

000 001 LATENT POSTERIOR-MEAN RECTIFIED FLOW FOR 002 HIGH-FIDELITY PERCEPTUAL FACE RESTORATION 003 004

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007 008 ABSTRACT 009

011 The perception-distortion (PD) tradeoff theory suggests that image restoration al-
012 gorithms must balance perceptual quality and fidelity. To achieve minimal dis-
013 tortion while maintaining perfect perceptual quality, Posterior-Mean Rectified
014 Flow (PMRF) proposes a flow-based approach where the source distribution is
015 minimal-distortion estimates. Although PMRF is shown to be effective, its pixel-
016 space modeling approach limits its ability to align with human perception. In
017 this work, we propose **Latent-PMRF**, which reformulates PMRF in the latent
018 space of a variational autoencoder (VAE), facilitating better alignment with hu-
019 man perception during optimization. By defining the source distribution on latent
020 representations of minimal-distortion estimates, we bound the minimum distor-
021 tion by the VAE’s reconstruction error. Moreover, we reveal that the design of
022 VAE is crucial, and our proposed **Sim-VAE** significantly outperforms existing
023 VAEs in both reconstruction and restoration. Extensive experiments on blind face
024 restoration demonstrate the superiority of Latent-PMRF, offering an improved PD-
025 tradeoff compared to existing methods, along with remarkable convergence effi-
026 ciency, achieving a $5.79 \times$ speedup over PMRF in terms of FID. Our code will be
027 publicly available.

028 1 INTRODUCTION 029

030 Face images are among the most common types of images, yet they often suffer from complex
031 degradations during formation, recording, processing, and transmission (Wang et al., 2022a). Typi-
032 cal degradations, such as blur (Zhang et al., 2022), noise (Elad et al., 2023), downsampling (Dong
033 et al., 2014; Luo et al., 2023; Liang et al., 2021), and JPEG compression (Jiang et al., 2021), can
034 significantly degrade visual quality. Perceptual face restoration aims to recover high-quality, visu-
035 ally pleasing face images from degraded inputs. The key challenge lies in enhancing perceptual
036 quality while maintaining fidelity. Recent studies show that generative models, particularly diffu-
037 sion models (Sohl-Dickstein et al., 2015; Ho et al., 2020; Li et al., 2022; Wang et al., 2024) and flow
038 matching models (Zhu et al., 2024; Ohayon et al., 2025), offer strong solutions for perceptual quality
039 by modeling the distribution of natural images. Although such posterior modeling approaches can
040 achieve perfect perceptual quality in theory, they do not guarantee minimal distortion under perfect
041 perceptual quality constraints (Blau & Michaeli, 2018; Freirich et al., 2021; Ohayon et al., 2025).
042 To minimize distortion, Posterior-Mean Rectified Flow (PMRF) (Ohayon et al., 2025) transports
043 minimum distortion estimation to the target distribution using a rectified flow model. This approach
044 can theoretically achieve minimal distortion (Freirich et al., 2021; Ohayon et al., 2025) under perfect
045 perceptual quality constraints.

046 In this work, we challenge the necessity of constructing PMRF in the pixel space. While perceptual
047 quality is formally defined as the statistical distance between the distributions of reconstructed and
048 original images (Blau & Michaeli, 2018), researchers have found that distances in feature space bet-
049 ter correlate with human perception (Heusel et al., 2017; Zhang et al., 2018; Szegedy et al., 2015;
050 Sauer et al., 2021; Kumari et al., 2022). For instance, the most widely used metric for evaluating
051 image generation models is the Fréchet Inception Distance (FID) (Heusel et al., 2017), which mea-
052 suring distribution difference within the feature space of the InceptionNet (Szegedy et al., 2015).
053 Additionally, many Generative Adversarial Networks (GANs) (Goodfellow et al., 2020) define dis-
054 criminator in the feature spaces of pre-trained networks, such as EfficientNet (Sauer et al., 2021)
055 and CLIP (Kumari et al., 2022). These findings suggest that measuring distribution distances in

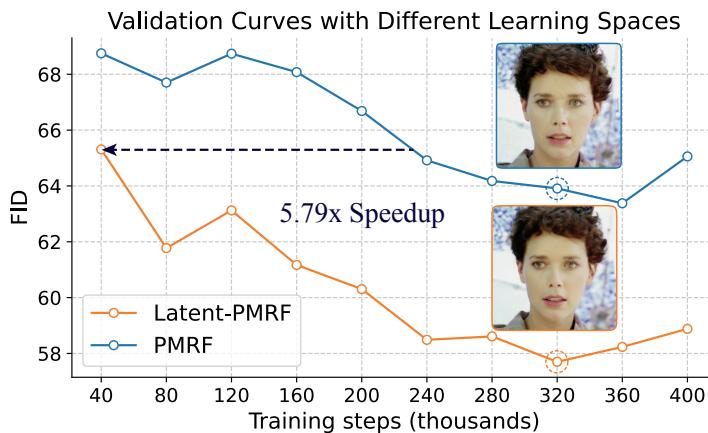


Figure 1: Illustration of perception optimization efficiency in latent space. We train PMRF and Latent-PMRF with the same compute budget. For VAEs with perceptual compression capabilities, differences in their latent space align better with human perception than those in pixel space, making latent space modeling more effective for perception optimization. Validation curves demonstrate the superior perceptual quality achieved by Latent-PMRF, with a $5.79 \times$ speedup over PMRF in terms of FID.

feature space is an effective approach. Motivated by this, we propose reformulating PMRF in the latent space of a variational autoencoder (VAE) (Kingma, 2014), where perceptual quality can be optimized more efficiently, as shown in Figure 1.

While the idea appears straightforward, its optimality in terms of distortion requires careful analysis. Analogous to PMRF, we consider two distinct source distributions: (1) the posterior mean of latent representations, and (2) the latent representations of the posterior mean. We show that the second approach offers several advantages and is preferable. Most notably, it achieves minimal distortion bounded by the VAE’s reconstruction error, which is not guaranteed by the first approach.

Overall, our **Latent-PMRF** can be understood as a rectified flow model (Liu et al., 2023b) in latent space, where the source distribution consists of the latent representations of the posterior mean and the target distribution consists of the latent representations of high-quality (HQ) images. While extensive research has explored latent space models for restoration tasks (Wang et al., 2024; Yue et al., 2023; Zhu et al., 2024; Lin et al., 2024; Gu et al., 2022; Liu et al., 2023a; Wang et al., 2022b), a fundamental question remains: are the commonly used VAEs sufficient for image restoration? We reveal that the VAEs employed in Stable Diffusion (SD) (Rombach et al., 2022), SDXL (Podell et al., 2024), and FLUX (Egger et al., 2024) are suboptimal for this task, as shown in Table 1. Unlike image generation, where increasing latent dimensionality often complicates optimization, restoration tasks benefit from a more informative latent space, as it reduces reconstruction error and thus lowers the minimal distortion bound.

To address this, we propose **Sim-VAE**, a simplified variant of SD-VAE, incorporating loss enhancements and architectural improvements that significantly improve both the VAE’s reconstruction ability and the restoration performance of the final model. Our contributions are summarized as follows:

- Latent-PMRF achieves better alignment with human perception during optimization, resulting in a $5.79 \times$ speedup over PMRF in terms of FID.
- The source distribution design of Latent-PMRF bounds the minimum distortion to the VAE’s reconstruction error, and our improved Sim-VAE significantly boosts restoration performance when integrated with Latent-PMRF.
- Extensive experiments show that our Latent-PMRF achieves an improved PD-tradeoff and produces visually appealing results with high consistency to the inputs.

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 Table 1: **Comparison of VAEs** in CelebA-Test (Wang et al., 2021a). We evaluate the reconstruction
 113 performance of various VAEs and their effectiveness as latent spaces for Latent-PMRF. Notably, our
 114 Sim-VAE demonstrates significantly improved reconstruction capabilities and enhances the per-
 115 formance of Latent-PMRF in restoration. The best results are denoted in **bold**, and the second-best
 116 results are underlined. **f8c4** denotes $8 \times$ downampling with 4 latent channels.

VAE	Reconstruction			Restoration		
	PSNR↑	LPIPS↓	MMD _{DINOv2} ↓	PSNR↑	LPIPS↓	MMD _{DINOv2} ↓
SD1.5-VAE f8c4	30.463	0.044	0.6931	25.875	0.231	1.0302
SD-XL-VAE f8c4	32.396	0.039	0.3973	26.481	0.263	1.0286
FLUX-VAE f8c16	38.763	<u>0.008</u>	<u>0.0675</u>	26.152	0.245	1.0565
SD-VAE f8c32	<u>40.398</u>	0.015	0.0986	25.265	<u>0.222</u>	0.8938
Sim-VAE f8c16	37.903	0.026	0.0966	<u>26.441</u>	0.219	0.8918
Sim-VAE f8c32	42.713	0.007	0.0511	26.382	0.224	0.8770

2 BACKGROUND

2.1 RECTIFIED FLOW

Rectified Flow (Liu et al., 2023b; Lipman et al., 2023; Albergo & Vanden-Eijnden, 2023) is a generative modeling approach that constructs a probability path $(p_t)_{0 \leq t \leq 1}$ from a source distribution p_0 to a target distribution p_1 . Sampling involves drawing $X_0 \sim p_0$ and solving an Ordinary Differential Equation (ODE) defined by a velocity field v_t , which guides the transformation:

$$\frac{d}{dt} \psi_t(x) = v_t(\psi_t(x)), \quad \psi_0(x) = x. \quad (1)$$

The velocity field v_t is parameterized by a neural network v_t^θ and trained via regression to match the conditional velocity field:

$$v_t(x_t | x_0, x_1) = x_1 - x_0, \quad (2)$$

where X_t follows a linear interpolation between $X_0 \sim p_0$ and $X_1 \sim p_1$. The training objective is to minimize the Conditional Flow Matching (CFM) loss:

$$\mathcal{L}_{\text{CFM}}(\theta) = \mathbb{E}_{t, X_t, x_0, X_1} \|v_t^\theta(X_t) - (x_1 - x_0)\|^2. \quad (3)$$

2.2 POSTERIOR-MEAN RECTIFIED FLOW

Let y denote a low-quality (LQ) image, which is a realization of a random vector Y with probability density function p_Y , and let x denote a high-quality image, which is a realization of a random vector X with probability density function p_X . Posterior-Mean Rectified Flow (PMRF) is an image restoration framework designed to minimize distortion while preserving perceptual quality. PMRF achieves minimum distortion through two key stages:

1. **Posterior Mean Estimation:** A regression model is trained to estimate the posterior mean $x^* = \mathbb{E}[X|Y = y]$ given a LQ image y . This initial estimation step is theoretically optimal for minimizing the expected distortion between the predicted and true high-quality images.

