurations result in different splits, as LLMs 1251 might produce different answers. All examples 1252 classified as *Rejected* were discarded from further 1253 experiments. We can observe that in most cases, 1254 datasets are imbalanced, underrepresenting non-1255 hallucinated examples. Only for TriviaOA, there is 1256 an approximately balanced number of examples or 1257 even more non-hallucinated ones, depending on the 1258 configuration used. We split each dataset into 80% training examples and 20% test examples. Splits were stratified according to hallucination labels.

F **Extended results**

F.1 Extended method comparison

In Tables 4 and 5, we present the extended results from Table 1 in the main part of this paper. These results cover probes trained with both all-layers and per-layer variants across all models, as well as lower temperature ($temp \in \{0.1, 1.0\}$). In all cases, the all-layers variant outperforms the perlayer variant, suggesting that hallucination-related information is distributed across multiple layers. Additionally, we observe a smaller generalisation gap (measured as the difference between test and training performance) for the LapEigvals method, indicating more robust features present in the Laplacian eigenvalues. Finally, as demonstrated in Section 6, increasing the temperature during answer generation improves probe performance, which is also evident in Table 4, where probes trained on answers generated with temp = 1.0 consistently outperform those trained on data generated with temp = 0.1.

F.2 Best found hyperparameters

We present the hyperparameter values corresponding to the results in Table 1 and Table 4. Table 6 shows the optimal hyperparameter k for selecting the top-k eigenvalues from either the attention maps in AttnEigvals or the Laplacian matrix in LapEigvals. While fewer eigenvalues were sufficient for optimal performance in some cases, the best results were generally achieved with the highest tested value, k = 100.

Table 7 reports the layer indices that yielded the highest performance for the per-layer models. Performance typically peaked in layers above the

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⁷https://github.com/alibaba/eigenscore (MIT license)

⁸https://github.com/lorenzkuhn/semantic_ uncertainty (MIT license)



Figure 8: Token count histograms for datasets used in the experiments. Number of tokens determined separately for *question* (left-hand-side plots) and gold *answer* (right-hand-side plots) of each example in the datasets with meta-llama/Llama-3.1-8B-Instruct tokeniser (whenever multiple possible answers occurred, they were flattened).

129610th, especially for Llama-3.1-8B, where atten-1297tion maps from the final layers more often led to1298better hallucination detection. Interestingly, the1299first layer's attention maps also produced strong1300performance in a few cases. Overall, no clear pat-1301tern emerges regarding the optimal layer, and as1302noted in prior work, selecting the best layer in the1303per-layer setup often requires a search.

F.3 Comparison with hidden-states-based baselines

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1306 We take approach considered in the previous works (Azaria and Mitchell, 2023; Orgad et al., 2025) 1307 and aligned to our evaluation protocol. Specifi-1308 cally, we trained a logistic regression classifier on PCA-projected hidden states to predict whether the 1310 model is hallucinating or not. To this end, we se-1311 lect the last token of the answer. While we also 1312 tested the last token of the prompt, we observed 1313 significantly lower performance, which aligns with 1314 results presented by (Orgad et al., 2025). We con-1315 sidered hidden states either from all layers or a 1316 single layer corresponding to the selected token. In 1317 all-layer scenario we use concatenation of hidden 1318 states of all layers, and in per-layer scenario we use 1319 hidden states of each layer separately and select the 1320 best performing layer. 1321

In Table 8 we show obtained results. The all-layer version is consistently worse than our LapEigvals. Although the per-layer version outperforms LapEigvals, we argue that it should be treated as a rough reference since it is not a fully fair comparison. Note that our LapEigvals is designed specifically to operate on attention maps and shows the best performance among all attentionbased methods. Our work is one of the first to detect hallucinations solely using attention maps, providing an important insight about behaviour of LLMs, and it motivates further theoretical research on information flow patterns inside these models.

G Extended results of ablations

1336In the following section, we extend the ablation1337results presented in Section 6.1 and Section 6.2.1338Figure 10 compares the top k eigenvalues across1339all five LLMs. In Figure 11 we present a layer-wise1340performance comparison for each model.

H Extended results of generalisation study

We present the complete results of the generalisa-1343 tion ablation discussed in Section 6.4 of the main 1344 paper. Table 9 reports the absolute Test AUROC 1345 values for each method and test dataset. Except 1346 for TruthfulQA, LapEigvals achieves the highest 1347 performance across all configurations. Notably, 1348 some methods perform close to random, whereas 1349 LapEigvals consistently outperforms this baseline. 1350 Regarding relative performance drop (Figure 12), 1351 LapEigvals remains competitive, exhibiting the 1352 lowest drop in nearly half of the scenarios. These 1353 results indicate that our method is robust but war-1354 rants further investigation across more datasets, par-1355 ticularly with a deeper analysis of TruthfulQA. 1356

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I QA prompts

Following, we describe all prompts for QA used to obtain the results presented in this work:

- prompt p₁ medium-length one-shot prompt 1360 with single example of QA task (Listing 1), 1361
 prompt p₂ medium-length zero-shot prompt 1362 without examples (Listing 2), 1363
- prompt p_3 long few-shot prompt; the main prompt used in this work; modification of prompt used by (Kossen et al., 2024) (Listing 3),
- prompt p_4 short-length zero-shot prompt without examples (Listing 4).

