SEMI-STRUCTURED LLM REASONERS CAN BE RIGOROUSLY AUDITED

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ABSTRACT

Although Large Language Models (LLMs) have become capable reasoners, the problem of faithfulness persists: their reasoning can contain errors and omissions that are difficult to detect and that may obscure biases in model outputs. To address this issue, we introduce Semi-Structured Reasoning Models (SSRMs), which are trained to produce semi-structured representations of reasoning. SSRMs generate reasoning traces in a *non-executable* Pythonic syntax that names each reasoning step and marks its inputs and outputs. This structure allows SSRM traces to be automatically *audited* to identify reasoning flaws. We evaluate three types of audits: hand-crafted *structured reasoning audits*, written in a domain-specific language (DSL) implemented in Python; LLM-generated *structured reasoning audits*; and learned *typicality audits*, which apply probabilistic models over reasoning traces. We show that all of these methods can be used to effectively flag probable reasoning errors. Importantly, the auditability of SSRMs does not appear to compromise overall accuracy: in evaluation on twelve benchmarks and two model families, SSRMs demonstrate strong performance and generalizability relative to other models of comparable size. We provide our code at Anonymous Github Link.

1 INTRODUCTION

Large Language Models (LLMs) often benefit from reasoning techniques such as short Chain-of-Thought (CoT) prompting (Wei et al., 2022) or long CoT reasoning (Chen et al., 2025a; Wang et al., 2025; Wang, 2025). Yet in many applications, LLMs may generate superficially plausible but incorrect reasoning that obscures biases in the output (Turpin et al., 2024); more generally, reasoning traces are not causally related to the final output (Bao et al., 2024). This problem of "unfaithful"LLM reasoning has been extensively investigated in short CoT settings (Lanham et al., 2023; Bentham et al., 2024; Parcalabescu & Frank, 2024), and is likely to be more problematic in long CoT reasoning.

As a concrete step toward demystifying reasoning LLMs and improving their reliability, we present methods for *rigorously checking LLM reasoning on specific tasks*. To illustrate and motivate this, consider the simplified medical question-answering (QA) task in Figure 1, adapted from the MedCalcBench (Khandekar et al., 2024). The "flawed" reasoning trace appears superficially plausible but is incomplete compared to the "ideal" trace: it contains one obvious omission, one subtler error, and one issue where the LLM fails to explicitly check the compatibility of the units for an extracted value. Although none of these affect the final answer in this example, such flaws are undesirable in consequential tasks. Human experts performing similar tasks are often expected to carefully follow explicit instructions—variously called rubrics, cookbooks, or policies depending on the domain—to ensure that reasoning is complete and decisions are made consistently. This observation motivates the central research question: *can we detect when an LLM deviates from a desired reasoning strategy?*

Since analyzing arbitrary reasoning traces is difficult, we begin by training an LLM to generate *semi-structured* reasoning traces, as shown in Figure 2 (Top). Following prior work (Cohen & Cohen, 2024), we adopt a Pythonic syntax that labels different types of reasoning steps using a restricted, task-specific vocabulary and specifies the *inputs and outputs* of each step, *without requiring the steps to be executable*. Because the steps can perform arbitrary computations and consume or produce arbitrary strings of text, this semi-structured format is highly general. In this paper we provide new evidence for this generality by showing that training models to generate *semi-structured* traces achieves performance comparable to similarly trained free-form reasoning models and other baselines.

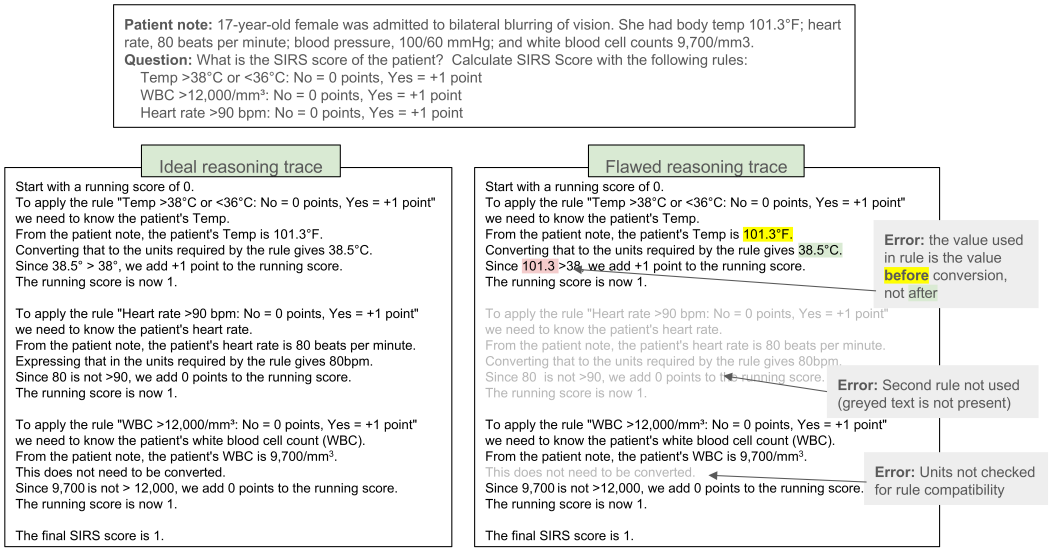

Figure 1: Overview of the problem addressed. Top: a question that requires the LLM to extract information and apply reasoning to answer correctly. Bottom left: a desired reasoinng trace. Bottom right: a flawed reasoning trace. The flawed trace differs from the desired one in three ways: (1) the incorrect patient measurement is used to determine applicability of the first rule; (2) the second rule is skipped; (3) the units associated with a patient measurement are not explicitly checked against those required by the third rule. In this example, none of these reasoning flaws affects the final answer, so this flawed reasoning trace will be reinforced during reinforcement learning with an outcome reward.

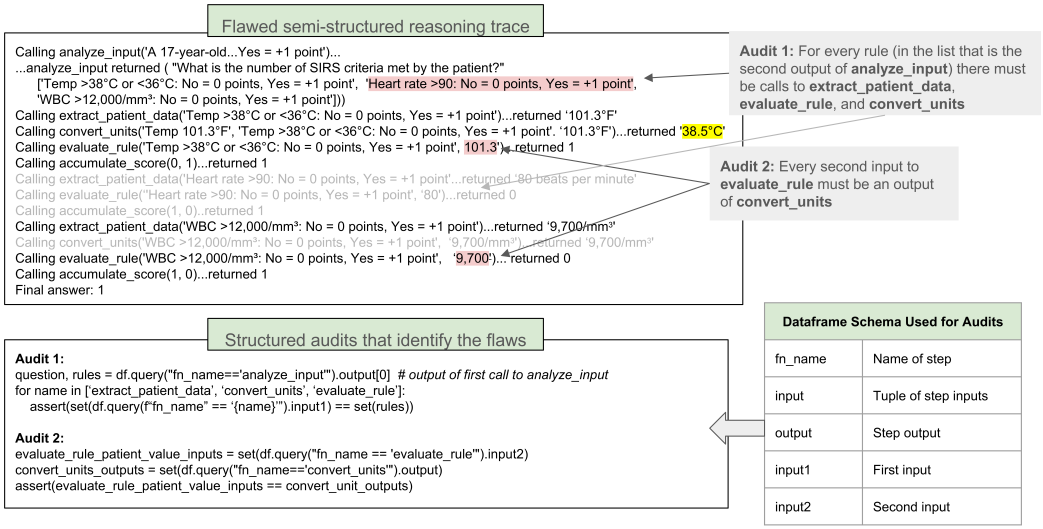

Figure 2: Overview of our approach. An LLM is trained to generate a *semi-structured trace* comprising a function name, its inputs, and its outputs for each reasoning step. Two plausible constraints on this semi-structured trace are also shown, given in natural language (gray boxes) and as executable tests (bottom left). The executable tests are *reasoning trace audits*, and in this case are hand-written. We also explore *typicality audits*, which are learned from a corpus of reasoning traces.

We further show that semi-structured reasoning *facilitates the scalable detection of reasoning flaws*. For example, in Figure 2, one can observe that a desired rule was skipped by comparing the number of `evaluate_rule` steps with the length of the rule list returned by `analyze_input`. We refer to such

checks as *reasoning audits*. Figure 2 also provides natural-language descriptions of two audits alongside their corresponding *structured reasoning audits*. Our results indicate that these manually implemented audits can identify potential reasoning flaws and flag outcomes that are likely incorrect.

Our DSL for structured queries uses trace that has been encoded as a Pandas DataFrame, and audits also look like Python unit tests—two widely-used programming constructs. Because of these design choices, we show that *structured reasoning audits are also easily generated automatically by modern LLMs given minimal guidance*, substantially reducing the cost of auditing reasoning in a new domain.

Beyond enabling structured reasoning queries, *semi-structured reasoning facilitates additional forms of analysis*. A recurring question in the literature (Kambhampati et al., 2025) is whether reasoning LLMs generate novel "reasoning patterns" or simply reproduce patterns that are seen during training. This issue is difficult to address without a formal definition of "reasoning patterns." In this work, we explore certain definitions of a "reasoning pattern" for semi-structured reasoners and use it to build *probabilistic models of reasoning patterns for specific tasks*. We evaluate the hypothesis that model accuracy correlates with the probability assigned to its reasoning patterns. By analogy with structured audits, we term this a *typicality audit*, and show that they can also identify potential reasoning errors.

