

INTRIGUING BIAS-VARIANCE TRADEOFF IN DIFFUSION MODELS

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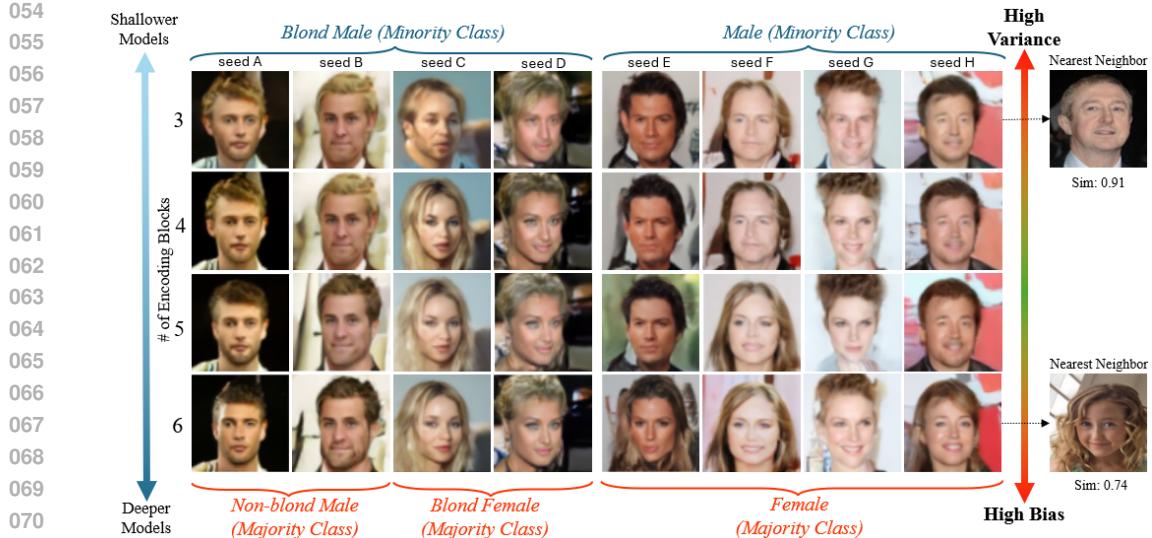


Figure 1: Deeper Diffusion Models (DMs) tend to generate samples of majority classes more compared to shallower models, while shallower models tend to give away information about their training data. Here, unconditional DMs with different depths are trained on CelebA (Liu et al., 2015) that contains “blond male” group and “male” class as minorities (less than 1% and 41.68%, respectively). **High Bias:** Starting from the same seeds on which shallow models generate samples of the minority, deeper models generate samples of the majority. Instead of “blond male”, deeper models generate “non-blond male” and “blond female”. Also, deeper models generate samples of the majority class “female” instead of “male”. **High Variance:** Shallower models are prone to generate images similar to the training dataset samples. The generated image of the shallow model resembles its nearest neighbor much more compared to the image generated by the deep model.

2 DIFFUSION MODELS AS IMPLICIT MEMORY MODULES

To facilitate our “bias-variance tradeoff” explanation, first we need to conceptualize the generative process of DMs as a “*memory recall*” task. As confirmed in (Carlini et al., 2023), DMs can memorize specific training samples and output them during generation. They implicitly store data in latent representations, enabling lossy recall via iterative refinement. This perspective emphasizes the role of the model’s learned score function as implicitly encoding a memory retrieval mechanism.

To further explain the memory retrieval behavior, we employ the analogy between energy-based Associative Memories (AMs) (Krotov & Hopfield, 2021) and DMs, inspired by (Hoover et al., 2023). Under this perspective, we view the latent space of DMs as an energy landscape, in which high energy corresponds to noisy signals and low energy represents images resembling the training data distribution. The denoising process starts with a high energy corrupted signal, and by following the direction of negative energy gradients, it eventually ends up in an *attractor*, i.e., a local minimum. In DMs, an attractor can be a “*memory*” or a “*superposition of memories*” (see Figure 2 (A)). The learning phase of DMs can be viewed analogously to traditional memory-based algorithms, such as k -NN, in the sense that they both store training data representations. However, it differs to k -NN in explicit-implicit dichotomy. In our interpretation of the training process of DMs, training data is *implicitly* memorized by DMs, as opposed to the *explicit* memorization in k -NN. Hence, we treat DMs as “Implicit Memory Modules”.

3 BIAS-VARIANCE TRADEOFF IN DIFFUSION MODELS

Given the analogy between the implicit memorization in DMs and explicit memorization in k -NNs, we can gain an insight on the bias-variance tradeoff in DMs. In k -NNs, the value of the hyperparameter k determines this tradeoff. Higher value of k leads the query point to attend to a larger number of neighbors, which smooths the decision landscape and reduces sensitivity to individual data points. Similarly, in the context of DMs, an initial corrupted noise (comparable to a query in k -NN) attends to a number of implicit memories (comparable to explicit memories in k -NN) to generate a sample

