ACHIEVING EXACT FEDERATED UNLEARNING WITH IMPROVED POST-UNLEARNING PERFORMANCE

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ABSTRACT

Federated learning is a machine learning paradigm that allows multiple clients to train aggregated model via sharing model updates to a central server without sharing their data. Even though the data is not shared, it can indirectly influence the aggregated model via the shared model updates. In many real-life scenarios, we need to completely remove a client's influence (unlearning) from the aggregated model, such as competitive clients who want to remove their influence from the aggregated model (e.g., large language models (LLMs) fine-tuned collaboratively by multiple clients for a specific downstream task) after leaving the coalition to ensure other clients do not benefit from their contributions. The influence removal is also needed when the adversarial client negatively affects the aggregated model. Though the aggregated model can be retrained from scratch to ensure exact unlearning (completely removing the client's influence from the aggregated model), it performs poorly just after the unlearning, which is undesirable during deployment. To overcome this challenge, this paper proposes federated unlearning algorithms that ensure exact unlearning while achieving better performance post-unlearning. The proposed algorithms are problem-agnostic, making them applicable across various domains. Our experimental results further validate the effectiveness of the proposed federated unlearning algorithms in fine-tuning LLMs and performing vision tasks within a federated learning framework using real-world datasets.

1 INTRODUCTION

Individual users often lack sufficient data to train a state-of-the-art machine learning model. However, combining data from multiple users can significantly enhance model performance. *Federated learning* (FL) (Zhang et al., 2021) allows such collaboration among users (client) while preserving their data privacy, making it especially valuable in sectors like finance (Li et al., 2020) and health care (Xu et al., 2021). In FL, clients train models locally on their own data, while a central server iteratively aggregates their updates to refine a aggregated model. This approach is well-suited for many commercial applications as FL can train accurate models collaboratively without sharing user data. For example, multiple companies from the same industrial sector, such as banking, insurance, or healthcare, can collaborate and train a more accurate model without sharing sensitive data (Aledhari et al., 2020). Another example is improving the performance of large language models (LLMs) on downstream tasks by fine-tuning them using federated learning (Kuang et al., 2024; Wu et al., 2024).

Although FL algorithms do not directly access client data, the aggregated model is still influenced by the local models trained on each client's data. When a client leaves the collaboration, it is necessary to update the aggregated model to remove the influence of its data – a process known as *federated unlearning* (FU),[1] for example, companies leaving the collaboration may demand the removal of their contributions to ensure their competitors do not benefit from them. FU is also desirable to remove the influence of adversarial clients, i.e., the adversary behaves like a client and degrades the model performance by contributing contaminated updates (Fang et al., 2020). Additionally, the development of FU facilitates the exercise of the *right to be forgotten* formalized in many regional or government data regulations such as GDPR (2016) and CCPA (2018).

[1]This differs from the typical FL setting, where clients may be intermittently active or inactive during the training process.

We can trivially achieve FU by retraining the collaboration from scratch without the target client's data (Liu et al., 2023). Despite its simplicity, the new server model suffers from low performance at the onset, as it is restarted with random initialization and takes time to recover its performance. As a result, it slows down the deployment of the unlearned model as training large models on the collaboration of many users can be time-consuming. In many real-world scenarios such as healthcare, security and financial services, maintaining uninterrupted services is of critical importance, as delaying will cause disruptions with severe consequences (Rostek et al., 2022; Wallace et al., 2003; Liu & Ren, 2024). When those services are powered by FL (Wang et al., 2023; Prajapat et al., 2024; Roth et al., 2022; Bharati et al., 2022), sometimes it is imperative to perform unlearning due to reasons such as data poisoning attack (Tolpegin et al., 2020; Shi et al., 2022b) or the clients discover post-hoc they inadvertently included low-quality or sensitive data in their training data, and it becomes critical to minimize the delay due to the unlearning process and resume the service as soon as possible. It naturally raises a question: *How can we guarantee the exact federated unlearning with minimal delay while ensuring better post-unlearning performance ?*

This paper proposes two novel methods for achieving exact FU *without delay* while improving post-unlearning performance. The first method, Bi-Models Training (BMT) (Section 3.1), preserves isolated copies of local models and reuses clients' existing knowledge residing in these models during unlearning for better aggregation. Despite being unlearning-friendly, these local models fail to capture the joint influence of multiple clients on the global model. Training the power set of clients can capture the influence of all possible influences of the clients but is computationally expensive and may lead to *double influence*, where a client affects multiple sub-FL models. As a result, we propose the second method, Multi-Models Training (MMT) (Section 3.2), that trains each sub-FL model on disjoint subsets of clients to avoid double influence and aggregates the best sub-FL models upon unlearning to achieve improved initialization of the aggregated model. Our experimental results further validate the effectiveness of BMT and MMT in fine-tuning LLMs and performing vision tasks within a federated learning framework using real-world language and vision datasets (Section 4).

1.1 Related Works

In this section, we review the most relevant work in machine unlearning, federated unlearning, and unlearning in federated LLMs.

Machine unlearning. MU aims to remove the influence of a selected subset of data from the trained ML model. Based on the guarantee of removal, MU methods are broadly categorized into exact unlearning and approximate unlearning (Nguyen et al., 2022; Wang et al., 2024). In exact unlearning, we aim for an identical model to one that would have been obtained by retraining without that data to be erased. Retraining is a method that trivially achieves exact unlearning but is computationally expensive with large models and datasets. Existing works can exactly unlearn for support vector machines (Cauwenberghs & Poggio, 2000), k-means (Ginart et al., 2019), random forests (Brophy & Lowd, 2021). Bourtoule et al. (2021) partitions the entire training data set into a few disjoint subsets and trains one base model with each of these subsets. Since each base model is only trained with a subset of the original training data, the performance may be sub-optimal. Approximate unlearning aims for a model whose distribution closely resembles that of the retrained model. Guo et al. (2020) proposed a certified removal method to approximately unlearn linear model by Newton-like update. Nguyen et al. (2020) minimizes the KL divergence between the approximate posterior of the unlearned model and the retrained model under the variational inference framework.