J LLM-as-Judge prompt

During hallucinations dataset construction we lever-
aged *llm-as-judge* approach to label answers gen-
erated by the LLMs. To this end, we utilised
gpt-4o-mini with prompt in Listing 5, which is
an adapted version of the prompt used by (Orgad
et al., 2025).1371
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Figure 9: Number of examples per each label in generated datasets (Hallucination - number of hallucinated examples, Non-Hallucination - number of truthful examples, Rejected - number of examples unable to evaluate).

Table 4: (Part I) Performance comparison of methods evaluated in this work on an extended set of configurations. We marked results for AttentionScore in gray as it is unsupervised approach, not directly comparable to the other ones. In **bold**, we highlight the best performance on the test split of data, individually for each dataset, LLM, and temperature.

LLM	Temp	Feature	all-layers	per-layer			Train	Train AUROC					Test.	Test AUROC		
				_	CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA	CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Llama3.1-8B	0.1	AttentionScore		~	0.509	0.667	0.607	0.556	0.567	0.563	0.541	0.653	0.631	0.575	0.571	0.650
Llama3.1-8B	0.1	AttentionScore	>		0.494	0.614	0.568	0.522	0.522	0.489	0.504	0.587	0.558	0.521	0.511	0.537
Llama3.1-8B	0.1	AttnLogDet		>	0.574	0.776	0.702	0.688	0.739	0.709	0.606	0.770	0.713	0.708	0.741	0.777
Llama3.1-8B	0.1	AttnLogDet	>		0.843	0.884	0.851	0.839	0.861	0.913	0.770	0.837	0.768	0.758	0.827	0.820
Llama3.1-8B	0.1	AttnEigvals		>	0.764	0.828	0.713	0.742	0.793	0.680	0.729	0.799	0.728	0.749	0.773	0.790
Llama3.1-8B	0.1	AttnEigvals	>		0.861	0.895	0.878	0.858	0.867	0.979	0.776	0.838	0.755	0.781	0.822	0.819
Llama3.1-8B	0.1	LapEigvals	`	>	0.758	0.817	0.698	0.707	0.781	0.708	0.757	0.793	0.711	0.733	0.780	0.764
Llama3.1-8B	0.1	LapEigvals	>		0.809	106.0	0.804	CC8.U	0.890	0.903	0.830	0.86/	./93	0./82	0.8/2	0.822
Llama3.1-8B	1.0	AttentionScore		>	0.514	0.640	0.607	0.558	0.578	0.533	0.525	0.642	0.607	0.572	0.602	0.629
Llama3.1-8B	1.0	AttentionScore	>		0.507	0.602	0.580	0.534	0.535	0.546	0.493	0.589	0.556	0.538	0.532	0.541
Llama3.1-8B	1.0	AttnLogDet		>	0.596	0.755	0.704	0.697	0.750	0.757	0.597	0.763	0.757	0.686	0.754	0.771
Llama3.1-8B	1.0	AttnLogDet	>		0.848	0.882	0.856	0.846	0.867	0.930	0.769	0.827	0.793	0.748	0.842	0.814
Llama3.1-8B	1.0	AttnEigvals		>	0.762	0.820	0.758	0.754	0.800	0.796	0.723	0.784	0.732	0.728	0.796	0.770
Llama3.1-8B	1.0	AttnEigvals	>		0.867	0.889	0.873	0.867	0.876	0.972	0.782	0.819	0.790	0.768	0.843	0.833
Llama3.1-8B	1.0	LapEigvals		>	0.760	0.803	0.732	0.722	0.795	0.751	0.743	0.789	0.725	0.724	0.794	0.764
Llama3.1-8B	1.0	LapEigvals	>		0.879	0.896	0.866	0.857	0.901	0.918	0.830	0.874	0.827	0.791	0.889	0.829
I lama3 2-3B	0 1	AttentionScore		, ,	0 526	0 697	0 597	0.570	0.570	0 569	0 547	0 714	0 643	0 582	0 551	0 564
