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Pincer Movements are Always Better Than Same-Direction Search

Roee M. Francos[1] and Alfred M. Bruckstein[1]

***Abstract*—Given a planar circular region, containing an unknown number of smart mobile evaders, our aim is to detect all of them, or at least to confine them by sweeping the region's boundary, using a team of sweeping agents. We assume all sweepers are identical and have line sensors of equal length. By deriving conditions on the speed of the sweepers and their paths, sweepers can ensure the successful completion of the confinement task implying that evaders with a known limit on their speed cannot escape the initial domain. The critical speed is the minimal speed ensuring sweepers confine all evaders to their original domain. If sweepers move at higher speeds, they can succeed in the complete detection task as well. The prevailing idea in multi-agent based search protocols is to distribute sweepers equally across the domain of interest in order to divide the search effort among cooperating sweepers and thereby obtain better performance as the number of sweepers increases. Previous works suggested confinement and complete detection search protocols for groups of agents based on distributing searchers uniformly around the region and having them move in the same (clockwise or counterclockwise) direction. Recent work suggested pincer strategies for the same purpose. However, no sufficient quantitative comparison was done to prove pincer-based strategies are always better in terms of performance metrics such as minimal sweeper speed for confinement, and time of complete detection, for both of which a lower value is better. In this paper we provide a complete analysis of this problem yielding exact results proving pincer-based strategies are always better in all aspects when an even number of sweepers are working together. We do this for the case of sweepers having linear detectors, but we believe similar results can be obtained in general, for any number of sweepers, more general sensor geometries and different environments.**

I. Introduction

The goal of this research is to analytically prove that pincer-based guaranteed search strategies outperform their same-direction counterparts. Employing pincer movement strategies implies that sweepers move out in opposite directions along the boundary of the evader region to detect evaders while performing same-direction protocols implies that sweepers are deployed at equally spaced intervals along the boundary and all sweep in the same-direction. Each developed strategy provides a "must-win" search protocol in which a team of n identical sweepers ensures the detection of all smart evaders that are initially in a circular region of radius R_0.

The evaders are smart mobile agents capable of detecting and responding to the motions of searchers by performing evasive maneuvers, to avoid detection. Evaders attempt to escape the searching team and move out of the initial region, at a maximal speed of V_T. The evader region is defined as the region where potential evaders may be currently located. All sweepers move at a speed $V_s > V_T$ and detect evaders with linear sensors of length $2r$. Every evader that intersects a sweeper's field of view is immediately detected.

There can be any number of evaders inside the region, and this number as well as the evaders' locations is unknown to the sweepers. Every "must-win" strategy requires a minimal speed that depends on the trajectory of the sweepers. Different strategies are evaluated by using two metrics, total search time until all evaders are detected and a minimal critical speed required for a successful search.

To facilitate the comparison between pincer-based and same-direction protocols, we develop two types of cooperative same-direction search protocols circular and spiral. We compare the two types of pincer-movement search processes, circular and spiral, developed in [1] with their same-direction counterparts, for any even number of sweeping agents.

We prove pincer-based strategies provide superior results in all scenarios and that the spiral pincer sweep process allows detection of all evaders while sweeping at nearly theoretically optimal speeds. We present a quantitative and experimental comparison between the total search time of same-direction and pincer-movement search strategies for the case of even numbers of searchers showing that pincer-based strategies provide superior results in all considered scenarios.

A. Overview of Related Research

Literature on detection of smart opponents is classified into guaranteed detection strategies and probabilistic detection methods. Probabilistic approaches aim to develop algorithms that maximize the probability to detect a set of targets being searched and are often referred to as pursuit-evasion games, in which the pursuers' goal is to detect and catch the evaders and the evaders goal is to avoid being detected and caught by the pursuing team. There are many variants for pursuit-evasion games that may range from a single pursuer-evader setting to combinations of single and multiple pursuers-evaders settings.

Search for static agents requires to fully scan an area containing the agents, however a more critical question is how to efficiently search for smart dynamic agents. Planning against smart opponents is a long standing question that has been investigated for centuries, with the most notable execution of a pincer-movement maneuver carried out by the forces of Hannibal at the battle of Cannae. In modern times, search missions are typically considered to be carried out by flying entities such as manned planes, UAVs or drones, with first works considered by Koopman, see [2]. Patrolling a corridor by utilizing a multi-agent search team aimed at ensuring the detection of smart agents was also investigated in [3] with provably optimal protocols provided in [4].

Roee M. Francos and Alfred M. Bruckstein are with the Department of Computer Science, Technion- Israel Institute of Technology, Haifa, Israel, 320003, Emails:(roee.francos@cs.technion.ac.il, alfred.bruckstein@cs.technion.ac.il).

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In [5], the problem of optimally building a barrier around an advancing fire in order to confine its spread is considered. The objective is to find the minimal barrier construction speed allowing containment of the fire and assessing the optimality of the solution.

A non-escape search protocol aimed at guaranteeing detection of evaders that are initially inside a convex region of the plane from which they can escape is investigated in [6]. The idea is to use a collaborative protocol between members of the search team that move with an inwards spiral pattern in a leader-follower formation. In [7], McGee et al. also investigate a search problem for smart targets under different assumptions and applied search patterns consisting of spiral and linear sections. In [8], searching for smart evaders using concentric arc trajectories is considered with a goal of detecting submarines in a channel or in a half plane. In these cited works, the searchers' sensors detect targets within a disk shaped area around the searcher's location.

In [9], implicit cooperation between pairs of defenders moving in pincer-movements is used to detect intruders prior to their entry to the guarded region. In [10], related "reach-avoid games" problems are explored. In [11], pursuit–evasion problems involving multiple pursuers and multiple evaders are studied. Pursuers and evaders are all assumed to be identical, and pursuers follow either a constant bearing or a pure pursuit strategy. Recent surveys on pursuit evasion problems are [12]–[14].

In [15], the confinement and cleaning tasks for a line formation of agents or alternatively for a single agent with a linear sensor are analyzed. In [1], teams of agents perform pincer sweep search strategies with linear sensors. The comparison in this article stemmed from reviews of [1] that sought rigorous proofs that pincer strategies are always better than same-direction sweeps. The same-direction protocols developed in this work are different than those performed in [1], result in better performance, and therefore offer a more precise and proven comparison.

B. Contributions

In this paper, we provide several theoretical and experimental contributions to multi-agent search and coordinated motion planning literature by proving that in contrast to the prevalent concept of deploying searching agents equally around a domain of interest, a different distribution of the searchers improves the performance of the search protocols. To facilitate this proof and quantify that pincer-based search protocols are always better than same-direction protocols, we propose two types of same-direction search protocols that are extension to prevailing search techniques for guaranteeing detection of all smart evaders. The smart agents are initially inside a given circular region from which they try to escape the team of searching agents. A detailed theoretical analysis of trajectories, critical speeds and search times for same-direction sweep protocols performed by a team of n cooperative agents are developed in order to quantitatively compare these methods to the state-of-the-art pincer-based protocols developed in [1], and to the theoretical lower bound. The purpose for developing the same-direction protocols is therefore only to prove and emphasize the benefits of using pincer-based search strategies compared to same-direction protocols, and to analytically quantify, using teams of sweepers with exactly the same capabilities the improvement in the considered search metrics.

- We propose two types of same-direction sweep protocols:
  - Same-direction circular sweep pincer sweep strategy
  - Same-direction spiral sweep pincer sweep strategy
- We prove that for both same-direction sweep protocol types, the corresponding pincer-based protocols yield a lower critical speed.
- We show that circular and spiral pincer-based sweep protocols always result in shorter sweep times compared to their same-direction counterparts.
- Results show that for an increasing number of sweepers, circular pincer-based protocols require a smaller critical speed even when compared to spiral same-direction protocols that can only be implemented with sweepers that have more advanced capabilities.
- The theoretical analysis is complemented by simulation experiments in MATLAB and NetLogo [16] that verify the theoretical results and illustrates them graphically in the figures embedded throughout the text and in the attached video.
- We discuss considerations for deployment of multi-robot searching teams for guaranteed evader detection in practical robotic applications.

II. Same-direction Versus Pincer-based Sweeps

The complete search time until all evaders are detected depends on the search protocol performed by the team of sweepers. Two types of search patterns are investigated, circular and spiral. The desired outcome is that after each sweep around the region, the radius of the circle bounding the evader region is reduced by a strictly positive value. This guarantees complete detection of all evaders, by decreasing in finite time the potential area where evaders may be located to zero. At the start of the circular search protocol only half the footprint of the sweepers' sensors is inside the evader region, i.e. a footprint of length r, while the other half is outside the region with the intention of detecting evaders that attempt to escape outside of the region. At the start of the spiral search protocol the entire length of the sweepers' sensors is inside the evader region, i.e. a footprint of length $2r$.

In [15], it is proven that a smart evader may escape from point $P = (0, R_0)$ (shown in Fig. 1 (a)), when basing a single sweeper's speed only on a single traversal around the evader region. Hence, the sweeper's critical speed must increase to cope with such a potential adversarial escape plan. Point P is considered as the "most dangerous point", meaning that evaders located there, have the maximum time to spread during sweeper movement. Hence, if evaders spreading from this point are detected, evaders attempting to escape from all other points will also be detected. If we choose to distribute a multi-agent search team equally along the boundary of

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the initial evader region, we would have the same problem of possible escape from the points adjacent to the starting locations of every sweeper.

In [1], an alternative method for multi-agent search strategies in which pairs of sweepers move out in opposite directions along the boundary of the evader region and sweep in pincer-movements instead of deploying sweepers at equally spaced intervals along the boundary and requiring them to move in the same-direction, is proposed. The search protocol can be employed by a search team with any even number of sweepers. At the start of each sweep, sweepers are positioned in pairs back-to-back. In each pair, one sweeper moves counter-clockwise while the other moves clockwise. Every time sweepers meet, implying their sensors are back-to-back again, they exchange their movements directions. The search region is partitioned into a number of non-overlapping sections depending on the number of sweepers in the search team, such that every sweeper sweeps a particular angular sector of the region.

Sweeping with pincer-based search protocols removes the need to sweep additional areas to detect evaders from these additional ”most dangerous points” since in pincer-based protocols the ”most dangerous points” are now located at the tips of their sensors closest to the evader region's center.

The search can be either 2 dimensional where sweepers travel on a plane or 3 dimensional implying that sweepers are drone-like agents that fly over the evader region. In case the search is planar, exchanging of movement directions occurs after the completion of each sweep when a sweeper “bumps” into a sweeper that scans the adjacent section. If the search is 3 dimensional, sweepers fly at different altitudes above the evader region, and every time a sweeper is directly above another, they exchange the angular section they sweep between them, and continue the search. The analysis of 2 and 3 dimensional search protocols is similar.

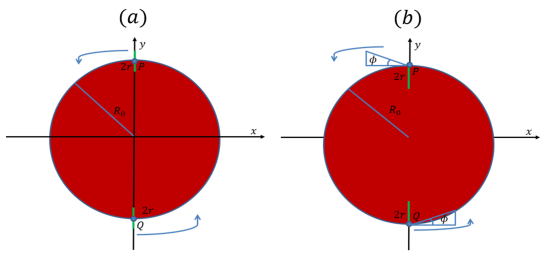

Fig. 1. Initial placement of 2 agents employing same-direction sweep protocols (a) - same-direction circular sweep. (b) - same-direction spiral sweep. The sweepers sensors are shown in green. ϕ is the angle between the tip of a sweeper's sensor and the normal of the evader region and depends on the ratio between the sweeper and evader speeds.