2. **Rectified Flow:** Subsequently, a rectified flow model transforms the posterior mean estimation to match the true high-quality data distribution. This is achieved by learning a velocity field $v_t^\theta(\cdot)$ that guides the transformation through time t , enabling the model to recover fine details and natural variations present in the true data distribution.

The synergy between posterior mean estimation and flow-based modeling enables PMRF to achieve superior performance in image restoration tasks. By combining a distortion-optimal initial estimate with learned continuous transformations, PMRF successfully reconstructs high-fidelity images that are both perceptually pleasing and faithful to the original content.

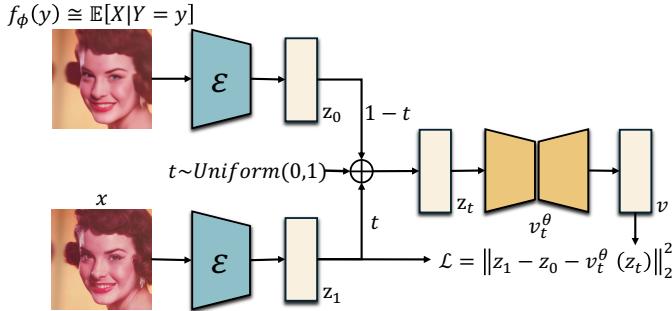


Figure 2: **Training Framework of Latent-PMRF.** We first estimate the posterior mean $\mathbb{E}[X|Y = y]$ from low-quality input y using a pretrained estimator $f_\phi(\cdot)$. The posterior mean and the high-quality input x are then encoded into latent representations z_0 and z_1 . A flow network $v_t^\theta(\cdot)$ is trained to predict the velocity field along their linear interpolation: $z_t = (1 - t)z_0 + tz_1$.

3 LATENT POSTERIOR-MEAN RECTIFIED FLOW

In this section, we introduce **Latent Posterior-Mean Rectified Flow (Latent-PMRF)**, a framework designed to achieve efficient perceptual quality optimization while preserving the distortion-minimizing property of PMRF. We first illustrate the Latent-PMRF framework and its implementation. Then, we provide the theoretical underpinnings, analyzing how our method addresses both perceptual quality and distortion. We prove that under ideal conditions, Latent-PMRF recovers the theoretical optimum of PMRF, and in practice, its performance is directly linked to the VAE’s reconstruction fidelity.

3.1 THE LATENT-PMRF FRAMEWORK

The core idea of Latent-PMRF is to reframe image restoration as a transport problem in a learned latent space. Let $(\mathcal{E}, \mathcal{D})$ be the encoder and decoder of a pre-trained VAE, the latent representations of posterior mean $Z^* = \mathcal{E}(\mathbb{E}[X|Y])$ is defined as source distribution, and $Z = \mathcal{E}(X)$ is defined as target distribution. Then a rectified flow will be learned to transport Z^* to Z as shown in Figure 2. This simple framework is both efficient at perceptual quality optimization and able to achieve minimum distortion upper bounded by the reconstruction error of VAE, as illustrated in the following sections.

3.2 ANALYSIS OF PERCEPTUAL QUALITY AND DISTORTION

We now analyze how the Latent-PMRF design addresses the dual objectives of high perceptual quality and low distortion.

3.2.1 EFFICIENT PERCEPTUAL QUALITY OPTIMIZATION

Operating in the latent space is particularly advantageous for optimizing perceptual quality, as supported by several established practices in the field. First, perceptual metrics like LPIPS Zhang et al. (2018), FID Heusel et al. (2017) typically measure differences in the feature space of pretrained networks. Second, GAN-based image generation methods Sauer et al. (2021); Kumari et al. (2022) successfully employ feature-space discriminators for improved visual quality. Furthermore, state-of-the-art diffusion models Rombach et al. (2022); Podell et al. (2024); Esser et al. (2024); Labs (2024) increasingly operate in VAE latent space, demonstrating the effectiveness of latent-space learning for perceptual quality optimization.

3.2.2 THEORETICAL BOUNDS ON DISTORTION

While optimizing for perception, maintaining low distortion is paramount. Our choice of the posterior mean latent distribution $p(\mathcal{E}(\mathbb{E}[X|Y]))$ as the source is the key to achieving this. The posterior mean $\mathbb{E}[X|Y]$ is the optimal estimator under the MSE metric, establishing the theoretical lower bound on distortion. By initiating the transport from the latent representation of this optimal estimate, Latent-PMRF inherits this low-distortion property.

216 The connection to the theoretical optimum is formalized by the following theorems. Theorem 1
 217 establishes this link under ideal conditions.

218 **Theorem 1 (Asymptotic Equivalence of Latent-PMRF and PMRF).** *Let $(\mathcal{E}, \mathcal{D})$ be a VAE that forms
 219 an isometry between the data space and the latent space. Then, the optimal estimator $\hat{X}_{lat} =$
 220 $\mathcal{D}(T_{lat}(\mathcal{E}(X^*)))$ derived from Latent-PMRF is identical to the optimal estimator \hat{X}_{PMRF} derived
 221 from PMRF, thus achieving the theoretical minimum distortion $D^* + W_2^2(p_{X^*}, p_X)$.*

223 *Proof.* The proof is provided in Appendix A.3. □

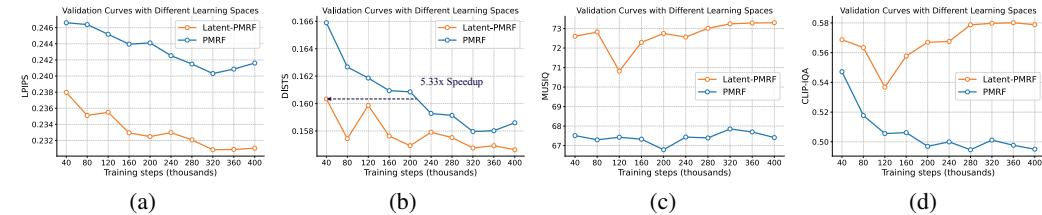
226 Theorem 1 shows that PMRF is a special case of Latent-PMRF under a perfect, isometric VAE.
 227 In practice, however, VAEs are imperfect and inevitably introduce reconstruction error. A natural
 228 question is how this imperfection affects the framework’s total distortion. We show that the
 229 expected distortion decomposes into two terms: one inherent to the VAE’s reconstruction error,
 230 and another governed by the transport process optimized by rectified flow. Consequently, reducing
 231 Latent-PMRF’s distortion requires improving the VAE’s reconstruction fidelity, which directly motivates
 232 our proposal of Sim-VAE. In addition, our framework can exploit pre-trained image-space
 233 estimators for the posterior mean, offering a practical advantage. See Appendix A.4 for details.

234 3.3 IMPROVED VARIATIONAL AUTOENCODER

236 For Latent-PMRF, the VAE not only sets the upper bound of restoration quality but also influences
 237 flow optimization. To this end, we introduce Sim-VAE, a streamlined variational autoencoder de-
 238 signed for high-fidelity reconstruction and better resolution generalization.

239 Sim-VAE departs from the widely used SD-VAE by: 1. **Simplifying ResBlocks** to remove redundant
 240 normalization/activation layers. 2. **Replacing group normalization with pixel-wise layer normal-
 241 ization** to avoid imbalanced feature activations. 3. **Eliminating self-attention in middle layers**,
 242 improving resolution generalization. 4. **Redistributing computation in resizing layers** by combin-
 243 ing resolution and channel adjustments for greater efficiency. Full architectural details, comparisons,
 244 and training loss are provided in Appendix B.

246 4 EXPERIMENTS



256 Figure 3: **Convergence Efficiency of Latent-PMRF.** We train both PMRF and Latent-PMRF using
 257 Sim-VAE for 400k iterations on FFHQ with a batch size of 64. Latent-PMRF significantly acceler-
 258 ates convergence, achieving a 5.33× speedup in DISTS. It also outperforms PMRF in LPIPS,
 259 MUSIQ, and CLIP-IQA, achieving scores that PMRF cannot achieve within training. Furthermore,
 260 Latent-PMRF demonstrates strong performance even early in training, highlighting the importance
 261 of optimizing in a well-structured latent space.

262 4.1 EXPERIMENT SETUP

264 **Datasets.** We use two primary datasets: LSDIR (Li et al., 2023), containing 84,991 high-quality
 265 natural images, and FFHQ (Karras et al., 2019), which has 70,000 high-quality face images. For
 266 preprocessing, we crop LSDIR images into 512×512 patches and filter them using Q-Align (Wu
 267 et al., 2024) with a minimum score threshold of 3.5. FFHQ images are resized to 512×512 .

268 **Implementation Details.** Sim-VAE is trained on a combination of the filtered LSDIR dataset and
 269 the first 10,000 images from FFHQ, using 256×256 image patches for 150,000 iterations with a
 batch size of 64. The Adam optimizer (Kingma & Ba, 2015) with default parameters and a cosine

270 Table 2: **Impact of VAE architectures** on CelebA-Test (Wang et al., 2021a). All VAEs use 32
 271 channels. The results show that Sim-VAE significantly outperforms SD-VAE in both reconstruction
 272 and restoration tasks. Replacing 3×3 convolutions with self-attention causes training instability,
 273 making results unavailable.

VAE	Reconstruction			Restoration		
	PSNR↑	LPIPS↓	MMD _{DINOv2} ↓	PSNR↑	LPIPS↓	MMD _{DINOv2} ↓
Sim-VAE	42.7129	0.0073	0.0511	26.3823	0.2236	0.8770
- layernorm	43.0518	0.0063	0.0619	26.1698	0.2270	0.8928
- 3×3 conv	N/A	N/A	N/A	N/A	N/A	N/A
- interpolate	42.9766	0.0075	0.0556	26.2465	0.2245	0.8817
SD-VAE	40.3979	0.0145	0.0986	25.2646	0.2224	0.8938

280 Table 3: **Impact of Latent Channels** on CelebA-Test (Wang et al., 2021a). Latent-PMRF benefits
 281 from richer latent representations, with 32 channels achieving a good balance across various metrics.

Channel	Reconstruction			Restoration		
	PSNR↑	LPIPS↓	MMD _{DINOv2} ↓	PSNR↑	LPIPS↓	Q-Align↑
16	37.9034	0.0261	0.0966	26.4412	0.2191	4.1006
24	40.8142	0.0116	0.0603	26.3911	0.2251	4.1934
32	42.7129	0.0073	0.0511	26.3823	0.2236	4.2934
48	45.0554	0.0033	0.0485	26.4600	0.2264	4.3055

288 learning rate schedule is used, decaying from 10^{-4} to 10^{-6} after a 500-step warm-up at 10^{-5} . We
 289 set the latent channel to 32, unless specified otherwise.