Llama3 2-3B	0.1	AttentionScore	\ \	>	0.506	0.635	0.523	0.515	0.534	0.473	0.519	0.644	0.573	0.561	0.510	0.489
I lama3 2-3B	1.0	AttnLogDet	>	``	0.573	0.762	0.697	0.687	0.719	0.725	0.579	0.774	0.735	0.698	0.711	0.674
Llama3 2-3B	01	AttnLogDet	`	•	0.782	0.868	0.845	0.877	0.824	0.918	0.695	0.843	0.763	0.749	0 796	0.678
Llama3 2-3B	1.0	AttnFiovals	>	``	0.675	0.782	0.750	0.725	0.755	0.727	0.626	0.797	0.734	0.695	0.724	0.720
Llama3 2-3B	1.0	Attn Fiovals	`	>	0.814	0.73	0.872	0.852	0.842	0.963	0.703	0.844	777 U	0.744	0.788	0.688
Llama3 2-3B	0.1	LanFiovals	•	``	0.681	0.774	0.733	0 708	0.733	0.727	0.676	0.781	0.736	0.697	0.732	0.690
Llama3.2-3B	0.1	LapEigvals	>		0.831	0.875	0.837	0.832	0.852	0.895	0.801	0.857	0.779	0.736	0.826	0.743
I lama 22-2B	1 0	AttentionScore			0 537	0.668	0 588	0 578	0 553	0 555	0 557	0 637	0 507	0 503	0 558	0.675
Llama3.2-3B	1.0	AttentionScore	>	>	0.512	0.606	0.554	0.529	0.517	0.484	0.509	0.588	0.546	0.530	0.515	0.581
Llama3.2-3B	1.0	AttnLogDet		>	0.578	0.738	0.677	0.720	0.716	0.739	0.597	0.724	0.678	0.707	0.711	0.742
Llama3.2-3B	1.0	AttnLogDet	>		0.784	0.869	0.816	0.839	0.831	0.924	0.700	0.801	0.690	0.734	0.789	0.795
Llama3.2-3B	1.0	AttnEigvals		>	0.642	0.777	0.716	0.747	0.763	0.735	0.641	0.756	0.696	0.703	0.746	0.748
Llama3.2-3B	1.0	AttnEigvals	>	```	0.819	0.878	0.847	0.876	0.847	0.978	0.724	0.819	0.694	0.749	0.804	0.723
Llama3.2-3B Llama3 2-3B	0.1	LapEigvals LanFiovals	`	>	0.842	0.764	0.083	0.719	0.127	0.082	0.812	0.734 0.828	0.693	0.757	0.832	0.787
		om Orador							10000							
C.6104	1.0	AttentionScore	``	>	/10/0	9002 0	COC.U	0.000	CZ0.U	100.0	272.U	100.0	0.03/	0.021	0.624	0.03/
Dhi3 5	1.0	AttnLogDat	>	``	0.583	0.730	0 741	0.410	75L 0	0022.0	0 585	110.0	0.785	0.776	+cc.0	0 765
Phi3.5	1.0	AttnLogDet	>	>	0.845	0.863	0.905	0.852	0.875	0.981	0.723	0.802	0.802	0.759	0.842	0.716
Phi3.5	0.1	AttnEigvals		>	0.760	0.781	0.793	0.745	0.802	0.854	0.678	0.764	0.790	0.747	0.791	0.774
Phi3.5	0.1	AttnEigvals	>		0.862	0.867	0.904	0.861	0.881	0.999	0.728	0.802	0.787	0.740	0.838	0.761
Phi3.5	0.1	LapEigvals		>	0.734	0.758	0.737	0.704	0.775	0.759	0.716	0.757	0.761	0.732	0.768	0.741
Phi3.5	0.1	LapEigvals	>	_	0.856	0.860	0.897	0.841	0.884	0.965	0.810	0.819	0.815	0.791	0.858	0.717
Phi3.5	1.0	AttentionScore		~	0.499	0.567	0.615	0.626	0.637	0.618	0.533	0.581	0.630	0.645	0.642	0.626
Phi3.5	1.0	AttentionScore	>		0.489	0.540	0.566	0.469	0.553	0.541	0.520	0.541	0.594	0.504	0.540	0.554
Phi3.5	1.0	AttnLogDet		>	0.587	0.733	0.773	0.722	0.766	0.753	0.557	0.762	0.784	0.736	0.772	0.763
Phi3.5	1.0	AttnLogDet	>		0.842	0.868	0.921	0.859	0.879	0.971	0.745	0.818	0.815	0.769	0.848	0.755
Phi3.5	1.0	AttnEigvals	``	>	0.755	0.794	0.820	0.790	0.809	0.864	0.710	0.795	0.787	0.752	0.799	0.747
2.5idd	1.0	AttnEigvals LonFiguals	>		0.828	0.871	0.924	0.8/6	0.88/0	0.998	0.773	0.829	0.755	0.737	0.820	0.802
C.CIIIT Phi3.5	0.1	LapEigvais LanFiovals	7	>	0.856	0.863	cc/.0 116.0	0.710	0.880	0.061	0.821	0.836	0.826	0.795	0.872	727.0 777.0
		0-1-J														

