Optimistic Verifiable Training by Controlling Hardware Nondeterminism

Megha Srivastava[*]
Department of Computer Science
Stanford University
megha@cs.stanford.edu

Simran Arora
Department of Computer Science
Stanford University
simarora@stanford.edu

Dan Boneh
Department of Computer Science
Stanford University
dabo@cs.stanford.edu

Abstract

The increasing compute demands of AI systems have led to the emergence of services that train models on behalf of clients lacking necessary resources. However, ensuring correctness of training and guarding against potential training-time attacks, such as data poisoning and backdoors, poses challenges. Existing works on verifiable training largely fall into two classes: proof-based systems, which are difficult to scale, and "optimistic" methods that consider a third-party auditor who can replicate the training process and contest the trainer. A key challenge with the latter is that nondeterminism between GPU types during training prevents exact replication of the training process, resulting in schemes that are non-robust. We propose a method that combines training in a higher precision than the target, rounding after intermediate computations, and sharing rounding decisions based on an adaptive thresholding procedure, to successfully control for nondeterminism. Across three different NVIDIA GPUs (A40, Titan XP, RTX 2080 Ti), we achieve exact training replication at FP32 precision for both full-training and fine-tuning of ResNet-50 (23M) and GPT-2 (117M) models. Our verifiable training scheme significantly decreases the storage and time costs compared to proof-based systems, and is publicly released at https://github.com/meghabyte/verifiable-training.

1 Introduction

We are currently in the "large-scale era" of machine learning (ML), where the exciting capabilities of modern AI systems have required a dramatic increase in training compute needs [Sevilla et al., 2022]. In turn, several model training services, such as Replicate, OpenAI's Finetuning API, Together AI, Amazon Sagemaker, MosaicML Training, and Gensyn, have been created to support clients who lack the resources to train a model themselves. However, these services require clients to place a significant degree of trust in them to train the model correctly, without introducing a training-time attack such as data poisoning or undetectable backdoors [Wan et al., 2023, Goldwasser et al., 2022]. How can we help a client, such as an individual or a small company, hold the service provider accountable in case of misbehavior during training?

Consider an education start-up that wishes to finetune the Llama-70b language model (70B parameters) on their own curated dataset to support student learning. This task requires significant resources,

---

[*]Correspondence to megha@cs.stanford.edu.

and the company might even lack the necessary expertise. Instead, they might choose to pay a trainer with vast computing resources to perform the training task (Figure 1). However, what if the trainer adds data points that spread misinformation, introduces a backdoor that advances a political agenda for specific prompts, or tries to save work by under-training the model? If the client starts to notice suspicious model behavior, is there any action they can take? We study this problem of *verifiable training*, or ensuring that the training of an ML model was performed correctly.

One possibility is for the trainer to provide the client with a cryptographic proof that the model was trained according to the specification. However, proof-based systems require cryptographic techniques that can be difficult to scale to the complexity of real-world ML systems. For instance, recent work based on zero-knowledge proof systems for verifiable *inference*, a much simpler task than training, requires more than 8 minutes to generate proofs for only 20 images [Liu et al., 2021]. Thus, practical proof-based methods for verifiable training have only been implemented for simple tasks such as logistic and linear regression [Garg et al., 2023, Ames et al., 2022].

An alternative "optimistic" approach is to consider a third-pary auditor (Figure 1). This could be a trusted 3rd party, such as a non-profit organization that may not have sufficient computing resources to provide training as a service beyond auditing, or a different provider that the client approaches and wishes to compare with the original model trainer. When a client suspects foul play, they can ask the auditor to challenge the trainer by training the model using the auditor's own compute, and demonstrate
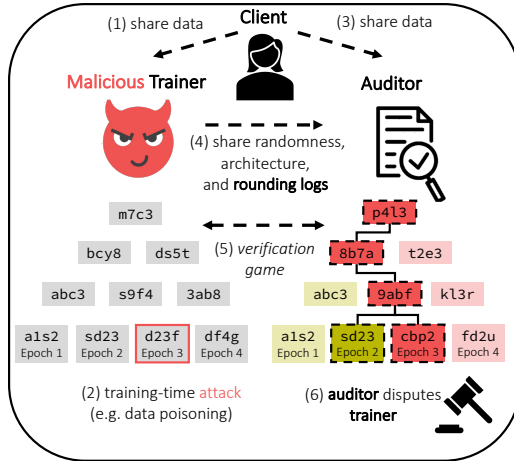

Figure 1: Overview of our scheme. After an auditor challenges a trainer, they train the model, storing weights in a Merkle tree, and enter a binary search procedure to identify the exact steps of the dispute. We show how to control GPU nondeterminism between auditor and trainer, expanding the set of potential auditors.

that the trainer did not train correctly. Based on the provided evidence required from the auditor (i.e. the precise timesteps model training diverged, as shown in Figure 1), the client can then choose to refuse the trainer's model, pursue legal action against the trainer, or even dispute a potentially corrupt auditor if the client deems such evidence as incorrect, or another auditor disagrees. This protocol can be efficiently carried out using techniques from the literature on verifiable computing, such as the "verification game" method of Teutsch and Reitwießner [2019], which uses an interactive binary-search procedure to identify the exact intermediate computation step (e.g. training epoch) where the two parties diverged. Applying verifiable computation techniques to model training is particularly important given the increase in decentralized machine learning services like Gensyn, which seek to make ML compute more accessible by creating a network of many untrusted GPUs.

Unfortunately, the issue with such "optimistic" approaches is nondeterminism during model training: two models trained on different GPU types, even with same data order and random seed, can learn different weights (Figure 2). The auditor cannot simply compare their model weights with the trainer's, and recent work has shown that protocols based on comparing model weights, such as Jia et al. [2021]'s "proof of learning," are not robust and can be forged due to errors from nondeterminism [Thudi et al., 2022, Fang et al., 2023].

Our work addresses this limitation by asking: can the trainer provide additional information to the auditor that removes the effects of hardware nondeterminism? Our starting point is the observation that hardware nondeterminism occurs due to the accumulation of errors from floating point operations. For example, a matrix-vector multiply often results in different floating point values on different GPUs, since GPUs often accumulate in different orders. To address this issue, a natural approach is to perform training using a *higher* precision (e.g. FP32) than the target precision of the model weights (e.g. FP16), and periodically round back to the target precision. The hope is that all floating point errors will be confined to the higher precision bits, so that the rounded values *are* deterministic. However, this fails because computed values can occasionally straddle the "rounding boundary": i.e.,

the trainer can round up while the auditor rounds down, quickly causing them to diverge. Instead, we propose a solution where the trainer *records* the rounding direction for certain intermediate computation so that auditor can stay in sync with the trainer. As this requires the trainer to record a large number of bits, we also show how to reduce the amount of data needed to eliminate errors.

We use this strategy to adapt the verification game described by Teutsch and Reitwießner [2019] for verifiable training. The game's efficiency lies in our ability to store hashes of model checkpoints in a Merkle tree [Merkle, 1988]. To determine if training was performed according to the specification, the auditor needs to reconstruct the Merkle tree and compare the resulting Merkle root hash with the Merkle root hash provided by the trainer's – if they do not match, the two parties can enter an interactive binary search procedure to identify the exact training step of the dispute. The purpose of the binary search game is to hold both parties accountable: an auditor should not be able to simply claim that a model was improperly trained, but convince a third-party (e.g., the public, or a judge) by showing at what point during training the trainer misbehaved. We show our verifiable training scheme can scale to tasks such as full training of ResNet-50 (23M parameters) and finetuning of GPT-2 (117M parameters), significantly outperforming existing methods with respect to both time and storage cost, while eliminating statistical error due to non-determinism. For example, the proposal in prior work Jia et al. [2021] would require $> 140\times$ more storage cost than our method by comparing model weights at every step in order to achieve low (yet still non-zero) statistical error.

Concretely, our contributions include: (1) A method for two parties, training the same model on different GPU types, to achieve identical training results by logging and sharing rounding decisions; (2) A verifiable training scheme based on the verification game from Teutsch and Reitwießner [2019], which stores model weights in a Merkle tree for efficient comparison between a trainer and auditor; (3) Experiments showing the ability of our approach to scale to large models such as ResNet-50 and GPT-2 between three different NVIDIA GPU architectures (A40, Titan XP, RTX 2080 Ti); (4) Methods to reduce the storage cost of our approach via efficient encoding of rounding logs and an adaptive threshold mechanism to reduce the amount of rounding decisions logged; and (5) Comparisons with existing methods, including proof-based systems, that highlight the improved storage and time efficiency of our method. [2]

2 Related Works

Without any verifiable training scheme in place, significant trust is placed in the trainer, leaving a client vulnerable to many different attacks, such as the "poisoning" of data samples to cause undesirable behavior (e.g., generating unsafe text [Carlini et al., 2023, Koh et al., 2021, Wan et al., 2023]) and planting backdoors triggered by certain inputs [Goldwasser et al., 2022]. Therefore, training ML models in trusted environments has been an exciting direction explored by many researchers. One line of work consists of proof-based systems, where a proof of correctness (for a desired specification) is provided using cryptographic techniques such as succinct non-interactive arguments (SNARKs) [Micali, 1994, Bitansky et al., 2012, Lee et al., 2020, Liu et al., 2021, Garg et al., 2023, Kang et al., 2022]. However, even the most recent proof-based systems for verifiable training suffer extreme latencies, such as 22 minutes for training VGG-11 on one batch of 16 data inputs [Abbaszadeh et al., 2024], and have therefore primarily been developed for simpler models (e.g., logistic regression) that are less likely to be delegated to others in the first place [Garg et al., 2023, Ames et al., 2022]. Meanwhile, an alternative solution of training models in a trusted execution environment (TEE), such as NVIDIA's H100, incurs a performance penalty due to the cost of running inside a TEE [Dhanuskodi et al., 2023]. Furthermore, clients lose all security guarantees if an attacker can extract the attestation key from even one GPU [Nilsson et al., 2020, Bulck et al., 2018].

Our approach is most similar to proof-of-learning protocols, which consider a trusted 3rd party that compares checkpointing during the course of training with the original training sequence [Jia et al., 2021]. However, such methods not only incur high storage cost by requiring model weights to be stored frequently, but are non-robust due to errors from training nondeterminism. Several works have shown that proof-of-learning protocols can be spoofed and fail to verify correctness in several important contexts [Fang et al., 2023, Kong et al., 2023, Thudi et al., 2022]. Although Choi et al. [2023] recently proposed a verification procedure that is immune to several known proof-of-learning

---

[2]Our method is implemented entirely within the `pytorch` framework (compatible with version 2.3.1), and is available at `https://github.com/meghabyte/verifiable-training`.

Test Input	Accuracy	90.2 %	89.9 %	90.7 %
	Scores	A40	TitanXP	RTX Ti
	dog	0.09	0.26	**0.75**
	cat	**0.69**	0.28	0.20
	deer	0.08	**0.29**	0.01

(a) CIFAR-10 Classification (Res-Net 50)

Test Input	Perplexity	4.18 ppl	4.18 ppl	4.22 ppl
This above all: to thine own self be	Scores	A40	TitanXP	RTX Ti
	true	**0.0101**	0.0091	0.0096
	my	0.0098	**0.0100**	0.0083
	thou	0.0097	0.0095	**0.0106**

(b) Shakespeare Text Finetuning (GPT-2)

Figure 2: Even after ensuring the same software version, random seed, and use of deterministic algorithms via library flags, training nondeterminism persists between three GPU types.

attacks, their method is not only limited to supervised learning algorithms, but also based on an assumption that models temporarily overfit data during training, which may not always hold true.

GPU Nondeterminism: Prior work has investigated software patches for deterministic training, for instance by enforcing FP accumulation ordering, at a significant cost to efficiency Jooybar et al. [2013], Defour and Collange [2015], Chou et al. [2020], TensorFlow [2021], Zhuang et al. [2021]. While these options address deterministic computation on a *single* GPU architecture, achieving deterministic results across multiple GPU architectures remains challenging Crane [2018a], NVIDIA [2022]. We control hardware nondeterminism across GPUs in order to design an efficient and reliable verifiable training scheme. However, our method's impact extends beyond verifiable training, as training nondeterminism can have several negative consequences including bias, reproducibility, and downstream effects on ML pipelines [Zhuang et al., 2021, Crane, 2018b, Srivastava et al., 2020].

3 Set-Up: The Verification Game

Our method for verifiable training is based on the interactive verification game proposed by Teutsch and Reitwießner [2019] in the context of blockchains. The core idea is to resolve a dispute between a challenger, in our case the auditor, and a solver, in our case the trainer, for an expensive computation (e.g., model training). In order for the auditor to take any meaningful action (e.g., pursue legal action), they need to prove the exact source of the dispute (e.g., training time-step where an attack occurred). If we can save model weights at different time steps into a compact data structure such as a Merkle tree, then identifying the source of disagreement can be done efficiently using binary search [Merkle, 1988]. More precisely, the verification game consists of the following parties:

1. trainer, who has putatively trained a model according to a client's specifications. In our example, this is a service provider with sufficient compute power to train a model.
2. client, who receives a model from the trainer and approaches an auditor.
3. auditor, who officially challenges the trainer on behalf of a client. This is a trusted 3rd-party that has sufficient resources but does not necessarily provide training as a service. The client can choose several auditors to audit the trainer's model.
4. judge: Sometimes a judge may need to arbitrate a legal claim. The judge can only perform minimal computations (e.g., one training epoch), but can examine the auditor's claims and enforce a penalty against either the trainer, for incorrect training, or the auditor, for a false alarm.

When the trainer is approached by an auditor, they would need to share training parameters, model architecture, and randomness, as shown in Figure 1. The auditor would then replicate the training process, storing model weights in a Merkle tree at the same checkpointing interval as the trainer (every leaf node in a Merkle tree is a hash of the data and every non-leaf node is a hash of its children). The main loop of the verification game starts when both parties have the root of their respective Merkle trees. If training was performed correctly, then the trainer's root should match the auditor's. Otherwise, a binary search procedure is performed, where the auditor iteratively descends the Merkle tree until it identifies two consecutive leaf nodes, i and $i + 1$, where the hash at i matches that of the trainer, but the hash at leaf $i + 1$ does not. This identifies the point in the computation of the dispute.

This interactive verification game requires the cooperation of the trainer. If the trainer refuses to share the value at a certain node of their Merkle tree within a given time frame, they can be considered to have failed the audit. Additionally, the trainer and auditor use a Merkle tree to store model weights, requiring far less storage than prior work, if correct training produces identical weights (and

Example	Sum Order	FP32	Rounded to FP16
$a, b, c = 0.1, -0.1, 0.2$	$a + b + c$	0011111001001100110011001101	0011001001100110
	$a + c + b$	0011111001001100110011001110	0011001001100110
$a, b, c = 10.02, 13.162813186645508, 0.2$	$a + b + c$	01000001101110110001000000000001	0100110111011001
	$a + c + b$	01000001101110110001000000000000	0100110111011000

Table 1: Two examples of floating point accumulation error when rounding arithmetic performed higher precision (e.g. FP32) down to lower precision (e.g. FP16). In the second example, the error in the FP32 result transfers to the rounded FP16 result.

identical hash values).The problem is that training nondeterminism leads to weight divergence, and causes this verification game to always fail. This why we seek to prevent divergence in training.

4 The Nondeterminism Challenge

Although there are user-side controls for forcing deterministic operations within a single GPU architecture , these controls do not prevent nondeterminism between GPU architectures (e.g., NVIDIA H100 and V100), where trained models can have similar aggregate performance (e.g., accuracy) yet yield very different predictions, as shown in Figure 2 Crane [2018a], NVIDIA [2022]. There are three main sources of nondeterminism between GPU types:

1. Floating-Point Arithmetic: Computers represent real values using integer and FP representations, typically the IEEE 754 standard (Figure 5). There is a tradeoff between the approximation fidelity and the # of bits used to represent the real values. The chosen precision controls the representable numerical range (e.g., 32-bit FP values can represent values between $1.17549435e - 38$ and $3.40282347e + 38$). Because computers round to representable FP values, changing the order in which FP values are accumulated can change the resulting sum Kahan [1965], Whitehead and Fit-Florea [2011]. Over the course of the many operations during training, this can lead to a large difference in the end result between the trainer and auditor.

2. Parallel Computation: In a GPU, a single operation (called a *kernel*) is executed by thousands of threads in parallel. GPUs contain a set of *streaming multiprocessors* (SMs), which run the *thread blocks* required for the kernel. At the hardware level, these blocks are divided into *warps* that are assigned to the available cores. Because different GPUs have a different number and size of compute units, applications partition arithmetic workloads (e.g., batch matrix multiplies) differently to achieve high performance NVIDIA [2022], thus changing the order of FP operations.

3. Memory Hierarchy and Variable Delays: The time taken for memory access by each thread depends on the physical location of the data, which can create variable delays Jooybar et al. [2013], Defour and Collange [2015], Chou et al. [2020]. The GPU memory hierarchy consists of large amounts of high bandwidth memory (HBM) and small amounts of fast SRAM memory, and maintains an L1 and L2 cache to improve access times. The caches sizes and access times differ across GPU architectures (e.g. an NVIDIA A100 has 192KB / 40 MB of L1/L2 cache memory, while the H100 has 256KB / 50MB). This affects warp scheduling, leading to changes in operation ordering resulting in nondeterminism. Finally, to compute primitives such as GEMMs ($D = A \cdot B + C$), the workhorse of machine learning, GPUs split the work of computing the tiles of D across a thread block NVIDIA [2023], resulting in nondeterminism that a robust verification method needs to control.

5 Method Overview

5.1 Accumulation Errors Start at Higher Precision Bits

Our key idea is that if nondeterminism of training between GPU types occurs due to FP operations, then any error will initially be introduced in the lower bits. Suppose that both trainer and auditor train at a *higher* FP (e.g., $b_{tr} = 64$) precision than the client's target model precision (e.g., $b_m = 32$) and then periodically *round* (e.g., $b_r = 32$) after intermediate computation steps (e.g., a convolution layer). One might hope that this will "erase" the errors due to nondeterminism, and prevent them from accumulating. Unfortunately, simply rounding to the nearest FP32 after each computation during training is insufficient for determinism. The problem is due to rounding errors that straddle
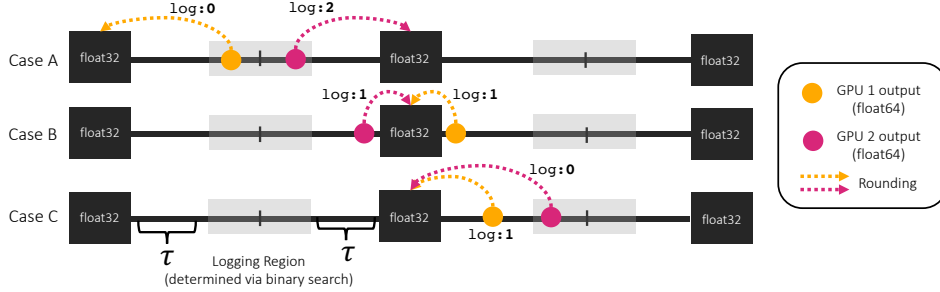
Figure 3: Divergence between outputs on two different GPUs (in FP64) for a given function and input can result in different rounding choices when rounding to the nearest FP32. We only wish to log rounding decisions for Case A, where the auditor should copy the trainer's rounding choice in order to reach the same value. This requires defining a logging region, determined by a threshold τ,

the *rounding boundary*. Consider Case A in Figure 3, which shows a divergence in the output of a computation using FP64 on two different GPUs. Because the outputs of GPU 1 and 2 are on different sides of the boundary, rounding to the nearest FP32 results in different values, introducing error.

What if the trainer records their rounding choice (e.g., up, down, none) for every intermediate computation? The auditor could then copy the trainer's choice, and therefore round to the exact same value and successfully control for nondeterminism. However, the auditor should not copy the trainer's behavior for every output (see Cases B & C, Figure 3). If a computation output on GPU 1 is too close to the rounded value, then it is possible that GPU 2 is also close in distance but from the opposite direction. In this case, the auditor should ignore the trainer's choice. We therefore need to introduce a threshold τ under which the trainer does not record their rounding choice.

Our method requires upper bounding the divergence d_{div} between any two different GPUs for any intermediate computation f (i.e. difference in outputs for the same input). Let ϵ_b represent the distance between two FP32 values, after rounding to b_r bits of the mantissa (Figure 5) and controlling for the exponent. We need to select b_r and τ such that $d_{div} < \epsilon_{b_r}$ and $d_{div} < 2\tau$ (Figure 3). Because the set of possible FP numbers is finite, there exist optimal bounds for b_r and τ. In practice, we find that $b_r \leq 32$ and $\tau > 0.25 \cdot \epsilon_{32}$ are sufficient for standard intermediate computations in neural network training (e.g., convolution, layer norm) in FP64. We study different values for b_r in Section 6.

5.2 Primitives

We assume both trainer and auditor train models using the IEEE-754 standard FP numbers (Figure 5). Besides requiring read and write disk I/O operations, we define the following functions:

1. $\mathsf{rnd}_{b_r}(x)$: rounds input x to the nearest FP up to b_r bits of the mantissa, as shown in Figure 5.
2. $\log(x, b_r, \tau, f)$: logs to file f a logging direction c, which is either 0 (down), 1 (ignore), or 2 (up) depending on threshold τ and rounding amount b_r, as shown in Algorithm 4.
3. $\mathsf{rev}(x, b_r, c)$: reverses rounding of input x based on logging direction c. If $x < \mathsf{rnd}_{b_r}(x)$ & $c = 0$, then return x rounded to the nearest float *below* x with b_r precision. If $x > \mathsf{rnd}_{b_r}(x)$ & $c = 2$, then return x rounded to the nearest float *above* x with b_r precision. Otherwise, do not correct.
4. $\mathsf{threshold}(l, b_r, b_{tr})$: identifies the optimal threshold to log rounding directions (0 or 2) instead of 1, which the rev function ignores, based on the binary search procedure in Section 5.4.
5. $\mathsf{hash}_{\mathsf{sha256}}(\theta)$: creates a SHA-256 hash of provided model weights θ (in b_m precision).
6. $\mathsf{tree}(leaf_1, leaf_2..., leaf_n)$: create a Merkle tree where each *leaf* node is the output of $\mathsf{hash}_{\mathsf{sha256}}(\theta)$ for model weights θ at a given checkpoint, with a checkpointing interval k [Merkle, 1988].

5.3 Training and Auditing

The trainer's task begins when a client approaches them with dataset D, training specifications (epochs E, loss function loss, etc.), and a requested model precision b_m. The trainer can then choose a training precision $b_{tr} > b_m$, a rounding amount $b_r \leq b_m$, and a checkpointing interval k to periodically store small $\mathsf{hash}_{\mathsf{sha256}}(\theta)$ of model weights θ in a Merkle tree, for efficient comparison with an eventual auditor. Then, as detailed in Algorithm 1, the trainer can perform training as normal, but after every intermediate computation (e.g., convolution) perform the rnd_{b_r} operation on

each output. Rounding is applied to computations in both the forward and backward passes. Finally, either using a fixed threshold τ or a layer-specific optimal τ from the threshold function described in Section 5.4, the trainer applies log, which logs rounding choices *only for the computations an auditor should copy*. The output of the algorithm includes a rounding log file F and the root of the Merkle tree which, along with the shared randomness R and all training parameters, the trainer can share with any trusted third-pary auditor who challenges them.

After a client approaches them, the auditor initiates the verification game described in Section 3. To avoid penalty, the trainer must cooperate by sharing the rounding amount b_r, randomness R used in training (e.g., a pseudo-random number generator), the checkpointing interval k, and set of rounding logs F. The auditor then follows the training procedure and corrects their rounding choice (e.g., up or down) to match those logged in F using the rev operation, as detailed in Algorithm 2 (Appendix). By correcting each rounding mismatch during the course of training, the auditor is able to prevent nondeterminism errors from accumulating. Therefore, the auditor can store the $\text{hash}_{\text{sha256}}(\theta)$ of model weights θ in a Merkle tree at interval k, knowing that if training was done correctly, the model weights should be identical to the trainer's at any timestep. The output of Algorithm 2 is the root of the auditor's Merkle tree, which they can use to compare with the trainer's root.

5.4 Reducing storage cost

Logging rounding decisions for every neural network layer output during training incurs a large baseline storage cost, and is our main limitation. For dataset D, batch size B, training epochs E, and model layers L_θ, the upper bound on the total storage cost for verifiable training with our method is:

$$\text{storage cost (B)} = |D| \times E \times B \times \left(\sum_{l=1}^{L} o_{l,f} + \sum_{l=1}^{L} o_{l,b}\right) \quad (1)$$

where $o_{l,f}$ and $o_{l,f}$ represent the size of outputs of the forward pass and backward pass of layer l. Note that the log entries do not need to be kept around in the RAM and can be written straight to the disk. Moreover, this cost is a one-time cost incurred by the trainer, who in our context is likely to be a powerful commercial provider with access to such storage capacity. Furthermore, as we later show in Section 6, for models with many linear layers like Transformer-based language models (e.g., GPT-2), where parameters significantly outnumber intermediate computations, this storage cost is significantly smaller than alternative approaches that require saving model weights [Jia et al., 2021]. Nevertheless, we now describe our method for reducing storage cost by (i) efficiently encoding rounding logs and (ii) adaptive selection of the threshold τ to reduce the storage costs.

Efficient Encoding: Each log entry is a value from the set $0, 1, 2$, as opposed to the FP model weights. We pack sub-sequences of five log entries into a single byte via a fast GPU-based radix-3 to radix-2 conversion, yielding 1.6 bits/entry storage that is close to the best possible packing of 1.58 bits/entry, and yields a 77% storage reduction relative to naively storing one log entry per byte.

Adaptive Threshold: Recall that we need to select a threshold τ that controls for whether the trainer logs a rounding choice, or instead logs 1 which the auditor ignores. The more one can increase τ, the more 1 values are recorded, which can make rounding logs more compressible (due to long sequences of 1s). Furthermore, it is possible that the divergence d_{div} between outputs on two different GPUs, given the same input, is function-specific. For example, while convolution requires several matrix multiplications that might result in a large FP accumulation error, normalization operations are unlikely to result in large d_{div}, and a larger τ can be applied. We develop an efficient algorithm (Algorithm 3 in the Appendix) to find the optimal value for τ given a particular layer and data of output values that led to different rounding choices between any two GPUs (e.g., Case A in Figure 3). For a given rounding amount b_r and training precision b_{tr}, the algorithm performs a binary search between $\tau = 0.25 \cdot \epsilon_{32}$ (our upper bound on the d_{div} between two GPUs for any function) and $\tau = 0.5 \cdot \epsilon_{b_r}$ (the rounding boundary). By performing this procedure for the different intermediate computations in a model, the trainer can hope to better compress the rounding log F.

Merkle Tree Storage: Storing SHA-256 hashes of model weights during training in a Merkle tree creates an efficient mechanism for the verification game described in Section 3, with negligible storage requirements. The audit ends when either the trainer withdraws, the auditor confirms that training was performed correctly, or the auditor can present paths to the two leaves of their Merkle tree where divergence starts, providing evidence to dispute the trainer.
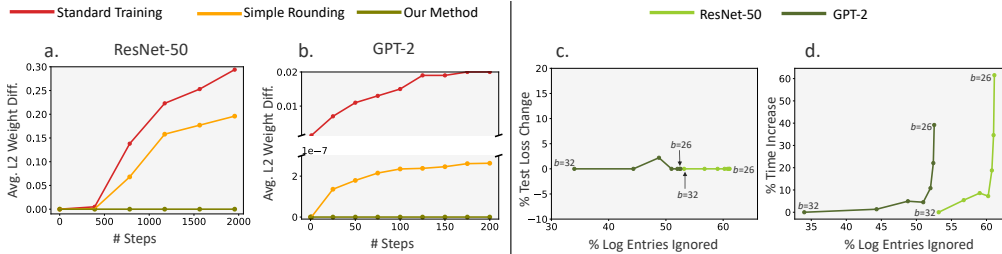
Figure 4: We successfully control for nondeterminism between GPU types for both ResNet-50 (a.) and GPT-2 (b.) tasks, while standard training and simple rounding without performing rev corrections result in model divergence over the course of training. Stronger rounding has minimal affect to model performance (c.), but at the cost of increasing time for trainer (d.).

