
A Green Granular Convolutional Neural Network with Software-FPGA Co-designed Learning

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Abstract

1 Different from traditional tedious CPU-GPU-based training algorithms using
2 gradient descent methods, the software-FPGA co-designed learning algorithm
3 is created to quickly solve a system of linear equations to directly calculate
4 optimal values of hyperparameters of the green granular neural network (GGNN).
5 To reduce both CO_2 emissions and energy consumption effectively, a novel
6 green granular convolutional neural network (GGCNN) is developed by using
7 a new classifier that uses GGNNs as building blocks with new fast software-
8 FPGA co-designed learning. Initial simulation results indicates that the FPGA
9 equation solver code ran faster than the Python equation solver code. Therefore,
10 implementing the GGCNN with software-FPGA co-designed learning is feasible.
11 In the future, The GGCNN will be evaluated by comparing with a convolutional
12 neural network (CNN) with the traditional software-CPU-GPU-based learning in
13 terms of speeds, model sizes, accuracy, CO_2 emissions and energy consumption
14 by using popular datasets. New algorithms will be created to divide the inputs
15 to different input groups that will be used to build different small-size GGNNs
16 to solve the curse of dimensionality.

17 1 Introduction

18 In recent years, deep neural networks such as a Convolutional Neural Network (CNN) have been
19 effectively used in different applications. However, a major problem is that traditional tedious
20 CPU-GPU-based training algorithms using gradient descent methods take huge amount of training
21 time, generate much CO_2 emissions and waste a lot of energy. For instance, a popular CNN needs
22 a large number of training epochs for very slow hyperparameter optimization. Thus, traditional
23 neural network software systems with very slow hyperparameter optimization algorithms are not
24 suitable for high-speed real-time learning and fast real-time prediction applications. In addition
25 to the long training time problem, the conventional neural networks have the black-box problem
26 (i.e., hyperparameters such as weights are not meaningful). How to build explainable open-box
27 machine learning systems with low CO_2 emissions and low energy consumption is an important
28 long-term problem.

29 In recent years, new green machine learning (ML) systems have been made to reduce both
30 CO_2 emissions and computational energy consumption. For instance, the AutoML framework
31 for different methods such as neural architecture search (NAS), and automated pruning and
32 quantization is used to build efficient on-device ML systems with low energy consumption and
33 low CO_2 emissions by measuring GPU hours and the estimated CO_2 emission amount CO_{2e} [1].
34 Since CO_{2e} is proportional to the total computational power p_t : $CO_{2e} = 0.954p_t$ [2], effectively
35 reducing training times results in greatly reducing both energy consumption and CO_2 emissions.

36 Currently, popular ML systems running on CPUs and GPUs generate a lot of CO_2 emissions
 37 and also waste much energy because (1) tedious traditional training algorithms such as gradient
 38 descent algorithms and genetic algorithms take huge amount of time to optimize billions of
 39 hyperparameters, and (2) CPUs and GPUs are not green effective and not energy efficient. In
 40 summary, an urgent challenge is developing a novel ML system with high-speed non-traditional
 41 training algorithms running on the green and energy efficient hardware to significantly reduce
 42 both CO_2 emissions and energy consumption.

43 Based on the successful implementation of the FPGA-based direct linear equation solver [3-5],
 44 the high-speed FPGA-based direct linear equation solver can be used to quickly generate optimal
 45 hyperparameters in just one epoch for the new GGNN in a real-time manner. For example,
 46 the Questa*-Intel FPGA Edition Software provides the FPGA design simulation that involves
 47 generating simulation files, compiling simulation models, running the simulation, and viewing the
 48 results. We will use FPGA software simulation systems to implement the high-speed FPGA-based
 49 direct linear equation solver. The goal is to develop more effective and faster hardware-based
 50 hyperparameter optimization algorithms with a fast direct linear equation solver for training a
 51 new GGNN. We will develop the novel Green Granular Convolutional Neural Network (GGCNN)
 52 with new fast FPGA-based training algorithm to effectively reduce both CO_2 emissions and
 53 energy consumption more effectively than the CPU-GPU-based training algorithms.

54 2 A Building Block: an Efficient FPGA-based GGNN

55 2.1 Granular Sets

56 Different sets dealing with uncertainty of data and information, such as the fuzzy set [6], the
 57 neutrosophic fuzzy set [7], the intuitionistic fuzzy set [8], and Pythagorean fuzzy set [9], were
 58 defined. A new granular set is defined as follows to be used to build a new granular neural
 59 network.

60 Definition 1. Let X be a nonempty set. A granular set A in X is defined as $A =$
 61 $\{x, \mu_A(x), \nu_A(x), \phi_A(x), \varphi_A(x), \theta_A(x), \vartheta_A(x) : x \in X\}$, where (1) $\mu_A(x)$ is degree of mem-
 62 bership of x for $0 \leq \mu_A(x) \leq 1$, (2) $\nu_A(x)$ is degree of non-membership of x for $0 \leq \nu_A(x) \leq 1$,
 63 (3) $\phi_A(x)$ is certain degree of $\mu_A(x)$ for $0 \leq \phi_A(x) \leq 1$, (4) $\varphi_A(x)$ is uncertain degree of
 64 $\mu_A(x)$ for $0 \leq \varphi_A(x) \leq 1$, (5) $\theta_A(x)$ is certain degree of $\nu_A(x)$ for $0 \leq \theta_A(x) \leq 1$, and
 65 (6) $\vartheta_A(x)$ is uncertain degree of $\nu_A(x)$ for $0 \leq \vartheta_A(x) \leq 1$, where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$,
 66 $0 \leq \phi_A(x) + \varphi_A(x) \leq 1$, and $0 \leq \theta_A(x) + \vartheta_A(x) \leq 1$.

67 Meaningful linguistic values, such as *very slow*, *about 25*, *around 200*, can be represented by
 68 the granular sets that are used to build interpretable granular fuzzy If-Then rules. For example,
 69 an explainable granular If-Then rule is If x_1 is *low* and x_2 is *around -1000*, Then an output is
 70 *high*.

71 2.2 Software-FPGA Co-designed Learning

72 The green granular neural network (GGNN) with new fast software-FPGA co-designed learning iss
 73 designed using granular sets and the software-FPGA co-designed learning algorithm. It uses the
 74 software-based learning system to compute the coefficients for a linear system of hyperparameter
 75 equations, then uses the fast FPGA-based learning system to optimize the hyperparameters, and
 76 finally builds a GGNN model for prediction.

77 For convenience, an N -record relational database has n numerical input fields x_i for $i = 1, 2, \dots, n$,
 78 and one numerical output field y . Now the problem is how to build a GGNN using given N records
 79 in the relational database. Here, granular sets are used as basic granules in granular partitions of
 80 the input variables x_i for $i = 1, 2, \dots, n$ and the output variable y . The interval $[a_i, b_i]$ of x_i are
 81 partitioned into $m_i - 1$ intervals ($a_{i1} \leq x_i \leq a_{i2}$, $a_{i2} \leq x_i \leq a_{i3}$, ..., $a_{i(m_i-1)} \leq x_i \leq a_{im_i}$). So
 82 m_i granules A_{ij} are used to cover the $m_i - 1$ intervals for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m_i$. The
 83 granules A_{ij} are defined by granular sets such as a fuzzy set [6]. After the above granulation of
 84 x_i for $i = 1, 2, \dots, n$, there are G data base granules for $G = \prod_{i=1}^n (m_i - 1)$. For each data base
 85 granule, a GGNN with an output $g(x_1, x_1, \dots, x_n)$ is constructed by using two input granular sets
 86 covering one interval of x_i and 2^n output granular sets of y . So y has 2^n granular sets B_k for
 87 $k = 1, 2, \dots, 2^n$.

88 The granular rule base has 2^n granular IF-THEN rules for one database granule such that
 89 IF x_1 is A_{1j_1} and ... x_n is A_{nj_n} THEN y is B_k for $j_i \in 1, 2, i = 1, 2, \dots, n$, and $k = 1, 2, \dots, 2^n$.
 90 A database granule has $K = \prod_{i=1}^n k_i$ records totally if an input x_i has k_i values for $i =$
 91 $1, 2, \dots, n$ in the database granule, and an output y has K corresponding values y_k for $k =$
 92 $1, 2, \dots, K$. The optimization function for the database granule is to minimize $Q = \frac{1}{2} \sum_{k=1}^K [y_k -$
 93 $g(x_{1k}, x_{2k}, \dots, x_{nk})]^2$. Based on $\frac{\partial Q}{\partial p_j} = 0$ for the GGNN, we can get a linear system of $M-$
 94 hyperparameter equations for $k = 1, 2, \dots, M$ for $M = 2^{n+1}$:

$$T_1^k q_1 + T_2^k q_2 + \dots + T_M^k q_M = \psi_k \quad (1)$$

95 Now we can solve the linear system of M -hyperparameter equations to directly get optimal M
 96 hyperparameters q_k of the GGNN for $k = 1, 2, \dots, M$.

97 Based on the successful design of the FPGA-based linear equation solver [3-5], it is feasible to
 98 use the same architecture of the FPGA-based linear equation solver to solve equation (1) to get
 99 optimized hyperparameters q_k for $k = 1, 2, \dots, M$.

100 The major merits of the granular constructive learning method are (1) quickly optimize parameters
 101 using predefined formulas, and (2) discover meaningful granular rules from training data.

102 We develop the novel GGNN with new fast FPGA-based training algorithm to reduce CO_2
 103 emissions more effectively than the CPU-GPU-based training algorithms. Popular CPUs and
 104 GPUs generate much more CO_2 emissions and run less efficiently than the field programmable
 105 gate array (FPGA) [10, 11]. For instance, the new FPGA-based massive parallel data processing
 106 system can reduce CO_2 emissions by around 50% [11]. FPGA is a light-weight hardware with low
 107 CO_2 emissions and low energy consumption [12] for quickly solving a system of linear equations.
 108 For example, on a Xilinx Vertex 6 FPGA (200MHz), the minimum latency of the FPGA-based
 109 direct linear equation solver was lower than 5 microseconds for a linear system of equations
 110 of order 32 [3]. Thus, it is feasible to use FPGA to implement the new FPGA-based training
 111 algorithm.

112 3 Simulations for FPGA-based Learning Methods

113 Once we calculated coefficients as $T_1^k, T_2^k, \dots, T_M^k$, we can solve equation (1) by simply using
 114 matrix inversion method. However, matrix inversion is, by its nature, not hardware-friendly. Many
 115 algorithms rely on division which requires huge resources on FPGA. Also, we usually need to
 116 fix the matrix size in prior to feeding numbers to the hardware. The first problem has been a
 117 hot topic in the FPGA community, and the second problem can be solved by HLS (High-Level
 118 Synthesis) [13].

119 Based on previous sections, if we have n input parameters, T_1^k, \dots, T_M^k will form a square matrix
 120 with $2^{n+1} \times 2^{n+1}$. There have been some researches focusing on FPGA-based matrix inversion
 121 for the past decades [3, 14, 15], such as steepest descent method [16], QR decomposition [15]
 122 and Gauss Jordan method [17], etc. The current method we use is LUP (LU factorization with
 123 partial pivoting). Fig. 2 shows an example for a 4×4 matrix. Following the color order, we can
 124 easily decompose a given matrix in $A = LU$. And the inverse of an upper or lower triangular
 125 matrix is easy to compute, since U^{-1} is also an upper triangular matrix [18].

126 Furthermore, we used a FPGA code and a Python code to test their running times on for the
 127 matrix inversion with different linear systems of hyperparameter equations of different orders (i.e.,
 128 8, 16, 32, 64, and 100). For each case, we create 20 complex square matrices of different orders.
 129 Table 1 shows running times that are measured in 10^{-4} s. All the tests are running on the same
 130 computer. The FPGA code ran faster than the Python code.

Table 1: Running times of the Python code and the FPGA code

Method	8	16	32	64	100
Python	1.139	1.581	7.057	30.147	76.581
FPGA	0.638	1.313	2.803	6.207	12.284

131 In addition, the FPGA code is effective to reduce both energy consumption and CO_2 emissions
 132 because the short execution time of the FPGA code results in a small computational power

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix} \\
= \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} & l_{21}u_{14}+u_{24} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} & l_{31}u_{14}+l_{32}u_{24}+u_{34} \\ l_{41}u_{11} & l_{41}u_{12}+l_{42}u_{22} & l_{41}u_{13}+l_{42}u_{23}+l_{43}u_{33} & l_{41}u_{14}+l_{42}u_{24}+l_{43}u_{34}+u_{44} \end{bmatrix} \\
\text{●} \rightarrow \text{●} \rightarrow \text{●} \rightarrow \text{●} \rightarrow \text{●} \rightarrow \text{●} \rightarrow \text{●}$$

Figure 1: LU factorization on a 4×4 matrix [19]

133 consumption p_t for $CO_2e = 0.954p_t$ [2]. Importantly, a FPGA hardware will be much faster than
134 a software-based equation solver to reduce both energy consumption and CO_2 emissions more
135 effectively.

136 Based on LUP, we can write the corresponding C program. To generate feasible Verilog scripts,
137 we can use Vivado HLS to transform the C program to Verilog code and simulate it in the
138 software. Therefore, the new software-FPGA-based learning method is feasible and useful for
139 implementing the new fast FPGA-based GGNNs.

140 To compare an artificial neural network (ANN) and the GGNN using a fuzzy set (a special granular
141 set), simulations using two different functions are done. The first 3-input-1-output benchmark
142 function f_k^1 [20-23] is given below:

$$f_k^1 = (1 + x_k^{0.5} + y_k^{-1} + z_k^{-1.5})^2. \quad (2)$$

143 The training data set with 8,000 training data is generated by f_k^1 shown in equation (2) such
144 that $x_k^{tr} = 1.0 + \lfloor \frac{k}{400} \rfloor$, $y_k^{tr} = 1.0 + \lfloor \frac{k}{20} \rfloor$, $z_k^{tr} = 1.0 + k \bmod 20$, where the operator *mod* is
145 used, $f_k^1 \in [4.248, 55.833]$, and $k = 0, 1, \dots, 7,999$. A testing data set with 6,859 testing data is
146 generated by f_k^1 such that $x_j^{te} = 1.5 + \lfloor \frac{j}{361} \rfloor$, $y_j^{te} = 1.5 + \lfloor \frac{j}{19} \rfloor$, $z_j^{te} = 1.5 + j \bmod 19$, where the
147 operator *mod* is used, $j = 0, 1, \dots, 6,858$. 8,000 training data are distributed in 27 subspaces,
148 but data in 16 subspaces are used to train both ANNs and *GGNN* (i.e., no training data in 11
149 other subspaces like missing data in the subspaces). 6,858 testing data are distributed in all the
150 27 subspaces to compare ANNs and *GGNN*.

151 Tables 2 to 4 show that *GGNN* outperforms both 10-Layer *ANN* and 20-Layer *ANN* in terms
152 of the prediction Mean Square Error (MSE), and the prediction Root Mean Square Error (RMSE)
153 for 100, 500, and 1,000 training epochs.