													1			
LLM	Temp	Feature	all-layers	per-layer			Train	Train AUROC					Test .	Test AUROC		
					CoQA I	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA	CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Mistral-Nemo	0.1	AttentionScore		>	0.504	0.574	0.591	0.509	0.550	0.546	0.515	0.559	0.587	0.527	0.545	0.681
Mistral-Nemo	0.1	AttentionScore	>		0.508	0.536	0.537	0.507	0.520	0.535	0.484	0.523	0.533	0.495	0.505	0.631
Mistral-Nemo	0.1	AttnLogDet		>	0.584	0.716	0.702	0.675	0.689	0.744	0.583	0.723	0.688	0.668	0.722	0.731
Mistral-Nemo	0.1	AttnLogDet	>		0.828	0.842	0.861	0.858	0.854	0.963	0.734	0.786	0.752	0.709	0.822	0.776
Mistral-Nemo	0.1	AttnEigvals		>	0.708	0.751	0.749	0.749	0.747	0.797	0.672	0.740	0.701	0.704	0.738	0.717
Mistral-Nemo	0.1	AttnEigvals	>		0.845	0.842	0.878	0.864	0.859	0.996	0.768	0.789	0.743	0.716	0.809	0.752
Mistral-Nemo Mistral-Nemo	$0.1 \\ 0.1$	LapEigvals LapEigvals	>	>	0.763 0.868	$0.772 \\ 0.862$	$0.732 \\ 0.875$	0.723 0.869	$0.781 \\ 0.886$	0.725 0.977	0.759 0.823	0.760 0.821	0.697 0.755	0.696 0.767	0.769 0.858	$0.710 \\ 0.737$
Mistral-Nemo	1.0	AttentionScore			0.502	0.586	0.606	0.546	0.553	0.570	0.525	0.587	0.588	0.564	0.570	0.632
Mistral-Nemo	1.0	AttentionScore	>		0.493	0.541	0.552	0.503	0.521	0.531	0.493	0.531	0.529	0.510	0.532	0.494
Mistral-Nemo	1.0	AttnLogDet		>	0.591	0.723	0.716	0.717	0.717	0.741	0.581	0.730	0.703	0.711	0.707	0.801
Mistral-Nemo	1.0	AttnLogDet	>		0.829	0.851	0.870	0.860	0.857	0.963	0.728	0.798	0.769	0.772	0.812	0.852
Mistral-Nemo	1.0	AttnEigvals		>	0.704	0.762	0.742	0.757	0.752	0.806	0.670	0.749	0.742	0.719	0.737	0.804
Mistral-Nemo	1.0	AttnEigvals	>		0.844	0.851	0.893	0.864	0.862	0.996	0.778	0.781	0.761	0.758	0.821	0.802
Mistral-Nemo	1.0	LapEigvals		<u> </u>	0.765	0.790	0.749	0.740	0.804	0.779	0.738	0.763	0.708	0.723	0.785	0.818
Mistral-Nemo	1.0	LapEigvals	 		0.876	0.877	0.884	0.881	0.901	0.978	0.835	0.833	0.795	0.812	0.865	0.828
Mistral-Small-24B	0.1	AttentionScore		>	0.520	0.538	0.517	0.577	0.535	0.571	0.525	0.552	0.592	0.625	0.533	0.724
Mistral-Small-24B	0.1	AttentionScore	>		0.520	0.472	0.449	0.510	0.449	0.491	0.493	0.493	0.467	0.556	0.461	0.645
Mistral-Small-24B	0.1	AttnLogDet		>	0.585	0.674	0.659	0.724	0.685	0.698	0.586	0.684	0.695	0.752	0.682	0.721
Mistral-Small-24B	0.1	AttnLogDet	>		0.851	0.817	0.799	0.820	0.861	0.898	0.762	0.760	0.725	0.763	0.778	0.767
Mistral-Small-24B	0.1	AttnEigvals		>	0.734	0.722	0.667	0.745	0.757	0.732	0.720	0.707	0.697	0.773	0.758	0.765
Mistral-Small-24B	1.0	AttnEigvals	>		0.8/2	0.8/3	0.925	0.903	0.777	0.993	0.793	0.710	0.731	0.803	0.809	06/.0
Mistral-Small-24B	0.1	LapEigvals	>		0.887	0.870	0.901	0.887	0.905	0.979	0.852	0.808	0.722	0.821	0.831	0.757
Mistral-Small-24B	1.0	AttentionScore		>	0.511	0.555	0.582	0.561	0.562	0.542	0.535	0.566	0.576	0.567	0.574	0.606
Mistral-Small-24B	1.0	AttentionScore	>		0.497	0.503	0.463	0.519	0.451	0.493	0.516	0.504	0.462	0.455	0.463	0.451
Mistral-Small-24B	1.0	AttnLogDet		>	0.591	0.727	0.710	0.732	0.720	0.677	0.600	0.771	0.714	0.726	0.734	0.687
Mistral-Small-24B	1.0	AttnLogDet	>		0.850	0.847	0.827	0.856	0.853	0.877	0.766	0.842	0.747	0.753	0.833	0.735
Mistral-Small-24B	1.0	AttnEigvals		>	0.757	0.743	0.728	0.764	0.779	0.741	0.723	0.780	0.733	0.734	0.780	0.718
Mistral-Small-24B	1.0	AttnEigvals	>		0.877	0.878	0.923	0.911	0.895	0.997	0.805	0.848	0.751	0.760	0.844	0.765
Mistral-Small-24B	0.1	LapEigvals		>	0.814	0.702	0.000	06/00	00/00	0./05	CU8.U	06/ 0	0./12	18/.0	0.1/9	07/.0
MISURAL-AURINI	1.0	Laprigvais	>	_	CK0.0	0.020	0.070	N14.U	102.0	נטליט	100.0	700'N	1.171	0.040	0.0/0	0./40