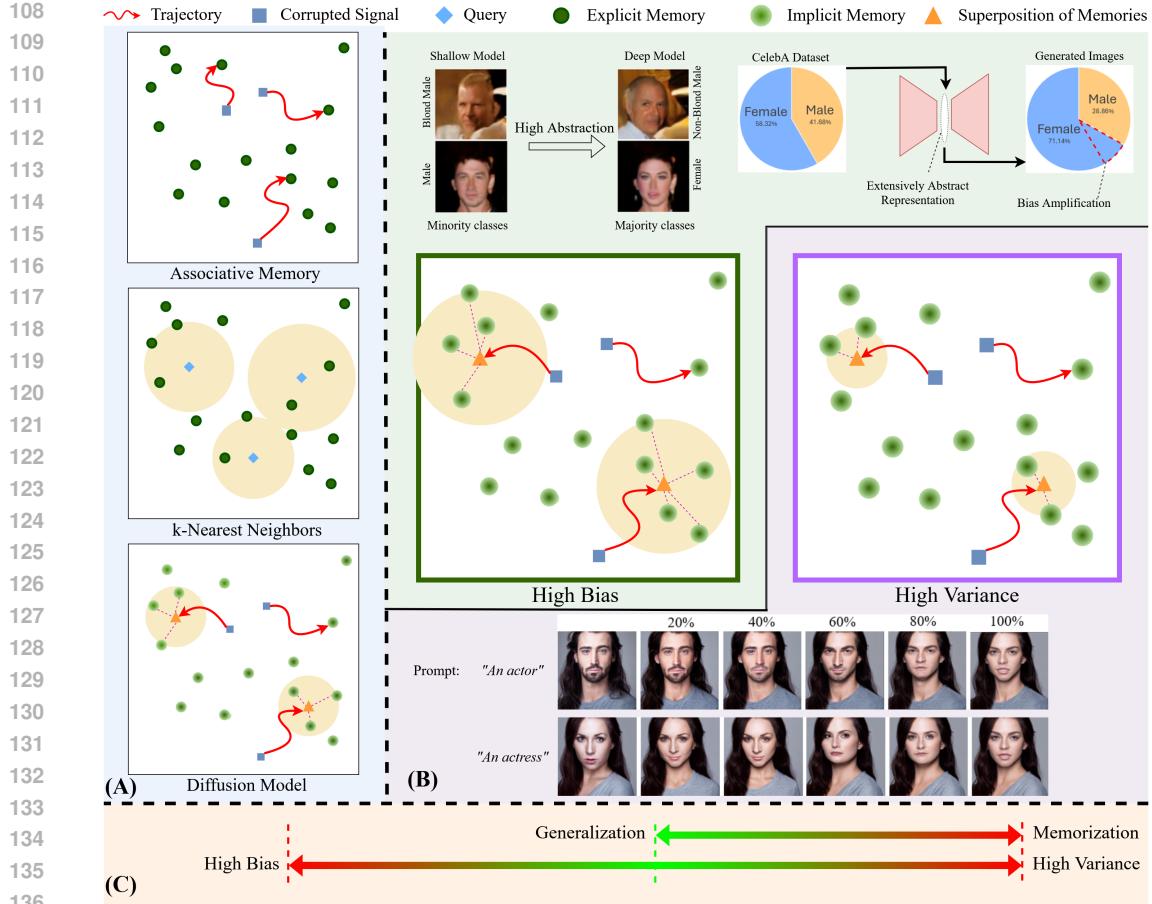


Figure 2: Illustration of the key concepts. (A) Comparison of how Associative Memory (AM), k -NNs, and DMs store and attend to the training data. In AMs and k -NNs, data is stored explicitly, as opposed to implicit storing in DMs. Corrupted signals (noise) in DMs converge to attractors like AMs. However, in DMs, there is no guarantee to converge to a memory, but the attractors are potentially *superpositions of memories*. Like k -NNs, DMs attend to a number of training samples during inference. (B) Top: For smoother energy landscapes, initial corrupted signals tend to attend to a larger number of memories (High Bias), which leads to amplification of the inherent dataset bias. We show actual bias amplification of absolute 12.82% in CelebA (Liu et al., 2015) model. This leads to more frequent generation of the majority class. E.g., ‘‘Non-Blond Male’’ and ‘‘Female’’ are more frequently generated for CelebA by *deeper* models, which naturally inherit higher bias as per our analysis. Bottom: For less smooth energy landscapes, initial corrupted signals are likely to attend to a smaller number of memories (High Variance). High variance restricts initial corrupted signals to converge to attractors corresponding to a small number of memories, leading to non-diverse generation, potentially revealing training data that is private. We show examples generated by the prompts ‘‘An actor’’ and ‘‘An actress’’. Starting from the same gaussian seed we gradually push SD V1.5 (Rombach et al., 2022) to a higher variance state during the inference. Both cases eventually converge to the same/similar person. (C) Our explanation of ‘‘bias-variance tradeoff’’ for DMs covers the contrasting concepts of ‘‘memorization’’ and ‘‘generalization’’. However, it establishes a more complete perspective beyond ‘‘generalization’’. It captures the risk of falling into ‘‘high bias’’ state by getting too far from ‘‘memorization’’.

resembling the distribution of a class ¹ (comparable to categorization to a class in k -NN.). Analogous to the higher values of k in k -NNs, which lead to the smoothening of the decision landscape, the energy landscape of DMs smoothes with higher levels of abstraction in the DM representations (see Figure 2). Hence, we can expect the bias-variance trade-off in DMs to be linked with the abstraction level of representations.

¹We intentionally treat ‘‘concepts’’ as ‘‘classes’’ to relate our discussion to k -NNs.

162 Table 1: Analogies between k -NNs and Diffusion Models (DMs).
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	k -NN	DM
164 Storing training data	165 explicit	166 implicit
167 Inference initialized with	168 a query	169 a corrupted signal
168 Inference process	169 attends to nearest neighbors	170 converges to a minimum in nearby memories
169 Smoothing factor	170 k defined as a hyperparameter	171 τ defined by abstraction level of representation
170 Larger smoothing factor	171 smoothens decision boundaries	172 smoothens the energy landscape
171 Inference goal	172 classify the query	173 refine corrupted signal to resemble a class

171 Consider a dataset $D = \{x_i, y_i\}_{i=1}^N \subset \mathbb{R}^d$, where each x_i is a training sample and y_i is its corresponding label, and $Q \in \mathbb{R}^d$ denote a query point (a corrupted signal). In k -NN, prediction is based on the average over the k closest training points to Q , denoted as $\mathcal{N}_k(Q) \subset D$. The k -NN prediction can be written as:

$$175 \quad 176 \quad 177 \quad \hat{y}(Q) = \frac{1}{k} \sum_{x_i \in \mathcal{N}_k(Q)} y_i. \quad (1)$$

178 Alternatively, using a soft weighting scheme based on distances, the prediction becomes:

$$179 \quad 180 \quad 181 \quad \hat{y}_\tau(Q) = \sum_{i=1}^N w_i^{(\tau)}(Q) y_i, \quad \text{where} \quad w_i^{(\tau)}(Q) = \frac{\exp\left(-\frac{\|Q-x_i\|^2}{\tau}\right)}{\sum_{j=1}^N \exp\left(-\frac{\|Q-x_j\|^2}{\tau}\right)}. \quad (2)$$

182 Here, $\tau > 0$ acts as a temperature that controls how broadly the query attends to the dataset: small τ approximates hard k -NN with small k , while large τ induces a smoother, more global averaging.

185 Now, consider an energy-based interpretation of DMs where the learned memories define a landscape $E : \mathbb{R}^d \rightarrow \mathbb{R}$, whose local minima correspond to the data points x_i . Given a corrupted input Q , the 186 reconstruction follows the gradient flow:

$$187 \quad 188 \quad 189 \quad \tau \frac{dx}{dt} = -\nabla E(x), \quad x(0) = Q, \quad (3)$$

190 where τ controls the rate of change, iteratively mapping Q to a nearby energy minimum. The 191 smoothness of the energy landscape, defined by the number, sharpness, and separation of local 192 minima, determines how many minima influence the trajectory of Q . In a rugged landscape, the flow 193 quickly collapses into a nearby basin, analogous to small- k behavior in k -NN, where only the most 194 immediate neighbors contribute. In contrast, a smoothed energy landscape causes the flow to be 195 influenced by multiple nearby minima, resembling a k -NN with large k aggregating information from 196 many neighbors. Thus, τ has the same role in DMs as k in k -NNs. A comprehensive set of analogies 197 between DMs and k -NNs is given in Table 1.

198 Our definition of ‘‘bias-variance tradeoff’’ aligns with its traditional counterpart, specifically with 199 k -NN classification. The bias and the variance of the model $\hat{f}(x)$ is defined by

$$200 \quad 201 \quad 202 \quad \text{bias}(\hat{f}(x_0)) = \mathbb{E}[\hat{f}(x_0)] - f(x_0), \quad (4)$$

203 and

$$204 \quad 205 \quad 206 \quad \text{variance}(\hat{f}(x_0)) = \mathbb{E}[(\hat{f}(x_0) - \mathbb{E}[\hat{f}(x_0)])^2], \quad (5)$$

207 where specifically in our definition of bias-variance in diffusion models, $f(x_0)$ is the class of the 208 training image that is encoded to x_0 and $\hat{f}(x_0)$ is the class of the generated image decoded from x_0 .