In summary, this paper makes the following contributions:

- We introduce two-stage training recipes for SSRMs that produce semi-structured reasoning traces.
- We illustrate that both manually-generated and LLM-generated structured audits can effectively reveal potential reasoning flaws, and that failing certain audits increases the probability of error.
- We show that typicality audits can reveal common reasoning patterns linked to better outcomes.
- We demonstrate that auditability comes without cost in generalization performance, as SSRMs achieve results comparable to similarly trained unstructured reasoning models and other baselines.

2 Related Work

Faithfulness and Process Models. CoT prompting has been shown to sometimes produce predictions that preserve underlying LLM biases, accompanied by explanations that obscure those biases (Turpin et al., 2024). This observation has motivated extensive research on explanation faithfulness in CoT prompting Jacovi & Goldberg (2020); Turpin et al. (2024); Lanham et al. (2023); Bao et al. (2024). Nevertheless, defining and measuring faithfulness remains challenging, with some prior studies advocating quantitative approaches that assess mechanistic influence in neural networks through numerical metrics (Parcalabescu & Frank, 2024; Bentham et al., 2024; Chen et al., 2025b). In this work, we propose *reasoning audits* as a concrete and testable alternative to measuring faithfulness.

Other studies have proposed methods verifying reasoning chains using *process reward models* (Paul et al., 2024; Sun et al., 2024b) and *step reward models* Viteri et al. (2024); Wang et al. (2023); Saparov & He (2022); Lai et al. (2024). However, these reward models are typically tailored to specific domains—such as mathematics (Paul et al., 2024; Sun et al., 2024b) or theorem-proving (Saparov & He, 2022; Lai et al., 2024)—and often rely on Monte Carlo Tree Search (Kocsis & Szepesvári, 2006) to explore and evaluate multiple candidate reasoning chains, a computationally expensive procedure. While our work is largely orthogonal, the symbolic and statistical audits we propose could provide complementary signals for future reward-model training. In particular, the statistical audits we proposed refine the notion of reasoning patterns, which have previously been identified either through task-specific analyses (Zhang et al., 2025) or via LLM pipeline methods (Zhou et al., 2025).

Semi-structured LLM Reasoning. Various prompting strategies—such as CoT (Wei et al., 2022), Tree-of-Thought (ToT) (Yao et al., 2023), Chain-of-Code (CoC) (Li et al., 2023), and Program-of-Thought (PoT) (Chen et al., 2022)—have been widely employed to enhance the reasoning capabilities of LLMs. More recently, research has shifted from prompting toward inference-time scaling by incorporating search algorithms, particularly tree-based search (including Monte Carlo Tree Search variants) and beam search, into the sampling process (Feng et al., 2023; Trinh et al., 2024; Xin et al., 2024; Kocsis & Szepesvári, 2006); by ensembling multiple reasoning trajectories through self-consistency (Wang et al., 2022; Huang et al., 2025; Aggarwal et al., 2023); and by applying reinforcement learning (RL) to extend the length of reasoning (OpenAI, 2024; Shao et al., 2024; Guo et al., 2025; Qwen, 2024; Kumar et al., 2024; Yang et al., 2025). Despite these, LLMs frequently produce reasoning traces that appear plausible yet incorrect, and such errors can be difficult to detect.

Previous studies have proposed that faithfulness can be improved by using a code-like format for LLM outputs. Prior work assumes this format is either fully executable Python programs (Chen et al., 2022; Gao et al., 2023; Lyu et al., 2023; Paranjape et al., 2023) or partially executable pseudocode (Li et al., 2023; Weir et al., 2024; Chae et al., 2024). While enabling the use of Python as a tool often improves performance, the reasoning process used to generate the pseudocode remains obscured. These works have also argued (sometimes implicitly) that faithfulness is qualitatively improved with code-based outputs. In contrast, we pursue the more concrete goal of auditing the reasoning process.

We build most on the reasoning-chain syntax used in Program Trace Prompting (PTP) (Cohen & Cohen, 2024). While PTP uses few-shot prompting to extrapolate "partial programs" and sample traces for novel inputs, SSRMs achieve strong performance without task-specific few-shot prompts.

3 Training Methods

We use a two-stage training recipe for a Semi-Structured Reasoning Model (SSRM). The first stage performs SFT to teach the model to produce the semi-structured reasoning traces, while the second stage uses reinforcement learning with verifiable rewards (RLVR) to enhance the reasoning ability.

Supervised Fine-Tuning. To collect SFT data for semi-structured reasoning, we follow the PTP approach (Cohen & Cohen, 2024). We generate semi-structured reasoning traces with PTP using both Claude Sonnet 3.5 and 3.7 (Anthropic, 2024; 2025) on all BBH tasks (Suzgun et al., 2022) as well as subsets of the training data from GSM8K (Cobbe et al., 2021), MATH500 (Lightman et al., 2023), and MedCalcBenchV2 (please see Section 4). Only traces that yield correct final answer are retained.

Chain-of-Thought Baseline. To establish a fair baseline for comparison, we construct a standard CoT dataset. We generate CoT traces on BBH using the original few-shot prompts applied to Claude Sonnet 3.5, and augment the training data with ground-truth CoT solutions from GSM8K, MATH500, and MedCalcBenchV2, for the same problem instances used in the semi-structured SFT training data.

Training Template. We structure each example using a consistent markup format. In the semi-structured setting, partial programs are wrapped in `<partial_program>` tags, reasoning traces in `<program_trace>` tags, both enclosed within a `<think>` tag. The final answer is placed inside the `<answer>` tag for easy parsing. For the CoT baseline, only `<think>` and `<answer>` tags are used.

RLVR Dataset. In the second stage, we enhance the SFT model with RLVR. We construct the RLVR dataset by sampling eight responses per problem from the English subset of DAPO-Math-17K (Yu et al., 2025), using the SFT checkpoint. We randomly discard half of the samples with pass rates of either zero or one. We further include a held-out subset of MedCalcBenchV2 excluded from SFT.

Reward Design. We adopt a rule-based reward combining outcome accuracy and structural validity. Outcome accuracy measures the correctness of the final answer, while format rewards are assigned if the reasoning trace conforms to the semi-structured or CoT format, evaluated via regular expressions.

RL Algorithm. We optimize with the Group Relative Policy Optimization (GRPO) (Shao et al., 2024), which estimates token-level advantages without requiring a critic. For a specific question-answer pair (q, a), the policy model first samples a group of G individual responses $\{\mathbf{o}_i\}_{i=1}^{G}$. Subsequently, the advantage of the i-th response is calculated as $A_{i,t} = \frac{r_i - \text{mean}\left(\{R_i\}_{i=1}^{G}\right)}{\text{std}\left(\{R_i\}_{i=1}^{G}\right)}$. And the training objective is

$$\mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{(q,a)\sim\mathcal{D},\{\mathbf{o}_i\}_{i=1}^{G}\sim\pi_{\theta_{\text{old}}}(\cdot|q)} \left[\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}\left(\min\left(r_{i,t}(\theta)A_{i,t}, \text{clip}\left(r_{i,t}(\theta), 1-\varepsilon, 1+\varepsilon\right)A_{i,t}\right) - \beta D_{\text{KL}}\left(\pi_\theta \| \pi_{\text{ref}}\right)\right)\right],$$
$$\text{where} \quad r_{i,t}(\theta) = \frac{\pi_\theta\left(\mathbf{o}_{i,t} \mid q, \mathbf{o}_{i,<t}\right)}{\pi_{\theta_{\text{old}}}\left(\mathbf{o}_{i,t} \mid q, \mathbf{o}_{i,<t}\right)} \tag{1}$$

Differ from standard GRPO, we adopt fully on-policy training and token-level loss (Yu et al., 2025).

4 EXPERIMENTS

In this section, we present a series of experiments conducted across diverse benchmarks—including mathematics, medical, and health domains—covering both in-domain datasets and those outside the training mixture. We also compare SSRMs to strong prompted baselines. Our goal is to address three key questions: (1) Can the reasoning traces of SSRMs be audited, either through structured queries or statistical methods? (2) Can prompted models be audited in a similar manner? (3) Is semi-structured reasoning more difficult to learn? Detailed setups and dataset descriptions are listed in Appendix E.

Experimental Setup. We use Qwen2.5-7B (Yang et al., 2024) as the base model for SSRM and conduct auditability analysis on its generated semi-structured reasoning traces. To further validate the performance, we also train an SSRM based on Llama3.1-8B (Grattafiori et al., 2024). All models are trained using `verl` (Sheng et al., 2024) on 8 H100 GPUs, with evaluations conducted on 1 H100.

In addition to similarly trained unstructured baselines, we compare SSRMs against baselines of comparable size. Non-reasoning baselines include Llama3.1-8B-Instruct (Grattafiori et al., 2024), Medical Llama (ContactDoctor, 2024) (fine-tuned for biomedical knowledge), and the Qwen series (Yang et al., 2024). Reasoning baselines include the DeepSeek-Distilled series (Guo et al., 2025). For prompted baselines, we evaluate Claude Sonnet 3.5 (Anthropic, 2024) and Qwen2.5-7B-Instruct.

We use greedy decoding and report accuracy for all tasks, except for AIME24, where we sample 32 responses and report Pass@1 with a temperature of 0.7. The maximum generation length is set to 32,768 tokens. All tasks are evaluated in a zero-shot setting, except for prompted baselines, which use two-shot prompts. Reasoning baselines follow the recommended setting (temperature 0.6, top-p 0.95) (Guo et al., 2025). For Qwen2.5-7B, we omit the chat template following Liu et al. (2025).