Table 5: (Part II) Performance comparison of methods evaluated in this work on an extended set of configurations. We marked results for AttentionScore in gray as it is unsupervised approach, not directly comparable to the other ones. In **bold**, we highlight the best performance on the test split of data, individually for each dataset, LLM, and temperature.

LLM	Temp	Feature	all-layers	per-layer			top-k e	eigenvalues		
					CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Llama3.1-8B	0.1	AttnEigvals		\checkmark	50	100	25	100	100	10
Llama3.1-8B	0.1	AttnEigvals	\checkmark		100	100	100	100	50	100
Llama3.1-8B	0.1	LapEigvals		\checkmark	50	100	10	100	100	100
Llama3.1-8B	0.1	LapEigvals	\checkmark		10	100	100	100	100	100
Llama3.1-8B	1.0	AttnEigvals		\checkmark	100	100	100	100	100	100
Llama3.1-8B	1.0	AttnEigvals	\checkmark		100	100	100	100	100	100
Llama3.1-8B	1.0	LapEigvals		\checkmark	100	100	100	100	100	100
Llama3.1-8B	1.0	LapEigvals	\checkmark		100	25	100	100	100	100
Llama3.2-3B	0.1	AttnEigvals		\checkmark	100	100	100	100	100	10
Llama3.2-3B	0.1	AttnEigvals	\checkmark		100	25	100	100	100	100
Llama3.2-3B	0.1	LapEigvals		\checkmark	100	100	100	100	50	5
Llama3.2-3B	0.1	LapEigvals	\checkmark		25	100	100	100	100	100
Llama3.2-3B	1.0	AttnEigvals		\checkmark	100	100	100	100	100	50
Llama3.2-3B	1.0	AttnEigvals	\checkmark		100	100	100	100	100	100
Llama3.2-3B	1.0	LapEigvals		\checkmark	100	100	10	100	100	25
Llama3.2-3B	1.0	LapEigvals	\checkmark		25	100	100	100	100	100
Phi3.5	0.1	AttnEigvals		\checkmark	100	100	100	100	100	100
Phi3.5	0.1	AttnEigvals	\checkmark		100	10	10	25	100	50
Phi3.5	0.1	LapEigvals		\checkmark	100	100	100	100	100	100
Phi3.5	0.1	LapEigvals	\checkmark		10	50	100	100	100	100
Phi3.5	1.0	AttnEigvals		\checkmark	100	100	100	100	100	100
Phi3.5	1.0	AttnEigvals	\checkmark		100	100	10	100	100	50
Phi3.5	1.0	LapEigvals		\checkmark	100	100	100	100	100	50
Phi3.5	1.0	LapEigvals	√		10	100	100	100	100	100
Mistral-Nemo	0.1	AttnEigvals		\checkmark	100	100	100	100	100	100
Mistral-Nemo	0.1	AttnEigvals	\checkmark		100	100	100	100	100	100
Mistral-Nemo	0.1	LapEigvals		\checkmark	100	100	100	100	100	10
Mistral-Nemo	0.1	LapEigvals	✓		10	25	100	50	100	100
Mistral-Nemo	1.0	AttnEigvals		\checkmark	100	100	100	100	100	100
Mistral-Nemo	1.0	AttnEigvals	\checkmark		100	100	100	100	50	100
Mistral-Nemo	1.0	LapEigvals		\checkmark	100	100	50	100	100	100
Mistral-Nemo	1.0	LapEigvals	\checkmark		10	50	100	100	100	100
Mistral-Small-24B	0.1	AttnEigvals		\checkmark	100	100	10	100	50	25
Mistral-Small-24B	0.1	AttnEigvals	\checkmark		100	100	100	100	100	25
Mistral-Small-24B	0.1	LapEigvals		\checkmark	100	100	50	100	100	10
Mistral-Small-24B	0.1	LapEigvals	✓		25	100	100	100	10	100
Mistral-Small-24B	1.0	AttnEigvals		\checkmark	100	100	100	100	100	100
Mistral-Small-24B	1.0	AttnEigvals	\checkmark		100	100	100	100	100	100
Mistral-Small-24B	1.0	LapEigvals		\checkmark	100	100	100	50	100	50
Mistral-Small-24B	1.0	LapEigvals	\checkmark		10	50	10	10	100	50

Table 6: Values of k hyperparameter, denoting how many highest eigenvalues are taken from the Laplacian matrix, corresponding to the best results in Table 1 and Table 4.