209 3.1 IMPLICATIONS OF LATENT SPACE ABSTRACTION LEVEL

210 Here, we examine how the energy landscape corresponding to DMs’ learned memories gets affected 211 by the abstraction level of their latent representation.

212 As formally depicted in Theorem 1, increasing the level of abstraction in latent representations induces 213 smoother energy landscapes. To support this claim, we analyze the local and global smoothness 214 of the energy function by examining two key quantities: the Hessian norm, which captures local 215 curvature, and the Lipschitz constant of the gradient field, which reflects how rapidly gradients can 216 vary across the space. We show that both the Hessian norm and the Lipschitz constant decrease with 217 higher levels of abstraction.

216 **Theorem 1** (Higher abstraction in representations induces smoother energy landscapes). *Let $E : \mathcal{H} \rightarrow \mathbb{R}$ be an energy function defined over a space of representations \mathcal{H} , $\phi^{(a)} : \mathcal{H} \rightarrow Z^{(a)}$ 217 be a hierarchy of abstraction maps indexed by level a , where each $\phi^{(a)}$ is a smooth, contractive 218 transformation, and $Z^{(a)}$ be a vector space of latent representations after some level of abstraction. 219 Define the energy at abstraction level a as*

$$221 \quad E := E \circ (\phi^{(a)})^{-1} : Z^{(a)} \rightarrow \mathbb{R}. \quad (6)$$

223 *Then the energy landscapes $\{E^a\}_a$ become progressively smoother with increasing a in the following 224 sense:*

$$226 \quad \|\nabla^2 E^{(a+1)}\| < \|\nabla^2 E^{(a)}\|, \quad \text{and} \quad L^{(a+1)} < L^{(a)}, \quad (7)$$

228 *where $\|\nabla^2 E^{(a)}\|$ denotes the Hessian norm (curvature), and $L^{(a)}$ is the Lipschitz constant of the 229 gradient $\nabla E^{(a)}$.*

230 The complete proof of Theorem 1 is given in the Appendix. As a result of smoothing energy 231 landscapes, some of the nearby minima will be merged into a wider yet shallower minimum basin, 232 which is defined in Definition 1.

234 **Definition 1** (Merged Local Minimum). *Let $E : \mathbb{R}^d \rightarrow \mathbb{R}$ be an energy function corresponding to a 235 latent representation. If the energy landscape becomes smoother, then a set of nearby local minima 236 $\{x_i^*\}_{i=1}^n$ of E may merge into a single, wider and shallower local minimum \tilde{x}^* of the smoothed 237 energy function \tilde{E} . Formally, for some small $\varepsilon > 0$,*

$$238 \quad \|\phi(x_i^*) - \tilde{x}^*\| \leq \varepsilon, \quad \forall i = 1, \dots, n,$$

240 *where ϕ is an abstraction (encoding) map. Hence, \tilde{x}^* is a representative of the local minima in E 241 under the abstraction i.e. in \tilde{E} . We refer to such a minimum \tilde{x}^* as a “Merged Local Minimum”.*

242 The local minima that form a merged local minimum can potentially be from different classes. The 243 question that arises here is what happens if the high-energy corrupted signal ends up in such a new 244 merged minimum. In other words, what is the probability of generating each of the classes residing 245 in the merged minimum? Answering this, we show in Theorem 2 that the generation gets heavily 246 biased in favor of the majority class.

247 **Theorem 2** (Merged local minima are biased representatives). *Let $E : \mathbb{R}^d \rightarrow \mathbb{R}$ be an energy 248 function, and let $\{x_i^*\}_{i=1}^n \subset \mathbb{R}^d$ be local minima of E , each associated with a class label $y_i \in \mathcal{Y}$. 249 Define the convex hull of these minima as:*

$$251 \quad H_C(\{x_i^*\}_{i=1}^n) := \left\{ \sum_{i=1}^n \alpha_i x_i^* \mid \alpha_i \geq 0, \sum_{i=1}^n \alpha_i = 1 \right\}. \quad (8)$$

254 Assume that an abstracted or smoothed version of the energy function \tilde{E} satisfies:

$$256 \quad \tilde{E}(x) \approx E(x) \quad \text{for } x \in H_C(\{x_i^*\}), \quad (9)$$

257 and

$$258 \quad \tilde{x}^* := \arg \min_{x \in H_C(\{x_i^*\}_{i=1}^n)} \tilde{E}(x), \quad (10)$$

260 *where \tilde{x}^* is a global minimum of \tilde{E} restricted to the convex hull. Define the empirical class distribution 261 among the constituents as*

$$263 \quad P_n(c) := \frac{1}{n} \sum_{i=1}^n I[y_i = c], \quad \text{for each } c \in \mathcal{Y}, \quad (11)$$

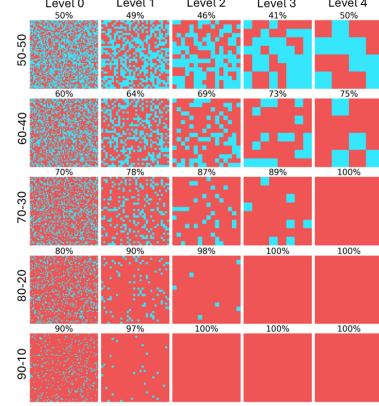
265 *and let $c_{maj} := \arg \max_{c \in \mathcal{Y}} P_n(c)$ denote the majority class. Then the classification of the merged 266 minimum \tilde{x}^* is highly biased toward c_{maj} , i.e.,*

$$268 \quad \mathbb{P}[f(\tilde{x}^*) = c_{maj}] \gg \mathbb{P}[f(\tilde{x}^*) = c], \quad \forall c \in \mathcal{Y} \setminus \{c_{maj}\}, \quad (12)$$

269 *where $f : \mathbb{R}^d \rightarrow \mathcal{Y}$ is a labeling function such that $f(x_i^*) = y_i$.*

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Figure 3: (Illustration of bias amplification.) Five different energy landscapes containing minima for two classes: “red” and “blue” with corresponding proportions of 50%-50%, 60%-40%, 70%-30%, 80%-20%, and 90%-10% are shown (left column). Each of the landscapes gets gradually more abstract, and the class of the minima is decided based on Theorem 2 by getting a local average over classes using a 2x2 window (random in case of tie in the average). The proportion of minima for the 50%-50% case remains almost the same for every level of abstraction (see first row). However, for biased cases, the bias gets amplified, and the amplification rate gets exacerbated for higher levels of initial bias (see rows 2 to 5). The percentage of the “red” (majority) class is given above each landscape.



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 286 *Proof.* Let \tilde{x}^* be a merged local minimum basin formed by smoothing the energy landscape containing n local minima, with the number of local minima corresponding to classes A and B being p and q , respectively. The initial probability of convergence to classes A and B before smoothing is

$$\mathbb{P}_{\text{init}}(A) = \frac{p}{n}, \quad \mathbb{P}_{\text{init}}(B) = \frac{q}{n}. \quad (13)$$

291 The initial odds ratio of class A to class B is given by

$$\lambda_{\text{init}}(A) = \frac{p}{q}. \quad (14)$$

292 After merging the minima, the generated image will belong to class A , class B , or a mixture of both.
 293 However, as long as classes A and B are distinguishable and the model is well-trained, the generation
 294 of a mixture of classes is not possible. Therefore, the generated image must either be from class A or
 295 from class B .