Primary Evaluation: MedCalcBenchV2. Our primary evaluation benchmark is MedCalcBenchV2, a cleaned version of MedCalcBench (Khandekar et al., 2024) (See Appendix E.1). MedCalcBenchV2 measures an LLM's ability to extract information from clinical text (*patient note*) and perform calculations using either explicit rules or formulas provided in the prompt. We observe that rule-based tasks are substantially more challenging than formula-based tasks. Errors in formula-based problems primarily arise from computation or extraction mistakes, whereas errors in rule-based problems more often involve failures to follow explicit instructions, consistent with prior findings on rule-following tasks (Sun et al., 2024a). To account for this discrepancy, we treat the two categories as two distinct tasks: MedCalcV2 Rules and MedCalcV2 Formulas. Evaluation follows the original MedCalcBench criteria, which allow small numeric deviations and employ rule-based checks for date-based answers.

Domain-Specific Language for structured audits. Our DSL for structured audits looks like Python unit tests: they are class methods, can be called without arguments, and contain assertion statements invoked by the class method `self.assertTrue`. An audit fails if it raises an exception or if an `assertTrue` call does not hold. The method can access a Pandas DataFrame `self.df` that represents the semi-structured trace, and assertions usually operate on this data structure using Pandas operations.

Additional Evaluation. To evaluate the generalizability of the SSRMs beyond in-domain data, we conduct additional experiments on a range of benchmarks: general reasoning (GPQA-Diamond (Rein et al., 2024)), mathematical reasoning (AIME24), commonsense reasoning (CommonsenseQA (Talmor et al., 2018)), truthfulness (TruthfulQA (Lin et al., 2021)), as well as several medical and health-related tasks, namely MedQA (Jin et al., 2020), the biology and health subsets of MMLU-Pro (Wang et al., 2024), and PubMedQA (Jin et al., 2019), which we convert to multiple-choice.

4.1 EXPERIMENTAL RESULTS

Both hand-crafted and LLM-generated structured audits are effective for auditing semi-structured reasoning traces generated by SSRMs. To validate that semi-structured reasoning can be systematically audited, we first apply hand-crafted audits for the two MedCalcV2 tasks based on the analysis of the training examples. Table 1 reports results for all individual audits that are applied with sufficient frequency[1] and are sufficiently discriminative—specifically, audits that succeed at least 5% of the time and fail at least 5% of the time. The second audit for MedCalcV2 Formulas (e.g., "math is correct") uses Python's `eval` function; whereas all other audits inspect only trace structure.

[1]Audits may not be applied to all traces—for example, one cannot confirm that number of rules evaluated is the same as the number of rules extracted if rule extraction fails to produce a legal output.

Table 1: Hand-crafted structured audits for Qwen SSRM generated semi-structured traces on two MedCalcV2 tasks. For each, we report the failure rate, the outcome accuracy conditioned on audit failing or passing, the accuracy difference (Δ) between passing and failing cases, and the p-value for testing $\Delta \neq 0$. One star (*) for statistical significance at $p < 0.1$ and two stars (**) for $p < 0.05$.

		– accuracy and difference –				
	%Failed	Failing	Passing	Δ	p-val	description of audit
MedCalcV2 Formulas	22.0	0.77	0.86	0.09		`solve_formula` output is formatted well
	49.0	0.84	0.83	-0.01		`solve_formula` math is correct$^{\text{math}}$
MedCalcV2 Rules	13.2	0.22	0.46	0.24	**	one `get_data` step per extracted rule
	13.4	0.22	0.47	0.25	**	`get_data` called on all rules
	14.0	0.21	0.47	0.26	**	one `eval_rule` step per rule
	20.3	0.26	0.48	0.22	**	all rule outputs summed correctly

Table 2: LLM-generated structured audits on the same set of Qwen SSRM traces for MedCalcV2.

		– accuracy and difference –				
	%Failed	Failing	Passing	Δ	p-val	description of audit
MedCalcV2 Formulas	5.7	0.76	0.84	0.08		step 4 output feeds into step 5 input
	6.4	0.45	0.86	0.41	**	step 3 output feeds into step 4 input
	8.3	0.62	0.86	0.24		sstep 2 output feeds into step 3 input
	9.3	0.82	0.84	0.02		convert_units called once per datapoint
	10.2	0.81	0.84	0.03		convert_units receives formula as first input
	12.6	0.80	0.84	0.05		convert_units correct second input
	14.4	0.37	0.92	0.55	**	get_data receives formula from analyze_input
MedCalcV2 Rules	13.4	0.22	0.47	0.25	**	get_data called for each rule
	13.9	0.21	0.47	0.26	**	consistent rules across get_data steps

As suggested in Figure 2, reasoning flaws do not always yield incorrect outcomes. In Table 1, for each audit a, we present test accuracy when a fails ("Failing" column), when a passes ("Passing" column), the accuracy difference Δ, and the statistical significance of the difference being non-zero.

The results suggest that reasoning errors are more frequent in MedCalcV2 Rules than in Formulas. While math errors in Formulas occur frequently, they do not correlate with outcome errors.[2] In contrast, reasoning errors in Rules are common and associated with substantial accuracy losses. The most common failure is mis-summing rule contributions, followed by skipping a rule. Other failing audits indicate mismatches between the counts of patient data extraction and rule application steps.

Because manually generating audits is expensive, we also explore automatic generation of structured audits using LLMs. We manually write audits for three additional tasks from BBH, and use those as few-shot examples to prompt Claude Sonnet 4.0 to output structured audits given a set of three correct sample traces. The results, shown in Table 2, show that LLM-generated structured audits are comparably useful to hand-crafted ones. (For more results and details, please check Appendix E.4)

Typicality audits are also applicable for auditing semi-structured reasoning traces generated by SSRMs. Typicality audits provide a complementary use of the semi-structured format by analyzing *abstract versions of reasoning processes*, aka "reasoning patterns" (Zhang et al., 2025). Prior work has conjectured that LLMs predominantly reproduce "reasoning patterns" observed in the training data and struggle to generate novel sequences—i.e., LLM reasoning often relies on retrieving previously seen reasoning examples (Kambhampati et al., 2025). If this holds, reasoning within a given task should exhibit regularity, thereby enabling statistical analyses to flag outlier traces as potential errors.

In past work, "reasoning patterns" are typically identified heuristically or by LLMs (Zhang et al., 2025; Zhou et al., 2025). Here we define "reasoning patterns" as the sequence of step names. For example, in Figure 2, the pattern is "`analyze_input extract_patient_data convert_units evaluate_rule accumulate_score extract_patient_data evaluate_rule accumulate_score`". We then construct a probabilistic model M over these sequences, treating them as language tokens. This formulation yields a precise version of the conjecture above: *LLM correctness is positively correlated with the*

[2]MedCalcV2 numerical answers are soft-matched to the target, whereas the implemented audits check exact equivalence before and after simplification.

probability of the required reasoning pattern under M. To test this, we compute the correlation between outcome correctness and the probability of the reasoning pattern generated by the SSRMs.

We consider the following types of *reason pattern typicality* models M: a *unigram* language model smoothed with a Dirichlet prior (referred to as *multinomial* in the tables below); *bigram* and *trigram* models, implemented simply by extending the base vocabulary to consider all n-grams of step names for $n = 2, 3$; an HMM with three hidden states over trigrams, denoted *HMM(3,3)* in the table; and a final model, *HMM**, in which we perform a grid search over different n-gram sizes and numbers of hidden states, selecting the configuration that optimizes the BIC criterion (Please see Appendix E.2).

Table 3 summarizes the results obtained by fitting these models to the test data.[3] We use Kendall's τ to measure correlation because it makes no parametric assumptions and observe only moderate correlations ranging from 0.08 to 0.26. As another way of testing if highly typical reasoning patterns correspond to higher accuracies than atypical ones, we sort all test predictions by pattern probability and compare the accuracy of the least probable third with that of the most probable third, excluding the middle third. Using this method, we observe significant differences in many models. For example, under the *HMM(3,3)* model, the accuracy difference (Δ) between the least and most probable tertiles is approximately 25% for both MedCalcV2 tasks. These results suggest that *a typicality model* M can approximate the behavior of a structured audit by: (a) sorting predictions by typicality, (b) splitting predictions into quantiles, and (c) interpreting the top quantile as audit-passed cases, the bottom quantile as audit-failed cases, and any other intermediate quantiles as unevaluable. Following this approach, the typicality audits function as an *abstaining classifier* Pietraszek (2005) for evaluating outcome correctness, abstaining specifically on intermediate scores. The abstention rate can be adjusted by splitting the predictions into different numbers of quantiles: for instance, dividing predictions into two groups yields no abstentions, whereas dividing them into eight groups results in abstention for the middle six octiles—i.e., $\frac{6}{8} = \frac{3}{4}$ of the time. As shown in Figure 3, higher abstention rates are associated with larger accuracy difference (Δ) across both MedCalcV2 tasks.

Table 3: The results prove that atypical reasoning pattern in the MedCalcV2 tasks are more likely to result in errors. We evaluate several typicality/probability models, all of which correlate with correctness, though the correlation is weaker on MedCalcV2 Rules. In additional to Kendall's τ for correlation, we also partition the test data into three equal groups by probability and report accuracy in the lowest and highest tertiles, the accuracy difference (Δ), and the p-value of this difference.