Table 2: Efficient encoding reduces storage requirements by 77%, and rounding to $b = 26$ improves the compression further between 5-20% (values reported for 1 step of training). The original proof-of–learning protocol from Jia et al. [2021] requires storing 2.78 GB of model weights for GPT-2, or more than **140x** our storage cost, while still incurring statistical error.

	ResNet-50 $b = 32$	**ResNet-50** $b = 26$	**GPT-2** $b = 32$	**GPT-2** $b = 26$
Naive Encoding	456 MB	456 MB	92 MB	92 MB
Efficient Encoding	105 MB	105 MB	22 MB	22 MB
+ Zip Compression	96 MB	91 MB	20 MB	18 MB

6 Empirical Results

We evaluate our verifiable training method on the two large-scale models listed below with all possible trainer and auditor pairs across NVIDIA GPUs A40, TITAN Xp, and RTX 2080 Ti (see Appendix B for more details). In Section 6.2, we compare our method with recent proof-based systems.

1. **ResNet-50**: We train (from random initialization) ResNet-50 (23M) on CIFAR-10 with dataset size 50K & batch size B=64. Test accuracy = 90.7% after 100 epochs training on Titan RTX Ti.
2. **GPT-2**: We finetune GPT-2 (117M) on a corpus of Shakespeare text with dataset size 1.1M tokens, batch size B=8, and sequence length 64. Perplexity = 4.22 after 1 epoch training on Titan RTX Ti.

Figure 2 shows that nondeterminism due to GPU architecture exists for both tasks. While we can repeatedly obtain identical results across training runs on the same GPU architecture, training on different GPU architectures results in fundamentally different models.

6.1 Implementation and Findings

We implement our verifiable training method entirely on top of the `pytorch` framework, with `torch` version 1.13.1 and CUDA version 11.7. The intermediate computations we apply rnd_b to are layers (e.g., `torch.nn.Conv2D`) in the model's computation graph. Rounding-related operations (rnd and rev) either using casting or FP functions (e.g., `torch.nextafter`) that can run on the GPU, thus having little impact on computational speed. Because we observed that the `torch.randn` operation used for dropout in GPT-2 is non-deterministic for long inputs (even for the same seed, see Appendix I), we implement our own dropout as our method requires shared randomness R.

Successful control for non-determinism: Our method completely eliminates non-determinism between full training runs of both for both the ResNet-50 training and GPT-2 fine-tuning tasks across all possible trainer and auditor pairs between the A40, Titan XP, and RTX 2080 Ti GPUs. As Figure 4 shows, standard FP32 training results in an increasing divergence (l2-distance of weights) between models on different GPUs over the course of training. Furthermore, we show the simple approach of training in FP64 and rounding to FP32 after every intermediate computation, but without the auditor correcting rounding decisions with rev, fails to mitigate this issue. Only our method, in which the auditor follows the rounding decisions ($b_r = 32$) made by the trainer for every intermediate computation, eliminates non-determinism and persists over time. Our implementation, which requires disk I/O during training to store the rounding decisions, results in a small increase in training time for the trainer (1.2-1.4x) and auditor (1.3-1.7x) using a non-optimized, prototype implementation (Table 5). We report the storage requirements of our method in Table 2, showing

Table 3: Average # of rev corrections performed by auditor per training step. Even at $b = 32$, auditing only requires 20-25 corrections (**2e-6 to 9e-6%** of samples) per training step.

ResNet-50	$b = 32$	$b = 31$	$b = 30$	$b = 29$	$b = 28$	$b = 27$	$b = 26$
Forward	15 ± 3	6 ± 2	3 ± 1	3 ± 1	0	0	0
Backward	10 ± 0.6	6 ± 0.6	2 ± 1	0.7 ± 0.7	0 ± 0	0 ± 0	0 ± 0
GPT-2	$b = 32$	$b = 31$	$b = 30$	$b = 29$	$b = 28$	$b = 27$	$b = 26$
Forward	2 ± 0.7	2.3 ± 0.8	2.2 ± 0.4	0.2 ± 0.2	0.4 ± 0.2	0 ± 0	0 ± 0
Backward	19 ± 13	0.75 ± 0.3	1.2 ± 0.4	0.2 ± 0.2	$0. \pm 0.0$	0 ± 0	0 ± 0

Table 4: Adaptive thresholds identified for different operations using Algorithm 3 with $b = 32$.

	2D Convolution	**Batch Norm**	**Linear**	**Layer Norm**
Dimension	*256 (1,1)*	*(128, 128, 16, 16)*	*(768,768)*	*(768,1)*
τ	$0.305 * 2^{-23}$	$0.499 * 2^{-23}$	$0.465 * 2^{-23}$	$0.499 * 2^{-23}$

that our efficient encoding scheme reduces the size of the trainer's rounding logs by 77%, relative to naive logging. Because the Merkle tree stores 32-byte SHA-256 hashes, its overall size (KBs) and creation time are negligible and not reported. Finally, we show that decreasing the rounding amount b to values even as low as 26 has little effect on model performance (we observe no change in accuracy, so report test loss), but increase training time (Figure 4). We observe that smaller values of b do allow more log entries to be ignored, improving compression of the file, which we discuss next.

Compression with adaptive threshold: Our approach outperforms (Table 2) the storage costs of proof-of-learning protocols that save model weights for GPT-2 (2.78GB), which has many linear layers – we observe more than **140x** reduction relative to the approach in Jia et al. [2021]. We further reduce the storage cost of our method by decreasing the rounding amount b and implementing the adaptive thresholding strategy (Section 5.4). Table 4 reports adaptive thresholds τ for four different pytorch layers at rounding amount $b_r = 32$. Convolutions require the lowest τ, indicating larger divergence in outputs between GPU types, which is expected due to the large # of matrix multiplications. Meanwhile, τ is higher for normalization layers, likely due to smaller divergences between GPU types. Because adaptive thresholding seeks to reduce the # of times rounding decisions (0 and 2) are logged and improve log file compression, we report storage cost after zip compression in Table 2. As expected, more aggressive rounding (which results in a higher τ) improves the compression rate. Although the compression gains are mild in comparison to our encoding step, they build-up over the course of training. Finally, we report the average # of rev corrections an auditor needs to perform for one training step in our two tasks (Table 3). These values are surprisingly small in comparison to the # of operations logged – only a maximum of **2e-6%** (ResNet-50) and **9e-6%** (GPT-2) of logged values, are actually needed by the auditor! We also observe that severe rounding (e.g., $b = 27$) completely eliminated the hardware non-determinism for our tasks, requiring no corrections from the auditor. This shows a huge gap between the # of values currently saved by the trainer and those needed by the auditor, motivating an exciting future possibility of significantly reducing the storage cost of our method if we could reliably predict when a divergence will not occur.

6.2 Comparison with alternative approaches

Logistic Regression: Garg et al. [2023] recently proposed a zero-knowledge proof-based system for verifiable training of a logistic regression, which importantly does not leak information about the client's data or require a trusted third-party auditor, unlike our work. However, since verifiable training itself is motivated by a client not having sufficient resources to train the model, it is crucial to consider the implications of scale. The authors report the prover time and proof size requirements for one training pass of logistic regression on a dataset of 2^{18} items, with 1024 dimensions and a batch size of 2014, as **72 seconds** (training and proof generation time) and **350 MB** respectively. We replicate this training task, and find that our method significantly improves upon both storage and time requirements, requiring only **106 KB** and **7 seconds** (both training and auditing). Furthermore, because Garg et al. [2023] do not report the duration of "offline phase" of their method, their reported value is a lower bound on the actual time required. Finally, we note that the original proof-of-learning protocol from Jia et al. [2021], which also considers a trusted third-party, would require **9.2 MB per training step** to store all model weights. Therefore, our method is at least **85x** more space efficient.

VGG-11: Concurrent to this work, Abbaszadeh et al. [2024] introduce a zero-knowledge proof-of-training protocol for deep neural networks, presenting results for one batch step of training for

a simplified version of the VGG-11 model with 10M parameters, which is less than the original VGG-11 network and ResNet-50 [Simonyan and Zisserman, 2015]. While the authors do not provide architectural details, we can assume that increasing the # of parameters to the original VGG-11 would only increase their reported proof time and size. We, therefore, compare their reported values with an implementation of our method for the same task of verifying the training of VGG-11 on CIFAR-10 with a batch size of 16. While their use of incrementally verifiable computation leads to tractable proof size (1.36MB vs. the 1.2MB per iteration cost of our method), Abbaszadeh et al. [2024]'s method requires **22 min. per training iteration**. In comparison, our method requires training and auditing times of only 6 sec. per iteration and is significantly more efficient (factor of **220x**), an important consideration for model training as a commercial service.

Finally, in Appendix Section J, we compare our results with an adaption of Gupta et al. [2023]'s protocol for secure inference of GPT-2. Compared with our method's storage cost (18MB) and training time (11s for training, 13.5s for auditing), scaling Gupta et al. [2023]'s protocol for training would introduce around a **10,000x** data and **40x** time overhead. While proof-based systems provide strong security guarantees without a third party, they do so at the cost of relying on hard-to-scale cryptographic techniques, as well as approximating non-linear functions that can harm performance.