296 To avoid generating a mixture, the model must select all exclusive semantic features from one class.
 297 Thus, to generate an image from class A , all exclusive features must be selected from class A , each
 298 with an odds ratio of $\frac{p}{q}$. Let S be the number of exclusive semantic features. The odds ratio of
 299 generating an image from class A to class B after smoothing is

$$\lambda_{\text{smooth}}(A) = \left(\frac{p}{q}\right)^S. \quad (15)$$

300 If $p > q$, for $S > 1$ we have

$$\lambda_{\text{smooth}}(A) = \left(\frac{p}{q}\right)^S > \frac{p}{q} = \lambda_{\text{init}}(A), \quad (16)$$

301 and for a sufficiently large S ,

$$\lambda_{\text{smooth}}(A) \gg \lambda_{\text{init}}(A). \quad (17)$$

302 So,

$$\mathbb{P}_{\text{smooth}}(A) \gg \mathbb{P}_{\text{smooth}}(B), \quad (18)$$

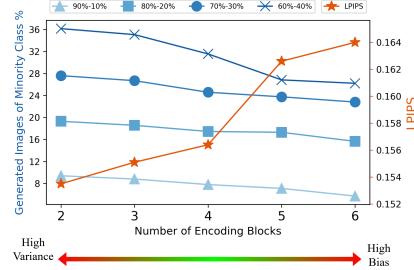
303 where $\mathbb{P}_{\text{smooth}}(A)$ and $\mathbb{P}_{\text{smooth}}(B)$ are probabilities of convergence to classes A and B after smoothing,
 304 respectively. \square

305 Figure 3 illustrates how biased representatives exacerbate bias. It shows five energy landscapes with
 306 different proportions of classes, getting gradually more abstract (left to right). Based on Theorem 2,
 307 we represent nearby classes as the majority class (random in case of a tie) in the more abstract
 308 representations. It can be seen that while class proportions remain almost identical in the balanced
 309 case, the majority class dominates with higher levels of abstraction in unbalanced cases. Also, the
 310 stronger the initial bias, the greater the bias amplification induced by representation abstraction.

320 3.2 LATENT SPACE ABSTRACTION LEVEL INFLUENCES BIAS-VARIANCE TRADEOFF IN 321 DIFFUSION MODELS

322 Based on Theorem 1 and Theorem 2, we claim that the bias-variance tradeoff in diffusion models is
 323 directly influenced by the level of abstraction of the latent space representation.

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 Figure 4: % of generated images from the minority class, and the average LPIPS metric to measure their diversity. Five DMs with 2 to 6 encoding blocks each trained on four custom MNIST datasets contained only “6”s and “9”s with biased proportions (see legend). Deeper models, that make more abstract representations, generate more diverse images (higher average LPIPS) and amplify the bias. “High variance” and “high bias” states can be identified on “shallow” and “deep” models, respectively.



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 Table 2: Percentage of images generated by unconditional DMs trained on CelebA. Higher abstraction of representations that is achieved by a larger number of encoding blocks leads to amplification of bias, as well as more diversity in generating images.

	Dataset Proportions	# of Encoding Blocks					
		2	2 (large)	3	4	5	6
Male	41.68%	40.33%	40.85%	39.18%	37.22%	30.36%	28.85%
Female	58.32%	59.67%	59.15%	60.82%	62.78%	69.64%	71.15%
Average LPIPS		22.91	23.26	23.17	25.65	28.03	29.12

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 346 **High abstraction levels of representations lead to “high bias”.** Based on Theorem 1, highly abstracted representation makes the energy landscape corresponding to the DM’s learned memories smoother, and this smoothness results in the emergence of merged minima basins. Theorem 2 establishes that converging to those merged minima basins amplifies the probability of generating majority class samples considerably. Consequently, the generated images may belong to the majority class with a higher probability than the original proportion of that class. That is, the bias gets amplified. **Low abstraction levels of representations lead to “high variance”.** Low abstraction levels lead to uneven energy landscapes with numerous individual local minima, which are representatives of less aggregated and more individual data points. The convergence behavior of the corrupted signal is very sensitive to the presence or absence of individual training samples in this case. As a result, this will be the state of “high variance”.

3.3 “HIGH BIAS STATE” LEADS TO AMPLIFICATION OF THE INHERENT BIAS IN DATASETS

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 360 To verify the established relationship between high levels of abstraction and “high bias”, we designed and conducted a series of experiments on MNIST and CelebA datasets to further examine this connection.

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 362 First, we trained a series of unconditional diffusion models on custom MNIST datasets containing only digits “6” and “9” with different proportions. The base case is a diffusion model with two encoding
 363 blocks trained on a biased dataset containing 90% samples of class “6” and 10% samples of class “9”.
 364 We further trained increasingly deeper models on the same biased dataset. The models consist of
 365 three, four, five, and six encoding blocks. Interestingly, we observed that while the percentage of
 366 generated images for the base model almost aligns with the proportions of the training dataset, deeper
 367 models; which leverage increasingly more abstract representations, tend to amplify the bias. The
 368 same experiments are also conducted on datasets consisting 80-20, 70-30, and 60-40 percent of “6”s
 369 and “9”s, respectively, and the results were consistent. We also used Learned Perceptual Image Patch
 370 Similarity (LPIPS) metric Zhang et al. (2018) to measure the diversity of generated images. For each
 371 setting, we generated 1000 images and calculated the LPIPS metric for each pair. Eventually, the
 372 average LPIPS is reported, which showed higher diversity for deeper architectures (see Figure 4).

373 To validate our observation further, we trained a series of increasingly deeper DMs on CelebA
 374 dataset Liu et al. (2015) and generated 10,000 images. As presented in Table 2, the proportions
 375 of generated images across the two classes of “Male”, “Female” support our claims. Although the
 376 proportion of “Female” samples to “Male” samples is 58.32% to 41.68% in the training data, the bias
 377 amplifies as the number of encoding blocks increases. Skeptically, this observation could be caused
 by over-parameterization. Hence, we repeated the experiment on a large 2-encoding-block model that

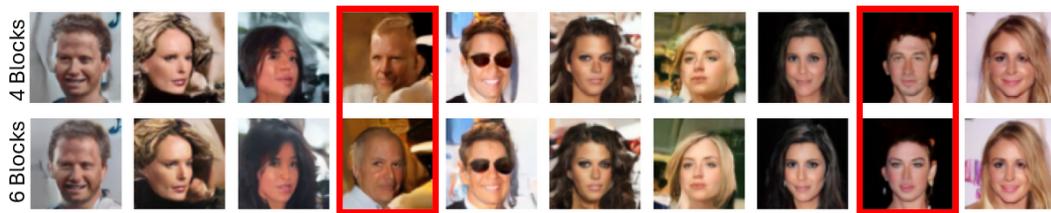


Figure 5: 10 unconditional consecutive generations of two DMs trained on CelebA, starting from the same seeds: one with 6-encoding blocks and one with 4-encoding blocks. The deeper model tends to follow the biased data distribution in CelebA. While most outputs are visually similar, two highlighted cases differ: the 6-encoding block model favors the majority classes, producing non-blond males over blond males (40.81 % vs. 0.86 %) and females over males (58.32 % vs. 41.68 %).