MedCalcV2 Formulas		– accuracy and difference –			
	τ	Tertile 1	Tertile 3	Δ	p-val
multinomial	0.25	0.72	0.95	0.22	*
bigram	0.25	0.72	0.95	0.23	*
trigram	0.26	0.72	0.95	0.23	*
HMM(3,3)	0.26	0.72	0.97	**0.25**	**
HMM*	0.21	0.74	0.97	0.07	
MedCalcV2 Rules		– accuracy and difference –			
	τ	Tertile 1	Tertile 3	Δ	p-val
multinomial	0.17	0.32	0.57	**0.25**	**
bigram	0.17	0.32	0.57	**0.25**	**
trigram	0.17	0.32	0.57	**0.25**	**
HMM(3,3)	0.17	0.32	0.57	**0.25**	**
HMM*	0.08	0.43	0.52	0.09	

Given the effectiveness of both structured and typicality audits in identifying potential reasoning errors, a straightforward extension is to apply apply at inference time. For example, when combining typicality audits with self-consistency, reasoning traces in the lowest tertile can be resampled more extensively, whereas those in the highest tertile—more likely to be correct—might require no additional sampling. This strategy can help concentrate the sampling budget on the most error-prone cases. We evaluate this approach on the MedCalcV2 Rules tasks and report the results in Appendix E.7. Our findings show that audit-guided self-consistency reduces computational cost while maintaining comparable or slightly improved performance relative to vanilla self-consistency with a fixed sampling budget. However, we do not observe significant improvements over greedy decoding with a

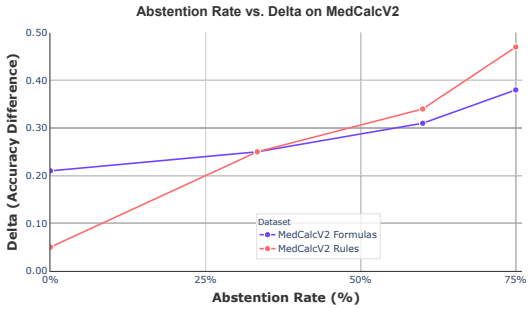

Figure 3: Abstention Rate vs. Δ (accuracy difference between the highest- and lowest- probability quantiles) on MedCalcV2 under a typicality audit.

[3]Note that the correctness label is not used in this step. Moreover, the distribution of reasoning patterns in the training data may differ for tasks with ground-truth reasoning chains.

Table 4: Results of applying structured audits to Claude Sonnet 3.5 with semi-structured prompting on both MedCalcV2 Formulas and Rules. Overall, the results resemble those of SSRM, although the prompted MedCalcV2 Formulas system does have some rarely-failing audits that impact accuracy.

		– accuracy and difference –				
	%Failed	Failing	Passing	Δ	p-val	description of audit
MedCalcV2 Formulas	1.7	0.000	0.662	0.662		one `get_datastep`
	2.1	0.000	0.664	0.664	*	one `insert_variables` step
Claude Sonnet 3.5	3.8	0.091	0.673	0.582	**	`solve_formula` output is a number^{math}
(65.1% acc)	9.2	0.593	0.657	0.064		`solve_formula` output is formatted correctly
	47.3	0.667	0.636	-0.030		`solve_formula` math is correct^{math}
MedCalcV2 Rules	5.8	0.182	0.399	0.218		analyze_input returns correct # values
	14.7	0.196	0.420	0.223	**	one `convert_units` step per rule
Claude Sonnet 3.5	14.7	0.196	0.420	0.223	**	one `get_data` step per rule
(38.7% acc)	15.8	0.183	0.425	0.242	**	one `evaluate_rule` step per rule
	17.1	0.169	0.432	0.263	**	one `accumulate_score` step per rule

single sample. We hypothesize that traces flagged as incorrect by audits may correspond to problems that the model struggles to solve even with additional sampling budget. We leave this to future work.

Both structured and typicality audits can be applied to semi-structured traces from few-shot prompted models. Table 4 presents the results of structured audits applied to few-shot prompted Claude Sonnet 3.5 on both MedCalcV2 tasks, while Table 5 shows typicality audit results for the same model (limited to three representative typicality models for brevity). Overall, Claude Sonnet 3.5 behaves similarly to SSRMs under these audits, except for the typicality audit for Rules, where more typical reasoning traces exhibit *higher* error rates on average (although not significantly so). This may be attributed to the high error rate of the prompted model itself: even typical reasoning processes often lead to errors. In Appendix E.4, we present results for Qwen2.5-7B-Instruct—a weaker prompted model and the instruction-tuned version of the Qwen2.5-7B base used as the backbone for SSRMs—as well as LLM-generated structured audit results on prompted Claude.

Table 5: Results of applying typicality audits to semi-structured reasoning traces from few-shot prompted Claude Sonnet 3.5 on both MedCalcV2 Formulas and Rules. Overall, the results indicate that the hypothesis—that atypical reasoning patterns correspond to higher error rates—holds for MedCalcV2 Formulas but not for the noisier Rules.

		– accuracy and difference –			
	τ	Tertile 1	Tertile 3	Δ	p-val
MedCalcV2 Formulas					
Claude Sonnet 3.5 (65.1% acc)					
trigram	0.13	0.56	0.67	0.11	
HMM(3,3)	0.21	0.56	0.70	0.14	
HMM*	0.30	0.54	0.87	0.33	**
MedCalcV2 Rules					
Claude Sonnet 3.5 (38.7% acc)					
trigram	-0.06	0.47	0.33	-0.14	
HMM(3,3)	-0.06	0.46	0.32	-0.14	
HMM*	-0.05	0.43	0.33	0.00	

Semi-structured reasoning is learnable and achieves performance and generalization on par with unstructured reasoning. We consider two different model families for training SSRMs: a stronger one based on Qwen2.5-7B and a weaker one based on Llama3.1-8B. Our trained Qwen SSRM achieves strong results, as shown in Table 6. On average, *it outperforms the unstructured reasoning baseline trained with the same procedure*. On the two challenging MedCalcV2 tasks, it exceeds six other strong baselines of comparable size. Within the training mixture, it outperforms the best baseline by nearly ten points. Moreover, it generalizes effectively to tasks outside the training mixture: while it does not match the top math-specialized reasoning models, it outperforms all non-reasoning baselines. On a range of medical QA benchmarks, it achieves performance comparable to reasoning models, lagging only slightly behind BioMedical-Llama-3-8B, a specialized model for biomedical knowledge. By contrast, although the Llama SSRM is based on a weaker backbone, it delivers performance comparable to similarly trained unstructured reasoning baselines on both in-domain and out-of-domain benchmarks, further supporting that semi-structured reasoning does not compromise performance.

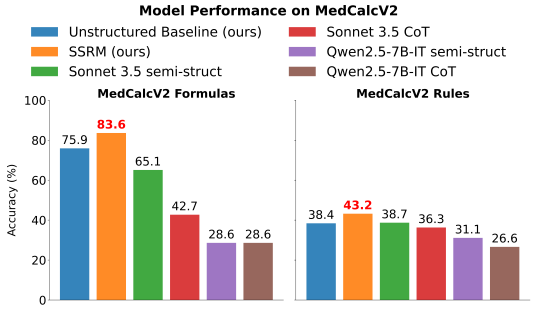

Figure 4: Comparison of SSRM against similarly trained and prompted baselines on MedCalcV2.

Table 6: Our models are initialized from Qwen2.5-7B and Llama3.1-8B, trained with SFT followed by RLVR on a mix of MedCalcV2 and other math tasks. Underlined results indicate the best performance among our comparably trained models; starred results denote best among non-reasoning models; and bold results are best overall. On average, SSRM outperforms the unstructured CoT format and six strong, comparably sized baseline models.[4] We sample 32 times and report Pass@1 for AIME24.

	SSRM from Qwen2.5-7B (ours)					SSRM from Llama3.1-8B (ours)				Instr/reasoning LLMs (Qwen2.5-7B)			Instr/reasoning LLMs (Llama3/3.1-8B)		
	Base	unstr. +SFT	unstr. ++RL	semi-str. +SFT	semi-str. ++RL	unstr. +SFT	unstr. ++RL	semi-str. +SFT	semi-str. ++RL	Instr	*OpR1	*DSeek	BioL	Instr	*DSeek
MedCalcV2 Formulas	3.0	52.4	75.9	63.3	***83.6**	48.7	58.9	56.9	75.0	56.4	44.8	36.9	12.6	10.0	29.7
MedCalcV2 Rules	0.0	27.4	38.4	38.9	***43.2**	27.9	28.4	20.8	36.3	32.1	22.6	14.2	16.6	9.5	9.2
GSM8k	85.4	74.4	90.5	76.6	*90.9	42.3	43.8	57.4	36.2	*90.9	**94.8**	89.2	51.9	81.1	75.7
MATH500	69.2	44.6	77.0	45.4	75.2	15.6	15	22.6	12.4	***78.8**	91.0	94.0	17.4	46.2	87.2
train mix avg	*39.4*	*49.7*	*70.5*	*56.1*	****73.2***	33.6	36.5	39.4	40	*64.6*	*63.3*	*58.6*	*24.6*	*36.7*	*50.5*
AIME24	9.1	1.1	12.1	3.7	*12.4	0.3	0.5	0.2	0.9	11.8	45.3	**53.4**	0.1	2.1	45.2
GPQA-D	31.8	31.8	*38.4	30.3	34.3	25.8	33.8	22.2	26.3	32.8	41.4	**50.5**	26.8	31.8	43.9
TruthfulQA	49.7	57.3	56.3	41.1	54.3	41.6	39.8	31.3	32.9	***55.6**	42.6	47.5	53.0	54.3	52.6
CommonsenseQA	70.5	70.1	72.8	70.8	***75.7**	52.7	52.4	58.6	60.4	66.8	54.0	52.3	39.3	50.4	63.1
MedQA	57.4	62.4	62.0	55.9	61.4	55.4	57	50.4	53	*62.8	31.1	36.4	**76.9**	68.9	58.1
MMLU Pro Bio	64.6	68.6	71.8	59.3	69.9	58	58.4	55.4	59.3	***73.5**	50.9	66.7	64.6	67.8	73.1
MMLU Pro Health	42.1	53.2	53.1	40.5	51.7	40.1	41.6	40.1	39.7	*54.8	22.0	33.4	53.1	**58.3**	46.5
PubmedQA	66.3	73.4	71.4	70.2	*76.2	73.9	75.5	68.2	73.4	73.5	73.3	72.7	**77.1**	75.6	73.8
med/health avg	*57.6*	*64.4*	*64.6*	*56.5*	*64.8*	56.9	58.1	53.5	56.4	*66.2*	*44.3*	*52.3*	****67.9***	*67.7*	*62.9*
overall avg	*45.3*	*51.3*	*60.8*	*50.2*	****61.7***	*39.7*	*41.7*	*40.3*	*42*	*58.0*	*52.1*	*54.3*	*39.5*	*45.6*	*54.6*

[4]All accuracies are percentages. "Instr" models are instruction-trained, "DSeek" are distilled from DeepSeek-R1, and OpR1 is OpenR1-Qwen-7B. BioL is Bio-Medical-Llama-3-8B.