Federated unlearning (FU). Many recent works adapt machine unlearning to the federated learning settings (Liu et al., 2020; Wang et al., 2021; Gong et al., 2021). Liu et al. proposed FedEraser, which involves using historical updates from the server and local calibration training on the unlearned client. The federated unlearning protocol proposed in this work can be used to unlearn an arbitrary subset of clients without any constraint on the type of data each client possesses. At the same time, it requires no participation of the unlearned client. Wang et al. proposed a channel pruning-based method to selectively forget a specific class from the trained ML model. Such an approach has limited scope as it is impractical to assume that each participant in the FL setting possesses precisely one class of data. Gong et al. concerned with the setting where no centralized party/server is present, which does not apply to the centralized FL setting. In terms of exact federated unlearning, Xiong et al. (2023) and Tao et al. (2024) use quantization and sampling strategies, respectively, to get a checkpoint during the FL training where the unlearned client's data have not made a quantifiable impact and use it as initialization for model retraining and since speed up the retraining process. On the other hand, Qiu et al. (2023) proposed to cluster the clients and train a few intermediate FL models and then subsequently obtain the global FL model through one-shot aggregation. At the unlearning stage, only the intermediate FL model where the unlearning client is present is retrained (and hence reducing the retraining cost). Our proposed method touches on both ideas and uses aggregation of a few sub-FL models to obtain a good initialization for much more efficient retraining. The way we obtained our sub-FL models trades-off between computation budget and post-unlearning performance, played an essential role in ensuring its effectiveness.

Unlearning in Federated LLMs. Federated learning has emerged as a powerful paradigm for training or fine-tuning LLMs collaboratively across organizations without exposing raw data (Sani et al., 2024; Kuang et al., 2024; Wu et al., 2024; Chen et al., 2023). Concurrently, recent advances in machine unlearning have demonstrated promising techniques for selectively removing unwanted or sensitive information from LLMs (Liu et al., 2024; Yao et al., 2023; Hu et al., 2024), laying the groundwork for both approximate and exact unlearning methodologies. Building on these complementary areas, recent research such as (Zuo et al., 2024) has begun to explore unlearning within federated LLM frameworks, integrating efficient mechanisms like Low-Rank Adaptation with blockchain-enabled transparency to address the unique challenges of cross-organizational collaboration and regulatory compliance. Due to the demand for powerful and collaborative language models with strict data privacy and accountability requirements, there is a need for efficient unlearning algorithms that can ensure exact unlearning while having better post-unlearning performance.

2 Problem Setting

Federated learning. This paper considers the centralized federated learning (FL) setting with a trusted central server and multiple clients. In this setting, a central server shared an aggregated model with the clients and then each client trains this model on his dataset and send model updates (weights or gradients) to the central server, which then aggregates these updates to get a better aggregated model. In our setting, we assume that the number of clients participating in FL process varies over time. Let $\mathcal{C}_t$ denote the set of participating clients at the beginning of the FL communication round t. An FL communication round (communication round for brevity) represents one cycle of model sharing by the central server with clients and then receiving the updated aggregated model.

Each client $c \in \mathcal{C}_t$ has training dataset $\mathcal{D}_{c,t}$ with $n_{c,t}$ labeled samples, where each sample is drawn from the distribution ν_c over $\mathcal{X} \times \mathcal{Y}$. Here, $\mathcal{X}$ represents the input space, and $\mathcal{Y}$ represents the label space. The learning model is denoted by $h_\theta : \mathcal{X} \to \mathcal{Y}$ with parameters $\theta \in \mathbb{R}^d$, where d is the number of model parameters. The loss incurred by the learning model h_θ on a sample $(x, y) \in \mathcal{X} \times \mathcal{Y}$ is denoted by $l(h_\theta(x), y)$, which can be the root mean squared error (for regression) or cross-entropy loss (for classification).

After the communication round t, the loss incurred by the client c for model parameters θ is the average loss of the model h_θ on the samples in $\mathcal{D}_{c,t}$ and defined by $f_{c,t}(\theta) := \frac{1}{n_{c,t}} \sum_{s=1}^{n_{c,t}} l(h_\theta(x_{c,s}), y_{c,s})$, where $(x_{c,s}, y_{c,s})$ is the s-th sample in $\mathcal{D}_{c,t}$. The central server aims to find a learning model with the minimum average loss for each client. The server updates θ by solving the following weighted optimization problem in round t:

$$\operatorname*{argmin}_{\theta} \frac{1}{n_t} \sum_{c \in C_t} n_{c,t} f_{c,t}(\theta), \tag{1}$$

where $n_t = \sum_{c=1}^{C_t} n_{c,t}$. Since the clients can not share their private local data $D_{c,t}$ with the server, the optimization problem given in Eq. (1) must be solved in a federated manner by using the suitable FL algorithm, such as FedAvg (McMahan et al., 2017)).

Exact federated unlearning. Let client c influence be completely removed from the aggregated model. Exact federated unlearning is the process of completely removing the influence of client c training data from the aggregated model, resulting in a model that is equivalent to the models trained without the training data of client c. However, the aggregated model resulting from retraining without the data of client c may have a poor performance in the initial round, which may not be expected when these models are deployed in practice. Therefore, our goal is to design methods that ensure exact federated unlearning while leading to an aggregated model with as high accuracy as possible.

3 Exact Federated Unlearning Methods

Due to multiple communication rounds of the FL training, it becomes impossible to completely remove a client's data influence from the trained aggregated model. Therefore, the most straightforward way to achieve the exact federated unlearning is to restart the federated learning process from scratch with the remaining clients. This method of retraining the aggregated model from scratch is called *retraining from scratch* (RFS) (Bourtoule et al., 2021; Liu et al., 2023). Although RFS is a simple method, the new model may have very low accuracy in the initial rounds after unlearning compared to the aggregated model before unlearning due to restarting the FL process with random initialization of the aggregated model. Such performance reduction of the aggregated model may not be desirable during deployment in practice involving critical applications such as healthcare (Prayitno et al., 2021; Dhade & Shirke, 2024) and finance (Long et al., 2020). In many real-world scenarios such as healthcare, security and financial services, maintaining uninterrupted services is of critical importance, as delaying will cause disruptions with severe consequences (Rostek et al., 2022; Wallace et al., 2003; Liu & Ren, 2024). This shortcoming of RFS raises a question: How to guarantee the exact federated unlearning while ensuring better post-unlearning performance?

We can delay exact unlearning by training a new model from scratch, while using the old model until a certain performance threshold is reached. However, if the reason for unlearning is due to sensitive data leakage or malicious adversarial client attack (Jin et al., 2021; Rashid et al., 2023; Fang et al., 2020; Shi et al., 2022a), further deployment of the old model will not be a viable option. Therefore, it becomes crucial to minimize the delay due to the unlearning process in order to obtain a new unlearned model as soon as possible, especially for the services that have low tolerance for disruption. Naturally, it raises a question: How to guarantee the exact federated unlearning with minimal delay while ensuring better post-unlearning performance? To answer this question, we propose two novel methods for achieving exact federated unlearning that completely remove the client's influence while giving better post-unlearning performance and shorter delay than RFS and other existing methods.