Table 3: Average cosine similarity in Inception feature space and average Euclidean distance in pixel space between generated images and their corresponding k nearest neighbors in the CelebA training dataset. The generated images from shallower models consistently show higher cosine similarity and lower Euclidean distance in the Inception feature and pixel spaces, respectively, indicating their greater tendency to expose training data information.

# of Encoding Blocks	Avg. cosSim in Inception Feature Space for k Nearest Neighbors				Avg. Euclidean Distance in Pixel Space for k Nearest Neighbors			
	$k = 1$	$k = 2$	$k = 5$	$k = 10$	$k = 1$	$k = 2$	$k = 5$	$k = 10$
	3	0.8751	0.8612	0.8567	0.8543	97.78	99.89	113.01
4	0.8644	0.8541	0.8506	0.8420	106.15	106.25	113.11	115.03
5	0.8428	0.8355	0.8277	0.8265	106.62	106.87	107.25	117.01
6	0.8142	0.8112	0.8056	0.8016	121.41	121.84	129.91	143.05

has almost the same number of parameters as the 5-encoding-block model (see Table A in Appendix). The results remain largely similar to the small 2-encoding-block model we experimented with earlier which confirms that it is not the number of parameters, but the abstraction level of representations that is responsible for the observed bias amplification. Please note that the results for the classes that are combinations of *genders* and *hair colors* are not reported, because in that case the minority class (blond male) contributes to less than 1% of the training data, and getting a meaningful result of its changes required generating millions of samples, so we reported the results for “gender” classes.

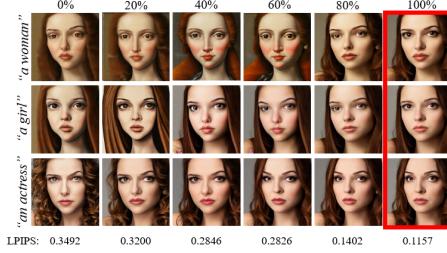
Figure 5 shows 10 consecutive generations of two models with 4 and 6 encoding blocks, trained on CelebA starting from the same seed. In the two highlighted sample pairs, the generated images differ: the deeper model generates examples from majority classes - a “non-blond male” that makes up 40.81% of the dataset, instead of a “blond male”, which represents only 0.86% of the dataset; and a “female” that constitutes 58.32% of the dataset, instead of a “male” that accounts for 41.68%. These results are perfectly explained by our bias amplification insight.

3.4 “HIGH VARIANCE STATE” UNDERMINES PRIVACY PRESERVATION IN DIFFUSION MODELS

We also investigated the state of “high variance” in which the model generates images that more closely resemble specific samples of the training data. Table 3 gives the average result of measured similarities between the generated images and their nearest neighbors in the training data. For this comparison, cosine similarity in Inception feature space and Euclidean distance in pixel space are used. The experiments are carried on for $k = 1, 2, 5$, and 10. The results clearly show in general, shallower models tend to generate images resembling some specific samples of the training data more than deeper ones, as they yield larger cosine similarities between their features in Inception feature space and their nearest neighbors, while having smaller Euclidean distance in pixel space. This observation emphasizes the vulnerability of shallower models with respect to the privacy of training data in diffusion models.

Also, relevant to privacy preserving of training data is the capability of models in generating diverse images. Figure 4 and Table 2 give average mutual LPIPS scores for generated images by models trained on MNIST and CelebA, respectively. The results consistently show lower LPIPS for shallower models, indicating their limited ability to generate diverse outputs.

432
 433 Figure 6: Images generated with SD v1.5 for three sim-
 434 ilar prompts: “a woman”, “a girl”, and “an actress”,
 435 using the same seed (left column). Gradually bypassing
 436 the model’s mid-block makes the outputs more similar
 437 and realistic, as the model relies on less abstract rep-
 438 resentations, inducing “high-variance” state, where the
 439 noisy signal attends to only a few attractors, reflecting
 440 specific memorized patterns.
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442 As a qualitative experiment, we also tried to force the pretrained model to work with less abstract data
 443 representations to simulate the “high bias” state. To do so, we bypassed the mid-block of the U-Net of
 444 SD V1.5. This way, the model relies on the less abstract representations flowing via skip connections.
 445 Figure 6 shows the images generated by SD V1.5 for three prompts: “a woman”, “a girl”, and “an
 446 actress”. We show the results as percentage bypassed mid-block activations in gradually increasing
 447 proportions, up to complete bypassing (100%). As can be observed, the complete bypass results in
 448 generating almost similar images for different prompt. This follows naturally from our bias-variance
 449 analysis in the previous sections. It entails that promoting higher variance of pre-trained models by
 450 bypassing can put the (presumed) privacy of DMs at risk.
 451

4 RELATED WORK

452
 453 **Explanation of Diffusion Models** Many recent studies have focused on explanation of the underlying
 454 mechanisms of DMs. Yi *et al.* (2024) showed that the generation of images in DMs starts
 455 with low frequency signals in initial steps, followed by the construction of high frequency signals
 456 in later steps. This behavior leads to the generation of the overall image first and details later in
 457 the denoising process. The memorization-generalization properties of DMs are explored in Li *et al.*
 458 (2023), Li *et al.* (2024), and Yoon *et al.* (2023). Furthermore, Vastola (2025) explored the
 459 elements influencing the generalization of DMs. Kwon *et al.* (2023) uncovered the
 460 semantic latent space in DMs, which is further explored in Park *et al.* (2023) and Wang *et al.* (2025).
 461 As a more intuitive alternative to the common explanation of DM models’ generation as *iterative*
 462 *denoising*, Hoover *et al.* (2023) proposed to conceptualize the inference process as
 463 *memory retrieval*.
 464

465 **Bias Mitigation in Diffusion Models** DMs are accused of amplifying the biases inherent in their
 466 training data Friedrich et al. (2023); Bianchi et al. (2023). However, Seshadri *et al.* (2023) challenged this concern and argued that in the text-to-image case, bias exacerbation stems
 467 from distribution shift in training captions and prompts. Conversely, as suggested by Chen *et al.*, the
 468 bias amplification is more serious than presumed and can potentially result in the cascading of bias
 469 towards future models Chen et al. (2024b).
 470

471 **Privacy of Diffusion Models** To preserve the confidentiality of potentially private dataset samples,
 472 Jahanian *et al.* (2022) proposed using generative models to have variants of samples
 473 instead of explicitly storing and working with real datasets. However, Carlini *et al.* (2023) revealed that DMs are prone to generate images highly resembling their training data. Also, it
 474 is argued that DMs can be pushed to give away their training samples using membership inference
 475 method Duan et al. (2023). In response to the challenge, robust DMs that address privacy concerns
 476 are proposed Dockhorn et al. (2023); Wang et al. (2024); Chen et al. (2024a). Moreover, the inherent
 477 privacy guarantees and conditions affecting privacy-preserving properties of datasets generated by
 478 DMs are analyzed in Wei et al. (2024).
 479