In Figure 4, we show that Qwen SSRM not only outperforms the Claude Sonnet 3.5, which is used to seed the SFT training data, but also significantly outperforms the Qwen instruction-tuned variant. For Claude Sonnet 3.5 and Qwen2.5-7B-Instruct, we employ two-shot prompting, using two fixed demonstrations across all MedCalc "calculators".[5] For each prompted model, we evaluate two prompt variants: one with unstructured free-form CoT prompts and one with the semi-structured format.

We also analyze the token usage of Qwen SSRM and unstructured reasoning baselines (see Appendix E.5 for details). In summary, SSRM consume more tokens than the unstructured reasoning baselines on MedCalcV2 Tasks, while token usage is comparable on MATH500 and GPQA-D. One factor contributing to the increased usage is redundant argument and variable referencing, as shown in Figure 2. We leave the development of a more efficient referencing mechanism to future work.

5 Conclusion

We have presented methods for scalably testing whether an LLM adheres to a prescribed reasoning strategy on specific critical tasks. Our methods combine a Semi-Structured Reasoning Model (SSRM), which outputs reasoning steps in a semi-structured format, with methods for *auditing* these reasoning traces. We consider two challenging tasks: (a) extracting information from clinical text and (b) performing a series of calculations using the extracted values, based on either predefined rules or given formulas. These tasks are adapted from MedCalcBench, which has been cleaned, deduplicated, and restructured to separate the simpler formula-based tasks from the more complex rule-based ones.

We show that *structured reasoning audits* can reveal meaningful classes of likely reasoning errors for these tasks and qualitatively distinguish between the types of errors made across tasks and models. We further introduce *typicality audits*, which are probabilistic models trained on a corpus of semi-structured reasoning traces. Typicality audits approximate structured audits by (a) sorting predictions by typicality, (b) splitting predictions into quantiles, and (c) interpreting the top quantile as a pass and the bottom quantile as a fail. Both types of audits can be applied to few-shot prompted models.

Importantly, auditability appears to come without a cost in accuracy: overall, our Qwen SSRM model outperforms plausible baselines, including strong closed-source prompted models, an identically-trained unstructured baseline, and many other strong comparably-sized models. Likewise, the Llama SSRM demonstrates comparable performance relative to its identically-trained unstructured baseline.

[5]This also diverges from the MedCalcBench few-shot evaluation, which selects a single demonstration from the same calculator as the test instance.

Reproducibility Statement

To facilitate reproducibility, we provide detailed information on the datasets used (please see Appendix E), implementation details (please see Appendix D), and code (Anonymous Github Link).

References

Pranjal Aggarwal, Aman Madaan, Yiming Yang, et al. Let's sample step by step: Adaptive-consistency for efficient reasoning and coding with llms. *arXiv preprint arXiv:2305.11860*, 2023.

Anthropic. Claude 3.5 sonnet. https://www.anthropic.com/news/claude-3-5-sonnet, 2024.

Anthropic. Claude 3.7 sonnet. https://www.anthropic.com/news/claude-3-7-sonnet, 2025.

Guangsheng Bao, Hongbo Zhang, Linyi Yang, Cunxiang Wang, and Yue Zhang. Llms with chain-of-thought are non-causal reasoners. *arXiv preprint arXiv:2402.16048*, 2024.

Oliver Bentham, Nathan Stringham, and Ana Marasović. Chain-of-thought unfaithfulness as disguised accuracy, 2024. URL https://arxiv.org/abs/2402.14897.

Hyungjoo Chae, Yeonghyeon Kim, Seungone Kim, Kai Tzu-iunn Ong, Beong-woo Kwak, Moohyeon Kim, Seonghwan Kim, Taeyoon Kwon, Jiwan Chung, Youngjae Yu, et al. Language models as compilers: Simulating pseudocode execution improves algorithmic reasoning in language models. *arXiv preprint arXiv:2404.02575*, 2024.

Qiguang Chen, Libo Qin, Jinhao Liu, Dengyun Peng, Jiannan Guan, Peng Wang, Mengkang Hu, Yuhang Zhou, Te Gao, and Wangxiang Che. Towards reasoning era: A survey of long chain-of-thought for reasoning large language models. *arXiv preprint arXiv:2503.09567*, 2025a.

Wenhu Chen, Xueguang Ma, Xinyi Wang, and William W Cohen. Program of thoughts prompting: Disentangling computation from reasoning for numerical reasoning tasks. *arXiv preprint arXiv:2211.12588*, 2022.

Yanda Chen, Joe Benton, Ansh Radhakrishnan, Jonathan Uesato, Carson Denison, John Schulman, Arushi Somani, Peter Hase, Misha Wagner, Fabien Roger, et al. Reasoning models don't always say what they think. *arXiv preprint arXiv:2505.05410*, 2025b.

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.

Cassandra A Cohen and William W Cohen. Watch your steps: Observable and modular chains of thought. *arXiv preprint arXiv:2409.15359*, 2024.

ContactDoctor. Bio-medical: A high-performance biomedical language model. https://huggingface.co/ContactDoctor/Bio-Medical-Llama-3-8B, 2024.

Noura Dridi and Melita Hadzagic. Akaike and Bayesian information criteria for hidden Markov models. *IEEE Signal processing letters*, 26(2):302–306, 2018.

Xidong Feng, Ziyu Wan, Muning Wen, Stephen Marcus McAleer, Ying Wen, Weinan Zhang, and Jun Wang. Alphazero-like tree-search can guide large language model decoding and training. *arXiv preprint arXiv:2309.17179*, 2023.

Luyu Gao, Aman Madaan, Shuyan Zhou, Uri Alon, Pengfei Liu, Yiming Yang, Jamie Callan, and Graham Neubig. Pal: Program-aided language models. In *International Conference on Machine Learning*, pp. 10764–10799. PMLR, 2023.

Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Alex Vaughan, et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024.

Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025.

Chengsong Huang, Langlin Huang, Jixuan Leng, Jiacheng Liu, and Jiaxin Huang. Efficient test-time scaling via self-calibration. *arXiv preprint arXiv:2503.00031*, 2025.

Alon Jacovi and Yoav Goldberg. Towards faithfully interpretable NLP systems: How should we define and evaluate faithfulness? In Dan Jurafsky, Joyce Chai, Natalie Schluter, and Joel Tetreault (eds.), *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, pp. 4198–4205, Online, July 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.acl-main.386. URL https://aclanthology.org/2020.acl-main.386.

Di Jin, Eileen Pan, Nassim Oufattole, Wei-Hung Weng, Hanyi Fang, and Peter Szolovits. What disease does this patient have. *A Large-scale Open Domain Question Answering Dataset from Medical Exams. arXiv [cs. CL]*, 2020.

Qiao Jin, Bhuwan Dhingra, Zhengping Liu, William Cohen, and Xinghua Lu. Pubmedqa: A dataset for biomedical research question answering. In *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP)*, pp. 2567–2577, 2019.

Subbarao Kambhampati, Kaya Stechly, and Karthik Valmeekam. (how) do reasoning models reason? *Annals of the New York Academy of Sciences*, 2025.

Nikhil Khandekar, Qiao Jin, Guangzhi Xiong, Soren Dunn, Serina Applebaum, Zain Anwar, Maame Sarfo-Gyamfi, Conrad Safranek, Abid Anwar, Andrew Zhang, et al. Medcalc-bench: Evaluating large language models for medical calculations. *Advances in Neural Information Processing Systems*, 37:84730–84745, 2024.

Levente Kocsis and Csaba Szepesvári. Bandit based monte-carlo planning. In *European conference on machine learning*, pp. 282–293. Springer, 2006.

Aviral Kumar, Vincent Zhuang, Rishabh Agarwal, Yi Su, John D Co-Reyes, Avi Singh, Kate Baumli, Shariq Iqbal, Colton Bishop, Rebecca Roelofs, et al. Training language models to self-correct via reinforcement learning. *arXiv preprint arXiv:2409.12917*, 2024.

Xin Lai, Zhuotao Tian, Yukang Chen, Senqiao Yang, Xiangru Peng, and Jiaya Jia. Step-dpo: Step-wise preference optimization for long-chain reasoning of llms. *arXiv preprint arXiv:2406.18629*, 2024.

Tamera Lanham, Anna Chen, Ansh Radhakrishnan, Benoit Steiner, Carson Denison, Danny Hernandez, Dustin Li, Esin Durmus, Evan Hubinger, Jackson Kernion, et al. Measuring faithfulness in chain-of-thought reasoning. *arXiv preprint arXiv:2307.13702*, 2023.