7 Security Analysis

Our work makes a *1-of-n* honesty assumption, i.e., as long as one of n auditors is honest, any attack from a malicious trainer that results in diverging model weights will be detected. One consideration is the potential manipulation of the rounding logs by an adversarial trainer who could select rounding decisions that achieve a desired outcome, and which the auditor would follow. Concretely, let us define our threat model so that the trainer knows an auditor's GPU a priori. Recall that an auditor only copies the trainer's rounding decision in Case A in Figure 3, when both GPUs compute values close to the rounding boundary. Under this threat model, the trainer can identify the n steps where the auditor is close to the boundary (as in Case A), enumerate the set of 2^n different models that result from different rounding decisions, and selectively pick a model that exhibits a desired property.

However, the trainer cannot use this strategy to embed an arbitrary property (e.g., a specific backdoor). It can only select from the set of models that differ in certain rounding decisions, which all require the trainer to use the correct training specifications accepted by the client (such as exact training data & hyperparameters). Furthermore, since the expected # of divergences between the trainer and the auditor is extremely small (see Table 3), the set of possible models where an auditor would not detect an attack (e.g., many rev ops) is limited. Finally, we show in Table 6 in the appendix that the divergence (measured both as ℓ_2-norm between model weights and output distributions) due to GPU non-determinism is significantly less than the divergence due to data ordering during training. Therefore, if a client will accept a model trained with *any* random ordering of the data during training, then it is unlikely that an adversarial trainer — that can only alter rounding decisions — could produce a model that the client would not accept. Nevertheless, fully understanding the model properties obtained by manipulating rounding logs adversarially is an important future direction.

8 Limitations and Future Work

Our verifiable training scheme successfully controls for hardware nondeterminism. It expands the pool of potential auditors of a model training service, allowing us to envision a world where a client can even use two competing service providers it trusts to audit each other. Relative to proof-based systems, a limitation is the need for all parties to trust the third-party auditor. If the trainer provides finetuning services on top of closed-source models (e.g., OpenAI), then our scheme will only work for the third-party auditors that the trainer is willing to share model weights with. Other limitations included the added latency of training in higher precision and the storage cost. While we have shown that our method requires significantly less storage than alternatives, the vast majority of stored rounding decisions are not used by the auditor and are therefore unnecessary (Section 6). Therefore, an exciting direction for future work is to mitigate this gap by better predicting when GPU divergence between computations occurs. Recent work has similarly argued for a stronger profile of noise during training in the context of verification [Fang et al., 2023]. Finally, another direction for future work includes adapting our method for distributed training [Li et al., 2020].

9 Acknowledgements

References

Jaime Sevilla, Lennart Heim, Anson Ho, Tamay Besiroglu, Marius Hobbhahn, and Pablo Villalobos. Compute trends across three eras of machine learning. In *2022 International Joint Conference on Neural Networks (IJCNN)*. IEEE, July 2022. doi: 10.1109/ijcnn55064.2022.9891914. URL `http://dx.doi.org/10.1109/IJCNN55064.2022.9891914`.

Alexander Wan, Eric Wallace, Sheng Shen, and Dan Klein. Poisoning language models during instruction tuning, 2023.

Shafi Goldwasser, Michael P. Kim, Vinod Vaikuntanathan, and Or Zamir. Planting undetectable backdoors in machine learning models, 2022.

Tianyi Liu, Xiang Xie, and Yupeng Zhang. zkcnn: Zero knowledge proofs for convolutional neural network predictions and accuracy. Cryptology ePrint Archive, Paper 2021/673, 2021. URL `https://eprint.iacr.org/2021/673`. `https://eprint.iacr.org/2021/673`.

Sanjam Garg, Aarushi Goel, Somesh Jha, Saeed Mahloujifar, Mohammad Mahmoody, Guru-Vamsi Policharla, and Mingyuan Wang. Experimenting with zero-knowledge proofs of training. Cryptology ePrint Archive, Paper 2023/1345, 2023. URL `https://eprint.iacr.org/2023/1345`. `https://eprint.iacr.org/2023/1345`.

Scott Ames, Carmit Hazay, Yuval Ishai, and Muthuramakrishnan Venkitasubramaniam. Ligero: Lightweight sublinear arguments without a trusted setup. Cryptology ePrint Archive, Paper 2022/1608, 2022. URL `https://eprint.iacr.org/2022/1608`. `https://eprint.iacr.org/2022/1608`.

Jason Teutsch and Christian Reitwießner. A scalable verification solution for blockchains. *CoRR*, abs/1908.04756, 2019. URL `http://arxiv.org/abs/1908.04756`.

Hengrui Jia, Mohammad Yaghini, Christopher A. Choquette-Choo, Natalie Dullerud, Anvith Thudi, Varun Chandrasekaran, and Nicolas Papernot. Proof-of-learning: Definitions and practice. *CoRR*, abs/2103.05633, 2021. URL `https://arxiv.org/abs/2103.05633`.

Anvith Thudi, Hengrui Jia, Ilia Shumailov, and Nicolas Papernot. On the necessity of auditable algorithmic definitions for machine unlearning. In *31st USENIX Security Symposium (USENIX Security 22)*, pages 4007–4022, Boston, MA, August 2022. USENIX Association. ISBN 978-1-939133-31-1. URL `https://www.usenix.org/conference/usenixsecurity22/presentation/thudi`.

Congyu Fang, Hengrui Jia, Anvith Thudi, Mohammad Yaghini, Christopher A. Choquette-Choo, Natalie Dullerud, Varun Chandrasekaran, and Nicolas Papernot. Proof-of-learning is currently more broken than you think, 2023.

Ralph C. Merkle. A digital signature based on a conventional encryption function. In Carl Pomerance, editor, *Advances in Cryptology — CRYPTO '87*, pages 369–378, Berlin, Heidelberg, 1988. Springer Berlin Heidelberg. ISBN 978-3-540-48184-3.

Nicholas Carlini, Matthew Jagielski, Christopher A. Choquette-Choo, Daniel Paleka, Will Pearce, Hyrum Anderson, Andreas Terzis, Kurt Thomas, and Florian Tramèr. Poisoning web-scale training datasets is practical, 2023.

Pang Wei Koh, Jacob Steinhardt, and Percy Liang. Stronger data poisoning attacks break data sanitization defenses, 2021.

Silvio Micali. CS proofs (extended abstracts). In *35th Annual Symposium on Foundations of Computer Science, Santa Fe, New Mexico, USA, 20-22 November 1994*, pages 436–453. IEEE Computer Society, 1994. doi: 10.1109/SFCS.1994.365746. URL `https://doi.org/10.1109/SFCS.1994.365746`.

Nir Bitansky, Ran Canetti, Alessandro Chiesa, and Eran Tromer. From extractable collision resistance to succinct non-interactive arguments of knowledge, and back again. In *Innovations in Theoretical Computer Science (ITCS)*, pages 326–349. ACM, 2012.

Seunghwa Lee, Hankyung Ko, Jihye Kim, and Hyunok Oh. vcnn: Verifiable convolutional neural network based on zk-snarks. Cryptology ePrint Archive, Paper 2020/584, 2020. URL `https://eprint.iacr.org/2020/584`. `https://eprint.iacr.org/2020/584`.

Daniel Kang, Tatsunori Hashimoto, Ion Stoica, and Yi Sun. Scaling up trustless dnn inference with zero-knowledge proofs, 2022.

Kasra Abbaszadeh, Christodoulos Pappas, Dimitrios Papadopoulos, and Jonathan Katz. Zero-knowledge proofs of training for deep neural networks. Cryptology ePrint Archive, Paper 2024/162, 2024. URL `https://eprint.iacr.org/2024/162`. `https://eprint.iacr.org/2024/162`.

Gobikrishna Dhanuskodi, Sudeshna Guha, Vidhya Krishnan, Aruna Manjunatha, Rob Nertney, Michael O'Connor, and Phil Rogers. Creating the first confidential gpus. *Commun. ACM*, 67(1):60–67, dec 2023. ISSN 0001-0782. doi: 10.1145/3626827. URL `https://doi.org/10.1145/3626827`.

Alexander Nilsson, Pegah Nikbakht Bideh, and Joakim Brorsson. A survey of published attacks on intel sgx, 2020.

Jo Van Bulck, Marina Minkin, Ofir Weisse, Daniel Genkin, Baris Kasikci, Frank Piessens, Mark Silberstein, Thomas F. Wenisch, Yuval Yarom, and Raoul Strackx. Foreshadow: Extracting the keys to the intel SGX kingdom with transient Out-of-Order execution. In *27th USENIX Security Symposium (USENIX Security 18)*, page 991–1008, Baltimore, MD, August 2018. USENIX Association. ISBN 978-1-939133-04-5. URL `https://www.usenix.org/conference/usenixsecurity18/presentation/bulck`.

Zhifeng Kong, Amrita Roy Chowdhury, and Kamalika Chaudhuri. Can membership inferencing be refuted?, 2023.

Dami Choi, Yonadav Shavit, and David Duvenaud. Tools for verifying neural models' training data. In *Neural Information Processing Systems*, 2023.

Hadi Jooybar, Wilson W. L. Fung, Mike O'Connor, Joseph Devietti, and Tor M. Aamodt. Gpudet: a deterministic gpu architecture. In *ASPLOS '13: Proceedings of the eighteenth international conference on Architectural support for programming languages and operating systems*, 2013.

