AEGIS: AUTHENTIC EDGE GROWTH IN SPARSITY FOR LINK PREDICTION IN EDGE-SPARSE BIPARTITE KNOWLEDGE GRAPHS

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ABSTRACT

Bipartite knowledge graphs in niche domains are typically data-poor and edge-sparse, which hinders link prediction. We introduce AEGIS (Authentic Edge Growth In Sparsity), an edge-only augmentation framework that resamples existing training edges—either uniformly simple or with inverse-degree bias degree_aware—thereby preserving the original node set and sidestepping fabricated endpoints. To probe authenticity across regimes, we consider naturally sparse graphs (game design pattern's game–pattern network) and induce sparsity in denser benchmarks (Amazon, MovieLens) via high-rate bond percolation. We evaluate augmentations on two complementary metrics: AUC-ROC (higher is better) and the Brier score (lower is better), using two-tailed paired t-tests against sparse baselines. On Amazon and MovieLens, copy-based AEGIS variants match the baseline while the semantic KNN augmentation is the only method that restores AUC and calibration; random and synthetic edges remain detrimental. On the text-rich GDP graph, semantic KNN achieves the largest AUC improvement and Brier score reduction, and simple also lowers the Brier score relative to the sparse control. These findings position authenticity-constrained resampling as a data-efficient strategy for sparse bipartite link prediction, with semantic augmentation providing an additional boost when informative node descriptions are available.

1 INTRODUCTION

Bipartite graphs are two-mode structures; single-relation bipartite graphs (Newman, 2018; Latapy et al., 2008) naturally capture many knowledge-centric applications (e.g., movie–genre, product–category), where the task is to decide whether a single relation exists between two node types. In niche domains, these graphs are often extremely sparse: many nodes have only a handful of incident edges, supervision becomes scarce, and link prediction must proceed with very limited evidence.

This study tackles edge sparsity by comparing five edge-augmentation strategies (uniform authentic, inverse-degree-biased authentic, random ER-like, perturbation-based synthetic, and semantic-KNN) and contributes in three ways:

- We design a stress test for edge-limited bipartite link prediction—applying high-rate bond percolation, augmenting edges solely within the training split, and evaluating with threshold-independent metrics (AUC and Brier score)—without claiming causal disentanglement of sparsity factors.
- We introduce Authentic Edge Growth in Sparsity (AEGIS), an edge-only augmentation that replicates observed links in a structure-consistent manner (uniform or inverse-degree biased) while preserving the original node set.
- We provide an empirical study on two benchmarks (MovieLens, Amazon) and a domain case study (GDP), showing how authenticity-constrained copies act as a strong sparsity baseline and deliver calibration gains in text-rich settings, while semantic augmentation becomes essential when richer node descriptions are available.

2 RELATED WORK

In this section, we briefly review related work that forms the background for our study. We begin by describing single-relation bipartite knowledge graphs and link prediction as a core task, with particular attention to the imbalanced degree distributions that commonly arise in practice and motivate our augmentation strategies. Next, we survey graph data augmentation methods, especially those relevant to edge-level augmentation, to contextualize our approach. Finally, we introduce the concepts of edge sparsity, percolation, and homophily in graph structures, which underpin our workflow: edge dropping is applied only to benchmark datasets to induce sparsity, while both benchmark and case study graphs are subsequently augmented via multiple edge-level policies.

Single-relation Bipartite (two-mode) Knowledge Graphs and Link Prediction. Knowledge graph completion (KGC) broadly covers inferring missing entities and relations. Usually, a knowledge graph defines a tuple in the form of "(head entity) $\rightarrow$ (relation) $\rightarrow$ (tail entity)". And KGC aims to predict the relation among a given head and entity, or the entity given the other, and the relation. A special case of KGC is the binary link prediction - estimate the probability that a link exists between a head and a tail entity - in the context of single-relation bipartite (two-mode) knowledge graphs. The working definition of single-relation bipartite (two-mode) knowledge graphs in this study is a graph that only has two kinds of nodes (e.g., A and B), and there is only one directed relation that exists in this graph (A $\rightarrow$ B). Bipartite (two-mode) network analysis highlights side-specific degree patterns and component structure (Latapy et al., 2008; Newman, 2018). In practice, bipartite graphs often exhibit imbalanced degree distributions across modes (Latapy et al., 2008; Newman, 2018). These imbalanced degree distributions shape component structure and intensify cold-start behavior for low-degree nodes, especially under high-rate edge dropping (Schein et al., 2002; Rong et al., 2019). This motivates inverse-degree authentic resampling: a conservative way to allocate limited augmentation budget toward sparsity-affected endpoints without inventing new nodes or altering the two-mode constraint. While we do not claim causal disentanglement, this design aligns with observed failure modes in edge-sparse, long-tail regimes (Newman, 2018; Steck, 2011).

Graph Data Augmentation. Graph data augmentation creates plausible variants of graph data without extra labeling to expand training signals (Zhao et al., 2022a). Methods can be organized along two orthogonal axes: (i) whether the policy is learned vs. rule-based, (ii) the task level (node/edge/graph), and (iii) the operation modality (structure, features, or labels) (Zhao et al., 2022a; Zhou et al., 2025; Ding et al., 2022). Rule-based structural regularizers such as DropEdge (Rong et al., 2019) and DropNode (Feng et al., 2020) randomly remove components during training and work well as anti-overfitting in dense settings, but can be counterproductive under edge sparsity where supervision is already limited. Beyond subtractive policies, additive strategies aim to increase effective connectivity or introduce informative structure. Interpolation-based methods (e.g., GraphSMOTE (Zhao et al., 2021), FG-SMOTE (Wang et al., 2025b)) adapt oversampling to graphs by interpolating features or ties, while generative/counterfactual approaches (e.g., CFLP (Zhao et al., 2022b), CLBR (Zhu et al., 2023), AGGG (Wang et al., 2025a)) synthesize training instances by modeling causal or distributional structure. As null baselines, random edge additions resemble two-mode Erdős–R'enyi draws (ERDdS & R&wi, 1959; Newman, 2018), and synthetic index perturbations play the role of stress tests rather than realistic augmentation. Our work focuses on a rule-based, edge-only, train-only policy —authenticity-constrained edge resampling — that replicates observed ties under type constraints to densify supervision around real patterns, contrasting with attribute-similarity completion and null additions. In the edge sparsity regime studied here, we observe that such authenticity constraints offer more reliable improvements in both ROC-AUC and Brier score compared to random or synthetic additions; semantic-only completion can raise ROC-AUC but does not consistently improve calibration as measured by the Brier score under class imbalance.

Edge sparsity, Percolation and Homophily. Random high-rate edge dropping corresponds to bond percolation on networks, which linearly scales mean degree and induces component fragmentation (Newman, 2002; 2018). Attribute-similarity (homophily) is a common mechanism for tie formation (McPherson et al., 2001), informing semantic-KNN completions. However, precision–recall behavior under imbalance can diverge from ROC improvements (Davis & Goadrich, 2006; Bi et al., 2024).

3 PROBLEM FORMULATION

Let $G = (U, V, E)$ be a single-relation bipartite graph with U and V disjoint node sets, and E is a set of edges. We consider binary link prediction on a bipartite graph $G = (U, V, E)$ as estimating, for each candidate pair (u, v) where $u \in U$ and $v \in V$, the probability $P(u, v) = \sigma(s(u, v))$, where $s(u, v)$ is a learned scoring function (e.g., dot-product $s(u, v) = \mathbf{h}_u^\top \mathbf{h}_v$, bilinear $s(u, v) = \mathbf{h}_u^\top \mathbf{W} \mathbf{h}_v$, or cosine similarity). We train with a class-balanced binary cross-entropy over observed positives and sampled negatives; evaluation reports threshold-independent metrics (AUC-ROC, Brier score).

Edge sparsity regime (bond percolation). We study a scenario where sparsity is induced by random edge dropping at a high rate (bond percolation; retain rate q is $0 \ll q \ll 1$), which proportionally reduces side-specific mean degrees, lowers global edge density, and fragments component structure (Newman, 2002; 2018). Our goal is a scenario-driven evaluation of augmentation policies under edge sparsity, not a causal decomposition of which attribute drives performance. A full causal decomposition of which specific graph attributes drive performance is out of scope for this paper, but may be explored in future work; here, our focus is on scenario-driven evaluation of augmentation policies under edge sparsity.

Evaluation. We report two complementary metrics: (i) AUC-ROC, where higher values indicate better ranking across thresholds, and (ii) the Brier score, where lower values indicate better probabilistic calibration and overall predictive reliability (Glenn et al., 1950; Bi et al., 2024). This combination lets us assess whether an augmentation both separates positives from negatives and assigns calibrated link probabilities. Following APA guidelines[1], we present each method's $M \pm SD$ along with two-tailed paired Student t-tests (df $= 31$) against the sparse baseline. Tables include the t-statistic, p-value, and Cohen's d, with significance levels ($p < .05$, $p < .01$, $p < .001$) flagged by asterisks to show when observed differences are unlikely to arise by chance.

4 METHODOLOGY

4.1 AUTHENTICITY-CONSTRAINED EDGE RESAMPLING

We define authentic edge growth in sparsity (AEGIS) as empirical tie resampling: duplicating observed training edges (with replacement) under type constraints, without introducing new nodes or synthetic endpoints. AEGIS preserves observed relational patterns and respects the two-mode structure, contrasting with two-mode ER-like random additions (ERDdS & R&wi, 1959) or interpolation-based synthesis (Chawla et al., 2002). To avoid leakage, augmentation applies only to the training graph's edge index; validation/test graphs and labels remain unchanged. We instantiate AEGIS with two sampling policies: (i) uniform resampling ("simple"), sampling existing edges uniformly; and (ii) low-degree-biased resampling ("degree-aware"), sampling with probability inversely proportional to endpoint degrees to prioritize low-degree nodes (cold-start mitigation). The procedures below cover both authentic policies and the contrastive baselines.

4.2 AUGMENTATION METHODS

All augmentations operate only on the *training* subgraph's forward edge index; we do not add nodes, and we do not modify validation/test graphs or labels, avoiding leakage. In this study, we compare five distinct edge augmentation policies to address sparsity in bipartite knowledge graphs: AEGIS-Simple uniformly resamples observed edges from the training set, duplicating existing links without creating new endpoints. AEGIS-Degree applies an inverse-degree bias to resampling, preferentially augmenting edges for low-degree nodes to mitigate cold-start issues. The Random ER-like policy introduces edges between randomly selected node pairs, simulating two-mode Erdős–Rényi random graphs (ERDdS & R&wi, 1959). Perturbation-based synthetic augmentation generates new edges by perturbing the indices of existing edges in a SMOTE-style fashion (Chawla et al., 2002). Semantic-KNN completion introduces edges between nodes with high semantic similarity (e.g.,

[1] https://apastyle.apa.org/style-grammar-guidelines/tables-figures/sample-tables

high cosine similarity between node features), reflecting homophily-driven tie formation (McPherson et al., 2001).

Algorithm 1 AEGIS-Simple: Uniform Authentic Resampling

Require: An edge $e_i(u, v) \in E$ where i is a unique edge index and augmentation factor $\phi \geq 1$
Ensure: Augmented set of edges E_{aug}

$n_e \leftarrow |E|, n'_e \leftarrow \lfloor(\phi - 1)n_e\rfloor$
Initialize $E' \leftarrow \emptyset$
while $|E'| < n'_e$ **do**
  $e_x(u, v) \sim \mathcal{U}(E)$, where $\mathcal{U}$ is the uniform distribution.
  $E' \leftarrow E' \cup \{e_x(u, v)\}$
end while
$E_{aug} \leftarrow E \cup E'$
return E_{aug}

Algorithm 2 AEGIS-Degree: Inverse-Degree-Biased Authentic Resampling

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Require: An edge $e_i(u, v) \in E$ where i is a unique edge index, augmentation factor $\phi \geq 1$, smoothing constant $\varepsilon > 0$
Ensure: Augmented set of edges E_{aug}

$n_e \leftarrow |E|, n'_e \leftarrow \lfloor(\phi - 1)n_e\rfloor$
$E_k \leftarrow e_i(u_k, v)$ and $E_l \leftarrow e_i(u, v_l)$ where $u_k \in U$ and $v_l \in V$
$deg(u_k) \leftarrow |E_k|, deg(v_l) \leftarrow |E_l|$
$w(e_i) \leftarrow \frac{1}{\deg(u_k)+\varepsilon} + \frac{1}{\deg(v_l)+\varepsilon}$
Normalize $P_E \leftarrow w(e_i)/\sum_i w(e_i)$
Initialize $E' \leftarrow \emptyset$
while $|E'| < n'_e$ **do**
  Sample $e_x(u, v) \sim P_E$
  $E' \leftarrow E' \cup \{e_x(u, v)\}$
end while
$E_{aug} \leftarrow E \cup E'$
return E_{aug}

Algorithm 3 Random ER-Like Augmentation

Require: An edge $e_i(u, v) \in E$ where i is a unique edge index, $n_u \leftarrow |U|$, $n_v \leftarrow |V|$, augmentation factor $\phi \geq 1$
Ensure: Augmented set of edges E_{aug}

$n_e \leftarrow |E|, n'_e \leftarrow \lfloor(\phi - 1)n_e\rfloor$
Initialize $E' \leftarrow \emptyset$
while $|E'| < n'_e$ **do**
  $e_x(u, v)$ where $u \sim \mathcal{U}(U), v \sim \mathcal{U}(V)$, where $\mathcal{U}$ is the uniform distribution.
  $E' \leftarrow E' \cup \{e_x(u, v)\}$
end while
$E_{aug} \leftarrow E \cup E'$
return E_{aug}

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5 EXPERIMENTS

5.1 DATASET STATISTICS AND EDGE SPARSITY CONSTRUCTION

We evaluate our methods on two widely used benchmark datasets—MovieLens (Harper & Konstan, 2015) (movie–genre) and Amazon (McAuley et al., 2015) (product–category)—as well as a domain-specific use case, GDP (game design patterns) (Björk & Holopainen, 2005). Details of GDP can be found in Appendix A. While the benchmark datasets are originally well-connected, we simulate extreme edge sparsity by applying high-rate random bond percolation (i.e., random edge removal) as described by Newman (2002). In contrast, the GDP dataset is inherently sparse and does not require additional edge removal. Table 1 summarizes the key characteristics of each dataset, including

Algorithm 4 Perturbation-based Synthetic Augmentation

Require: An edge $e_i(u, v) \in E$ where i is a unique edge index, augmentation factor $\phi \geq 1$, perturbation radius r, $n_u \leftarrow |U|$, $n_v \leftarrow |V|$

Ensure: Augmented set of edges E_{aug}

$n_e \leftarrow |E|$, $n'_e \leftarrow \lfloor(\phi - 1)n_e\rfloor$
Initialize $E' \leftarrow \emptyset$
while $|E'| < n'_e$ **do**
  $e_x(u_j, v_k) \sim \mathcal{U}(E)$, where $\mathcal{U}$ is the uniform distribution.
  $\delta_u, \delta_v \sim U\{-r, \ldots, r\}$
  $u' \leftarrow \min(\max(u_j + \delta_u, 0), n_u - 1)$
  $v' \leftarrow \min(\max(v_k + \delta_v, 0), n_v - 1)$
  $E' \leftarrow E' \cup \{e_x(u, v)\}$
end while
$E_{aug} \leftarrow E \cup E'$
return E_{aug}

Algorithm 5 Semantic-KNN Augmentation

Require: An edge $e_i(u, v) \in E$ where i is a unique edge index, $n_u \leftarrow |U|$, $n_v \leftarrow |V|$, semantic feature matrices $\mathbf{x}_U \in \mathbb{R}^{n_u \times d}$, $\mathbf{x}_V \in \mathbb{R}^{n_v \times d}$ (row-normalized), neighbour parameter k, similarity threshold τ, per-node cap α, augmentation factor $\phi \geq 1$

Ensure: Augmented set of edges E_{aug}

$n_e \leftarrow |E|$, $n'_e \leftarrow \lfloor(\phi - 1)n_e\rfloor$
$T_U \leftarrow \mathbf{x}_U \cdot \mathbf{x}_U^T$, $T_V \leftarrow \mathbf{x}_V \cdot \mathbf{x}_V^T$, where T_U and T_V are self-similarity tensors based on cosine distance.
$S_U \leftarrow Knn(T_U, k)$, $S_V \leftarrow Knn(T_V, k)$ where $Knn(T, k)$ selects k elements with the highest self-similarity in T.
$S_U \leftarrow S_U(i) \gg \tau$ and $S_V \leftarrow S_V(i) \gg \tau$ where τ is a threshold parameter.
Initialize $E' \leftarrow \emptyset$, $c_U(u_i) \leftarrow 0$, $c_V(v_i) \leftarrow 0$
$n \leftarrow n'_e$
for each $e_i(u_j, v_k) \in E$ **do**
  for each $v_{knn} \in \mathcal{S}_V(v_k)$ while $n > 0$ **do**
    if $(u_j, v_{knn}) \notin E \cup E'$ and $c_U(u_j) < \alpha$ and $c_V(v_{knn}) < \alpha$ **then**
      $E' \leftarrow E' \cup \{(u_j, v_{knn})\}$
      $c_U(u_j) \leftarrow c_U(u_j) + 1$, $c_V(v_{knn}) \leftarrow c_V(v_{knn}) + 1$
      $n \leftarrow n - 1$
    end if
  end for
  for each $u_{knn} \in \mathcal{S}_U(u_j)$ while $n > 0$ **do**
    if $(u_{knn}, v_k) \notin E \cup E'$ and $c_U(u_{knn}) < \alpha$ and $c_V(v_k) < \alpha$ **then**
      $E' \leftarrow E' \cup \{(u_{knn}, v_k)\}$
      $c_U(u_{knn}) \leftarrow c_U(u_{knn}) + 1$, $c_V(v_k) \leftarrow c_V(v_k) + 1$
      $n \leftarrow n - 1$
    end if
  end for
end for
$E_{aug} \leftarrow E \cup E'$
return E_{aug}

the cardinalities of the two node sets, the number of edges in the original graph, the percolation retain rate q used to generate sparse training scenarios, and the resulting number of edges after edge dropping.

Table 1: Dataset Statistics and Edge Sparsity Construction

Dataset	Mode U	$\lvert U\rvert$	Mode V	$\lvert V\rvert$	$\lvert E\rvert$ (orig)	retain q	$\lvert E\rvert$ (after)
Amazon	Products	1465	Categories	317	6307	0.01	67
MovieLens	Movies	9708	Genres	19	22050	0.01	213
GDP	Games	208	Patterns	296	715	N/A	715

5.2 Augmentation Budgets

On the training subgraph (benchmarks after edge dropping and GDP), we target 100× augmentation. Validation/test graphs remain unchanged. Reported significance is always per-dataset versus its own sparse baseline; original graphs of benchmarks are shown as upper bounds, not budget-matched baselines.

5.3 Pipeline

Benchmark Datasets Pipeline. Bond percolation (edge dropping) on benchmarks (e.g., keep ∼1% edges) → split via RandomLinkSplit (80/10/10; disjoint training ratio) with negative sampling (negative sampling ratio; allow adding negative train samples) → augmentation (AEGIS-Simple; AEGIS-Degree; Random ER-Like Augmentation; Perturbation-based Synthetic Augmentation; Semantic-KNN Augmentation) with the factor of 100 on *train* graph only → Graph Attention Network's Heterogeneous variant (hetero GAT) training with class-balanced binary cross-entropy loss → evaluation. **Domain Case Study Pipeline.** Naturally sparse two-mode graph (no edge drop) → customized training/valid set → augmentation (AEGIS-Simple; AEGIS-Degree; Random ER-Like Augmentation; Perturbation-based Synthetic Augmentation; Semantic-KNN Augmentation) with the factor of 100 on *train* graph only → Graph Attention Network's Heterogeneous variant (hetero GAT) training with class-balanced binary cross-entropy loss → evaluation

The implementation specifics for the results demonstrated in this manuscript can be found in Appendix B

6 Results

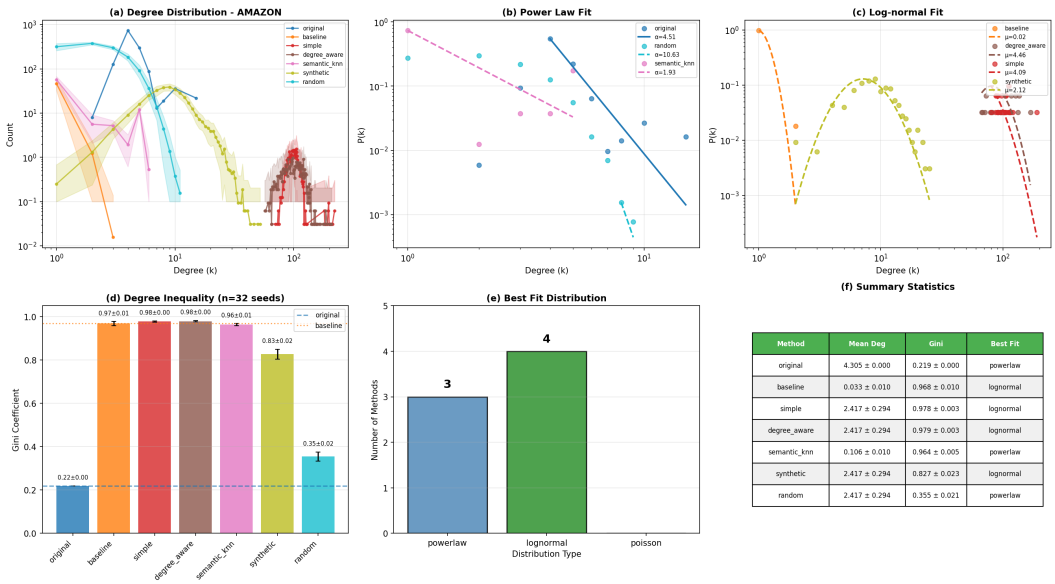
6.1 Benchmark Validation

To probe structural limits under extreme sparsity (q=0.01, ϕ=100), we inspected degree distributions for Amazon and MovieLens as a diagnostic (not the exact graphs used for AUC/Brier). As shown in Figure 1 and Figure 2, the copy-style augmentations (`simple`, `degree_aware`) substantially raise mean degree yet retain hub-dominated inequality (Amazon Gini ≈ 0.98, MovieLens ≈ 0.99), consistent with faithfully resampling the sparse baseline topology. `Random` flattens the distribution (Amazon Gini 0.36, MovieLens 0.48) but erodes structure; `synthetic` sits in between. `Semantic_knn` stays very sparse because similarity thresholds block most edges (mean degree 0.106 Amazon, 0.019 MovieLens). None recover the original moderate inequality (Amazon Gini 0.219, MovieLens 0.266), highlighting a trade-off between structural fidelity and connectivity when dropout is extreme.

Tables 2 and 3 show that the original graphs function as upper bounds and that copy-style AEGIS variants (`simple`, `degree_aware`) stay statistically indistinguishable from the sparse baselines. Only `semantic_knn` achieves meaningful gains (+0.091 on Amazon), while random and synthetic additions drive AUC down, especially on MovieLens. Tables 4 and 5 indicate that `semantic_knn` improves or preserves calibration (e.g., −0.015 on Amazon), the copy-based AEGIS options provide no consistent benefit, and random/synthetic edges raise the Brier score on every benchmark.

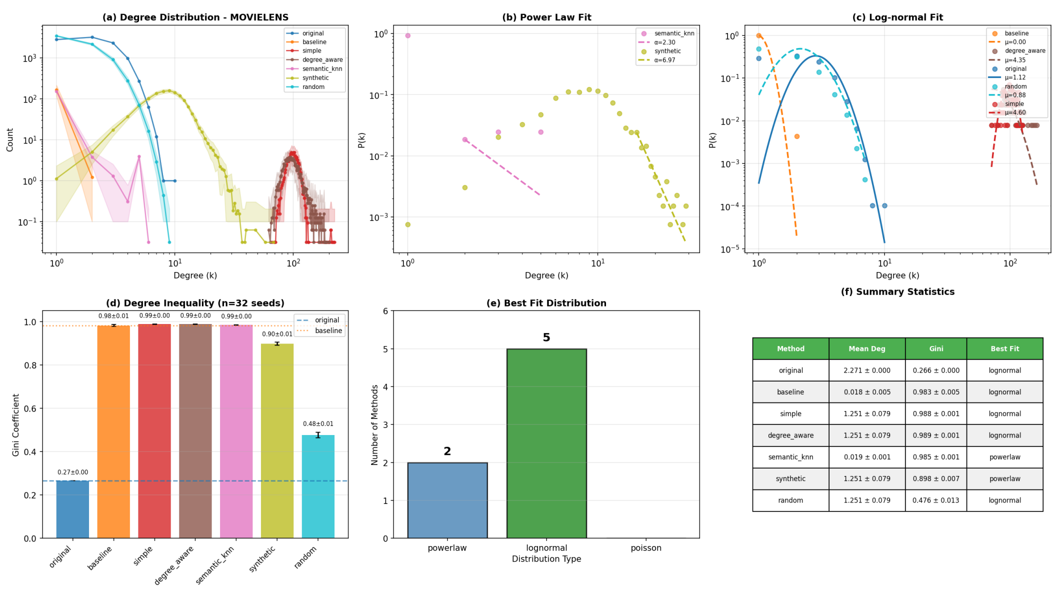
6.2 Domain Case Study: Game Design Pattern (GDP)

For the domain-specific GDP dataset (Figure 3), structural diagnostics (again on graphs used for topology inspection rather than evaluation) reflect its expert-curated sparsity. The original graph is already uneven (Gini 0.54). Copy-style augmentations preserve that pattern (Gini 0.978 for



Method	Mean Deg	Gini	Best Fit
original	4.305 ± 0.000	0.219 ± 0.000	powerlaw
baseline	0.033 ± 0.010	0.968 ± 0.010	lognormal
simple	2.417 ± 0.294	0.978 ± 0.003	lognormal
degree_aware	2.417 ± 0.294	0.979 ± 0.003	lognormal
semantic_knn	0.106 ± 0.010	0.964 ± 0.005	powerlaw
synthetic	2.417 ± 0.294	0.827 ± 0.023	lognormal
random	2.417 ± 0.294	0.355 ± 0.021	powerlaw

Figure 1: Amazon (product–category), GAT, q=0.01, ϕ=100: Comprehensive degree analysis ($M \pm$ SD, $n = 32$ seeds). Panel (a) shows degree distributions on log-log scale with $\pm 1\sigma$ confidence bands; (b) Power Law fits with exponent α (lower α = heavier tail); (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (0=perfect equality, 1=maximum inequality); (e) best-fit distribution counts (lower KS statistic wins); (f) summary statistics table.



Method	Mean Deg	Gini	Best Fit
original	2.271 ± 0.000	0.266 ± 0.000	lognormal
baseline	0.018 ± 0.005	0.983 ± 0.005	lognormal
simple	1.251 ± 0.079	0.988 ± 0.001	lognormal
degree_aware	1.251 ± 0.079	0.989 ± 0.001	lognormal
semantic_knn	0.019 ± 0.001	0.985 ± 0.001	powerlaw
synthetic	1.251 ± 0.079	0.898 ± 0.007	powerlaw
random	1.251 ± 0.079	0.476 ± 0.013	lognormal

Figure 2: MovieLens (movie–genre), GAT, q=0.01, ϕ=100: Comprehensive degree analysis ($M \pm$SD, $n = 32$ seeds). Panel (a) shows degree distributions on log-log scale with $\pm 1\sigma$ confidence bands; (b) Power Law fits with exponent α (lower α = heavier tail); (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (0=perfect equality, 1=maximum inequality); (e) best-fit distribution counts (lower KS statistic wins); (f) summary statistics table.

both `simple` and `degree_aware`) and even fit Poisson under extreme sparsity, suggesting they maintain the domain's intentional connectivity. `Semantic_knn` adds edges cautiously (mean degree 0.171), benefiting from richer text, while `random` collapses inequality (Gini 0.378) and `synthetic` lands in between (Gini 0.825). These results support using structure-preserving methods on curated graphs and caution against topology-altering augmentations that distort expert signals.

Table 2: Amazon (product–category), GAT, q=0.01, ϕ=100: AUC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A higher AUC is better.

Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
baseline	0.630 ± 0.162	+0.000	—	—	—
degree_aware	0.650 ± 0.204	+0.020^{ns}	−0.50	0.619	−0.09
simple	0.637 ± 0.199	+0.007^{ns}	−0.17	0.864	−0.03
semantic_knn	0.722 ± 0.197	+0.091*	−2.40	0.023	−0.42
synthetic	0.732 ± 0.181	+0.101*	−2.48	0.019	−0.44
random	0.626 ± 0.252	−0.004^{ns}	0.08	0.936	0.01
original	0.928 ± 0.008	+0.298***	−10.42	<0.001	−1.84

Table 3: MovieLens (movie–genre), GAT, q=0.01, ϕ=100: AUC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A higher AUC is better.

Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
baseline	0.710 ± 0.061	+0.000	—	—	—
degree_aware	0.713 ± 0.064	+0.003^{ns}	−0.42	0.681	−0.07
simple	0.717 ± 0.063	+0.007^{ns}	−1.33	0.195	−0.23
semantic_knn	0.708 ± 0.064	−0.002^{ns}	0.35	0.732	0.06
synthetic	0.679 ± 0.075	−0.031*	2.15	0.039	0.38
random	0.652 ± 0.089	−0.059***	3.66	0.001	0.65
original	0.811 ± 0.015	+0.101***	−9.95	<0.001	−1.76

Table 4: Amazon (product–category), GAT, q=0.01, ϕ=100: Brier score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A lower Brier score is better.

Method	Brier score $M \pm SD$	ΔBrier score	$t(31)$	p	d
baseline	0.249 ± 0.048	+0.000	—	—	—
degree_aware	0.248 ± 0.054	−0.001^{ns}	0.29	0.772	0.05
simple	0.248 ± 0.049	−0.001^{ns}	0.30	0.765	0.05
semantic_knn	0.233 ± 0.044	−0.015*	2.19	0.036	0.39
synthetic	0.244 ± 0.029	−0.005^{ns}	0.70	0.488	0.12
random	0.259 ± 0.040	+0.010^{ns}	−0.92	0.367	−0.16
original	0.135 ± 0.020	−0.114***	14.03	<0.001	2.48

Table 5: MovieLens (movie–genre), GAT, q=0.01, ϕ=100: Brier score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A lower Brier score is better.

Method	Brier score $M \pm SD$	ΔBrier score	$t(31)$	p	d
baseline	0.231 ± 0.016	+0.000	—	—	—
degree_aware	0.233 ± 0.014	+0.001^{ns}	−0.72	0.474	−0.13
simple	0.231 ± 0.012	−0.000^{ns}	0.12	0.907	0.02
semantic_knn	0.235 ± 0.014	+0.004^{ns}	−1.43	0.162	−0.25
synthetic	0.245 ± 0.008	+0.014***	−4.82	<0.001	−0.85
random	0.245 ± 0.009	+0.013***	−3.89	<0.001	−0.69
original	0.218 ± 0.004	−0.013***	5.22	<0.001	0.92

Unlike Amazon and MovieLens, GDP's original graph exhibits higher baseline inequality (Gini = 0.540) due to domain constraints where certain game design patterns (e.g., "Core Loop", "Feedback") are inherently more prevalent than specialized patterns. Under 99% dropout, AEGIS-Simple maintains this domain-informed structure with extreme inequality (Gini = 0.978 ± 0.008), though interestingly, GDP's copy-style methods show Poisson as the best-fit distribution rather than Power Law (panel e), suggesting that extreme sparsity disrupts typical scale-free characteristics in smaller graphs. AEGIS-Degree shows nearly identical behavior (Gini = 0.978 ± 0.008), indicating that inverse-degree bias provides limited benefit when the original topology already reflects expert knowledge rather than purely statistical bias. Random augmentation disrupts domain structure (Gini $= 0.378 \pm 0.092$; shifts to Power Law with $\alpha = 6.87 \pm 2.77$), explaining its poor performance on this expert-curated dataset. Notably, semantic-KNN performs relatively better on GDP (mean degree $= 0.171 \pm 0.064$ vs Amazon's 0.106 ± 0.010), as richer textual features from game descriptions provide stronger semantic signals for edge prediction. Synthetic augmentation achieves intermediate inequality (Gini $= 0.825 \pm 0.054$) with Power Law fitting ($\alpha = 5.41 \pm 2.07$). This domain-specific finding validates AEGIS-Simple's design: for knowledge graphs where topology encodes expert curation rather than random connectivity, authentic resampling preserves meaningful structure even at extreme sparsity, whereas topology-altering methods (random, synthetic) sacrifice domain validity for artificial connectivity.

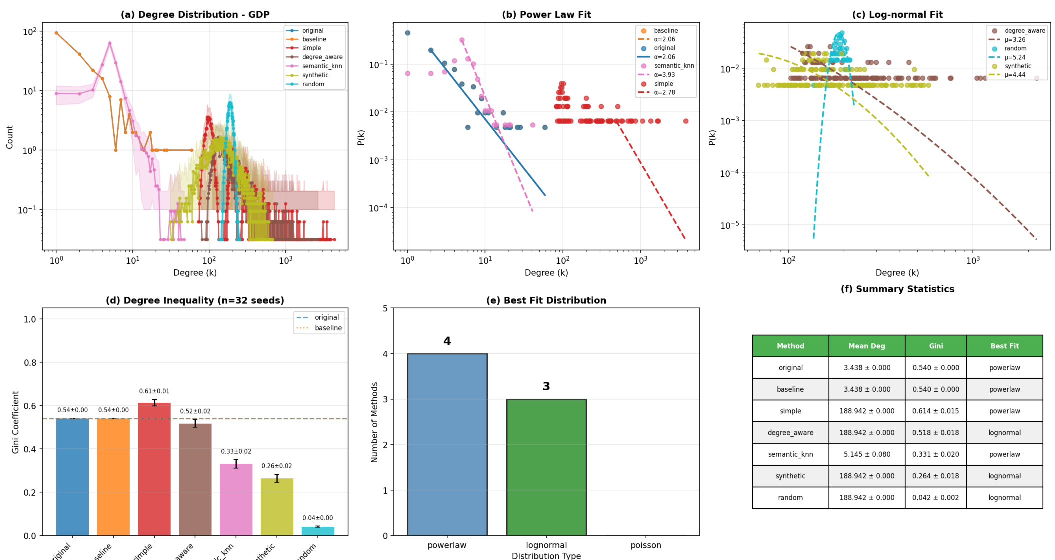


Figure 3: GDP (game–pattern), GAT, q=0.01, ϕ=100: Comprehensive degree analysis ($M \pm$ SD, $n = 32$ seeds). Panel (a) shows degree distributions on log-log scale with $\pm 1\sigma$ confidence bands; (b) Power Law fits with exponent α (lower α = heavier tail); (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (0=perfect equality, 1=maximum inequality); (e) best-fit distribution counts (lower KS statistic wins); (f) summary statistics table.

For AUC, Table 6 highlights that the strength of `semantic_knn` reaches even higher (+0.014, $p = 0.008$), whereas random and synthetic edges crater ranking quality. For authenticity-constrained augmentation on a richly described graph: `degree_aware` significantly decreases AUC (-0.028, $t = 5.29$, $p < 0.001$). Regarding the Brier score, Table 7 confirms that GDP's long-form textual descriptions let both AEGIS degrees and semantic completion improve calibration (Brier: `degree_aware` $+0.036$, `simple` -0.013, `semantic_knn` -0.054), while random/synthetic edges still degrade reliability.

Table 6: GDP (game–pattern), GAT, q=0.01, ϕ=100: AUC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A higher AUC is better.

Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
baseline	0.800 ± 0.022	+0.000	—	—	—
degree_aware	0.772 ± 0.026	-0.028^{***}	5.29	<0.001	0.93
simple	0.793 ± 0.023	-0.007^{ns}	1.67	0.104	0.30
semantic_knn	0.814 ± 0.017	$+0.014^{**}$	-2.83	0.008	-0.50
synthetic	0.645 ± 0.061	-0.155^{***}	13.42	<0.001	2.37
random	0.613 ± 0.076	-0.187^{***}	13.14	<0.001	2.32

Sensitivity analyses across other q/ϕ and augmentation methods (Appendix C) show the same pattern: copy-style AEGIS remains baseline-like, semantic_knn is the only consistent AUC/Brier lift when text is informative, random/synthetic degrade.

Table 7: GDP (game–pattern), GAT, q=0.01, ϕ=100: Brier score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$; ns = not significant; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$). A lower Brier score is better.

Method	Brier score $M \pm SD$	ΔBrier score	$t(31)$	p	d
baseline	0.302 ± 0.040	+0.000	—	—	—
degree_aware	0.337 ± 0.079	$+0.036^{*}$	-2.59	0.015	-0.46
simple	0.289 ± 0.018	-0.013^{*}	2.41	0.022	0.43
semantic_knn	0.247 ± 0.017	-0.054^{***}	7.06	<0.001	1.25
synthetic	0.269 ± 0.013	-0.032^{***}	4.76	<0.001	0.84
random	0.266 ± 0.017	-0.036^{***}	4.92	<0.001	0.87

7 Discussion

Authenticity beyond duplication. Across Amazon and MovieLens, the copy-style AEGIS variants (`simple`, `degree_aware`) match rather than exceed the sparse baseline in either AUC or Brier, whereas the semantic KNN augmenter (`semantic_knn`) is the only method that reliably lifts performance (e.g., $+0.091$ AUC and -0.015 Brier on Amazon) and at least prevents collapse on MovieLens. On GDP, copy-style variants chiefly aid calibration, while semantic completion adds the only AUC gains. This suggests that "authentic" augmentation hinges on injecting semantically plausible endpoints rather than merely duplicating surviving edges; even so, the assumption that higher feature similarity implies a greater likelihood of a true link should be validated on a domain-by-domain basis. Authenticity can thus be viewed as a spectrum—from strict edge resampling to high-confidence semantic completion—where tighter constraints suit semantics-poor graphs, and softer, well-filtered semantic links benefit semantics-rich settings.

Text richness matters. GDP's game–pattern descriptions are long and semantically rich, yielding the largest gains ($+0.014$ AUC and -0.054 Brier for `semantic_knn`), while `degree_aware` also improves calibration. Amazon's product metadata is shorter yet structured enough to benefit, whereas MovieLens's brief genre/synopsis features offer little semantic signal—suggesting that authenticity constraints deliver the most when node descriptions carry meaningful content.

Metric behavior. Amazon's synthetic augmenter shows that higher AUC can coexist with degraded calibration, underscoring the need to pair ROC analysis with Brier. On MovieLens, `semantic_knn` preserves both AUC and Brier relative to the sparse baseline, whereas random and synthetic additions worsen both. GDP exhibits the strongest recovery: only in this text-rich setting do copy-based AEGIS variants gain traction for calibration (Brier drops without AUC gains), and the semantic augmentation provides the largest joint improvements.

Limitations. The extreme sparsity stems from a single 0.99 bond-percolation pass, simultaneously altering degree, density, and component structure. Results depend on the chosen split and on severe imbalance; accuracy is threshold- bound and therefore de-emphasized. Future work should examine alternative sparsity regimes, adaptive drop ratios, and how textual richness governs augmentation gains.

8 Conclusion and Future Work

We presented AEGIS, an authenticity-constrained edge augmentation framework for sparse bipartite graphs, and evaluated it on Amazon, MovieLens, and the GDP domain case study. Copy-based variants (uniform or inverse-degree resampling) act as reliable sparsity baselines that avoid fabricating new endpoints; their efficacy nevertheless hinges on how much semantic information the domain provides (e.g., aiding calibration on GDP). The semantic KNN augmentation is indispensable for recovering performance on Amazon and MovieLens and delivers the largest gains on GDP, where richer textual descriptions unlock sizable improvements in AUC and Brier. Future work will explore density-preserving augmentation, adaptive authenticity constraints that couple semantics with edge resampling (including explicit semantic-threshold/filtration schemes), cost-aware policies, and expansion to additional sparse domains. **Reproduction Statement.** The code repo is available at `https://anonymous.4open.science/r/AEGIS-6BA3/` **Generative AI Statement.** Large language models are used in writing this manuscript only to aid or polish writing. An example of the used prompts is "Please polish these sentences in an academic way: [the actual contents]"

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A Appendix: The Game Design Pattern Dataset

The Game Design Patterns (GDP) dataset serves as a (semi-)ontology for the game design domain, offering formalized descriptions, properties, and constraints for each concept (Noy et al., 2001). Central to GDP is the notion of a "pattern"—a recurring interaction that can be instantiated in diverse games, independent of genre or theme. For example, the "Alignment" pattern refers to "the goal of forming a linear arrangement of game elements." [2] This pattern is exemplified in games such as Tic-Tac-Toe, Candy Crush Saga, and Tetris.

Patterns in GDP not only describe game mechanics but also exhibit structural relationships with other patterns: they can enable ("instantiate"), modify, or potentially conflict with the deployment of other patterns. As such, GDP provides a shared vocabulary for game designers to communicate, analyze, and create games.

[2] http://virt10.itu.chalmers.se/index.php/Alignment

However, the identification and verification of game design patterns is a highly specialized and expert-driven process. Despite the vast number of games, only a limited number of patterns have been formally proposed, and even fewer game–pattern relationships have been verified by experts. This results in an inherently sparse bipartite graph, making GDP an ideal testbed for evaluating augmentation strategies in edge-sparse domains.

B Appendix: Implementation Specifics

B.1 Experimental Setup

Hardware & Software: CUDA 11.8, NVIDIA A40 GPU, PyTorch 2.4.0+cu118, PyTorch Geometric 2.7.0, Torch Sparse 0.6.18+pt24cu118.

Table 8: Hyperparameter configuration. TF-IDF dimensions are dataset-specific: Amazon (512), MovieLens (1024), GDP (128). Sensitivity analysis varies GNN type, drop ratio, and augmentation factor.

Parameter	**Value**
Data Split & Loading	
Val / Test / Disjoint train ratio	0.1 / 0.1 / 0.3154
Negative sampling ratio	1.4875
Add negative train samples	True
Train / Val Neighbor sampling	[20, 10] / [20, 10]
Train / Val batch size	128 / 64
Train / Val shuffle	True / False
Architecture & Training	
Layers / Hidden dim / Link predictor	3 / 768 / Dot
LR / Max epochs / Eval freq	4.5×10^{-4} / 779 / 15
Early stopping (patience / δ)	10 / 10^{-3}
Sensitivity Analysis	
GNN architecture	GAT, GraphSAGE, GCN (GraphConv)
Edge drop ratio	0.99, 0.95
Augmentation factor	$1\times$, $5\times$
Semantic-KNN	
k_g, k_p / sim_thresh / cap	1, 1 / 0.6 / 4

C Appendix: Sensitivity Analysis

C.1 Analysis Design

C.1.1 Retain Rates, Augmentation Factors and GNN Architectures

The sensitivity analysis systematically evaluates edge augmentation methods across varying retain rates $q \in \{0.01, 0.05, 0.10\}$, augmentation factors $\phi \in \{1, 5, 100\}$, and GNN architectures (GAT, GraphSAGE, GCN). For the GCN baseline, we use `GraphConv` (Morris et al., 2019) [3] rather than the original spectral GCN (Kipf, 2016), as the latter's symmetric normalization is incompatible with PyTorch Geometric's heterogeneous graph conversion (`to_hetero()`). `GraphConv` performs mean aggregation and is functionally similar, as recommended by the PyG documentation for bipartite message passing[4].

Given the limited time and resource, we prioritize experiments that balance coverage across architectural variations and sparsity conditions. Table 9 summarizes the 36 completed experimental configurations across datasets.

[3] `https://github.com/pyg-team/pytorch_geometric`

[4] `https://pytorch-geometric.readthedocs.io/en/latest/notes/heterogeneous.html`

Table 9: Experimental configuration coverage across datasets, GNN architectures, retain rates (q), and augmentation factors (ϕ). Checkmarks indicate completed experiments. Amazon and MovieLens use artificial sparsity via edge dropout; GDP is naturally sparse (no dropout,indicated by –). Factor $\phi = 1$ (baseline-only) experiments exist for all configurations but are excluded from appendix tables per design.

Dataset	Architecture	q	$\phi = 100$	$\phi = 5$	$\phi = 2$
Amazon	GAT	0.01	✓[†]	✓	✓
	GAT	0.05	–	✓	✓
	GAT	0.10	–	✓	✓
	GraphSAGE	0.01	–	✓	✓
	GraphSAGE	0.05	–	✓	✓
	GraphSAGE	0.10	–	✓	✓
	GCN	0.01	–	✓	✓
	GCN	0.05	–	✓	✓
	GCN	0.10	–	✓	✓
MovieLens	GAT	0.01	✓[†]	✓	✓
	GAT	0.05	–	✓	✓
	GAT	0.10	–	✓	–
	GraphSAGE	0.01	–	✓	✓
	GCN	0.01	–	✓	✓
GDP	GAT	–	✓[†]	✓	✓
	GraphSAGE	–	–	✓	✓
	GCN	–	–	✓	✓

[†]Legacy 100× experiments (Sept 2025) - no runtime statistics.

C.1.2 DEGREE DISTRIBUTION ANALYSIS: METRICS AND RATIONALE

Beyond the primary metrics (AUC-ROC and Brier score), we report degree distribution analysis and runtime profiling to characterize how augmentation transforms graph structure and computational cost.

Edge dropping and augmentation fundamentally alters the graph's degree distribution, potentially affecting both model performance and the structural assumptions underlying GNN message passing. We quantify these changes using three complementary approaches: (i) summary statistics describing central tendency and dispersion, (ii) the Gini coefficient capturing degree inequality, and (iii) distribution fitting to distinguish scale-free from random graph structure.

Summary Statistics. For each original, sparsified (in the case of benchmark datasets) and augmented graph, we compute the mean degree $\bar{d} = \frac{1}{|V|} \sum_{v \in V} d_v$, standard deviation σ_d, median, minimum, and maximum degree, as well as the number of isolated nodes ($d_v = 0$). These statistics characterize how augmentation affects both the average connectivity and the presence of extreme values.

Gini Coefficient. The Gini coefficient (Ceriani & Verme, 2012), originally developed for income inequality measurement, has been adapted to quantify heterogeneity in network degree distributions (Hu & Wang, 2005; Badham, 2013). For a sorted degree sequence $(d_1, d_2, \ldots, d_n)$, the Gini coefficient is computed as:

$$G = \frac{2 \sum_{i=1}^{n} i \cdot d_i - (n+1) \sum_{i=1}^{n} d_i}{n \sum_{i=1}^{n} d_i} \tag{1}$$

where $G = 0$ indicates perfect equality (all nodes have identical degree) and $G = 1$ indicates maximal inequality (one node holds all edges). In the context of bipartite link prediction, a very high Gini coefficient ($G > 0.9$) indicates that most nodes have near-zero degree after percolation, concentrating supervision on a few hub nodes. AEGIS-Degree's inverse-degree bias explicitly targets this scenario by preferentially augmenting low-degree nodes, which should manifest as a reduction in G relative to the baseline. Conversely, if G becomes too low, over-smoothing may reduce discriminative structure.

Distribution Fitting. Many real-world networks exhibit heavy-tailed degree distributions, often claimed to follow power laws (Barabási & Albert, 1999). Following the rigorous statistical framework of Clauset et al. (2009), we fit three candidate distributions to the empirical degree sequence:

- **Power Law** (scale-free model): $P(d) \propto d^{-\alpha}$ for $d \geq d_{\min}$. The exponent α is estimated via maximum likelihood with automatic $d_{\min}$ selection. Scale-free networks exhibit hub nodes and preferential attachment dynamics.
- **Log-normal**: $P(d) \propto \frac{1}{d} \exp\left(-\frac{(\ln d - \mu)^2}{2\sigma^2}\right)$. Log-normal distributions arise from multiplicative growth processes and can fit many "scale-free" networks equally well or better (Broido & Clauset, 2019; Stumpf & Porter, 2012).
- **Poisson** (Erdős–Rényi model): $P(d) = \frac{\lambda^d e^{-\lambda}}{d!}$. The Poisson distribution characterizes classical random graphs $G(n, p)$ where $\lambda = np$ (ERDdS & R&wi, 1959). A good Poisson fit indicates augmentation produces ER-like random connectivity.

We compare fits using the Kolmogorov–Smirnov (KS) statistic, where lower values indicate better fit.

Aggregation Protocol. Since each of the 32 seeds produces a different sparse graph (due to distinct bond percolation realizations), we compute degree statistics independently for each seed and report aggregated results as $M \pm \mathrm{SD}$, consistent with our treatment of AUC-ROC and Brier scores. Distribution parameters (α, μ, σ, KS statistics) are likewise fitted per-seed and aggregated.

C.2 Overarching Findings

Across datasets and augmentation methods, augmentation helps selectively and the best method depends on the metric: `semantic_knn` and `synthetic` drive AUC gains under extreme sparsity for Amazon (GAT/GCN), but `semantic_knn` is often neutral or negative for MovieLens and mixed for GDP; `simple`/`degree_aware` offer small, stable lifts in several settings (MovieLens GCN at q=0.01, Amazon GAT/GCN at higher q), while `random` rarely wins on AUC. Brier leaders differ: `random`/`synthetic` improve calibration for Amazon GraphSAGE and MovieLens GCN, `semantic_knn` leads GDP GAT at high ϕ, and `degree_aware`/`simple` give modest Brier gains in multiple cases—highlighting a trade-off between ranking and calibration. Degree distributions show that most augmentations densify graphs and reduce isolation, with `random`/`synthetic` most effective at lowering Gini, whereas `semantic_knn` keeps graphs sparsest. Runtime overhead is negligible for augmentation (sub-0.16s) and training deltas stay small relative to dense-orig baselines, so performance differences are governed by graph quality rather than compute cost; practitioners should pick methods by desired metric (AUC vs. calibration) and acceptable shifts in degree structure.

C.3 Benchmark Dataset - Amazon

C.3.1 Summary

Across Amazon, graph augmentation helps when coupled with the right encoder: for GAT, `semantic_knn` and `synthetic` yield the largest AUC gains at extreme sparsity (q=0.01, ϕ=100), and `simple`/`degree_aware` offer modest, more stable lifts at q=0.10, ϕ=2. For GraphSAGE, AUC gains center on `synthetic` at q=0.01, while Brier is dominated by `random` (and `synthetic` at high dropout), revealing a trade-off between ranking quality and calibration. For GCN, `semantic_knn` is the clear AUC winner at q=0.05, whereas `degree_aware` and `simple`/`random`/`synthetic` drive the best Brier at higher q. Degree distributions remain sparse for most methods, with `random`/`synthetic` most effective at reducing inequality and isolated nodes, and `semantic_knn` keeping graphs sparsest. Runtime overheads are negligible across all augmentation methods (sub-0.07s augmentation; training within a few seconds of baseline), so observed gains or losses are driven by graph quality rather than compute cost.

C.3.2 GAT

Summary Analysis Across Amazon GAT, AUC gains concentrate at high sparsity: `synthetic` and `semantic_knn` lead at q=0.01, ϕ=100 (+0.101 and +0.091), while `simple`/`degree_aware` give smaller but significant lifts at q=0.10, ϕ=2 (+0.017/+0.004); `random` is consistently weakest. Brier improvements are largest for `semantic_knn` at q=0.01, ϕ=100 (-0.015) and for `simple` at q=0.10, ϕ=2 (-0.011), with mild gains for `degree_aware`/`simple` at q=0.10, ϕ=5 ($-0.009/-0.007$); `synthetic`/`random` often worsen Brier. Degree-wise, augmentations raise mean degree from the ultra-sparse baseline (0.033 at q=0.01, ϕ=100) to ≈ 2.4 at ϕ=100 (Gini 0.827–0.979) or to ≈ 0.6/0.48 at ϕ=5/2 (Gini remains high), while `semantic_knn` keeps graphs sparsest (mean degree 0.106 at ϕ=100) but slightly lowers inequality. Runtime overheads are negligible: augmentation < 0.064s and training within a few seconds of baseline (versus $\sim$ 180s for the dense original), so accuracy differences stem from graph quality rather than cost.

AUC and Brier Score

Table 10: Amazon (product–category) GAT: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline (n = 32 seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	100×	baseline	0.630 ± 0.162	+0.000	—	—	—
0.01	100×	degree_aware	0.650 ± 0.204	+0.020^{ns}	−0.50	0.619	−0.09
0.01	100×	simple	0.637 ± 0.199	+0.007^{ns}	−0.17	0.864	−0.03
0.01	100×	semantic_knn	0.722 ± 0.197	+0.091*	−2.40	0.023	−0.42
0.01	100×	**synthetic**	0.732 ± 0.181	+0.101*	−2.48	0.019	−0.44
0.01	100×	random	0.626 ± 0.252	-0.004^{ns}	0.08	0.936	+0.01
0.01	100×	original	0.928 ± 0.008	+0.298***	−10.42	<0.001	−1.84
0.01	5×	baseline	0.648 ± 0.188	+0.000	—	—	—
0.01	5×	degree_aware	0.635 ± 0.226	-0.013^{ns}	0.44	0.665	+0.08
0.01	5×	simple	0.625 ± 0.221	-0.023^{ns}	0.79	0.435	+0.14
0.01	5×	**semantic_knn**	0.726 ± 0.185	+0.078^{ns}	−1.80	0.081	−0.32
0.01	5×	synthetic	0.613 ± 0.248	-0.035^{ns}	0.83	0.410	+0.15
0.01	5×	random	0.611 ± 0.199	-0.037^{ns}	0.91	0.368	+0.16
0.01	5×	original	0.929 ± 0.009	+0.281***	−8.40	<0.001	−1.49
0.01	2×	**baseline**	0.648 ± 0.188	+0.000	—	—	—
0.01	2×	degree_aware	0.628 ± 0.237	-0.019^{ns}	0.70	0.488	+0.12
0.01	2×	simple	0.611 ± 0.231	-0.036^{ns}	1.39	0.174	+0.25
0.01	2×	semantic_knn	0.617 ± 0.237	-0.031^{ns}	0.95	0.347	+0.17
0.01	2×	synthetic	0.585 ± 0.232	-0.063^{ns}	1.53	0.136	+0.27
0.01	2×	random	0.565 ± 0.232	-0.083*	2.38	0.023	+0.42
0.01	2×	original	0.929 ± 0.009	+0.281***	−8.40	<0.001	−1.49
0.05	5×	baseline	0.752 ± 0.064	+0.000	—	—	—
0.05	5×	degree_aware	0.755 ± 0.063	+0.003^{ns}	−0.44	0.665	−0.08
0.05	5×	**simple**	0.757 ± 0.064	+0.005^{ns}	−0.79	0.437	−0.14
0.05	5×	semantic_knn	0.730 ± 0.071	-0.022**	3.11	0.004	+0.55
0.05	5×	synthetic	0.646 ± 0.074	-0.106***	9.10	<0.001	+1.61
0.05	5×	random	0.608 ± 0.086	-0.144***	9.68	<0.001	+1.71
0.05	5×	original	0.929 ± 0.009	+0.177***	−14.93	<0.001	−2.64
0.05	2×	**baseline**	0.752 ± 0.064	+0.000	—	—	—
0.05	2×	degree_aware	0.752 ± 0.067	-0.000^{ns}	0.06	0.953	+0.01
0.05	2×	simple	0.749 ± 0.070	-0.003^{ns}	0.57	0.573	+0.10
0.05	2×	semantic_knn	0.735 ± 0.070	-0.017*	2.61	0.014	+0.46
0.05	2×	synthetic	0.712 ± 0.067	-0.040***	4.91	<0.001	+0.87
0.05	2×	random	0.695 ± 0.085	-0.057***	4.78	<0.001	+0.84
0.05	2×	original	0.929 ± 0.009	+0.177***	−14.93	<0.001	−2.64

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.10	5×	baseline	0.730 ± 0.047	+0.000	—	—	—
0.10	5×	**degree_aware**	0.745 ± 0.051	+0.015**	−3.49	0.001	−0.62
0.10	5×	simple	0.745 ± 0.036	+0.014**	−2.85	0.008	−0.50
0.10	5×	semantic_knn	0.732 ± 0.043	+0.002^{ns}	−0.21	0.837	−0.04
0.10	5×	synthetic	0.673 ± 0.058	-0.057***	4.84	<0.001	+0.85
0.10	5×	random	0.661 ± 0.057	-0.070***	6.42	<0.001	+1.13
0.10	5×	original	0.929 ± 0.009	+0.199***	−22.72	<0.001	−4.02
0.10	2×	baseline	0.730 ± 0.047	+0.000	—	—	—
0.10	2×	degree_aware	0.734 ± 0.040	+0.004^{ns}	−0.74	0.463	−0.13
0.10	2×	**simple**	0.747 ± 0.047	+0.017**	−3.15	0.004	−0.56
0.10	2×	semantic_knn	0.732 ± 0.043	+0.002^{ns}	−0.21	0.837	−0.04
0.10	2×	synthetic	0.708 ± 0.048	-0.022**	3.19	0.003	+0.56
0.10	2×	random	0.692 ± 0.057	-0.038***	5.69	<0.001	+1.01
0.10	2×	original	0.929 ± 0.009	+0.199***	−22.72	<0.001	−4.02