LLM	temp	Feature			Lay	er index		
			CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Llama3.1-8B	0.1	AttentionScore	13	10	0	0	0	28
Llama3.1-8B	0.1	AttnLogDet	7	13	16	11	29	21
Llama3.1-8B	0.1	AttnEigvals	22	31	26	31	31	7
Llama3.1-8B	0.1	LapEigvals	15	14	20	29	31	20
Llama3.1-8B	1.0	AttentionScore	29	10	0	0	0	23
Llama3.1-8B	1.0	AttnLogDet	17	11	13	29	29	30
Llama3.1-8B	1.0	AttnEigvals	22	31	31	31	31	31
Llama3.1-8B	1.0	LapEigvals	15	14	31	29	29	29
Llama3.2-3B	0.1	AttentionScore	15	12	12	12	21	14
Llama3.2-3B	0.1	AttnLogDet	12	13	24	10	25	14
Llama3.2-3B	0.1	AttnEigvals	27	14	14	25	27	17
Llama3.2-3B	0.1	LapEigvals	11	8	12	25	12	14
Llama3.2-3B	1.0	AttentionScore	24	12	0	24	21	14
Llama3.2-3B	1.0	AttnLogDet	12	26	23	25	25	12
Llama3.2-3B	1.0	AttnEigvals	11	27	25	25	27	10
Llama3.2-3B	1.0	LapEigvals	11	18	12	25	25	11
Phi3.5	0.1	AttentionScore	7	15	0	0	0	19
Phi3.5	0.1	AttnLogDet	20	18	16	17	13	23
Phi3.5	0.1	AttnEigvals	18	19	15	19	18	28
Phi3.5	0.1	LapEigvals	18	28	28	19	31	28
Phi3.5	1.0	AttentionScore	19	0	1	0	0	19
Phi3.5	1.0	AttnLogDet	12	29	14	19	13	14
Phi3.5	1.0	AttnEigvals	18	30	17	31	31	31
Phi3.5	1.0	LapEigvals	18	28	15	19	31	31
Mistral-Nemo	0.1	AttentionScore	2	18	35	0	30	35
Mistral-Nemo	0.1	AttnLogDet	37	17	15	38	38	33
Mistral-Nemo	0.1	AttnEigvals	38	38	18	18	15	31
Mistral-Nemo	0.1	LapEigvals	16	37	37	18	37	8
Mistral-Nemo	1.0	AttentionScore	10	16	28	14	30	21
Mistral-Nemo	1.0	AttnLogDet	18	20	18	18	15	18
Mistral-Nemo	1.0	AttnEigvals	38	39	39	18	15	18
Mistral-Nemo	1.0	LapEigvals	16	37	37	18	37	18
Mistral-Small-24B	0.1	AttentionScore	14	39	33	35	0	30
Mistral-Small-24B	0.1	AttnLogDet	16	38	18	16	38	11
Mistral-Small-24B	0.1	AttnEigvals	36	36	19	16	38	20
Mistral-Small-24B	0.1	LapEigvals	21	35	24	36	35	34
Mistral-Small-24B	1.0	AttentionScore	15	1	0	1	0	30
Mistral-Small-24B	1.0	AttnLogDet	14	27	17	24	38	34
Mistral-Small-24B	1.0	AttnEigvals	36	27	21	24	36	23
Mistral-Small-24B	1.0	LapEigvals	21	36	16	21	35	34

Table 7: Values of a layer index (numbered from 0) corresponding to the best results for *per-layer* models in Table 4.