5 CONCLUSIONS

480 In this work, we explored the concept of “bias-variance trade-off” in the context of DMs, which ex-
 481 plains bias amplification at one extreme and privacy risks as well as reduced generative diversity at the
 482 other. Our arguments are aligned with the opposing concepts of “memorization” and “generalization”,
 483 but it further captures the state beyond generalization. This study revealed the potential drawbacks
 484 of extremely deep DMs, not limited to computational cost. We also showed the vulnerability of
 485 pre-trained DMs to be pushed to “high variance” state in order to get information of their training
 486 data.
 487

486 REFERENCES
487

488 Federico Bianchi, Pratyusha Kalluri, Esin Durmus, Faisal Ladhak, Myra Cheng, Debora Nozza,
489 Tatsunori Hashimoto, Dan Jurafsky, James Zou, and Aylin Caliskan. Easily accessible text-to-
490 image generation amplifies demographic stereotypes at large scale. In *Proceedings of the 2023*
491 *ACM Conference on Fairness, Accountability, and Transparency*, pp. 1493–1504, 2023.

492 Nicolas Carlini, Jamie Hayes, Milad Nasr, Matthew Jagielski, Vikash Sehwag, Florian Tramer, Borja
493 Balle, Daphne Ippolito, and Eric Wallace. Extracting training data from diffusion models. In *32nd*
494 *USENIX Security Symposium (USENIX Security 23)*, pp. 5253–5270, 2023.

495 Chen Chen, Daochang Liu, and Chang Xu. Towards memorization-free diffusion models. In
496 *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp.
497 8425–8434, 2024a.

498 Tianwei Chen, Yusuke Hirota, Mayu Otani, Noa Garcia, and Yuta Nakashima. Would deep generative
499 models amplify bias in future models? In *Proceedings of the IEEE/CVF Conference on Computer*
500 *Vision and Pattern Recognition*, pp. 10833–10843, 2024b.

502 Prafulla Dhariwal and Alexander Nichol. Diffusion models beat gans on image synthesis. *Advances*
503 *in neural information processing systems*, 34:8780–8794, 2021.

504 Tim Dockhorn, Tianshi Cao, Arash Vahdat, and Karsten Kreis. Differentially private diffusion models.
505 *Transactions on Machine Learning Research*, 2023.

507 Jinhao Duan, Fei Kong, Shiqi Wang, Xiaoshuang Shi, and Kaidi Xu. Are diffusion models vulnerable
508 to membership inference attacks? In *International Conference on Machine Learning*, pp. 8717–
509 8730. PMLR, 2023.

510 Patrick Esser, Robin Rombach, and Bjorn Ommer. Taming transformers for high-resolution image
511 synthesis. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*,
512 pp. 12873–12883, 2021.

514 Felix Friedrich, Manuel Brack, Lukas Struppek, Dominik Hintersdorf, Patrick Schramowski, Sasha
515 Luccioni, and Kristian Kersting. Fair diffusion: Instructing text-to-image generation models on
516 fairness. *arXiv preprint arXiv:2302.10893*, 2023.

517 Stuart Geman, Elie Bienenstock, and René Doursat. Neural networks and the bias/variance dilemma.
518 *Neural computation*, 4(1):1–58, 1992.

519 Benjamin Hoover, Hendrik Strobelt, Dmitry Krotov, Judy Hoffman, Zsolt Kira, and Polo Chau.
520 Memory in plain sight: A survey of the uncanny resemblances between diffusion models and
521 associative memories. In *Annual Conference on Neural Information Processing Systems*, 2023.

523 Rongjie Huang, Jiawei Huang, Dongchao Yang, Yi Ren, Luping Liu, Mingze Li, Zhenhui Ye, Jinglin
524 Liu, Xiang Yin, and Zhou Zhao. Make-an-audio: Text-to-audio generation with prompt-enhanced
525 diffusion models. In *International Conference on Machine Learning*, pp. 13916–13932. PMLR,
526 2023.

527 Ali Jahanian, Xavier Puig, Yonglong Tian, and Phillip Isola. Generative models as a data source
528 for multiview representation learning. In *International Conference on Learning Representations*,
529 2022.

530 Dmitry Krotov and John J Hopfield. Large associative memory problem in neurobiology and machine
531 learning. In *International Conference on Learning Representations*, 2021.

533 Mingi Kwon, Jaeseok Jeong, and Youngjung Uh. Diffusion models already have a semantic latent
534 space. In *The Eleventh International Conference on Learning Representations*, 2023.

535 Puheng Li, Zhong Li, Huishuai Zhang, and Jiang Bian. On the generalization properties of diffusion
536 models. *Advances in Neural Information Processing Systems*, 36:2097–2127, 2023.

538 Xiang Li, John Thickstun, Ishaan Gulrajani, Percy S Liang, and Tatsunori B Hashimoto. Diffusion-lm
539 improves controllable text generation. *Advances in neural information processing systems*, 35:
4328–4343, 2022.

540 Xiang Li, Yixiang Dai, and Qing Qu. Understanding generalizability of diffusion models requires
 541 rethinking the hidden gaussian structure. *Advances in Neural Information Processing Systems*, 37:
 542 57499–57538, 2024.

543

544 Haohe Liu, Zehua Chen, Yi Yuan, Xinhao Mei, Xubo Liu, Danilo Mandic, Wenwu Wang, and
 545 Mark D Plumbley. Audioldm: Text-to-audio generation with latent diffusion models. *arXiv*
 546 *preprint arXiv:2301.12503*, 2023.

547

548 Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In
 549 *Proceedings of International Conference on Computer Vision (ICCV)*, December 2015.

550

551 Yong-Hyun Park, Mingi Kwon, Jaewoong Choi, Junghyo Jo, and Youngjung Uh. Understanding the
 552 latent space of diffusion models through the lens of riemannian geometry. *Advances in Neural*
 553 *Information Processing Systems*, 36:24129–24142, 2023.

554

555 Dustin Podell, Zion English, Kyle Lacey, Andreas Blattmann, Tim Dockhorn, Jonas Müller, Joe
 556 Penna, and Robin Rombach. Sdxl: Improving latent diffusion models for high-resolution image
 557 synthesis. *arXiv preprint arXiv:2307.01952*, 2023.

558

559 Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, and Björn Ommer. High-
 560 resolution image synthesis with latent diffusion models. In *Proceedings of the IEEE/CVF Conference*
 561 *on Computer Vision and Pattern Recognition (CVPR)*, pp. 10684–10695, June 2022.

562

563 Preethi Seshadri, Sameer Singh, and Yanai Elazar. The bias amplification paradox in text-to-image
 564 generation. *arXiv preprint arXiv:2308.00755*, 2023.

565

566 John J Vastola. Generalization through variance: how noise shapes inductive biases in diffusion
 567 models. *arXiv preprint arXiv:2504.12532*, 2025.

568

569 Haichen Wang, Shuchao Pang, Zhigang Lu, Yihang Rao, Yongbin Zhou, and Minhui Xue. dp-promise:
 570 Differentially private diffusion probabilistic models for image synthesis. In *33rd USENIX Security*
 571 *Symposium (USENIX Security 24)*, pp. 1063–1080, 2024.