3.1 Bi-Models Training (BMT)

To have a better aggregated model after unlearning, we must design a new FL training process that allows exact federated unlearning while having a better initialization than random initialization. One way to achieve better initialization is to design methods that can exploit the remaining clients' existing knowledge. To do this, we propose a method named Bi-Models Training (BMT) that can be incorporated into any existing federated learning framework. The main idea of BMT is to have an additional model for each client that is only trained on its data, making these models unaffected by other clients' training data. We refer to this model as *local model*. We use the term *global model* for referring to the aggregated model, which is trained using all client's data and used for deployment. We now discuss how BMT can be incorporated into the different stages of any existing federated learning framework, namely: Initialization, FL Training, Unlearning, and New Client joining the FL process, whose details are given as follows.

Initialization. The central server starts the standard FL training process by randomly initializing the global aggregated model. This randomly initialized global model is then shared with all clients. Each client updates the global model using its local training data and then shares the model update (updated model or gradients) with the central server. As compared to the standard initialization in any FL training process, each client makes a copy of the locally updated global model[2] (i.e., local model). As the initial global model is randomly initialized, these local models are, by design, isolated from the influence of other clients' training data.

FL training. After receiving the first model updates, the central server aggregates them to get the aggregated global model as per the underlying FL algorithm (McMahan et al., 2017; Shlezinger et al., 2020; Zhang et al., 2021). In each subsequent communication round, each client receives the updated global model from the central server and then trains it using its training data. After updating the global model, each client shares the model update with the central server. Besides the standard FL training process, each client also updates their local model using their training data.

[2]The locally updated global model in the first communication rounds is the same as the model that is a copy of the initial global model and trained on client's training data.

Unlearning. Let c be the client whose influence needs to be completely removed from the global model after a communication round t and $\mathcal{C}_{t,r}$ be the set of remaining clients, i.e., $\mathcal{C}_{t,r} = \mathcal{C}_t \setminus \{c\}$. The central server first discards the current global model and requests each client to share their current copy of local models. Once the central server receives the local models from all remaining clients, the central server aggregates them to get the new initialization for the global model as per the underlying FL algorithm, e.g., for FedAvg, the central server performs weighted aggregation on the remaining client's local models, where each client's weight is proportional to their respective training data. Our experimental results (in Section 4) show that the resulting initialized global model performs better than random model initialization as done in RFS. Lastly, the central server restarts the FL training process with newly initialized global model, which is completely free from the influence of the unlearned client's data.

New client. When a new client wants to join the ongoing FL collaboration, the central server waits until the end of the ongoing communication round. Once it is over, the central server starts the FL training process with the new client by sharing the current global model with the new client, who then updates the current global model using its training data and shares the model update with the central server. Apart from this, the central client also shares the randomly initialized global model with the new client, who updates it, which then acts as the local model of the new client for subsequent rounds. Other clients do not influence this local model, as the initial global model is randomly initialized.

In summary, BMT has two models for each client: global and local. All clients train their local model on their data in isolation, whereas the global model is trained using the underlying FL training protocol. To completely remove a client influence from the global model, the central server first discards the global model and then uses the local models of the remaining clients to re-initialize the global model, which is further updated via FL training. This process ensures that BMT, by design, guarantees the exact federated unlearning. Further, using the remaining clients' local models leads to an initialization of the global model that is already influenced by the remaining clients to some extent, leading to a better performance than RFS, as shown in our experiments in Section 4. The trade-off for improved post-unlearning performance is the cost of pre-training local models. This trade-off is worthwhile for applications that require exact unlearning and fast deployment of good model.

3.2 Multi-Models Training (MMT)

The key insight of the previous section is that BMT achieves a better initial global model because it is influenced by the clients' local models. However, the local model only contains influence from an individual client and has no joint influence of multiple clients. Since all clients influence the global model, we should capture the joint influence of different clients and then use it to get a better initialization of the global model. To capture the joint influence, we can train FL models using only a subset of clients. We refer to these FL models as *sub-FL models*. Formally, a sub-FL model is an FL model that is trained via FL protocol using a subset of clients, where the size of the subset varies from 2 to $N - 1$. One can train all possible sub-FL models (power set of clients excluding global model) to capture the influence of all possible interactions of different subsets of clients. However, this approach is not computationally feasible as these sub-FL models increase exponentially with the number of clients (i.e., $2^n - n - 2$ for n clients). Another problem of training arbitrary sub-FL models leads to a situation of *double influence*, which is defined as follows:

Definition 1. *Let S_i be the set of clients whose data are used in training the i-th sub-FL model. The sub-models i and j leads to double influence if $S_i \cap S_j \neq \emptyset$, $S_i \setminus S_j \neq \emptyset$, and $S_j \setminus S_i \neq \emptyset$.*

When a client data is used to train two sub-FL models, it can lead to double influence if both are also trained using data from different clients, e.g., one is trained on clients $\{1, 2\}$ and another on clients $\{1, 3\}$; the client 1 data is used in both sub-FL models and hence having the double influence.

To avoid the double influence, each sub-FL model should be trained on disjoint subsets of clients, or the set of clients used for training sub-FL models is a proper superset of the set of clients used for another sub-FL model. One possible way to achieve this is to organize sub-FL models in a hierarchical tree structure. In this tree, the root node represents the global model while the leaf nodes correspond to the local models, and intermediate nodes represent sub-FL models, with each child node having disjoint sets of clients compared to its siblings. As we move from the root node to the leaf nodes, each sub-FL model branches into further subsets, maintaining either disjoint relationships or superset relations, thus ensuring a clear and systematic flow of influence throughout the hierarchy.

We refer to this hierarchical tree structure as an *influence tree*. After unlearning a client, we should aggregate the sub-FL models with higher influence (those influenced by a larger number of clients) and local models to get the initialization for the global model. If the number of models to aggregate is less, it implies that the initialization of the global model contains the most joint influence of clients. This relationship inspires our proposed metric *influence degradation score*, which measures how good is an influence tree. Next, we formally define the influence degradation score.

Definition 2 (Influence Degradation Score (IDS)). *Let $\mathcal{T}$ be any influence tree structure. The influence degradation score for $\mathcal{T}$, denoted by $s(\mathcal{T})$, is defined as the average number of sub-FL and local models that are aggregated to get the initial global modal after unlearning any client.*

Though the tree structure, by design, eliminates double influence, we do not know which tree structure gives the lowest IDS for given clients' different likelihood of requesting unlearning (as the probability of requesting unlearning may vary across the clients). As our goal is to construct an influence tree with minimum IDS, our following result shows that the binary influence tree constructed using Huffman coding has the lowest IDS among all n-ary influence tree structures, where $n > 2$.

Theorem 1. *Given an n-ary influence tree $\mathcal{T}$, there exists a binary influence tree $\mathcal{T}_2$ that has smaller IDS, i.e., $s(\mathcal{T}_2) < s(\mathcal{T})$. Let p_c be the unlearning probability of the client c. Then, Huffman coding with n symbols representing clients and weights $\{p_c\}_{c=1}^{n}$ gives the optimal binary influence tree such that $s(\mathcal{T}_{\textit{Huffman}}) \leq s(\mathcal{T}_2)$ for any influence tree $\mathcal{T}_2$ for the same group of clients.*

With Theorem 1, we can use Huffman coding (Huffman, 1952) to construct an influence tree that has the lowest IDS among all types of influence trees. In some real-life applications, the client's unlearning probability can be unknown. In such cases, we can assume that each client is equally likely to be unlearned, hence having the same unlearning probability. A client (on the leaf node) influences a sub-FL model if there is a path from the model to the leaf node representing that client. We next propose a method, named Multi-Models Training (MMT), that uses the sub-FL models to get better initialization for the global model. MMT can be easily incorporated into any existing federated learning framework, whose details are given as follows.

Initialization. Similar to BMT, the central server starts the standard FL training process by randomly initializing the global aggregated model. This randomly initialized global model is then shared with all clients. Each client updates the global model using its local training data and then shares the model update (updated model or gradients) with the central server. Each client makes a copy of the locally updated global model. Compared to BMT, MMT also initializes the sub-FL models using the model updates of clients corresponding to the sub-FL models.

FL training. After receiving the first model updates, the central server aggregates them to get the aggregated global and sub-FL models. In each subsequent communication round, each client receives the updated global and its sub-FL models from the central server and then trains them using its training data. After updating these models, each client shares the global and its sub-FL model updates with the central server. Apart from this, each client also updates their local model. By eliminating double influence in the *influence tree*, we achieve sub-linear scaling of training costs relative to the number of clients. This is because sub-FL models at the same depth in the tree have disjoint datasets, so their combined training cost is roughly equivalent to training the global FL model (assuming unchanged hyperparameters). Additionally, the depth of a balanced binary tree scales sub-linearly with the number of leaf nodes, provided the unlearning probability across clients remains approximately uniform. Unlike other exact FL unlearning methods (Xiong et al., 2023; Tao et al., 2024) that incur linear memory costs due to constant model checkpoint saving, our approach does not require saving extra checkpoints during training.

Unlearning. For unlearning a client, the central server first discards the current global model and related sub-FL models and then requests all clients not in any of the remaining sub-FL models to share their current copy of local models. Once the central server receives all requested local models, it aggregates them with sub-FL models (choosing only the most influential unaffected sub-FL model over its descendants) to get the new initialization for the global model as per the underlying FL algorithm. After removing the sub-FL models related to the unlearned client, the remaining influence tree may no longer have the lowest IDS for the remaining clients. It leads to two options: create a new influence tree while using earlier sub-FL models as much as possible or keep using the existing influence tree, which may not be the best but retains the sub-FL models trained over time. Lastly, the central server restarts the FL training process with the newly initialized global and sub-FL (if any) models, which are completely free from the influence of the unlearned client's data.

New client. Adding a new client to the ongoing FL collaboration can worsen the existing influence tree compared to the influence tree created using a new client. Like BMT, when a new client wants to join, the central server waits until the end of the ongoing communication round. Once it is over, the central server can create a new influence tree while using earlier sub-FL models as much as possible or keep using the existing influence tree to retain the existing sub-FL models, which are trained over time by adding new sub-FL models. After that, the central server starts FL training with the new client by sharing the current global and sub-FL models. The new client updates these models using its training data and shares the updates with the central server. Additionally, the central server shares a randomly initialized global model, which the new client updates and uses as its local model for future rounds. Overall, the initialization of the global model in MMT has the joint influence of multiple clients, which makes it better than BMT and hence leads to better post-unlearning performance, as corroborated by our experiments in Section 4. However, note that there is an additional computational cost for this improved performance over BMT as we need to train multiple sub-FL models in parallel.

4 Experiments

This section empirically verifies the effectiveness of BMT and MMT in two settings: (1) *sequential unlearning* setting, where multiple clients sequentially leave the federation, and (2) *continual learning and unlearning* setting, where clients can join and/or leave the federation at will. Next, we analyze the impact of the branching factor in the MMT structure on the model performance. Then, we consider special scenarios when the clients follow a fixed unlearning order (according to their subscription plans) and clients with non-uniform unlearning probabilities (clients from different demographics).

Baselines. We compare BMT and MMT against the following baselines: Standalone, where the centralized model trains on aggregated data from all remaining clients; Retraining from Scratch (RfS), where the federated model is retrained excluding data from the leaving client; FedCIO (Qiu et al., 2023); Exact-Fun (Xiong et al., 2023); and FATS (Tao et al., 2024).

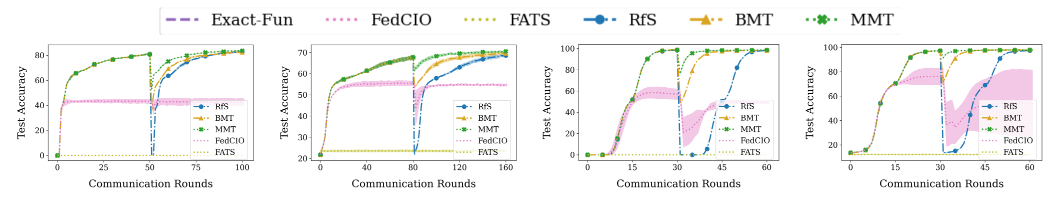
Sequential Unlearning

Language tasks. We also compare the performances of the proposed methods on two language tasks: 1) language identification, where the goal is to detect the language of the given text (Conneau, 2019), and 2) multilingual sentiment analysis, where the goal is to identify the sentiment of the given text using. We use the Huggingface papluca/language-identification dataset for the former task and Huggingface tyqiangz/multilingual-sentiments for the latter. We then randomly sample 200 and 500 data points separately for each class from top-8 classes with the most data. For both datasets, we finetune a pretrained GPT-2 Radford et al. (2019) model with the 200 data points set and Llama-3.2-3B model with the 500 data points set[3] to predict which language the input sequences belong to, with next-token prediction as the objective of getting the correct label. Fig 1 shows performance in unlearning settings for different NLP tasks. In all cases, MMT improves the fastest after unlearning, followed by BMT. In particular, for the larger model, both methods significantly outperformed other baselines, which corroborates our methods' scalability.[4] This experiment validates our method is effective across different modalities and model architectures.