Chengshu Li, Jacky Liang, Andy Zeng, Xinyun Chen, Karol Hausman, Dorsa Sadigh, Sergey Levine, Li Fei-Fei, Fei Xia, and Brian Ichter. Chain of code: Reasoning with a language model-augmented code emulator. *arXiv preprint arXiv:2312.04474*, 2023.

Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. Let's verify step by step. In *The Twelfth International Conference on Learning Representations*, 2023.

Stephanie Lin, Jacob Hilton, and Owain Evans. Truthfulqa: Measuring how models mimic human falsehoods. *arXiv preprint arXiv:2109.07958*, 2021.

Zichen Liu, Changyu Chen, Wenjun Li, Penghui Qi, Tianyu Pang, Chao Du, Wee Sun Lee, and Min Lin. Understanding r1-zero-like training: A critical perspective. *arXiv preprint arXiv:2503.20783*, 2025.

Qing Lyu, Shreya Havaldar, Adam Stein, Li Zhang, Delip Rao, Eric Wong, Marianna Apidianaki, and Chris Callison-Burch. Faithful chain-of-thought reasoning, 2023. URL https://arxiv.org/abs/2301.13379.

OpenAI. Learning to reason with llms, 2024. URL https://openai.com/index/learning-to-reason-with-llms/.

Bhargavi Paranjape, Scott Lundberg, Sameer Singh, Hannaneh Hajishirzi, Luke Zettlemoyer, and Marco Tulio Ribeiro. Art: Automatic multi-step reasoning and tool-use for large language models. *arXiv preprint arXiv:2303.09014*, 2023.

Letitia Parcalabescu and Anette Frank. On measuring faithfulness or self-consistency of natural language explanations, 2024. URL https://arxiv.org/abs/2311.07466.

Debjit Paul, Robert West, Antoine Bosselut, and Boi Faltings. Making reasoning matter: Measuring and improving faithfulness of chain-of-thought reasoning. *arXiv preprint arXiv:2402.13950*, 2024.

Tadeusz Pietraszek. Optimizing abstaining classifiers using roc analysis. In *Proceedings of the 22nd international conference on Machine learning*, pp. 665–672, 2005.

Qwen. Qwq: Reflect deeply on the boundaries of the unknown, 2024. URL https://qwenlm.github.io/blog/qwq-32b-preview/.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R Bowman. Gpqa: A graduate-level google-proof q&a benchmark. In *First Conference on Language Modeling*, 2024.

Abulhair Saparov and He He. Language models are greedy reasoners: A systematic formal analysis of chain-of-thought. *arXiv preprint arXiv:2210.01240*, 2022.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, YK Li, Y Wu, et al. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. *arXiv preprint arXiv:2402.03300*, 2024.

Guangming Sheng, Chi Zhang, Zilingfeng Ye, Xibin Wu, Wang Zhang, Ru Zhang, Yanghua Peng, Haibin Lin, and Chuan Wu. Hybridflow: A flexible and efficient rlhf framework. *arXiv preprint arXiv:2409.19256*, 2024.

Wangtao Sun, Chenxiang Zhang, XueYou Zhang, Xuanqing Yu, Ziyang Huang, Pei Chen, Haotian Xu, Shizhu He, Jun Zhao, and Kang Liu. Beyond instruction following: Evaluating inferential rule following of large language models. *arXiv preprint arXiv:2407.08440*, 2024a.

Zhiqing Sun, Longhui Yu, Yikang Shen, Weiyang Liu, Yiming Yang, Sean Welleck, and Chuang Gan. Easy-to-hard generalization: Scalable alignment beyond human supervision. *arXiv preprint arXiv:2403.09472*, 2024b.

Mirac Suzgun, Nathan Scales, Nathanael Schärli, Sebastian Gehrmann, Yi Tay, Hyung Won Chung, Aakanksha Chowdhery, Quoc V Le, Ed H Chi, Denny Zhou, et al. Challenging big-bench tasks and whether chain-of-thought can solve them. *arXiv preprint arXiv:2210.09261*, 2022.

Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. Commonsenseqa: A question answering challenge targeting commonsense knowledge. *arXiv preprint arXiv:1811.00937*, 2018.

Trieu H Trinh, Yuhuai Wu, Quoc V Le, He He, and Thang Luong. Solving olympiad geometry without human demonstrations. *Nature*, 625(7995):476–482, 2024.

Miles Turpin, Julian Michael, Ethan Perez, and Samuel Bowman. Language models don't always say what they think: unfaithful explanations in chain-of-thought prompting. *Advances in Neural Information Processing Systems*, 36, 2024.

Scott Viteri, Max Lamparth, Peter Chatain, and Clark Barrett. Markovian agents for truthful language modeling. *arXiv preprint arXiv:2404.18988*, 2024.

Jun Wang. A tutorial on llm reasoning: Relevant methods behind chatgpt o1. *arXiv preprint arXiv:2502.10867*, 2025.

Junlin Wang, Shang Zhu, Jon Saad-Falcon, Ben Athiwaratkun, Qingyang Wu, Jue Wang, Shuaiwen Leon Song, Ce Zhang, Bhuwan Dhingra, and James Zou. Think deep, think fast: Investigating efficiency of verifier-free inference-time-scaling methods. *arXiv preprint arXiv:2504.14047*, 2025.

Peiyi Wang, Lei Li, Zhihong Shao, RX Xu, Damai Dai, Yifei Li, Deli Chen, Y Wu, and Zhifang Sui. Math-shepherd: A label-free step-by-step verifier for llms in mathematical reasoning. *arXiv preprint arXiv:2312.08935*, 2023.

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models. *arXiv preprint arXiv:2203.11171*, 2022.

Yubo Wang, Xueguang Ma, Ge Zhang, Yuansheng Ni, Abhranil Chandra, Shiguang Guo, Weiming Ren, Aaran Arulraj, Xuan He, Ziyan Jiang, et al. Mmlu-pro: A more robust and challenging multi-task language understanding benchmark. *arXiv preprint arXiv:2406.01574*, 2024.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35:24824–24837, 2022.

Nathaniel Weir, Muhammad Khalifa, Linlu Qiu, Orion Weller, and Peter Clark. Learning to reason via program generation, emulation, and search. *arXiv preprint arXiv:2405.16337*, 2024.

Huajian Xin, ZZ Ren, Junxiao Song, Zhihong Shao, Wanjia Zhao, Haocheng Wang, Bo Liu, Liyue Zhang, Xuan Lu, Qiushi Du, et al. Deepseek-prover-v1. 5: Harnessing proof assistant feedback for reinforcement learning and monte-carlo tree search. *arXiv preprint arXiv:2408.08152*, 2024.

An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, et al. Qwen2. 5 technical report. *arXiv preprint arXiv:2412.15115*, 2024.

An Yang, Anfeng Li, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Gao, Chengen Huang, Chenxu Lv, et al. Qwen3 technical report. *arXiv preprint arXiv:2505.09388*, 2025.

Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Tom Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. *Advances in neural information processing systems*, 36:11809–11822, 2023.

Qiying Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Tiantian Fan, Gaohong Liu, Lingjun Liu, Xin Liu, et al. Dapo: An open-source llm reinforcement learning system at scale. *arXiv preprint arXiv:2503.14476*, 2025.

Yufeng Zhang, Xuepeng Wang, Lingxiang Wu, and Jinqiao Wang. Enhancing chain of thought prompting in large language models via reasoning patterns. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pp. 25985–25993, 2025.

Ben Zhou, Sarthak Jain, Yi Zhang, Qiang Ning, Shuai Wang, Yassine Benajiba, and Dan Roth. Self-supervised analogical learning using language models. *arXiv preprint arXiv:2502.00996*, 2025.

A USE OF LLMS

Large Language Models (LLMs) are not used in paper writing or method formulation. Their involvement is limited to the experimental phase, where we leverage LLMs to seed SFT training data.

B LIMITATIONS

We demonstrate SSRMs' effectiveness on Qwen2.5-7B and Llama3.1-8B. Experiments with different architectures and extend to larger scales could help clarify the generalizability of the technique.

While symbolic audits provide a novel mechanism for monitoring behavior of LLMs, they can only capture some aspects of intended behavior. If audit coverage is incomplete, a model might pass all audits while following a logically incorrect reasoning process. (This limitation is analogous to the use of unit tests in software development, where test coverage is often incomplete). Additionally, models can execute individual steps incorrectly—a failure mode that reasoning audits typically fail to detect.

Typicality audits identify reasoning traces that are unusual, which need not be correlated with traces that are incorrect (e.g., if a model has a high error rate, highly typical traces might still be incorrect.)

In this study, we conducted only preliminary experiments integrating test-time-scaling with audits. Further investigations into effectively combining audits with test-time-scaling methods—such as audit-based self-consistency—to show their utility during inference time are left for future work.

C BROADER IMPACTS

This paper introduces Semi-Structured Reasoning Models (SSRMs) and presents two types of audits to identify probable reasoning errors in the semi-structured reasoning traces: (1) hand-crafted or LLM-generated structured audits and (2) probabilistic model-based typicality audits. Our goal is to detect undesirable reasoning shortcuts for LLMs while maintaining good downstream performance.

D TRAINING DETAILS

Detailed hyperparameters configurations for both Stage 1 (SFT) and Stage 2 (RLVR) are provided in Table 7. We provide the detailed settings in subsequent subsections to support reproducibility.

D.1 SUPERVISED FINE-TUNING (SFT) CONFIGURATIONS

Table 5 presents the system prompt template we used for SSRMs. The same system prompt is used for both Stage 1 and 2. Table 6 shows a semi-structured reasoning trace from GSM8K used for SFT.