Table 11: Amazon (product–category) GAT: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	100×	baseline	0.249 ± 0.048	+0.000	—	—	—
0.01	100×	degree_aware	0.248 ± 0.054	-0.001^{ns}	0.29	0.772	+0.05
0.01	100×	simple	0.248 ± 0.049	-0.001^{ns}	0.30	0.765	+0.05
0.01	100×	**semantic_knn**	0.233 ± 0.044	-0.015*	2.19	0.036	+0.39
0.01	100×	synthetic	0.244 ± 0.029	-0.005^{ns}	0.70	0.488	+0.13
0.01	100×	random	0.259 ± 0.040	+0.010^{ns}	−0.92	0.367	−0.16
0.01	100×	original	0.135 ± 0.020	-0.114***	14.03	<0.001	+2.48
0.01	5×	baseline	0.251 ± 0.062	+0.000	—	—	—
0.01	5×	degree_aware	0.245 ± 0.053	-0.006^{ns}	1.85	0.074	+0.33
0.01	5×	simple	0.243 ± 0.058	-0.008^{ns}	1.55	0.132	+0.28
0.01	5×	**semantic_knn**	0.236 ± 0.048	-0.015^{ns}	1.61	0.117	+0.29
0.01	5×	synthetic	0.256 ± 0.041	+0.005^{ns}	−0.28	0.778	−0.05
0.01	5×	random	0.268 ± 0.063	+0.017^{ns}	−1.87	0.071	−0.34
0.01	5×	original	0.141 ± 0.016	-0.110***	10.52	<0.001	+1.86
0.01	2×	baseline	0.251 ± 0.062	+0.000	—	—	—
0.01	2×	degree_aware	0.254 ± 0.066	+0.003^{ns}	−0.35	0.730	−0.06
0.01	2×	simple	0.257 ± 0.064	+0.006^{ns}	−1.35	0.188	−0.24
0.01	2×	**semantic_knn**	0.247 ± 0.047	-0.004^{ns}	0.94	0.353	+0.17
0.01	2×	synthetic	0.256 ± 0.055	+0.005^{ns}	−0.26	0.798	−0.05
0.01	2×	random	0.280 ± 0.079	+0.029*	−2.38	0.024	−0.43
0.01	2×	original	0.141 ± 0.016	-0.110***	10.52	<0.001	+1.86
0.05	5×	baseline	0.238 ± 0.022	+0.000	—	—	—
0.05	5×	degree_aware	0.235 ± 0.021	-0.003^{ns}	0.71	0.481	+0.13
0.05	5×	**simple**	0.234 ± 0.027	-0.004^{ns}	1.09	0.285	+0.19
0.05	5×	semantic_knn	0.243 ± 0.029	+0.006^{ns}	−1.52	0.139	−0.27
0.05	5×	synthetic	0.256 ± 0.024	+0.018***	−4.52	<0.001	−0.80
0.05	5×	random	0.272 ± 0.028	+0.034***	−8.69	<0.001	−1.54
0.05	5×	original	0.141 ± 0.016	-0.097***	20.57	<0.001	+3.64
0.05	2×	**baseline**	0.238 ± 0.022	+0.000	—	—	—
0.05	2×	degree_aware	0.238 ± 0.028	-0.000^{ns}	0.08	0.939	+0.01
0.05	2×	simple	0.238 ± 0.031	-0.000^{ns}	0.02	0.984	+0.00
0.05	2×	semantic_knn	0.244 ± 0.028	+0.006^{ns}	−1.90	0.067	−0.34
0.05	2×	synthetic	0.242 ± 0.028	+0.004^{ns}	−1.02	0.317	−0.18

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.05	2×	random	0.245 ± 0.031	+0.007^{ns}	−1.50	0.145	−0.26
0.05	2×	original	0.141 ± 0.016	-0.097***	20.57	<0.001	+3.64
0.10	5×	baseline	0.235 ± 0.024	+0.000	—	—	—
0.10	5×	**degree_aware**	0.226 ± 0.023	-0.009*	2.29	0.029	+0.41
0.10	5×	simple	0.228 ± 0.023	-0.007*	2.25	0.032	+0.40
0.10	5×	semantic_knn	0.230 ± 0.022	-0.005^{ns}	1.16	0.255	+0.20
0.10	5×	synthetic	0.253 ± 0.024	+0.018**	−3.56	0.001	−0.63
0.10	5×	random	0.260 ± 0.022	+0.025***	−4.96	<0.001	−0.88
0.10	5×	original	0.141 ± 0.016	-0.094***	17.66	<0.001	+3.12
0.10	2×	baseline	0.235 ± 0.024	+0.000	—	—	—
0.10	2×	degree_aware	0.232 ± 0.024	-0.003^{ns}	0.83	0.411	+0.15
0.10	2×	**simple**	0.224 ± 0.027	-0.011***	3.72	<0.001	+0.66
0.10	2×	semantic_knn	0.230 ± 0.022	-0.005^{ns}	1.16	0.255	+0.20
0.10	2×	synthetic	0.240 ± 0.025	+0.005^{ns}	−1.33	0.193	−0.24
0.10	2×	random	0.243 ± 0.021	+0.008*	−2.10	0.044	−0.37
0.10	2×	original	0.141 ± 0.016	-0.094***	17.66	<0.001	+3.12

Degree Distribution Analysis

Table 12: Amazon (product–category) GAT: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	100×	baseline	0.0334 ± 0.0102	0.968 ± 0.010	1417.3 ± 14.4	lognormal
0.01	100×	degree_aware	2.4168 ± 0.2940	0.979 ± 0.003	1430.3 ± 4.4	lognormal
0.01	100×	simple	2.4168 ± 0.2940	0.978 ± 0.003	1430.3 ± 4.4	lognormal
0.01	100×	semantic_knn	0.1063 ± 0.0100	0.964 ± 0.005	1382.2 ± 10.3	powerlaw
0.01	100×	synthetic	2.4168 ± 0.2940	0.827 ± 0.023	1130.6 ± 41.5	lognormal
0.01	100×	**random**	2.4168 ± 0.2940	0.355 ± 0.021	136.8 ± 40.9	powerlaw
0.01	100×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.01	5×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	5×	degree_aware	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	simple	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	semantic_knn	0.1060 ± 0.0096	0.963 ± 0.005	1381.6 ± 10.1	powerlaw
0.01	5×	synthetic	0.1208 ± 0.0147	0.925 ± 0.010	1329.9 ± 17.3	powerlaw
0.01	5×	**random**	0.1208 ± 0.0147	0.893 ± 0.012	1298.8 ± 19.1	lognormal
0.01	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.01	2×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	2×	degree_aware	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	simple	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	semantic_knn	0.0483 ± 0.0059	0.978 ± 0.003	1419.5 ± 6.0	powerlaw
0.01	2×	synthetic	0.0483 ± 0.0059	0.960 ± 0.005	1401.4 ± 8.1	lognormal
0.01	2×	**random**	0.0483 ± 0.0059	0.954 ± 0.006	1396.2 ± 8.4	lognormal
0.01	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	5×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal
0.05	5×	degree_aware	0.5930 ± 0.0365	0.916 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	simple	0.5930 ± 0.0365	0.915 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	semantic_knn	0.2682 ± 0.0140	0.858 ± 0.009	1188.1 ± 16.1	powerlaw
0.05	5×	synthetic	0.5930 ± 0.0365	0.718 ± 0.016	910.9 ± 31.0	powerlaw
0.05	5×	**random**	0.5930 ± 0.0365	0.639 ± 0.018	811.7 ± 33.7	powerlaw
0.05	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	2×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.05	2×	degree_aware	0.2372 ± 0.0146	0.919 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	2×	simple	0.2372 ± 0.0146	0.920 ± 0.004	1301.9 ± 9.3	powerlaw
0.05	2×	semantic_knn	0.2372 ± 0.0146	0.876 ± 0.010	1221.0 ± 17.0	powerlaw
0.05	2×	synthetic	0.2372 ± 0.0146	0.829 ± 0.011	1174.2 ± 17.8	powerlaw
0.05	2×	**random**	0.2372 ± 0.0146	0.810 ± 0.012	1156.3 ± 18.2	lognormal
0.05	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	5×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	5×	degree_aware	1.1873 ± 0.0446	0.841 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	simple	1.1873 ± 0.0446	0.843 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	5×	synthetic	1.1873 ± 0.0446	0.576 ± 0.015	572.2 ± 27.0	powerlaw
0.10	5×	**random**	1.1873 ± 0.0446	0.489 ± 0.013	450.4 ± 26.1	powerlaw
0.10	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	2×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	2×	degree_aware	0.4749 ± 0.0179	0.848 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	simple	0.4749 ± 0.0179	0.850 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	2×	synthetic	0.4749 ± 0.0179	0.712 ± 0.012	941.3 ± 17.4	powerlaw
0.10	2×	**random**	0.4749 ± 0.0179	0.685 ± 0.014	911.9 ± 21.6	powerlaw
0.10	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw

Runtime Analysis

Table 13: Amazon (product–category) GAT: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	3.95 ± 0.41
0.01	5×	degree_aware	0.0013 ± 0.0010	3.98 ± 1.26
0.01	5×	simple	0.0010 ± 0.0001	3.83 ± 0.95
0.01	5×	semantic_knn	0.0116 ± 0.0056	3.85 ± 0.87
0.01	5×	synthetic	0.0011 ± 0.0004	3.74 ± 0.74
0.01	5×	**random**	0.0009 ± 0.0001	4.00 ± 0.96
0.01	5×	original	0.0000 ± 0.0000	178.54 ± 20.74
0.01	2×	baseline	0.0000 ± 0.0000	4.04 ± 0.43
0.01	2×	degree_aware	0.0013 ± 0.0005	3.93 ± 0.91
0.01	2×	simple	0.0011 ± 0.0001	3.94 ± 0.90
0.01	2×	semantic_knn	0.0066 ± 0.0084	3.80 ± 0.70
0.01	2×	synthetic	0.0013 ± 0.0013	3.81 ± 0.71
0.01	2×	**random**	0.0010 ± 0.0001	3.89 ± 0.78
0.01	2×	original	0.0000 ± 0.0000	180.52 ± 20.72
0.05	5×	baseline	0.0000 ± 0.0000	9.81 ± 4.34
0.05	5×	degree_aware	0.0014 ± 0.0007	9.93 ± 3.32
0.05	5×	simple	0.0011 ± 0.0001	10.23 ± 4.17
0.05	5×	semantic_knn	0.0358 ± 0.0058	9.66 ± 3.27
0.05	5×	synthetic	0.0012 ± 0.0003	12.38 ± 4.65
0.05	5×	**random**	0.0010 ± 0.0001	15.04 ± 5.33
0.05	5×	original	0.0000 ± 0.0000	181.93 ± 20.91
0.05	2×	baseline	0.0000 ± 0.0000	9.47 ± 4.17
0.05	2×	degree_aware	0.0014 ± 0.0012	8.61 ± 2.24
0.05	2×	**simple**	0.0010 ± 0.0000	9.77 ± 5.23
0.05	2×	semantic_knn	0.0219 ± 0.0068	9.74 ± 3.93

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.05	2×	synthetic	0.0011 ± 0.0006	11.43 ± 3.88
0.05	2×	random	0.0010 ± 0.0000	11.23 ± 3.91
0.05	2×	original	0.0000 ± 0.0000	178.24 ± 20.43
0.10	5×	baseline	0.0000 ± 0.0000	20.88 ± 6.92
0.10	5×	degree_aware	0.0013 ± 0.0001	24.83 ± 8.14
0.10	5×	**simple**	0.0011 ± 0.0001	24.59 ± 8.47
0.10	5×	semantic_knn	0.0615 ± 0.0026	22.58 ± 7.62
0.10	5×	synthetic	0.0011 ± 0.0001	24.20 ± 7.16
0.10	5×	random	0.0012 ± 0.0008	24.25 ± 6.23
0.10	5×	original	0.0000 ± 0.0000	178.61 ± 20.40
0.10	2×	baseline	0.0000 ± 0.0000	21.17 ± 7.02
0.10	2×	degree_aware	0.0012 ± 0.0001	23.30 ± 8.60
0.10	2×	**simple**	0.0010 ± 0.0001	23.62 ± 8.30
0.10	2×	semantic_knn	0.0636 ± 0.0026	22.85 ± 7.70
0.10	2×	synthetic	0.0011 ± 0.0001	23.27 ± 8.54
0.10	2×	random	0.0010 ± 0.0001	24.57 ± 9.45
0.10	2×	original	0.0000 ± 0.0000	179.84 ± 20.52

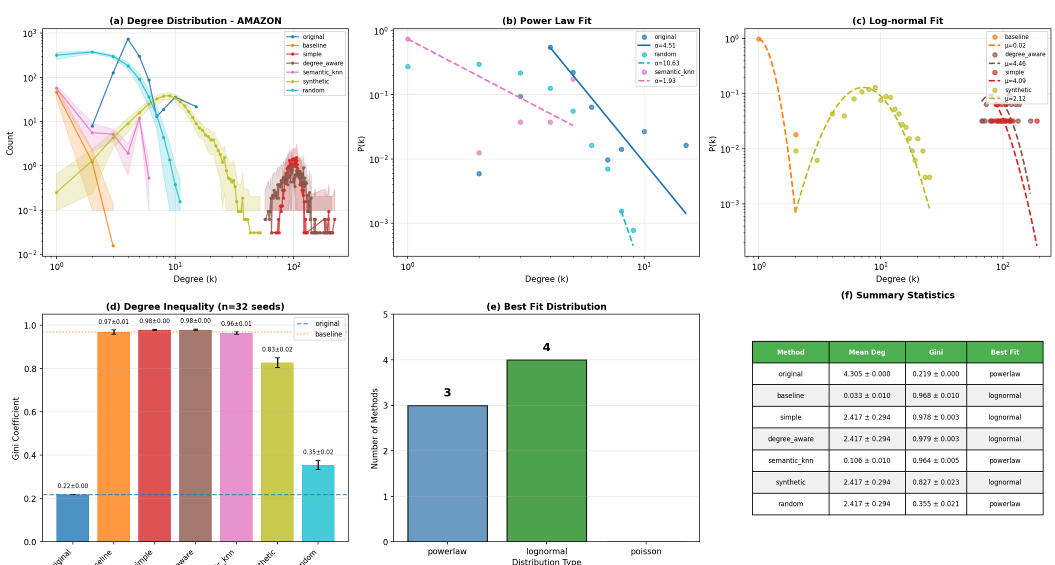

Figure 4: Amazon (product–category), GAT, q=0.01, ϕ=100: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

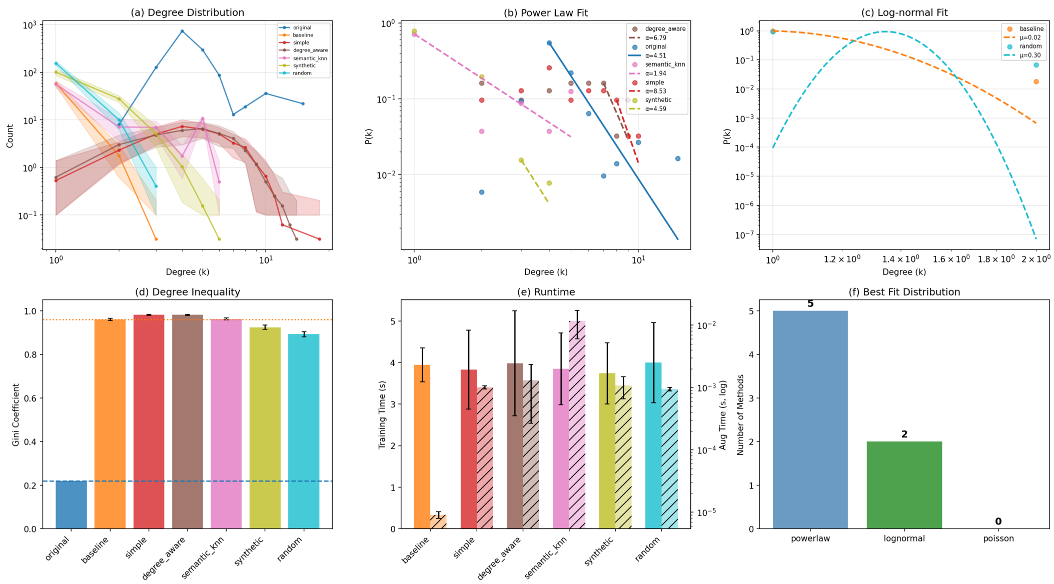

Figure 5: Amazon (product–category), GAT, q=0.01, ϕ=5: Comprehensive analysis ($M \pm \mathrm{SD}$, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

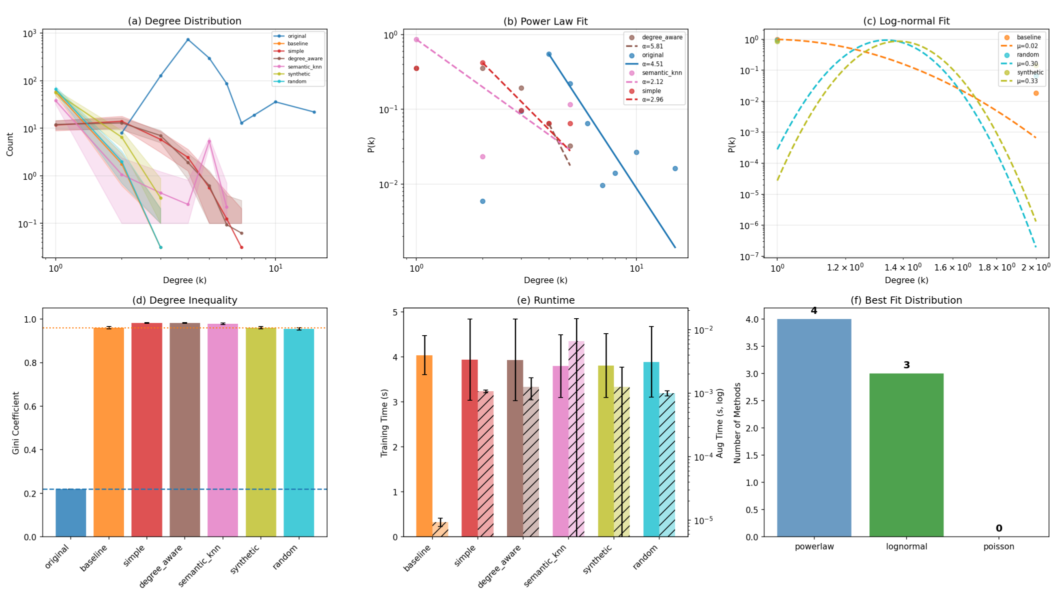

Figure 6: Amazon (product–category), GAT, q=0.01, ϕ=2: Comprehensive analysis ($M \pm \mathrm{SD}$, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

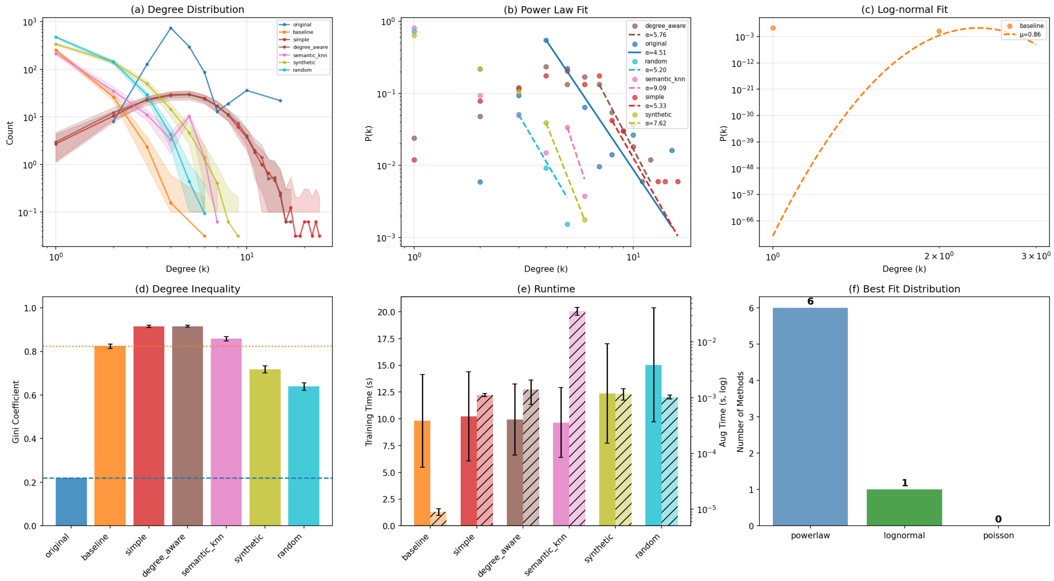

Figure 7: Amazon (product–category), GAT, q=0.05, ϕ=5: Comprehensive analysis ($M \pm$SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

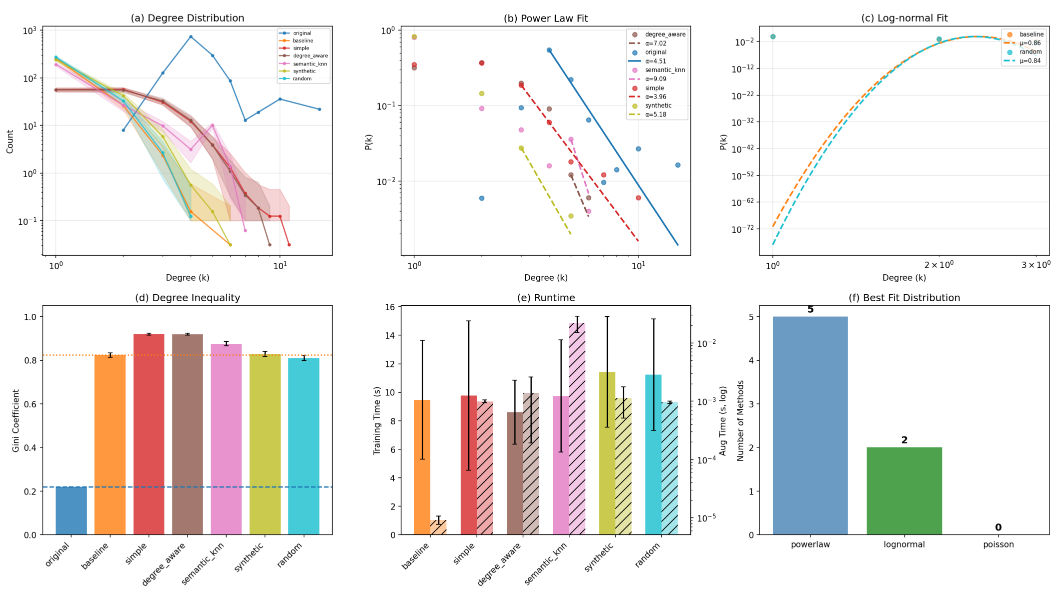

Figure 8: Amazon (product–category), GAT, q=0.05, ϕ=2: Comprehensive analysis ($M \pm$SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

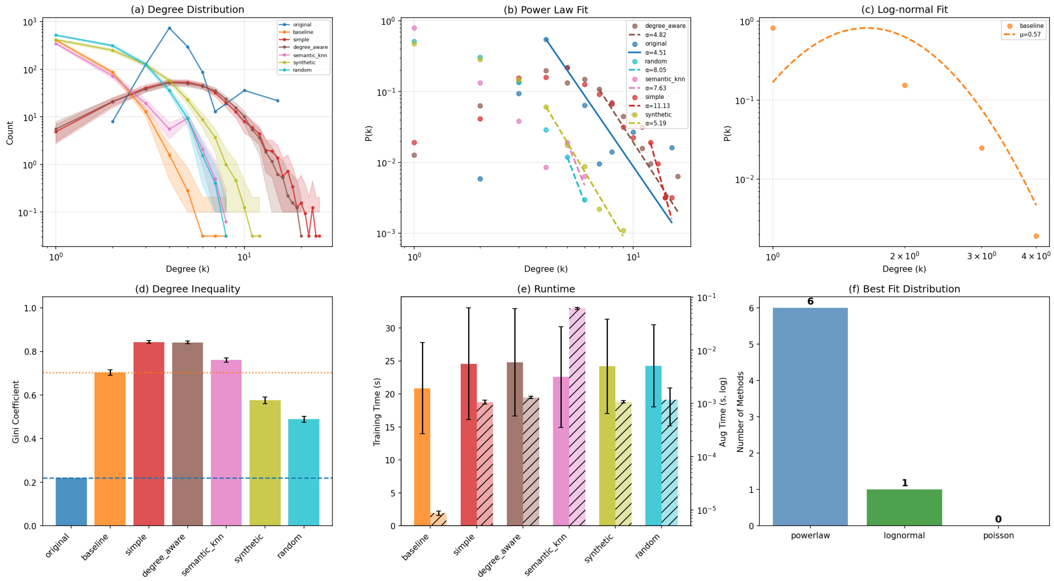

Figure 9: Amazon (product–category), GAT, q=0.10, ϕ=5: Comprehensive analysis ($M \pm$SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

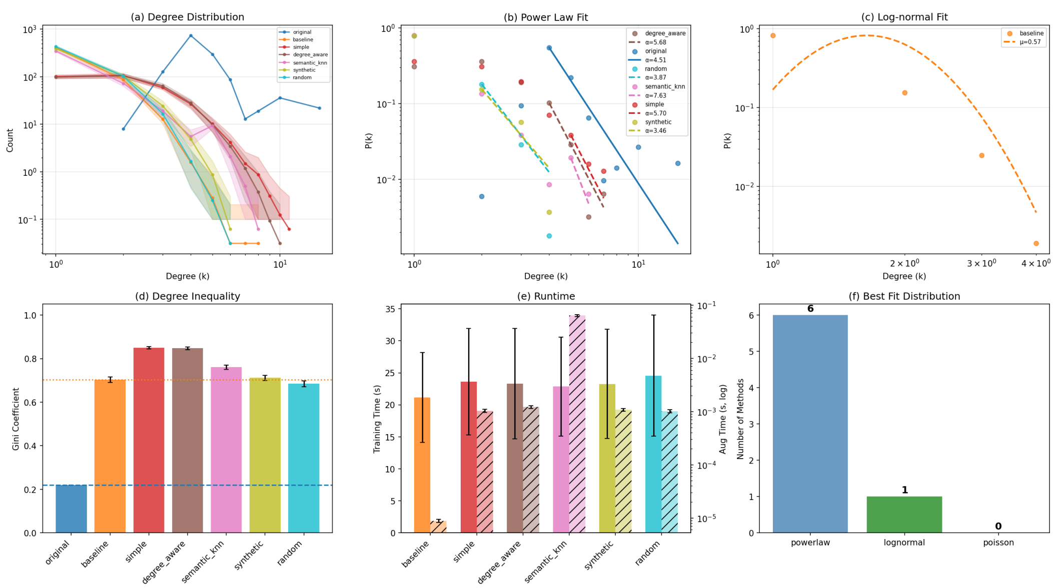

Figure 10: Amazon (product–category), GAT, q=0.10, ϕ=2: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.3.3 GRAPHSAGE

Summary Analysis For Amazon GraphSAGE, `synthetic` leads AUC at q=0.01, ϕ=5 (+0.077) and stays positive at ϕ=2 (+0.036), while most other augmentations hover near baseline and `random` is the weakest. Brier tells the opposite story: `random` delivers the largest reductions (e.g., q=0.10, ϕ=5, ΔBrier= -0.092; q=0.05, ϕ=5, -0.079), with `synthetic` also improving at q=0.10, ϕ=5 (-0.072); `semantic_knn` and `degree_aware` typically give modest or no Brier gains. Degree statistics show augmentations raising mean degree from the ultra-sparse baseline (0.043) to $\sim$ 0.12 at ϕ=5 and up to $\sim$ 1.19 at q=0.10, ϕ=5, while `random`/`synthetic` sharply lower Gini and isolated counts. Runtime overheads remain negligible: augmentation under 0.012s and training within a few seconds of baseline (vs. $\sim$ 134s for the dense original), so differences are driven by graph quality rather than cost.

