HeuriGym: An Agentic Benchmark for LLM-Crafted Heuristics in Combinatorial Optimization

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Abstract

While Large Language Models (LLMs) have demonstrated significant advancements in reasoning and agent-based problem-solving, current evaluation methodologies fail to adequately assess their capabilities: existing benchmarks either rely on closed-ended questions prone to saturation and memorization, or subjective comparisons that lack consistency and rigor. In this work, we introduce **HeuriGym**, an agentic framework designed for evaluating heuristic algorithms generated by LLMs for combinatorial optimization problems, characterized by clearly defined objectives and expansive solution spaces. HeuriGym empowers LLMs to propose heuristics, receive evaluative feedback via code execution, and iteratively refine their solutions. We evaluate nine state-of-the-art models on various problems across domains such as computer systems, logistics, and biology, exposing persistent limitations in tool use, planning, and adaptive reasoning. To quantify performance, we propose the Quality-Yield Index (QYI), a metric that captures both solution pass rate and quality. Even top models like GPT-o4-mini-high and Gemini-2.5-Pro attain QYI scores of only 0.6, well below the expert baseline of 1. Our open-source benchmark aims to guide the development of LLMs toward more effective and realistic problem-solving in scientific and engineering domains.

1 Introduction

Large Language Models (LLMs) now excel at complex reasoning and agent-based problem-solving, enabling applications from code generation [78, 97] to adaptive decision-making [122, 160]. However, current evaluation frameworks fail to capture these emergent abilities, leaving open the question of whether LLMs demonstrate genuine problem-solving ingenuity beyond pattern recognition. Current evaluation paradigms fall into two categories, each with clear limitations. **(1) Ground-truth-based benchmarks** (e.g., AIME [98], HumanEval [25], GPQA Diamond [111]) rely on closed-form questions but now suffer from ceiling effects, with models surpassing 80% accuracy [99, 5, 38]. Even new tests like Humanity's Last Exam (HLE) [107] saw performance jump from 3% to 25% within months [99]. These static datasets face both data contamination and a mismatch with real-world, open-ended problem solving. **(2) Judge-preference evaluations** (e.g., Chatbot Arena [27]) assess models via human or LLM-based comparisons [169], better capturing open-ended quality but introducing high variance. Judgments often hinge on style or superficial cues rather than reasoning [128, 164], and LLM-as-a-judge systems remain unreliable across domains, especially for technical expertise [71].

To address these limitations, we introduce **HeuriGym**, a new evaluation paradigm with an agentic framework centered on combinatorial optimization problems, which naturally combine *well-defined objectives* with *large solution spaces*. Rather than relying on well-known benchmarks such as SAT or

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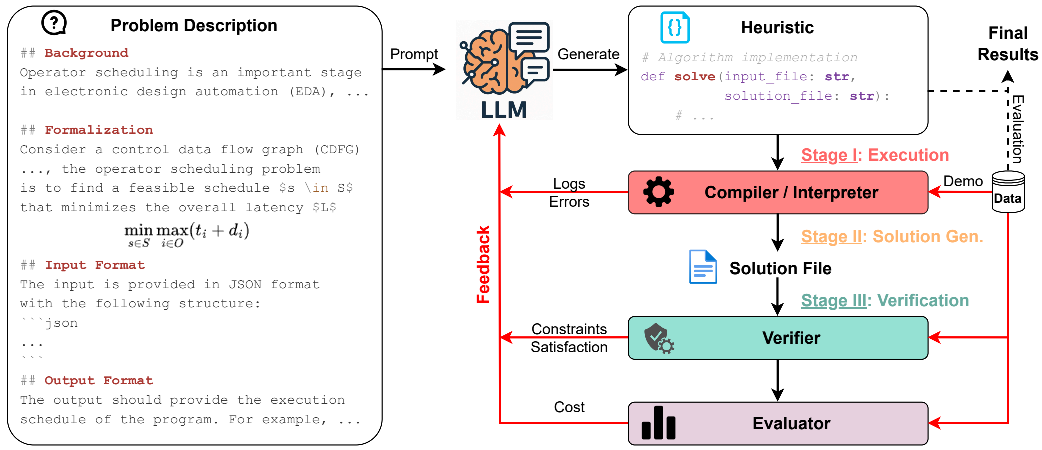

Figure 1: Overview of the HeuriGym agentic framework for heuristic program generation, execution, and verification. We use operator scheduling [31] as an example for the problem description.

TSP, we assess whether LLMs can produce high-quality solutions to novel yet foundational problems spanning computer systems [15, 94], scientific reasoning [21, 20], computational biology [37, 149], logistics [76, 50], and electronic design automation [57, 31]. They are well-suited for benchmarking LLMs because they resist memorization, provide clear quantitative metrics, and mirror real-world tasks where optimal solutions are tractable only for small cases. With no heuristic dominating all problems [151], the search space remains diverse, demanding algorithmic knowledge, heuristic reasoning, and creative problem-solving—capabilities underexplored in current evaluations. Our framework goes beyond static tests by creating an interactive loop where LLMs generate heuristics, receive execution feedback, and iteratively refine solutions, reflecting real engineering workflows and enabling deeper assessment of reasoning, tool use, and instruction following.

Our benchmark evaluates LLMs across (1) *tool-augmented reasoning* with external libraries, (2) *multi-step planning* for decomposing problems, (3) *instruction fidelity* in following constraints, and (4) *iterative refinement* from runtime feedback. It uniquely probes practical creativity, testing how models adapt textbook algorithms or devise new strategies when exact methods like Integer Programming fail. To assess both solution quality and yield, we propose the Quality-Yield Index (QYI), ranging from 0 (all outputs incorrect) to 1 (expert-level). Across nine optimization problems, even state-of-the-art LLMs such as GPT-o4-mini-high [99] and Gemini-2.5-Pro [38] score only 0.6 QYI, exposing their limitations in realistic problem solving that demands theory, tool use, and adaptive reasoning.

2 HeuriGym: An Agentic Framework for Heuristic Generation

As illustrated in Fig. 1, our framework begins by presenting a formal problem description to the LLM, which is then prompted to generate a complete heuristic algorithm. The generated program conforms to a standardized function signature and is subsequently compiled (for C++) or interpreted (for Python). Upon execution, the solution is verified for yield and evaluated for performance. Crucially, the framework incorporates a feedback loop: execution logs, verification outcomes, and evaluation costs from a small demonstration set are appended back to the prompt, enabling iterative refinement.

Problem Description As shown on the left of Fig. 1, we use operator scheduling [31, 87], a classic optimization problem in electronic design automation, as an example. Each benchmark task is accompanied by a structured problem description with three main parts: **(1) Background**: Introduces the optimization context and key terminology to help the LLM understand the problem setting. **(2) Formalization**: Defines the optimization objective and constraints using mathematical notation (e.g., minimizing latency under hardware resource constraints), guiding the LLM toward objective-oriented algorithm design. **(3) Input/Output Format**: Specifies the structure of input and output files, providing clear expectations for parsing and execution. More detailed information on the problem set can be found in Section 3.

Prompt Design Effective prompt engineering is crucial for leveraging LLMs' capabilities [148, 118]. We construct both system- and user-level prompts, tailored to each problem instance. A complete prompt example is provided in Appendix C.

System prompt. The system prompt includes machine configuration details (e.g., CPU cores, memory limits), available libraries with version numbers, and task-specific constraints such as execution timeouts. This environment specification instructs the LLM to avoid relying on unrealistic assumptions or producing inefficient solutions that violate runtime limits.

User prompt. In the initial iteration, the user prompt includes the problem description and a code skeleton with a predefined function signature. As shown in Fig. 1, the LLM is only provided the interface – function name, input path, and output path – without hints on data structures or algorithmic approache, contrasting with prior work [113, 84, 162] that often handcrafts partial implementations or restricts the design space. Here, LLMs must reason about the problem holistically: parsing inputs, constructing internal representations, and designing and implementing heuristics from scratch.

Feedback Loop To emulate a few-shot in-context learning setup [42, 85, 152], we partition the dataset into a small *demonstration set* (around five instances) and a larger *evaluation set*. Demonstration data is used during the refinement loop to provide timely, example-based feedback to the LLM; the evaluation set is withheld until the model stabilizes its performance.

Each problem includes a domain-specific verifier and evaluator. The verifier ensures constraint satisfaction (e.g., dependency preservation in operator scheduling), while the evaluator calculates the cost based on the given problem objective. If the verifier fails, diagnostic messages are recorded.

After each iteration, we log the LLM-generated solution, execution trace, verification result, and evaluation score. These logs are appended to the prompt with the demonstration data in the next iteration, enabling the LLM to learn from past attempts and incrementally improve its output.

Metric Design Traditional LLM benchmarks predominantly rely on the $\text{PASS}@k$ metric [25, 170, 65], which measures the probability of generating a ground-truth solution within the top-k samples. While $\text{PASS}@k$ is effective for single-turn tasks with deterministic ground truths, it falls short in capturing the iterative reasoning and problem-solving abilities required in our multi-round agentic setting. Specifically, it does not reflect whether the LLM can understand problem constraints, debug based on feedback, or iteratively refine its solutions over multiple attempts.

To better evaluate LLMs in this complex setting, we introduce a new metric, denoted as $\text{SOLVE}_s@i$, which tracks the LLM's ability to solve constrained problems within i iterations:

$$\text{SOLVE}_s@i := \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}(\text{pass stage } s \text{ in the \textit{first} } i\text{-th iteration}),$$

where N is the total number of test instances, and $s \in \{\text{I, II, III}\}$ indicates the specific stage of the pipeline that the solution must pass. Each stage reflects a key milestone in agentic reasoning:

- **Stage I: Execution**. The generated program must compile or interpret correctly with all necessary libraries included, and successfully perform basic I/O operations (e.g., reading and writing files).
- **Stage II: Solution Generation**. The program must produce a non-empty output within the predefined timeout and adhere to the expected output format.
- **Stage III: Verification**. The solution must satisfy all problem-specific constraints, as checked by a problem-specific verifier.

However, $\text{SOLVE}_s@i$ only indicates whether a *feasible* solution is eventually produced through the iterative process – it does not account for solution quality. To address this, we additionally define separate metrics for quality and yield as follows:

$$\texttt{Quality} = \frac{1}{\hat{N}}\sum_{n=1}^{\hat{N}} \min\left(1, \frac{c_n^\star}{c_n}\right) \qquad \texttt{Yield} = \frac{\hat{N}}{N},$$

where c_n and $c_n^\star$ represent the cost of the LLM-generated and expert-provided solutions, respectively, and $\hat{N}$ is the number of instances that pass verification (Stage III) in the *current* iteration. In this paper, we adopt the capped version of quality, which checks whether the LLM matches expert performance (up to a maximum of 1), though an uncapped version can also be used to measure cases where the LLM outperforms the expert. We define a unified metric, the *Quality-Yield Index (QYI)*, as the harmonic mean of quality and yield. This formulation, analogous to the F-score [140], penalizes imbalanced values more strongly than the arithmetic mean:

$$\texttt{QYI} = \frac{2 \cdot \texttt{Quality} \cdot \texttt{Yield}}{\texttt{Quality} + \texttt{Yield}}.$$

QYI captures both success rate and the relative quality of solutions, enabling holistic evaluation of an LLM's agentic reasoning capabilities, including its capacity for long-horizon planning and iterative refinement. Additionally, we can define a weighted QYI by averaging QYI scores across different problems, weighted by the number of instances in each, as an overall performance metric.

3 Benchmark Construction

This section outlines the construction of the combinatorial optimization benchmark as well as the principles behind. Our primary goal is to evaluate an LLM's capacity for reasoning rather than its ability to regurgitate well-known algorithms. To this end, we intentionally exclude ubiquitous problems such as the Traveling Salesman Problem [112] and canonical satisfiability (SAT) formulations [121] – problems that are so widely studied and frequently included in public datasets that they are likely memorized during pretraining. Instead, we focus on problems that meet the following criteria:

Limited exposure in the literature. For each candidate problem, we perform a Google Scholar search and retain it only if the most-cited paper has fewer than 1,000 citations (as of April 2025). This empirical threshold ensures that the problem is well-defined and supported by peer-reviewed work, yet not so well-known that an LLM could solve it through rote memorization or pattern matching.

Clear natural-language specification with well-defined objectives. Each problem must be clearly expressible using plain language without the need for visual aids. We encode mathematical objectives in LaTeX to eliminate ambiguity, ensuring the LLM receives well-specified instructions.

Large solution spaces. We focus on problems that admit vast solution spaces with many feasible outputs, encouraging creative exploration and reasoning rather than narrow pattern recognition [59].

Scalable data instances. Each problem includes two disjoint sets of instances: a small-scale demonstration set and a large-scale evaluation set, differing by at least an order of magnitude. The demonstration set supports few-shot prompting and iterative refinement, while the evaluation set is reserved for final performance testing, as discussed in Section 2.

Reproducible expert baselines. Reference implementations are bundled in the benchmark repository to ensure fair comparison across future studies. Where possible, we include both exact solvers (e.g., ILP) and high-quality heuristics to illuminate the performance gap.

We prioritize domains with real-world impact, where even small gains yield significant societal or industrial benefits. Many selected problems remain open, with heuristics far from theoretical bounds – offering a compelling testbed for LLMs.

Table 1: Existing combinatorial optimization problems in our HeuriGym benchmark.

Domain	Problem	References	Difficulty
Electronic Design Automation (EDA)	Operator scheduling	[87, 31]	★
	Technology mapping	[57, 19]	★★
	Global routing	[79, 81]	★★★
Compilers	E-graph extraction	[15, 150]	★
	Intra-operator parallelism	[94, 168]	★★
Computational Biology	Protein sequence design	[37, 69]	★
	Mendelian error detection	[89, 119]	★★
Logistics	Airline crew pairing	[70, 2]	★★
	Pickup and delivery w/ time windows	[137, 76]	★★★

The initial release of HeuriGym spans nine diverse real-world optimization problems, including fundamentally different types such as covering, scheduling, and routing (Table 1). A detailed description of each problem is provided in Appendix E. There are 218 realistically sourced instances released in total with hundreds of instances reserved as private test sets for future release. The instance distribution is detailed in Appendix H. Notably, most problems are NP-hard and feature complex constraints, resulting in a compact yet highly challenging problem suite. To ensure clarity and correctness, we prompt a weaker LLM [83] to identify any unclear or ambiguous statements after drafting the initial natural language description. The full prompt template used for refining problem descriptions is provided in Appendix C.4.

4 Evaluation

To evaluate the reasoning capabilities of LLMs on CO problems, we benchmark nine prominent models released in late 2024 and mid-2025. See Appendix D for more details about the models used. Full experimental settings and results of each problem can be found in Appendix A and F. For the major result, we fix the temperature to 0 for all LLMs to ensure deterministic outputs, following standard practice in recent benchmarks [102, 161, 107].

Table 2: Overall $\text{SOLVE}_{\text{III}}@i$ metric across models on the entire HeuriGym benchmark.

	DeepSeek-V3	DeepSeek-R1	Gemini-2.5-Flash	Gemini-2.5-Pro	LLaMA-4-Marverick	LLaMA-3.3	Qwen3-235B	Claude-3.7-Sonnet	GPT-o4-mini-high
$\text{SOLVE}_{\text{III}}@10$	46.8%	73.4%	67.4%	65.1%	35.8%	33.9%	45.9%	60.1%	**74.8%**
$\text{SOLVE}_{\text{III}}@5$	42.7%	**72.9%**	58.3%	64.2%	33.5%	33.9%	45.4%	58.7%	69.7%
$\text{SOLVE}_{\text{III}}@1$	14.2%	44.0%	25.2%	20.2%	6.0%	20.6%	38.5%	9.2%	**53.2%**

As shown in Table 2, most LLMs fail to fully solve a large fraction of test cases within a single attempt, as reflected in the $\text{SOLVE}_{\text{III}}@1$ score. Increasing the number of iterations generally improves performance across all models. Among all models, GPT-o4-mini-high and DeepSeek-R1 demonstrate high success rates across multiple iterations, highlighting their stronger program repair capabilities. Refer to Appendix A for the details of $\text{solve}_{\text{II}}@i$ and $\text{solve}_{\text{I}}@i$.

To assess solution quality, we compare the final LLM-generated programs to expert-designed solutions using the weighted QYI metric defined in Section 2. As illustrated in Figure 3, a substantial performance gap remains: even the best-performing model, Gemini-2.5-Pro, achieves a QYI of only 0.62, indicating that its solutions are, on average, just 60% as effective as expert-crafted ones. Several models, such as LLaMA-3.3 and LLaMA-4, produce results with QYI scores below 30%, highlighting their limited effectiveness on these tasks. We also estimate the API cost for each model and find that Gemini-2.5-Flash offers the best cost-efficiency relative to its achieved QYI. Additional ablation studies, extra experiments, and analyses are provided in the Appendix A and F.

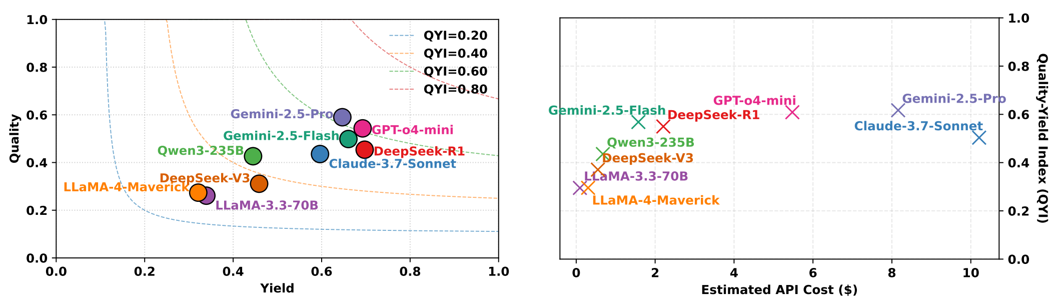

Figure 2: Quality-Yield Index and estimated API cost of different models.

5 Limitation and Future Work

We highlight two main challenges for future LLM development in combinatorial optimization: (1) **Correctness** – reducing hallucinations and improving constraint satisfaction, and (2) **Performance** – navigating large search spaces for high-quality solutions. While our framework establishes a foundation, two limitations remain. First, the current experimental pipeline is implemented in Python for accessibility. Although preliminary C++ results (Appendix F.8) are available, full integration of highly-efficient compiled languages remains challenging due to reliance on domain-specific libraries. Second, our iterative self-refinement agentic workflow can be interpreted as a form of Test-Time Scaling, analogous to compute-optimal scaling [130], which creates opportunities to incorporate techniques such as best-of-N sampling [134], beam search [154], and evolutionary algorithms [97, 162]. Such directions are not fully explored in our paper. Currently HeuriGym includes only nine problems. Although these have been carefully curated to test reasoning and generalization, they may eventually become saturated as LLM capabilities improve. Overall, we believe HeuriGym can serve as a shared testbed that guides and inspires future research toward greater LLM autonomy.

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Appendix

A Evaluation

To evaluate the reasoning capabilities of LLMs on CO problems, we benchmark nine prominent models released in late 2024 and mid-2025 (Appendix D). These models represent the current state-of-the-art in general-purpose LLMs and rank among the top entries on OpenRouter [100] and Chatbot Arena leaderboards [27]. We exclude smaller models due to the complexity of the benchmark tasks. All evaluations are conducted via official APIs to ensure reproducibility. We adopt the agentic workflow in Fig. 1, constraining each model to generate Python programs that solve the given problems under fixed resource limits: a maximum of 8 CPU cores and problem-specific timeouts. We also allow the models to access the given Python libraries for external tool use. Full details of the experimental settings and results of each problem can be found in Appendix F.

A.1 Overall Performance

Table 3: Overall $\text{SOLVE}_s@i$ metric of models on the whole HeuriGym benchmark.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	46.8%	42.7%	14.2%	87.6%	83.0%	66.1%	**100.0%**	**100.0%**	90.8%
DeepSeek-R1	73.4%	**72.9%**	44.0%	88.1%	88.1%	60.6%	**100.0%**	**100.0%**	71.6%
Gemini-2.5-Flash	67.4%	58.3%	25.2%	83.9%	79.4%	56.4%	**100.0%**	**100.0%**	72.9%
Gemini-2.5-Pro	65.1%	64.2%	20.2%	89.4%	89.0%	42.7%	**100.0%**	**100.0%**	51.4%
LLaMA-4-Maverick	35.8%	33.5%	6.0%	84.9%	74.3%	8.3%	85.3%	85.3%	13.3%
LLaMA-3.3	33.9%	33.9%	20.6%	78.4%	78.4%	40.4%	99.5%	99.5%	61.9%
Qwen3-235B	45.9%	45.4%	38.5%	86.2%	83.0%	56.0%	**100.0%**	**100.0%**	70.6%
Claude-3.7-Sonnet	60.1%	58.7%	9.2%	97.7%	97.7%	41.3%	**100.0%**	**100.0%**	60.1%
GPT-o4-mini-high	**74.8%**	69.7%	**53.2%**	**100.0%**	**100.0%**	**93.1%**	**100.0%**	**100.0%**	**100.0%**

For the overall evaluation, we fix the generation temperature at 0, following standard practice in recent LLM benchmarks [102, 161, 107]. This ensures deterministic outputs and eliminates randomness across runs. Notably, OpenAI's o-series models only support a fixed temperature of 1.0 [99]. We measure the multi-round performance using the $\text{SOLVE}_s@i$ metric, where $i \in \{1, 5, 10\}$ indicates the number of iterations allowed.

As shown in Table 3, most LLMs fail to solve a large fraction of test cases within a single attempt, as reflected in the $\text{SOLVE}_{\text{III}}@1$ score. Increasing the number of iterations generally improves performance across all models. For instance, the $\text{SOLVE}_{\text{III}}$ success rate rises from 53.2% to 74.8% for GPT-o4-mini-high as i increases, underscoring the importance of iterative refinement in improving LLM-generated solutions. Among all models, GPT-o4-mini-high and DeepSeek-R1 demonstrate high success rates across multiple iterations, highlighting their stronger program repair capabilities.

To assess solution quality, we compare the final LLM-generated programs to expert-designed solutions using the weighted QYI metric defined in Section 2. As illustrated in Fig. 3, a substantial performance gap remains: even the best-performing model, Gemini-2.5-Pro, achieves a QYI of only 0.62, indicating that its solutions are, on average, just 60% as effective as expert-crafted ones. Several models, such as LLaMA-3.3 and LLaMA-4-Maverick, produce results with QYI scores below 30%, highlighting their limited effectiveness on these tasks. We also estimate the API cost for each model and find that Gemini-2.5-Flash offers the best cost-efficiency relative to its achieved QYI.

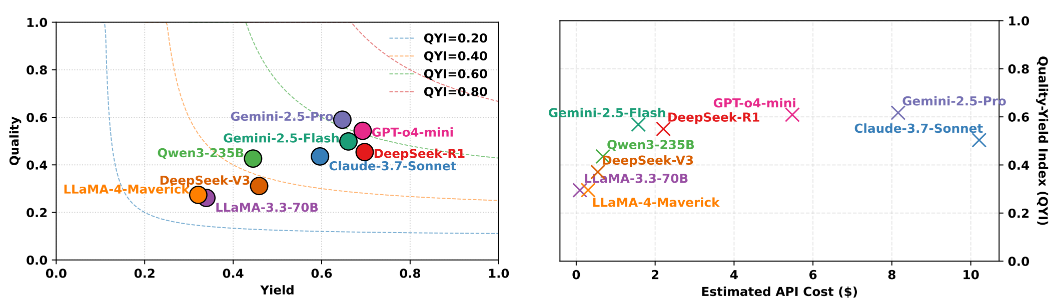

Figure 3: Quality-Yield Index and estimated API cost of different models.

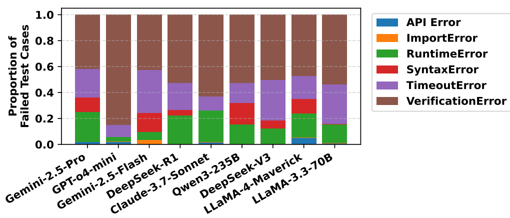

Figure 4: Error classifications.

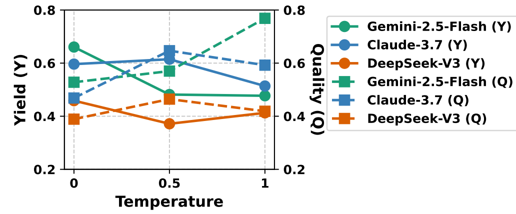

Figure 5: Quality-Yield tradeoff.

We further compare state-of-the-art open-source evolutionary frameworks under the same setting (a fixed budget of 10 iterations), using Gemini-2.5-Pro—the best-performing model in our benchmark—as the base LLM. The initial program is generated directly from the problem description. As shown in Table 4, these frameworks perform poorly, often worse than the baseline model. Their main weakness is the lack of incorporating program execution feedback, and their search process breaks context across iterations, stalling progress on repeatedly patching the initial flawed program. This highlights both the complexity of our benchmark and the need for LLMs to reason more deeply about problem-specific strategies. These frameworks originally only target toy problems under 20 lines of code (e.g., TSP, bin packing), while our benchmark typically requires 300+ lines, making strategy discovery essential rather than relying on prebuilt meta-heuristics.

Table 4: Performance of evolutionary frameworks.

Frameworks	SOLVE$_{\text{III}}$@10	QYI
Gemini-2.5-Pro	0.6514	0.6170
HSEvo [35]	0.5000	0.4491
ReEvo [162]	0.4771	0.4486
EoH [84]	0.4954	0.4492

Table 5: Ablation study on pickup and delivery with time windows.

# of Demos / # of Feedback Rounds	QYI
5/10	0.4196
3/10	0.2829
0/10	0.2351
5/5	0.3330
5/1	0.2350

To identify common failure modes, we analyze and categorize the most common error types produced by the evaluated models, as shown in Fig. 4. These include: (1) Hallucinated APIs: using nonexistent or outdated library calls. (2) Incorrect algorithmic logic: flawed implementation even when the general approach is reasonable. (3) Constraint misunderstanding: ignoring or misinterpreting problem constraints. (4) Timeouts: no output or the execution time exceeds the given constraints. Additional error cases and examples are listed in Appendix F.7.

To assess the robustness and sensitivity of LLM performance under different settings, we conduct a set of ablation experiments with full details in Appendix F.

Temperature. We evaluate three representative models across the QYI spectrum using temperatures $T \in \{0.0, 0.5, 1.0\}$. Fig. 5 shows that higher T increases diversity and quality but lowers yield due to more invalid outputs (Appendix F.3). Greedy decoding ($T = 0$) has maximum yield with suboptimal quality, while stochastic sampling ($T = 1$) achieves better quality at the cost of solving fewer problems. This reveals a fundamental trade-off between quality and yield that future LLMs must address.

Few-shot demonstrations. We assess the impact of in-context examples by comparing zero-shot, half-shot, and full-shot prompts. Due to budget constraints, these experiments are conducted on a few representative models. Specifically, we evaluate Gemini-2.5-Pro on the pickup and delivery problem—one of the most challenging tasks in our benchmark (full results in Appendix F.4). As shown in Table 5, providing more informative demonstrations significantly boosts the overall performance, especially for tasks involving unfamiliar domains or requiring long-horizon reasoning.

Feedback rounds. To evaluate the role of iterative refinement, we vary the number of feedback rounds given to LLMs (1, 5, and 10), keeping the temperature fixed at 0. The results in Table 5 show that later iterations frequently fix logic errors or constraint violations from earlier attempts,

Table 6: Comparison with other recent benchmarks.

Subjects	Benchmark	Well-Defined Objective	Large Solution Space	Agentic Setting	Evaluation Metrics
Frontier Knowledge	Humanity's Last Exam (HLE) [107]	✓	✗	✗	Accuracy
Software Engineering	HumanEval [25]	✓	✗	✗	PASS@k
	BigCodeBench [170]	✓	✗	✗	PASS@k
	LiveCodeBench [62]	✓	✗	✗	PASS@1
	SWE-Bench [65]	✓	✗	✗	PASS@1
	Commit0 [166]	✓	✗	✓	Pass rate
Performance Engineering	KernelBench [102]	✗	✓	✗	FAST$_p$
Daily-Life Tasks	Chatbot Arena [27]	✗	✓	✗	ELO
	τ-Bench [161]	✓	✓	✓	PASS^k
Combinatorial Optimization	NPHardEval [48]	✓	✗	✗	Accuracy
	GraphArena [136]	✓	✗	✗	Accuracy
	HeuriGym (This work)	✓	✓	✓	SOLVE$_s$@i, QYI

underscoring the value of multi-round reasoning. We provide further analysis in Appendix A.2 and Appendix F.5.

A.2 Case Study

We present a case study on technology mapping [19] to highlight both the promise and current limitations of LLMs, where the task is to cover a logic network with K-input lookup tables (LUTs) while minimizing their total count; we fix $K = 6$. As an expert baseline, we use ABC [14], a state-of-the-art logic synthesis tool that leverages optimized cut enumeration and dynamic programming (DP)-based covering. We find that top-performing LLMs, such as GPT-o4-mini-high and Gemini-2.5-Pro, can mimic similar heuristic strategies and iteratively refine them through feedback. Fig. 6 shows GPT-o4-mini-high explores a range of approaches over multiple iterations, evolving from naive mappings to sophisticated DP-based heuristics with pruning. It finally converges on a strategy that effectively balances yield and quality, achieving the highest QYI. The full generated programs across those iterations are listed in Appendix G.

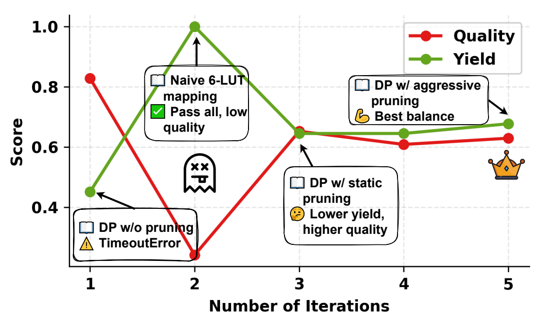

Figure 6: One iterative example of GPT-o4-mini-high on the technology mapping problem.

A.3 Ablation Study

Nonetheless, a substantial gap remains between LLMs and expert tools (∼60% of expert performance), due to the latter's extensive use of domain-specific optimizations and efficient implementations. This suggests that while LLMs can learn and refine heuristic algorithms, they are not yet capable of generating solutions with expert-level performance in real-world complex optimization tasks.

B Related Work

LLMs for Combinatorial Optimization. Recent LLM-based combinatorial optimization (CO) methods follow two main paradigms. The first emphasizes formalization—translating natural language into structured optimization problems. This direction was initiated by the NL4Opt Competition [110], with follow-up work improving domain-specific model training [153, 64, 77] and prompting strategies [157, 3, 61]. While effective on benchmarks, these methods struggle to scale due to their reliance on exact solvers [51]. The second paradigm focuses on heuristic discovery. FunSearch [113] and AlphaEvolve [97] use LLMs with evolutionary search to generate novel heuristics, but require evaluating thousands of candidates. Recent approaches [162, 84, 35] improve efficiency via metaheuristic templates, but still limit LLMs to filling in a small portion of the algorithm. In contrast, HeuriGym removes reliance on templates or scaffolds. It tasks LLMs with generating complete, self-contained optimization programs, including custom data structures and end-to-end pipelines—better reflecting real-world CO challenges, where success depends on uncovering problem-specific structure and designing bespoke algorithms [151].

Evaluation on LLMs. As shown in Table 6, existing LLM benchmarks expose key limitations. Many focus on closed-ended tasks in domains like mathematics [98], programming [25, 170, 86], and specialized knowledge [111, 107, 55], with fixed ground-truths that are prone to data contamination (see § 1). In contrast, open-ended benchmarks [27, 102] encourage diverse outputs but often lack clear objectives, resulting in inconsistent evaluations. Benchmarks like NPHardEval [48] and GraphArena [136] assess exact solutions to small NP-hard instances, limiting real-world relevance where heuristic solutions are often preferred for scalability. Our benchmark instead accepts any *feasible* solution that satisfies constraints, enabling broader evaluation of algorithmic reasoning. It tasks LLMs with synthesizing executable code, using external libraries, and refining solutions through execution feedback, mimicking realistic workflows. We also propose new evaluation metrics to quantify multi-round reasoning, as detailed in Section 2.

C Prompt Design

In this section, we detail the system and user prompts used by the LLM agent, as well as the auxiliary prompt employed to enhance our problem descriptions.

C.1 System Prompt

Each iteration of our benchmark begins with a task-agnostic system prompt that instructs the LLM to generate and iteratively refine executable heuristics for combinatorial optimization problems. This system prompt is followed by a task-specific problem statement and an input/output specification. The prompt includes placeholders – highlighted in red – that are dynamically instantiated at runtime for each task. For instance, {NUM_CPU_CORES} represents the CPU core limit for the task (default: 8), and {TIMEOUT} specifies the wall-clock time limit (default: 10 seconds).

System Prompt

You are a world-class optimization expert and algorithmic problem solver. Your task is to develop a highly efficient solution to the following optimization problem. Please analyze the problem background, mathematical formulation, and I/O specifications with extreme rigor and attention to detail.
Your mission is to devise and implement the most performant algorithm possible, optimizing for both computational efficiency and solution quality. You should leverage your deep knowledge of algorithms, data structures, and optimization techniques to craft a powerful solution. You have complete freedom in your algorithmic approach. Think systematically and creatively. Your goal is to push the boundaries of what's possible within the computational constraints. Please strictly follow the instructions below.

- A problem template is provided below. You only need to implement the solve function. Do NOT modify the function signature including the data types of the input arguments. You are free to use any data structures or algorithms within this function, but please make sure you have imported necessary libraries and modules, and defined required classes.

System Prompt (Continued)

- The evaluation machine has {NUM_CPU_CORES} CPU cores and sufficient memory to run your program. The time limit for this question is {TIMEOUT} seconds. You are free to implement parallel algorithms where appropriate to maximize performance.
- The Python version is 3.12. You may use any standard Python libraries and only the following third-party libraries:
  - numpy==2.2.5
  - networkx==3.4.2
  - pandas==2.2.3
- Your response should consist of a complete implementation of the 'solve' function. Do NOT include any explanations, comments, additional text, or Markdown formatting.
- You will receive execution feedback after the user runs your program, including runtime metrics and correctness evaluation.

C.2 User Prompt

For each problem, the first iteration begins with the following user prompt, which introduces the task and its objective to the LLM, along with a program template that the model is expected to complete.

User Prompt

\# Problem Information
{PROBLEM DESCRIPTION}

\# Program Template

```
def solve(input_file: str, solution_file: str):
    """
    Solve the optimization problem.

    Please do NOT change the function name and arguments.
    Inputs should be read from input_file
    and outputs should be written to solution_file.
    Input and output formats have been specified in the
    problem statement.
    """
    raise NotImplementedError(
        "This is a placeholder implementation you need to fill in."
    )
```

C.3 Prompts for Improvement Guidance

Based on the feasibility of the final outputs, we issue one of two improvement prompts in subsequent iterations. If any test cases fail, we provide the following prompt:

Improvement Guidance Case 1

Feedback from Previous Iteration (Iteration `{iteration-1}`)
These are the test cases and results from the previous iteration:
Test Case 1: `{test_name}`
Input File:
{content}
Result:
`{execution_message}`

Test Case 2: `{test_name}`
Input File:
{content}
Result:
`{execution_message}`

...

Improvement Guidance
The program failed to produce valid solutions for some test cases. Please fix the following issues:

1. Check for compilation errors or runtime exceptions.
2. Ensure the program handles all edge cases and meets the problem constraints correctly.
3. Verify that the input and output format match the expected format.
4. Make sure all required functions are implemented correctly, and no external forbidden libraries are used.
5. If the program is not able to produce valid solutions for any test case, please try to find the root cause and fix it.
6. If the program is able to produce valid solutions for some test cases, please try to improve the solution.

Otherwise, if all test cases pass verification, we issue the following prompt:

Improvement Guidance Case 2

Feedback from Previous Iteration (Iteration `{iteration-1}`)
...

Improvement Guidance
Please carefully observe the problem structure and improve upon this program by:

1. Addressing any weaknesses in the previous approach.
2. Introducing more advanced or efficient algorithms.
3. Focusing on improving performance for test cases.

Your goal is to improve the solution for as many test cases as possible, with special attention to those where the previous solution performed poorly.

C.4 Refinement Prompt for Problem Descriptions

To ensure clarity and correctness in problem specification, we employ a human-in-the-loop process. Specifically, we prompt a weaker LLM to flag any unclear or ambiguous statements in the task description. The following prompt is used for this purpose:

Refinement Prompt for Problem Descriptions

If you were to solve the programming task below, do you have any questions? Is there anything I should clarify before you begin writing code?

Problem Description
{PROBLEM DESCRIPTION}

C.5 Example Problem Description

The following provides an example problem description for operator scheduling. For other problems, please refer to our repository.

```
## Background

High-level synthesis (HLS) is an important stage in electronic design automation (EDA), aimed at
↪  translating a high-level program specification (e.g., written in C/C++ or SystemC) into a
↪  cycle-accurate hardware implementation. After the program is parsed and analyzed, it is typically
↪  transformed into an intermediate representation known as a Control Data Flow Graph (CDFG). This
↪  graph captures the operations (e.g., arithmetic, memory accesses) and their control/data
↪  dependencies. The CDFG can further be processed into a Directed Acyclic Graph (DAG) to facilitate
↪  scheduling and optimization.

One of the core challenges in HLS is operator scheduling, which determines the exact control step (or
↪  cycle) at which each operation is executed, while satisfying data dependencies and resource
↪  constraints. Efficient scheduling plays a critical role in optimizing design quality in terms of
↪  performance, area, and power.

## Formalization

Consider a CDFG with $n$ operation nodes $o_i$, where $i \in O = \{1, 2, \ldots, n\}$, and a precedence
↪  relation $\prec$ on $O$ that captures operation dependencies. Each operation $o_i$ is associated
↪  with a cycle delay $d_i \in \mathbb{Z}^+$ and a resource type $r_i \in R = \{1, 2, \ldots, k\}$. Let
↪  $T = \{0, 1, 2, \ldots, L\}$ represent the set of control steps (c-steps), and define a schedule as
↪  an $n$-tuple $s = (t_1, t_2, \ldots, t_n)$, where $t_i \in T$ denotes the start time (c-step) of
↪  operation $o_i$.

A schedule $s$ is feasible if it satisfies all data dependencies:
$\forall i, j \in O: i \prec j \Rightarrow t_i + d_i \leq t_j$.
Let $S$ denote the set of all feasible schedules. For a given schedule $s$, let $N_r(t)$ be the number
↪  of operations that use resource $r$ in control step $t$, and define the total usage of resource $r$
↪  as $N_r = \sum_{t \in T} N_r(t)$.

Given a bound $G_r$ on the number of available instances for each resource type $r \in R$, the operator
↪  scheduling problem is to find a feasible schedule $s \in S$ that minimizes the overall latency $L$,
↪  defined as
$\min_{s \in S} \max_{i \in O} (t_i + d_i)$,
subject to the resource constraints
$\forall r \in R, t \in T: N_r(t) \leq G_r$.

## Input Format
The input is provided in JSON format with the following structure:

```json
{
  "name": "input",
  "delay": {
    "mul": 3,
    "sub": 1
  },
  "resource": {
    "mul": 2,
    "sub": 1
  },
  "nodes": [
    ["n1", "mul"],
    ["n2", "mul"],
    ["n3", "sub"]
  ],
  "edges": [
    ["n1", "n3", "lhs"],
    ["n2", "n3", "rhs"]
  ]
}
```
```

```
Where:
- `name`: Name of the input graph
- `delay`: Maps each resource type to its execution delay in cycles
- `resource`: Maps each resource type to the number of available functional units
- `nodes`: List of nodes, where each node is represented as `[node_id, resource_type]`
- `edges`: List of edges, where each edge is represented as `[source_node, target_node, edge_name]`

## Output Format
The output should provide the execution schedule of the program, indicating the start cycle of each
↪  operation. For example, the following output means that `n1` and `n2` start at cycle 0, while `n3`
↪  starts at cycle 3:
```
n1:0
n2:0
n3:3
```
```

D Models

The LLMs used in our experiments are listed in Table 5. All models were accessed via official APIs provided by their respective organizations, except for the Meta models, which are accessed through the OpenRouter [100] API.

Table 7: Model specifications with API names and official pricing.

Organization	Model	API Name	Price ($In/$Out)	Type
OpenAI	GPT-o4-mini-high	o4-mini:high	1.1/4.4	Reasoning
Anthropic	Claude-3.7-Sonnet	claude-3-7-sonnet-20250219	3/15	Reasoning
DeepSeek	DeepSeek-V3	deepseek-chat(0324)	0.27/1.10	Base
DeepSeek	DeepSeek-R1	deepseek-reasoner	0.55/2.19	Reasoning
Google	Gemini-2.5-Flash	gemini-2.5-flash-preview-04-17	0.15/3.5	Reasoning
Google	Gemini-2.5-Pro	gemini-2.5-pro-preview-05-06	1.25/10.0	Reasoning
Meta	LLaMA-3.3	meta-llama/Llama-3.3-70B-Instruct	0.07/0.33	Base
Meta	LLaMA-4-Maverick	meta-llama/Llama-4-Maverick-17B-128E-Instruct	0.27/0.85	Base
Alibaba	Qwen3-235B	qwen3-235b-a22b	0.29/2.86	Reasoning

E Problem Set

In this section, we provide more details on the problems included in Table 1. For a representative problem description used in the prompts, please consult our repository for additional details.

E.1 Operator Scheduling

Operator scheduling is a critical stage in high-level synthesis (HLS) [32, 103], the process of converting behavioral hardware descriptions into register-transfer level (RTL) implementations. This task involves carefully assigning each operation to a specific clock cycle while managing a variety of constraints such as data dependencies, resource availability, and performance targets. The effectiveness of the scheduling process is vital, as it directly influences key design metrics including area, power consumption, and execution time, making it an important focus in the field of electronic design automation (EDA).

Over the years, researchers have developed a wide range of techniques to tackle the inherent challenges of operator scheduling in HLS. Exact methods, such as those based on integer linear programming (ILP) [60, 101], can provide optimal solutions but often suffer from scalability issues. As a result, many commercial and academic HLS tools [155, 16] rely on heuristics to achieve practical, near-optimal results. Traditional heuristic approaches, including priority-function-based methods [125, 105, 106], focus on balancing resource utilization with performance requirements. Notably, methods leveraging systems of difference constraints (SDC) enable an efficient formulation that captures a rich set of scheduling restrictions and casts the optimization objective into a linear programming (LP) framework [31, 34]. More recently, the incorporation of machine learning techniques [22, 87] has further advanced the state-of-the-art, enhancing both scheduling efficiency and solution quality in the face of increasingly complex hardware designs.

E.2 Technology Mapping

Technology mapping, in the context of logic synthesis for integrated circuits and field-programmable gate arrays (FPGAs), is the process of converting a logic network into an equivalent network of standard cells or logic resources from a specific technology library. The objective is to optimize key design metrics such as area, delay, and power consumption. It is a crucial step in the VLSI design flow and FPGA design flow, determining the actual physical implementation of a design.