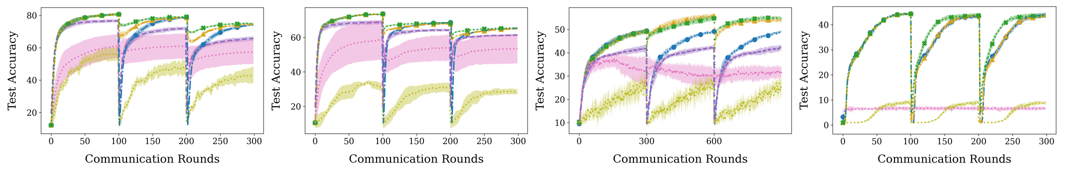
Vision tasks. We ran experiments on 4 popular vision datasets: MNIST (LeCun et al., 1998), FashionMNIST (Xiao et al., 2017), CIFAR-10 (Krizhevsky et al., 2009) and CIFAR-100 (Krizhevsky et al., 2009). We also consider language tasks with large language models. To simulate clients with realistic non-IID data, we let the client i receives the most data from the i-th class and the same amount of data from the remaining classes. We use ρ to denote the ratio between the majority class and minority class for all clients. Each client contains 200 training/test samples and $\rho = 0.02$ for MNIST and FashionMNIST; 1000 training samples, 300 test samples, $\rho = 0.2$ for CIFAR-10; 400 training samples, 100 test samples, $\rho = 0.1$ for CIFAR-100. Due to space constraints, details of the

[3]Due to the high computation and memory requirements to train multiple LLMs, we opted for GPT-2 and Llama-3.2-3B instead of the more popular and larger LLMs to show the performance of BMT and MMT.

[4]We did not include Exact-Fun for both LLM models and FATS for LLama-3.2-3B, as both methods have poor scalability w.r.t. model size due to their GPU memory requirement to save all intermediate model checkpoints.



(a) GPT-2 LI (b) GPT-2 MSA (c) Llama-3.2 LI (d) Llama-3.2 MSA



(e) MNIST (f) FashionMNIST (g) CIFAR-10 (h) CIFAR-100

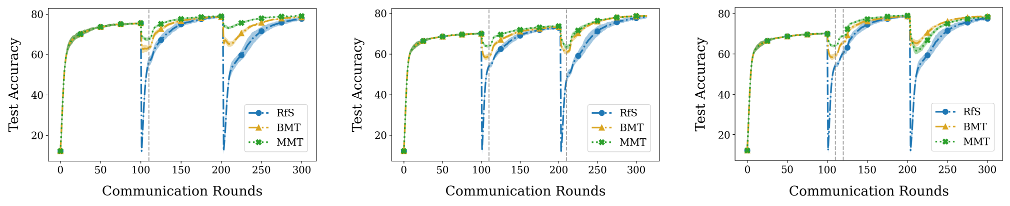
Figure 1: **Top row:** Sequential unlearning setting on two language tasks (language identification (LI) and multilingual sentiment analysis (MSA)) for federated fine-tuning of GPT-2 and Llama-3.2-3B. **Bottom row:** Test accuracy in the sequential unlearning setting for different real-world datasets.

models used for different vision tasks, training settings, and metrics are deferred to Appendix B. We also have additional experiments demonstrating the impact of data heterogeneity in Appendix B.

We simulate a practical scenario when clients gradually leave the federation. After each client leaves, we continue training the federated model on the remaining clients. Particularly, we simulate the leaving of 3 clients $\{1, 3, 5\}$. It is noteworthy that this unlearning order is to MMT's disadvantage as none of the sub-FL models can completely replace the server model. Hence, the server parameters must be aggregated from the parameters of other sub-FL models. Fig. 1 shows performance in the sequential unlearning setting on different datasets. As can be seen, BMT and MMT consistently outperform other methods by a large margin with better initialization and faster convergence after unlearning[5]. These results highlight the effectiveness of BMT and MMT, especially the advantage of sub-FL models in MMT for faster recovery after unlearning. Therefore, it is infeasible for large-scale experiments like CIFAR-100. In Appendix B.2 we also show the unlearning result when an adversarial client provides data with zero signal and demonstrate how our method improved the global model. See Table 1 for the comparisons across different tasks and baselines.

Ablation Studies

Continual learning and unlearning. This experiment aims to simulate a continual setting in which new clients can join, and existing clients can leave the federation at any time during training. We will consider three settings corresponding to varying learning and unlearning order: 1) **2U1N**: Unlearn - New client - Unlearn; 2) **2U2N**: Unlearn - New client - Unlearn - New client; 3) **3U2N**: Unlearn - New client - Unlearn - New client - Unlearn. For a fixed number of communication rounds k, a new client will be introduced at round $k + 10$, and an existing client will leave after every k round. We use the same k as the previous experiment. Fig. 2 shows performance in the continual learning and unlearning setting on the MNIST dataset. As can be seen, both BMT and MMT can seamlessly accommodate new clients and demonstrate that they can learn and unlearn rapidly.

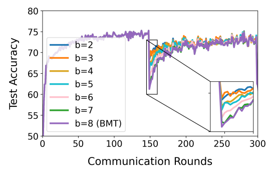


(a) Unlearn - Join - Unlearn (b) Unlearn - Join - Unlearn - Join (c) Unlearn - Join - Join - Unlearn

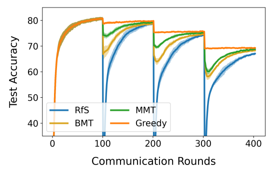
Figure 2: Test accuracy in the continual learning and unlearning setting on MNIST.

Branching factor in MMT structure. Recall that the default MMT uses a binary structure, i.e., a branching factor of $b = 2$ at each node in the tree of sub-FL modes. Therefore, we conduct

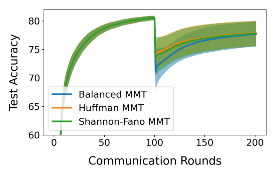
[5]We excluded the Exact-Fun baseline for CIFAR-100 due to its excessive GPU memory requirements, which are infeasible even with an H100 80GB GPU.



(a) Branching factor



(b) Subscription models



(c) Unlearning probabilities

Figure 3: **Left:** The effect of branching factor b in MMT structure. **Middle:** Performance of greedily constructed MMT given fixed unlearning order. **Right:** Performance of various tree construction methods given non-uniform unlearning probabilities.

experiments to analyze the impact of varying branching factors in MMT structure on the model performance. Particularly, for a federation consisting of n clients, the branching factor can range from 1 to n where $b = 1$ indicates a traditional setting with no sub-FL models while $b = n$ coincides with BMT, in which an auxiliary local model is maintained for each client. Note that we do not compare with $b = 1$ because it is equivalent to RFS.