AUC and Brier Score

Table 14: Amazon (product–category) GraphSAGE: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline (n = 32 seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	5×	baseline	0.507 ± 0.153	+0.000	—	—	—
0.01	5×	degree_aware	0.520 ± 0.192	+0.012^{ns}	−0.40	0.689	−0.07
0.01	5×	simple	0.494 ± 0.194	-0.014^{ns}	0.42	0.675	+0.07
0.01	5×	semantic_knn	0.526 ± 0.177	+0.019^{ns}	−0.65	0.521	−0.11
0.01	5×	**synthetic**	0.584 ± 0.170	+0.077*	−2.34	0.026	−0.41
0.01	5×	random	0.507 ± 0.183	-0.000^{ns}	0.01	0.990	+0.00
0.01	5×	original	0.921 ± 0.009	+0.413***	−15.34	<0.001	−2.71
0.01	2×	baseline	0.507 ± 0.153	+0.000	—	—	—
0.01	2×	degree_aware	0.516 ± 0.177	+0.009^{ns}	−0.30	0.766	−0.05
0.01	2×	simple	0.471 ± 0.176	-0.036^{ns}	1.38	0.177	+0.24
0.01	2×	semantic_knn	0.522 ± 0.163	+0.014^{ns}	−0.46	0.646	−0.08
0.01	2×	**synthetic**	0.543 ± 0.186	+0.036^{ns}	−1.10	0.279	−0.19
0.01	2×	random	0.528 ± 0.177	+0.020^{ns}	−0.54	0.593	−0.10
0.01	2×	original	0.921 ± 0.009	+0.413***	−15.34	<0.001	−2.71
0.05	5×	baseline	0.663 ± 0.086	+0.000	—	—	—
0.05	5×	degree_aware	0.649 ± 0.082	-0.014^{ns}	1.18	0.247	+0.21
0.05	5×	simple	0.658 ± 0.069	-0.005^{ns}	0.45	0.655	+0.08
0.05	5×	**semantic_knn**	0.677 ± 0.079	+0.014^{ns}	−1.31	0.198	−0.23
0.05	5×	synthetic	0.677 ± 0.075	+0.014^{ns}	−1.32	0.195	−0.23
0.05	5×	random	0.611 ± 0.113	-0.051**	2.98	0.006	+0.53
0.05	5×	original	0.921 ± 0.009	+0.258***	−17.13	<0.001	−3.03
0.05	2×	baseline	0.663 ± 0.086	+0.000	—	—	—
0.05	2×	degree_aware	0.646 ± 0.079	-0.016^{ns}	1.46	0.154	+0.26
0.05	2×	simple	0.632 ± 0.074	-0.031**	3.04	0.005	+0.54
0.05	2×	**semantic_knn**	0.668 ± 0.085	+0.005^{ns}	−0.54	0.591	−0.10
0.05	2×	synthetic	0.655 ± 0.096	-0.008^{ns}	0.69	0.497	+0.12
0.05	2×	random	0.648 ± 0.084	-0.014^{ns}	1.05	0.301	+0.19
0.05	2×	original	0.921 ± 0.009	+0.258***	−17.13	<0.001	−3.03
0.10	5×	**baseline**	0.743 ± 0.037	+0.000	—	—	—
0.10	5×	degree_aware	0.730 ± 0.038	-0.013^{ns}	1.95	0.061	+0.34
0.10	5×	simple	0.737 ± 0.043	-0.005^{ns}	0.73	0.469	+0.13
0.10	5×	semantic_knn	0.733 ± 0.038	-0.010^{ns}	1.49	0.147	+0.26
0.10	5×	synthetic	0.737 ± 0.057	-0.006^{ns}	0.51	0.615	+0.09
0.10	5×	random	0.698 ± 0.065	-0.045**	3.54	0.001	+0.63
0.10	5×	original	0.921 ± 0.009	+0.178***	−28.27	<0.001	−5.00
0.10	2×	**baseline**	0.743 ± 0.037	+0.000	—	—	—
0.10	2×	degree_aware	0.737 ± 0.045	-0.006^{ns}	0.83	0.411	+0.15
0.10	2×	simple	0.740 ± 0.049	-0.003^{ns}	0.32	0.752	+0.06
0.10	2×	semantic_knn	0.733 ± 0.038	-0.010^{ns}	1.49	0.147	+0.26
0.10	2×	synthetic	0.738 ± 0.038	-0.005^{ns}	0.65	0.521	+0.11
0.10	2×	random	0.730 ± 0.050	-0.013^{ns}	1.45	0.158	+0.26

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.10	2×	original	0.921 ± 0.009	+0.178***	−28.27	<0.001	−5.00

Table 15: Amazon (product–category) GraphSAGE: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	5×	baseline	0.397 ± 0.041	+0.000	—	—	—
0.01	5×	degree_aware	0.395 ± 0.050	-0.001^{ns}	−0.01	0.993	−0.00
0.01	5×	**simple**	0.391 ± 0.046	-0.006^{ns}	0.46	0.646	+0.08
0.01	5×	semantic_knn	0.399 ± 0.059	+0.002^{ns}	−0.29	0.777	−0.05
0.01	5×	synthetic	0.398 ± 0.039	+0.001^{ns}	−0.35	0.729	−0.06
0.01	5×	random	0.401 ± 0.033	+0.004^{ns}	−0.68	0.499	−0.12
0.01	5×	original	0.105 ± 0.011	-0.292***	37.24	<0.001	+6.58
0.01	2×	baseline	0.397 ± 0.041	+0.000	—	—	—
0.01	2×	degree_aware	0.394 ± 0.057	-0.003^{ns}	0.20	0.847	+0.04
0.01	2×	simple	0.408 ± 0.036	+0.012^{ns}	−1.72	0.095	−0.31
0.01	2×	semantic_knn	0.402 ± 0.044	+0.005^{ns}	−0.96	0.344	−0.17
0.01	2×	**synthetic**	0.393 ± 0.047	-0.004^{ns}	0.31	0.758	+0.06
0.01	2×	random	0.398 ± 0.046	+0.001^{ns}	−0.22	0.829	−0.04
0.01	2×	original	0.105 ± 0.011	-0.292***	37.24	<0.001	+6.58
0.05	5×	baseline	0.361 ± 0.032	+0.000	—	—	—
0.05	5×	degree_aware	0.367 ± 0.035	+0.005^{ns}	−0.82	0.419	−0.14
0.05	5×	simple	0.366 ± 0.031	+0.005^{ns}	−0.74	0.468	−0.13
0.05	5×	semantic_knn	0.361 ± 0.031	-0.000^{ns}	0.03	0.976	+0.01
0.05	5×	synthetic	0.330 ± 0.073	-0.031*	2.56	0.015	+0.45
0.05	5×	**random**	0.282 ± 0.072	-0.079***	6.62	<0.001	+1.17
0.05	5×	original	0.105 ± 0.011	-0.256***	41.94	<0.001	+7.41
0.05	2×	**baseline**	0.361 ± 0.032	+0.000	—	—	—
0.05	2×	degree_aware	0.367 ± 0.027	+0.006^{ns}	−1.01	0.322	−0.18
0.05	2×	simple	0.365 ± 0.024	+0.004^{ns}	−0.76	0.453	−0.13
0.05	2×	semantic_knn	0.372 ± 0.029	+0.011*	−2.16	0.038	−0.38
0.05	2×	synthetic	0.375 ± 0.028	+0.014*	−2.43	0.021	−0.43
0.05	2×	random	0.369 ± 0.036	+0.008^{ns}	−1.33	0.194	−0.23
0.05	2×	original	0.105 ± 0.011	-0.256***	41.94	<0.001	+7.41
0.10	5×	baseline	0.315 ± 0.050	+0.000	—	—	—
0.10	5×	degree_aware	0.315 ± 0.056	+0.000^{ns}	−0.02	0.986	−0.00
0.10	5×	simple	0.319 ± 0.041	+0.004^{ns}	−0.49	0.627	−0.09
0.10	5×	semantic_knn	0.305 ± 0.042	-0.009^{ns}	1.07	0.293	+0.19
0.10	5×	synthetic	0.243 ± 0.056	-0.072***	6.29	<0.001	+1.11
0.10	5×	**random**	0.223 ± 0.028	-0.092***	9.04	<0.001	+1.60
0.10	5×	original	0.105 ± 0.011	-0.210***	24.31	<0.001	+4.30
0.10	2×	baseline	0.315 ± 0.050	+0.000	—	—	—
0.10	2×	degree_aware	0.313 ± 0.045	-0.001^{ns}	0.13	0.898	+0.02
0.10	2×	simple	0.325 ± 0.045	+0.010^{ns}	−0.87	0.390	−0.15
0.10	2×	semantic_knn	0.305 ± 0.042	-0.009^{ns}	1.07	0.293	+0.19
0.10	2×	synthetic	0.286 ± 0.047	-0.029*	2.38	0.023	+0.42
0.10	2×	**random**	0.261 ± 0.051	-0.053***	3.95	<0.001	+0.70
0.10	2×	original	0.105 ± 0.011	-0.210***	24.31	<0.001	+4.30

Degree Distribution Analysis

Table 16: Amazon (product–category) GraphSAGE: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	5×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	5×	degree_aware	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	simple	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	semantic_knn	0.1060 ± 0.0096	0.963 ± 0.005	1381.6 ± 10.1	powerlaw
0.01	5×	synthetic	0.1208 ± 0.0147	0.925 ± 0.010	1329.9 ± 17.3	powerlaw
0.01	5×	**random**	0.1208 ± 0.0147	0.893 ± 0.012	1298.8 ± 19.1	lognormal
0.01	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.01	2×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	2×	degree_aware	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	simple	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	semantic_knn	0.0483 ± 0.0059	0.978 ± 0.003	1419.5 ± 6.0	powerlaw
0.01	2×	synthetic	0.0483 ± 0.0059	0.960 ± 0.005	1401.4 ± 8.1	lognormal
0.01	2×	**random**	0.0483 ± 0.0059	0.954 ± 0.006	1396.2 ± 8.4	lognormal
0.01	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	5×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal
0.05	5×	degree_aware	0.5930 ± 0.0365	0.916 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	simple	0.5930 ± 0.0365	0.915 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	semantic_knn	0.2682 ± 0.0140	0.858 ± 0.009	1188.1 ± 16.1	powerlaw
0.05	5×	synthetic	0.5930 ± 0.0365	0.718 ± 0.016	910.9 ± 31.0	powerlaw
0.05	5×	**random**	0.5930 ± 0.0365	0.639 ± 0.018	811.7 ± 33.7	powerlaw
0.05	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	2×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal
0.05	2×	degree_aware	0.2372 ± 0.0146	0.919 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	2×	simple	0.2372 ± 0.0146	0.920 ± 0.004	1301.9 ± 9.3	powerlaw
0.05	2×	semantic_knn	0.2372 ± 0.0146	0.876 ± 0.010	1221.0 ± 17.0	powerlaw
0.05	2×	synthetic	0.2372 ± 0.0146	0.829 ± 0.011	1174.2 ± 17.8	powerlaw
0.05	2×	**random**	0.2372 ± 0.0146	0.810 ± 0.012	1156.3 ± 18.2	lognormal
0.05	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	5×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	5×	degree_aware	1.1873 ± 0.0446	0.841 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	simple	1.1873 ± 0.0446	0.843 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	5×	synthetic	1.1873 ± 0.0446	0.576 ± 0.015	572.2 ± 27.0	powerlaw
0.10	5×	**random**	1.1873 ± 0.0446	0.489 ± 0.013	450.4 ± 26.1	powerlaw
0.10	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	2×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	2×	degree_aware	0.4749 ± 0.0179	0.848 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	simple	0.4749 ± 0.0179	0.850 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	2×	synthetic	0.4749 ± 0.0179	0.712 ± 0.012	941.3 ± 17.4	powerlaw
0.10	2×	**random**	0.4749 ± 0.0179	0.685 ± 0.014	911.9 ± 21.6	powerlaw
0.10	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw

Runtime Analysis

Table 17: Amazon (product–category) GraphSAGE: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	4.08 ± 1.40
0.01	5×	degree_aware	0.0013 ± 0.0007	3.97 ± 1.62
0.01	5×	simple	0.0012 ± 0.0009	3.65 ± 1.41
0.01	5×	semantic_knn	0.0117 ± 0.0067	4.11 ± 1.91
0.01	5×	synthetic	0.0011 ± 0.0004	4.55 ± 1.44
0.01	5×	**random**	0.0010 ± 0.0001	3.94 ± 1.23
0.01	5×	original	0.0000 ± 0.0000	133.71 ± 11.00
0.01	2×	baseline	0.0000 ± 0.0000	4.02 ± 1.36
0.01	2×	degree_aware	0.0013 ± 0.0008	3.64 ± 1.54
0.01	2×	simple	0.0010 ± 0.0000	3.83 ± 1.72
0.01	2×	semantic_knn	0.0050 ± 0.0011	3.68 ± 1.29
0.01	2×	synthetic	0.0011 ± 0.0004	3.70 ± 1.51
0.01	2×	**random**	0.0009 ± 0.0000	4.00 ± 1.84
0.01	2×	original	0.0000 ± 0.0000	132.37 ± 11.42
0.05	5×	baseline	0.0000 ± 0.0000	6.55 ± 1.25
0.05	5×	degree_aware	0.0012 ± 0.0001	7.16 ± 3.75
0.05	5×	**simple**	0.0010 ± 0.0000	7.62 ± 2.02
0.05	5×	semantic_knn	0.0343 ± 0.0023	7.68 ± 2.54
0.05	5×	synthetic	0.0010 ± 0.0000	7.71 ± 2.24
0.05	5×	random	0.0010 ± 0.0000	6.29 ± 1.14
0.05	5×	original	0.0000 ± 0.0000	133.17 ± 11.53
0.05	2×	baseline	0.0000 ± 0.0000	6.72 ± 1.28
0.05	2×	degree_aware	0.0014 ± 0.0008	7.56 ± 3.10
0.05	2×	simple	0.0011 ± 0.0001	7.42 ± 2.51
0.05	2×	semantic_knn	0.0224 ± 0.0082	8.51 ± 3.91
0.05	2×	synthetic	0.0014 ± 0.0016	8.11 ± 2.11
0.05	2×	**random**	0.0010 ± 0.0001	8.21 ± 2.37
0.05	2×	original	0.0000 ± 0.0000	134.67 ± 11.76
0.10	5×	baseline	0.0000 ± 0.0000	15.43 ± 5.57
0.10	5×	degree_aware	0.0013 ± 0.0001	16.61 ± 6.89
0.10	5×	simple	0.0011 ± 0.0001	15.24 ± 3.81
0.10	5×	semantic_knn	0.0624 ± 0.0026	13.80 ± 5.13
0.10	5×	synthetic	0.0011 ± 0.0001	13.90 ± 2.43
0.10	5×	**random**	0.0010 ± 0.0001	13.15 ± 1.35
0.10	5×	original	0.0000 ± 0.0000	133.62 ± 11.59
0.10	2×	baseline	0.0000 ± 0.0000	15.21 ± 5.53
0.10	2×	degree_aware	0.0014 ± 0.0016	14.29 ± 6.33
0.10	2×	**simple**	0.0010 ± 0.0001	15.29 ± 4.83
0.10	2×	semantic_knn	0.0626 ± 0.0052	13.57 ± 5.02
0.10	2×	synthetic	0.0011 ± 0.0005	12.98 ± 4.72
0.10	2×	random	0.0010 ± 0.0000	11.62 ± 3.01
0.10	2×	original	0.0000 ± 0.0000	132.70 ± 11.33

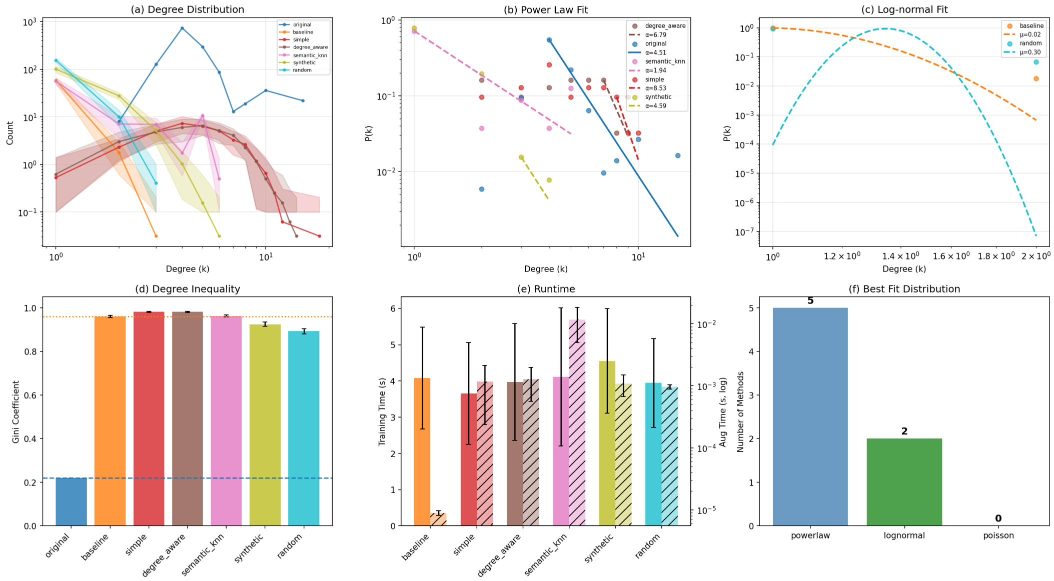

Figure 11: Amazon (product–category), GraphSAGE, q=0.01, ϕ=5: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

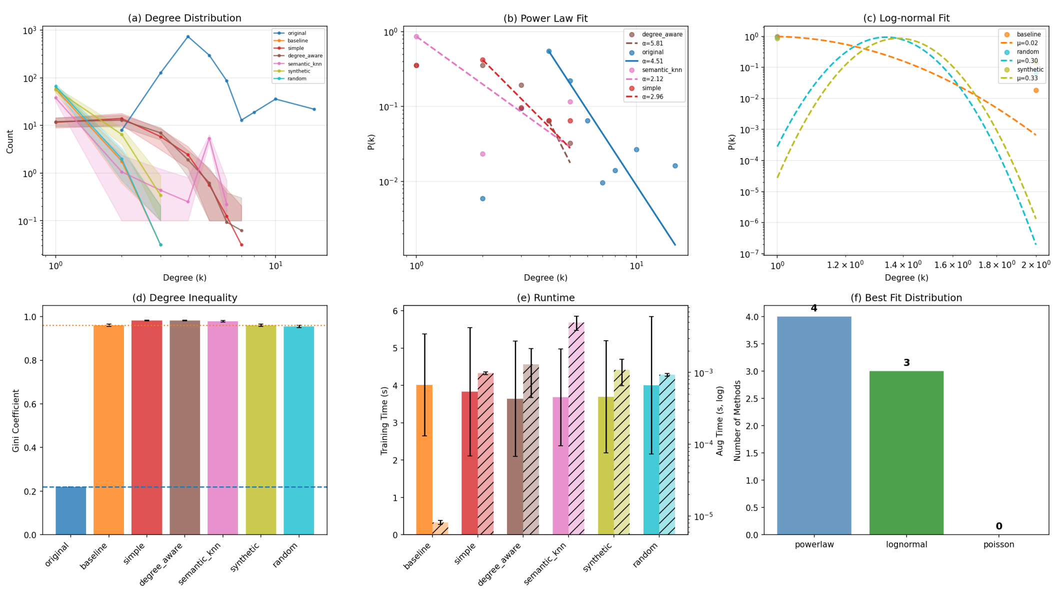

Figure 12: Amazon (product–category), GraphSAGE, q=0.01, ϕ=2: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

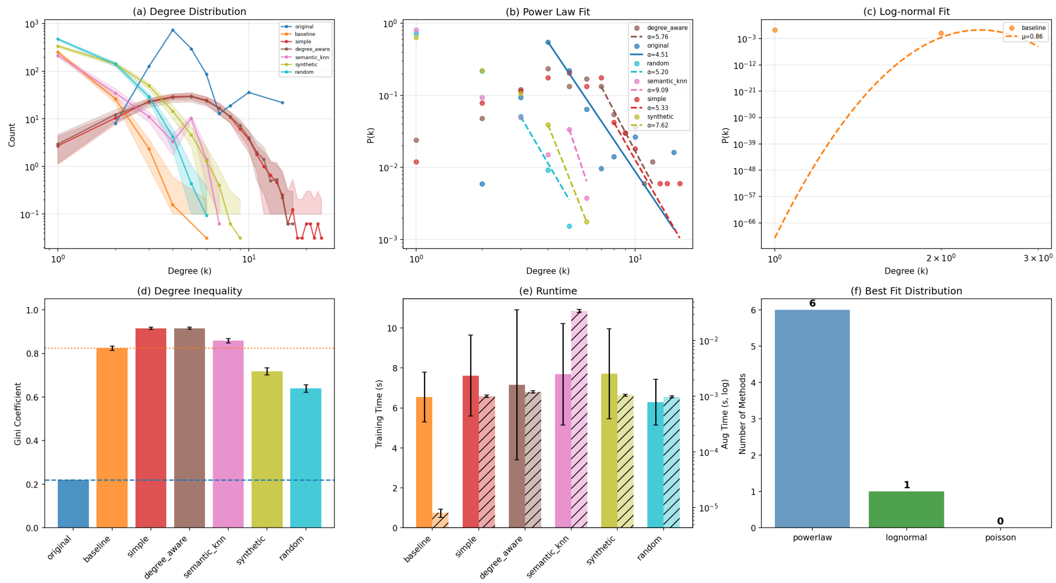

Figure 13: Amazon (product–category), GraphSAGE, q=0.05, ϕ=5: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

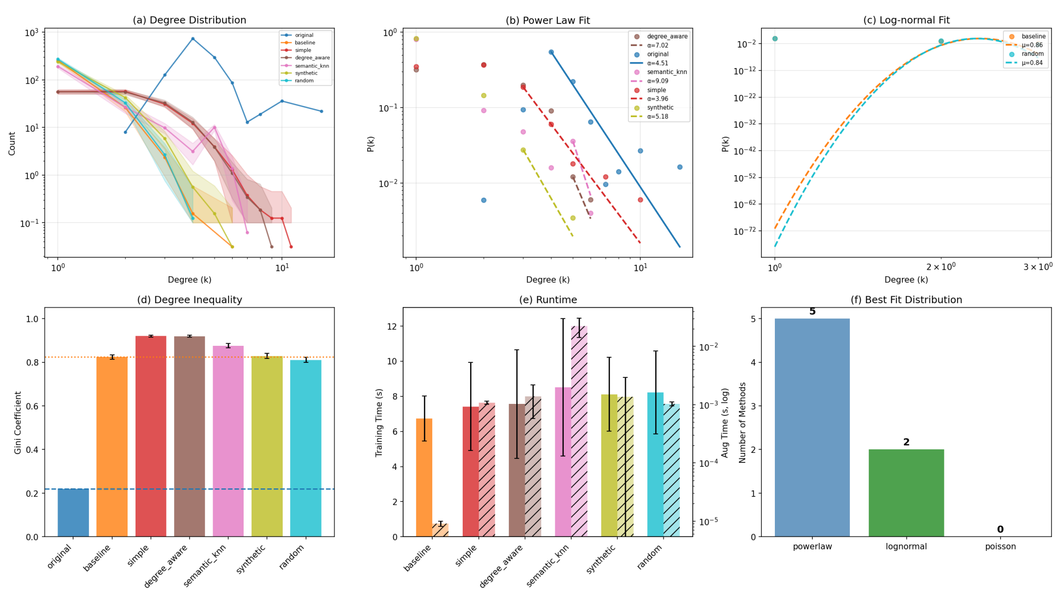

Figure 14: Amazon (product–category), GraphSAGE, q=0.05, ϕ=2: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

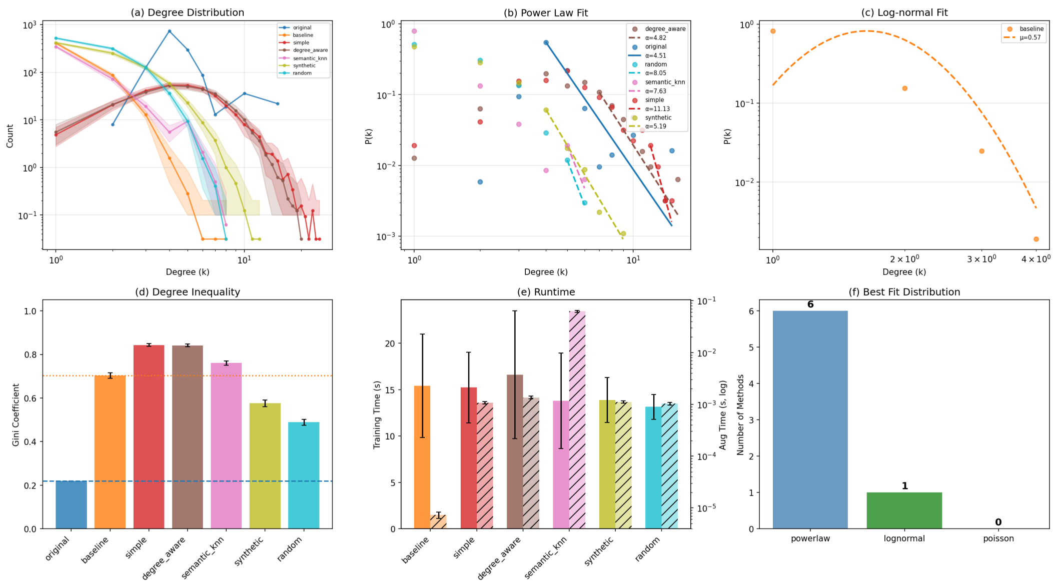

Figure 15: Amazon (product–category), GraphSAGE, q=0.10, ϕ=5: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

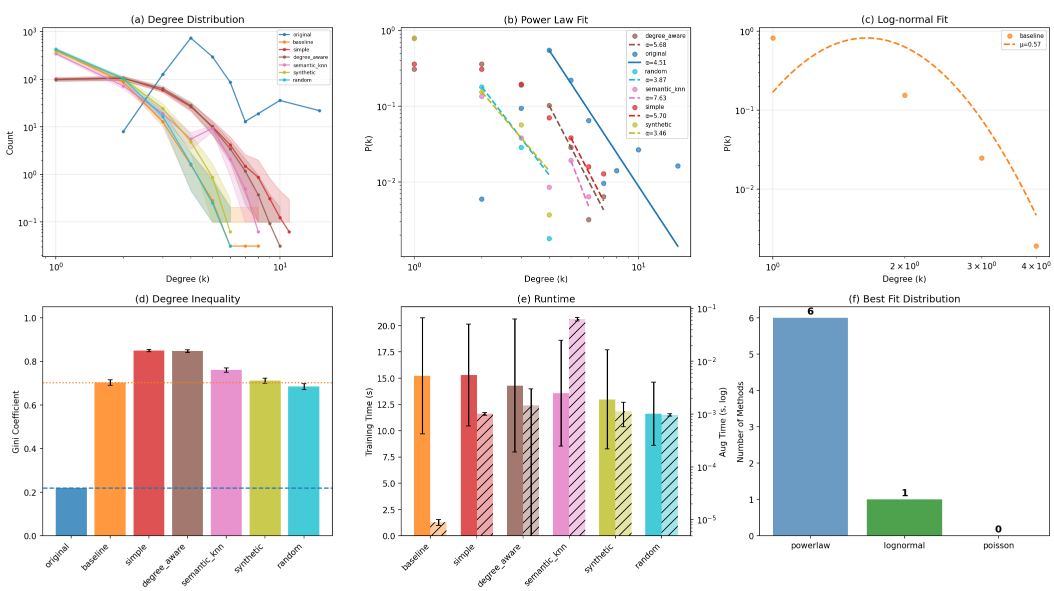

Figure 16: Amazon (product–category), GraphSAGE, q=0.10, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.3.4 GCN

Summary Analysis For Amazon GCN, `semantic_knn` provides the largest AUC lifts at q=0.05 (ϕ=2: +0.091; ϕ=5: +0.087), while `random` and `simple` often lag (e.g., `simple` −0.083 at

q=0.01, ϕ=2). Brier improves most with `degree_aware` at q=0.05, ϕ=5 (-0.031) and with `simple`/`random`/`synthetic` at q=0.10, ϕ=5 ($-0.023/-0.036/-0.032$); `semantic_knn` typically worsens Brier despite AUC gains. Degree distributions move from extreme sparsity (mean degree 0.043, Gini 0.960) to moderately denser graphs (up to ~ 0.59 at q=0.05, ϕ=5 and ~ 1.19 at q=0.10, ϕ=5), with `random`/`synthetic` sharply lowering Gini and isolated nodes. Runtime overheads remain negligible: augmentation < 0.063s and training within a few seconds of baseline, whereas the dense original costs ~ 219s, so accuracy changes are attributable to graph quality rather than compute.