Here in our problem setting, we focus on area-optimal technology mapping for lookup table (LUT)-based FPGAs. Given an input logic network, the goal is to cover the network with K-input subgraphs, each of which can be implemented by a K-LUT, while minimizing the number of LUTs representing the circuit area.

The most widely adopted approaches are cut-based methods, which operate in two stages: cut enumeration and cut selection. In this approach, all feasible K-input cuts—i.e., subgraphs with at most K inputs—are enumerated for each node in the boolean network. Then, a dynamic programming-based selection process chooses one cut per node to construct a full LUT cover of the circuit, optimizing for metrics such as area or delay [19, 29, 92]. A refinement of this approach is known as priority cut pruning, which retains only a limited set of the most promising cuts per node rather than considering all possible cuts. This significantly improves scalability for large circuits and is widely implemented in tools such as ABC [14].

E.3 Global Routing

The global routing problem addresses the challenge of planning signal paths across a chip after logic placement, determining how a set of nets should traverse the layout to ensure connectivity while reserving space for detailed routing. Rather than producing exact wire geometries, global routing generates abstract paths through routing regions. This step must account for routing congestion, layer limitations, and timing criticality, while managing a growing number of nets in modern designs like Very-Large-Scale Integration (VLSI). The quality of the global routing solution plays a critical role in determining the feasibility and effectiveness of downstream routing stages and can ultimately dictate the success or failure of physical design closure.

The problem has been studied extensively via sequential and ILP-based methods. Maze routing, introduced by [74], laid the groundwork for sequential approaches, with subsequent improvements such as the work by [132]. For multi-terminal nets, rectilinear Steiner tree methods were developed [30]. However, sequential routing lacks global coordination and often leads to congestion. ILP-based methods formulate routing as a 0-1 programming, concurrently optimizing over all nets with objectives like wire length and capacity constraints. While exact ILP solvers are computationally intensive, relaxation techniques such as randomized rounding [17] and multi-commodity network flow models [127, 4] have been employed. Interior-point methods for solving the LP relaxation [142, 12] have also proven effective for scalable and near-optimal routing.

[58] provided a comprehensive survey of global routing techniques for integrated circuits. [93] revisited the problem, offering a historical perspective and highlighting key open challenges that remain unresolved. More recently, to foster the development of advanced global routing methods, [79, 80] introduced an ISPD contest that encourages the use of GPU-based techniques to accelerate global routing.

E.4 E-Graph Extraction

E-graph [18, 96] is a data structure that compactly represents a set of expressions. Given an input program and a set of rewrite rules, an e-graph is constructed by applying the rules to the program, generating new expressions, and merging equivalent expressions. It has been widely used to represent and explore the huge number of equivalent program space in tensor graph transformation [158, 24], sparse linear algebra optimization [146], code optimization [72, 129], digital signal processor (DSP) compilation [141, 138], circuit datapath synthesis [139, 26], and floating-point arithmetic [104].

In an e-graph, all functionally equivalent terms are organized in the same equivalent classes, known as e-classes. Nodes within each e-class that represent values or operators are called e-nodes. E-classes are a partition of e-nodes, where each e-node belongs to exactly one e-class. Dependencies in e-graphs are directed, which point from e-nodes to their children e-classes, indicating the operator (e-node) requires the values (e-nodes) from the child e-classes to compute its value.

In e-graph extraction, an optimized term from an e-graph is extracted after rewrites, based on a user-defined cost model. The goal is to produce a functionally equivalent but improved implementation of the original input program. The e-graph extraction problem is proven to be NP-hard when common sub-expressions are considered [133, 165].

Existing e-graph extraction methods include exact methods employing ILP [26, 129]. Recently, there has been significant progress in employing heuristics for e-graph extraction. These include a simple working-list method [104], a relaxation method utilizing gradient descent [15], and a specialized method tailored for sparse e-graphs [49]. The dataset used in evaluation for this work primarily comes from SmoothE [15].

E.5 Intra-Operator Parallelism

Intra-Operator Parallelism (IOPDDL), an emerging challenge introduced in the ASPLOS'25 contest track [94], addresses the complexities of distributed deep learning. Leading teams in this competition have predominantly employed meta-heuristic approaches, distinguished by their unique pre-processing and optimization strategies.

The effective distribution of large machine learning models across multiple hardware accelerators is paramount for achieving desired performance in both training and serving applications [168, 167, 126, 109, 75, 44, 23]. This task necessitates sharding the computation graph to minimize communication overhead, a process made intricate by the vast number of operations and tensors involved. Specifically, for a given graph where nodes represent operations with distinct execution strategies (each possessing associated cost and memory usage), an optimal strategy must be chosen for every node. The objective is to minimize the aggregate sum of node and edge costs, without exceeding a strict memory usage constraint across all devices at any point. The inherent diversity in topological and memory characteristics of ML models across varied tasks and modalities renders this problem especially demanding.

E.6 Protein Sequence Design

Understanding how proteins fold into their native three-dimensional structures [66, 147] is a central problem in structural biology [91, 45], traditionally framed as a forward problem: predicting the structure a given amino acid sequence will adopt [82, 145]. In contrast, the protein sequence design or inverse folding problem starts from a fixed target structure and seeks sequences that are likely to fold into it. Many works have shown that this inverse formulation not only offers practical applications in protein engineering but also deepens our understanding of sequence–structure relationships [43, 163, 124, 40, 135, 73].

A common modeling approach treats sequence design as a global optimization problem over the space of amino acid sequences. Methods developed by [135], [124], and others define a fitness function to select sequences with favorable folding properties. These functions are designed to balance positive design (low free energy in the target structure) with negative design (high energy in competing folds), promoting both thermodynamic stability and structural specificity. More recently, people have been working on multi-state design with more or less general fitness functions [108, 8, 6, 95, 159, 52, 143].

In our benchmark, we focus on the Grand Canonical (GC) model [135] of protein sequence design. The GC model operates on (i) a detailed three-dimensional geometric representation of a target structure with n residues, (ii) a simplified binary alphabet distinguishing only hydrophobic (H) and polar (P) residues, and (iii) a fitness function Φ that favors sequences with densely packed hydrophobic cores while penalizing solvent-exposed hydrophobic residues. Despite its simplicity, the H/P model has been shown to capture key qualitative features of real protein structures [41, 67]. Several studies [90, 11] have explored the correspondence between sequences optimized under the GC model and those observed in natural proteins. However, a key obstacle has remained: computing an optimal sequence for a given structure is computationally challenging. The brute-force enumeration over all 2^n H/P sequences is infeasible for realistic protein sizes, and the algorithmic complexity of the problem was explicitly raised as an open question by [54]. An efficient algorithm that constructs an optimal sequence in polynomial runtime was introduced later [69] using network flow.

E.7 Mendelian Error Detection

Chromosomes encode an individual's genetic information, with each gene occupying a specific position known as a locus. At each locus, a diploid organism carries two alleles—one inherited from each parent—forming its genotype. When direct genotyping is not available, researchers rely on the observable traits or phenotypes, which represent sets of compatible genotypes. A group of related individuals, along with their phenotypes at a locus, is organized into a pedigree, where each individual is either a founder or has parents defined within the structure.

Due to experimental and human errors, pedigree data may contain inaccuracies. These errors are classified as either parental errors (incorrect parentage, which we assume do not occur here) or phenotype errors, which can lead to Mendelian errors. A Mendelian error arises when all genotype combinations compatible with observed phenotypes violate Mendel's law that each individual inherits one allele from each parent. Detecting such inconsistencies is computationally challenging; the number of possible genotype combinations grows exponentially with pedigree size, making full enumeration impractical. In fact, verifying consistency has been shown to be NP-complete [1].

Error detection and correction are crucial for downstream tasks like genetic mapping or disease gene localization. However, existing tools are often limited by scalability issues, strong assumptions, or incomplete analysis. To address these limitations, a soft constraint network framework for detecting Mendelian inconsistencies was proposed [119], estimating the minimum number of required corrections, and suggesting optimal modifications. These problems naturally align with weighted constraint satisfaction and provide a rich testbed for scalable and flexible inference in large, complex pedigrees.

E.8 Airline Crew Pairing

The airline crew pairing problem is a well-established topic in operations research. It involves constructing sequences of flight legs—known as pairings—that begin and end at a crew base, cover all scheduled flights, and satisfy a variety of regulatory and contractual constraints. The primary goal is to minimize total crew-related costs, such as wages, hotel accommodations, and deadhead travel, while ensuring legality and operational feasibility. This problem is typically formulated as a set partitioning model and addressed using column generation and branch-and-price techniques [39, 68]. Foundational systems developed for carriers like American Airlines demonstrated the effectiveness of these methods at scale [9]. More recent innovations include dynamic constraint aggregation [47] and machine learning-based pairing generation [156], which are now integral to commercial solvers such as Jeppesen [63] and Sabre [117], capable of processing monthly schedules with tens of thousands of flights.

In addition to exact methods, heuristic and metaheuristic techniques – such as genetic algorithms, simulated annealing, and local search – have been explored to improve scalability and reduce computation time, particularly for medium-sized instances or disruption recovery [88, 131]. These hybrid approaches aim to complement exact optimization methods by leveraging historical data and incorporating planner preferences, offering more flexible and adaptive solutions in practice.

E.9 Pickup and Delivery Problem with Time Windows

The Pick-up and Delivery Problem with Time Windows (PDPTW), originally proposed by [46], is generalized from a classical NP-hard combinatorial optimization problem—the Capacitated Vehicle Routing Problem (CVRP). It introduces additional complexity through precedence constraints, requiring pick-up locations to precede corresponding drop-off locations, and service time windows at each location. The problem can be seen in many logistic and public transportation systems, with the primary objective of minimizing the total travel cost.

Over the past three decades, a wide range of models and algorithms have been proposed to address the PDPTW, with most falling into the category of heuristic or metaheuristic approaches. Prominent works include simulated annealing [76, 13], large neighborhood search [33, 115], and iterated local search [120]. In contrast, research into exact solution methods has been relatively limited, with the most effective approaches relying on the set partitioning formulation combined with the branch-cut-and-price algorithm [114, 10]. [116] provided a comprehensive survey of PDPTW solvers developed up to 2007. [56] later reviewed more recent advancements up to 2018, with a particular emphasis on

PDPTW variants for people transportation, referred to as the Dial-a-Ride problem. [137] develop a mathematical model and applies heuristics (Genetic Algorithm, Simulated Annealing, Tabu Search) to analyze how time-window constraints affect urban pickup/delivery truck routing and scheduling.

To support algorithm development, several benchmark datasets have been created and maintained. The Li and Lim dataset [76] is widely used and includes instances ranging from 100 to 1000 locations. More recently, [120] released a larger-scale dataset generated from real-world spatial-temporal distributions.

F Additional Experiments

In this section, we provide more experimental results and analysis on our benchmark.

F.1 Experimental Settings

By default, we constrain LLMs to generate Python code for each problem and execute the code on a CPU server, with each instance allocated 8 CPU cores. The timeout for each problem is specified in Table 8.