David Defour and Caroline Collange. Reproducible floating-point atomic addition in data-parallel environment. In *Proc. of the Federated Conference on Computer Science and Information Systems*, 2015.

Yuan Hsi Chou, Christopher Ng, Shaylin Cattell, Jeremy Intan, Matthew D. Sinclair, Joseph Devietti, Timothy G. Rogers, and Tor M. Aamodt. Deterministic atomic buffering. In *2020 53rd Annual IEEE/ACM International Symposium on Microarchitecture (MICRO)*, 2020.

TensorFlow. Tensorflow 2.8.0-rc0, 2021. URL `https://github.com/tensorflow/tensorflow/releases/tag/v2.8.0-rc0`.

Donglin Zhuang, Xingyao Zhang, Shuaiwen Leon Song, and Sara Hooker. Randomness in neural network training: Characterizing the impact of tooling. In *arXiv:2106.11872v1*, 2021.

Matt Crane. Questionable answers in question answering research: Reproducibility and variability of published results. *Transactions of the Association for Computational Linguistics*, 6:241–252, 2018a. doi: 10.1162/tacl_a_00018. URL `https://aclanthology.org/Q18-1018`.

NVIDIA. Determinism across gpu architectures, 2022. URL `https://github.com/NVIDIA/framework-reproducibility/issues/28`.

Matt Crane. Questionable answers in question answering research: Reproducibility and variability of published results. *Transactions of the Association for Computational Linguistics*, 6:241–252, 2018b. doi: 10.1162/tacl_a_00018. URL `https://aclanthology.org/Q18-1018`.

Megha Srivastava, Besmira Nushi, Ece Kamar, Shital Shah, and Eric Horvitz. An empirical analysis of backward compatibility in machine learning systems, 2020.

William Kahan. Further remarks on reducing truncation errors, 1965. URL `https://dl.acm.org/doi/pdf/10.1145/363707.363723`.
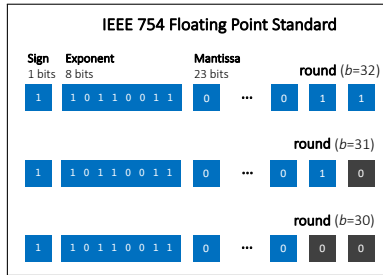
Figure 5: We define rounding to b bits as rounding to the nearest 32-bit FP number that has 0s in the last $32 - b$ bits of the mantissa, after accounting for the exponent.

Nathan Whitehead and Alex Fit-Florea. Precision & performance: Floating point and ieee 754 compliance for nvidia gpus, 2011. URL `https://developer.nvidia.com/sites/default/files/akamai/cuda/files/NVIDIA-CUDA-Floating-Point.pdf`.

NVIDIA. Cuda: Hopper tuning guide, 2023. URL `https://docs.nvidia.com/cuda/pdf/Hopper_Tuning_Guide.pdf`.

Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition, 2015.

Kanav Gupta, Neha Jawalkar, Ananta Mukherjee, Nishanth Chandran, Divya Gupta, Ashish Panwar, and Rahul Sharma. Sigma: Secure gpt inference with function secret sharing. Cryptology ePrint Archive, Paper 2023/1269, 2023. URL `https://eprint.iacr.org/2023/1269`. `https://eprint.iacr.org/2023/1269`.

Shen Li, Yanli Zhao, Rohan Varma, Omkar Salpekar, Pieter Noordhuis, Teng Li, Adam Paszke, Jeff Smith, Brian Vaughan, Pritam Damania, and Soumith Chintala. Pytorch distributed: experiences on accelerating data parallel training. *Proc. VLDB Endow.*, 13(12):3005–3018, August 2020. ISSN 2150-8097. doi: 10.14778/3415478.3415530. URL `https://doi.org/10.14778/3415478.3415530`.

A IEEE Floating Point Image

See Figure 5.

B GPU Details

All experiments reported in our paper are run with the following three GPUs:

- NVIDIA Titan XP: 3840 Cores, 12 GB
- NVIDIA RTX 2080 Ti: 4352 Cores, 11 GB
- NVIDIA A40: 10752 Cores, 48 GB

We are able to successfully replicate training runs between all pairs of these 3 GPUs.

C Logging Algorithm

See Algorithm 4

D Train Algorithm

See Algorithm 1.

E Audit Algorithm

See Algorithm 2.

F Adaptive Thresholding Algorithm

See Algorithm 3.

G Time Requirements

See Table 5.

H Model Divergence Comparison

See Table 6.

I Random Number Generation

Our verifiable training scheme requires shared randomness between the trainer and auditor, which is used for deciding input data batching, weight initialization, and operations such as dropout (randomly setting outputs to zero). More formally, our scheme requires sharing the same random seed and pseudo-random generator. However, in our implementation based on pytorch (assuming the same software version between trainer and auditor), we chose to rely on the the torch random seed functionality. While this successfully controls for batch input ordering and weight initialization, it is unfortunately not sufficient for random number generation, as operations such as torch.nn.randn() leverage parallelism when the requested # of values is higher than a certain amount. Specifically, we found that across T40, RTX 2080 Ti, V100, A40, and A100, given the same seed, torch.randint() produces identical tensors onlt up to size 40960. At size 40961, T40 (which is an older GPU) deviated from the rest. Likewise, at size 69633, 2080 Ti deviated from the rest, and so on. Based on these observations, we arranged for calls to torch.randint() in the dropout layer (which is the only operation using large random tensors in our tasks) to be replaced by generating and concatenating multiple random tensors of size 40960 or less. Specifically, a random tensor of size $n > 40960$ is generated by concatenating $(n//40960)$ random tensors of size 40960 and one random tensor of size $(n\%40960)$. However, we emphasize that it is therefore important in our scheme either for both parties to implement this change a priori, or simply use an external source for pseudorandomness.

J Comparison with GPT-2 Inference

The previously discussed proof-based systems for verifiable training by-pass the need for a third-party auditor, but very few efficient systems exist in the literature. Many more works study secure *inference* of deep neural networks, which could be used to construct verifiable training protocols with stronger security guarantees than ours (e.g., allowing a trainer to keep a proprietary model's weights private), but come at a significant cost to performance and resources. To demonstrate this, we consider adapting Gupta et al. [2023]'s protocol for secure inference of GPT-2 based on multi-party computation, to our context of verifiable training. Gupta et al. [2023] show how two parties, the client with private data and the trainer, can jointly compute the forward pass of a known model architecture without revealing additional information beyond the model output to each other. Because they report the the communication overhead $P = 0.37$GB and time $T = 0.96$ seconds for one forward pass on a single data input, we can calculate $2 \times P \times D \times E = $ **189 GB** and $2 \times T \times D \times E = $ **983 seconds** as estimated communication cost and time, respectively, for 1 step of training in out GPT-2 task, where 2 considers both the forward and backward pass. Compared with our method's required storage cost (18MB) and training time (11s for training, 13.5 seconds for auditing), scaling Gupta et al. [2023]'s protocol for training would introduce around a **10,000x** data and **40x** time overhead.

Algorithm 1 train

INPUT: dataset D, epochs E, batch size B, shared randomness R, model W_θ, loss function loss, rounding amount b_r, training precision b_{tr}, target model precision b_m, checkpointing interval k
OUTPUT: Merkle tree root M_{root}, rounding log file F

1: $F, M_{leaves} \leftarrow$ create empty file and leaf list
2: $W_\theta \leftarrow \mathsf{init}(R, b_{tr})$ //initialize weights
3: $T \leftarrow \frac{D*E}{B}$
4: **for** $t = 1...T$ **do**
5:     $input \leftarrow \mathsf{batch}(R, D, B)$ // get data batch
      // forward pass
6:     **for** layer $l_\theta \in W_\theta$.layers **do**
7:       $output \leftarrow l_\theta(input)$
8:       $\tau \leftarrow \mathsf{threshold}(l_\theta, b_r, b_{tr})$ //set threshold
9:       $\mathsf{log}(output, b_r, \tau, F)$
10:      $output \leftarrow \mathsf{rnd}_{b_r}(output)$
11:      $input \leftarrow output$
12:    **end for**
13:    $loss \leftarrow \mathsf{loss}(output)$
14:    // backward pass, reversed layers
15:    $grad\_output \leftarrow \nabla_{\mathsf{loss}}$
16:    **for** layer $l_\theta \in W_\theta$.layers **do**
17:      $grad\_input \leftarrow \nabla_{l_\theta}(grad\_output)$
18:      $\tau \leftarrow \mathsf{threshold}(\nabla_{l_\theta}, b_r, b_{tr})$
19:      $\mathsf{log}(grad\_input, b_r, \tau, F)$
20:      $grad\_input \leftarrow \mathsf{rnd}_{b_r}(grad\_input)$
21:      $grad\_output \leftarrow grad\_input$
22:    **end for**
23:    $\theta \leftarrow$ update update weights
24:    **if** $t \bmod k = 0$ **then**
25:      $M_{leaves}.\mathsf{append}(\mathsf{hash}_{\mathsf{sha256}}(\theta \text{ in precision } b_m))$
26:    **end if**
27: **end for**
28: $M_{root} \leftarrow \mathsf{tree}(M_{leaves})$ // create Merkle tree
29: **return** F, M_{root}, and model W_θ in target precision b_m

---

Table 5: Training time requirements, including Merkle tree operations (at $k = 5$), for 1 step of training broken down by stage of our verifiable training process. Note that reported times are specific to the particular dataset, batch size, and task, and using a non-optimized prototype codebase – therefore the relative increase is time is more important.