572

573 Li Wang, Boyan Gao, Yanran Li, Zhao Wang, Xiaosong Yang, David A Clifton, and Jun Xiao.
 574 Exploring the latent space of diffusion models directly through singular value decomposition.
 575 *arXiv preprint arXiv:2502.02225*, 2025.

576

577 Joseph L Watson, David Juergens, Nathaniel R Bennett, Brian L Trippe, Jason Yim, Helen E Eisenach,
 578 Woody Ahern, Andrew J Borst, Robert J Ragotte, Lukas F Milles, et al. De novo design of protein
 579 structure and function with rfdiffusion. *Nature*, 620(7976):1089–1100, 2023.

580

581 Rongzhe Wei, Eleonora Kreacic, Haoyu Peter Wang, Haoteng Yin, Eli Chien, Vamsi K Potluru, and
 582 Pan Li. On the inherent privacy properties of discrete denoising diffusion models. *Transactions on*
 583 *Machine Learning Research*, 2024.

584

585 Tong Wu, Zhihao Fan, Xiao Liu, Hai-Tao Zheng, Yeyun Gong, Jian Jiao, Juntao Li, Jian Guo, Nan
 586 Duan, Weizhu Chen, et al. Ar-diffusion: Auto-regressive diffusion model for text generation.
 587 *Advances in Neural Information Processing Systems*, 36:39957–39974, 2023.

588

589 Mingyang Yi, Aoxue Li, Yi Xin, and Zhenguo Li. Towards understanding the working mechanism of
 590 text-to-image diffusion model. In *The Thirty-eighth Annual Conference on Neural Information*
 591 *Processing Systems*, 2024.

592

593 TaeHo Yoon, Joo Young Choi, Sehyun Kwon, and Ernest K Ryu. Diffusion probabilistic models
 594 generalize when they fail to memorize. In *ICML 2023 workshop on structured probabilistic*
 595 *inference \& generative modeling*, 2023.

596

597 Richard Zhang, Phillip Isola, Alexei A Efros, Eli Shechtman, and Oliver Wang. The unreasonable
 598 effectiveness of deep features as a perceptual metric. In *Proceedings of the IEEE conference on*
 599 *computer vision and pattern recognition*, pp. 586–595, 2018.

594 **A APPENDIX**595 **A.1 PROOF OF THEOREM 1**

598 **Theorem 1** (Higher abstraction in representations induces smoother energy landscapes). *Let $E : \mathcal{H} \rightarrow \mathbb{R}$ be an energy function defined over a space of representations \mathcal{H} , $\phi^{(a)} : \mathcal{H} \rightarrow Z^{(a)}$ be a hierarchy of abstraction maps indexed by level a , where each $\phi^{(a)}$ is a smooth, contractive transformation, and $Z^{(a)}$ be a vector space of latent representations after some level of abstraction. Define the energy at abstraction level a as*

$$603 \quad E := E \circ (\phi^{(a)})^{-1} : Z^{(a)} \rightarrow \mathbb{R}. \quad (19)$$

605 *Then the energy landscapes $\{E^a\}_a$ become progressively smoother with increasing a in the following 606 sense:*

$$607 \quad \|\nabla^2 E^{(a+1)}\| < \|\nabla^2 E^{(a)}\|, \quad \text{and} \quad L^{(a+1)} < L^{(a)}, \quad (20)$$

609 *where $\|\nabla^2 E^{(a)}\|$ denotes the Hessian norm (curvature), and $L^{(a)}$ is the Lipschitz constant of the 610 gradient $\nabla E^{(a)}$.*

612 *Proof.* Let $J_{\phi^{(a)}-1}(z^{(a)})$ denote the Jacobian of $\phi^{(a)-1}$ at $z^{(a)}$. Then, by the chain rule, the gradient 613 of the energy function $E^{(a)}$ at $z^{(a)}$ is given by:

$$614 \quad \nabla E^{(a)}(z^{(a)}) = J_{\phi^{(a)}-1}(z^{(a)})^\top \nabla E(\phi^{(a)-1}(z^{(a)})). \quad (21)$$

617 Similarly, the Hessian of $E^{(a)}$ is given by:

$$619 \quad \nabla^2 E^{(a)}(z^{(a)}) = J_{\phi^{(a)}-1}(z^{(a)})^\top \nabla^2 E(\phi^{(a)-1}(z^{(a)})) J_{\phi^{(a)}-1} + \sum_i \frac{\partial E}{\partial x_i} \nabla^2 \phi^{(a)-1}(z^{(a)}). \quad (22)$$

621 Taking norms on both sides, we obtain:

$$623 \quad \|\nabla^2 E^{(a)}(z^{(a)})\| \leq \overbrace{\|J_{\phi^{(a)}-1}\|^2}^{\text{decreasing in } a} \cdot \overbrace{\|\nabla^2 E\|}^{\text{fixed}} + \overbrace{\left\| \sum_i \frac{\partial E}{\partial x_i} \nabla^2 \phi^{(a)-1}(z^{(a)}) \right\|}^{\text{decreasing in } a}. \quad (23)$$

627 Based on Lemma 1, $\phi^{(a)}$ is a contractive and smoothing map (i.e., the Jacobian norm decreases and 628 the second derivatives of its inverse diminish with increasing a), both terms in the bound decrease 629 with increasing a . Therefore, the norm of the Hessian satisfies

$$631 \quad \|\nabla^2 E^{(a+1)}\| < \|\nabla^2 E^{(a)}\|. \quad (24)$$

633 Moreover, let $L^{(a)}$ denote the Lipschitz constant of $\nabla E^{(a)}$, so that:

$$634 \quad \|\nabla E^{(a)}(z_1^{(a)}) - \nabla E^{(a)}(z_2^{(a)})\| \leq L^{(a)} \|z_1^{(a)} - z_2^{(a)}\|, \quad \forall z_1^{(a)}, z_2^{(a)} \in Z^{(a)}. \quad (25)$$

636 Since $\|\nabla^2 E^{(a)}\| \geq L^{(a)}$, Equation 25 implies:

$$638 \quad L^{(a+1)} < L^{(a)} \quad (26)$$

640 This shows that both the second-order and first-order variations of the energy function diminish with 641 increasing abstraction level. Hence, the energy landscapes become progressively smoother. \square

642 **Lemma 1** (Higher levels of abstraction in the representation induce smaller Jacobian norm and 643 second-order derivative of the representation mapping).