Table 8: Timeout for each problem.

Problem	**Timeout (sec)**
Operator scheduling	10
Technology mapping	10
Global routing	300
E-graph extraction	10
Intra-op parallelism	60
Protein sequence design	10
Mendelian error detection	10
Airline crew pairing	10
Pickup and delivery w/ time windows	60

F.2 Detailed Results on Each Problem

We provide the detailed $\text{SOLVE}_s@i$ values for each problem in Tables 9 through 17. The variation in $\text{SOLVE}_s@i$ across different problems highlights the diverse levels of difficulty, as summarized in Table 1. For instance, the global routing problem remains unsolved by all evaluated LLMs – even for generating a single feasible solution. In the case of the pickup and delivery problem, the low $\text{SOLVE}_{\text{III}}@10$ ratio also indicates that current LLMs struggle to consistently satisfy the problem's constraints.

Table 9: $\text{SOLVE}_s@i$ results on operator scheduling problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	**100.0%**	**100.0%**	4.2%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Gemini-2.5-Pro	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
LLaMA-4-Maverick	20.8%	0.0%	0.0%	**100.0%**	4.2%	0.0%	**100.0%**	**100.0%**	4.2%
LLaMA-3.3	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Qwen3-235B	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
GPT-o4-mini-high	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 10: $\text{SOLVE}_s@i$ results on technology mapping problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	87.1%	87.1%	**77.4%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	0.0%	0.0%	0.0%	93.5%	77.4%	67.7%	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	74.2%	74.2%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-4-Maverick	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-3.3	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	6.5%
Qwen3-235B	0.0%	0.0%	0.0%	**100.0%**	87.1%	0.0%	**100.0%**	**100.0%**	3.2%
Claude-3.7-Sonnet	87.1%	87.1%	0.0%	**100.0%**	**100.0%**	64.5%	**100.0%**	**100.0%**	**100.0%**
GPT-o4-mini-high	**100.0%**	**100.0%**	45.2%	**100.0%**	**100.0%**	51.6%	**100.0%**	**100.0%**	**100.0%**

Table 11: $\text{SOLVE}_s@i$ results on global routing problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	**0.0%**	**0.0%**	**0.0%**	33.3%	33.3%	0.0%	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	**0.0%**	**0.0%**	**0.0%**	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	**0.0%**	**0.0%**	**0.0%**	20.8%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	**0.0%**	**0.0%**	**0.0%**	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-4-Maverick	**0.0%**	**0.0%**	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
LLaMA-3.3	**0.0%**	**0.0%**	**0.0%**	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	4.2%
Qwen3-235B	**0.0%**	**0.0%**	**0.0%**	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	**0.0%**	**0.0%**	**0.0%**	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
GPT-o4-mini-high	**0.0%**	**0.0%**	**0.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 12: $\text{SOLVE}_s@i$ results on e-graph extraction problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	4.3%	0.0%	0.0%	**100.0%**	**100.0%**	82.6%	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Gemini-2.5-Pro	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
LLaMA-4-Maverick	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-3.3	39.1%	39.1%	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Qwen3-235B	87.0%	87.0%	87.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	39.1%	39.1%	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
GPT-o4-mini-high	**100.0%**	**100.0%**	39.1%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 13: $\text{SOLVE}_s@i$ results on intra-op parallelism problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	82.1%	53.6%	35.7%	82.1%	53.6%	35.7%	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	92.9%	92.9%	35.7%	92.9%	92.9%	35.7%	**100.0%**	**100.0%**	35.7%
Gemini-2.5-Flash	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	82.1%	82.1%	0.0%	82.1%	82.1%	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-4-Maverick	96.4%	96.4%	3.6%	**100.0%**	**100.0%**	3.6%	**100.0%**	**100.0%**	3.6%
LLaMA-3.3	75.0%	75.0%	3.6%	82.1%	82.1%	3.6%	**100.0%**	**100.0%**	**100.0%**
Qwen3-235B	75.0%	71.4%	67.9%	78.6%	75.0%	75.0%	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	82.1%	82.1%	71.4%	82.1%	82.1%	78.6%	**100.0%**	**100.0%**	96.4%
GPT-o4-mini-high	**100.0%**	**100.0%**	92.9%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 14: $\text{SOLVE}_s@i$ results on protein sequence design problem.

	$\text{SOLVE}_{\text{III}}$			SOLVE_{II}			SOLVE_{I}		
Model	@10	@5	@1	@10	@5	@1	@10	@5	@1
DeepSeek-V3	83.3%	83.3%	83.3%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	87.5%	87.5%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Gemini-2.5-Flash	95.8%	**95.8%**	**95.8%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	**100.0%**	**95.8%**	0.0%	**100.0%**	95.8%	0.0%	**100.0%**	**100.0%**	4.2%
LLaMA-4-Maverick	83.3%	83.3%	0.0%	95.8%	95.8%	0.0%	**100.0%**	**100.0%**	4.2%
LLaMA-3.3	12.5%	12.5%	12.5%	95.8%	95.8%	95.8%	95.8%	95.8%	95.8%
Qwen3-235B	87.5%	87.5%	87.5%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	58.3%	45.8%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
GPT-o4-mini-high	91.7%	91.7%	91.7%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 15: SOLVE_s@i results on mendelian error detection problem.

Model	$\text{SOLVE}_{\text{III}}$ @10	$\text{SOLVE}_{\text{III}}$ @5	$\text{SOLVE}_{\text{III}}$ @1	SOLVE_{II} @10	SOLVE_{II} @5	SOLVE_{II} @1	SOLVE_{I} @10	SOLVE_{I} @5	SOLVE_{I} @1
DeepSeek-V3	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
DeepSeek-R1	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Gemini-2.5-Flash	**100.0%**	10.0%	10.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	80.0%	80.0%	**80.0%**	80.0%	80.0%	80.0%	**100.0%**	**100.0%**	**100.0%**
LLaMA-4-Maverick	60.0%	60.0%	60.0%	60.0%	60.0%	60.0%	60.0%	60.0%	60.0%
LLaMA-3.3	55.0%	55.0%	55.0%	55.0%	55.0%	55.0%	**100.0%**	**100.0%**	**100.0%**
Qwen3-235B	55.0%	55.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Claude-3.7-Sonnet	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
GPT-o4-mini-high	**100.0%**	50.0%	35.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 16: SOLVE_s@i results on airline crew pairing problem.

Model	$\text{SOLVE}_{\text{III}}$ @10	$\text{SOLVE}_{\text{III}}$ @5	$\text{SOLVE}_{\text{III}}$ @1	SOLVE_{II} @10	SOLVE_{II} @5	SOLVE_{II} @1	SOLVE_{I} @10	SOLVE_{I} @5	SOLVE_{I} @1
DeepSeek-V3	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	14.3%
Gemini-2.5-Pro	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**
LLaMA-4-Maverick	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	35.7%	**100.0%**	**100.0%**	**100.0%**
LLaMA-3.3	42.9%	42.9%	42.9%	42.9%	42.9%	42.9%	**100.0%**	**100.0%**	**100.0%**
Qwen3-235B	21.4%	21.4%	0.0%	**100.0%**	85.7%	0.0%	**100.0%**	**100.0%**	0.0%
Claude-3.7-Sonnet	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
GPT-o4-mini-high	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

Table 17: SOLVE_s@i results on pickup and delivery with time windows problem.

Model	$\text{SOLVE}_{\text{III}}$ @10	$\text{SOLVE}_{\text{III}}$ @5	$\text{SOLVE}_{\text{III}}$ @1	SOLVE_{II} @10	SOLVE_{II} @5	SOLVE_{II} @1	SOLVE_{I} @10	SOLVE_{I} @5	SOLVE_{I} @1
DeepSeek-V3	0.0%	0.0%	0.0%	80.0%	73.3%	73.3%	**100.0%**	**100.0%**	**100.0%**
DeepSeek-R1	16.7%	13.3%	3.3%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Flash	**96.7%**	**90.0%**	6.7%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Gemini-2.5-Pro	30.0%	26.7%	**13.3%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
LLaMA-4-Maverick	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
LLaMA-3.3	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	0.0%	**100.0%**	**100.0%**	0.0%
Qwen3-235B	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**
Claude-3.7-Sonnet	0.0%	0.0%	0.0%	**100.0%**	**100.0%**	16.7%	**100.0%**	**100.0%**	**100.0%**
GPT-o4-mini-high	3.3%	0.0%	0.0%	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**	**100.0%**

F.3 Ablation on Temperature

We evaluate various models across different temperature settings, $T \in \{0.0, 0.5, 1.0\}$. For each model, we run 10 iterations per problem and report the highest QYI achieved across these iterations as the final QYI score for that problem. The overall benchmark score is then computed as the arithmetic mean of QYI across all problems. Detailed results are shown in Tables 18 to 20.

In general, improving the temperature can be beneficial to quality as the model becomes more creative, but may harm yield as it may not follow the constraints strictly. Note that yield emphasizes the best iteration that achieves the highest QYI, whereas $\text{SOLVE}_{\text{III}}$ reflects the cumulative success rate across iterations; therefore, their values may differ. Additionally, the weighted QYI is not the harmonic mean of weighted yield and weighted quality, as it is computed by aggregating metrics across different problems using a weighted approach.

We also report an uncapped version of the weighted QYI metric[2], which better reflects cases where LLM-generated programs outperform expert solutions on certain test instances. Improvements are underlined in the tables. While this variant achieves slightly higher scores for most models – indicating occasional superior performance – it also confirms that, in the majority of cases, LLMs still lag significantly behind expert solutions.

[2]The uncapped version of quality is computed as $1/\hat{N}\sum_{n=1}^{\hat{N}} c_n^{\star}/c_n$, and the uncapped QYI is derived by substituting the original quality metric with this uncapped variant.

Table 18: Performance of different models on Temperature = 0.

Model	Weighted Yield	Weighted Quality	Weighted QYI (Capped)	Weighted QYI (Uncapped)
Claude-3.7-Sonnet	0.5963	0.4686	0.5034	0.5034
DeepSeek-R1	**0.6972**	0.5775	0.5498	<u>0.5553</u>
DeepSeek-V3	0.4587	0.3890	0.3707	0.3707
Gemini-2.5-Flash	0.6606	0.5281	0.5682	<u>0.5753</u>
Gemini-2.5-Pro	0.6468	**0.6700**	**0.6170**	<u>**0.6228**</u>
LLaMA-3.3	0.3394	0.3521	0.2951	0.2953
LLaMA-4-Maverick	0.3211	0.3383	0.2955	0.2955
Qwen3-235B	0.4450	0.4513	0.4355	<u>0.4423</u>

Table 19: Performance of different models on Temperature = 0.5.