	ResNet-50	GPT-2
Original (No Rounding or Disk I/O)	24s	8s
Trainer	28s	11s
Auditor	31s	13.5

Table 6: Comparison of model divergence due to data ordering versus GPU non-determinism. Reported numbers are averaged between 10 pairs of models, error bars are standard deviation.

Metric	Data Ordering	GPU Non-determinism
l2 weight difference	133.2 ± 9	1.1 ± 0.07
l2 output distance	5.3 ± 0.03	0.26 ± 0.02

Algorithm 2 audit

INPUT: dataset D, epochs E, batch size B, shared randomness R, model W_θ, loss function loss, rounding amount b_r, training precision b_{tr}, target model precision b_m, checkpointing interval k, log file F from trainer

OUTPUT: Merkle tree root M_{root}

  1: $M_{leaves} \leftarrow$ create empty leaf list
  2: $W_\theta \leftarrow$ init(R, b_{tr}) //initialize weights
  3: $T \leftarrow \frac{D*E}{B}$
  4: **for** $t = 1...T$ **do**
  5:    $input \leftarrow$ batch(R, D, B) // get data batch
        // forward pass
  6:    **for** layer $l_\theta \in W_\theta$.layers **do**
  7:      $output \leftarrow l_\theta(input)$
  8:      **for** $output_i \in output$ **do**
  9:        // Match trainer rounding
10:        $c \leftarrow$ read$(output_i, F)$
11:        $output_i \leftarrow$ rev$(output_i, b_r, c)$
12:      **end for**
13:      $input \leftarrow output$
14:    **end for**
15:    $loss \leftarrow$ loss$(output)$
16:    // backward pass
17:    $grad\_output \leftarrow \nabla_{\mathsf{loss}}$
18:    **for** layer $l_\theta \in W_\theta$.layers **do**
19:      $grad\_input \leftarrow \nabla_{l_\theta}(grad\_output)$
20:      **for** $grad\_input_i \in grad\_input$ **do**
21:        // Match trainer rounding
22:        $c \leftarrow$ read$(grad\_input_i, F)$
23:        $grad\_input_i \leftarrow$ rev$(grad\_input_i, b_r, c)$
24:      **end for**
25:      $grad\_output \leftarrow grad\_input$
26:    **end for**
27:    $\theta \leftarrow$ update update weights
28:    **if** $t \bmod k = 0$ **then**
29:      M_{leaves}.append(hash$_{\mathsf{sha256}}(\theta$ in precision $b_m))$
30:    **end if**
31: **end for**
32: $M_{root} \leftarrow$ tree(M_{leaves}) // create Merkle tree
33: **return** M_{root}

Algorithm 3 threshold

INPUT: layer l, rounding amount b_r, training precision b_{tr}
OUTPUT: threshold τ

1: $P \leftarrow$ initialize empty list
2: $N, T \leftarrow$ initialize large # of data points and iterations
3: **for** i=1...N **do**
4:   $GPU1, GPU2 \leftarrow$ select two different GPU architectures
5:   $x \leftarrow$ select random input for layer l in b_{tr} floating-point precision
6:   $y_1 \leftarrow l_{GPU1}(x), y_2 \leftarrow l_{GPU2}(x)$, apply layer l on input x on each GPU
7:   **if** $\mathsf{rnd}_{b_r}(y_1) \neq \mathsf{rnd}_{b_r}(y_2)$ **then**
8:     **if** $y_1 > \mathsf{rnd}_{b_r}(y_1)$ and $y_2 < \mathsf{rnd}_{b_r}(y_2)$ **then**
9:       $P.\mathsf{append}(|y_1 - \mathsf{rnd}_{b_r}(y_1)|)$
10:      $P.\mathsf{append}(|y_2 - \mathsf{rnd}_{b_r}(y_2)|)$
11:     **end if**
12:     **if** $y_1 < \mathsf{rnd}_{b_r}(y_1)$ and $y_2 > \mathsf{rnd}_{b_r}(y_2)$ **then**
13:       $P.\mathsf{append}(|y_1 - \mathsf{rnd}_{b_r}(y_1)|)$
14:      $P.\mathsf{append}(|y_2 - \mathsf{rnd}_{b_r}(y_2)|)$
15:     **end if**
16:   **end if**
17: **end for**
18: //binary search to select threshold
19: $lower, upper, \tau \leftarrow 0.25 * (2^{-23}), 0.5 * (2^{9-b_r}), 0$
20: **for** t=1...T **do**
21:   $\tau \leftarrow (lower + upper)/2$
22:   $success \leftarrow True$
23:   **for** $p_i \in P$ **do**
24:     $\exp \leftarrow$ get exponent of p_i
25:     **if** $p_i < \exp * \tau$ **then**
26:       $success \leftarrow False$
27:     **end if**
28:   **end for**
29:   **if** $success$ **then**
30:     $lower \leftarrow \tau$
31:   **else**
32:     $upper \leftarrow \tau$
33:   **end if**
34: **end for**
35: **return** τ

Algorithm 4 log

INPUT: value x, rounding amount b_r, threshold τ, file F

1: $\exp \leftarrow$ get exponent of x
2: **if** $|x - \mathsf{rnd}_{b_r}(x)| > \exp * \tau$ **and** $x < \mathsf{rnd}_{b_r}(x)$ **then**
3:   $\mathsf{write}(2, F)$ // log rounding up
4: **else if** $|x - \mathsf{rnd}_{b_r}(x)| > \exp * \tau$ **and** $x > \mathsf{rnd}_{b_r}(x)$ **then**
5:   $\mathsf{write}(0, F)$ // log rounding down
6: **else**
7:   $\mathsf{write}(1, F)$ // log rounding ignore
8: **end if**

NeurIPS Paper Checklist

1. **Claims**

   Question: Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope?

   Answer: [Yes]

   Justification: See abstract and introduction.

   Guidelines:

   - The answer NA means that the abstract and introduction do not include the claims made in the paper.
   - The abstract and/or introduction should clearly state the claims made, including the contributions made in the paper and important assumptions and limitations. A No or NA answer to this question will not be perceived well by the reviewers.
   - The claims made should match theoretical and experimental results, and reflect how much the results can be expected to generalize to other settings.
   - It is fine to include aspirational goals as motivation as long as it is clear that these goals are not attained by the paper.

2. **Limitations**

   Question: Does the paper discuss the limitations of the work performed by the authors?

   Answer:[Yes]

   Justification: See Limitations.

   Guidelines:

   - The answer NA means that the paper has no limitation while the answer No means that the paper has limitations, but those are not discussed in the paper.
   - The authors are encouraged to create a separate "Limitations" section in their paper.
   - The paper should point out any strong assumptions and how robust the results are to violations of these assumptions (e.g., independence assumptions, noiseless settings, model well-specification, asymptotic approximations only holding locally). The authors should reflect on how these assumptions might be violated in practice and what the implications would be.
   - The authors should reflect on the scope of the claims made, e.g., if the approach was only tested on a few datasets or with a few runs. In general, empirical results often depend on implicit assumptions, which should be articulated.
   - The authors should reflect on the factors that influence the performance of the approach. For example, a facial recognition algorithm may perform poorly when image resolution is low or images are taken in low lighting. Or a speech-to-text system might not be used reliably to provide closed captions for online lectures because it fails to handle technical jargon.
   - The authors should discuss the computational efficiency of the proposed algorithms and how they scale with dataset size.
   - If applicable, the authors should discuss possible limitations of their approach to address problems of privacy and fairness.
   - While the authors might fear that complete honesty about limitations might be used by reviewers as grounds for rejection, a worse outcome might be that reviewers discover limitations that aren't acknowledged in the paper. The authors should use their best judgment and recognize that individual actions in favor of transparency play an important role in developing norms that preserve the integrity of the community. Reviewers will be specifically instructed to not penalize honesty concerning limitations.

3. **Theory Assumptions and Proofs**

   Question: For each theoretical result, does the paper provide the full set of assumptions and a complete (and correct) proof?

   Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper does not include theoretical results.
- All the theorems, formulas, and proofs in the paper should be numbered and cross-referenced.
- All assumptions should be clearly stated or referenced in the statement of any theorems.
- The proofs can either appear in the main paper or the supplemental material, but if they appear in the supplemental material, the authors are encouraged to provide a short proof sketch to provide intuition.
- Inversely, any informal proof provided in the core of the paper should be complemented by formal proofs provided in appendix or supplemental material.
- Theorems and Lemmas that the proof relies upon should be properly referenced.

4. **Experimental Result Reproducibility**

Question: Does the paper fully disclose all the information needed to reproduce the main experimental results of the paper to the extent that it affects the main claims and/or conclusions of the paper (regardless of whether the code and data are provided or not)?

Answer:[Yes]

Justification: See Experiments and Appendix.