290 Following PMRF, we utilize the posterior mean predictor trained by (Yue & Loy, 2024), and adopt
 291 HDiT (Crowson et al., 2024) as the velocity model of Latent-PMRF. The patch size is set to 1, and
 292 the transformer blocks are arranged as 2, 4, and 6 from high- to low-resolution. Depth-wise con-
 293 volutions (Chollet, 2017) are incorporated into both the attention and feed-forward layers. Training
 294 is performed on FFHQ for 400,000 iterations with a batch size of 64. LQ images are synthesized
 295 following (Ohayon et al., 2025; Wang et al., 2021a). We use the Adam optimizer (Kingma & Ba,
 296 2015) with $\beta_1 = 0.9$, $\beta_2 = 0.95$, and a fixed learning rate of 5×10^{-4} .

297 **Evaluation Metrics.** We evaluate our methods using a range of metrics grouped into four categories:

1. Reconstruction Fidelity: PSNR and MS-SSIM (Wang et al., 2003) assess reconstruction accuracy. For face restoration, we also include identity-related metrics like Deg (ArcFace embedding angle (Deng et al., 2019)) and landmark distance LMD (Yue & Loy, 2024).
2. Perceptual Similarity: LPIPS (Zhang et al., 2018) and DISTs (Ding et al., 2020) measure perceptual similarity between two images.
3. Non-Reference Metrics: CLIP-IQA (Wang et al., 2023), MUSIQ (Ke et al., 2021) and Q-Align (Wu et al., 2024) assess image quality without ground truth.
4. Statistical Distance: In addition to the commonly used FID (Heusel et al., 2017) for measuring distributional differences, we also consider FID_{DINOv2} (Stein et al., 2024) and MMD_{DINOv2} (Jayasumana et al., 2024). These metrics improve alignment with human perception using DINOv2 (Oquab et al., 2023) features, while MMD_{DINOv2} further enhances sample efficiency using Maximum Mean Discrepancy (MMD) with an RBF kernel.

311 4.2 CONVERGENCE EFFICIENCY OF LATENT-PMRF

312 In this section, we demonstrate that constructing the PMRF in the latent space of Sim-VAE facilitates
 313 perception optimization, thus significantly accelerating convergence. As shown in Figure 1 and
 314 Figure 3, Latent-PMRF accelerates convergence by $5.79 \times$ in terms of FID and $5.33 \times$ in terms of
 315 DISTs. It also achieves significantly better LPIPS, MUSIQ, and CLIP-IQA scores, outperforming
 316 standard PMRF, which fails to reach similar performance within 400k training steps. The improved
 317 convergence efficiency of Latent-PMRF allows us to achieve strong results using relatively fewer
 318 computational resources during training.

321 4.3 IMPROVING LATENT-PMRF WITH BETTER VAE

322 **Effects of Architecture Design.** As illustrated in Section 3.3, we propose a series of architec-
 323 tural modifications aimed at improving the learning ability of the VAE and boosting restoration

Table 4: Quantitative comparisons on **CelebA-Test** (Wang et al., 2021a) benchmark. Our approach achieves the best PD-tradeoff, significantly reducing distortion while preserving top-tier perceptual quality. PMRF* denotes PMRF trained under the same compute budget as ours. Runtime is measured on NVIDIA A100. #Params (M) is reported as A + B, where A represents trainable parameters and B denotes frozen parameters.

Method	PSNR↑	MS-SSIM↑	LPIPS↓	DISTS↓	Deg.↓	LMD↓	MUSIQ↑	Q-Align↑	FID↓	FID _{DINOv2} ↓	MMD _{DINOv2} ↓	Runtime(s)	#Params(M)
GFP-GAN	24.9861	0.8640	0.2407	0.1720	34.5372	2.4509	75.2940	4.7009	14.8021	223.0202	1.1638	0.0218	86.4
RestoreFormer	24.6157	0.8443	0.2416	0.1639	30.9218	1.9389	73.8584	4.5320	13.4083	152.1276	1.0003	0.0402	72.7
CodeFormer	25.1464	0.8589	0.2271	0.1700	35.7124	2.1389	75.5546	4.5835	15.3959	184.0517	1.1041	0.0349	94.1
VQFR	23.7626	0.8278	0.2391	0.1683	40.9100	3.0436	73.8407	4.5285	13.6547	199.7024	1.1287	0.0621	83.5
DiffFace	24.7964	0.8233	0.2723	0.1679	44.1442	2.7230	69.0060	4.0769	13.5138	184.1844	1.0441	3.7054	159.7 + 15.7
DiffBIR (v2)	25.3946	0.8668	0.2654	0.1911	31.2931	1.5646	76.1659	4.8782	20.9181	156.9969	1.0692	6.3952	363.1 + 1319.3
ResShift	26.0359	0.8734	0.2464	0.1692	32.2866	1.8718	67.9784	4.2413	19.1850	167.3501	1.0534	0.6230	118.9 + 77.0
FlowIE	24.8349	0.8505	0.2312	0.1585	32.2254	1.7757	74.1167	4.6108	17.5334	164.6910	1.0733	0.3877	398.6 + 191.3
PMRF	26.3321	0.8740	0.2232	0.1476	29.4504	1.5138	70.4967	4.2227	10.7225	96.8752	0.7214	0.5247	159.8 + 15.7
PMRF*	26.6431	0.8729	0.2407	0.1596	28.9294	1.3799	64.9143	3.7261	15.1663	140.6601	0.8578		
Latent-PMRF	26.3887	0.8789	0.2207	0.1576	29.0961	1.5217	73.1496	4.3325	10.9447	110.4742	0.8108	0.5745	151.2 + 106.8

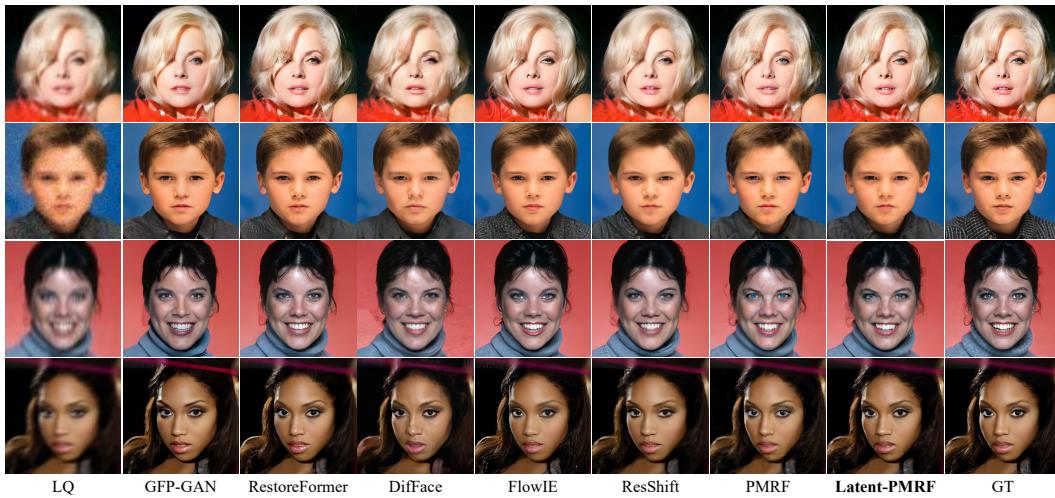


Figure 4: Qualitative comparisons on **CelebA-Test** (Wang et al., 2021a) benchmark. Our method produces visually appealing details while maintaining exceptionally high face identity preservation.

of Latent-PMRF. In this section, we demonstrate the practical implications of these modifications through controlled experiments. As shown in Table 2, we progressively remove various modifications to assess their impact on the reconstruction ability of the VAE and the restoration performance of Latent-PMRF trained on it. From the second row of the table, we observe that while replacing layer normalization with group normalization improves VAE fidelity, it degrades distributional faithfulness, and more importantly, severely hampers the restoration performance of Latent-PMRF. This suggests that group normalization negatively influences the learning of smooth features. The fourth row shows that using non-optimal resizing layers leads to poorer reconstruction and, consequently, worse restoration performance. Finally, when all modifications are removed, we obtain SD-VAE, which, while achieving good LPIPS in restoration, performs poorly in all other aspects.

Impact of Latent Channels. It is well known that increasing latent channels enhances the latent space representation and improves the VAE’s reconstruction ability. However, the effect of latent channels on the restoration performance of Latent-PMRF remains unclear. As shown in Table 3, Latent-PMRF benefits from a richer latent space, with Q-Align scores consistently improving as the number of latent channels increases. We find that 32 channels strike a good balance across various metrics, so we set the default to 32.

4.4 COMPARISONS WITH STATE-OF-THE-ART METHODS

We primarily compare our method with PMRF (Ohayon et al., 2025), as our goal is to construct it in the latent space. Additionally, we compare with traditional approaches such as GFP-GAN (Wang et al., 2021a), RestoreFormer (Wang et al., 2022b), CodeFormer (Liu et al., 2023a), and VQFR (Gu et al., 2022), as well as recent diffusion-based methods like DifFace (Yue & Loy, 2024), ResShift (Yue et al., 2023; 2025), and DiffBIR (Lin et al., 2024). For a fair comparison, we reproduce ResShift using their official code but exclude the LPIPS loss used in their journal ver-

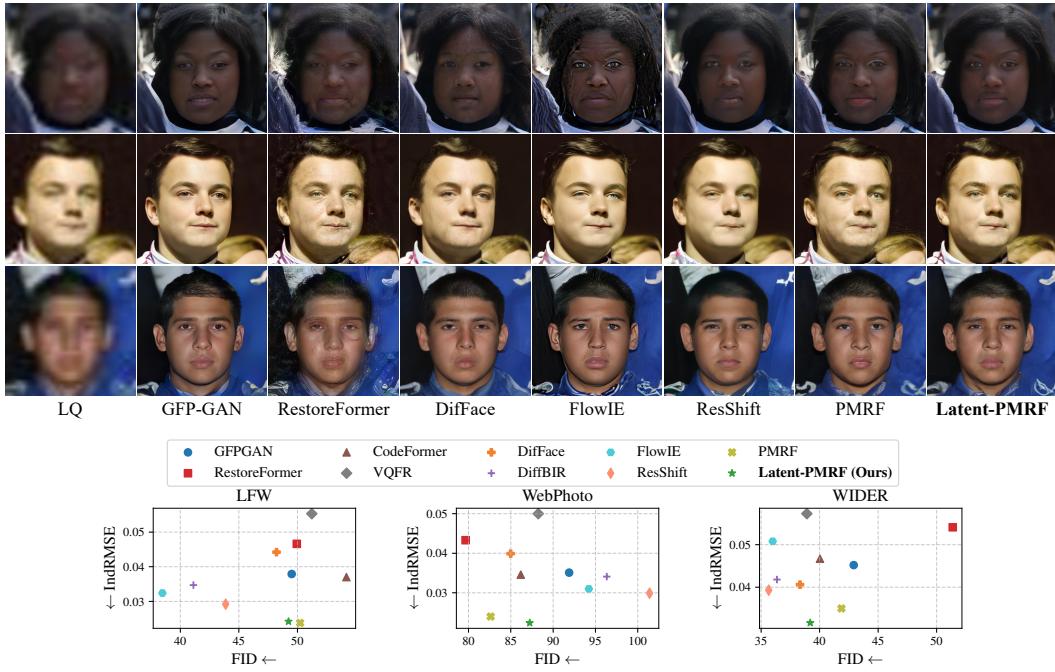


Figure 5: Comparisons on real-world datasets. Top: Qualitative results on the **WIDER-Test** (Zhou et al., 2022) dataset. Bottom: Comparison on the "distortion"-perception plane (IndRMSE vs. FID), where IndRMSE represents the RMSE of each method (Ohayon et al., 2025). Our method outperforms all others in IndRMSE, while achieving perceptual quality on par with the state-of-the-art.