Model	Weighted Yield	Weighted Quality	Weighted QYI (Capped)	Weighted QYI (Uncapped)
Claude-3.7-Sonnet	**0.6147**	**0.6468**	**0.5437**	<u>**0.5451**</u>
DeepSeek-R1	0.5138	0.5751	0.4743	<u>0.4812</u>
DeepSeek-V3	0.3716	0.4645	0.3322	0.3322
Gemini-2.5-Flash	0.4817	0.5700	0.4760	<u>0.4828</u>
Gemini-2.5-Pro	0.4817	0.5609	0.4767	<u>0.4789</u>
LLaMA-3.3	0.3991	0.4407	0.4108	0.4108
LLaMA-4-Maverick	0.3349	0.3712	0.3050	<u>0.3646</u>
Qwen3-235B	0.4128	0.4798	0.4269	<u>0.4327</u>

Table 20: Performance of different models on Temperature = 1.

Model	Weighted Yield	Weighted Quality	Weighted QYI (Capped)	Weighted QYI (Uncapped)
Claude-3.7-Sonnet	0.5138	0.5924	0.4828	<u>0.4841</u>
DeepSeek-R1	0.5688	0.5625	0.5313	<u>0.5383</u>
DeepSeek-V3	0.4128	0.4188	0.3839	<u>0.3841</u>
GPT-o4-mini-high	**0.6927**	0.6440	**0.6089**	<u>**0.6158**</u>
Gemini-2.5-Flash	0.4771	**0.7688**	0.5030	<u>0.5047</u>
Gemini-2.5-Pro	0.5229	0.4893	0.4921	<u>0.4981</u>
LLaMA-3.3	0.3028	0.3627	0.2868	<u>0.2916</u>
LLaMA-4-Maverick	0.2982	0.3271	0.2667	<u>0.2672</u>
Qwen3-235B	0.5459	0.5228	0.5294	<u>0.5364</u>

F.4 Few-Shot Demonstration

Table 21 highlights the impact of few-shot demonstrations on LLM performance across the entire HeuriGym benchmark. Introducing only a small number of demonstrations (e.g., three) can negatively affect solution quality and success rate, as these examples may not be representative of the overall dataset, leading the model to overfit to them. However, providing a larger set of demonstrations can potentially improve QYI, as the model benefits from greater diversity and can learn more generalizable patterns.

Table 21: Impact of few-shot demonstrations on performance (Model: Gemini-2.5-Pro).

# of Demos	Weighted Yield	Weighted Quality	Weighted QYI
Zero-shot	0.5872	0.7159	0.5999
Half-shot	0.5092	0.6526	0.5361
Full-shot	0.6468	0.6700	0.6170

F.5 Feedback Rounds

Table 22 shows that increasing the number of feedback rounds has a nuanced impact on performance. While a moderate number of rounds (e.g., five) can enhance overall quality by guiding the model to refine its solutions, excessive feedback may lead to diminishing returns or even degrade performance. This suggests that too many rounds can overwhelm the model, making it harder to identify and prioritize the most critical information from the feedback.

Table 22: Impact of feedback rounds on performance (Model: Gemini-2.5-Pro).

# of Feedback Rounds	Weighted Yield	Weighted Quality	Weighted QYI
1	0.6193	0.7290	0.6253
5	0.6055	0.7313	0.6259
10	0.6468	0.6700	0.6170

F.6 Iterative Best-of-N Sampling

To investigate the benefits of test-time search strategies, we sample k candidate programs in each iteration, evaluate them, and return feedback for all k programs to the LLM. After a fixed number of iterations, we select the best-performing program from the entire pool – a process we refer to as *iterative best-of-N sampling*. The total number of sampled programs is held constant across different values of k. This strategy allows the model to explore diverse candidate solutions in parallel and evolve the program based on evaluative feedback.

As shown in Table 23, increasing k leads to better quality of results, indicating that aggregating feedback across multiple candidates allows the LLM to better explore the solution space and improve sampling efficiency by allocating computational budget toward more informative evaluations.

Table 23: Impact of iterative best-of-N sampling on performance (Model: Gemini-2.5-Pro).

# of Samples @ Iteration	Weighted Yield	Weighted Quality	Weighted QYI
2@5	0.5688	0.7698	0.6160
1@10	0.6468	0.6700	0.6170

F.7 Error Analysis

In the following, we present representative examples of common errors made by LLMs during heuristic generation. These errors highlight current limitations in code reliability and execution:

- **Import error:** This type of error occurs when the generated code relies on external libraries that are not available in the environment. In the example below, the model attempts to import the `ortools` library, which results in a `ModuleNotFoundError`. Such errors suggest that the model does not strictly follow the instructions given in the prompt.

```
  File "operator_scheduling/gemini-2.5-flash-preview-04-17/iteration4/sol
  ↪  ver.py", line 2, in <module>
    from ortools.sat.python import cp_model
ModuleNotFoundError: No module named 'ortools'
```

- **API misuse error:** LLMs often misuse APIs due to a misunderstanding of library interfaces. In the following case, the model tries to call `random()` directly from the `random` module, which is not callable.

```
  File "intra_op_parallel/o4-mini/iteration3/solver.py", line 64, in
  ↪  init_jitter
    if len(ci) > 1 and random() < 0.1:
                       ^^^^^^^^
TypeError: 'module' object is not callable
```

- **Syntax error:** Syntax errors are common when the model fails to adhere to basic language rules. In this example, there is an unmatched parenthesis in a `while` loop condition, leading to a `SyntaxError`. Such mistakes typically indicate a lack of code completion validation in the generation process.

```
  File "crew_pairing/deepseek-chat/iteration7/solver.py", line 60
    while len(used_legs) < len(df)):
```

```
SyntaxError: unmatched ')'
```

- **Runtime error:** Even syntactically and semantically correct code can fail at runtime. In this case, the model modifies a dictionary while iterating over it, which raises a `RuntimeError`. This highlights the model's difficulty in reasoning about the actual executable code in a long context.

```
  File "technology_mapping/llama-4-maverick/iteration2/solver.py", line
  ↪  104, in technology_mapping
    for successor in G.successors(node):
RuntimeError: dictionary changed size during iteration
```

F.8 C++ Example

We conduct preliminary experiments on the technology mapping problem by modifying the prompt to instruct the LLM to generate a C++ solution, using the provided function template: `void solve(const std::string& input_file, const std::string& output_file)`.

Integrating C++ into our agentic feedback loop remains challenging due to dependencies on domain-specific libraries and the complexity of parallel execution. As a result, our preliminary experiment with C++ involves only a single iteration of prompting.

Table 24 presents a performance comparison between the Python solution with 10 iterations and the C++ solution with just one iteration. Although the C++ solution does not produce high-quality output in its initial attempt, it already achieves a better yield than the Python solution after 10 iterations – an unexpectedly strong outcome. Notably, the Python solution fails to generate any valid result in its first iteration. This is attributed to the significantly faster execution speed of C++ code, which enables it to avoid the timeout errors frequently encountered by Python in this task.

We expect to see further performance improvement with C++ after we integrate it into the feedback loop in our framework.

Table 24: Impact of C++ code on technology mapping performance (Model: `Gemini-2.5-pro`).

Language	# of Iterations	Yield	Quality	QYI
Python	10	0.7419	0.6423	0.6885
C++	1	0.7742	0.3493	0.4814

F.9 Token Usage

Table 25 presents an example of token usage when running the complete HeuriGym benchmark across different models. Among them, Gemini-2.5-Pro consumes the most tokens for prompt and completion.

Table 25: Token counts from a single run of HeuriGym across different models.

Model	Prompt Tokens	Completion Tokens
Claude-3.7-Sonnet	1911947	298369
DeepSeek-R1	1215661	702746
DeepSeek-V3	1536084	122058
GPT-o4-mini-high	1063688	978891
Gemini-2.5-Flash	2742487	332569
Gemini-2.5-Pro	2880737	455739
LLaMA-3.3	969887	77523
LLaMA-4-Maverick	1497730	77080
Qwen3-235B	1089888	127447

G Detailed Analysis of Case Study

Here we include a detailed analysis of the case study in Section A.2. We append the entire code of each iteration and analyze the generated solution.