Table 2: Function Prediction Performance Comparison between ANNs and the GGNN for f^1 for 100 training epochs.

Neural Network	MSE	RMSE
10-Layer <i>ANN</i>	58.22	7.63
20-Layer <i>ANN</i>	63.44	6.88
<i>GGNN</i>	47.31	6.88

154 4 A New GGCNN with Software-FPGA Co-Designed Learning

155 Since the previous simulations indicate that the new software-FPGA-based learning method is
156 feasible and useful to quickly train the GGNN, the GGNN can be used to build a new machine
157 learning model as a basic building block. A CNN consists of convolutional layers, activation

Table 3: Function Prediction Performance Comparison between ANNs and the GGNN for f^1 for 500 training epochs.

Neural Network	MSE	RMSE
10-Layer <i>ANN</i>	55.68	7.46
20-Layer <i>ANN</i>	76.99	8.77
<i>GGNN</i>	46.38	6.81

Table 4: Function Prediction Performance Comparison between ANNs and the GGNN for f^1 for 1,000 training epochs.

Neural Network	MSE	RMSE
10-Layer <i>ANN</i>	51.16	7.15
20-Layer <i>ANN</i>	53.66	7.33
<i>GGNN</i>	46.71	6.83

158 layers, pooling layers, and a classifier such as a MLP. The new GGCNN consists of convolutional
 159 layers, activation layers, pooling layers, a new FPGA-based GGNN layer called a FPGA learner,
 160 and a hybrid decision model for making a final decision. The new GGCNN with software-FPGA
 161 co-designed learning is shown in Fig. 3.

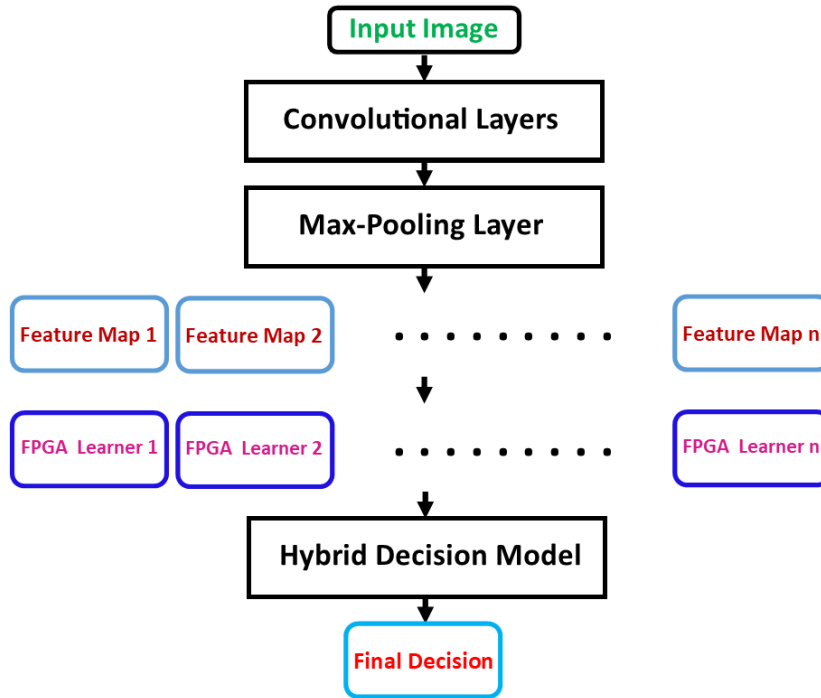


Figure 2: A GGCNN framework with Software-FPGA Co-Designed Learning

162 The FPGA-based direct hierarchical hyperparameter optimization algorithm for a GGCNN, a
 163 hybrid software-hardware-based algorithm, is given below as Algorithm 1. We assume that the
 164 linear equation solver can quickly solve a linear system of L hyperparameter equations. A $n \times n$

165 Feature Map (FM) has $n \times n$ features. N $n \times n$ feature maps FM_p for $p = 1, 2, \dots, N$ are
 166 generated by the last pooling layer. $K = N \times n \times n$.

for $k = 1$ **to** n **do**

Step 1: Use software to partition $n \times n$ feature map FM_p into M_j $n_j \times n_j$ sub-feature maps for for $M_j = l_j \times l_j$, and $n_j < L$).