Guidelines:

- The answer NA means that the paper does not include experiments.
- If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
- If the contribution is a dataset and/or model, the authors should describe the steps taken to make their results reproducible or verifiable.
- Depending on the contribution, reproducibility can be accomplished in various ways. For example, if the contribution is a novel architecture, describing the architecture fully might suffice, or if the contribution is a specific model and empirical evaluation, it may be necessary to either make it possible for others to replicate the model with the same dataset, or provide access to the model. In general. releasing code and data is often one good way to accomplish this, but reproducibility can also be provided via detailed instructions for how to replicate the results, access to a hosted model (e.g., in the case of a large language model), releasing of a model checkpoint, or other means that are appropriate to the research performed.
- While NeurIPS does not require releasing code, the conference does require all submissions to provide some reasonable avenue for reproducibility, which may depend on the nature of the contribution. For example
  (a) If the contribution is primarily a new algorithm, the paper should make it clear how to reproduce that algorithm.
  (b) If the contribution is primarily a new model architecture, the paper should describe the architecture clearly and fully.
  (c) If the contribution is a new model (e.g., a large language model), then there should either be a way to access this model for reproducing the results or a way to reproduce the model (e.g., with an open-source dataset or instructions for how to construct the dataset).
  (d) We recognize that reproducibility may be tricky in some cases, in which case authors are welcome to describe the particular way they provide for reproducibility. In the case of closed-source models, it may be that access to the model is limited in some way (e.g., to registered users), but it should be possible for other researchers to have some path to reproducing or verifying the results.

5. **Open access to data and code**

Question: Does the paper provide open access to the data and code, with sufficient instructions to faithfully reproduce the main experimental results, as described in supplemental material?

Answer: [Yes]

Justification: Code will be released in public version.

Guidelines:

- The answer NA means that paper does not include experiments requiring code.
- Please see the NeurIPS code and data submission guidelines (`https://nips.cc/public/guides/CodeSubmissionPolicy`) for more details.
- While we encourage the release of code and data, we understand that this might not be possible, so "No" is an acceptable answer. Papers cannot be rejected simply for not including code, unless this is central to the contribution (e.g., for a new open-source benchmark).
- The instructions should contain the exact command and environment needed to run to reproduce the results. See the NeurIPS code and data submission guidelines (`https://nips.cc/public/guides/CodeSubmissionPolicy`) for more details.
- The authors should provide instructions on data access and preparation, including how to access the raw data, preprocessed data, intermediate data, and generated data, etc.
- The authors should provide scripts to reproduce all experimental results for the new proposed method and baselines. If only a subset of experiments are reproducible, they should state which ones are omitted from the script and why.
- At submission time, to preserve anonymity, the authors should release anonymized versions (if applicable).
- Providing as much information as possible in supplemental material (appended to the paper) is recommended, but including URLs to data and code is permitted.

6. **Experimental Setting/Details**

Question: Does the paper specify all the training and test details (e.g., data splits, hyperparameters, how they were chosen, type of optimizer, etc.) necessary to understand the results?

Answer: [Yes]

Justification: See Experiments and Appendix.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The experimental setting should be presented in the core of the paper to a level of detail that is necessary to appreciate the results and make sense of them.
- The full details can be provided either with the code, in appendix, or as supplemental material.

7. **Experiment Statistical Significance**

Question: Does the paper report error bars suitably and correctly defined or other appropriate information about the statistical significance of the experiments?

Answer: [NA]

Justification: No experiment results have variation necessitating statistical significance.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The authors should answer "Yes" if the results are accompanied by error bars, confidence intervals, or statistical significance tests, at least for the experiments that support the main claims of the paper.
- The factors of variability that the error bars are capturing should be clearly stated (for example, train/test split, initialization, random drawing of some parameter, or overall run with given experimental conditions).
- The method for calculating the error bars should be explained (closed form formula, call to a library function, bootstrap, etc.)
- The assumptions made should be given (e.g., Normally distributed errors).
- It should be clear whether the error bar is the standard deviation or the standard error of the mean.

- It is OK to report 1-sigma error bars, but one should state it. The authors should preferably report a 2-sigma error bar than state that they have a 96% CI, if the hypothesis of Normality of errors is not verified.
- For asymmetric distributions, the authors should be careful not to show in tables or figures symmetric error bars that would yield results that are out of range (e.g. negative error rates).
- If error bars are reported in tables or plots, The authors should explain in the text how they were calculated and reference the corresponding figures or tables in the text.

8. **Experiments Compute Resources**

Question: For each experiment, does the paper provide sufficient information on the computer resources (type of compute workers, memory, time of execution) needed to reproduce the experiments?

Answer: [Yes]

Justification: See Appendix for GPUs.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The paper should indicate the type of compute workers CPU or GPU, internal cluster, or cloud provider, including relevant memory and storage.
- The paper should provide the amount of compute required for each of the individual experimental runs as well as estimate the total compute.
- The paper should disclose whether the full research project required more compute than the experiments reported in the paper (e.g., preliminary or failed experiments that didn't make it into the paper).

9. **Code Of Ethics**

Question: Does the research conducted in the paper conform, in every respect, with the NeurIPS Code of Ethics `https://neurips.cc/public/EthicsGuidelines`?

Answer: [Yes]

Justification: Yes

Guidelines:

- The answer NA means that the authors have not reviewed the NeurIPS Code of Ethics.
- If the authors answer No, they should explain the special circumstances that require a deviation from the Code of Ethics.
- The authors should make sure to preserve anonymity (e.g., if there is a special consideration due to laws or regulations in their jurisdiction).

10. **Broader Impacts**

Question: Does the paper discuss both potential positive societal impacts and negative societal impacts of the work performed?

Answer: [Yes]

Justification: See Security Analysis.

Guidelines:

- The answer NA means that there is no societal impact of the work performed.
- If the authors answer NA or No, they should explain why their work has no societal impact or why the paper does not address societal impact.
- Examples of negative societal impacts include potential malicious or unintended uses (e.g., disinformation, generating fake profiles, surveillance), fairness considerations (e.g., deployment of technologies that could make decisions that unfairly impact specific groups), privacy considerations, and security considerations.
- The conference expects that many papers will be foundational research and not tied to particular applications, let alone deployments. However, if there is a direct path to any negative applications, the authors should point it out. For example, it is legitimate to point out that an improvement in the quality of generative models could be used to

generate deepfakes for disinformation. On the other hand, it is not needed to point out that a generic algorithm for optimizing neural networks could enable people to train models that generate Deepfakes faster.

- The authors should consider possible harms that could arise when the technology is being used as intended and functioning correctly, harms that could arise when the technology is being used as intended but gives incorrect results, and harms following from (intentional or unintentional) misuse of the technology.
- If there are negative societal impacts, the authors could also discuss possible mitigation strategies (e.g., gated release of models, providing defenses in addition to attacks, mechanisms for monitoring misuse, mechanisms to monitor how a system learns from feedback over time, improving the efficiency and accessibility of ML).

11. **Safeguards**

Question: Does the paper describe safeguards that have been put in place for responsible release of data or models that have a high risk for misuse (e.g., pretrained language models, image generators, or scraped datasets)?

Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper poses no such risks.
- Released models that have a high risk for misuse or dual-use should be released with necessary safeguards to allow for controlled use of the model, for example by requiring that users adhere to usage guidelines or restrictions to access the model or implementing safety filters.
- Datasets that have been scraped from the Internet could pose safety risks. The authors should describe how they avoided releasing unsafe images.
- We recognize that providing effective safeguards is challenging, and many papers do not require this, but we encourage authors to take this into account and make a best faith effort.

12. **Licenses for existing assets**

Question: Are the creators or original owners of assets (e.g., code, data, models), used in the paper, properly credited and are the license and terms of use explicitly mentioned and properly respected?

Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper does not use existing assets.
- The authors should cite the original paper that produced the code package or dataset.
- The authors should state which version of the asset is used and, if possible, include a URL.
- The name of the license (e.g., CC-BY 4.0) should be included for each asset.
- For scraped data from a particular source (e.g., website), the copyright and terms of service of that source should be provided.
- If assets are released, the license, copyright information, and terms of use in the package should be provided. For popular datasets, `paperswithcode.com/datasets` has curated licenses for some datasets. Their licensing guide can help determine the license of a dataset.
- For existing datasets that are re-packaged, both the original license and the license of the derived asset (if it has changed) should be provided.
- If this information is not available online, the authors are encouraged to reach out to the asset's creators.

13. **New Assets**

Question: Are new assets introduced in the paper well documented and is the documentation provided alongside the assets?

Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper does not release new assets.
- Researchers should communicate the details of the dataset/code/model as part of their submissions via structured templates. This includes details about training, license, limitations, etc.
- The paper should discuss whether and how consent was obtained from people whose asset is used.
- At submission time, remember to anonymize your assets (if applicable). You can either create an anonymized URL or include an anonymized zip file.

14. **Crowdsourcing and Research with Human Subjects**

Question: For crowdsourcing experiments and research with human subjects, does the paper include the full text of instructions given to participants and screenshots, if applicable, as well as details about compensation (if any)?

Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Including this information in the supplemental material is fine, but if the main contribution of the paper involves human subjects, then as much detail as possible should be included in the main paper.
- According to the NeurIPS Code of Ethics, workers involved in data collection, curation, or other labor should be paid at least the minimum wage in the country of the data collector.

15. **Institutional Review Board (IRB) Approvals or Equivalent for Research with Human Subjects**

Question: Does the paper describe potential risks incurred by study participants, whether such risks were disclosed to the subjects, and whether Institutional Review Board (IRB) approvals (or an equivalent approval/review based on the requirements of your country or institution) were obtained?

Answer: [NA]

Justification: [NA]

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Depending on the country in which research is conducted, IRB approval (or equivalent) may be required for any human subjects research. If you obtained IRB approval, you should clearly state this in the paper.
- We recognize that the procedures for this may vary significantly between institutions and locations, and we expect authors to adhere to the NeurIPS Code of Ethics and the guidelines for their institution.
- For initial submissions, do not include any information that would break anonymity (if applicable), such as the institution conducting the review.