As can be seen in Fig. 3a, a smaller branching factor generally results in higher test accuracy. This improvement occurs because MMT with a smaller branching factor aggregates fewer sub-FL models during unlearning. Furthermore, each sub-FL model is more likely to converge to the global optimum due to training on more local datasets. Thus, MMT with the default binary structure is the most suitable configuration for unlearning.

Subscription models. The unlearning order can be fixed in certain circumstances, e.g. clients may subscribe to the service that allows them to participate in the federated process for a fixed duration and will leave once their subscription expires. In such scenarios, it is possible to construct an optimal MMT structure that maximizes learning performance when clients leave the federation in a fixed order. Particularly, the optimal structure is the one that arranges the clients by their expiration date, with the soon-to-expire client at the top and greedily building the tree until reaching those with the farthest expiration. As observed in Fig. 3b, the greedy implementation of MMT achieves the best unlearning performance and outperforms the default MMT that assumes uniform probabilities of unlearning for all clients. Therefore, the greedy structure is preferable if the unlearning order of the clients is known in advance, e.g., in subscription models.

Non-uniform unlearning probabilities. The default MMT implementation assumes uniform unlearning probabilities for all clients. However, it is practical to consider the case where these probabilities are non-uniform. In fact, we will demonstrate that given the unlearning probabilities of each client, it is possible to construct improved MMT structures that achieve better unlearning performance through two strategies based on Shannon-Fano coding (Shannon, 1948) and Huffman coding (Huffman, 1952). Fig. 3c shows the performance for different tree construction methods. This result is obtained by sampling the client to be removed for 100 times according to pre-defined non-uniform unlearning probabilities of all clients. MMT structures that follow Shannon-Fano coding and Huffam coding obtain visibly improved results over the default MMT. Furthermore, Huffman-MMT obtains slightly better results than the Shannon-Fano counterpart, which aligns with the classical information theory results that Huffman coding is more optimal than Shannon-Fano coding for prefix-free code (Thomas & Joy, 2006).

5 CONCLUSION

This work proposes two methods, BMT and MMT, for exact federated unlearning in the ongoing federated learning collaboration. Our methods ensure the complete removal of an unlearned client's data while having better performance post-unlearning with the remaining clients than retraining from scratch. Our methods are particularly useful in practical scenarios where model updation in collaborative environments cannot afford long delays, with minimal tolerance for interruptions. Our extensive experimental results demonstrate the effectiveness of the proposed methods. A few interesting future research directions include proposing a principal approach to design an influence tree under a resource constraint (i.e., the number of sub-FL models that can be trained is limited) and how to change the influence tree post-unlearning or after a client joins the collaboration while having the lowest IDS value and maximizing the use the existing trained sub-FL models.

Impact Statement

The primary contribution of this paper is advancing machine learning techniques for federated unlearning, and we do not anticipate any immediate negative societal impacts.

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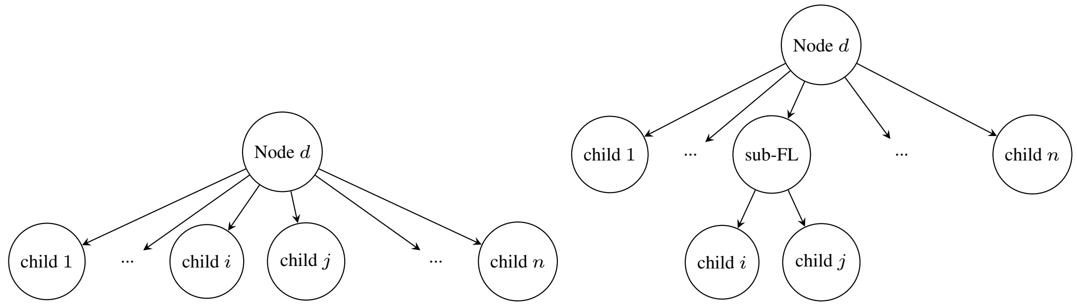
A Proof of Theorem 1

Proof. We first define the k-split influence node, which is a node in an influence tree with $k > 2$ leaf nodes. We first consider the influence tree $\mathcal{T}$, where k-split influence node only has leaf nodes as children. We denote this node as d, and the set of its leaf nodes is denoted by $\mathcal{C}$. We now follow the following procedure. First, we remove the edge between the node d and any two of its leaf nodes (siblings), denoted by i and j. We create a sub-FL model with these two removed nodes and then add the node for this sub-FL model as a child to the node d, as shown in Fig. 4. We denote the resulting tree as $\mathcal{T}'$. Let $f(\mathcal{T}, c)$ represent the number of sub-FL and local models that are aggregated to get the initial global modal after unlearning client c in the given influence tree $\mathcal{T}$.



(a) Influence tree $\mathcal{T}$ before making any change. (b) Influence tree $\mathcal{T}'$ after making the change.

Figure 4: Changes in influence tree structure.

Note that $f(\mathcal{T}', c) = f(\mathcal{T}, c) - 1$ for $c \in C \setminus \{i, j\}$ as one less leaf node to aggregate due to sub-FL model for $\{i, j\}$ leaf nodes, and $f(\mathcal{T}', c) = f(\mathcal{T}, c)$ for $c \in \{i, j\}$ as sub-FL model is no longer useful due to influence of leaf node i or j. With this, we have following IDS due to the node d:

$$\begin{aligned}
s_d(\mathcal{T}) &= \sum_{c \in \mathcal{C}} p_c f(\mathcal{T}, c) = \sum_{c \in \mathcal{C} \setminus \{i,j\}} p_c(f(\mathcal{T}', c) + 1) + \sum_{c \in \{i,j\}} p_c(f(\mathcal{T}', c) \\
&= k - 2 + \sum_{c \in \mathcal{C} \setminus \{i,j\}} p_c f(\mathcal{T}', c) + \sum_{c \in \{i,j\}} p_c(f(\mathcal{T}', c) \qquad \text{(node } d \text{ had } k \text{ leaf nodes)} \\
&= k - 2 + \sum_{c \in \mathcal{C}} p_c f(\mathcal{T}', c) \\
&= s_d(\mathcal{T}') \\
\implies s_d(\mathcal{T}) &> s_d(\mathcal{T}'). \qquad \text{(as } k > 2\text{)} \qquad (2)
\end{aligned}$$