Step 2: Use software to pre-calculate coefficients for M_j linear systems of hyperparameter equations for $n_j \times n_j$ sub-feature maps): Use software to calculate coefficients such as $P_1^{1k}, \dots, P_m^{1k}, P_1^{2k}, \dots, P_m^{2k}, P_1^{3k}, \dots, P_m^{3k}$, and then calculate $P_1^{1k} = P_1^{1k} + S_1^{1k}, \dots, P_m^{1k} = P_m^{1k} + S_m^{1k}, P_1^{2k} = P_1^{2k} + S_1^{2k}, \dots, P_m^{2k} = P_m^{2k} + S_m^{2k}, P_1^{3k} = T_1^{3k} + S_1^{3k}, \dots, P_m^{3k} = P_m^{3k} + S_m^{3k}$ of a linear system of hyperparameter equations.

167 *Step 3:* Use the FPGA-based linear equation solver to solve M_j linear systems of hyperparameter equations using M_j $l \times l$ sub-feature maps): Use the FPGA-based linear equation solver to calculate optimal hyperparameters such as $(c_i, \eta_i, \text{ and } \delta_i)$ for $i = 1, 2, \dots, m$ of each linear system of hyperparameter equations. The optimized hyperparameters are used to build new M_j GGNNs with relevant granular knowledge bases with meaningful granular If-Then rules.

Step 4: Use M_j Use the M_j FPGA-based GGNNs to make M_j decisions D_j^p for a new test feature map.

Step 5: Use all individual decisions D_j^p to make a hybrid decision.

end

Algorithm 1: Software-FPGA Co-Designed Learning Algorithm for the GGCNN

168 5 Conclusions

169 Initial simulation results indicates that the FPGA equation solver code ran faster than the Python
 170 equation solver code. In addition, the GGNN can perform more accurately than a traditional
 171 neural network. Therefore, it is feasible to make a novel software-FPGA co-designed GGNN to
 172 reduce both CO_2 emissions and energy consumption more effectively than the CPU-GPU-based
 173 neural networks. Since FPGA is a high-speed light-weight hardware with low CO_2 emissions
 174 and low energy consumption, the FPGA is used to quickly solve a system of linear equations to
 175 directly calculate optimal values of hyperparameters of the shallow GGNN. It is feasible to build
 176 the GGCNN using the GGNNs as basic building blocks to solve image recognition problems.

177 6 Future Works

178 In the future, the GGCNN with the software-FPGA co-designed learning will be evaluated by
 179 comparing with other machine learning models with traditional software-CPU-GPU co-designed
 180 learning in terms of speeds, model sizes, accuracy, CO_2 emissions and energy consumption by
 181 using popular datasets. New intelligent algorithms will be developed to find out optimal or near
 182 optimal sub-spaces on which accurate GGCNN models will be built.

183 A GGCNN with a large number of inputs has the curse of dimensionality. New algorithms will be
 184 created to divide the inputs to different input groups that will be used to build different small-size
 185 GGCNNs to solve the problem.

186 We will use different granular sets with different nonlinear membership functions, and then select
 187 the best one to improve performance (accuracy, AUC, F1-score, etc.) of the GGCNN.

188 After the software-FPGA co-designed learning is successful, a special high-speed FPGA hardware
 189 based direct linear equation solver as a fast learner will be implemented for building an efficient
 190 GGCNN with high classification accuracy to significantly reduce both CO_2 emissions and energy
 191 consumption.

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