AUC and Brier Score

Table 18: Amazon (product–category) GCN: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	5×	baseline	0.478 ± 0.196	+0.000	—	—	—
0.01	5×	degree_aware	0.445 ± 0.181	-0.033^{ns}	0.81	0.425	+0.14
0.01	5×	simple	0.446 ± 0.177	-0.032^{ns}	0.73	0.468	+0.13
0.01	5×	**semantic_knn**	0.481 ± 0.183	+0.003^{ns}	−0.07	0.946	−0.01
0.01	5×	synthetic	0.418 ± 0.158	-0.060^{ns}	1.35	0.188	+0.24
0.01	5×	random	0.399 ± 0.179	-0.079^{ns}	1.78	0.084	+0.32
0.01	5×	original	0.907 ± 0.015	+0.430***	−12.34	<0.001	−2.18
0.01	2×	**baseline**	0.478 ± 0.196	+0.000	—	—	—
0.01	2×	degree_aware	0.465 ± 0.199	-0.012^{ns}	0.34	0.736	+0.06
0.01	2×	simple	0.394 ± 0.174	-0.083*	2.09	0.045	+0.37
0.01	2×	semantic_knn	0.453 ± 0.184	-0.024^{ns}	0.53	0.601	+0.09
0.01	2×	synthetic	0.429 ± 0.193	-0.049^{ns}	1.22	0.230	+0.22
0.01	2×	random	0.454 ± 0.193	-0.024^{ns}	0.55	0.583	+0.10
0.01	2×	original	0.907 ± 0.015	+0.430***	−12.34	<0.001	−2.18
0.05	5×	baseline	0.500 ± 0.102	+0.000	—	—	—
0.05	5×	degree_aware	0.598 ± 0.116	+0.098***	−3.72	<0.001	−0.66
0.05	5×	**simple**	0.611 ± 0.119	+0.111***	−4.79	<0.001	−0.85
0.05	5×	semantic_knn	0.587 ± 0.102	+0.087***	−4.65	<0.001	−0.82
0.05	5×	synthetic	0.567 ± 0.100	+0.066**	−3.10	0.004	−0.55
0.05	5×	random	0.476 ± 0.087	-0.024^{ns}	1.18	0.247	+0.21
0.05	5×	original	0.907 ± 0.015	+0.407***	−21.62	<0.001	−3.82
0.05	2×	baseline	0.500 ± 0.102	+0.000	—	—	—
0.05	2×	degree_aware	0.511 ± 0.128	+0.011^{ns}	−0.49	0.629	−0.09
0.05	2×	simple	0.508 ± 0.126	+0.008^{ns}	−0.43	0.670	−0.08
0.05	2×	**semantic_knn**	0.592 ± 0.111	+0.091***	−5.15	<0.001	−0.91
0.05	2×	synthetic	0.481 ± 0.107	-0.019^{ns}	1.07	0.294	+0.19
0.05	2×	random	0.480 ± 0.100	-0.020^{ns}	1.10	0.279	+0.19
0.05	2×	original	0.907 ± 0.015	+0.407***	−21.62	<0.001	−3.82
0.10	5×	baseline	0.604 ± 0.078	+0.000	—	—	—
0.10	5×	**degree_aware**	0.648 ± 0.080	+0.044**	−3.10	0.004	−0.55
0.10	5×	simple	0.616 ± 0.083	+0.013^{ns}	−0.66	0.513	−0.12
0.10	5×	semantic_knn	0.602 ± 0.072	-0.001^{ns}	0.09	0.927	+0.02
0.10	5×	synthetic	0.640 ± 0.092	+0.036^{ns}	−1.87	0.071	−0.33
0.10	5×	random	0.617 ± 0.082	+0.014^{ns}	−0.79	0.438	−0.14
0.10	5×	original	0.907 ± 0.015	+0.304***	−21.01	<0.001	−3.71
0.10	2×	**baseline**	0.604 ± 0.078	+0.000	—	—	—
0.10	2×	degree_aware	0.554 ± 0.081	-0.050**	3.31	0.002	+0.58
0.10	2×	simple	0.594 ± 0.089	-0.009^{ns}	0.61	0.544	+0.11
0.10	2×	semantic_knn	0.602 ± 0.072	-0.001^{ns}	0.09	0.927	+0.02
0.10	2×	synthetic	0.578 ± 0.058	-0.026^{ns}	1.80	0.081	+0.32
0.10	2×	random	0.577 ± 0.073	-0.027^{ns}	1.75	0.091	+0.31

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.10	2×	original	0.907 ± 0.015	+0.304***	−21.01	<0.001	−3.71

Table 19: Amazon (product–category) GCN: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	5×	baseline	0.361 ± 0.084	+0.000	—	—	—
0.01	5×	**degree_aware**	0.340 ± 0.081	-0.022^{ns}	1.56	0.130	+0.28
0.01	5×	simple	0.353 ± 0.071	-0.008^{ns}	0.70	0.492	+0.13
0.01	5×	semantic_knn	0.371 ± 0.064	+0.010^{ns}	−0.30	0.767	−0.05
0.01	5×	synthetic	0.372 ± 0.052	+0.011^{ns}	−0.38	0.704	−0.07
0.01	5×	random	0.378 ± 0.050	+0.017^{ns}	−0.83	0.411	−0.15
0.01	5×	original	0.121 ± 0.018	-0.240***	15.58	<0.001	+2.75
0.01	2×	baseline	0.361 ± 0.084	+0.000	—	—	—
0.01	2×	**degree_aware**	0.351 ± 0.081	-0.010^{ns}	1.17	0.253	+0.21
0.01	2×	simple	0.360 ± 0.061	-0.001^{ns}	0.38	0.709	+0.07
0.01	2×	semantic_knn	0.369 ± 0.062	+0.008^{ns}	−0.22	0.826	−0.04
0.01	2×	synthetic	0.391 ± 0.045	+0.030^{ns}	−1.61	0.117	−0.29
0.01	2×	random	0.363 ± 0.061	+0.002^{ns}	0.16	0.875	+0.03
0.01	2×	original	0.121 ± 0.018	-0.240***	15.58	<0.001	+2.75
0.05	5×	baseline	0.326 ± 0.054	+0.000	—	—	—
0.05	5×	**degree_aware**	0.295 ± 0.042	-0.031**	2.85	0.008	+0.50
0.05	5×	simple	0.302 ± 0.045	-0.024*	2.19	0.036	+0.39
0.05	5×	semantic_knn	0.333 ± 0.053	+0.007^{ns}	−0.60	0.554	−0.11
0.05	5×	synthetic	0.339 ± 0.045	+0.012^{ns}	−0.97	0.342	−0.17
0.05	5×	random	0.324 ± 0.049	-0.003^{ns}	0.28	0.780	+0.05
0.05	5×	original	0.121 ± 0.018	-0.205***	18.98	<0.001	+3.36
0.05	2×	baseline	0.326 ± 0.054	+0.000	—	—	—
0.05	2×	**degree_aware**	0.311 ± 0.048	-0.015^{ns}	1.23	0.230	+0.22
0.05	2×	simple	0.321 ± 0.050	-0.005^{ns}	0.51	0.615	+0.09
0.05	2×	semantic_knn	0.336 ± 0.055	+0.009^{ns}	−0.75	0.459	−0.13
0.05	2×	synthetic	0.323 ± 0.045	-0.003^{ns}	0.32	0.755	+0.06
0.05	2×	random	0.332 ± 0.044	+0.006^{ns}	−0.54	0.593	−0.10
0.05	2×	original	0.121 ± 0.018	-0.205***	18.98	<0.001	+3.36
0.10	5×	baseline	0.291 ± 0.053	+0.000	—	—	—
0.10	5×	degree_aware	0.273 ± 0.030	-0.019^{ns}	1.68	0.103	+0.30
0.10	5×	simple	0.268 ± 0.023	-0.023*	2.42	0.022	+0.43
0.10	5×	semantic_knn	0.307 ± 0.050	+0.015^{ns}	−1.31	0.199	−0.23
0.10	5×	synthetic	0.260 ± 0.046	-0.032**	2.96	0.006	+0.52
0.10	5×	**random**	0.255 ± 0.035	-0.036***	4.28	<0.001	+0.76
0.10	5×	original	0.121 ± 0.018	-0.170***	16.88	<0.001	+2.98
0.10	2×	baseline	0.291 ± 0.053	+0.000	—	—	—
0.10	2×	degree_aware	0.286 ± 0.028	-0.006^{ns}	0.60	0.556	+0.11
0.10	2×	**simple**	0.277 ± 0.029	-0.015*	2.23	0.033	+0.39
0.10	2×	semantic_knn	0.307 ± 0.050	+0.015^{ns}	−1.31	0.199	−0.23
0.10	2×	synthetic	0.289 ± 0.045	-0.002^{ns}	0.28	0.785	+0.05
0.10	2×	random	0.292 ± 0.051	+0.001^{ns}	−0.08	0.938	−0.01
0.10	2×	original	0.121 ± 0.018	-0.170***	16.88	<0.001	+2.98

Degree Distribution Analysis

Table 20: Amazon (product–category) GCN: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	5×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	5×	degree_aware	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	simple	0.1208 ± 0.0147	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	5×	semantic_knn	0.1060 ± 0.0096	0.963 ± 0.005	1381.6 ± 10.1	powerlaw
0.01	5×	synthetic	0.1208 ± 0.0147	0.925 ± 0.010	1329.9 ± 17.3	powerlaw
0.01	5×	**random**	0.1208 ± 0.0147	0.893 ± 0.012	1298.8 ± 19.1	lognormal
0.01	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.01	2×	baseline	0.0426 ± 0.0054	0.960 ± 0.005	1404.4 ± 7.7	lognormal
0.01	2×	degree_aware	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	simple	0.0483 ± 0.0059	0.982 ± 0.002	1430.3 ± 4.4	powerlaw
0.01	2×	semantic_knn	0.0483 ± 0.0059	0.978 ± 0.003	1419.5 ± 6.0	powerlaw
0.01	2×	synthetic	0.0483 ± 0.0059	0.960 ± 0.005	1401.4 ± 8.1	lognormal
0.01	2×	**random**	0.0483 ± 0.0059	0.954 ± 0.006	1396.2 ± 8.4	lognormal
0.01	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	5×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal
0.05	5×	degree_aware	0.5930 ± 0.0365	0.916 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	simple	0.5930 ± 0.0365	0.915 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	5×	semantic_knn	0.2682 ± 0.0140	0.858 ± 0.009	1188.1 ± 16.1	powerlaw
0.05	5×	synthetic	0.5930 ± 0.0365	0.718 ± 0.016	910.9 ± 31.0	powerlaw
0.05	5×	**random**	0.5930 ± 0.0365	0.639 ± 0.018	811.7 ± 33.7	powerlaw
0.05	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.05	2×	baseline	0.2153 ± 0.0133	0.824 ± 0.011	1181.3 ± 16.7	lognormal
0.05	2×	degree_aware	0.2372 ± 0.0146	0.919 ± 0.005	1301.9 ± 9.3	powerlaw
0.05	2×	simple	0.2372 ± 0.0146	0.920 ± 0.004	1301.9 ± 9.3	powerlaw
0.05	2×	semantic_knn	0.2372 ± 0.0146	0.876 ± 0.010	1221.0 ± 17.0	powerlaw
0.05	2×	synthetic	0.2372 ± 0.0146	0.829 ± 0.011	1174.2 ± 17.8	powerlaw
0.05	2×	**random**	0.2372 ± 0.0146	0.810 ± 0.012	1156.3 ± 18.2	lognormal
0.05	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	5×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	5×	degree_aware	1.1873 ± 0.0446	0.841 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	simple	1.1873 ± 0.0446	0.843 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	5×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	5×	synthetic	1.1873 ± 0.0446	0.576 ± 0.015	572.2 ± 27.0	powerlaw
0.10	5×	**random**	1.1873 ± 0.0446	0.489 ± 0.013	450.4 ± 26.1	powerlaw
0.10	5×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw
0.10	2×	baseline	0.4323 ± 0.0163	0.703 ± 0.012	950.6 ± 19.3	lognormal
0.10	2×	degree_aware	0.4749 ± 0.0179	0.848 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	simple	0.4749 ± 0.0179	0.850 ± 0.006	1154.6 ± 11.2	powerlaw
0.10	2×	semantic_knn	0.4346 ± 0.0171	0.760 ± 0.010	1007.2 ± 17.0	powerlaw
0.10	2×	synthetic	0.4749 ± 0.0179	0.712 ± 0.012	941.3 ± 17.4	powerlaw
0.10	2×	**random**	0.4749 ± 0.0179	0.685 ± 0.014	911.9 ± 21.6	powerlaw
0.10	2×	original	4.3051 ± 0.0000	0.219 ± 0.000	114.0 ± 0.0	powerlaw

Runtime Analysis

Table 21: Amazon (product–category) GCN: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	3.72 ± 1.07
0.01	5×	degree_aware	0.0012 ± 0.0001	3.36 ± 1.13
0.01	5×	**simple**	0.0010 ± 0.0001	3.39 ± 1.17
0.01	5×	semantic_knn	0.0108 ± 0.0012	3.89 ± 1.70
0.01	5×	synthetic	0.0010 ± 0.0001	3.88 ± 1.61
0.01	5×	random	0.0010 ± 0.0001	3.73 ± 1.25
0.01	5×	original	0.0000 ± 0.0000	219.23 ± 37.81
0.01	2×	baseline	0.0000 ± 0.0000	3.72 ± 1.07
0.01	2×	degree_aware	0.0012 ± 0.0001	3.73 ± 1.78
0.01	2×	simple	0.0010 ± 0.0001	3.45 ± 1.68
0.01	2×	semantic_knn	0.0049 ± 0.0009	3.64 ± 1.51
0.01	2×	synthetic	0.0010 ± 0.0001	3.79 ± 1.47
0.01	2×	**random**	0.0009 ± 0.0001	3.33 ± 1.08
0.01	2×	original	0.0000 ± 0.0000	220.21 ± 38.20
0.05	5×	baseline	0.0000 ± 0.0000	7.02 ± 2.10
0.05	5×	degree_aware	0.0014 ± 0.0009	6.50 ± 1.43
0.05	5×	**simple**	0.0010 ± 0.0000	7.05 ± 2.29
0.05	5×	semantic_knn	0.0340 ± 0.0028	9.95 ± 3.79
0.05	5×	synthetic	0.0012 ± 0.0006	10.06 ± 3.76
0.05	5×	random	0.0010 ± 0.0000	8.04 ± 3.33
0.05	5×	original	0.0000 ± 0.0000	218.97 ± 37.68
0.05	2×	baseline	0.0000 ± 0.0000	7.06 ± 2.10
0.05	2×	degree_aware	0.0012 ± 0.0005	6.55 ± 1.70
0.05	2×	**simple**	0.0010 ± 0.0000	6.92 ± 3.42
0.05	2×	semantic_knn	0.0217 ± 0.0073	8.86 ± 3.56
0.05	2×	synthetic	0.0011 ± 0.0004	7.50 ± 2.72
0.05	2×	random	0.0010 ± 0.0000	8.40 ± 2.97
0.05	2×	original	0.0000 ± 0.0000	219.24 ± 37.79
0.10	5×	baseline	0.0000 ± 0.0000	14.04 ± 7.23
0.10	5×	degree_aware	0.0013 ± 0.0001	14.82 ± 4.01
0.10	5×	**simple**	0.0010 ± 0.0001	14.02 ± 5.16
0.10	5×	semantic_knn	0.0624 ± 0.0028	16.59 ± 7.88
0.10	5×	synthetic	0.0011 ± 0.0001	14.54 ± 3.93
0.10	5×	random	0.0010 ± 0.0001	13.90 ± 5.33
0.10	5×	original	0.0000 ± 0.0000	219.50 ± 37.87
0.10	2×	baseline	0.0000 ± 0.0000	13.91 ± 7.15
0.10	2×	degree_aware	0.0012 ± 0.0001	11.99 ± 4.25
0.10	2×	simple	0.0011 ± 0.0001	11.94 ± 5.13
0.10	2×	semantic_knn	0.0615 ± 0.0026	16.35 ± 7.63
0.10	2×	synthetic	0.0011 ± 0.0001	12.52 ± 3.77
0.10	2×	**random**	0.0010 ± 0.0001	13.85 ± 6.61
0.10	2×	original	0.0000 ± 0.0000	218.81 ± 38.02

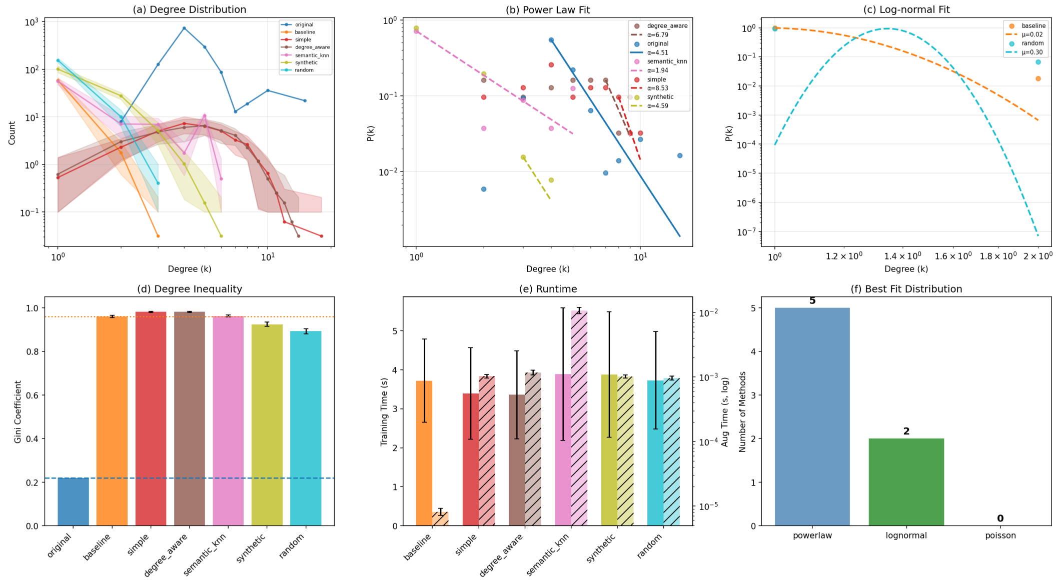

Figure 17: Amazon (product–category), GCN, q=0.01, ϕ=5: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

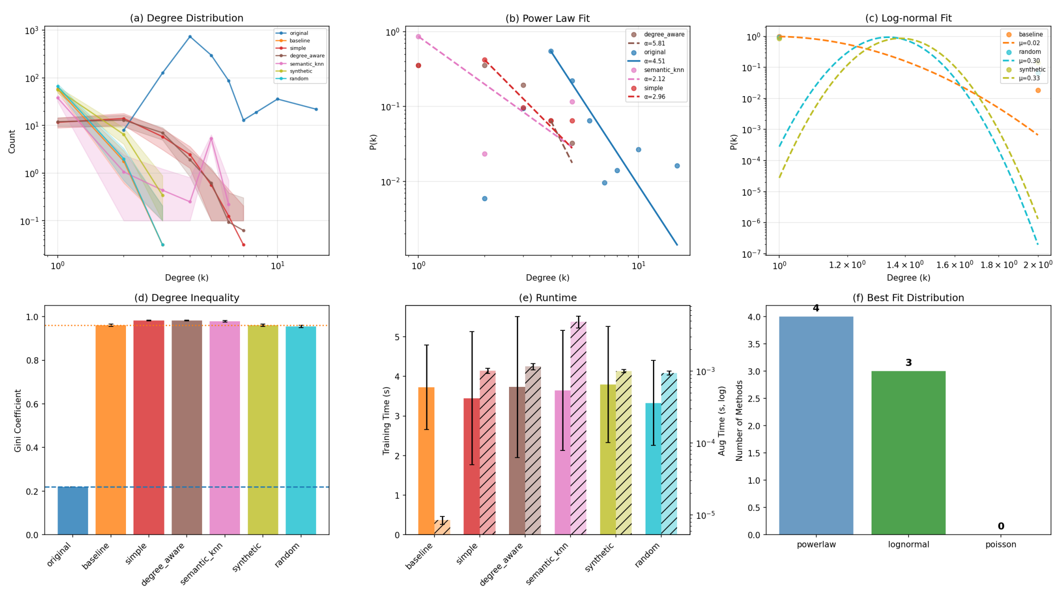

Figure 18: Amazon (product–category), GCN, q=0.01, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

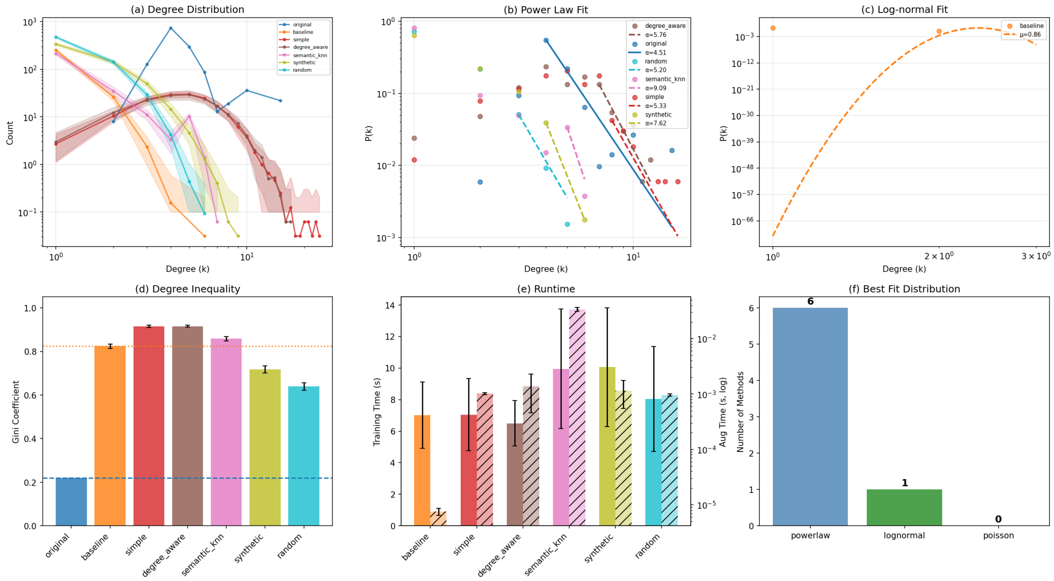

Figure 19: Amazon (product–category), GCN, q=0.05, ϕ=5: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

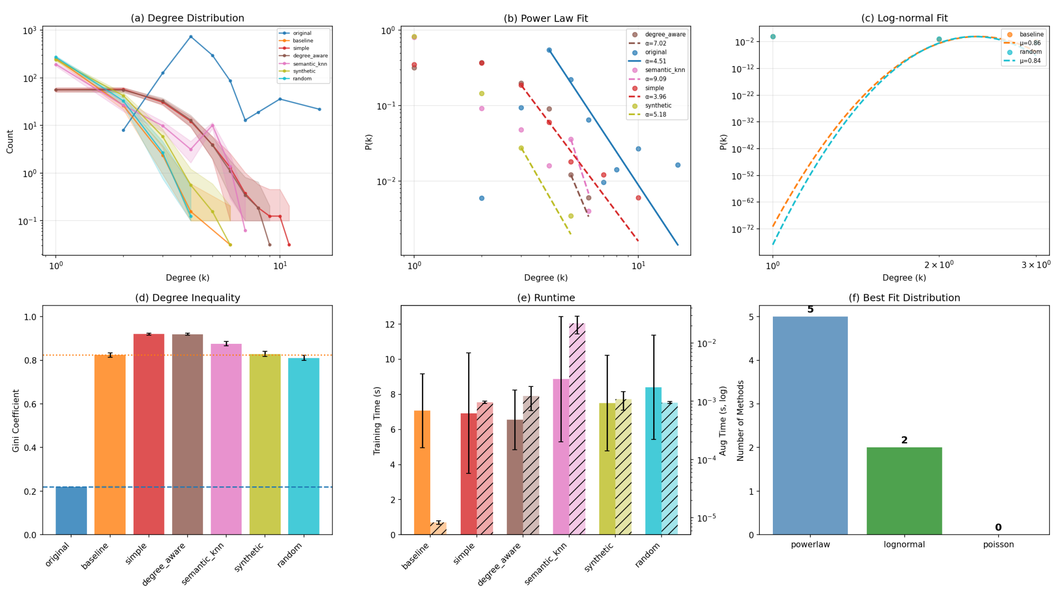

Figure 20: Amazon (product–category), GCN, q=0.05, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

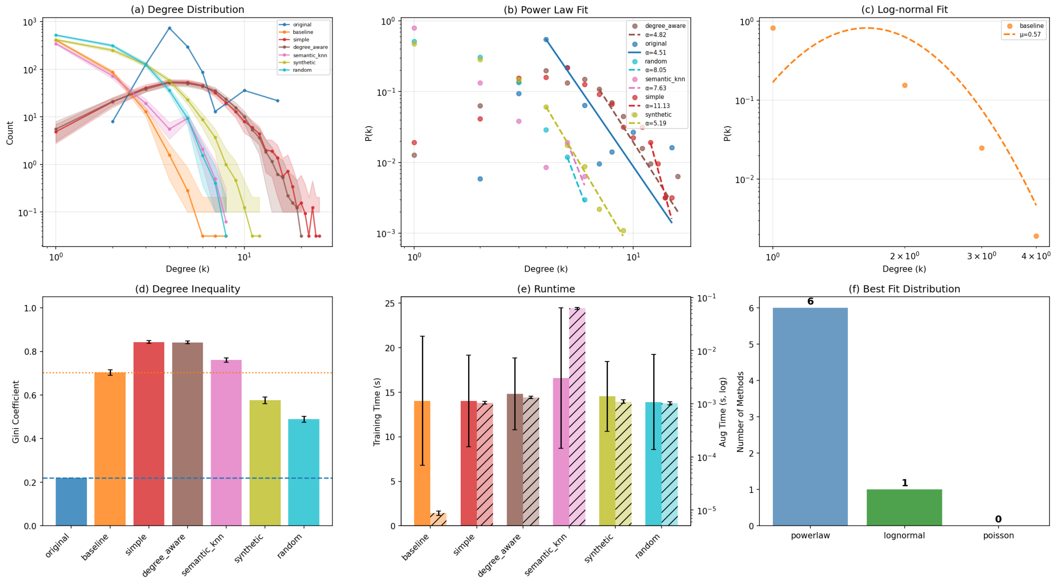

Figure 21: Amazon (product–category), GCN, q=0.10, ϕ=5: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

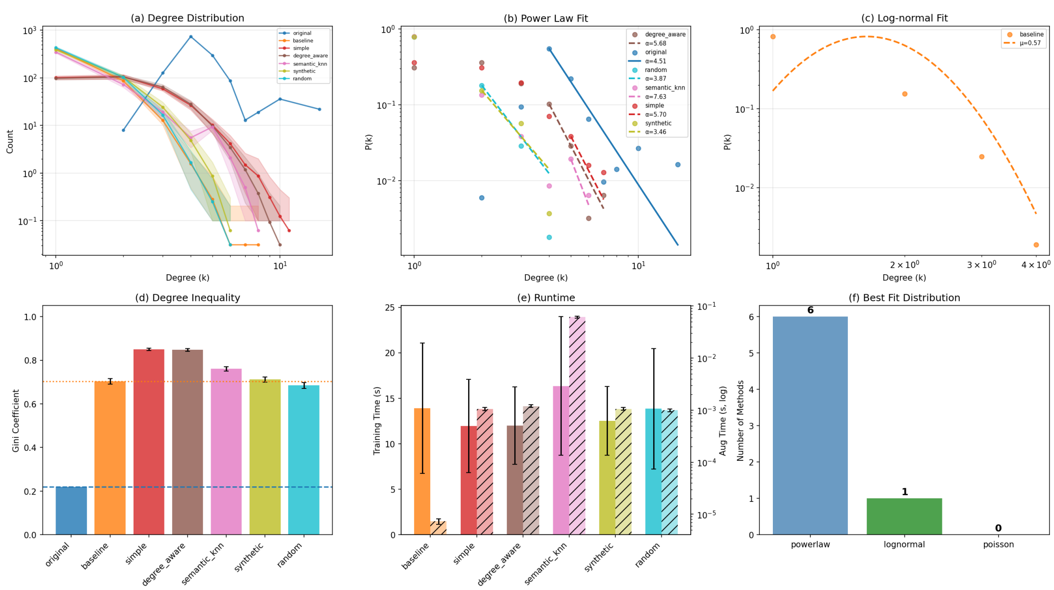

Figure 22: Amazon (product–category), GCN, q=0.10, ϕ=2: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.4 BENCHMARK DATASET - MOVIELENS

C.4.1 SUMMARY

On MovieLens, gains are modest and depend on encoder and objective: GAT sees only small AUC changes (best at q=0.01 with minor lifts from simple/degree_aware; semantic_knn often hurts), while Brier improves slightly for degree_aware/simple at low q. GraphSAGE benefits most in AUC from degree_aware at q=0.01, and Brier from synthetic at the same setting, reflecting calibration gains without large ranking shifts. GCN shows the clearest AUC gains from simple/degree_aware/semantic_knn at q=0.01 (up to +0.075), with the strongest Brier drops from synthetic (down to -0.082). Degree distributions remain very sparse for semantic_knn but rise toward mean degree ~ 1.25 for other augmentations, lowering isolated nodes; runtime overheads stay negligible (sub-0.16s augmentation, training within a couple seconds of baseline), so quality differences are not driven by cost.

C.4.2 GAT

Summary Analysis MovieLens GAT shows very small AUC shifts: best cases are minor lifts from simple/degree_aware at q=0.01 (e.g., ϕ=100: +0.007) and stability near baseline elsewhere, while semantic_knn often decreases AUC (notably q=0.05) and random stays flat or negative. Brier improvements are modest and concentrated at q=0.01, ϕ=5/2, where simple/degree_aware reduce error by up to -0.006; semantic_knn raises Brier at q=0.05, and synthetic/random frequently worsen it. Degree-wise, most augmentations keep graphs sparse (mean degree $\approx$ 0.04–0.12 at ϕ=5/2) with high Gini, while semantic_knn remains closest to the ultra-sparse baseline. Runtime overhead is negligible (aug $\leq$ 0.062s, training within $\sim \pm$2s of baseline), so differences reflect graph quality rather than cost.

AUC and Brier Score

Table 22: MovieLens (movie–genre) GAT: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline (n = 32 seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	100×	baseline	0.710 ± 0.061	+0.000	—	—	—
0.01	100×	degree_aware	0.713 ± 0.064	+0.003^{ns}	−0.42	0.681	−0.07
0.01	100×	**simple**	0.717 ± 0.063	+0.007^{ns}	−1.33	0.195	−0.23
0.01	100×	semantic_knn	0.708 ± 0.064	-0.002^{ns}	0.35	0.732	+0.06
0.01	100×	synthetic	0.679 ± 0.075	-0.031*	2.15	0.039	+0.38
0.01	100×	random	0.652 ± 0.089	-0.059***	3.66	<0.001	+0.65
0.01	100×	original	0.811 ± 0.015	+0.101***	−9.95	<0.001	−1.76
0.01	5×	baseline	0.712 ± 0.052	+0.000	—	—	—
0.01	5×	**degree_aware**	0.718 ± 0.060	+0.006^{ns}	−1.23	0.230	−0.22
0.01	5×	simple	0.714 ± 0.050	+0.002^{ns}	−0.47	0.640	−0.08
0.01	5×	semantic_knn	0.701 ± 0.063	-0.011^{ns}	1.63	0.112	+0.29
0.01	5×	synthetic	0.712 ± 0.057	-0.000^{ns}	0.06	0.955	+0.01
0.01	5×	random	0.705 ± 0.066	-0.007^{ns}	1.04	0.308	+0.18
0.01	5×	original	0.812 ± 0.014	+0.100***	−11.34	<0.001	−2.01
0.01	2×	**baseline**	0.712 ± 0.052	+0.000	—	—	—
0.01	2×	degree_aware	0.708 ± 0.058	-0.004^{ns}	0.99	0.328	+0.18
0.01	2×	simple	0.711 ± 0.059	-0.001^{ns}	0.16	0.877	+0.03
0.01	2×	semantic_knn	0.701 ± 0.063	-0.011^{ns}	1.63	0.112	+0.29
0.01	2×	synthetic	0.710 ± 0.077	-0.002^{ns}	0.21	0.834	+0.04
0.01	2×	random	0.710 ± 0.057	-0.002^{ns}	0.29	0.773	+0.05
0.01	2×	original	0.812 ± 0.014	+0.100***	−11.34	<0.001	−2.01
0.05	5×	baseline	0.719 ± 0.034	+0.000	—	—	—
0.05	5×	degree_aware	0.717 ± 0.035	-0.002^{ns}	0.45	0.655	+0.08
0.05	5×	simple	0.718 ± 0.033	-0.001^{ns}	0.29	0.777	+0.05

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.05	5×	semantic_knn	0.701 ± 0.034	-0.018***	3.97	<0.001	+0.70
0.05	5×	**synthetic**	0.723 ± 0.037	+0.003^{ns}	−0.53	0.600	−0.09
0.05	5×	random	0.710 ± 0.042	-0.009^{ns}	1.54	0.133	+0.27
0.05	5×	original	0.812 ± 0.014	+0.092***	−14.43	<0.001	−2.55
0.05	2×	**baseline**	0.719 ± 0.034	+0.000	—	—	—
0.05	2×	degree_aware	0.716 ± 0.028	-0.004^{ns}	0.78	0.440	+0.14
0.05	2×	simple	0.716 ± 0.030	-0.004^{ns}	0.80	0.432	+0.14
0.05	2×	semantic_knn	0.701 ± 0.034	-0.018***	3.97	<0.001	+0.70
0.05	2×	synthetic	0.706 ± 0.039	-0.013*	2.31	0.027	+0.41
0.05	2×	random	0.705 ± 0.045	-0.015^{ns}	1.87	0.071	+0.33
0.05	2×	original	0.812 ± 0.014	+0.092***	−14.43	<0.001	−2.55
0.10	5×	baseline	0.713 ± 0.029	+0.000	—	—	—
0.10	5×	degree_aware	0.714 ± 0.027	+0.002^{ns}	−0.75	0.456	−0.13
0.10	5×	**simple**	0.715 ± 0.025	+0.002^{ns}	−0.75	0.461	−0.13
0.10	5×	semantic_knn	0.706 ± 0.028	-0.006*	2.44	0.021	+0.43
0.10	5×	synthetic	0.709 ± 0.026	-0.003^{ns}	0.79	0.434	+0.14
0.10	5×	random	0.709 ± 0.025	-0.004^{ns}	0.84	0.407	+0.15
0.10	5×	original	0.812 ± 0.014	+0.099***	−17.98	<0.001	−3.18

Table 23: MovieLens (movie–genre) GAT: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	100×	**baseline**	0.231 ± 0.016	+0.000	—	—	—
0.01	100×	degree_aware	0.233 ± 0.014	+0.001^{ns}	−0.72	0.474	−0.13
0.01	100×	simple	0.231 ± 0.012	-0.000^{ns}	0.12	0.907	+0.02
0.01	100×	semantic_knn	0.235 ± 0.014	+0.004^{ns}	−1.43	0.162	−0.25
0.01	100×	synthetic	0.245 ± 0.008	+0.014***	−4.82	<0.001	−0.85
0.01	100×	random	0.245 ± 0.009	+0.013***	−3.89	<0.001	−0.69
0.01	100×	original	0.218 ± 0.004	-0.013***	5.22	<0.001	+0.92
0.01	5×	baseline	0.235 ± 0.015	+0.000	—	—	—
0.01	5×	degree_aware	0.230 ± 0.017	-0.005**	3.01	0.005	+0.53
0.01	5×	**simple**	0.229 ± 0.014	-0.006**	3.34	0.002	+0.59
0.01	5×	semantic_knn	0.236 ± 0.015	+0.001^{ns}	−0.49	0.627	−0.09
0.01	5×	synthetic	0.238 ± 0.015	+0.003^{ns}	−1.35	0.188	−0.24
0.01	5×	random	0.236 ± 0.013	+0.001^{ns}	−0.64	0.526	−0.11
0.01	5×	original	0.218 ± 0.004	-0.017***	6.33	<0.001	+1.12
0.01	2×	baseline	0.235 ± 0.015	+0.000	—	—	—
0.01	2×	**degree_aware**	0.231 ± 0.016	-0.004^{ns}	1.94	0.061	+0.34
0.01	2×	simple	0.232 ± 0.015	-0.003^{ns}	1.87	0.071	+0.33
0.01	2×	semantic_knn	0.236 ± 0.015	+0.001^{ns}	−0.49	0.627	−0.09
0.01	2×	synthetic	0.235 ± 0.013	+0.000^{ns}	−0.11	0.917	−0.02
0.01	2×	random	0.238 ± 0.010	+0.003^{ns}	−1.41	0.168	−0.25
0.01	2×	original	0.218 ± 0.004	-0.017***	6.33	<0.001	+1.12
0.05	5×	baseline	0.240 ± 0.008	+0.000	—	—	—
0.05	5×	degree_aware	0.239 ± 0.006	-0.001^{ns}	0.90	0.378	+0.16
0.05	5×	simple	0.240 ± 0.009	+0.000^{ns}	−0.46	0.650	−0.08
0.05	5×	semantic_knn	0.245 ± 0.007	+0.005***	−4.83	<0.001	−0.85
0.05	5×	**synthetic**	0.237 ± 0.011	-0.003^{ns}	1.76	0.088	+0.31
0.05	5×	random	0.239 ± 0.008	-0.001^{ns}	0.55	0.589	+0.10
0.05	5×	original	0.218 ± 0.004	-0.022***	14.99	<0.001	+2.65
0.05	2×	**baseline**	0.240 ± 0.008	+0.000	—	—	—

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.05	2×	degree_aware	0.241 ± 0.007	+0.001^{ns}	−1.24	0.224	−0.22
0.05	2×	simple	0.242 ± 0.007	+0.002^{ns}	−1.95	0.060	−0.34
0.05	2×	semantic_knn	0.245 ± 0.007	+0.005***	−4.83	<0.001	−0.85
0.05	2×	synthetic	0.244 ± 0.008	+0.004*	−2.55	0.016	−0.45
0.05	2×	random	0.244 ± 0.009	+0.004**	−3.10	0.004	−0.55
0.05	2×	original	0.218 ± 0.004	-0.022***	14.99	<0.001	+2.65
0.10	5×	baseline	0.238 ± 0.007	+0.000	—	—	—
0.10	5×	degree_aware	0.238 ± 0.005	-0.000^{ns}	0.51	0.613	+0.09
0.10	5×	simple	0.238 ± 0.006	+0.000^{ns}	−0.20	0.842	−0.04
0.10	5×	semantic_knn	0.240 ± 0.006	+0.002*	−2.36	0.025	−0.42
0.10	5×	**synthetic**	0.237 ± 0.007	-0.001^{ns}	0.41	0.682	+0.07
0.10	5×	random	0.238 ± 0.009	+0.000^{ns}	−0.17	0.870	−0.03
0.10	5×	original	0.218 ± 0.004	-0.020***	12.93	<0.001	+2.29

Degree Distribution Analysis

Table 24: MovieLens (movie–genre) GAT: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	100×	baseline	0.0176 ± 0.0052	0.983 ± 0.005	9538.7 ± 49.7	lognormal
0.01	100×	degree_aware	1.2506 ± 0.0794	0.989 ± 0.001	9587.2 ± 7.8	lognormal
0.01	100×	simple	1.2506 ± 0.0794	0.988 ± 0.001	9587.2 ± 7.8	lognormal
0.01	100×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	100×	synthetic	1.2506 ± 0.0794	0.898 ± 0.007	8452.6 ± 80.0	powerlaw
0.01	100×	**random**	1.2506 ± 0.0794	0.476 ± 0.013	2784.9 ± 217.0	lognormal
0.01	100×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.01	5×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	5×	degree_aware	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	5×	simple	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	5×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	5×	synthetic	0.0625 ± 0.0040	0.959 ± 0.003	9227.0 ± 31.4	powerlaw
0.01	5×	**random**	0.0625 ± 0.0040	0.941 ± 0.004	9120.0 ± 36.3	lognormal
0.01	5×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.01	2×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	2×	degree_aware	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	simple	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	2×	synthetic	0.0250 ± 0.0016	0.978 ± 0.002	9482.6 ± 14.9	lognormal
0.01	2×	**random**	0.0250 ± 0.0016	0.976 ± 0.002	9468.2 ± 15.8	lognormal
0.01	2×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.05	5×	baseline	0.1144 ± 0.0038	0.895 ± 0.004	8647.3 ± 35.3	lognormal
0.05	5×	degree_aware	0.3139 ± 0.0105	0.953 ± 0.002	9114.0 ± 20.2	powerlaw
0.05	5×	simple	0.3139 ± 0.0105	0.953 ± 0.002	9114.0 ± 20.2	lognormal
0.05	5×	semantic_knn	0.0704 ± 0.0022	0.938 ± 0.002	9065.5 ± 21.8	powerlaw
0.05	5×	synthetic	0.3139 ± 0.0105	0.824 ± 0.005	7528.5 ± 66.1	powerlaw
0.05	5×	**random**	0.3139 ± 0.0105	0.763 ± 0.007	7090.3 ± 80.0	lognormal
0.05	5×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.05	2×	baseline	0.1144 ± 0.0038	0.895 ± 0.004	8647.3 ± 35.3	lognormal
0.05	2×	degree_aware	0.1256 ± 0.0042	0.955 ± 0.002	9114.0 ± 20.2	powerlaw
0.05	2×	simple	0.1256 ± 0.0042	0.955 ± 0.002	9114.0 ± 20.2	powerlaw
0.05	2×	semantic_knn	0.0704 ± 0.0022	0.938 ± 0.002	9065.5 ± 21.8	powerlaw

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.05	2×	synthetic	0.1256 ± 0.0042	0.900 ± 0.004	8629.1 ± 36.0	lognormal
0.05	2×	**random**	0.1256 ± 0.0042	0.888 ± 0.004	8559.1 ± 36.5	lognormal
0.05	2×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.10	5×	baseline	0.2285 ± 0.0050	0.808 ± 0.004	7684.8 ± 39.8	lognormal
0.10	5×	degree_aware	0.6263 ± 0.0136	0.908 ± 0.002	8550.8 ± 25.1	lognormal
0.10	5×	simple	0.6263 ± 0.0136	0.909 ± 0.002	8550.8 ± 25.1	lognormal
0.10	5×	semantic_knn	0.1331 ± 0.0027	0.884 ± 0.003	8503.1 ± 24.7	lognormal
0.10	5×	synthetic	0.6263 ± 0.0136	0.703 ± 0.006	5837.7 ± 72.7	lognormal
0.10	5×	**random**	0.6263 ± 0.0136	0.625 ± 0.006	5181.8 ± 75.1	lognormal
0.10	5×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal

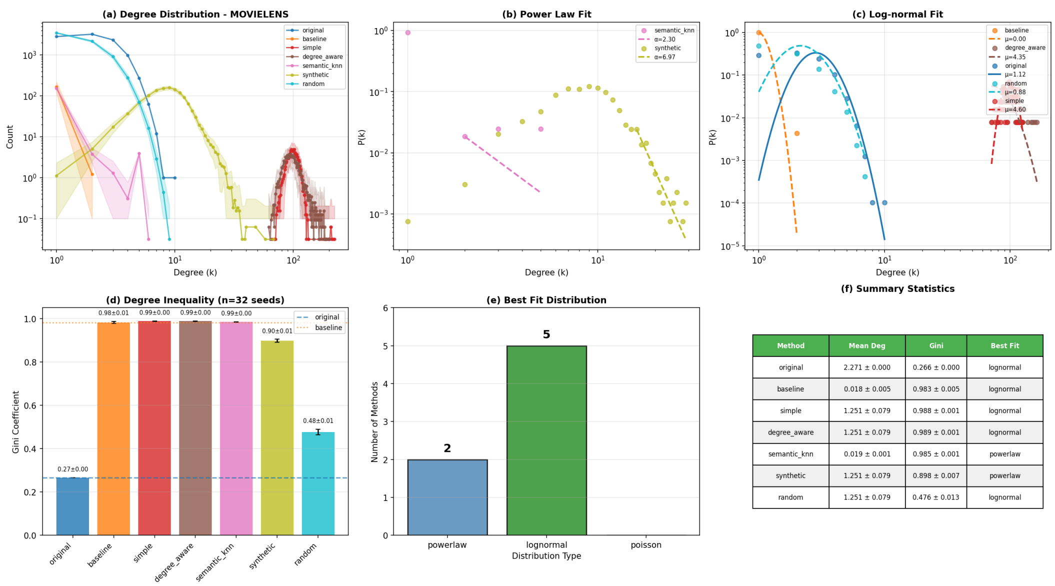

Figure 23: MovieLens (movie–genre), GAT, q=0.01, ϕ=100: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

Runtime Analysis

Table 25: MovieLens (movie–genre) GAT: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	10.75 ± 3.96
0.01	5×	degree_aware	0.0025 ± 0.0002	10.79 ± 4.00
0.01	5×	simple	0.0023 ± 0.0002	10.38 ± 3.83
0.01	5×	semantic_knn	0.1278 ± 0.0128	9.82 ± 3.43
0.01	5×	synthetic	0.0024 ± 0.0005	11.32 ± 4.33
0.01	5×	**random**	0.0022 ± 0.0002	12.13 ± 5.32
0.01	5×	original	0.0000 ± 0.0000	1139.45 ± 329.59
0.01	2×	baseline	0.0000 ± 0.0000	10.55 ± 3.97
0.01	2×	degree_aware	0.0024 ± 0.0003	10.17 ± 3.84

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	2×	**simple**	0.0022 ± 0.0002	10.48 ± 3.56
0.01	2×	semantic_knn	0.1284 ± 0.0104	9.64 ± 3.38
0.01	2×	synthetic	0.0023 ± 0.0002	10.89 ± 5.18
0.01	2×	random	0.0022 ± 0.0001	11.70 ± 5.32
0.01	2×	original	0.0000 ± 0.0000	1126.47 ± 323.34
0.05	5×	baseline	0.0000 ± 0.0000	37.68 ± 13.98
0.05	5×	degree_aware	0.0029 ± 0.0008	41.96 ± 14.37
0.05	5×	**simple**	0.0023 ± 0.0003	44.30 ± 15.61
0.05	5×	semantic_knn	0.2163 ± 0.0124	31.06 ± 11.65
0.05	5×	synthetic	0.0025 ± 0.0004	42.68 ± 13.99
0.05	5×	random	0.0023 ± 0.0002	42.32 ± 12.28
0.05	5×	original	0.0000 ± 0.0000	1109.65 ± 315.29
0.05	2×	baseline	0.0000 ± 0.0000	38.14 ± 14.05
0.05	2×	degree_aware	0.0025 ± 0.0002	34.76 ± 9.98
0.05	2×	**simple**	0.0022 ± 0.0002	37.27 ± 11.95
0.05	2×	semantic_knn	0.2098 ± 0.0120	31.42 ± 11.68
0.05	2×	synthetic	0.0025 ± 0.0003	38.47 ± 11.55
0.05	2×	random	0.0023 ± 0.0004	37.69 ± 14.78
0.05	2×	original	0.0000 ± 0.0000	1120.95 ± 318.64
0.10	5×	baseline	0.0000 ± 0.0000	68.52 ± 28.41
0.10	5×	degree_aware	0.0031 ± 0.0010	89.01 ± 36.01
0.10	5×	**simple**	0.0022 ± 0.0003	79.85 ± 26.68
0.10	5×	semantic_knn	0.3095 ± 0.0100	71.80 ± 25.47
0.10	5×	synthetic	0.0025 ± 0.0004	83.07 ± 31.44
0.10	5×	random	0.0023 ± 0.0002	82.97 ± 22.81
0.10	5×	original	0.0000 ± 0.0000	1123.86 ± 319.62

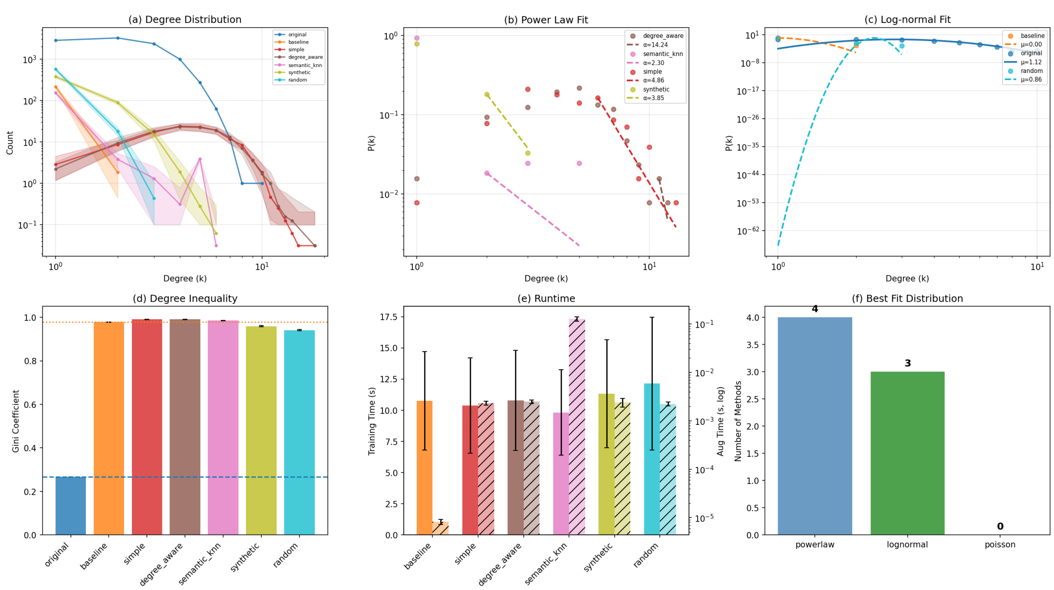

Figure 24: MovieLens (movie–genre), GAT, q=0.01, ϕ=5: Comprehensive analysis ($M \pm$SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

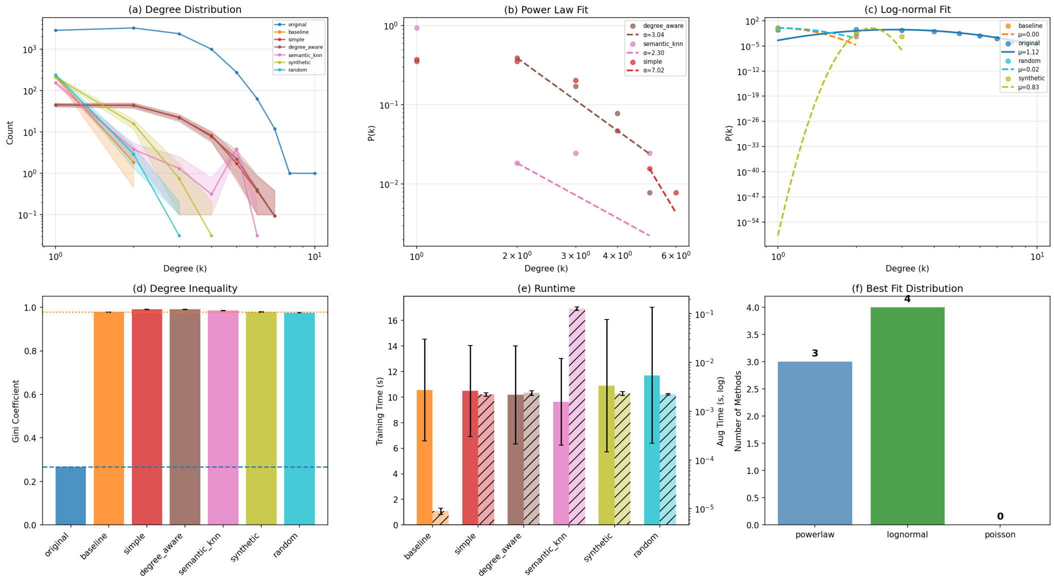

Figure 25: MovieLens (movie–genre), GAT, q=0.01, ϕ=2: Comprehensive analysis ($M \pm \mathrm{SD}$, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panels (a)–(f) follow the same structure as Fig. 24.

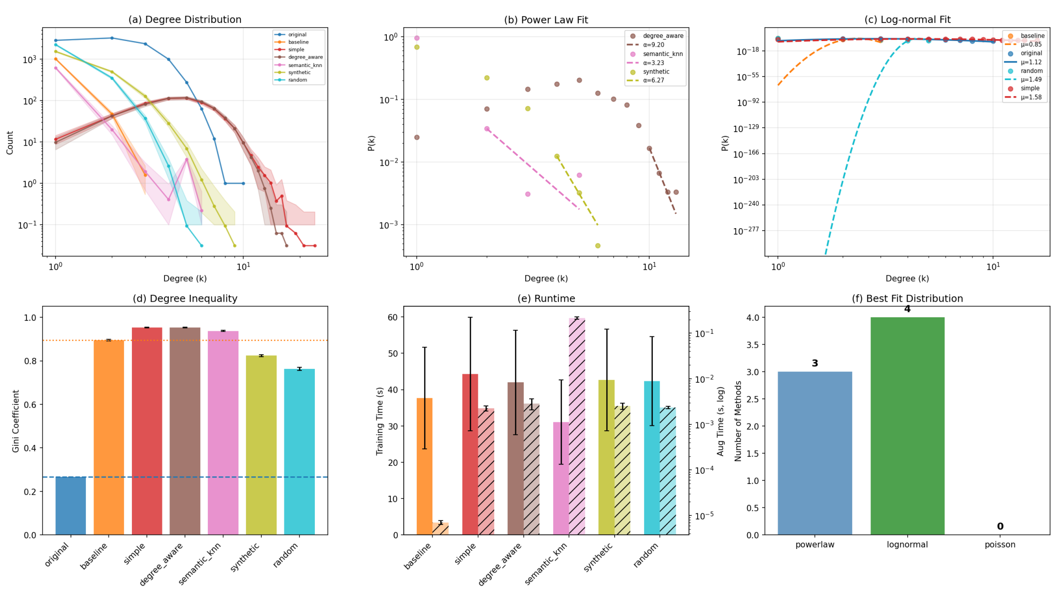

Figure 26: MovieLens (movie–genre), GAT, q=0.05, ϕ=5: Comprehensive analysis ($M \pm \mathrm{SD}$, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

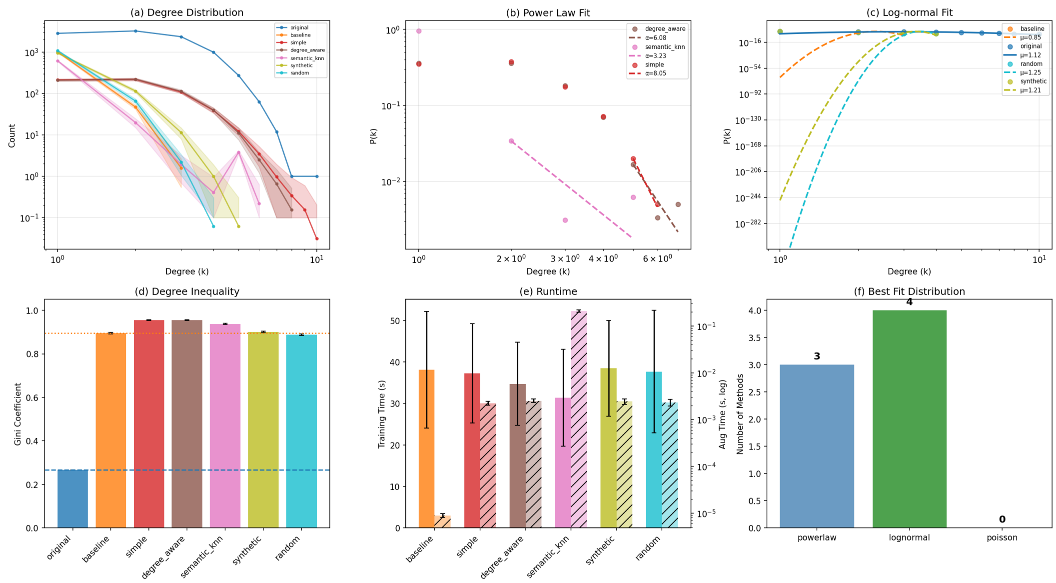

Figure 27: MovieLens (movie–genre), GAT, q=0.05, ϕ=2: Comprehensive analysis ($M \pm \text{SD}$, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panels (a)–(f) follow the same structure as Fig. 26.

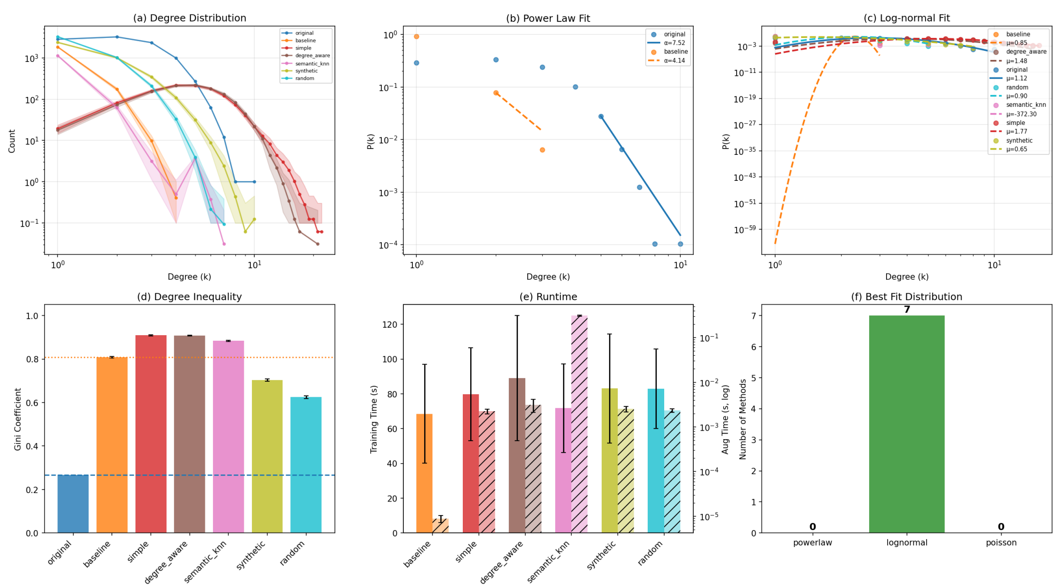

Figure 28: MovieLens (movie–genre), GAT, q=0.10, ϕ=5: Comprehensive analysis ($M \pm \text{SD}$, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panels (a)–(f) follow the standard analysis layout.

C.4.3 GRAPHSAGE

Summary Analysis MovieLens GraphSAGE sees its main AUC gains from `degree_aware` at q=0.01 (up to +0.024), with other methods near baseline and `synthetic` negative. Brier is led by `synthetic` at q=0.01, ϕ=5 (−0.014), while `semantic_knn` shows mixed or negligible effects and `random` offers little benefit. Degree statistics mirror the ultra-sparse regime: augmentations at ϕ=5/2 raise mean degree to $\sim$ 0.12 with high Gini; `semantic_knn` keeps graphs closest to baseline, and `random`/`synthetic` reduce isolated counts. Runtime overhead is minimal (aug

$\leq$ 0.012s, training within $\sim \pm$1s of baseline vs. dense-original cost), so performance differences stem from graph quality rather than compute.

AUC and Brier Score

Table 26: MovieLens (movie–genre) GraphSAGE: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	5×	baseline	0.637 ± 0.072	+0.000	—	—	—
0.01	5×	**degree_aware**	0.653 ± 0.063	+0.015^{ns}	−1.33	0.192	−0.24
0.01	5×	simple	0.642 ± 0.064	+0.005^{ns}	−0.36	0.724	−0.06
0.01	5×	semantic_knn	0.652 ± 0.054	+0.015^{ns}	−1.55	0.132	−0.27
0.01	5×	synthetic	0.608 ± 0.062	-0.030*	2.41	0.022	+0.43
0.01	5×	random	0.620 ± 0.074	-0.017^{ns}	1.08	0.287	+0.19
0.01	5×	original	0.677 ± 0.016	+0.040**	−3.13	0.004	−0.55
0.01	2×	baseline	0.637 ± 0.072	+0.000	—	—	—
0.01	2×	**degree_aware**	0.661 ± 0.056	+0.024^{ns}	−1.63	0.112	−0.29
0.01	2×	simple	0.642 ± 0.070	+0.005^{ns}	−0.34	0.738	−0.06
0.01	2×	semantic_knn	0.652 ± 0.054	+0.015^{ns}	−1.55	0.132	−0.27
0.01	2×	synthetic	0.613 ± 0.066	-0.025^{ns}	1.59	0.121	+0.28
0.01	2×	random	0.625 ± 0.078	-0.013^{ns}	0.87	0.390	+0.15
0.01	2×	original	0.677 ± 0.016	+0.040**	−3.13	0.004	−0.55

Table 27: MovieLens (movie–genre) GraphSAGE: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	5×	baseline	0.387 ± 0.037	+0.000	—	—	—
0.01	5×	degree_aware	0.374 ± 0.044	-0.013^{ns}	1.90	0.066	+0.34
0.01	5×	simple	0.387 ± 0.037	+0.001^{ns}	−0.08	0.937	−0.01
0.01	5×	semantic_knn	0.389 ± 0.037	+0.003^{ns}	−0.61	0.548	−0.11
0.01	5×	**synthetic**	0.373 ± 0.047	-0.014^{ns}	1.90	0.066	+0.34
0.01	5×	random	0.383 ± 0.035	-0.004^{ns}	0.51	0.612	+0.09
0.01	5×	original	0.273 ± 0.026	-0.114***	13.32	<0.001	+2.36
0.01	2×	baseline	0.387 ± 0.037	+0.000	—	—	—
0.01	2×	degree_aware	0.391 ± 0.033	+0.005^{ns}	−1.12	0.272	−0.20
0.01	2×	simple	0.388 ± 0.034	+0.001^{ns}	−0.13	0.899	−0.02
0.01	2×	semantic_knn	0.389 ± 0.037	+0.003^{ns}	−0.61	0.548	−0.11
0.01	2×	synthetic	0.377 ± 0.043	-0.010^{ns}	1.06	0.299	+0.19
0.01	2×	**random**	0.368 ± 0.044	-0.019*	2.17	0.038	+0.38
0.01	2×	original	0.273 ± 0.026	-0.114***	13.32	<0.001	+2.36

Degree Distribution Analysis

Table 28: MovieLens (movie–genre) GraphSAGE: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	5×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	5×	degree_aware	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	5×	simple	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	5×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	5×	synthetic	0.0625 ± 0.0040	0.959 ± 0.003	9227.0 ± 31.4	powerlaw
0.01	5×	**random**	0.0625 ± 0.0040	0.941 ± 0.004	9120.0 ± 36.3	lognormal
0.01	5×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.01	2×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	2×	degree_aware	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	simple	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	2×	synthetic	0.0250 ± 0.0016	0.978 ± 0.002	9482.6 ± 14.9	lognormal
0.01	2×	**random**	0.0250 ± 0.0016	0.976 ± 0.002	9468.2 ± 15.8	lognormal
0.01	2×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal

Runtime Analysis

Table 29: MovieLens (movie–genre) GraphSAGE: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	7.20 ± 3.13
0.01	5×	degree_aware	0.0027 ± 0.0010	7.81 ± 4.17
0.01	5×	**simple**	0.0022 ± 0.0004	7.59 ± 3.63
0.01	5×	semantic_knn	0.1358 ± 0.0137	8.38 ± 3.66
0.01	5×	synthetic	0.0028 ± 0.0019	6.96 ± 3.09
0.01	5×	random	0.0023 ± 0.0005	8.23 ± 4.69
0.01	5×	original	0.0000 ± 0.0000	355.69 ± 27.09
0.01	2×	baseline	0.0000 ± 0.0000	7.19 ± 3.15
0.01	2×	degree_aware	0.0024 ± 0.0005	8.23 ± 3.96
0.01	2×	**simple**	0.0021 ± 0.0003	7.74 ± 3.43
0.01	2×	semantic_knn	0.1349 ± 0.0100	8.38 ± 3.66
0.01	2×	synthetic	0.0023 ± 0.0003	7.41 ± 2.77
0.01	2×	random	0.0022 ± 0.0005	7.12 ± 3.06
0.01	2×	original	0.0000 ± 0.0000	355.45 ± 26.42

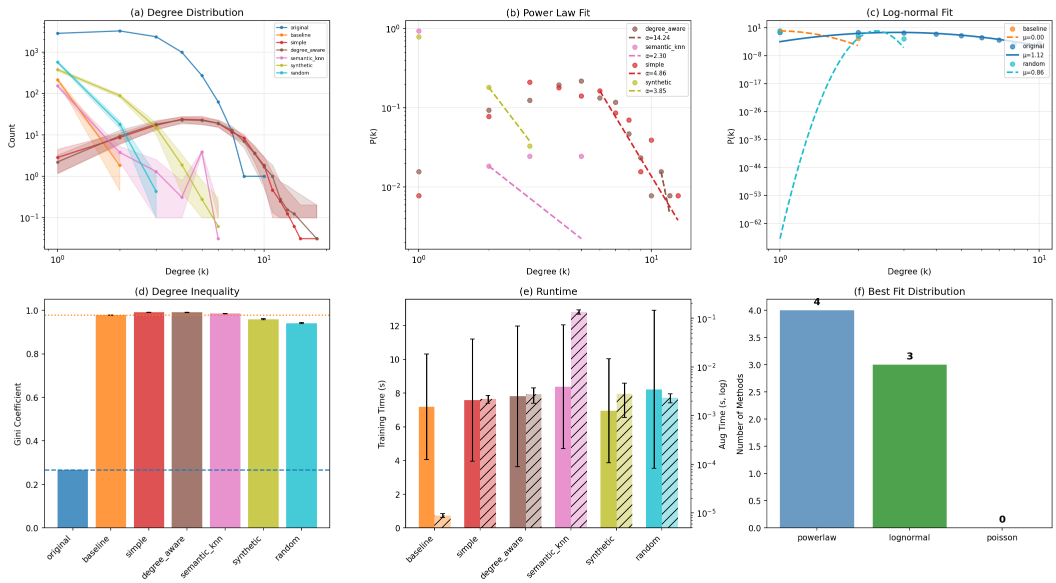

Figure 29: MovieLens (movie–genre), GraphSAGE, q=0.01, ϕ=5: Comprehensive analysis ($M \pm$SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

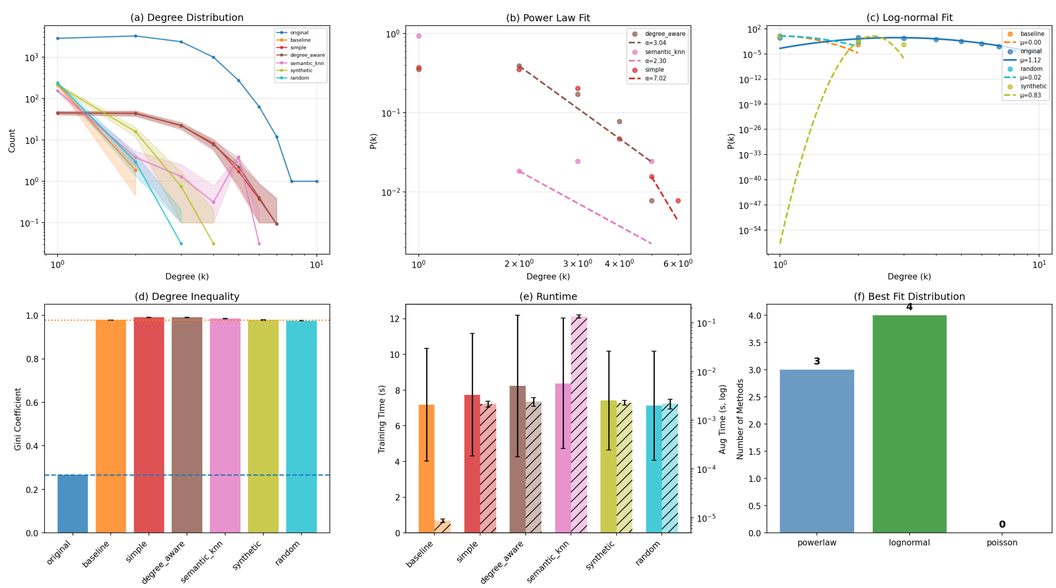

Figure 30: MovieLens (movie–genre), GraphSAGE, q=0.01, ϕ=2: Comprehensive analysis ($M \pm$SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.4.4 GCN

Summary Analysis MovieLens GCN shows the clearest AUC gains among the three augmentation methods: `simple` (and `degree_aware`/`synthetic`) improve ranking at

q=0.01 (ϕ=5: +0.075; ϕ=2: +0.027), with `semantic_knn` also positive but smaller; `random` trails. Brier improves most with `synthetic` at q=0.01, ϕ=5 (-0.082) and with `degree_aware`/`simple`/`random`/`synthetic` at q=0.10, ϕ=5 (up to -0.036), while `semantic_knn` often raises Brier. Degree-wise, augmentations lift mean degree from the ultra-sparse baseline (≈ 0.018) to ~ 0.12 at q=0.01 and up to ~ 1.19 at q=0.10, reducing isolated nodes and Gini most for `random`/`synthetic`; `semantic_knn` keeps graphs sparsest. Runtime overhead stays tiny (aug $\leq$ 0.062s; training within a few seconds of baseline vs. $\sim$ 219s for the dense original), so effectiveness hinges on graph quality rather than cost.

AUC and Brier Score

Table 30: MovieLens (movie–genre) GCN: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds). A higher AUC is better.

q	ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
0.01	5×	baseline	0.412 ± 0.088	+0.000	—	—	—
0.01	5×	degree_aware	0.473 ± 0.089	+0.062**	−3.46	0.002	−0.61
0.01	5×	**simple**	0.487 ± 0.099	+0.075**	−3.40	0.002	−0.60
0.01	5×	semantic_knn	0.455 ± 0.093	+0.044*	−2.22	0.034	−0.39
0.01	5×	synthetic	0.479 ± 0.116	+0.067**	−2.88	0.007	−0.51
0.01	5×	random	0.447 ± 0.102	+0.036$^{\text{ns}}$	−1.62	0.116	−0.29
0.01	5×	original	0.644 ± 0.017	+0.232***	−14.58	<0.001	−2.58
0.01	2×	baseline	0.412 ± 0.088	+0.000	—	—	—
0.01	2×	degree_aware	0.434 ± 0.081	+0.022$^{\text{ns}}$	−1.20	0.240	−0.21
0.01	2×	simple	0.439 ± 0.079	+0.027$^{\text{ns}}$	−1.37	0.179	−0.24
0.01	2×	**semantic_knn**	0.455 ± 0.093	+0.044*	−2.22	0.034	−0.39
0.01	2×	synthetic	0.435 ± 0.078	+0.023$^{\text{ns}}$	−1.17	0.249	−0.21
0.01	2×	random	0.443 ± 0.096	+0.032$^{\text{ns}}$	−1.86	0.072	−0.33
0.01	2×	original	0.644 ± 0.017	+0.232***	−14.58	<0.001	−2.58

Table 31: MovieLens (movie–genre) GCN: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

q	ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
0.01	5×	baseline	0.395 ± 0.022	+0.000	—	—	—
0.01	5×	degree_aware	0.372 ± 0.040	-0.023**	3.63	0.001	+0.64
0.01	5×	simple	0.376 ± 0.067	-0.019$^{\text{ns}}$	1.73	0.094	+0.31
0.01	5×	semantic_knn	0.366 ± 0.041	-0.029***	3.84	<0.001	+0.68
0.01	5×	**synthetic**	0.313 ± 0.051	-0.082***	7.45	<0.001	+1.32
0.01	5×	random	0.347 ± 0.052	-0.047***	4.67	<0.001	+0.83
0.01	5×	original	0.240 ± 0.008	-0.154***	38.96	<0.001	+6.89
0.01	2×	baseline	0.395 ± 0.022	+0.000	—	—	—
0.01	2×	degree_aware	0.377 ± 0.035	-0.017*	2.39	0.023	+0.42
0.01	2×	simple	0.391 ± 0.036	-0.004$^{\text{ns}}$	0.50	0.618	+0.09
0.01	2×	semantic_knn	0.366 ± 0.041	-0.029***	3.84	<0.001	+0.68
0.01	2×	synthetic	0.363 ± 0.035	-0.032***	4.54	<0.001	+0.80
0.01	2×	**random**	0.358 ± 0.037	-0.036***	4.98	<0.001	+0.88
0.01	2×	original	0.240 ± 0.008	-0.154***	38.96	<0.001	+6.89

Degree Distribution Analysis

Table 32: MovieLens (movie–genre) GCN: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
0.01	5×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	5×	degree_aware	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	5×	simple	0.0625 ± 0.0040	0.990 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	5×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	5×	synthetic	0.0625 ± 0.0040	0.959 ± 0.003	9227.0 ± 31.4	powerlaw
0.01	5×	**random**	0.0625 ± 0.0040	0.941 ± 0.004	9120.0 ± 36.3	lognormal
0.01	5×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal
0.01	2×	baseline	0.0226 ± 0.0014	0.978 ± 0.001	9490.3 ± 13.9	lognormal
0.01	2×	degree_aware	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	simple	0.0250 ± 0.0016	0.991 ± 0.001	9587.2 ± 7.8	powerlaw
0.01	2×	semantic_knn	0.0190 ± 0.0009	0.985 ± 0.001	9546.3 ± 8.7	powerlaw
0.01	2×	synthetic	0.0250 ± 0.0016	0.978 ± 0.002	9482.6 ± 14.9	lognormal
0.01	2×	**random**	0.0250 ± 0.0016	0.976 ± 0.002	9468.2 ± 15.8	lognormal
0.01	2×	original	2.2713 ± 0.0000	0.266 ± 0.000	0.0 ± 0.0	lognormal

Runtime Analysis

Table 33: MovieLens (movie–genre) GCN: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

q	ϕ	Method	Aug. Time (s)	Train Time (s)
0.01	5×	baseline	0.0000 ± 0.0000	8.53 ± 3.69
0.01	5×	degree_aware	0.0028 ± 0.0004	6.95 ± 3.18
0.01	5×	**simple**	0.0024 ± 0.0002	7.38 ± 4.11
0.01	5×	semantic_knn	0.1441 ± 0.0025	7.30 ± 3.77
0.01	5×	synthetic	0.0026 ± 0.0004	5.17 ± 1.42
0.01	5×	random	0.0026 ± 0.0006	6.28 ± 2.98
0.01	5×	original	0.0000 ± 0.0000	349.84 ± 24.68
0.01	2×	baseline	0.0000 ± 0.0000	8.62 ± 3.74
0.01	2×	degree_aware	0.0025 ± 0.0004	7.44 ± 3.23
0.01	2×	**simple**	0.0023 ± 0.0002	7.29 ± 3.16
0.01	2×	semantic_knn	0.1270 ± 0.0135	7.37 ± 3.82
0.01	2×	synthetic	0.0023 ± 0.0004	8.00 ± 4.02
0.01	2×	random	0.0023 ± 0.0005	7.50 ± 3.64
0.01	2×	original	0.0000 ± 0.0000	352.65 ± 24.33

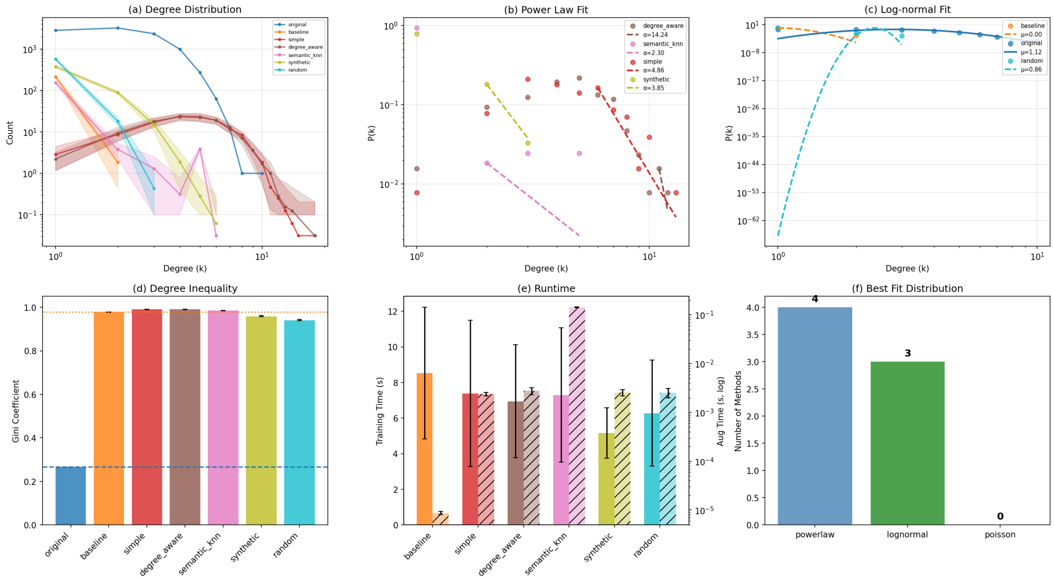

Figure 31: MovieLens (movie–genre), GCN, q=0.01, ϕ=5: Comprehensive analysis ($M \pm \mathrm{SD}$, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

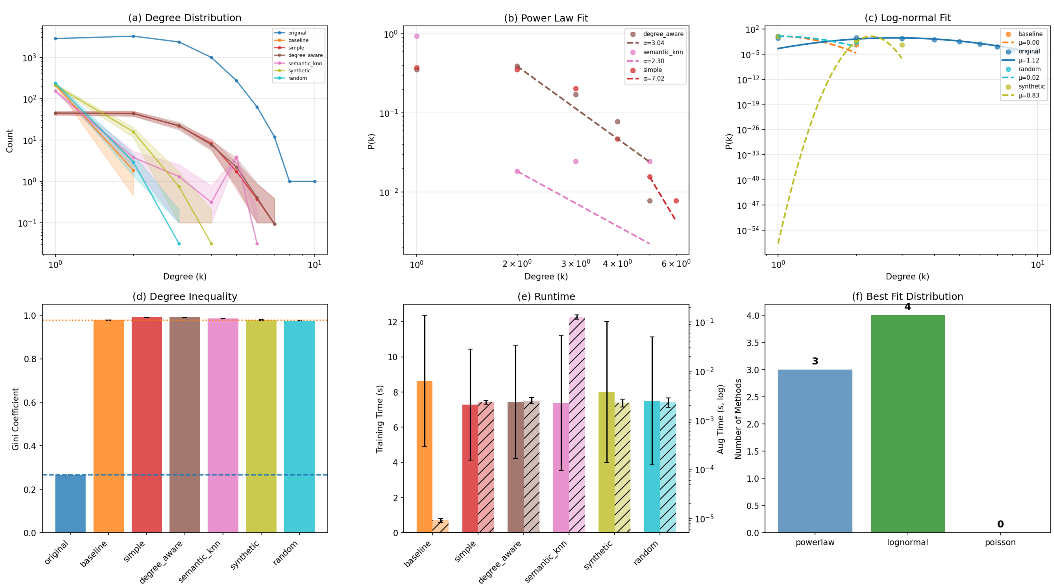

Figure 32: MovieLens (movie–genre), GCN, q=0.01, ϕ=2: Comprehensive analysis ($M \pm \mathrm{SD}$, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.5 DOMAIN CASE STUDY - GDP

C.5.1 SUMMARY

On GDP, effectiveness depends heavily on encoder and objective. For GAT, `semantic_knn` is the only method with clear AUC gains (at ϕ=100: +0.014), while `synthetic`/`random` degrade AUC and most methods hurt at lower ϕ. Brier flips: `semantic_knn` yields the largest reduction at ϕ=100 (-0.054), with `simple`/`degree_aware` giving mild gains at the same setting; lower ϕ generally raises Brier. GraphSAGE shows slight AUC upticks only from `simple` at ϕ=2 (+0.002) and consistent AUC/Brier drops from `semantic_knn`, while `degree_aware` is near baseline. GCN favors neither augmentation strongly: AUC edges up modestly for `simple`/`semantic_knn` at $\phi=5/2$, but Brier often worsens except small gains from `degree_aware` and `simple` at ϕ=5. Degree-wise, augmentations at ϕ=100 massively densify graphs (mean degree $\sim$ 189 for several methods), while lower ϕ still raise connectivity and cut isolated nodes; `semantic_knn` tends to retain higher Gini but reduces isolation. Runtime overheads stay small (aug $\leq$ 0.10s) though training increases vs. baseline, so accuracy changes reflect graph quality, not augmentation cost.

C.5.2 GAT

Summary Analysis GDP GAT shows limited AUC gains: only `semantic_knn` improves at ϕ=100 (+0.014), while `simple`/`degree_aware` are near baseline at ϕ=5 and all methods decline at ϕ=2. Brier improvements are concentrated at ϕ=100 with `semantic_knn` leading (-0.054) and `simple`/`synthetic`/`random` also lowering error; at lower ϕ, most methods raise Brier. Degree distributions at ϕ=100 explode in density for most augmentations (mean degree $\sim$ 189) with reduced inequality for `random`/`synthetic`, while `semantic_knn` keeps graphs closer to the sparse baseline (mean 5.14, lower Gini). Runtime overheads are small for augmentation ($<$ 0.10s) though training is higher than baseline; effects are driven by graph changes, not compute.

AUC and Brier Score

Table 34: GDP (game–pattern) GAT: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline (n = 32 seeds). A higher AUC is better.

ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
100×	baseline	0.800 ± 0.022	+0.000	—	—	—
100×	degree_aware	0.772 ± 0.026	-0.028***	5.29	<0.001	+0.93
100×	simple	0.793 ± 0.023	-0.007^{ns}	1.67	0.104	+0.30
100×	**semantic_knn**	0.814 ± 0.017	+0.014**	−2.83	0.008	−0.50
100×	synthetic	0.645 ± 0.061	-0.155***	13.42	<0.001	+2.37
100×	random	0.613 ± 0.076	-0.187***	13.14	<0.001	+2.32
5×	baseline	0.781 ± 0.020	+0.000	—	—	—
5×	degree_aware	0.782 ± 0.017	+0.001^{ns}	−0.32	0.754	−0.06
5×	**simple**	0.786 ± 0.016	+0.005^{ns}	−1.17	0.253	−0.21
5×	semantic_knn	0.772 ± 0.025	-0.009^{ns}	1.59	0.122	+0.28
5×	synthetic	0.745 ± 0.023	-0.036***	6.79	<0.001	+1.20
5×	random	0.752 ± 0.026	-0.029***	4.42	<0.001	+0.78
2×	**baseline**	0.781 ± 0.020	+0.000	—	—	—
2×	degree_aware	0.777 ± 0.021	-0.004^{ns}	0.91	0.368	+0.16
2×	simple	0.777 ± 0.020	-0.004^{ns}	0.73	0.469	+0.13
2×	semantic_knn	0.769 ± 0.021	-0.012*	2.18	0.037	+0.39
2×	synthetic	0.769 ± 0.027	-0.012*	2.59	0.014	+0.46
2×	random	0.773 ± 0.025	-0.008^{ns}	1.35	0.187	+0.24

Table 35: GDP (game–pattern) GAT: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
100×	baseline	0.302 ± 0.040	+0.000	—	—	—
100×	degree_aware	0.337 ± 0.079	+0.036*	−2.59	0.015	−0.46
100×	simple	0.289 ± 0.018	-0.013*	2.41	0.022	+0.43
100×	**semantic_knn**	0.247 ± 0.017	-0.054***	7.06	<0.001	+1.25
100×	synthetic	0.269 ± 0.013	-0.032***	4.76	<0.001	+0.84
100×	random	0.266 ± 0.017	-0.036***	4.92	<0.001	+0.87
5×	baseline	0.200 ± 0.010	+0.000	—	—	—
5×	**degree_aware**	0.197 ± 0.011	-0.003$^{\text{ns}}$	1.16	0.253	+0.21
5×	simple	0.199 ± 0.011	-0.001$^{\text{ns}}$	0.56	0.580	+0.10
5×	semantic_knn	0.213 ± 0.011	+0.013***	−6.02	<0.001	−1.06
5×	synthetic	0.288 ± 0.022	+0.088***	−20.21	<0.001	−3.57
5×	random	0.310 ± 0.027	+0.110***	−24.36	<0.001	−4.31
2×	**baseline**	0.200 ± 0.010	+0.000	—	—	—
2×	degree_aware	0.202 ± 0.008	+0.002$^{\text{ns}}$	−1.35	0.187	−0.24
2×	simple	0.200 ± 0.008	+0.000$^{\text{ns}}$	−0.00	0.997	−0.00
2×	semantic_knn	0.210 ± 0.009	+0.010***	−4.51	<0.001	−0.80
2×	synthetic	0.212 ± 0.014	+0.012***	−3.98	<0.001	−0.70
2×	random	0.219 ± 0.013	+0.019***	−6.67	<0.001	−1.18