While incorporating this additional loss term is feasible, we omit it as it is not the focus of our work and requires computationally expensive VAE decoding during training. We also include FlowIE (Zhu et al., 2024), which also utilizes flow models. Notably, both DiffBIR and FlowIE leverage facial priors from large-scale Stable Diffusion (Rombach et al., 2022), whereas other methods use relatively smaller models.

Results on Synthetic Dataset. We evaluate our method on the CelebA-Test benchmark (Wang et al., 2021a). As shown in Table 4, PMRF and Latent-PMRF strike the best balance between distortion and perceptual quality. Specifically, only PMRF and Latent-PMRF achieve a PSNR above 26.3 dB and demonstrate superior face identity preservation, as evaluated by Deg. and LMD. In terms of statistical distance, PMRF, and our method learn more accurate distributions, outperforming others in FID, FID_{DINOv2} and MMD_{DINOv2} . Notably, methods leveraging pretrained facial priors, such as GFP-GAN, DiffBIR, and FlowIE, achieve higher non-reference metric scores but tend to produce faces with lower faithfulness. In contrast, Latent-PMRF retains the high fidelity of PMRF while surpassing it in non-reference metrics. Moreover, Latent-PMRF demonstrates improved convergence properties—when the compute budget is reduced to match ours (scaling down from a batch size 256 and 3850 epochs (Ohayon et al., 2025)), PMRF experiences a significant performance drop. Overall, Latent-PMRF not only outperforms other methods but also converges much faster than PMRF.

We also present visual results in Figure 4. Compared to PMRF, our results generally exhibit better perceptual quality, which is reflected in the higher non-reference metrics we achieve. In contrast to other methods, which suffer from lower fidelity to the ground truth and consequently degrade face identity, our method preserves fine facial details while maintaining strong perceptual quality.

Results on Real-world Datasets. We evaluate the generalizability of Latent-PMRF on real-world datasets, including LFW (Huang et al., 2008; Wang et al., 2021a), WebPhoto (Wang et al., 2021a), and WIDER (Zhou et al., 2022). Since these datasets lack ground truth, we follow Ohayon et al. (2025) and use a pretrained posterior-mean estimator as a proxy for fidelity measurement. As shown in Figure 5, both Latent-PMRF and PMRF significantly outperform other methods in terms of fidelity, as indicated by IndRMSE. In terms of perceptual quality, Latent-PMRF outperforms PMRF on LFW and WIDER, while maintaining comparable performance to other methods. Overall, Latent-PMRF achieves a better perception-distortion tradeoff, offering comparable perceptual quality with superior distortion reduction. Visually, RestoreFormer produces poorly structured im-

432 ages, and FlowIE with the Stable Diffusion backbone shows artifacts with overly sharp details. In
 433 contrast, our method generates visually appealing images that remain consistent with the input.
 434

435 5 RELATED WORK

436 **Generative Models in Latent Space.** Diffusion-based generative models (Ho et al., 2020; Dhariwal
 437 & Nichol, 2021) achieve impressive image synthesis but are computationally expensive, particularly
 438 for high-resolution images. Latent Diffusion (Rombach et al., 2022) mitigates this by learning dis-
 439 tributions in a pretrained VAE’s latent space, retaining only perceptually important information to
 440 enhance efficiency and scalability. Large-scale text-to-image models (Rombach et al., 2022; Podell
 441 et al., 2024; Esser et al., 2024) follow this paradigm, with VAE design playing a crucial role. Esser
 442 et al. (2024) show that increasing latent channels improves performance but requires larger gen-
 443 erative models—for instance, even FLUX (12B parameters) (Labs, 2024) is limited to 16 latent
 444 channels. However, our Latent-PMRF framework greatly benefits from a more powerful VAE, since
 445 a stronger VAE enriches the source distribution with more information, thus alleviating the burden
 446 on the restoration process.

447 **Blind Face Restoration.** Blind face restoration aims to recover high-quality facial details from im-
 448 ages degraded by unknown and complex factors while maintaining fidelity. From a training objective
 449 perspective, existing methods mainly fall into two categories: (1) GAN-based approaches (Wang
 450 et al., 2021a; 2022b; Gu et al., 2022; Liu et al., 2023a) optimize a weighted combination of dis-
 451 tortion losses (e.g., L1, L2) and perceptual losses (e.g., adversarial loss (Goodfellow et al., 2020),
 452 perceptual loss (Johnson et al., 2016)), where the tradeoff between fidelity and perceptual quality is
 453 controlled by loss weighting (Blau & Michaeli, 2018; Ledig et al., 2017). (2) Posterior sampling-
 454 based methods (Lin et al., 2024; Zhu et al., 2024; Yue et al., 2023; Yue & Loy, 2024; Ohayon et al.,
 455 2025; Chen et al., 2024), particularly diffusion models, model the conditional posterior distribution
 456 of HQ images given degraded inputs. While these methods theoretically ensure superior perceptual
 457 quality, they often lead to suboptimal distortion (Ohayon et al., 2025).

458 PMRF (Ohayon et al., 2025) is the first approach to ensure optimal distortion under a perfect per-
 459 ceptual quality constraint. It first predicts the posterior mean (minimum distortion estimation) and
 460 then transports it to the HQ image distribution. However, we argue that distribution discrepancy in
 461 pixel space does not faithfully align with human perception. To address this, we propose construct-
 462 ing PMRF in the latent space of a VAE, which better optimizes perceptual quality. Furthermore, we
 463 design the source distribution to preserve PMRF’s distortion-minimum properties in latent space.

464 **Concurrent works.** ELIR (Cohen et al., 2025) independently extends PMRF to the latent space
 465 of VAE. However, their focus is on improving testing-time efficiency via Consistency Flow Match-
 466 ing (Yang et al., 2024), while we aim to enhance optimization efficiency for perceptual quality.
 467 Furthermore, they use the posterior mean of latent representations as the source distribution, which,
 468 as discussed in Appendix A.2, is suboptimal. This choice leads to significant fidelity degradation in
 469 their model, whereas our Latent-PMRF preserves the high fidelity of PMRF.

470 6 CONCLUSION

471
 472
 473 We propose Latent-PMRF, which retains the minimal distortion property of PMRF while achieving
 474 better perceptual quality optimization. Our theoretical analysis shows that the latent representation
 475 of the posterior mean achieves a minimum distortion determined by the VAE’s reconstruction error.
 476 Based on this insight, we introduce our Sim-VAE, with a series of modifications to enhance the
 477 reconstruction capability of the VAE, leading to a notable performance boost for Latent-PMRF.
 478 Latent-PMRF demonstrates remarkable convergence efficiency, achieving a $5.79 \times$ speedup over
 479 PMRF in FID convergence. Furthermore, Latent-PMRF exhibits a better PD-tradeoff compared to
 480 existing methods in blind face restoration, with improved perceptual quality compared to PMRF.
 481 Although Latent-PMRF achieves strong performance, we observe a slight decrease in test speed
 482 compared to PMRF (see Table 4). This is because, while the velocity prediction in the latent space is
 483 faster, the encoding and decoding processes of the VAE are inherently slow. Improving the efficiency
 484 of the VAE could be a potential area for further enhancement.

486 REFERENCES
487

- 488 Michael Samuel Albergo and Eric Vanden-Eijnden. Building normalizing flows with stochas-
489 tic interpolants. In *ICLR*, 2023. URL <https://openreview.net/forum?id=li7qeBbCR1t>.
- 490
- 491 Yochai Blau and Tomer Michaeli. The perception-distortion tradeoff. In *Proceedings of the IEEE*
492 *conference on computer vision and pattern recognition*, pp. 6228–6237, 2018.
- 493
- 494 Junyu Chen, Han Cai, Junsong Chen, Enze Xie, Shang Yang, Haotian Tang, Muyang Li, and Song
495 Han. Deep compression autoencoder for efficient high-resolution diffusion models. In *ICLR*,
496 2025. URL <https://openreview.net/forum?id=wH8XXUOUZU>.
- 497
- 498 Liangyu Chen, Xiaojie Chu, Xiangyu Zhang, and Jian Sun. Simple baselines for image restoration.
499 In *European conference on computer vision*, pp. 17–33. Springer, 2022.
- 500
- 501 Xiaoxu Chen, Jingfan Tan, Tao Wang, Kaihao Zhang, Wenhan Luo, and Xiaochun Cao. Towards
502 real-world blind face restoration with generative diffusion prior. *IEEE Transactions on Circuits*
503 and *Systems for Video Technology*, 2024.
- 504
- 505 François Fleuret. Xception: Deep learning with depthwise separable convolutions. In *Proceedings*
506 *of the IEEE conference on computer vision and pattern recognition*, pp. 1251–1258, 2017.
- 507
- 508 Elad Cohen, Idan Achituv, Idit Diamant, Arnon Netzer, and Hai Victor Habi. Efficient image
509 restoration via latent consistency flow matching. *arXiv preprint arXiv:2502.03500*, 2025.
- 510
- 511 Katherine Crowson, Stefan Andreas Baumann, Alex Birch, Tanishq Mathew Abraham, Daniel Z
512 Kaplan, and Enrico Shippole. Scalable high-resolution pixel-space image synthesis with hourglass
513 diffusion transformers. In *Forty-first International Conference on Machine Learning*, 2024.
- 514
- 515 Jiankang Deng, Jia Guo, Niannan Xue, and Stefanos Zafeiriou. Arcface: Additive angular margin
516 loss for deep face recognition. In *Proceedings of the IEEE/CVF conference on computer vision*
517 and *pattern recognition*, pp. 4690–4699, 2019.
- 518
- 519 Prafulla Dhariwal and Alexander Nichol. Diffusion models beat gans on image synthesis. *Advances*
520 *in neural information processing systems*, 34:8780–8794, 2021.
- 521
- 522 Keyan Ding, Kede Ma, Shiqi Wang, and Eero P Simoncelli. Image quality assessment: Unifying
523 structure and texture similarity. *IEEE transactions on pattern analysis and machine intelligence*,
524 44(5):2567–2581, 2020.
- 525
- 526 Chao Dong, Chen Change Loy, Kaiming He, and Xiaoou Tang. Learning a deep convolutional net-
527 work for image super-resolution. In *Computer Vision–ECCV 2014: 13th European Conference,*
528 *Zurich, Switzerland, September 6–12, 2014, Proceedings, Part IV 13*, pp. 184–199. Springer,
529 2014.
- 530
- 531 drhead. The vae used for stable diffusion 1.x/2.x and other models (kl-f8) has a critical flaw.
532 https://www.reddit.com/r/StableDiffusion/comments/1ag5h5s/the_vae_used_for_stable_diffusion_1x2x_and_other/?utm_source=share&utm_medium=web3x&utm_name=web3xcss&utm_term=1&utm_content=share_button, 2024. Accessed: 2025-02-07.
- 533
- 534 Michael Elad, Bahjat Kawar, and Gregory Vaksman. Image denoising: The deep learning revolution
535 and beyond—a survey paper. *SIAM Journal on Imaging Sciences*, 16(3):1594–1654, 2023.
- 536
- 537 Patrick Esser, Robin Rombach, and Bjorn Ommer. Taming transformers for high-resolution image
538 synthesis. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recogni-*
539 *tion*, pp. 12873–12883, 2021.
- 540
- 541 Patrick Esser, Sumith Kulal, Andreas Blattmann, Rahim Entezari, Jonas Müller, Harry Saini, Yam
542 Levi, Dominik Lorenz, Axel Sauer, Frederic Boesel, et al. Scaling rectified flow transformers for
543 high-resolution image synthesis. In *Forty-first International Conference on Machine Learning*,
544 2024.

- 540 Dror Freirich, Tomer Michaeli, and Ron Meir. A theory of the distortion-perception tradeoff in
 541 wasserstein space. *Advances in Neural Information Processing Systems*, 34:25661–25672, 2021.
 542
- 543 Peng Gao, Le Zhuo, Dongyang Liu, Ruoyi Du, Xu Luo, Longtian Qiu, Yuhang Zhang, Rongjie
 544 Huang, Shijie Geng, Renrui Zhang, Junlin Xie, Wenqi Shao, Zhengkai Jiang, Tianshuo Yang,
 545 Weicai Ye, Tong He, Jingwen He, Junjun He, Yu Qiao, and Hongsheng Li. Lumina-t2x: Scalable
 546 flow-based large diffusion transformer for flexible resolution generation. In *ICLR*, 2025. URL
 547 <https://openreview.net/forum?id=EbWf36quzd>.
- 548 Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair,
 549 Aaron Courville, and Yoshua Bengio. Generative adversarial networks. *Communications of the*
 550 *ACM*, 63(11):139–144, 2020.
- 551 Yuchao Gu, Xintao Wang, Liangbin Xie, Chao Dong, Gen Li, Ying Shan, and Ming-Ming Cheng.
 552 Vqfr: Blind face restoration with vector-quantized dictionary and parallel decoder. In *European*
 553 *Conference on Computer Vision*, pp. 126–143. Springer, 2022.
- 554 Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recogni-
 555 tion. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp.
 556 770–778, 2016.
- 557 Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, and Sepp Hochreiter.
 558 Gans trained by a two time-scale update rule converge to a local nash equilibrium. *Advances in*
 559 *neural information processing systems*, 30, 2017.
- 560 Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. *Advances in*
 561 *neural information processing systems*, 33:6840–6851, 2020.
- 562 Gary B Huang, Marwan Mattar, Tamara Berg, and Eric Learned-Miller. Labeled faces in the wild:
 563 A database forstudying face recognition in unconstrained environments. In *Workshop on faces*
 564 *in'Real-Life' Images: detection, alignment, and recognition*, 2008.
- 565 Sadeep Jayasumana, Srikumar Ramalingam, Andreas Veit, Daniel Glasner, Ayan Chakrabarti, and
 566 Sanjiv Kumar. Rethinking fid: Towards a better evaluation metric for image generation. In *Pro-*
 567 *ceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 9307–
 568 9315, 2024.
- 569 Jiaxi Jiang, Kai Zhang, and Radu Timofte. Towards flexible blind jpeg artifacts removal. In *Pro-*
 570 *ceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 4997–5006, 2021.
- 571 Justin Johnson, Alexandre Alahi, and Li Fei-Fei. Perceptual losses for real-time style transfer and
 572 super-resolution. In *Computer Vision–ECCV 2016: 14th European Conference, Amsterdam, The*
 573 *Netherlands, October 11–14, 2016, Proceedings, Part II 14*, pp. 694–711. Springer, 2016.
- 574 Tero Karras, Samuli Laine, and Timo Aila. A style-based generator architecture for generative
 575 adversarial networks. In *Proceedings of the IEEE/CVF conference on computer vision and pattern*
 576 *recognition*, pp. 4401–4410, 2019.
- 577 Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Analyz-
 578 ing and improving the image quality of stylegan. In *Proceedings of the IEEE/CVF conference on*
 579 *computer vision and pattern recognition*, pp. 8110–8119, 2020.
- 580 Junjie Ke, Qifei Wang, Yilin Wang, Peyman Milanfar, and Feng Yang. Musiq: Multi-scale im-
 581 age quality transformer. In *Proceedings of the IEEE/CVF international conference on computer*
 582 *vision*, pp. 5148–5157, 2021.
- 583 Diederik P Kingma. Auto-encoding variational bayes. In *ICLR*, 2014. URL <https://openreview.net/forum?id=33X9fd2-9FyZd>.
- 584 Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *ICLR*, 2015.
- 585 Nupur Kumari, Richard Zhang, Eli Shechtman, and Jun-Yan Zhu. Ensembling off-the-shelf models
 586 for gan training. In *Proceedings of the IEEE/CVF conference on computer vision and pattern*
 587 *recognition*, pp. 10651–10662, 2022.

- 594 Black Forest Labs. Flux. <https://github.com/black-forest-labs/flux>, 2024. Accessed: 2025-02-07.
 595
 596
- 597 Christian Ledig, Lucas Theis, Ferenc Huszár, Jose Caballero, Andrew Cunningham, Alejandro
 598 Acosta, Andrew Aitken, Alykhan Tejani, Johannes Totz, Zehan Wang, et al. Photo-realistic sin-
 599 gle image super-resolution using a generative adversarial network. In *Proceedings of the IEEE*
 600 *conference on computer vision and pattern recognition*, pp. 4681–4690, 2017.
- 601 Jimmy Lei Ba, Jamie Ryan Kiros, and Geoffrey E Hinton. Layer normalization. In *NeurIPS*, 2016.
 602
 603 Haoying Li, Yifan Yang, Meng Chang, Shiqi Chen, Huajun Feng, Zhihai Xu, Qi Li, and Yueting
 604 Chen. Srdiff: Single image super-resolution with diffusion probabilistic models. *Neurocomputing*,
 605 479:47–59, 2022.
- 606 Yawei Li, Kai Zhang, Jingyun Liang, Jiezhang Cao, Ce Liu, Rui Gong, Yulun Zhang, Hao Tang, Yun
 607 Liu, Denis Demandolx, et al. Lsdir: A large scale dataset for image restoration. In *Proceedings of*
 608 *the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 1775–1787, 2023.
 609
- 610 Jingyun Liang, Jiezhang Cao, Guolei Sun, Kai Zhang, Luc Van Gool, and Radu Timofte. Swinir:
 611 Image restoration using swin transformer. In *Proceedings of the IEEE/CVF international confer-
 612 ence on computer vision*, pp. 1833–1844, 2021.
- 613 Xinqi Lin, Jingwen He, Ziyan Chen, Zhaoyang Lyu, Bo Dai, Fanghua Yu, Yu Qiao, Wanli Ouyang,
 614 and Chao Dong. Diffbir: Toward blind image restoration with generative diffusion prior. In
 615 *European Conference on Computer Vision*, pp. 430–448. Springer, 2024.
 616
- 617 Yaron Lipman, Ricky T. Q. Chen, Heli Ben-Hamu, Maximilian Nickel, and Matthew Le. Flow
 618 matching for generative modeling. In *The Eleventh International Conference on Learning Repre-
 619 sentations*, 2023. URL <https://openreview.net/forum?id=PqvMRDCJT9t>.
- 620 Guangming Liu, Xin Zhou, Jianmin Pang, Feng Yue, Wenfu Liu, and Junchao Wang. Codeformer:
 621 A gnn-nested transformer model for binary code similarity detection. *Electronics*, 12(7):1722,
 622 2023a.
 623
- 624 Xingchao Liu, Chengyue Gong, and qiang liu. Flow straight and fast: Learning to generate and
 625 transfer data with rectified flow. In *The Eleventh International Conference on Learning Repre-
 626 sentations*, 2023b. URL <https://openreview.net/forum?id=XVjTT1nw5z>.
- 627 Zhuang Liu, Hanzi Mao, Chao-Yuan Wu, Christoph Feichtenhofer, Trevor Darrell, and Saining Xie.
 628 A convnet for the 2020s. In *Proceedings of the IEEE/CVF conference on computer vision and*
 629 *pattern recognition*, pp. 11976–11986, 2022.
 630
- 631 Xin Luo, Yunan Zhu, Shunxin Xu, and Dong Liu. On the effectiveness of spectral discriminators
 632 for perceptual quality improvement. In *Proceedings of the IEEE/CVF International Conference*
 633 *on Computer Vision*, pp. 13243–13253, 2023.
- 634 Guy Ohayon, Tomer Michaeli, and Michael Elad. Posterior-mean rectified flow: Towards minimum
 635 MSE photo-realistic image restoration. In *The Thirteenth International Conference on Learning*
 636 *Representations*, 2025. URL <https://openreview.net/forum?id=hPOt3yUXii>.
 637
- 638 Maxime Oquab, Timothée Darcet, Théo Moutakanni, Huy Vo, Marc Szafraniec, Vasil Khalidov,
 639 Pierre Fernandez, Daniel Haziza, Francisco Massa, Alaaeldin El-Nouby, et al. Dinov2: Learning
 640 robust visual features without supervision. *arXiv preprint arXiv:2304.07193*, 2023.
- 641 Dustin Podell, Zion English, Kyle Lacey, Andreas Blattmann, Tim Dockhorn, Jonas Müller, Joe
 642 Penna, and Robin Rombach. SDXL: Improving latent diffusion models for high-resolution image
 643 synthesis. In *The Twelfth International Conference on Learning Representations*, 2024. URL
 644 <https://openreview.net/forum?id=di52zR8xgf>.
 645
- 646 Ofir Press, Noah Smith, and Mike Lewis. Train short, test long: Attention with linear biases enables
 647 input length extrapolation. In *ICLR*, 2022. URL <https://openreview.net/forum?id=R8sQPpGCv0>.

- 648 Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, and Björn Ommer. High-
 649 resolution image synthesis with latent diffusion models. In *Proceedings of the IEEE/CVF confer-
 650 ence on computer vision and pattern recognition*, pp. 10684–10695, 2022.
- 651
- 652 Seyedmorteza Sadat, Jakob Buhmann, Derek Bradley, Otmar Hilliges, and Romann M. Weber.
 653 LiteVAE: Lightweight and efficient variational autoencoders for latent diffusion models. In
 654 *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL
 655 <https://openreview.net/forum?id=mTAb18kUzq>.
- 656
- 657 Axel Sauer, Kashyap Chitta, Jens Müller, and Andreas Geiger. Projected gans converge faster.
 658 *Advances in Neural Information Processing Systems*, 34:17480–17492, 2021.
- 659
- 660 Jascha Sohl-Dickstein, Eric Weiss, Niru Maheswaranathan, and Surya Ganguli. Deep unsupervised
 661 learning using nonequilibrium thermodynamics. In *International conference on machine learn-
 662 ing*, pp. 2256–2265. PMLR, 2015.
- 663
- 664 George Stein, Jesse Cresswell, Rasa Hosseinzadeh, Yi Sui, Brendan Ross, Valentin Villecroze,
 665 Zhaoyan Liu, Anthony L Caterini, Eric Taylor, and Gabriel Loaiza-Ganem. Exposing flaws of
 666 generative model evaluation metrics and their unfair treatment of diffusion models. *Advances in
 667 Neural Information Processing Systems*, 36, 2024.
- 668
- 669 Christian Szegedy, Wei Liu, Yangqing Jia, Pierre Sermanet, Scott Reed, Dragomir Anguelov, Du-
 670 mitru Erhan, Vincent Vanhoucke, and Andrew Rabinovich. Going deeper with convolutions. In
 671 *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1–9, 2015.
- 672
- 673 Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez,
 674 Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural informa-
 675 tion processing systems*, 30, 2017.
- 676
- 677 Jianyi Wang, Kelvin CK Chan, and Chen Change Loy. Exploring clip for assessing the look and
 678 feel of images. In *Proceedings of the AAAI conference on artificial intelligence*, volume 37, pp.
 679 2555–2563, 2023.
- 680
- 681 Jianyi Wang, Zongsheng Yue, Shangchen Zhou, Kelvin CK Chan, and Chen Change Loy. Exploiting
 682 diffusion prior for real-world image super-resolution. *International Journal of Computer Vision*,
 683 132(12):5929–5949, 2024.
- 684
- 685 Tao Wang, Kaihao Zhang, Xuanxi Chen, Wenhan Luo, Jiankang Deng, Tong Lu, Xiaochun Cao,
 686 Wei Liu, Hongdong Li, and Stefanos Zafeiriou. A survey of deep face restoration: Denoise,
 687 super-resolution, deblur, artifact removal. *arXiv preprint arXiv:2211.02831*, 2022a.
- 688
- 689 Xintao Wang, Yu Li, Honglun Zhang, and Ying Shan. Towards real-world blind face restoration
 690 with generative facial prior. In *Proceedings of the IEEE/CVF conference on computer vision and
 691 pattern recognition*, pp. 9168–9178, 2021a.
- 692
- 693 Xintao Wang, Liangbin Xie, Chao Dong, and Ying Shan. Real-esrgan: Training real-world blind
 694 super-resolution with pure synthetic data. In *Proceedings of the IEEE/CVF international confer-
 695 ence on computer vision*, pp. 1905–1914, 2021b.
- 696
- 697 Zhou Wang, Eero P Simoncelli, and Alan C Bovik. Multiscale structural similarity for image quality
 698 assessment. In *The Thirly-Seventh Asilomar Conference on Signals, Systems & Computers*, 2003,
 699 volume 2, pp. 1398–1402. Ieee, 2003.
- 700
- 701 Zhouxia Wang, Jiawei Zhang, Runjian Chen, Wenping Wang, and Ping Luo. Restoreformer: High-
 702 quality blind face restoration from undegraded key-value pairs. In *Proceedings of the IEEE/CVF
 703 conference on computer vision and pattern recognition*, pp. 17512–17521, 2022b.
- 704
- 705 Chenfei Wu, Jiahao Li, Jingren Zhou, Junyang Lin, Kaiyuan Gao, Kun Yan, Sheng-ming Yin, Shuai
 706 Bai, Xiao Xu, Yilei Chen, et al. Qwen-image technical report. *arXiv preprint arXiv:2508.02324*,
 707 2025.

- 702 Haoning Wu, Zicheng Zhang, Weixia Zhang, Chaofeng Chen, Liang Liao, Chunyi Li, Yixuan Gao,
 703 Annan Wang, Erli Zhang, Wenxiu Sun, Qiong Yan, Xiongkuo Min, Guangtao Zhai, and Weisi
 704 Lin. Q-align: Teaching LMMs for visual scoring via discrete text-defined levels. In Ruslan
 705 Salakhutdinov, Zico Kolter, Katherine Heller, Adrian Weller, Nuria Oliver, Jonathan Scarlett, and
 706 Felix Berkenkamp (eds.), *Proceedings of the 41st International Conference on Machine Learning*,
 707 volume 235 of *Proceedings of Machine Learning Research*, pp. 54015–54029. PMLR, 21–27 Jul
 708 2024. URL <https://proceedings.mlr.press/v235/wu24ah.html>.
- 709 Yuxin Wu and Kaiming He. Group normalization. In *Proceedings of the European conference on*
 710 *computer vision (ECCV)*, pp. 3–19, 2018.
- 711 Ling Yang, Zixiang Zhang, Zhilong Zhang, Xingchao Liu, Minkai Xu, Wentao Zhang, Chenlin
 712 Meng, Stefano Ermon, and Bin Cui. Consistency flow matching: Defining straight flows with
 713 velocity consistency. *arXiv preprint arXiv:2407.02398*, 2024.
- 714 Zongsheng Yue and Chen Change Loy. Difface: Blind face restoration with diffused error contrac-
 715 tion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2024.
- 716 Zongsheng Yue, Jianyi Wang, and Chen Change Loy. Resshift: Efficient diffusion model for im-
 717 age super-resolution by residual shifting. In *Thirty-seventh Conference on Neural Information*
 718 *Processing Systems*, 2023. URL <https://openreview.net/forum?id=ZIyAHaLsn>.
- 719 Zongsheng Yue, Jianyi Wang, and Chen Change Loy. Efficient diffusion model for image restoration
 720 by residual shifting. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 47(1):
 721 116–130, 2025. doi: 10.1109/TPAMI.2024.3461721.
- 722 Kaihao Zhang, Wenqi Ren, Wenhan Luo, Wei-Sheng Lai, Björn Stenger, Ming-Hsuan Yang, and
 723 Hongdong Li. Deep image deblurring: A survey. *International Journal of Computer Vision*, 130
 724 (9):2103–2130, 2022.
- 725 Richard Zhang, Phillip Isola, Alexei A Efros, Eli Shechtman, and Oliver Wang. The unreasonable
 726 effectiveness of deep features as a perceptual metric. In *Proceedings of the IEEE conference on*
 727 *computer vision and pattern recognition*, pp. 586–595, 2018.
- 728 Shangchen Zhou, Kelvin Chan, Chongyi Li, and Chen Change Loy. Towards robust blind face
 729 restoration with codebook lookup transformer. *Advances in Neural Information Processing Sys-
 730 tems*, 35:30599–30611, 2022.
- 731 Yixuan Zhu, Wenliang Zhao, Ao Li, Yansong Tang, Jie Zhou, and Jiwen Lu. Flowie: Efficient image
 732 enhancement via rectified flow. In *Proceedings of the IEEE/CVF Conference on Computer Vision*
 733 *and Pattern Recognition*, pp. 13–22, 2024.
- 734

735 A THEORETICAL ANALYSIS OF LATENT-PMRF

736 A.1 PRELIMINARIES: THE DISTORTION-PERCEPTION TRADE-OFF

737 Our theoretical analysis is situated within the distortion-perception (DP) framework, using Mean
 738 Squared Error (MSE) as the distortion metric. We begin by defining the key random variables and
 739 the theoretical optimum for an ideal estimator.

- 740
- 741 Let $X \sim p_X$ be the ground-truth image from the true data distribution.
 - 742 Let Y be the corresponding degraded observation.
 - 743 Let \hat{X} be an estimator for X , produced by a restoration model.
 - 744 Let $X^* = \mathbb{E}[X|Y]$ be the posterior mean estimate. X^* is the optimal estimator of X given
 745 Y in the MSE sense, as it minimizes $\mathbb{E}[\|X - \hat{X}\|^2]$ over all possible functions of Y .

746 The total MSE distortion of any estimator \hat{X} can be decomposed using the orthogonality principle
 747 of the posterior mean:

$$748 \mathbb{E}[\|X - \hat{X}\|^2] = \mathbb{E}[\|X - X^*\|^2] + \mathbb{E}[\|X^* - \hat{X}\|^2] \quad (4)$$

The first term, $D^* = \mathbb{E}[\|X - X^*\|^2]$, is the **irreducible error**. It represents the minimum possible distortion achievable by any estimator that only has access to the observation Y and is independent of our specific choice of model for \hat{X} . The second term, $\mathbb{E}[\|X^* - \hat{X}\|^2]$, is the **optimizable error**, which our model aims to minimize.

We are interested in the case of **perfect perceptual quality**, where the distribution of the estimator's outputs must match the true data distribution, i.e., $p_{\hat{X}} = p_X$. As shown by Theorem 2 of Freirich et al. (2021), the problem of finding the minimum optimizable error under this constraint is equivalent to solving the optimal transport problem between the distributions p_{X^*} and p_X . This minimum value is, by definition, the squared Wasserstein-2 distance, $W_2^2(p_{X^*}, p_X)$.

Therefore, the theoretical minimum total distortion for any estimator that satisfies the perfect perception constraint is given by:

$$D(0) = D^* + W_2^2(p_{X^*}, p_X) \quad (5)$$

This value serves as the gold standard against which we evaluate our Latent-PMRF framework under ideal conditions.

A.2 DETERMINING OF SOURCE DISTRIBUTION

When extending PMRF into the latent space, two natural candidates for the source distribution arise:

1. **Mean of latent codes:** $Z_{opt1}^* = \mathbb{E}[\mathcal{E}(X)|Y = y]$
2. **Latent code of the mean:** $Z_{opt2}^* = \mathcal{E}(\mathbb{E}[X|Y]) = \mathcal{E}(X^*)$

In our Latent-PMRF framework, we adopt the latter, $Z^* = \mathcal{E}(X^*)$. The motivation is straightforward: decoding this choice yields the posterior mean X^* up to the VAE's reconstruction error, thereby anchoring the transport process to the distortion-minimizing reference.

Specifically, the deviation introduced by Option 2 is simply

$$\|\mathcal{D}(\mathcal{E}(X^*)) - X^*\|^2,$$

which corresponds exactly to the **Inherent Reconstruction Error** in Theorem 2. Under a perfect isometric VAE (1), this error vanishes, and the source distribution coincides with the posterior mean.

By contrast, Option 1 introduces an additional, unavoidable error due to the nonlinearity of \mathcal{D} . Because

$$\mathcal{D}(\mathbb{E}[\mathcal{E}(X)|Y]) \neq \mathbb{E}[\mathcal{D}(\mathcal{E}(X))|Y],$$

even a perfect VAE would yield a non-zero deviation

$$\|\mathcal{D}(\mathbb{E}[\mathcal{E}(X)|Y]) - \mathbb{E}[X|Y]\|^2,$$

preventing the recovery of the theoretical optimum $D(0)$ guaranteed by Theorem 1.

Thus, choosing $Z^* = \mathcal{E}(X^*)$ cleanly isolates the VAE's reconstruction error as the *only* source of initial distortion—an error that is both theoretically principled and practically manageable. Moreover, this choice enables us to directly leverage powerful pre-trained estimators of the posterior mean in image space (X^*), eliminating the need to separately estimate latent posterior means.

A.3 PROOF OF THEOREM 1: ASYMPTOTIC EQUIVALENCE

Theorem 1 (Asymptotic Equivalence of Latent-PMRF and PMRF). *Let $(\mathcal{E}, \mathcal{D})$ be a VAE that forms an isometry between the data space and the latent space. Then, the optimal estimator $\hat{X}_{lat} = \mathcal{D}(T_{lat}(\mathcal{E}(X^*)))$ derived from Latent-PMRF is identical to the optimal estimator \hat{X}_{PMRF} derived from PMRF, thus achieving the theoretical minimum distortion $D^* + W_2^2(p_{X^*}, p_X)$.*

810 *Proof.* The proof proceeds by establishing the theoretical minimum distortion under the perfect
 811 perception constraint and then showing that the Latent-PMRF estimator achieves this minimum.
 812 An isometry implies that the encoder \mathcal{E} is a distance-preserving bijection with inverse \mathcal{D} , such that
 813 $\mathcal{D}(\mathcal{E}(x)) = x$ and $\|x_1 - x_2\| = \|\mathcal{E}(x_1) - \mathcal{E}(x_2)\|$. The Latent-PMRF estimator is defined as
 814 $\hat{X}_{lat} = \mathcal{D}(T_{lat}(\mathcal{E}(X^*)))$, where T_{lat} is the optimal transport map between the latent distributions
 815 $p_{Z^*} = p_{\mathcal{E}(X^*)}$ and $p_Z = p_{\mathcal{E}(X)}$.

816 We first analyze the optimizable error term for this estimator, $\mathbb{E}[\|X^* - \hat{X}_{lat}\|^2]$. By applying the
 817 properties of the isometry, we can translate this pixel-space distortion into the equivalent transport
 818 cost in the latent space:
 819

$$\begin{aligned}
 821 \mathbb{E}[\|X^* - \hat{X}_{lat}\|^2] &= \mathbb{E}[\|X^* - \mathcal{D}(T_{lat}(\mathcal{E}(X^*)))\|^2] \\
 822 &= \mathbb{E}[\|\mathcal{D}(\mathcal{E}(X^*)) - \mathcal{D}(T_{lat}(\mathcal{E}(X^*)))\|^2] \quad (\text{Since } \mathcal{D}(\mathcal{E}(X^*)) = X^*) \\
 823 &= \mathbb{E}[\|\mathcal{E}(X^*) - T_{lat}(\mathcal{E}(X^*))\|^2] \quad (\text{Since } \mathcal{D} \text{ is an isometry}) \\
 824 &= \mathbb{E}[\|Z^* - T_{lat}(Z^*)\|^2] \\
 825 &= W_2^2(p_{Z^*}, p_Z) \quad (\text{By definition of } T_{lat})
 \end{aligned} \tag{6}$$

826 This is, by definition, the cost minimized by the optimal transport map T_{lat} in the latent space,
 827 which equals $W_2^2(p_{Z^*}, p_Z)$. Furthermore, because the isometry preserves the cost function for
 828 transport, the Wasserstein distance itself is preserved between the spaces, meaning $W_2^2(p_{Z^*}, p_Z) =$
 829 $W_2^2(p_{X^*}, p_X)$.

830 Combining these results, the optimizable distortion of the Latent-PMRF estimator is equal to the
 831 theoretical minimum, $W_2^2(p_{X^*}, p_X)$. Therefore, the total distortion achieved by Latent-PMRF is
 832 $\mathbb{E}[\|X - \hat{X}_{lat}\|^2] = D^* + W_2^2(p_{X^*}, p_X) = D(0)$. This demonstrates that under the ideal condition
 833 of an isometric VAE, Latent-PMRF achieves the theoretical optimum distortion-perception trade-off.
 834 \square

835 A.4 PROOF OF THEOREM 2: THE INHERENT RECONSTRUCTION ERROR

836 **Theorem 2** (The Inherent Reconstruction Error in Latent-PMRF). *Let $(\mathcal{E}, \mathcal{D})$ be any VAE and $\hat{X} =$
 837 $\mathcal{D}(T_{lat}(\mathcal{E}(X^*)))$ be the estimator from Latent-PMRF. The total expected distortion (omit constant
 838 D^*) with respect to the posterior mean can be decomposed as:*

$$\mathbb{E}[\|X^* - \hat{X}\|^2] = \underbrace{\mathbb{E}[\|X^* - \mathcal{D}(\mathcal{E}(X^*))\|^2]}_{\text{Inherent Reconstruction Error}} + \underbrace{\mathbb{E}[\|\mathcal{D}(\mathcal{E}(X^*)) - \hat{X}\|^2]}_{\text{Transport-Related Error}} + 2\mathbb{E}[\langle \dots \rangle]$$

839 *Proof.* The proof is based on an algebraic decomposition of the total error, which isolates the con-
 840 tribution of the VAE’s reconstruction infidelity from the contribution of the transport process.
 841

842 We define an intermediate term, $\tilde{X} = \mathcal{D}(\mathcal{E}(X^*))$, which represents the direct, non-transported
 843 reconstruction of the posterior mean. The optimizable error vector, $X^* - \hat{X}$, is then decomposed by
 844 adding and subtracting this term:

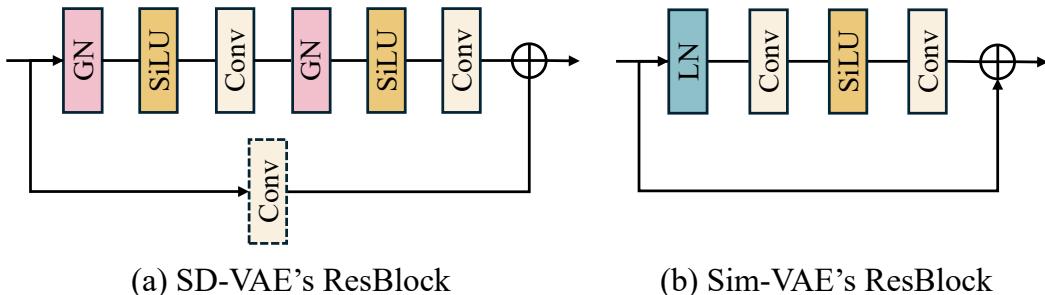
$$X^* - \hat{X} = (X^* - \tilde{X}) + (\tilde{X} - \hat{X}) \tag{7}$$

845 The vector $(X^* - \tilde{X})$ represents the error introduced solely by the VAE’s imperfect reconstruc-
 846 tion. The vector $(\tilde{X} - \hat{X})$ represents the change introduced by the transport process, viewed in the pixel
 847 space.

848 We compute the expected squared norm of the total error vector. Applying the identity $\|A + B\|^2 =$
 849 $\|A\|^2 + \|B\|^2 + 2\langle A, B \rangle$ and the linearity of expectation, we obtain:

$$\begin{aligned}
 850 \mathbb{E}[\|X^* - \hat{X}\|^2] &= \mathbb{E}[\|(X^* - \tilde{X}) + (\tilde{X} - \hat{X})\|^2] \\
 851 &= \mathbb{E}[\|X^* - \tilde{X}\|^2] + \mathbb{E}[\|\tilde{X} - \hat{X}\|^2] + 2\mathbb{E}[\langle X^* - \tilde{X}, \tilde{X} - \hat{X} \rangle]
 \end{aligned} \tag{8}$$

864 Substituting the definitions of \tilde{X} and \hat{X} yields the expression in the theorem. This identity proves
 865 the decomposition. The first term, the **Inherent Reconstruction Error**, is non-negative and de-
 866 pends only on the VAE and the distribution $p(X^*)$; it is independent of the transport map T and
 867 strictly positive for any imperfect VAE. The remaining terms constitute the **Transport-Related Er-
 868 rror**, as they depend on the final estimator \hat{X} and thus on the transport map T . This decomposition
 869 shows that the VAE’s reconstruction fidelity imposes a fundamental component to the total distor-
 870 tion, which cannot be eliminated by the subsequent transport model. This concludes the proof. \square
 871



882 Figure 6: Comparison of ResBlock designs between SD-VAE and Sim-VAE. Sim-VAE simplifies
 883 the ResBlock architecture by removing redundant components.
 884

886 B IMPROVED VARIATIONAL AUTOENCODER

888 In this section, we detail the design of **Sim-VAE**. For the Latent-PMRF model, the VAE not only
 889 defines the upper bound for restoration performance but also affects the optimization of flow model.
 890 We first outline several architectural improvements aimed at enhancing both the reconstruction abil-
 891 ity of the VAE and the distortion lower bound of Latent-PMRF. Next, We overview our training
 892 loss, where we propose eliminating the adversarial loss when VAE is strong enough, simplifying the
 893 training procedure.

