# Dynamics and Diversity of a Cross-feeding Microbial Ecosystem

*Keywords: microbial community, cross-feeding, bipartite network, number of coexisting species, population dynamics*

## Extended Abstract

In microbial community ecosystems, interspecies interactions through the leakage and uptake of metabolites (cross-feeding) are thought to be playing an important role. From a coupled-cell-model, it has been suggested that interactions through such leak-uptake interaction may contribute to the diversity and productivity of the entire system [1]. To study the effect of cross-feeding on larger systems, however, the model is computationally costly. A network-based approach to the bipartite topology of cross-feeding has also been developped and that showed the emergence of bistability in diversity of the system [2]. However, this framework can not explicitly treat the competitive effect on metabolites.

A population dynamics model shall be an ideal resolution to test the effect of cross-feeding in larger systems. For this purpose, we introduce a dynamical model to study communities with cross-feeding interaction, based on the standard setting for resource competition [3, 4]. Under a minimal setting (see top-left panel of the figure for the schematic of the system.), the population dynamics of this model is given as

$$\dot{x}_\mu = \left( -\chi_\mu + \sum_i^C \frac{b_i \tau_{\mu i} \bar{c}_i}{1+U_i} \right) x_\mu, \tag{1}$$

where $x_\mu, \chi_\mu, b_i, U_i$, and $\tau_{\mu i}$ are the population of species $\mu$, the base cost of living of the species $\mu$', the maximum benefit of the uptake of chemical species $i$, the rate of total exchange (leakage and uptake) of chemical $i$, and the ability of the species $\mu$ to take chemical $i$, respectively.

$$\bar{c}_i = \frac{\sum_\upsilon^N x_\upsilon \lambda_{\upsilon i}}{\sum_\upsilon^N x_\upsilon \lambda_{\upsilon i} + \sum_\nu^N x_\nu \tau_{\nu i}}. \tag{2}$$

is the equilibrium concentration of chemical $i$ in the commonly shared solution, where $\lambda_{\upsilon i}$ represents the species $\upsilon$'s ability to leak chemical $i$.

As shown in the figure (bottom), this model exhibits diverse behaviors including fixed point, limit cycle, low-dimensional chaos, and high-dimensional chaos depending upon the character of the cross-feeding network, namely, the degree of uptake links $m_{\text{take}}$ and the degree of leakage links $m_{\text{leak}}$. The relationship between the dynamics of the system and the number of species that can coexist will be reported and discussed.

## References

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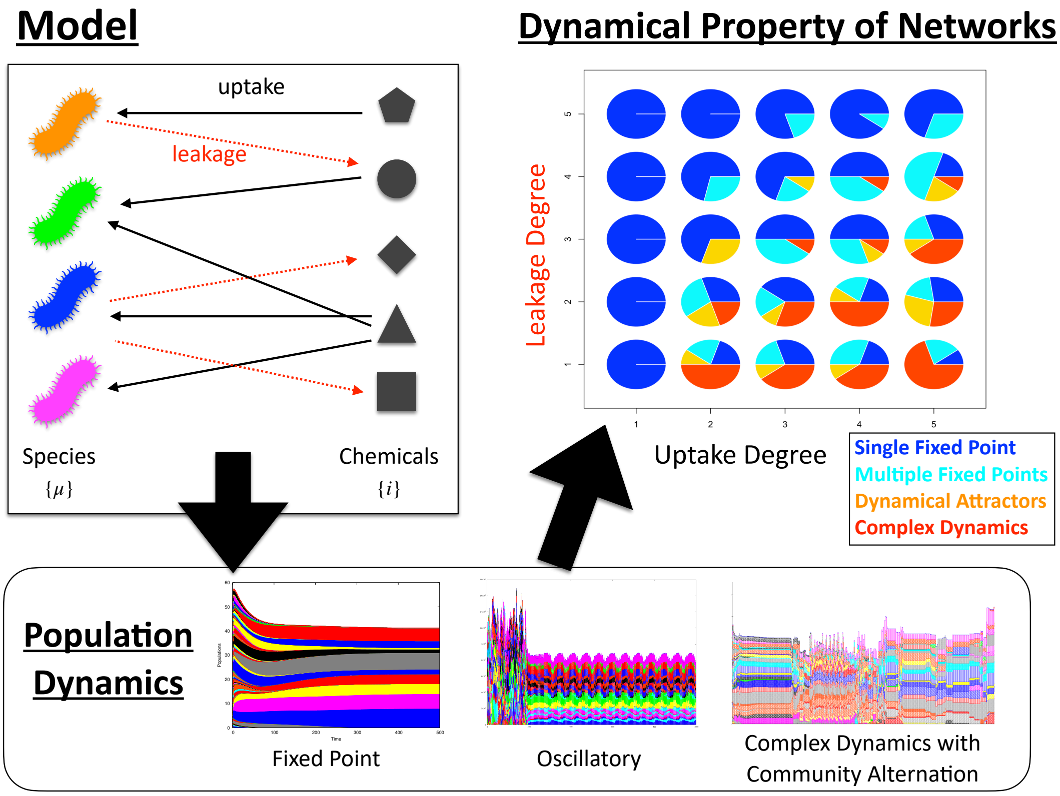


Figure 1: **The model and behaviors of cross-feeding system.** (Top-left) Schematic of the model. (Bottom) Typical examples of the stacked timeseries of the populations of microbial species $\{x_\mu\}$. (Top-right) Phase diagram of the system's dynamics to the change in the degrees of uptake and intake, $(m_{\text{take}}, m_{\text{leak}})$.