D.2 REINFORCEMENT LEARNING WITH VERIFIABLE REWARDS (RLVR) CONFIGURATIONS

Reward Design. We employ two types of rule-based rewards functions for reinforcement learning:

- **Outcome Rewards:** The generated response will be assigned a reward of 1 for correct answers and 0 for incorrect answers. For the MedCalcV2 data, we follow the original proposed protocol and allow for a range-based evaluation; for the DAPO math data, we use an exact-match criterion.
- **Format Rewards:** We require all models to format its responses using tags such as `<think>` and `<answer>`. For SSRMs specifically, additionally require the tags `<partial_program>` and `<program_trace>`, define at least three functions within the `<partial_program>` block, and exclusively invoke these functions within the `<program_trace>` block. Given the regular syntax of semi-structured reasoning traces, these constraints can be enforced via regular expressions.

Table 7: Hyperparameter settings for supervised fine-tuning (SFT) and reinforcement learning with verifiable rewards (RLVR). Both the semi-structured reasoning and CoT baseline settings use the same set of hyperparameters. †: max sequence length for SFT and max generation length for RLVR.

Hyperparameter	**SFT**	**RLVR**
Optimizer	AdamW	AdamW
Actor Learning Rate	1e-5	1e-6
Weight Decay	1e-4	0.1
Warmup Ratio	0.1	0.01
Prompt Length	-	2048
Max Length†	16384	4096
Loss Agg Mode	-	token_mean
Grad Clip	0.2	1.0
Batch Size	128	256
MiniBatch Size	-	256 (On-Policy)
Num Responses Per Prompt	-	8
Temperature	-	1.0
Sequence Packing	False	True
Entropy Coeff	-	0.0
KL Loss Coeff	-	0.0
Epochs	5	10

System Prompt Template for Semi-Structured Reasoning Models (SSRMs)

```
A conversation between User and Assistant. The User asks a question, and the Assistant solves it. The
↪  assistant first reasons through the problem by generating high-level partial programs with key parts
↪  hidden using "..." markers. It then simulates programs trace based on the incomplete partial programs.
↪  The partial program must be general enough to solve all instances of the problem type, not just
↪  specific examples. The partial programs and traces are enclosed within <partial_program>
↪  </partial_program> and <program_trace> </program_trace> tags, while the overall reasoning process and
↪  final answer are enclosed within <think> </think> and <answer> </answer> tags, respectively. You
↪  should also wrap your final answer in $\\boxed{{ANSWER}}$ if it is a mathematical expression.

Format:
<think>
<partial_program>
[Partial Program here]
</partial_program>
<program_trace>
[Program Trace here]
</program_trace>
</think>
<answer>
[Final Answer here]
</answer>
```

Figure 5: System Prompt Template for Semi-Structured Reasoning Models (SSRMs).

Semi-Structured Reasoning Trace for GSM8K

```
<think>
<partial_program>
@traced
def analyze_input(input_str: str) -> tuple[str, ...]:
 """From an input, extract a tuple of individual questions and given values, as well as the final
 ↪  question to be answered.
 """
 ...



@traced
def convert_to_equations(input_questions: tuple[str, ...]) -> tuple[str, ...]:
 """Takes a list of questions and converts them to a list of mathematical equations.
 """
 ...



@traced
def simplify_equation(input_equation: str, given_values: tuple[str, ...]) -> str:
 """Takes an input equation and simplifies it by replacing a variable with the value of the variable as
 ↪  found in a list of given values if possible or performing algebraic operations if not, returning the
 ↪  simplified equation.
 """
 ...

</partial_program>

<program_trace>
Calling analyze_input("Two-fifths of the seats in an auditorium that holds 500 people are currently taken.
↪  It was found that 1/10 of the seats are broken. How many seats are still available?")...
...analyze_input returned ('An auditorium holds 500 people.', 'Two-fifths of the seats are currently
↪  taken.', '1/10 of the seats are broken.', 'How many seats are still available?')
Calling convert_to_equations(('An auditorium holds 500 people.', 'Two-fifths of the seats are currently
↪  taken.', '1/10 of the seats are broken.', 'How many seats are still available?'))...
...convert_to_equations returned ('total_seats = 500', 'occupied_seats = total_seats * 2/5',
↪  'broken_seats = total_seats * 1/10', 'available_seats = total_seats - occupied_seats - broken_seats')
Calling simplify_equation('available_seats = total_seats - occupied_seats - broken_seats', ('total_seats
↪  = 500', 'occupied_seats = total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 500 - occupied_seats - broken_seats'
Calling simplify_equation('available_seats = 500 - occupied_seats - broken_seats', ('total_seats = 500',
↪  'occupied_seats = total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 500 - (500 * 2/5) - broken_seats'
Calling simplify_equation('available_seats = 500 - (500 * 2/5) - broken_seats', ('total_seats = 500',
↪  'occupied_seats = total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 500 - 200.0 - broken_seats'
Calling simplify_equation('available_seats = 500 - 200.0 - broken_seats', ('total_seats = 500',
↪  'occupied_seats = total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 500 - 200.0 - (500 * 1/10)'
Calling simplify_equation('available_seats = 500 - 200.0 - (500 * 1/10)', ('total_seats = 500',
↪  'occupied_seats = total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 500 - 200.0 - 50.0'
Calling simplify_equation('available_seats = 500 - 200.0 - 50.0', ('total_seats = 500', 'occupied_seats =
↪  total_seats * 2/5', 'broken_seats = total_seats * 1/10'))...
...simplify_equation returned 'available_seats = 250.0'
</program_trace>
</think>
<answer>
250
</answer>
```

Figure 6: Semi-Structured Reasoning Trace for GSM8K.

E EXPERIMENTAL DETAILS

E.1 MEDCALCBENCH V2

The original MedCalcBench (Khandekar et al., 2024) contains examples from 55 distinct *calculators*, including target quantities such as the SIRS score from Figure 1. In the original study, average scores were reported across all calculators: 37.9% in the zero-shot setting with GPT-4 and 50.9% in the one-shot setting. In the latter, the demonstration always used the same calculator as the test case, thereby evaluating the model's ability to extract data and reproduce an identical reasoning chain.

For our input, we concatenate the patient note and the original question, followed by a concise definition of the relevant formulas or rules. In the long-context CoT setting (for SSRMs), this concatenation serves as the sole input. In prompt-based settings, we employ a single two-shot CoT demonstration involving calculations of the same *type* (formula or rules), though not necessarily the same *calculator*, thereby testing the LLM's ability to extract data and perform a potentially different calculation. Therefore, MedCalcV2 scores are not directly comparable to those of MedCalcBench.

We implement two additional changes. First, we remove training samples in the original MedCalcBench that overlap with the test data to ensure a clean evaluation. Second, during testing, we discovered errors in results for the Glasgow Coma Scale Calculator: each ground-truth explanation duplicates the verbal-response rule and erroneously adds its value twice, leading to incorrect final scores. We manually correct these errors by deleting the duplicate lines and adjusting the final values in both the ground-truth explanations and the expected outputs. MedCalcV2 will be made available.

E.2 TYPICALITY AUDIT CONFIGURATION

Results labeled HMM* are obtained via a grid search over hidden-state counts (1, 2, 5, 10) and n-gram sizes (1, 2, 3, 10, 25, 50), selecting the model with the lowest Bayesian Information Criterion (BIC) score (Dridi & Hadzagic, 2018). HMM are implemented using the `CategoricalHMM` class from `hmmlearn`, with preprocessing to convert sequences into n-gram representations. Each sequence is augmented with start and end tokens, an unknown-word token, and padded to a uniform length. We use the Fisher's exact test in `scipy.stats` for statistical significance of proportional differences.

E.3 PROMPT FOR LLM-GENERATED AUDITS

Generated audits are created by prompting Claude-Sonnet-4-20250514 using the following prompt, replacing the label [TASKNAME] with the name of the task the audits are being generated for.

Prompt for LLM-Generated Audits

```
The attached file 'Example Audits' contains examples of audit functions which run on the traced outputs
↪ of functions called mocks. Each audit function tests the output to ensure that the mock has been run
↪ correctly by testing individual parts of the traced output, ensuring that each function the mock
↪ expects has been called, that the correct outputs lead to the correct inputs, and so on.

The attached file 'audit.py' contains the code which runs audit functions. Use this file to reference the
↪ expected structure of the dataframe that audit functions call on.

The attached file 'Audit Targets for [TASKNAME]' contains several traced outputs for a mock function ,
↪ [TASKNAME]. Generate a set of audit functions matching the format and construction of the examples
↪ from 'Example Audits', which will test other traced outputs of the function [TASKNAME]. Your
↪ generated audits should not programmatically generate the messages for success or failure.

Return only the python code for your output, with no extraneous introduction or afterward. Do not encase
↪ your output in backticks. Make sure to include imports and an if-main function.
```

Figure 7: Prompt for LLM-Generated Audits.

E.4 ADDITIONAL RESULTS

Table 8 presents the comparison between the prompted Sonnet 3.5 model and a smaller prompted model, `Qwen2.5-7B-Instruct`, which is similar to the model we trained. The structured audits reveal

Table 8: Results of applying hand-coded structured audits to prompted models for MedCalcV2 tasks.