LLM	Temp	Features	per-layer	all-layers			Test A	UROC (†)		
					CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
Llama3.1-8B	0.1	HiddenStates	\checkmark		0.835	0.840	0.766	0.736	0.820	0.834
Llama3.1-8B	0.1	HiddenStates		\checkmark	0.821	0.825	0.728	0.723	0.791	0.785
Llama3.1-8B	0.1	LapEigvals	\checkmark		0.757	0.793	0.711	0.733	0.780	0.764
Llama3.1-8B	0.1	LapEigvals		\checkmark	0.836	0.867	0.793	0.782	0.872	0.822
Llama3.1-8B	1.0	HiddenStates	\checkmark		0.836	0.850	0.786	0.754	0.850	0.823
Llama3.1-8B	1.0	HiddenStates		\checkmark	0.835	0.847	0.757	0.749	0.838	0.808
Llama3.1-8B	1.0	LapEigvals	\checkmark		0.743	0.789	0.725	0.724	0.794	0.764
Llama3.1-8B	1.0	LapEigvals		\checkmark	0.830	0.874	0.827	0.791	0.889	0.829
Llama3.2-3B	0.1	HiddenStates	\checkmark		0.800	0.808	0.732	0.750	0.782	0.760
Llama3.2-3B	0.1	HiddenStates		\checkmark	0.790	0.784	0.709	0.721	0.760	0.770
Llama3.2-3B	0.1	LapEigvals	\checkmark		0.676	0.774	0.730	0.727	0.712	0.690
Llama3.2-3B	0.1	LapEigvals		\checkmark	0.801	0.844	0.771	0.778	0.821	0.743
Llama3.2-3B	1.0	HiddenStates	\checkmark		0.778	0.758	0.679	0.719	0.773	0.716
Llama3.2-3B	1.0	HiddenStates		\checkmark	0.773	0.753	0.657	0.681	0.761	0.618
Llama3.2-3B	1.0	LapEigvals	\checkmark		0.715	0.765	0.696	0.696	0.738	0.767
Llama3.2-3B	1.0	LapEigvals		\checkmark	0.812	0.857	0.798	0.751	0.836	0.787
Phi3.5	0.1	HiddenStates	\checkmark		0.841	0.845	0.813	0.781	0.886	0.73
Phi3.5	0.1	HiddenStates		\checkmark	0.833	0.840	0.806	0.774	0.878	0.68
Phi3.5	0.1	LapEigvals	\checkmark		0.716	0.757	0.761	0.732	0.768	0.74
Phi3.5	0.1	LapEigvals		\checkmark	0.810	0.819	0.815	0.791	0.858	0.717
Phi3.5	1.0	HiddenStates	\checkmark		0.872	0.850	0.821	0.806	0.891	0.822
Phi3.5	1.0	HiddenStates		\checkmark	0.853	0.844	0.804	0.790	0.887	0.752
Phi3.5	1.0	LapEigvals	\checkmark		0.723	0.769	0.755	0.732	0.792	0.732
Phi3.5	1.0	LapEigvals		\checkmark	0.821	0.836	0.826	0.795	0.872	0.777
Mistral-Nemo	0.1	HiddenStates	\checkmark		0.818	0.814	0.734	0.731	0.821	0.792
Mistral-Nemo	0.1	HiddenStates		\checkmark	0.805	0.784	0.722	0.730	0.793	0.699
Mistral-Nemo	0.1	LapEigvals	\checkmark		0.759	0.760	0.697	0.696	0.769	0.710
Mistral-Nemo	0.1	LapEigvals		\checkmark	0.823	0.821	0.755	0.767	0.858	0.737
Mistral-Nemo	1.0	HiddenStates	\checkmark		0.793	0.777	0.738	0.719	0.783	0.722
Mistral-Nemo	1.0	HiddenStates		\checkmark	0.771	0.771	0.706	0.685	0.779	0.644
Mistral-Nemo	1.0	LapEigvals	\checkmark		0.738	0.763	0.708	0.723	0.785	0.818
Mistral-Nemo	1.0	LapEigvals		\checkmark	0.835	0.833	0.795	0.812	0.865	0.828
Mistral-Small-24B	0.1	HiddenStates	\checkmark		0.838	0.744	0.680	0.700	0.749	0.73
Mistral-Small-24B	0.1	HiddenStates		\checkmark	0.815	0.703	0.632	0.629	0.726	0.589
Mistral-Small-24B	0.1	LapEigvals	\checkmark		0.800	0.719	0.674	0.784	0.757	0.823
Mistral-Small-24B	0.1	LapEigvals		\checkmark	0.852	0.808	0.722	0.821	0.831	0.757
Mistral-Small-24B	1.0	HiddenStates	\checkmark		0.801	0.720	0.665	0.603	0.684	0.58
Mistral-Small-24B	1.0	HiddenStates		\checkmark	0.770	0.703	0.617	0.575	0.659	0.485
Mistral-Small-24B	1.0	LapEigvals	\checkmark		0.805	0.790	0.712	0.781	0.779	0.725
Mistral-Small-24B	1.0	LapEigvals		\checkmark	0.861	0.882	0.791	0.820	0.876	0.74

Table 8: Results of the probe trained on the hidden state features from the last generated token.

Table 9: Full results of the generalisation study. By gray color we denote results obtained on test split from the same QA dataset as training split, otherwise results are from test split of different QA dataset. We highlight the best performance in **bold**.