894 B.1 ARCHITECTURE IMPROVEMENTS

895 Our VAE architecture builds upon the classical VQGAN Esser et al. (2021), which has been widely
 896 adopted in numerous influential works Rombach et al. (2022); Podell et al. (2024); Esser et al.
 897 (2024); Labs (2024). We refer to this architecture as SD-VAE, reflecting its widespread adoption
 898 since Stable Diffusion. The encoder and decoder share a symmetric architecture, so we focus on
 899 describing the encoder, as the decoder follows an analogous structure in reverse.

900 **Simplified ResBlock:** Inspired by recent efficient convnet designs Liu et al. (2022); Chen et al.
 901 (2022), we propose a simplified ResBlock He et al. (2016) (Figure 6) that uses only one activation
 902 function and one normalization layer per block, improving efficiency without sacrificing perfor-
 903 mance.

904 **Pixel-wise Layer Norm:** The SD-VAE has been shown to produce imbalanced feature representa-
 905 tions, where certain regions in intermediate feature maps exhibiting disproportionately high magni-
 906 tudes Sadat et al. (2024); drhead (2024), as illustrated in Figure 8. While these local outliers in the
 907 feature maps serve to preserve global information drhead (2024), they may complicate latent diffu-
 908 sion model training. Inspired by Sadat et al. (2024); Karras et al. (2020), we propose replacing group
 909 normalization Wu & He (2018) with pixel-wise layer normalization Lei Ba et al. (2016); Chen et al.
 910 (2022), which normalizes each spatial location independently and promotes more balanced feature
 911 representations.

912 **Removing Self-Attention in Middle Layers:** SD-VAE uses self-attention Vaswani et al. (2017)
 913 in middle layers to capture global context, but this introduces a key limitation: resolution gen-
 914 eralization issues. VAEs are usually trained on fixed low-resolution inputs, and global operators
 915 like self-attention often struggle to maintain performance across different resolutions during infer-
 916 ence Press et al. (2022); Gao et al. (2025). While fine-tuning on high-resolution data is a common
 917 solution Sadat et al. (2024); Chen et al. (2025), it complicates training with additional optimization
 918 stages. To address this, we propose a simple modification: replacing self-attention with standard

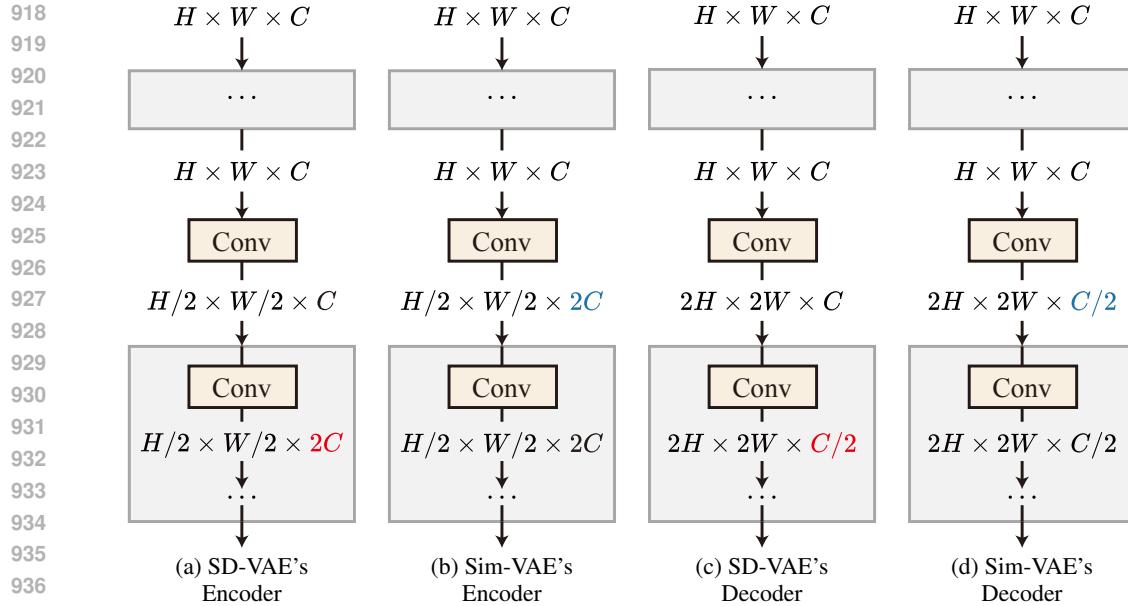


Figure 7: Illustration of the resizing layer design. Sim-VAE redistributes computation to ensure that channel dimension adjustments occur immediately with resolution changes.



Figure 8: Two examples of the latent representations. Using pixel-wise layer normalization instead of group normalization allows the model to learn more balanced feature maps.

3×3 convolutional layers, which offer better generalization across different resolutions.

Redistribute Parameters between Resizing Layers: In SD-VAE, resizing layers are responsible for handling stage transitions, but the original design separates resolution changes from channel adjustments (Figure 7a): resizing layers maintain channel dimensions, while later convolutional layers handle channel modifications. This creates bottlenecks during downsampling and retains inefficiently high-dimensional features during upsampling. We propose integrating channel adjustments directly into the resizing layers—expanding channels during downsampling and reducing them during upsampling. This change improves information preservation and computational efficiency without increasing parameter count or complexity.

B.2 TRAINING LOSS

The training objective for autoencoders typically comprises three components Esser et al. (2021): a reconstruction loss $\mathcal{L}_{\text{recon}}(\mathcal{D}(\mathcal{E}(x)), x)$ that measures the similarity between input and reconstructed images, a regularization term $\mathcal{L}_{\text{reg}}(\mathcal{E}(x))$ that constrains the latent space, and an adversarial loss Goodfellow et al. (2020) \mathcal{L}_{adv} that encourages photorealistic reconstructions by discriminating between real images x and their reconstructions $\mathcal{D}(\mathcal{E}(x))$. We observe that with sufficient model capacity, the adversarial loss becomes unnecessary without compromising performance, as also observed in Qwen-Image Wu et al. (2025). Thus, the training loss simplifies to:

972
973
974

$$\mathcal{L}_{\text{train}} = \mathcal{L}_{\text{recon}} + \lambda_{\text{reg}} \mathcal{L}_{\text{reg}} \quad (9)$$

The reconstruction loss $\mathcal{L}_{\text{recon}}$ combines ℓ_1 distance with perceptual loss Johnson et al. (2016), following the weighting scheme of Real-ESRGAN Wang et al. (2021b). For regularization, we use the Kullback-Leibler (KL) divergence as \mathcal{L}_{reg} , with λ_{reg} set to 10^{-6} as in Rombach et al. (2022).

C FURTHER EXPERIMENTS

To further validate the convergence of our Sim-VAE, we directly compare latent-PMRF (based on FLUX-VAE) and Latent-PMRF (based on Sim-VAE). As shown in Figure 9, even with the same number of latent channels (aligned computational cost), Sim-VAE exhibits superior convergence property.

In addition, we conduct comparative experiments on real-world datasets. As shown in Figure 10, Latent-PMRF achieves significantly better results than PMRF, while attaining comparable MUSIQ scores and substantially improved IndRMSE compared to state-of-the-art methods.

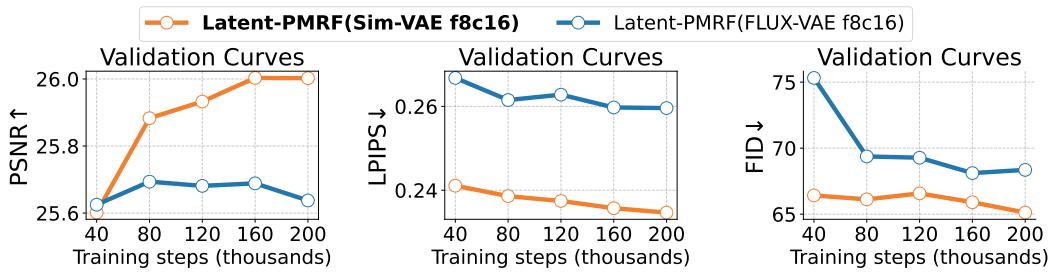


Figure 9: Convergence comparison of VAEs.

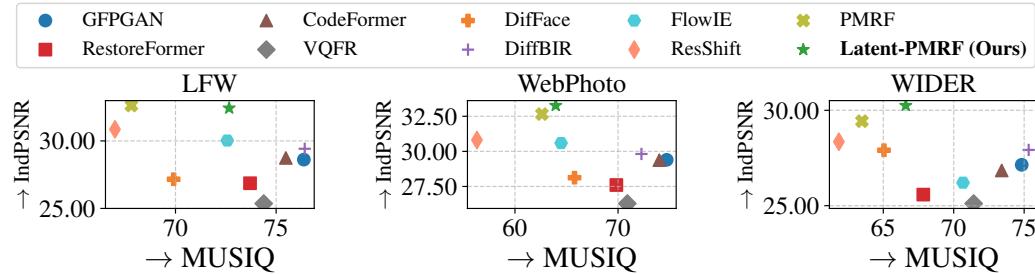


Figure 10: Comparisons on real-world datasets.

D STATEMENT ON THE USE OF LARGE LANGUAGE MODELS (LLMs)

In adherence to the ICLR 2026 submission guidelines, this section details the use of a Large Language Model (LLM) assistant during the preparation of this manuscript. The LLM, acting as a research and writing co-pilot, played a significant role in refining the manuscript's structure, language, and presentation. The authors maintained full intellectual control throughout the process and take complete responsibility for all content.

The precise role of the LLM can be categorized as follows:

- **Manuscript Writing and Polishing:** The authors wrote the initial drafts for all sections of the paper, providing the key technical details, experimental results, and core arguments. The LLM was then used extensively as an interactive writing assistant to:
 - **Enhance Clarity and Conciseness:** Rephrasing long or complex sentences to improve readability and flow.

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- **Improve Academic Tone:** Suggesting more formal and professional vocabulary and sentence structures appropriate for a top-tier conference submission.
 - **Correct Grammar and Syntax:** Performing comprehensive proofreading to identify and correct grammatical errors, typos, and awkward phrasing.
 - **Suggest Alternative Phrasing:** Providing multiple options for expressing a single idea to avoid repetitive language).
- **Technical Formalization:** For complex mathematical sections, such as the derivation of the Theorem 1,2, the authors provided the core mathematical steps and overall idea. The LLM assisted in translating these steps into a clear, well-structured narrative and formatting them professionally in LaTeX. The LLM did not generate novel mathematical proofs but rather helped in their presentation.

1038 Throughout this collaborative process, every suggestion and piece of text generated by the LLM was
1039 critically reviewed, edited, and approved by the human authors. The authors are solely responsible
1040 for the scientific validity, originality, and all claims made in this paper. The LLM is not considered
1041 an author.

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