645 **Proof. Diminishing Jacobian:** Define the sensitivity to perturbations at level a as the norm of the 646 Jacobian as:

$$647 \quad J(a) := \left\| \frac{\partial z^{(a)}}{\partial x} \right\|, \quad J(a+1) := \left\| \frac{\partial z^{(a+1)}}{\partial x} \right\|. \quad (27)$$

648 By first-order Taylor expansion, the effect of the perturbation on the representation at level a is:
 649

$$650 \quad z^{(a)}(x + \delta) \approx z^{(a)}(x) + \frac{\partial z^{(a)}}{\partial x} \delta, \quad (28)$$

651 and thus the magnitude of the change is:
 652

$$653 \quad \left\| z^{(a)}(x + \delta) - z^{(a)}(x) \right\| \approx \left\| \frac{\partial z^{(a)}}{\partial x} \delta \right\| \leq J(a) \cdot \|\delta\|. \quad (29)$$

654 Similarly, for level $a + 1$:
 655

$$656 \quad z^{(a+1)}(x + \delta) \approx z^{(a+1)}(x) + \frac{\partial z^{(a+1)}}{\partial x} \delta, \quad (30)$$

657 and
 658

$$659 \quad \left\| z^{(a+1)}(x + \delta) - z^{(a+1)}(x) \right\| \approx \left\| \frac{\partial z^{(a+1)}}{\partial x} \delta \right\| \leq J(a+1) \cdot \|\delta\|. \quad (31)$$

660 As increasing abstraction decreases sensitivity to perturbations:
 661

$$662 \quad \left\| z^{(a+1)}(x + \delta) - z^{(a+1)}(x) \right\| < \left\| z^{(a)}(x + \delta) - z^{(a)}(x) \right\|. \quad (32)$$

663 Using the approximations in Equation (29) and Equation (31), this gives:
 664

$$665 \quad J(a+1) \cdot \|\delta\| < J(a) \cdot \|\delta\| \implies J(a+1) < J(a). \quad (33)$$

666 **Diminishing second derivative of $\phi^{(a)^{-1}}$:** Consider a small perturbation δ added at level $a + 1$:
 667

$$668 \quad z^{(a+1)} + \delta. \quad (34)$$

669 The corresponding change at level a is approximated by a first and second order Taylor expansion of
 670 the inverse map $\phi^{(a+1)^{-1}}$:
 671

$$672 \quad z^{(a)} + \Delta z^{(a)} \approx \phi^{(a+1)^{-1}}(z^{(a+1)} + \delta) = z^{(a)} + J_{\phi^{(a+1)^{-1}}} \delta + \frac{1}{2} \delta^\top \nabla^2 \phi^{(a+1)^{-1}} \delta, \quad (35)$$

673 where $J_{\phi^{(a+1)^{-1}}}$ is the Jacobian matrix and $\nabla^2 \phi^{(a+1)^{-1}}$ is the Hessian tensor of $\phi^{(a+1)^{-1}}$. The
 674 change in the energy at level $a + 1$ due to δ is:
 675

$$676 \quad \Delta^2 E^{(a+1)} \approx \delta^\top \nabla^2 E^{(a+1)} \delta. \quad (36)$$

677 At level a , the change is:
 678

$$679 \quad \Delta^2 E^{(a)} \approx (J_{\phi^{(a+1)^{-1}}} \delta)^\top \nabla^2 E^{(a)} (J_{\phi^{(a+1)^{-1}}} \delta), \quad (37)$$

680 ignoring higher order terms involving $\nabla^2 \phi^{(a+1)^{-1}}$. As increasing abstraction implies reduction in
 681 sensitivity, we have $|J_{\phi^{(a+1)^{-1}}}| < 1$. So,
 682

$$683 \quad \|\nabla^2 E^{(a+1)}\| \leq \|J_{\phi^{(a+1)^{-1}}}\|^2 \cdot \|\nabla^2 E^{(a)}\|. \quad (38)$$

684 Thus,
 685

$$686 \quad \|\nabla^2 E^{(a+1)}\| < \|\nabla^2 E^{(a)}\| \quad (39)$$

687 \square
 688

689 A.2 IMPLEMENTATION DETAILS

690 All the experiments are implemented in Pytorch and they used three NVIDIA RTX 3090 GPUs each
 691 with 24G RAM. The details of the trained models are summarized in Table A.
 692

693 A.3 BIAS AMPLIFICATION

694 Figure A and Figure B show qualitative examples of bias amplification in deeper models. Figure A
 695 shows representative examples where shallower models generated images of “blond-male” minority
 696 group, but the images got changed to the majority groups (classes) of “non-blond male” and “blond
 697 female” for the deeper models when starting from the same seeds. Similarly, Figure B presents
 698 cases in which shallow models generate “male” images that are minority group, while deeper models
 699 converted the same seeds to the majority group, i.e., “female”.
 700



724 Figure A: Bias amplification in deeper models. Top row shows samples of generated images
725 resembling “blond male” by a relatively shallow model consisting of three encoding blocks. Lower
726 rows show generated images of increasingly deeper models starting from the same seeds. It can be
727 be seen that the deeper models prefer generating samples from the majority groups (non-blond male and
728 blond-female) instead of the minority group (blond-male), thereby amplifying the data bias.
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752 Figure B: Representative cases where shallow models generate samples of minority group “male”,
753 but deeper models generate majority group “female” starting from the same seeds.
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 757 Table A: Details of trained diffusion models. Please note the number of parameters on the “large 2
 758 encoding blocks” model, which is at the same scale of deeper models (“5 and 6 encoding blocks”).
 759 However, its bias-amplification behavior is more similar to shallower models (“2 and 3 encoding
 760 blocks”) (see Table 2 in the main paper).

Number of encoding blocks	Architecture					# of parameters
	Down-block types	Up-block types	Block out channels	Layers per block		
2	DownBlock2D, AttnDownBlock2D	AttnUpBlock2D, UpBlock2D	32, 64	2	1002883	
2 (Large)	DownBlock2D, AttnDownBlock2D	AttnUpBlock2D, UpBlock2D	128, 256	2	15930883	
3	DownBlock2D, AttnDownBlock2D, DownBlock2D	UpBlock2D, AttnUpBlock2D, UpBlock2D	32, 64, 128	2	3692867	
4	DownBlock2D, DownBlock2D, AttnDownBlock2D, DownBlock2D	UpBlock2D, UpBlock2D, AttnUpBlock2D, UpBlock2D	32, 32, 64, 128	2	3859203	
5	DownBlock2D, DownBlock2D, DownBlock2D, AttnDownBlock2D, DownBlock2D	UpBlock2D, AttnUpBlock2D, UpBlock2D, UpBlock2D, UpBlock2D	32, 64, 128, 128, 256	2	16887619	
6	DownBlock2D, DownBlock2D, DownBlock2D, DownBlock2D, AttnDownBlock2D, DownBlock2D	UpBlock2D, AttnUpBlock2D, UpBlock2D, UpBlock2D, UpBlock2D, UpBlock2D	32, 64, 128, 128, 256, 256	2	27290691	



783 Figure C: A model pushed towards “high variance” state by bypassing its mid-block generates similar
 784 looking images for slightly different prompts. The resulting images not only have high perceptual
 785 similarity across the prompts, but also better realism. This behavior is consistent with our arguments
 786 about “high variance” state, in which the model attends to fewer memories.

A.4 PUSHING MODELS TO HIGH VARIANCE STATE

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 791 Illustrated in Figure C, the model generates slightly different images for prompts “an actress”, “a
 792 queen”, , and “a woman”. Then, by gradually bypassing the mid-block, the model gets pushed to
 793 “high variance” state. Correspondingly, the generated images begin to look more alike and realistic
 794 in the “high variance” state, supporting our claim of attending to lower number of memories in this
 795 state.

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