III. Deployment of Multi-Robot Searching Teams for Guaranteed Evader Detection in Practical Robotic Applications

A vast breadth of real-world problems that are currently solved by human-controlled machines are expected to be replaced by partially autonomously operated robots in the nearby future. Search and rescue missions, airborne and underwater surveillance applications, various monitoring tasks for security applications, wildlife tracking, fire control as well as inspection tasks in hazardous zones can all benefit from the theoretical and experimental results developed in this work.

In many of the mentioned applications, planning for the worst-case scenario can guarantee solving the task for all other scenarios as well. The performed analysis in this work is mathematically sound and guarantees that in a continuous domain all smart evaders are detected. It further develops analytical formulae for the time at which all evaders are detected, the minimal speed that guarantees detection of all smart evaders and compares the obtained results to state-of-the-art methods investigating guaranteed detection without full state information, often the case in real-world settings.

The searching agents considered in this work do not assume knowledge of the number of evaders present in the region, their locations, or their escape plan and despite that they are able to detect all of them. This significantly differs from many previous works that assume such knowledge. Therefore, this work is of prime theoretical and practical importance as is in many pursuit-evasion games the searching team does not have complete information about the evaders it needs to detect, as is often assumed by many papers. Results are insensitive to locations of evaders or their numbers.

Since multi-agent pursuit-evasion search protocols mainly utilize multi-agent UAVs, sweepers fly over the environment containing the evaders, therefore investigating issues such as obstacles is not the main focus of the work, because the sweeping team flies over them. Obstacles limit the movements and locations of ground-moving evaders, and therefore their presence assists the searching team to detect them since it limits the escape options of evaders, and thus does not impact our “worst-case” analysis.

Search protocols can use a vast suite of onboard sensors to detect evaders, depending on the domain of application. Potential choices vary from visual sensors such as cameras which have both a high resolution and are lightweight. Therefore, detecting evaders with cameras requires a smaller battery in order to accomplish the desired task compared to other sensing modalities such as radars that increase the weight of the payload and hence limit the duration of the search mission due to increased energy consumption. Actual detection of evaders can utilize a vast number of computer-vision detection algorithms such as [17], [18].

An issue in implementing the mentioned protocols in real-world settings is the ability to coordinate the movements of the sweeping pairs comprising the team. This manifests in the sweepers' ability to maintain their speed throughout the sweep process and advance together toward the center of the evader region while reducing it. It is possible to account for coordination imperfections by indirect communication between searching pairs, through the means of sensing and observing the location of a sweeper's partner and advancing together when the partner robot reaches the desired location, or through direct wireless communication between the robots.

The discussed comparison between protocols can be applied in other convex environments as well, by using slight

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modifications to the explored sweeping strategies.

IV. Same-direction Circular Sweep

A. Circular Sweep Time Calculation

Previously in [15], the tightest lower bound for a searcher's speed is found by constructing a function of 2 variables $f(t, V_s)$, by demanding that the furthest possible spread of the evader region is cleaned by the furthest tip of the sweeper's line sensor. A lesser requirement is to demand that by the time the most problematic point in the evader region, point P, spreads to a possible circle of radius of r around point P, the sweeping swarm completes in addition to a sweep of $\frac{2\pi}{n}$ around the evader region an additional angular traversal that is proportional to traversing an arc of length r. This means that the agent travels an angle of $\frac{2\pi}{n} + \beta_0$ where β_0 is marked in Fig. $2(a)$. This assumption results in a simplified expression for the critical speed that bounds the previously found critical speed of [15] from above for all choices of geometric parameters. Denote the time it takes the most problematic points to spread a distance of r as T_e. These points are adjacent to the starting locations of the sweepers, and 2 such points P_1 and P_2, exist in case the search is performed with 2 sweepers, as shown in Figure 2. We have that $T_e = \frac{r}{V_T}$. We can see from Figure 2. that $\sin\beta_0 = \frac{r}{R_0}$, therefore $\beta_0 = \arcsin\frac{r}{R_0}$. The time it takes the sweeper to travel an angle of $\frac{2\pi}{n} + \beta_0$ is therefore given by $T_s = \frac{\left(\frac{2\pi}{n} + \arcsin\left(\frac{r}{R_0}\right)\right)R_0}{V_s}$. To guarantee no escape, we demand that $T_s \leq T_e$. Therefore, rearranging terms in the previous equation and plugging T_e instead of T_s yields,

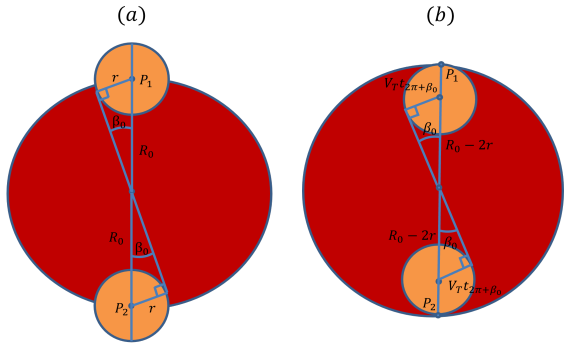

Fig. 2. Geometric representation required for critical speed calculation. Red areas indicate locations where potential evaders may be located. Orange circles denote spread of potential evaders around the most problematic points P_1 and P_2 during a traversal of $2\pi + \beta_0$ around the evader region. (a) - Same-direction circular sweep. (b) - Same-direction spiral sweep.

$$V_c \geq \frac{\left(\frac{2\pi}{n} + \arcsin\left(\frac{r}{R_0}\right)\right) R_0 V_T}{r} \quad (1)$$

The lower bound on a sweeper's speed ensuring confinement is obtained when (1) is satisfied with equality. To enable the construction of analytical results for the sweep times of the evader region, in future derivations we use the first order Taylor approximation for the arcsine function in (1). Such an approximation is valid since in all practical scenarios the ratio between $\frac{r}{R_0}$ is sufficiently small. Applying this approximation to (1) allows to define $V_{c_{circ}}$, the chosen critical speed,

$$V_{c_{circ}} = \frac{2\pi R_0 V_T}{rn} + V_T \quad (2)$$

For the sweeper team to advance inward toward the center of the evader region, it must travel in a speed greater than the critical speed. Denote by $\Delta V > 0$ the increment in the sweeping agents' speed that is above the critical speed. Each agent's speed V_s is therefore given by the sum of the critical speed and ΔV, namely $V_s = V_{c_{circ}} + \Delta V$. The total sweep times required for the sweeper team to reduce the evader region to a region bounded by a circle with a radius smaller or equal to r is given by the sum of the circular motions and inward advancements that are performed after the completion of each circular sweep. Denote the number of sweeps required by the sweeper team to complete this motion by N_n, where n indicates that the number of sweeps depends on the number of sweepers performing the search. The time it takes the sweepers to perform the circular sweeps is given by,

$$T_{circular} = -\frac{R_0(V_s+V_T)}{V_s V_T} + \frac{r(V_s-V_T)(n(V_s+V_T)+2\pi V_T N_n)}{2\pi {V_T}^2 V_s} + \left(1 + \frac{2\pi V_T}{n(V_s+V_T)}\right)^{N_n} (V_s + V_T)\left(\frac{2\pi R_0 V_T - rn(V_s-V_T)}{2\pi {V_T}^2 V_s}\right) + \frac{2\pi r}{n V_s} \quad (3)$$

The time required to perform the inward advancement is,

$$T_{in} = \frac{R_0}{V_s} + \left(1 + \frac{2\pi V_T}{V_s+V_T}\right)^{N-1}\left(\frac{2\pi R_0 V_T - r(V_s-V_T)}{V_s(V_s+V_T)}\right) \quad (4)$$

Full analytical development is provided in Appendix A of [19].

B. Same-direction Circular Sweep End-game

In order to entirely clean the evader region the sweepers need to change the scanning method when the evader region is bounded by a circle of radius r. This is due to the fact that a smart evader that is very close to the center of the evader region can travel at a very high angular velocity compared to the angular velocity of the pursuing agents. This constraint is described by the following two equations, $\omega_s = \frac{V_s}{r}, \omega_T = \frac{V_T}{\varepsilon}$. The first describes the searcher's angular velocity and the second the evader's angular velocity. Since ε can be arbitrarily small the evader can move just behind a sweeper's sensor and never be detected. Thus a slight modification to the sweep process needs to be applied in order to clean the entire evader region with the sweeper team that employs a circular scan.

After completing sweep number $N_n - 1$ the sweepers move toward the center of the evader region until the tip of the sweepers' sensors closest to the center of the evader region are placed at the center of the evader region. Following this motion the sweepers perform a circular sweep of radius r around the center of the evader region. Following the motion, the sweeper team travels to the right until cleaning the wavefront propagating from the right portion of the remaining evader region and then travels to the left until cleaning the remaining evader region. The time required to complete this motion is denoted by T_{linear}, and assumes that during the

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linear movement the margin between the tip of the sensor in each direction to the evader region boundaries satisfies,

$$\frac{2r - R_{last}}{V_T} > T_{linear} \tag{5}$$

A depiction of the scenario at the beginning of the end-game is presented in Fig. 3. Theorem 1 states the conditions for this demand to hold.

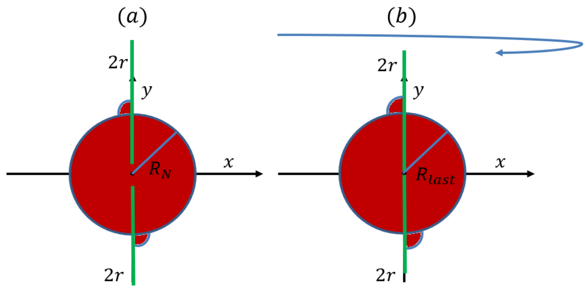

Fig. 3. Depiction of the end-game steps for the same-direction circular sweep performed by 2 sweepers. The sweepers sensors' are shown in green and red areas indicate locations where potential evaders may still be located. (a) - Evader region status and sweepers' locations prior to the last inward advancement. (b) - Evader region status and sweepers' locations prior to the linear sweep.

Theorem 1. *When defining $\alpha = \frac{R_0}{r}$, if ΔV satisfies that,*

$$\Delta V \geq \frac{-4\pi V_T \alpha + \pi V_T + V_T\sqrt{\pi^2 + 8\pi n}}{2n} \tag{6}$$

then the evader region will be completely cleaned by n sweepers that employ the linear scan after $N_n + 1$ iterations.

Therefore, the total scan time until a complete cleaning of the evader region is given by $T_{total} = T_{circular} + T_{in} + T_{linear}$. For the one dimensional scan to be valid and ensure a non escape search and complete cleaning of the evader region (5) must be satisfied. This demand implies that for a given α, the designer of the sweep process can infer which ΔV needs to be chosen in order to satisfy (6) and thus completely clean the evader region using the final linear sweeping motion. For a complete derivation see section $III(B)$ of [19].

Theorem 2. *For a valid circular search process the total search time until a complete cleaning of the evader region is given by, $T = T_{circular} + T_{in} + T_{linear}$, or as,*

$$T = -\frac{R_0}{V_T} + \frac{r(V_s - V_T)(n(V_s+V_T) + 2\pi V_T N_n)}{2\pi {V_T}^2 V_s} + \left(1 + \frac{2\pi V_T}{n(V_s+V_T)}\right)^{N_n - 1} \left(\frac{2\pi R_0 V_T - rn(V_s - V_T)}{V_s}\right) \left(\frac{1}{n(V_s+V_T)} + \frac{V_s}{2\pi {V_T}^2} + \frac{1}{2\pi V_T} + \frac{1}{nV_T}\right) + \frac{2\pi r}{nV_s} + \frac{6\pi r V_T V_s - 2\pi r {V_T}^2}{nV_s(V_s - V_T)^2} \tag{7}$$

The total search times until complete cleaning of the evader region are shown in Fig. 5.

V. Same-direction Spiral Sweep

Since our aim is to provide a sweep protocol that improves the same-direction circular sweep protocol, we desire that sweepers move in a more efficient trajectory to detect evaders during the search protocol. Hence, we require that throughout the motion of the sweepers, their sensors' footprint maximally intersects the evader region. This is achieved by using a spiral scan, in which the sweepers' sensors track the expanding evader region's wavefront, while trying to keep its shape nearly circular. An illustration of the initial placement of 2 sweepers that employ the same-direction spiral sweep process is presented in Fig. $1(b)$. The sweepers start with a sensor length of $2r$ inside the evader region. If the sweeper agents' speed is above the scenario's critical speed, the sweepers reduce the evader region's area after completing a traversal around the region. Each sweeper starts its spiral sweep with its sensor's tip that is furthest from the center of the evader region, in a position that is tangent to the boundary of the evader region. To preserve their sensors in an orientation that is tangent to the evader region, the sweepers move at angle ϕ to the normal of the evader region. ϕ is calculated from $\sin\phi = \frac{V_T}{V_s}$. This method of traveling at angle ϕ preserves the evader region's circular shape.

Contrary to the pincer-based strategy where each sweeper travels only an angle of $\frac{2\pi}{n}$ at each sweep iteration, in same-direction sweeps, each sweeper travels a larger angle than $\frac{2\pi}{n}$ at each iteration around the evader region in order to detect all escaping smart evaders. The additional angle, denoted by β, needs to be traversed in order to detect all evaders that may have spread from the "most dangerous points" at the beginning of each sweep. Such points are adjacent to the starting locations of every sweeper. The angle β depends on the radius of the circle that bounds the evader region. After a sweeper traverses the additional angle β, the evader region's boundary is due to spread from points that resided at the lower tips of the sensors. When the tips of the sensors leave these points, evaders may spread from them in all directions at a speed of V_T. The time it takes a sweeper to travel an angle of $\frac{2\pi}{n} + \beta_0$, where β_0 is shown in Fig. $2(b)$ is given by,

$$t_{\frac{2\pi}{n}+\beta_0} = \frac{(R_0 - r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_0\right)V_T}{\sqrt{{V_s}^2 - {V_T}^2}}} - 1\right)}{V_T} \tag{8}$$

The subscript 0 in β_0 denotes the sweep cycle number, indicating that the value of β changes as the sweep process progresses. After a sweeper completes a traversal of $\frac{2\pi}{n} + \beta_0$ around the evader region it moves towards the center of the evader region. During this motion its lower tip points to the center of the region. β_0 is given by,

$$\sin\beta_0 = \frac{V_T t_{\frac{2\pi}{n}+\beta_0}}{R_0 - 2r} \tag{9}$$

After a sweeper traverses $\frac{2\pi}{n} + \beta_0$ around the evader region the evader region's boundary is due to evaders that originated from the next "most dangerous points". The critical speed that satisfies the confinement task is computed numerically using the Newton method. When the sweepers travel towards the center of the evader region after completing a spiral sweep they have to meet the evader wavefront travelling outwards the region with a speed of V_T at the previous radius R_0.

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Therefore, the expansion of the evader region after the first sweep at time $t_{\frac{2\pi}{n}+\beta_0}$, has to satisfy that,

$$V_T t_{\frac{2\pi}{n}+\beta_0} \leq \frac{2rV_s}{V_s+V_T} \tag{10}$$

The critical speed is obtained when we have equality in (10). In order to calculate β_0 that is obtained when the sweepers move at the critical speed, the expression of $V_T t_{\frac{2\pi}{n}+\beta_0}$ in (10) is substituted with its equivalent expression in (9). Hence,

$$\beta_0 = \arcsin\left(\frac{2rV_s}{(V_s+V_T)(R_0-2r)}\right) \tag{11}$$

Substituting the expression for $t_{\frac{2\pi}{n}+\beta_0}$, yields

$$(R_0-r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_0\right)V_T}{\sqrt{V_s{}^2-V_T{}^2}}}-1\right)=\frac{2rV_s}{V_s+V_T} \tag{12}$$

In order to solve for V_s we write (12) as,

$$F(V_s)=\frac{2rV_s}{V_s+V_T}-(R_0-r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_0\right)V_T}{\sqrt{V_s{}^2-V_T{}^2}}}-1\right) \tag{13}$$

From (13) we find V_s using the Newton iterative root finding method whose equation is given by,

$$V_{s_{n+1}} = V_{s_n} - \frac{F(V_{s_n})}{\frac{\partial F(V_{s_n})}{\partial V_{s_n}}} \tag{14}$$

We choose as our initial estimate the lower bound on the sweeper speed (following a proof that derives a lower bound for a sweeper's critical speed in [1]) given by $V_{s_0} = \frac{\pi R_0 V_T}{nr} = V_{LB}$. By using the described iterative convergence, we obtain a solution for V_s, which is the same-direction spiral sweep's critical speed. Denote this speed as $V_{c_{spiral_{same}}}$. The solution converges to a result only slightly larger than the lower bound on the sweeper speed, V_{LB}.

Denote by $\Delta V > 0$ the addition to the sweeper's speed above the critical speed. The speed is therefore given by, $V_s = V_{c_{spiral_{same}}} + \Delta V$. If a sweeper moves with a speed greater than the critical speed, after each spiral sweep it can advance inwards towards the center of the evader region and sweep around an evader region that is bounded by a circle with a smaller radius. The total search time until the evader region is bounded by a circle with a radius that is less than or equal to $2r$ is given by the sum of the total spiral sweep times and the times of the inward advances. Namely,

$$T = T_{in} + T_{spiral} \tag{15}$$

After each iteration, the sweepers move inwards towards the center of the evader region and the radius of the circle that bounds the region decreases. Consequently, the angle β_i after which the sweepers move inwards changes as well. Therefore, after each sweep β_i is calculated with respect to the new radius of the circle that bounds the evader region,

$$\beta_i = \arcsin\left(\frac{2rV_s}{(V_s+V_T)(R_i-2r)}\right) \tag{16}$$

The time it takes to complete a spiral sweep of $\frac{2\pi}{n}+\beta_i$ around a region bounded by a circle of radius R_i is given by,

$$T_{spiral_i} = \frac{(R_i-r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_i\right)V_T}{\sqrt{V_s{}^2-V_T{}^2}}}-1\right)}{V_T} \tag{17}$$

Denote the distance an agent can advance towards the center of the evader region by $\delta_i(\Delta V)$. In the term $\delta_i(\Delta V)$, ΔV denotes the increase in the agent speed relative to the critical speed, and i denotes the number of sweep iterations the sweepers perform around the evader region, where i starts from sweep number 0. This results in a new evader region bounded by a circle with a radius of $R_{i+1} = R_i - \delta_i(\Delta V)$. We have that,

$$\delta_i(\Delta V) = 2r - V_T T_{spirali}\ ,\ 0 \leq \delta_i(\Delta V) \leq 2r \tag{18}$$

As a function of the iteration number, the distance a sweeper can advance inwards after completing an iteration is given by,

$$\delta_i(\Delta V) = 2r - (R_i-r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_i\right)V_T}{\sqrt{V_s{}^2-V_T{}^2}}}-1\right) \tag{19}$$

After sweepers complete a sweep, they move inward toward the region's center with the inner tips of their sensors pointing toward the center of the evader region with a speed of V_s, until they reach the position where they start their next sweep at the moment they meet the evader region's expanding wavefront.

During inwards advancements no cleaning is performed, while the evader region continues to spread. The time it takes a sweeper to move inwards until its entire sensor is over the evader region depends on the relative speed between the sweeper's inwards entry speed and the evader region outwards expansion speed. As the sweepers progress toward the center of the evader region, the evader region continuous to expand.

Therefore, sweepers can only advance by a smaller distance than $\delta_i(\Delta V)$, denoted by $\delta_{i_{eff}}(\Delta V)$, which depends on the ratio between the speed in which the sweeper progresses towards the center of the region and the sum of velocities of sweeper and evader region spread. $\delta_{i_{eff}}(\Delta V)$ is the actual distance the sweeper moves at each iteration in order to meet the wavefront of the evader region when its entire sensor overlaps the evader region. Therefore, the distance sweepers can advance inwards after completing an iteration is given by,

$$\delta_{i_{eff}}(\Delta V) = \delta_i(\Delta V)\left(\frac{V_s}{V_s+V_T}\right) \tag{20}$$

The evader region is therefore bounded by a circle of radius,

$$R_{i+1} = R_i - \delta_i(\Delta V)\left(\frac{V_s}{V_s+V_T}\right) \tag{21}$$

In the accompanying video, the inwards motion is not observed, however the equations that govern the motion of the sweepers and evaders in simulation consider the time required for sweepers to advance inwards and dictate the new radius of the evader region after the sweep. This process continues until the evader region is bounded by a circle with a radius that is smaller than $2r$. We denote this radius as R_N. Once the evader region is contained inside a circular domain with a radius of $2r < R_i < 4r$, β_i is,

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$$\beta_i = \arcsin\left(\frac{(R_i - 2r)\,V_s}{(V_s + V_T)\,(R_i - 2r)}\right) \quad (22)$$

The inwards advancement time depends on the iteration number. It is denoted by T_{in_i} and its expression is given by

$$T_{in_i} = \frac{\delta_{i_{eff}}(\Delta V)}{V_s} = \frac{2r - (R_i - r)\left(e^{\frac{\left(\frac{2\pi}{n}+\beta_i\right)V_T}{\sqrt{V_s{}^2 - V_T{}^2}}} - 1\right)}{V_s + V_T} \quad (23)$$

During the inward advancements only the tip of the sensor, that has zero width, is inserted into the evader region. Therefore, no evaders are detected until the sweeper completes its inward advance and starts sweeping again. This search methodology continues until the evader region is bounded by a circle with a radius that is less than or equal to $2r$.

To entirely clean the evader region, the sweepers need to change the scanning method when the evader region is bounded by a circle of radius $2r$, due to the same consideration that are described in the end-game of the same-direction circular sweep process. A detailed analysis of the end-game is provided in section IV of [19]. Hence, the total scan time until a complete cleaning of the evader region is given by,

$$T_{total} = T_{spiral} + T_{in} + T_{end} \quad (24)$$

VI. Comparative Analysis of Pincer Movement Search Strategies and Same-direction Sweeps

The purpose of this section is to compare between the obtained results for the circular and spiral same-direction sweep processes developed in the previous sections and the pincer sweep processes developed in [1]. In all forthcoming figures the number of sweepers is even, and ranges from 2 to 20 agents, that employ the spiral and circular pincer sweep processes and the same-direction sweep protocols. The chosen values of the parameters are $r = 10$, $V_T = 1$ and $R_0 = 100$. The top plot of Fig. 4 presents the comparison between critical speeds required to perform each sweep protocol. The bottom plot of Fig. 4 presents the ratio between the critical speeds of each protocol and the lower bound, V_{LB}.

The resulting conclusion is that critical speeds of sweepers implementing same-direction circular or same-direction spiral protocols is higher than the minimal critical speed of their pincer sweep counterparts. Requiring a higher critical speed implies that there are entire regions of operation where an evader region with a given radius could be cleaned by a sweeper team that performs the same-direction spiral sweep process but cannot be cleaned by a sweeper team that performs the same-direction circular sweep process. This also implies that sweeping teams performing pincer movement search strategies can successfully sweep larger regions than their same-direction sweeps counterparts.

Furthermore, results show that as the number of sweepers increases, circular pincer-based protocols require a smaller critical speed even when compared to spiral same-direction protocols. This result indicates that although implementing pincer-based circular protocols requires sweepers with more basic capabilities compared to spiral protocols, the cooperation between the sweepers considerably improves the overall performance of the sweeper team.

Hence, since pincer sweep processes require a smaller critical speed compared to same-direction sweep processes, in order to make a fair comparison, all sweepers in the team move at speeds above the critical speed of 2 sweepers that perform the corresponding same-direction sweep. The critical speed of 2 sweepers that perform the same-direction circular sweep is the highest amongst the compared search protocols.

The right plot of Fig. 5 shows the complete search times of teams performing circular pincer sweeps. The left plot of Fig. 5 shows the complete search times of teams performing the circular same-direction protocol. In both figures the sweepers move at the same speeds above the same-direction circular critical speed of 2 sweepers, since this speed is greater than the critical speed of search processes performed with more sweepers. We see that for all choices of speeds above the same-direction circular critical speed of 2 sweepers, the complete search times of teams performing same-direction circular sweeps are longer compared to teams performing their circular pincer sweep counterparts. Hence, from these results we conclude that performing circular pincer sweeps is always better than performing same-direction circular sweeps.

The right plot of Fig. 6 shows the complete search times of teams performing spiral pincer sweeps. The left plot of Fig. 6 shows the complete search times of teams performing the spiral same-direction protocol. In both figures sweepers move at equal speeds above the same-direction spiral critical speed of 2 sweepers, since this speed is greater than the critical speed of protocols performed with more sweepers.

We see that for all choices of speeds above the same-direction spiral critical speed of 2 sweepers, the complete search times of teams performing same-direction spiral protocols are longer compared to teams performing their spiral pincer sweep counterparts. This result is expected since as the number of sweepers increases, the gain in utilizing the cooperation between sweeping pairs in pincer-based sweep processes decreases the sweeping time more significantly compared to sweepers that perform the same-direction spiral sweeps. This occurs since same-direction sweepers must sweep larger angular sections at each iteration to ensure no evader escapes, while in pincer-based spiral search strategies, sweeping these additional sections is unnecessary due to the complementary trajectories of the sweepers. Hence, from these results we conclude that performing spiral pincer sweeps is always better than performing same-direction sweeps.

VII. Conclusions

This work compares same-direction and pincer-movement sweep protocols demonstrating and proving the superiority of the latter. We perform a quantitative comparison between pincer-based and same-direction sweep protocols for any number of even sweepers where the sensing capabilities and speeds of the sweeping teams are equivalent. We prove that critical speeds for pincer based search methods are lower than

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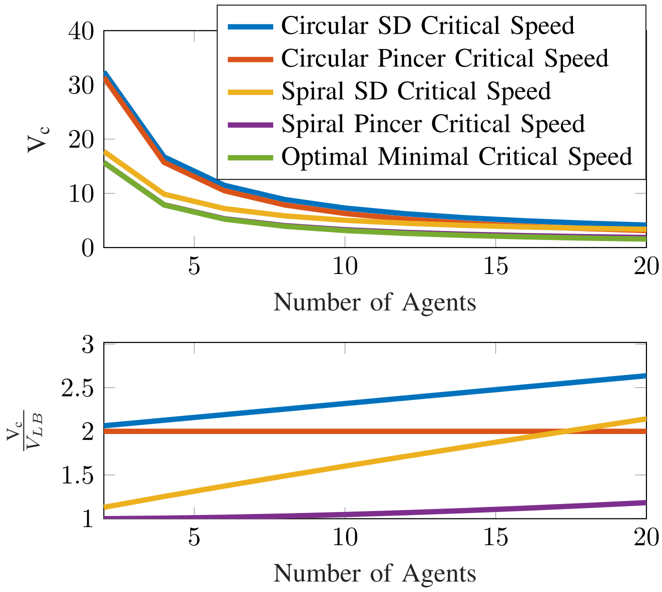

Fig. 4. Critical speeds as a function of sweepers' numbers.

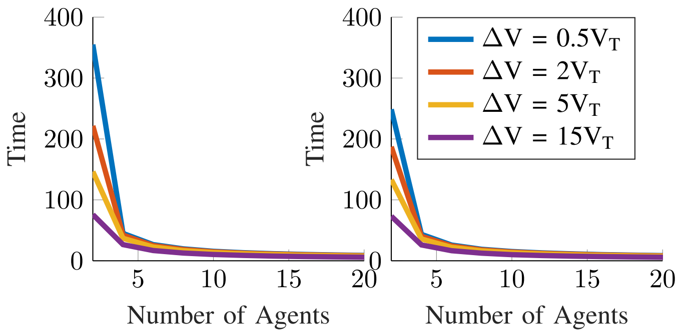

Fig. 5. The left plot shows sweep times of teams performing circular same-direction sweeps. The right plot shows complete search times of teams performing circular pincer sweeps. In both plots the sweepers' speeds are above the same-direction circular critical speed of 2 sweepers.

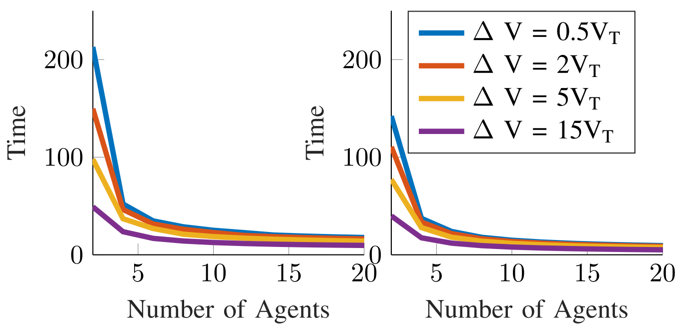

Fig. 6. The left plot presents sweep times of teams performing spiral same-direction protocols. The right plot shows the complete search times of teams performing spiral pincer sweep protocols. In both plots the sweepers' speeds are above the spiral same-direction critical speed of 2 sweepers.

their same-direction counterparts and therefore allow to sweep successfully larger regions.

Afterwards, we provide a quantitative comparison between the different search methods in terms of completion times of the sweep processes and show that circular pincer-based approaches are always better than their same-direction counterparts. Furthermore, we show that pincer-based spiral sweep search times are shorter for all choices of search parameters compared to their same-direction counterparts, as well.

Hence, for all search parameters and protocols choices, pincer-based protocols are best. Thus, the goal to prove and quantify that pincer-based strategies outperform by a large margin the most prevalent strategies for multi-agent search against adversarial opponents is achieved. We therefore hope, that following the proofs provided in this paper, the multi-agent research community will leverage the usage of pincer-movement based strategies in other important applications and topics of interest.

References

[1] R. M. Francos and A. M. Bruckstein, "Search for smart evaders with swarms of sweeping agents," *IEEE Transactions on Robotics*, vol. 38, no. 2, pp. 1080–1100, 2021.

[2] B. O. Koopman, *Search and screening: general principles with historical applications*. Pergamon Press, 1980.

[3] P. Vincent and I. Rubin, "A framework and analysis for cooperative search using uav swarms," in *Proceedings of the 2004 ACM symposium on Applied computing*. ACM, 2004, pp. 79–86.

[4] Y. Altshuler, V. Yanovsky, I. A. Wagner, and A. M. Bruckstein, "Efficient cooperative search of smart targets using uav swarms," *Robotica*, vol. 26, no. 4, pp. 551–557, 2008.

[5] A. Bressan and T. Wang, "On the optimal strategy for an isotropic blocking problem," *Calculus of Variations and partial differential equations*, vol. 45, no. 1-2, pp. 125–145, 2012.

[6] Z. Tang and U. Ozguner, "On non-escape search for a moving target by multiple mobile sensor agents," in *2006 American Control Conference*. IEEE, 2006, pp. 6–pp.

[7] T. G. McGee and J. K. Hedrick, "Guaranteed strategies to search for mobile evaders in the plane," in *2006 American Control Conference*. IEEE, 2006, pp. 6–pp.

[8] P. C. Hew, "Linear and concentric arc patrols against smart evaders," *Military Operations Research*, vol. 20, no. 3, pp. 39–48, 2015.

[9] D. Shishika and V. Kumar, "Local-game decomposition for multiplayer perimeter-defense problem," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 2093–2100.

[10] M. Chen, Z. Zhou, and C. J. Tomlin, "Multiplayer reach-avoid games via pairwise outcomes," *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1451–1457, 2016.

[11] V. R. Makkapati and P. Tsiotras, "Optimal evading strategies and task allocation in multi-player pursuit–evasion problems," *Dynamic Games and Applications*, vol. 9, no. 4, pp. 1168–1187, 2019.

[12] T. H. Chung, G. A. Hollinger, and V. Isler, "Search and pursuit-evasion in mobile robotics," *Autonomous robots*, vol. 31, no. 4, pp. 299–316, 2011.

[13] S. S. Kumkov, S. Le Ménec, and V. S. Patsko, "Zero-sum pursuit-evasion differential games with many objects: survey of publications," *Dynamic games and applications*, vol. 7, no. 4, pp. 609–633, 2017.

[14] I. E. Weintraub, M. Pachter, and E. Garcia, "An introduction to pursuit-evasion differential games," in *2020 American Control Conference (ACC)*. IEEE, 2020, pp. 1049–1066.

[15] R. M. Francos and A. M. Bruckstein, "Search for smart evaders with sweeping agents," *Robotica*, vol. 39, no. 12, pp. 2210–2245, 2021.

[16] S. Tisue and U. Wilensky, "Netlogo: A simple environment for modeling complexity," in *International conference on complex systems*, vol. 21. Boston, MA, 2004, pp. 16–21.

[17] M. Sandler, A. Howard, M. Zhu, A. Zhmoginov, and L.-C. Chen, "Mobilenetv2: Inverted residuals and linear bottlenecks," in *Proceedings of the IEEE conference on computer vision and pattern recognition*, 2018, pp. 4510–4520.

[18] A. Bochkovskiy, C.-Y. Wang, and H.-Y. M. Liao, "Yolov4: Optimal speed and accuracy of object detection," *arXiv preprint arXiv:2004.10934*, 2020.

[19] R. M. Francos and A. M. Bruckstein, "Pincer-based vs. same-direction search strategies after smart evaders by swarms of agents," *arXiv preprint arXiv:2104.06940*, 2021.

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