Degree Distribution Analysis

Table 36: GDP (game–pattern) GAT: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
100×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw
100×	degree_aware	188.9423 ± 0.0000	0.518 ± 0.018	53.8 ± 4.4	lognormal
100×	simple	188.9423 ± 0.0000	0.614 ± 0.015	53.8 ± 4.4	powerlaw
100×	semantic_knn	5.1447 ± 0.0796	0.331 ± 0.020	19.2 ± 4.2	powerlaw
100×	synthetic	188.9423 ± 0.0000	0.264 ± 0.018	0.0 ± 0.0	lognormal
100×	**random**	188.9423 ± 0.0000	0.042 ± 0.002	0.0 ± 0.0	lognormal
5×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw
5×	degree_aware	17.1875 ± 0.0000	0.442 ± 0.008	0.0 ± 0.0	powerlaw
5×	simple	17.1875 ± 0.0000	0.559 ± 0.008	0.0 ± 0.0	lognormal
5×	semantic_knn	7.1106 ± 0.0000	0.296 ± 0.000	0.0 ± 0.0	powerlaw
5×	synthetic	17.1875 ± 0.0000	0.296 ± 0.007	0.0 ± 0.0	lognormal
5×	**random**	17.1875 ± 0.0000	0.181 ± 0.006	0.0 ± 0.0	powerlaw
2×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw
2×	degree_aware	6.8750 ± 0.0000	0.498 ± 0.009	0.0 ± 0.0	powerlaw
2×	simple	6.8750 ± 0.0000	0.567 ± 0.009	0.0 ± 0.0	powerlaw
2×	**semantic_knn**	6.8750 ± 0.0000	0.315 ± 0.000	0.0 ± 0.0	powerlaw
2×	synthetic	6.8750 ± 0.0000	0.384 ± 0.009	0.0 ± 0.0	powerlaw
2×	random	6.8750 ± 0.0000	0.340 ± 0.008	0.0 ± 0.0	powerlaw

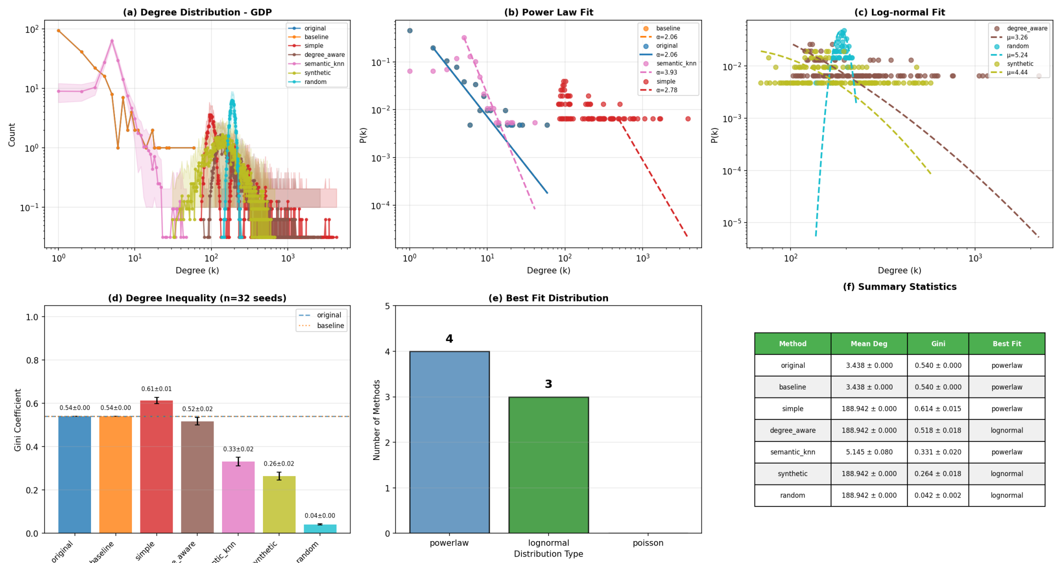

Figure 33: GDP (game–pattern), GAT, ϕ=100: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

Runtime Analysis

Table 37: GDP (game–pattern) GAT: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

ϕ	Method	Aug. Time (s)	Train Time (s)
5×	baseline	0.0008 ± 0.0000	70.48 ± 2.32
5×	degree_aware	0.0029 ± 0.0077	114.87 ± 4.60
5×	**simple**	0.0010 ± 0.0001	109.93 ± 3.83
5×	semantic_knn	0.0992 ± 0.0004	95.71 ± 20.63
5×	synthetic	0.0012 ± 0.0004	247.25 ± 85.61
5×	random	0.0010 ± 0.0000	259.04 ± 82.96
2×	baseline	0.0008 ± 0.0000	69.11 ± 2.07
2×	degree_aware	0.0014 ± 0.0015	79.00 ± 1.28
2×	**simple**	0.0009 ± 0.0000	79.37 ± 2.71
2×	semantic_knn	0.0795 ± 0.0020	89.45 ± 7.87
2×	synthetic	0.0010 ± 0.0004	100.68 ± 15.59
2×	random	0.0009 ± 0.0000	102.15 ± 24.68

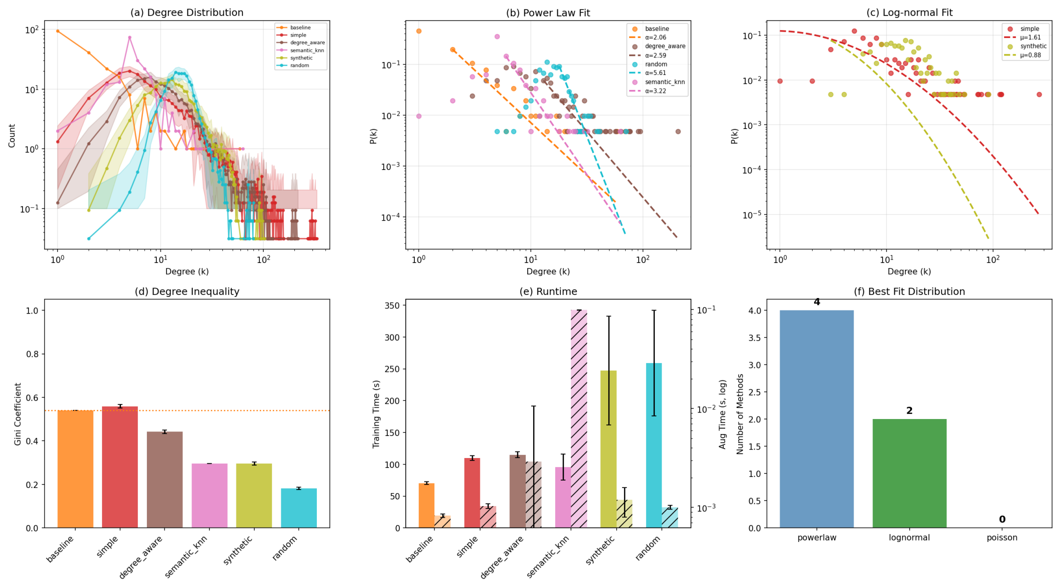

Figure 34: GDP (game–pattern), GAT, ϕ=5: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

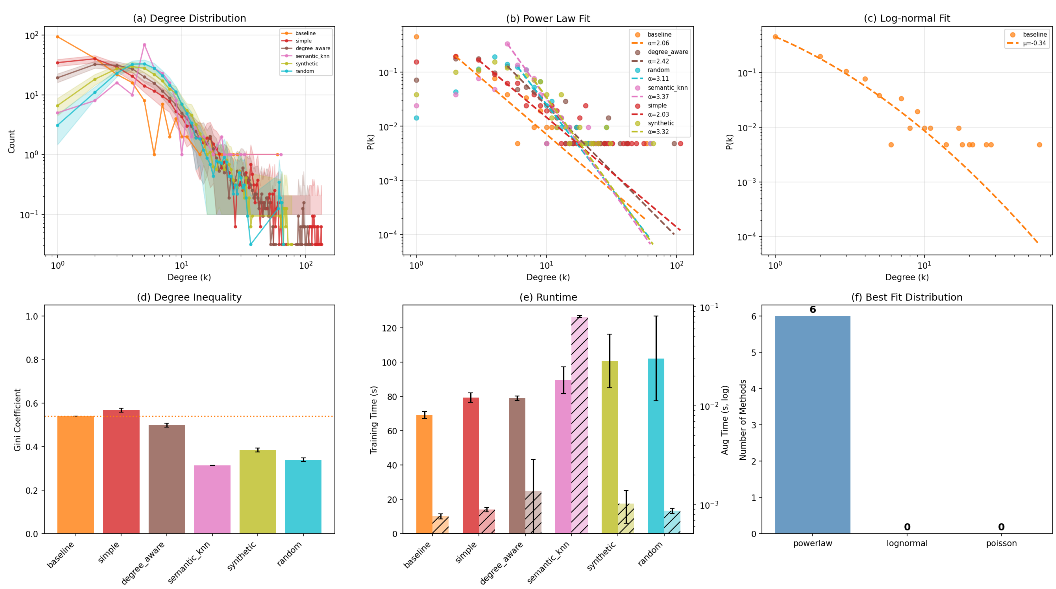

Figure 35: GDP (game–pattern), GAT, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.5.3 GRAPHSAGE

Summary Analysis GDP GraphSAGE largely tracks baseline: `simple` offers a small AUC lift at ϕ=2 (+0.002), while `semantic_knn` consistently lowers AUC (down to -0.040 at ϕ=5) and `synthetic`/`random` also degrade. Brier favors `simple`/`degree_aware` with minor reductions, whereas `semantic_knn` and especially `synthetic`/`random` sharply increase error. Degree distributions show augmentations densify graphs (mean degree 17 at ϕ=5, 6.9 at ϕ=2) with varied inequality: `random`/`synthetic` reduce Gini and isolate counts most, while `semantic_knn` yields lower Gini but modest density. Augmentation costs remain negligible (< 0.10s); training grows vs. baseline but differences stem from graph structure changes.

AUC and Brier Score

Table 38: GDP (game–pattern) GraphSAGE: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds). A higher AUC is better.

ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
5×	**baseline**	0.802 ± 0.012	+0.000	—	—	—
5×	degree_aware	0.799 ± 0.013	-0.003^{ns}	1.07	0.292	+0.19
5×	simple	0.798 ± 0.013	-0.004^{ns}	1.90	0.067	+0.34
5×	semantic_knn	0.763 ± 0.016	-0.040***	15.41	<0.001	+2.72
5×	synthetic	0.780 ± 0.019	-0.022***	5.37	<0.001	+0.95
5×	random	0.789 ± 0.021	-0.013***	3.66	<0.001	+0.65
2×	baseline	0.802 ± 0.012	+0.000	—	—	—
2×	degree_aware	0.801 ± 0.014	-0.001^{ns}	0.51	0.611	+0.09
2×	**simple**	0.805 ± 0.015	+0.002^{ns}	−0.76	0.454	−0.13
2×	semantic_knn	0.775 ± 0.017	-0.028***	7.77	<0.001	+1.37
2×	synthetic	0.767 ± 0.017	-0.036***	9.70	<0.001	+1.72
2×	random	0.764 ± 0.018	-0.039***	11.53	<0.001	+2.04

Table 39: GDP (game–pattern) GraphSAGE: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
5×	baseline	0.175 ± 0.013	+0.000	—	—	—
5×	degree_aware	0.173 ± 0.016	-0.002^{ns}	0.42	0.676	+0.07
5×	**simple**	0.172 ± 0.012	-0.003^{ns}	0.88	0.383	+0.16
5×	semantic_knn	0.192 ± 0.013	+0.017***	−5.95	<0.001	−1.05
5×	synthetic	0.233 ± 0.012	+0.059***	−18.91	<0.001	−3.34
5×	random	0.229 ± 0.012	+0.055***	−17.04	<0.001	−3.01
2×	baseline	0.175 ± 0.013	+0.000	—	—	—
2×	degree_aware	0.170 ± 0.012	-0.005^{ns}	1.75	0.090	+0.31
2×	**simple**	0.168 ± 0.011	-0.007**	2.82	0.008	+0.50
2×	semantic_knn	0.188 ± 0.013	+0.013***	−4.25	<0.001	−0.75
2×	synthetic	0.191 ± 0.013	+0.016***	−5.52	<0.001	−0.98
2×	random	0.197 ± 0.015	+0.022***	−6.13	<0.001	−1.08

Degree Distribution Analysis

Table 40: GDP (game–pattern) GraphSAGE: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
5×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw	
5×	degree_aware	17.1875 ± 0.0000	0.442 ± 0.008	0.0 ± 0.0	powerlaw	
5×	simple	17.1875 ± 0.0000	0.559 ± 0.008	0.0 ± 0.0	lognormal	
5×	semantic_knn	7.1106 ± 0.0000	0.296 ± 0.000	0.0 ± 0.0	powerlaw	
5×	synthetic	17.1875 ± 0.0000	0.296 ± 0.007	0.0 ± 0.0	lognormal	

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
5×	**random**	17.1875 ± 0.0000	0.181 ± 0.006	0.0 ± 0.0	powerlaw	
2×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw	
2×	degree_aware	6.8750 ± 0.0000	0.498 ± 0.009	0.0 ± 0.0	powerlaw	
2×	simple	6.8750 ± 0.0000	0.567 ± 0.009	0.0 ± 0.0	powerlaw	
2×	**semantic_knn**	6.8750 ± 0.0000	0.315 ± 0.000	0.0 ± 0.0	powerlaw	
2×	synthetic	6.8750 ± 0.0000	0.384 ± 0.009	0.0 ± 0.0	powerlaw	
2×	random	6.8750 ± 0.0000	0.340 ± 0.008	0.0 ± 0.0	powerlaw	

Runtime Analysis

Table 41: GDP (game–pattern) GraphSAGE: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

ϕ	Method	Aug. Time (s)	Train Time (s)
5×	baseline	0.0007 ± 0.0000	50.46 ± 3.40
5×	degree_aware	0.0015 ± 0.0008	89.71 ± 8.85
5×	**simple**	0.0009 ± 0.0001	84.11 ± 5.51
5×	semantic_knn	0.0976 ± 0.0031	64.55 ± 9.27
5×	synthetic	0.0011 ± 0.0006	285.53 ± 59.60
5×	random	0.0017 ± 0.0046	294.93 ± 50.21
2×	baseline	0.0008 ± 0.0001	51.52 ± 3.29
2×	degree_aware	0.0015 ± 0.0010	59.70 ± 3.63
2×	simple	0.0016 ± 0.0039	58.55 ± 3.02
2×	semantic_knn	0.0882 ± 0.0350	65.54 ± 12.46
2×	synthetic	0.0014 ± 0.0018	69.10 ± 16.66
2×	**random**	0.0009 ± 0.0001	73.48 ± 23.85

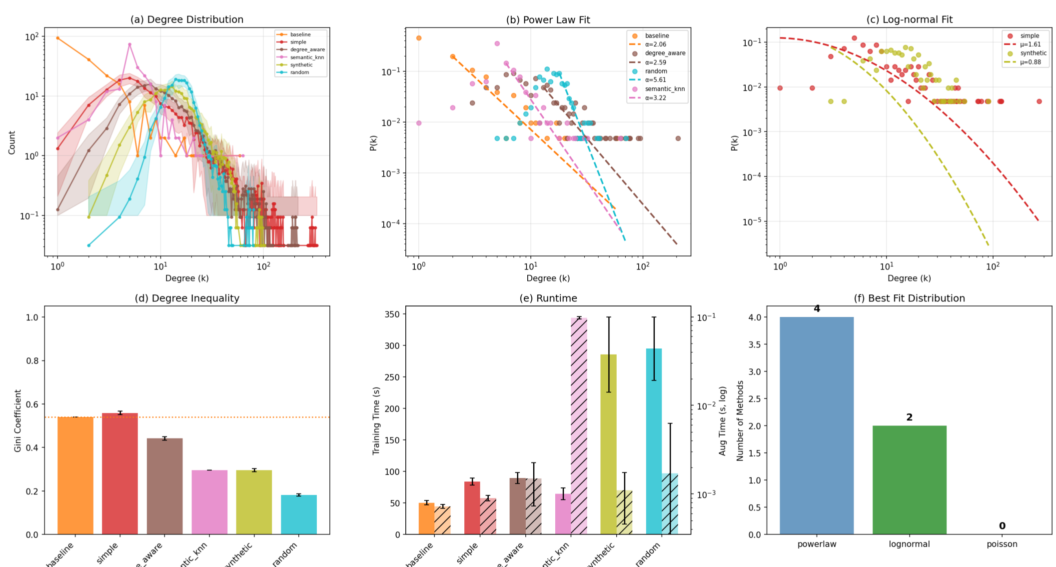

Figure 36: GDP (game–pattern), GraphSAGE, ϕ=5: Comprehensive analysis ($M \pm$ SD, $n = 32$ seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

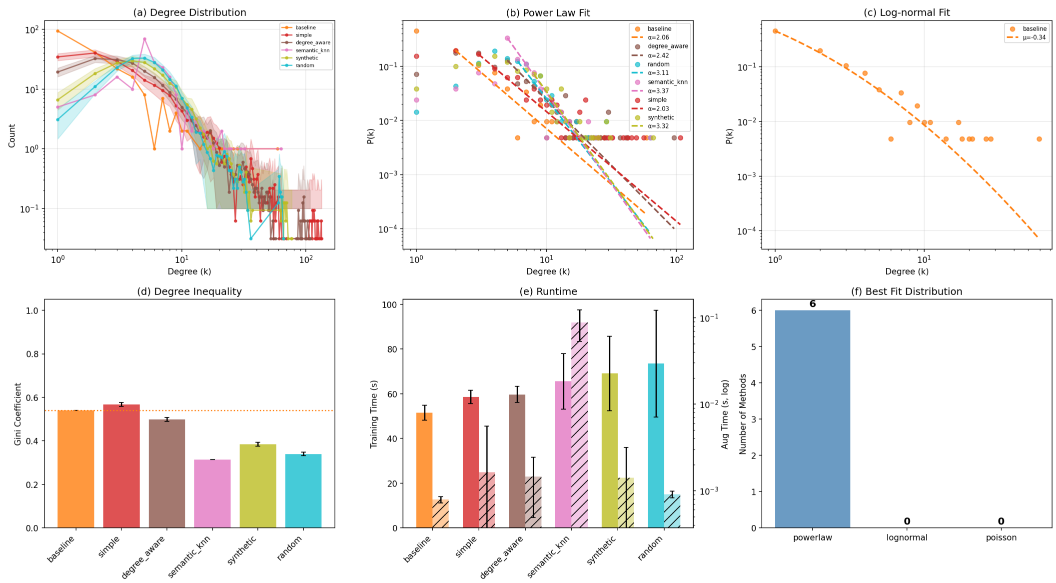

Figure 37: GDP (game–pattern), GraphSAGE, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, augmentation methods, and original graph. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

C.5.4 GCN

Summary Analysis GDP GCN yields limited gains: `simple` and `semantic_knn` provide modest AUC at ϕ=5/2 (up to +0.020), while `degree_aware` lags. Brier improves slightly for `degree_aware`/`simple` at ϕ=5 ($-0.031/-0.024$) and for `semantic_knn` at ϕ=2 (-0.009); `synthetic`/`random` generally worsen calibration. Degree distributions densify with larger ϕ (raising mean degree and reducing isolates), with `random`/`synthetic` lowering Gini most, while `semantic_knn` keeps graphs relatively sparse. Augmentation costs stay tiny (<0.063s); training is higher than baseline but differences reflect graph quality, not augmentation overhead.

AUC and Brier Score

Table 42: GDP (game–pattern) GCN: AUC-ROC ($M \pm SD$) with paired t-tests vs. sparse baseline (n = 32 seeds). A higher AUC is better.

ϕ	Method	AUC $M \pm SD$	ΔAUC	$t(31)$	p	d
5×	**baseline**	0.767 ± 0.028	+0.000	—	—	—
5×	degree_aware	0.605 ± 0.049	-0.162***	16.14	<0.001	+2.85
5×	simple	0.620 ± 0.043	-0.147***	15.89	<0.001	+2.81
5×	semantic_knn	0.733 ± 0.036	-0.034***	3.91	<0.001	+0.69
5×	synthetic	0.634 ± 0.062	-0.132***	10.84	<0.001	+1.92
5×	random	0.605 ± 0.079	-0.161***	11.05	<0.001	+1.95
2×	**baseline**	0.767 ± 0.028	+0.000	—	—	—
2×	degree_aware	0.700 ± 0.027	-0.067***	9.52	<0.001	+1.68
2×	simple	0.705 ± 0.031	-0.062***	7.93	<0.001	+1.40
2×	semantic_knn	0.746 ± 0.029	-0.020**	2.83	0.008	+0.50
2×	synthetic	0.731 ± 0.035	-0.035***	5.01	<0.001	+0.89
2×	random	0.732 ± 0.030	-0.034***	4.90	<0.001	+0.87

Table 43: GDP (game–pattern) GCN: Brier Score ($M \pm SD$) with paired t-tests vs. sparse baseline ($n = 32$ seeds, lower is better).

ϕ	Method	Brier $M \pm SD$	ΔBrier	$t(31)$	p	d
5×	baseline	0.209 ± 0.012	+0.000	—	—	—
5×	degree_aware	0.249 ± 0.001	+0.041***	−19.14	<0.001	−3.38
5×	simple	0.249 ± 0.002	+0.040***	−18.89	<0.001	−3.34
5×	**semantic_knn**	0.203 ± 0.016	-0.005^{ns}	1.47	0.152	+0.26
5×	synthetic	0.233 ± 0.022	+0.025***	−5.75	<0.001	−1.02
5×	random	0.240 ± 0.020	+0.031***	−7.44	<0.001	−1.32
2×	baseline	0.209 ± 0.012	+0.000	—	—	—
2×	degree_aware	0.229 ± 0.011	+0.020***	−7.84	<0.001	−1.39
2×	simple	0.225 ± 0.009	+0.016***	−6.40	<0.001	−1.13
2×	**semantic_knn**	0.200 ± 0.015	-0.009*	2.64	0.013	+0.47
2×	synthetic	0.202 ± 0.014	-0.007*	2.15	0.040	+0.38
2×	random	0.203 ± 0.016	-0.006^{ns}	1.64	0.112	+0.29

Degree Distribution Analysis

Table 44: GDP (game–pattern) GCN: Degree Distribution Statistics ($M \pm SD$, $n = 32$ seeds). Lower Gini coefficient indicates more uniform degree distribution.

q	ϕ	Method	Mean Degree	Gini Coeff.	Num. Isolated	Best Fit
5×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw	
5×	degree_aware	17.1875 ± 0.0000	0.442 ± 0.008	0.0 ± 0.0	powerlaw	
5×	simple	17.1875 ± 0.0000	0.559 ± 0.008	0.0 ± 0.0	powerlaw	
5×	semantic_knn	7.1106 ± 0.0000	0.296 ± 0.000	0.0 ± 0.0	powerlaw	
5×	synthetic	17.1875 ± 0.0000	0.296 ± 0.007	0.0 ± 0.0	lognormal	
5×	**random**	17.1875 ± 0.0000	0.181 ± 0.006	0.0 ± 0.0	powerlaw	
2×	baseline	3.4375 ± 0.0000	0.540 ± 0.000	0.0 ± 0.0	powerlaw	
2×	degree_aware	6.8750 ± 0.0000	0.498 ± 0.009	0.0 ± 0.0	powerlaw	
2×	simple	6.8750 ± 0.0000	0.567 ± 0.009	0.0 ± 0.0	powerlaw	
2×	**semantic_knn**	6.8750 ± 0.0000	0.315 ± 0.000	0.0 ± 0.0	powerlaw	
2×	synthetic	6.8750 ± 0.0000	0.384 ± 0.009	0.0 ± 0.0	powerlaw	
2×	random	6.8750 ± 0.0000	0.340 ± 0.008	0.0 ± 0.0	powerlaw	

Runtime Analysis

Table 45: GDP (game–pattern) GCN: Runtime Statistics ($M \pm SD$, seconds, $n = 32$ seeds). Lower times are better.

ϕ	Method	Aug. Time (s)	Train Time (s)
5×	baseline	0.0009 ± 0.0008	47.42 ± 3.34
5×	degree_aware	0.0016 ± 0.0012	175.64 ± 74.79
5×	**simple**	0.0009 ± 0.0001	169.94 ± 85.77
5×	semantic_knn	0.0980 ± 0.0033	90.26 ± 18.84
5×	synthetic	0.0010 ± 0.0003	260.38 ± 118.23
5×	random	0.0009 ± 0.0001	243.10 ± 114.05
2×	baseline	0.0008 ± 0.0000	47.79 ± 3.36
2×	degree_aware	0.0012 ± 0.0000	120.59 ± 27.40
2×	**simple**	0.0009 ± 0.0000	114.75 ± 18.65
2×	semantic_knn	0.0812 ± 0.0010	87.45 ± 23.11

ϕ	Method	Aug. Time (s)	Train Time (s)
2×	synthetic	0.0010 ± 0.0000	88.50 ± 16.09
2×	random	0.0009 ± 0.0000	88.82 ± 15.28

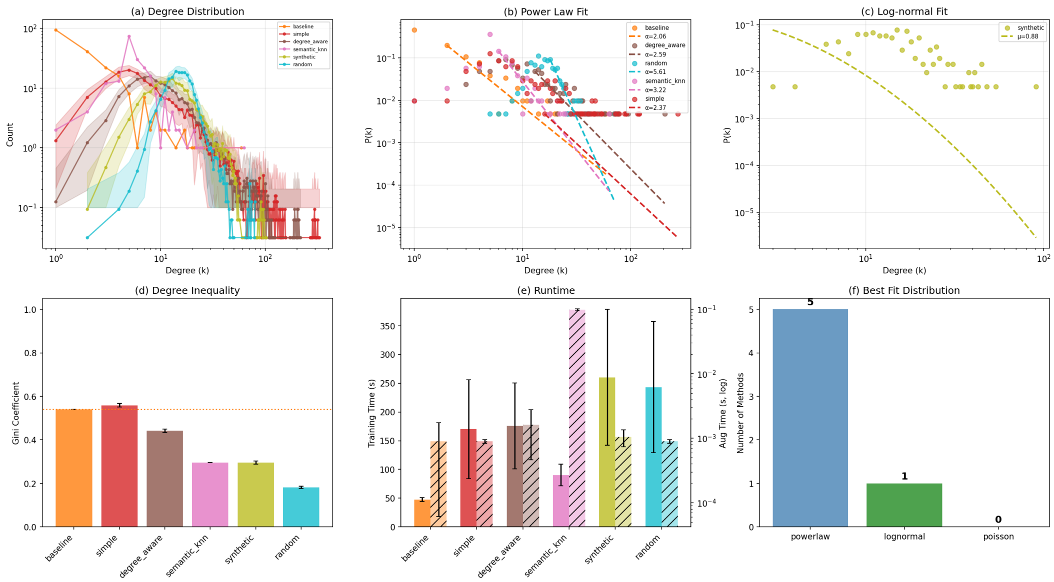

Figure 38: GDP (game–pattern), GCN, ϕ=5: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline, and augmentation methods. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.

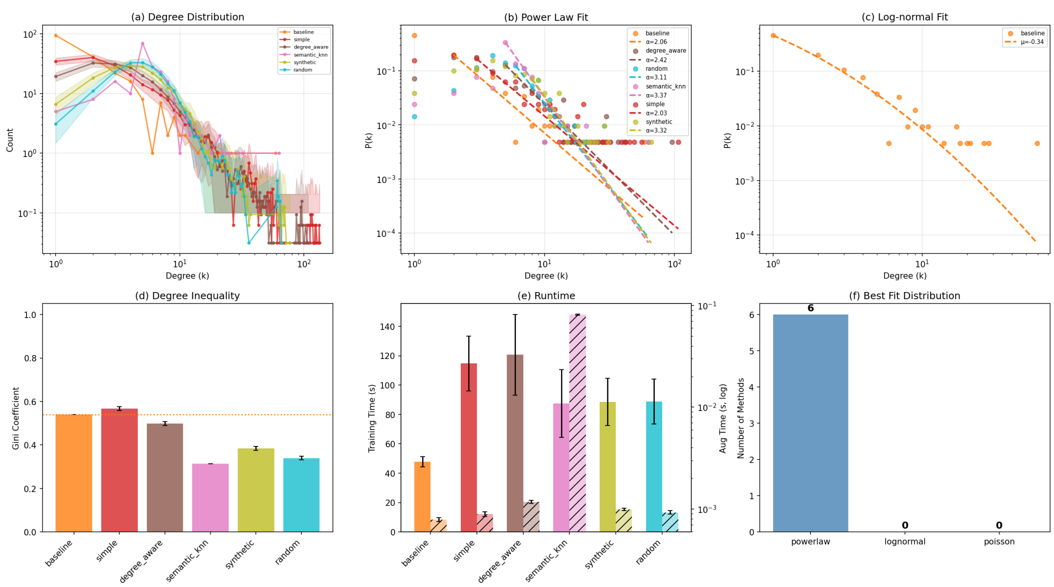

Figure 39: GDP (game–pattern), GCN, ϕ=2: Comprehensive analysis ($M \pm$ SD, n = 32 seeds) comparing baseline and augmentation methods. Panel (a) shows degree distributions on log-log scale with confidence bands; (b) Power Law fits with exponent α; (c) Log-normal fits with parameters μ and σ; (d) Gini coefficients quantifying degree inequality (lower = more uniform); (e) runtime comparison showing training time (left axis) and augmentation time (right axis, log scale); (f) best-fit distribution counts across methods.