Feature	Train Dataset			Test A	UROC (†)		
		CoQA	HaluevalQA	NQOpen	SQuADv2	TriviaQA	TruthfulQA
AttnLogDet	CoQA	0.758	0.687	0.644	0.646	0.640	0.587
AttnEigvals	CoQA	0.782	0.726	0.696	0.659	0.702	0.560
LapEigvals	CoQA	0.830	0.790	0.748	0.743	0.786	0.629
AttnLogDet	HaluevalQA	0.580	0.823	0.750	0.727	0.787	0.668
AttnEigvals	HaluevalQA	0.579	0.819	0.792	0.743	0.803	0.688
LapEigvals	HaluevalQA	0.685	0.873	0.796	0.778	0.848	0.595
AttnLogDet	NQOpen	0.552	0.720	0.794	0.717	0.766	0.597
AttnEigvals	NQOpen	0.546	0.725	0.790	0.714	0.770	0.618
LapEigvals	NQOpen	0.656	0.792	0.827	0.748	0.843	0.564
AttnLogDet	SQuADv2	0.553	0.716	0.774	0.746	0.757	0.658
AttnEigvals	SQuADv2	0.576	0.730	0.737	0.768	0.760	0.711
LapEigvals	SQuADv2	0.673	0.801	0.806	0.791	0.841	0.625
AttnLogDet	TriviaQA	0.565	0.761	0.793	0.736	0.838	0.572
AttnEigvals	TriviaQA	0.577	0.770	0.786	0.742	0.843	0.616
LapEigvals	TriviaQA	0.702	0.813	0.818	0.773	0.889	0.522
AttnLogDet	TruthfulQA	0.550	0.597	0.603	0.604	0.662	0.811
AttnEigvals	TruthfulQA	0.538	0.600	0.595	0.646	0.685	0.833
LapEigvals	TruthfulQA	0.590	0.552	0.529	0.569	0.631	0.829



Figure 10: Probe performance across different topk eigenvalues: $k \in \{5, 10, 25, 50, 100\}$ for TriviQA dataset with temp = 1.0 and 5 considered LLMs.



Figure 11: Analysis of model performance across different layers for and 5 considered LLMs and TriviaQA dataset with temp = 1.0 and k = 100 top eigenvalues (results for models operating on all layers provided for reference).



Figure 12: Generalisation across datasets measured as a per cent performance drop in Test AUROC (less is better) when trained on one dataset and tested on the other. Training datasets are indicated in the plot titles, while test datasets are shown on the x-axis. Results computed on Llama-3.1-8B with k = 100 top eigenvalues and temp = 1.0.

Listing 1: One-shot QA (prompt p_1)

Listing 2: Zero-sho QA (prompt p_2).

```
Please provide a concise and direct response to the following question, keeping your

→ answer as brief and to-the-point as possible while ensuring clarity. Avoid

→ any unnecessary elaboration or additional details.

Question: {question}

Answer:
```

Listing 3: Few-shot QA prompt (prompt p_3), modified version of prompt used by (Kossen et al., 2024).

Answer the following question as briefly as possible. Here are several examples: Question: What is the capital of France? Answer: Paris Question: Who wrote *Romeo and Juliet*? Answer: William Shakespeare Question: What is the boiling point of water in Celsius? Answer: 100°C Question: How many continents are there on Earth? Answer: Seven Question: What is the fastest land animal? Answer: Cheetah Question: {question} Answer:

Listing 4: Zero-shot shor QA prompt (prompt p_4).

```
Answer the following question as briefly as possible.
Question: {question}
Answer:
```

Listing 5: Prompt used in *llm-as-judge* approach for determining halucination labels. Prompt is a modified version of the one used by (Orgad et al., 2025).

You will evaluate answers to questions. For each question, I will provide a model's $\,\hookrightarrow\,$ answer and one or more correct reference answers. You would have to determine if the model answer is correct, incorrect, or model \hookrightarrow refused to answer. The model answer to be correct has to match from one to \hookrightarrow all of the possible correct answers. If the model answer is correct, write 'correct' and if it is not correct, write ' \hookrightarrow incorrect'. If the Model Answer is a refusal, stating that they don't have ← enough information, write 'refuse'. For example: Question: who is the young guitarist who played with buddy guy? Ground Truth: [Quinn Sullivan, Eric Gales] Model Answer: Ronnie Earl Correctness: incorrect Question: What is the name of the actor who plays Iron Man in the Marvel movies? Ground Truth: [Robert Downey Jr.] Model Answer: Robert Downey Jr. played the role of Tony Stark/Iron Man in the Marvel ↔ Cinematic Universe films. Correctness: correct Question: what is the capital of France? Ground Truth: [Paris] Model Answer: I don't have enough information to answer this question. Correctness: refuse Question: who was the first person to walk on the moon? Ground Truth: [Neil Armstrong] Model Answer: I apologise, but I cannot provide an answer without verifying the \hookrightarrow historical facts. Correctness: refuse Question: {{question}} Ground Truth: {{gold_answer}}
Model Answer: {{predicted_answer}} Correctness: