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# A novel stochastic model based on echo state networks for hydrological time series forecasting

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## Abstract

1 The Stochastic Streamflow Models (SSMS) are time series models for precise prediction of  
2 hydrological data useful in hydrologic risk management. Nowadays, deep learning networks  
3 get many considerations in time series forecasting. However, despite their theoretical  
4 benefits, they fail due to their drawbacks, such as complex architectures, slow convergence  
5 and the vanishing gradient problem. In order to alleviate these drawbacks, we propose a  
6 new stochastic model applied in problems that involve stochastic behavior and periodic  
7 characteristics. The new model has two components, the first one, a type of recurrent  
8 neural network embedding the echo-state (ESN) learning mechanism instead of conventional  
9 backpropagation. The last component adds the uncertainty associated with stationary  
10 processes. This model is called Stochastic Streamflow Model ESN (SSMESN). It was  
11 calibrated with time series of monthly discharge data from MOPEX data set. Preliminary  
12 results show that the SSMESN can achieve a significant prediction performance, learning  
13 speed. This model, can be considered a first attempt that applies the echo state network  
14 methodology to stochastic process.

## 15 1 Motivation

16 In probability theory, an stochastic process is defined as a set of models that allow the study of problems  
17 with random components. Natural phenomena such as precipitation and streamflow discharge have nonlinear,  
18 complex and chaotic characteristics. In order to model the behavior of these phenomena, initially linear  
19 approximation was used [2, 11]. Afterwards, were developed methods using self-correcting models such  
20 as the PAR(p) model [12, 4]. However, these models are statistical, linear and they cannot capture real  
21 chaotic characteristics of hydrometeorological time series, being they sometimes inadequate [13]. Currently,  
22 Deep learning (DL) approaches [17], attempt to model this complex non-linear behavior. In fact, DL have  
23 been widely used in the recent literature from simplest feedforward Neural Network (ANN) to the most  
24 complex Recurrent architecture LSTM. Studies on forecasting performance, show that Recurrent Neural  
25 Networks(RNN) [6] are better than their peers ANN, in virtually all tests [3]. However, literature on stochastic  
26 models shows preference to use feedforward ANN than RNN, because the last one generates greater complexity  
27 in the training process, slow convergence rate, as well as vanishing gradient problems [15]. All it is added to  
28 the complexity that uncertainty analysis and stochastic simulation requires [4]. This motivated the development  
29 of a stochastic process model using RNN and the echo-state (ESN) learning mechanism instead of conventional  
30 backpropagation, the interesting property of ESN is that only the readout layer is trained, whereas the recurrent  
31 topology has fixed connection weights [10]. ESN is a training approach attractive as simple and fast compared  
32 to other approaches used in RNN, all in order to reduce complexity, and leverage its proven ability to represents  
33 the characteristics of time series.

## 34 2 Proposal: Stochastic Streamflow model ESN (SSMESN)

35 The figure 1 details our model, it generates scenarios  $Y_{v,t}$  of hydrological synthetic data , in terms of monthly  
36 intervals, and can be resume by

$$Y_{v,t} = f(R_{v,t} + E_{v,t}) \quad (1)$$

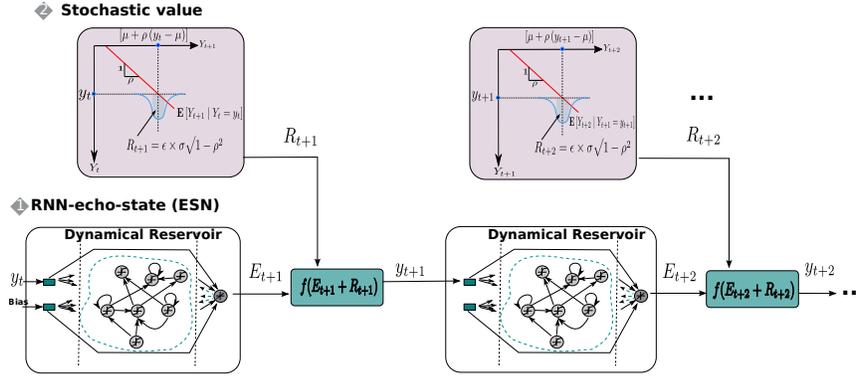


Figure 1: Generation of synthetic scenarios, the new SSMESN model.

		Horizon											
		12 months					24 months						
		Metric											
Model		RMSE	MSE	MAD	NRMSE	MPE	NSE	RMSE	MSE	MAD	NRMSE	MPE	NSE
01541500	TF	0.68528	0.471	0.59818	0.74254	123.67	0.39673	0.643840	415010.53775	0.73835	98.6120	43049	
	<b>SSMESN</b>	<b>0.61774</b>	<b>0.3818</b>	<b>0.5057</b>	<b>0.66936</b>	<b>98.937</b>	<b>0.51098</b>	<b>0.559250</b>	<b>312820.44525</b>	<b>0.64134</b>	<b>76.1760</b>	<b>57072</b>	
	NSP	0.68959	0.47714	0.5934	0.74721	120.21	0.38887	0.699450	489960.58354	0.80212	106.030	32763	
	LSTM	0.68863	0.475590	0.59665	0.74617	120.97	0.39085	0.642360	41309	0.5323	0.73665	96.0570	43312
MOPEX basin 12413000	TF	1.4658	2.1549	1.0567	1.4564	156.42	-1.3207	1.3793	1.9048	0.99965	0.72012	160.2	0.45817
	<b>SSMESN</b>	<b>1.26</b>	<b>1.5903</b>	<b>0.93179</b>	<b>1.2519</b>	<b>154.03</b>	<b>-0.71271</b>	<b>1.306</b>	<b>1.7064</b>	<b>0.89157</b>	<b>0.68185</b>	<b>131.670</b>	<b>0.51463</b>
	NSP	1.3454	1.8235	0.98366	1.3368	150.53	-0.9639	1.365	1.8667	0.98498	0.71267	157.320	0.46902
	LSTM	1.2965	1.6952	0.94809	1.2881	142.2	-0.82572	1.3736	1.8903	0.99941	0.71717	162	0.4623
03054500	TF	1.1956	1.4316	1.045	0.69638	475.29	0.47014	1.1604	1.3471	0.94575	0.68206	312.910	0.51439
	<b>SSMESN</b>	<b>1.0688</b>	<b>1.1425</b>	<b>0.90358</b>	<b>0.62255</b>	<b>363.55</b>	<b>0.57716</b>	<b>1.1167</b>	<b>1.2471</b>	<b>0.87742</b>	<b>0.65638</b>	<b>225.150</b>	<b>0.55042</b>
	NSP	1.1793	1.3926	1.0242	0.68688	447.6	0.48458	1.1744	1.3796	0.95197	0.69026	316.540	0.50268
	LSTM	1.2063	1.457	1.0463	0.70264	457.65	0.46073	1.1611	1.3484	0.93855	0.68244	299.540	0.51391
01541000	TF	0.78917	0.6235	0.71423	0.77839	138.14	0.33828	0.7325	0.537150	0.62133	0.80215	97.8990	0.32783
	<b>SSMESN</b>	<b>0.68499</b>	<b>0.469330</b>	<b>0.59049</b>	<b>0.67563</b>	<b>107.960</b>	<b>0.50191</b>	<b>0.6488</b>	<b>0.421020</b>	<b>0.52178</b>	<b>0.7105</b>	<b>76.5720</b>	<b>0.47315</b>
	NSP	0.77286	0.598140	0.695860	0.76229	130.29	0.36519	0.73998	0.5481	0.62001	0.81034	96.0870	0.31413
	LSTM	0.78557	0.617970	0.70885	0.77484	133.77	0.34415	0.732090	0.53651	0.6172	0.80171	95.8670	0.32863

Table 1: Summary of results (RMSE, MSE, MAD, NRMSE, MPE and NSE) of all methods in four series datasets (discharge) MOPEX [7], 1) each column has the results of a specific stochastic model NSP[4][1], TF[16], LSTM [9] and our *SSMESN*, in a particular metric; 2) each row compares the results of all the methods in a particular data set with a specific horizon value (Monthly Forecasts); 3) Bold rows indicate the best result of each column in a particular metric.

37 where  $E_{v,t}$  is the value produced by the RNN with the echo-state (ESN) learning mechanism, (see equation  
38 2), where  $W^{out}$  is the weight matrix between the internal states  $x(t+1)$  added to the input signals  $y_t$  and  
39 the output neurons,  $\delta(\cdot)$  the activation function.  $R_{v,t}$  is the stochastic value (see equation 3), where  $\sigma_{t+1}$ ,  
40 is the standard deviation in month  $t+1$ , the correlation coefficient between months  $t+1$  and  $t$  is  $r_t$  and  
41  $\epsilon = N(0, 1)$ , a normally distributed random noise with zero mean and standard deviation one.

$$E_{t+1} = \delta \left( W^{out} (x(t+1) + y_t) + \theta_t \right) \quad (2)$$

42

$$R_{t+1} = \epsilon \times \sigma_{t+1} \times \sqrt{(1 - r_t^2)} \quad (3)$$

43 The above equations are concatenated, (2, 3), to obtain the extended equation of our model:

$$Y_{t+1} = f \left( \delta \left( W^{out} \times \left( \vartheta \left[ W^{in} y_t + \theta_t + W x(t-1) \right] + y_t \right) + \theta_t \right) + R_t \right) \quad (4)$$

### 44 3 Preliminary Results

45 Experiments were made using the well-known MOPEX data set [7]. Table 1 shows *SSMESN* as a promising  
46 stochastic model, it outperforms the feedforward models (NSP[4][1], LSTM [9]) and the shallow statistical  
47 model (TF[16]) in forecasting performance, learning speed and short-term memory capacity [15]. The main  
48 model component "RNN-Echo State Network (ESN)" has a highly inter-linked recurrent topology and random  
49 initialize. ESN has two interesting properties; the first is that only the last layer is trained, the second is thanks  
50 to its internal memory which is the result of recurrent connections it is not necessary to embed previous input  
51 signals (sliding windows). Apparently it may seem surprising that a recurrent neural network with random  
52 connections may be effective, but randomized parameters have been successful in several domains [8, 14, 5].

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