		− accuracy and difference −				
	%Failed	Failing	Passing	Δ	p-val	description of audit
MedCalcV2 Formulas	1.712	0.000	0.662	0.662	0.162	one "get_data" step
	2.055	0.000	0.664	0.664	0.086	one "insert_variables" step
Claude 3.5	3.767	0.091	0.673	0.582	0.033	"solve_formula" output is a number
(65.1% acc)	9.247	0.593	0.657	0.064	0.870	"solve_formula" output is formatted correctly
	47.260	0.667	0.636	-0.030	0.852	"solve_formula" math is correct
	3.425	0.000	0.296	0.296	0.126	"solve_formula" output is a string
	5.479	0.188	0.292	0.104	0.777	"solve_formula" output is a number
	7.192	0.381	0.279	-0.102	0.487	"solve_formula" output is formatted correctly
Qwen2.5-7B-Instruct	29.110	0.376	0.249	-0.128	0.140	"solve_formula" math is correct
(28.6% acc)	29.795	0.103	0.365	0.261	0.000	one "get_data" step
	30.137	0.102	0.366	0.264	0.000	one "insert_variables" step
	33.219	0.165	0.347	0.182	0.015	one "analyze_input" step
MedCalcV2 Rules	5.789	0.182	0.399	0.218	0.181	analyze_input returns two values
	14.737	0.196	0.420	0.223	0.028	one step per rule with step_fn of "convert_units"
Claude 3.5	14.737	0.196	0.420	0.223	0.028	one step per rule with step_fn of "get_data"
(38.7% acc)	15.789	0.183	0.425	0.242	0.015	one step per rule with step_fn of "check_rule"
	17.105	0.169	0.432	0.263	0.005	one step per rule with step_fn of "accumulate_score"
	1.316	0.400	0.309	-0.091	0.672	one step per rule with step_fn of "get_data"
	1.579	0.333	0.310	-0.023	1.000	one step per rule with step_fn of "convert_units"
Qwen2.5-7B-Instruct	1.579	0.333	0.310	-0.023	1.000	one step per rule with step_fn of "accumulate_score"
(31.1% acc)	1.579	0.333	0.310	-0.023	1.000	one step per rule with step_fn of "check_rule"
	2.632	0.500	0.305	-0.195	0.363	one step with step_fn of "analyze_input"
	4.737	0.389	0.307	-0.082	0.630	analyze_input returns two values

Table 9: Summary of LLM-generated audits on BBH tasks, using a prompted Claude Sonnet 3.5.

Task	Task Acc	Avg Audits/Example	Code Lines	Min p-value
geometric shapes	37.89%	14.50	128	< 0.001
formal fallacies	46.31%	10.75	107	< 0.001
causal judgement	57.48%	11.75	88	< 0.001
dyck languages	64.00%	27.00	89	< 0.001
disambiguation qa	82.63%	10.98	93	
ruin names	83.16%	10.00	105	
penguins in a table	87.21%	11.01	114	< 0.05
multistep arithmetic two	87.89%	12.00	95	
snarks	91.53%	85.72	109	
date understanding	87.89%	11.33	88	
logical deduction three objects	87.89%	10.99	94	
movie recommendation	91.05%	13.98	90	
reasoning about colored objects	94.21%	14.00	95	
word sorting	95.26%	19.82	110	< 0.05
boolean expressions	95.26%	6.09	92	< 0.05
temporal sequences	96.84%	12.98	90	
sports understanding	97.37%	7.00	71	< 0.05
hyperbaton	97.89%	7.00	69	< 0.001
tracking shuffled objects	98.95%	17.00	105	< 0.05
object counting	100.00%	9.00	70	
web of lies	100.00%	15.00	94	

that `Qwen2.5-7B-Instruct`'s performance diverges significantly from the larger Sonnet model. In the Formula task, Sonnet 3.5 exhibits no significant reasoning errors, whereas `Qwen2.5-7B-Instruct` frequently commits errors in the initial reasoning steps, resulting in substantially poorer outcomes. In the Rule task, `Qwen2.5-7B-Instruct` demonstrates a distinct failure mode than Sonnet model: it generates correctly structured solution traces, but then fails to execute each individual step correctly.

Table 9 shows results with LLM-generated audits on 21 tasks from the BBH benchmark suite. We report the number of lines of code in the generated audits, and the average number of audits that are run on each example. As a concise measure of the utility of the audits, we report the smallest p-value of any audit, as computed in Table 1 (i.e., for the null hypothesis that audit failure is not

associated with incorrect outputs.) A small p-value indicates that some LLM-generated audit does indeed provide information about an "interesting" reasoning failure. Nearly half of the generated audits have p-values less than 0.05, including all four of the tasks with the highest error rates.

Table 10: LLM-generated structured audits Claude Sonnet 3.5 Prompted Models for MedCalcV2.

		– accuracy and difference –				
	%Failed	Failing	Passing	Δ	p-val	description of audit
Formulas	1.71	0.00%	44.60%	44.60%	*	one get_data step
	0.34	0.00%	43.99%	43.99%		one analyze_input step
	0.68	0.00%	44.14%	44.14%		analyze_input returns tuple with 2 elements
	2.05	0.00%	44.76%	44.76%	*	one insert_variables step
	4.79	35.71%	44.24%	8.53%		convert_units called on each datapoint
	3.42	50.00%	43.62%	-6.38%		convert_units' second input is a datapoint
	0.68	50.00%	43.79%	-6.21%		convert_units's first input is the formula
	0.68	50.00%	43.79%	-6.21%		insert_variables' first input is the formula
	3.08	44.44%	43.82%	-0.63%		insert_variables' second input is an output of convert_units
	2.74	37.50%	44.01%	6.51%		get_data's inputs match the output of analyze_input
	0.34	0.00%	43.99%	43.99%		solve_formula's input is an output of insert_variables
	92.81	42.80%	57.14%	14.34%		final answer matches last solve_formula output
Rules	5.79	18.18%	39.94%	21.76%	*	analyze_input returns tuple with 2 elements
	14.74	19.64%	41.98%	22.33%	**	get_data called for each rule
	14.74	19.64%	41.98%	22.33%	**	consistent rules across get_data steps
	1.05	0.00%	39.10%	39.10%		convert_units inputs are outputs of get_data
	14.74	19.64%	41.98%	22.33%	**	convert_units called for each rule
	14.74	19.64%	41.98%	22.33%	**	consistent rules across convert_units steps
	2.63	0.00%	39.73%	39.73%	**	check_rule inputs are outputs of convert_units
	15.79	18.33%	42.50%	24.17%	**	check_rule called for each rule
	15.79	18.33%	42.50%	24.17%	**	consistent rules across check_rule steps
	0.79	0.00%	38.99%	38.99%		accumulate_score inputs are outputs of check_rule
	17.11	16.92%	43.17%	26.25%	**	accumulate_score called for each rule

Table 10 shows results of generated structured audits on the same reasoning traces used in Table 4.

E.5 Token Usage Analysis

As shown in Table 11, SSRM consumes more tokens than the unstructured reasoning baselines on MedCalcV2 Rules and Formulas, whereas token usage is comparable on MATH500 and GPQA-Diamond. The higher token consumption primarily results from redundant arguments and variable referencing, as illustrated in Figure 2. Developing a more efficient variable referencing mechanism is left for future work.

Dataset	Qwen Unstructured	Qwen SSRM
GSM8K	319.78	841.72
Math500	909.27	978.89
MedCalcV2 Formulas	411.80	1778.14
MedCalcV2 Rules	425.70	2260.87
GPQA Diamond	1608.33	1411.29
MedQA	359.25	1065.34

Table 11: Token usage of Qwen SSRM and corresponding unstructured baseline across datasets.

E.6 Evaluation Configurations

We use `Lighteval` for all evaluations. For non-reasoning models, we report accuracy using greedy decoding. For reasoning models, we set the temperature to 0.6 and top-p to 0.95. For the AIME24 dataset—where we observe high variance—we sample 32 responses using a temperature of 0.7 for non-reasoning models, while retaining the configurations for reasoning models, and report Pass@1.

E.7 Test-Time-Scaling with Typicality Audits

To investigate the effectiveness of combining test-time-scaling with audits, we apply typicality audits (HMM*). We perform a grid search using the first half of the generated responses from the benchmark; to ensure data integrity, we evaluate the model only on the second half. We consider two variants here: vanilla self-consistency and audit-based self-consistency. Given a sampling budget of k responses per question, in vanilla self-consistency we sample k times per question and use majority voting to determine the final answer. In audit-based self-consistency, we divide the model-generated traces into tertiles: for traces in top tertile we perform no additional sampling, for those in the middle tertile we sample $k-3$ additional times, and for those in the bottom tertile we sample $k-1$ additional times.

Table 12: Comparison of Self-Consistency and Audit-Based Self-Consistency on MedcalcV2 Rule.

Sampling Budget	**Self-Consistency**	**Audit-Based Self-Consistency**	**Effective Samples**
Greedy (Temp = 0)	44.2	44.2	-
Sampling (Temp = 0.7)	44.2	44.2	-
3	46.3	45.3	306 (53.68%)
5	45.3	46.8	522 (54.95%)
7	45.3	45.3	764 (57.44%)
9	45.3	45.3	1002 (58.60%)
15	45.3	44.2	1702 (59.72%)
30	45.2	46.3	3501 (61.42%)
60	44.7	45.8	7071 (62.03%)

We report accuracy on the MedCalcV2 Rule tasks, along with the effective number of samples—i.e., the actual number generated under the audit-based procedure. For vanilla self-consistency, the total number of samples is $k \times n$, where n is the number of questions in the corresponding benchmark.

As shown in Table 12, audit-based self-consistency consistently outperforms vanilla self-consistency given the same per-question sampling budget. More specifically, when $k = 5$, audit-based self-consistency outperforms vanilla self-consistency by 1.5 percentage points while using only 54.95% of the total sampling budget. These preliminary experiments demonstrate the effectiveness of combining typicality audits with test-time-scaling methods and suggest a promising direction for future research.