Iteratively apply the same procedure on the rest of the child nodes until every node only has two children. After this, we obtain a binary tree. Since each operation strictly reduces IDS, $s_d(\mathcal{T}_2) < s_d(\mathcal{T})$. When the original tree already has some child nodes L that already belong to a binary subtree $\mathcal{T}_L$, we treat this subtree as a single child node c'_k and apply the aforementioned operations on the child nodes that do not yet belong to a binary subtree. If all the child nodes belong to some binary subtree, we check from bottom-up to find the largest binary subtrees and treat them as a single child node to apply the aforementioned operations. Following this procedure, we can transform any arbitrary tree into a binary tree. In general,

$$f(\mathcal{T}', l) = f(\mathcal{T}_L, l) + f(\mathcal{T}', c') = f(\mathcal{T}_L, l) + f(\mathcal{T}, c') - 1 = f(\mathcal{T}, l) - 1$$

for $c' \in \mathcal{C} \setminus \{i, j\}$, $l \in L$ and $f(\mathcal{T}', l) = f(\mathcal{T}_L, l) + f(\mathcal{T}', c') = f(\mathcal{T}_L, l) + f(\mathcal{T}, c') = f(\mathcal{T}, l)$ for $c' \in \{i, j\}$, $l \in L$. Notice that $f(\mathcal{T}', l) = f(\mathcal{T}_L, l) + f(\mathcal{T}', c')$ always holds. Therefore, the inequality in Eq. (2) generalizes for any tree structure with the generalized operation. After applying this procedure on any arbitrary tree $\mathcal{T}$ with at least one k-split influence node, the resulting binary tree $\mathcal{T}_2$ always has a strictly smaller value of IDS, i.e., $s(\mathcal{T}_2) < s(\mathcal{T})$. Now we will proof the second part of theorem. Assume N is the number of non-root nodes, q_d is the probability of reaching a non-root node d starting from the root node, and N_d^s is the number of siblings of a non-root node i. Note that for a node d, $q_d = \sum_{c \in \mathcal{C}_d} p_c$ where $\mathcal{C}_d$ is the set of all the client nodes (i.e., leaf nodes) that are descendants of node d and p_c is the probability of unlearning of the c-th descendant. Given an influence binary tree $\mathcal{T}_2$ having n clients with known unlearning probability of each client, the IDS is given as follows:

$$s(\mathcal{T}) = \sum_{c=1}^{n} p_c f(S, c) = \sum_{d=1}^{N} q_d * N_d^s. \tag{3}$$

For $\sum_{c=1}^{n} p_c f(S,c)$, we can group all leaf nodes that share some common ancestor node d into a collection, with $\mathcal{C}_d$ denoting this set. Since the same node has the same N_c^s, we can sum p_c of all such leaf nodes and rewrite $\sum_{c=1}^{n} p_c f(S,c)$ as $\sum_{d=1}^{N} \sum_{c\in\mathcal{C}_d} p_c * N_d^s = \sum_{d=1}^{N} q_d * N_d^s$. Since each node of the binary tree has only one sibling, we have

$$\sum_{d=1}^{N} q_d * N_d^s = \sum_{d=1}^{N} q_d * 1 = \sum_{d=1}^{N} \sum_{c\in\mathcal{C}_d} p_c = \sum_{c=1}^{n} p_c * l_c \tag{4}$$

where n is the number of leaf nodes, l_c is the depth of the c-th leaf node, and p_c is the probability of reaching a non-root node c starting from the root node. As splitting the q_d of each non-leaf node into $\mathcal{C}_d = \{p_1, p_2, ..., p_\tau\}$, where $q_d = \sum_{c\in\mathcal{C}_d} p_c$ and each element in $\mathcal{C}_d$ corresponds to a p_c of a leaf node, which is a descendant of that non-leaf node. Therefore, we can write $\sum_{d=1}^{N} q_d * 1$ as the sum

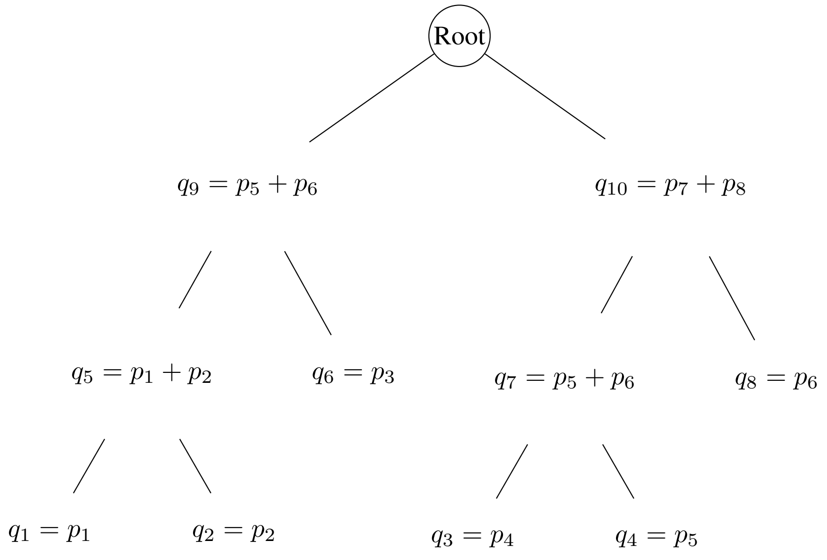


Figure 5: Visualization for $\sum_{d=1}^{N} q_d * 1 = \sum_{c=1}^{n} p_c * l_c$. One can easily see the equality holds.

of $p_c * l_c$ of all possible branches that reach each leaf node, as all the ancestor nodes of all leaf nodes have exactly one sibling (refer to Fig 5 for intuition). Finally, notice that $\sum_{c=1}^{n} p_c * l_c$ is the expected code word length, if the same binary tree is used to represent a binary prefix-free code encoding scheme. Since Huffman coding is optimal for minimizing $\sum_{c=1}^{n} p_c * l_c$, $s(\mathcal{T}_{\text{Huffman}}) \le s(\mathcal{T}_2)$ where $\mathcal{T}_{\text{Huffman}}$ is an influence tree constructed following Huffman coding and $\mathcal{T}_2$ is any binary influence trees for the same set of p_c. $\square$

B Additional experimental results

Models. For MNIST and FashionMNIST, we use simple MLP networks with 30 and 80 hidden units, respectively. For CIFAR-10, we use a CNN network with 5×5 convolutional layers followed by 2×2 max pool layer for feature extraction and two fully connected layers with 32 hidden units and ReLU for classification. For CIFAR-100, we use a VGG-16 model (Simonyan, 2014).

Training. We use FedAvg (McMahan et al., 2017) to train FL models for 100 rounds with 1 local epoch on MNIST and FashionMNIST, 300 rounds with 1 local epoch on CIFAR-10 and 100 rounds with 10 local epoch on CIFAR-100. We use the SGD optimizer with a learning rate 0.01, weight decay 0.1, batch size 20, and gradient clipping 10 for MNIST and FashionMNIST. We use the AdamW optimizer with batch size 64 and the same hyperparameters for CIFAR-10. We use the SGD optimizer with a learning rate 0.005, momentum 0.9, weight decay 10^{-5}, and batch size 64 for CIFAR-100. Our experiments are conducted on NVIDIA L40 46GB and NVIDIA H100 80GB GPUs.

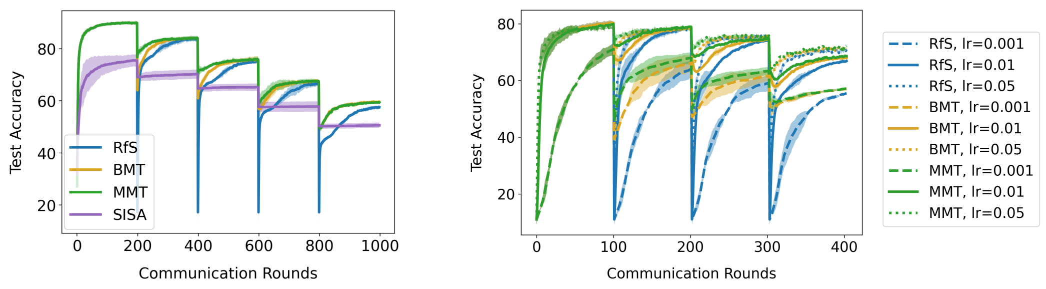
Metrics. We report test accuracy measured on a fixed test set that combines local test sets of all possible clients, including those that join/leave the federation in later stages. The combined test set allows us to observe the visible trend of the performance after one client is removed.

Dataset	Acc. (%)	RfS	Exact-Fun	FedCIO	FATS	BMT	MMT
MNIST	75	51	–	–	–	34	**16**
FashionMNIST	65	26	–	–	–	16	**3**
CIFAR-10	45	162	–	–	–	**3**	**3**
CIFAR-100	40	54	–	–	–	54	**37**
GPT-2 Lang. Identification	70	16	–	–	–	11	**6**
GPT-2 Multilingual Analysis	65	49	–	–	–	22	**7**
Llama-3 Lang. Identification	85	21	–	–	–	6	**2**
Llama-3 Multilingual Analysis	85	19	–	–	–	4	**1**

Table 1: The number of rounds required to reach a threshold accuracy after the first unlearning, as shown in Fig. 1, varies across different algorithms, show that MMT outperforms all other algorithms. In the table, '-' denotes the method did not reach the threshold.

B.1 Benchmarking against SISA

SISA is a model-agnostic exact unlearning method proposed in Bourtoule et al. (2021). It involves partitioning the training dataset into disjoint subsets and training isolated models on each subset, whose predictions are aggregated during inference time. In our context, we can train isolated models on each client's data and remove only the influenced model when a client leaves the collaboration to achieve exact unlearning. Fig. 6a shows SISA performance in the sequential unlearning setting on MNIST. Even though SISA obtains a better initialization than RfS, it incurs the worst post-unlearning performance compared to FL methods. This result suggests that the SISA training paradigm may not be well-suited for collaborative training among heterogeneous clients with limited data. Therefore, it serves as a less competitive exact FU benchmark.



(a) Benchmarking against SISA

(b) Different learning rates

Figure 6: **Left:** Sequential unlearning benchmark against SISA. **Right:** Performance for different learning rates in the sequential unlearning setting.

B.2 Additional Ablation Studies

Varying learning rates. Fig. 6b shows performance for three learning rates $\{0.001, 0.01, 0.05\}$ in the sequential unlearning setting on the MNIST dataset. In all cases, MMT converges the fastest, followed by BMT and RfS. This result validates the effectiveness of BMT and MMT compared to RfS across varying learning rates.

Removing noisy data or adversarial client. When a client detects noise in their data and requests unlearning, or when the server identifies an adversarial client, unlearning such clients improves model performance, as shown in Fig. 7.

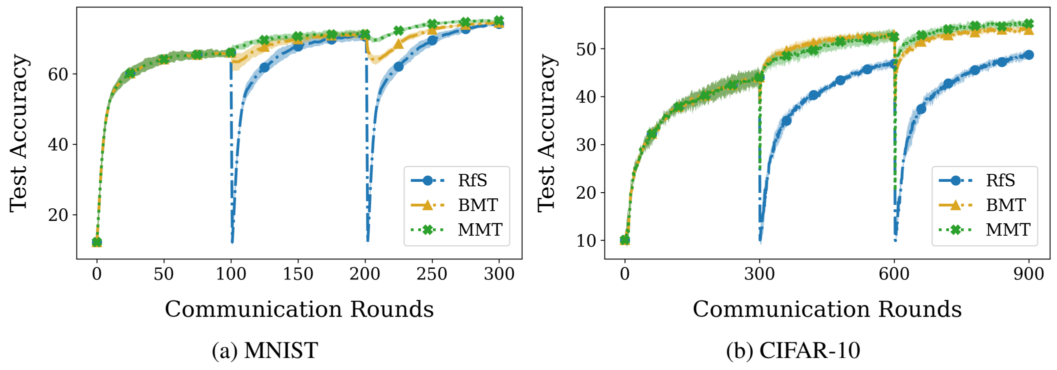


Figure 7: Performance after removing adversarial client or a client with noisy data.

Data heterogeneity. As mentioned earlier, the data heterogeneity ratio ρ defines the ratio between the number of samples in the majority and minority classes within a client's dataset. $\rho = 1$ indicates an IID dataset while $\rho \approx 0$ indicates an extremely non-IID dataset. As shown in Fig. 8, MMT consistently obtains the best performance across different heterogeneity ratios. The gap to other methods is more pronounced with lower ρ, suggesting MMT is more favorable when we have extremely non-IID data.

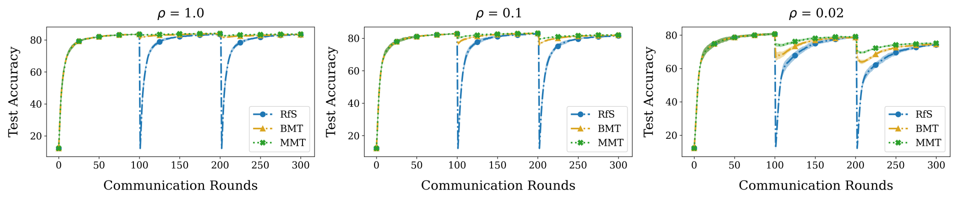


Figure 8: The effect of data heterogeneity on sequential unlearning performance on MNIST.