In the first iteration, the GPT-o4-mini-high model generates an algorithm based on cut enumeration and dynamic programming (DP)-based cut selection, which is the common approach used in technology mapping tools. The core logic is highlighted in the following code snippet. However, it does not prune the cut enumeration space and computes all possible K-feasible cuts for each node, which leads to an exponential explosion in computation time. While this achieves high quality, it frequently triggers `TimeoutError` for larger datasets, resulting in low yield.

```python
# iteration 1: base algorithm without pruning the search space

def solve(input_file: str, solution_file: str):
    from collections import deque
    class Node:
        __slots__ = ('name','fanin_names','fanins','fanouts','patterns_one','patterns_zero','const_val')
        def __init__(self,name):
            self.name = name
            self.fanin_names = []
            self.fanins = []
            self.fanouts = []
            self.patterns_one = []
            self.patterns_zero = []
            self.const_val = None
    raw_lines = open(input_file).read().splitlines()
    lines = []
    i = 0
    while i < len(raw_lines):
        line = raw_lines[i].rstrip()
        if not line or line.lstrip().startswith('#'):
            i += 1
            continue
        if line.endswith('\\'):
            merged = line[:-1].rstrip()
            i += 1
            while i < len(raw_lines):
                nxt = raw_lines[i].rstrip()
                if nxt.endswith('\\'):
                    merged += ' ' + nxt[:-1].rstrip()
                    i += 1
                else:
                    merged += ' ' + nxt
                    i += 1
                    break
            lines.append(merged)
        else:
            lines.append(line)
            i += 1
    model_name = ''
    input_list = []
    output_list = []
    nodes = {}
    idx = 0
    while idx < len(lines):
        l = lines[idx].strip()
        if not l or l.startswith('#'):
            idx += 1
            continue
        if l.startswith('.model'):
            parts = l.split()
            if len(parts) > 1:
                model_name = parts[1]
            idx += 1
        elif l.startswith('.inputs'):
            parts = l.split()
            for name in parts[1:]:
                input_list.append(name)
                if name not in nodes:
                    nodes[name] = Node(name)
            idx += 1
        elif l.startswith('.outputs'):
            parts = l.split()
            for name in parts[1:]:
                output_list.append(name)
            idx += 1
        elif l.startswith('.names'):
            parts = l.split()
            in_names = parts[1:-1]
```

```
            out_name = parts[-1]
            if out_name not in nodes:
                nodes[out_name] = Node(out_name)
            node = nodes[out_name]
            node.fanin_names = in_names[:]
            patterns = []
            j = idx + 1
            while j < len(lines):
                lj = lines[j].strip()
                if not lj or lj.startswith('.') or lj.startswith('#'):
                    break
                patterns.append(lj)
                j += 1
            if len(in_names) == 0:
                if patterns and patterns[0].strip() == '1':
                    node.const_val = 1
                else:
                    node.const_val = 0
            else:
                for pline in patterns:
                    parts2 = pline.split()
                    if len(parts2) == 1:
                        mask = parts2[0]; bit = '1'
                    else:
                        mask = parts2[0]; bit = parts2[1]
                    if bit == '1':
                        node.patterns_one.append(mask)
                    else:
                        node.patterns_zero.append(mask)
            idx = j
        elif l.startswith('.end'):
            break
        else:
            idx += 1
    for node in nodes.values():
        node.fanins = []
        for fn in node.fanin_names:
            if fn in nodes:
                node.fanins.append(nodes[fn])
        node.fanouts = []
    for node in nodes.values():
        for f in node.fanins:
            f.fanouts.append(node)
    reachable = set()
    dq = deque()
    for out in output_list:
        if out in nodes:
            dq.append(out)
    while dq:
        u = dq.pop()
        if u in reachable:
            continue
        reachable.add(u)
        for f in nodes[u].fanins:
            if f.name not in reachable:
                dq.append(f.name)
    nodes = {name:node for name,node in nodes.items() if name in reachable}
    for node in nodes.values():
        node.fanins = [f for f in node.fanins if f.name in nodes]
        node.fanouts = [f for f in node.fanouts if f.name in nodes]
    indeg = {name: len(node.fanins) for name,node in nodes.items()}
    dq = deque([name for name,d in indeg.items() if d == 0])
    topo_names = []
    while dq:
        u = dq.popleft()
        topo_names.append(u)
        for w in nodes[u].fanouts:
            indeg[w.name] -= 1
            if indeg[w.name] == 0:
                dq.append(w.name)
    topo_list = [nodes[name] for name in topo_names]
    K = 6
    def prune_cuts(cset):
        cuts = list(cset)
        res = []
        for c in cuts:
            skip = False
            for d in cuts:
                if d is not c and d.issubset(c):
                    skip = True
                    break
            if not skip:
                res.append(c)
```

```
        return res
    cuts = {}      # Cut Enumeration
    for n in topo_list:
        if not n.fanins:
            cuts[n.name] = [frozenset([n.name])]
        else:
            cuts_n = None
            for f in n.fanins:
                cf = cuts[f.name]
                if cuts_n is None:
                    cuts_n = cf[:]
                else:
                    newset = set()
                    for c1 in cuts_n:
                        for c2 in cf:
                            u = c1 | c2
                            if len(u) <= K:
                                newset.add(u)
                    cuts_n = prune_cuts(newset)
            if cuts_n is None:
                cuts_n = []
            cuts_set = set(cuts_n)
            cuts_set.add(frozenset([n.name]))
            cuts[n.name] = prune_cuts(cuts_set)
    cost = {}
    best_cut = {}
    INF = 10**18
    for n in topo_list:     # DP-based Cut Selection
        if not n.fanins:
            cost[n.name] = 0
            best_cut[n.name] = None
        else:
            bcut = None
            bval = INF
            for c in cuts[n.name]:
                if c == frozenset([n.name]):
                    continue
                s = 0
                for m in c:
                    s += cost.get(m,0)
                cval = s + 1
                if cval < bval:
                    bval = cval; bcut = c
            if bcut is None:
                fl = frozenset([f.name for f in n.fanins])
                bcut = fl
                s = 0
                for m in bcut:
                    s += cost.get(m,0)
                bval = s + 1
            cost[n.name] = bval
            best_cut[n.name] = bcut
    mapping_nodes = set()
    dq = deque()
    for out in output_list:
        if out in best_cut and best_cut[out] is not None:
            dq.append(out)
    while dq:
        u = dq.pop()
        if u in mapping_nodes:
            continue
        mapping_nodes.add(u)
        c = best_cut[u]
        if c:
            for m in c:
                if m in nodes:
                    nm = nodes[m]
                    if m not in input_list and nm.const_val is None:
                        dq.append(m)
    mapping_topo = [n for n in topo_list if n.name in mapping_nodes]
    f = open(solution_file,'w')
    f.write('.model '+model_name+'\n')
    f.write('.inputs '+' '.join(input_list)+'\n')
    f.write('.outputs '+' '.join(output_list)+'\n')
    for n in mapping_topo:
        leaves = best_cut[n.name]
        inputs = sorted(leaves)
        k = len(inputs)
        N = 1<<k
        full_mask = (1<<N) - 1
        masks = []
        for i in range(k):
            m = 0
```

```
            for j in range(N):
                if (j>>i)&1:
                    m |= (1<<j)
            masks.append(m)
        vis = set()
        queue2 = [n.name]
        while queue2:
            u = queue2.pop()
            if u in vis or u in inputs:
                continue
            vis.add(u)
            for fn in nodes[u].fanins:
                if fn.name not in vis:
                    queue2.append(fn.name)
        cone_nodes = [nodes[name] for name in topo_names if name in vis]
        val = {}
        for i,name in enumerate(inputs):
            val[name] = masks[i]
        for v in cone_nodes:
            if v.const_val is not None:
                val[v.name] = full_mask if v.const_val==1 else 0
            else:
                if v.patterns_zero:
                    zero_mask = 0
                    for p in v.patterns_zero:
                        m = full_mask
                        for idx,ch in enumerate(p):
                            vm = val[v.fanin_names[idx]]
                            if ch=='1':
                                m &= vm
                            elif ch=='0':
                                m &= (~vm)&full_mask
                        zero_mask |= m
                    val[v.name] = (~zero_mask)&full_mask
                else:
                    one_mask = 0
                    for p in v.patterns_one:
                        m = full_mask
                        for idx,ch in enumerate(p):
                            vm = val[v.fanin_names[idx]]
                            if ch=='1':
                                m &= vm
                            elif ch=='0':
                                m &= (~vm)&full_mask
                        one_mask |= m
                    val[v.name] = one_mask
        root_mask = val[n.name]
        f.write('.names '+' '.join(inputs)+' '+n.name+'\n')
        for j in range(N):
            if (root_mask>>j)&1:
                line = ''.join('1' if (j>>i)&1 else '0' for i in range(k))
                f.write(line+' 1\n')
    f.write('.end\n')
    f.close()

```

In the second iteration, the model overcorrects by adopting a naive 6-LUT mapping strategy that trivially assigns each node to a separate LUT. While this approach passes all test cases within the timeout, it produces poor solution quality. Specifically, it redundantly maps each non-primary-input node as an individual LUT by directly copying inputs to outputs, resulting in excessive and inefficient resource usage.

```
# iteration 2: a very naive mapping strategy. Simply copy the input to the output.

def solve(input_file: str, solution_file: str):
    raw = open(input_file).read().splitlines()
    merged = []
    i = 0
    while i < len(raw):
        line = raw[i].rstrip()
        if not line or line.lstrip().startswith('#'):
            i += 1
            continue
        if line.endswith('\\'):
            accum = line[:-1].rstrip()
            i += 1
            while i < len(raw):
                nxt = raw[i].rstrip()
                if nxt.endswith('\\'):
                    accum += ' ' + nxt[:-1].rstrip()
```

```
                    i += 1
                else:
                    accum += ' ' + nxt
                    i += 1
                    break
            merged.append(accum)
        else:
            merged.append(line)
            i += 1
    with open(solution_file, 'w') as f:
        for l in merged:
            f.write(l + '\n')
```

In the third iteration, the model proposes a more sophisticated strategy that first applies static pruning to the cut enumeration space – limiting the number of cuts per node to 20 – followed by a dynamic programming-based cut selection. This approach improves solution quality, though full yield is still not achieved. Additionally, truth table computation is optimized using bitwise operations.

```
# iteration 3: static pruning by capping the number of cuts per node to M = 20; efficient bitwise operations

def solve(input_file: str, solution_file: str):
    from collections import deque
    raw_lines = open(input_file).read().splitlines()
    lines = []
    i = 0
    while i < len(raw_lines):
        line = raw_lines[i].rstrip()
        if not line or line.lstrip().startswith('#'):
            i += 1
            continue
        if line.endswith('\\'):
            accum = line[:-1].rstrip()
            i += 1
            while i < len(raw_lines):
                nxt = raw_lines[i].rstrip()
                if nxt.endswith('\\'):
                    accum += ' ' + nxt[:-1].rstrip()
                    i += 1
                else:
                    accum += ' ' + nxt
                    i += 1
                    break
            lines.append(accum)
        else:
            lines.append(line)
            i += 1
    class Node:
        def __init__(self, name):
            self.name = name
            self.fanin_names = []
            self.patterns_one = []
            self.patterns_zero = []
            self.const_val = None
            self.fanins = []
            self.fanouts = []
    model_name = ''
    input_list = []
    output_list = []
    nodes = {}
    idx = 0
    while idx < len(lines):
        l = lines[idx].strip()
        if not l:
            idx += 1
            continue
        if l.startswith('.model'):
            parts = l.split()
            if len(parts) > 1:
                model_name = parts[1]
            idx += 1
        elif l.startswith('.inputs'):
            parts = l.split()
            for name in parts[1:]:
                input_list.append(name)
                if name not in nodes:
                    nodes[name] = Node(name)
            idx += 1
        elif l.startswith('.outputs'):
            parts = l.split()
```

```
            for name in parts[1:]:
                output_list.append(name)
            idx += 1
        elif l.startswith('.names'):
            parts = l.split()
            in_names = parts[1:-1]
            out_name = parts[-1]
            if out_name not in nodes:
                nodes[out_name] = Node(out_name)
            node = nodes[out_name]
            node.fanin_names = in_names[:]
            patterns = []
            j = idx + 1
            while j < len(lines) and not lines[j].startswith('.'):
                s = lines[j].strip()
                if s:
                    patterns.append(s)
                j += 1
            if len(in_names) == 0:
                if patterns and patterns[0] == '1':
                    node.const_val = 1
                else:
                    node.const_val = 0
            else:
                for pline in patterns:
                    parts2 = pline.split()
                    if len(parts2) == 1:
                        mask = parts2[0]; bit = '1'
                    else:
                        mask = parts2[0]; bit = parts2[1]
                    if bit == '1':
                        node.patterns_one.append(mask)
                    else:
                        node.patterns_zero.append(mask)
            idx = j
        elif l.startswith('.end'):
            break
        else:
            idx += 1
    for node in nodes.values():
        node.fanins = [nodes[nm] for nm in node.fanin_names if nm in nodes]
        node.fanouts = []
    for node in nodes.values():
        for f in node.fanins:
            f.fanouts.append(node)
    reachable = set()
    stack = [nm for nm in output_list if nm in nodes]
    while stack:
        u = stack.pop()
        if u in reachable: continue
        reachable.add(u)
        for p in nodes[u].fanin_names:
            if p in nodes and p not in reachable:
                stack.append(p)
    nodes = {nm:nd for nm,nd in nodes.items() if nm in reachable}
    for nd in nodes.values():
        nd.fanins = [f for f in nd.fanins if f.name in nodes]
        nd.fanouts = [f for f in nd.fanouts if f.name in nodes]
    indeg = {nm: len(nd.fanins) for nm,nd in nodes.items()}
    dq = deque([nm for nm,d in indeg.items() if d == 0])
    topo = []
    while dq:
        u = dq.popleft()
        topo.append(u)
        for w in nodes[u].fanouts:
            indeg[w.name] -= 1
            if indeg[w.name] == 0:
                dq.append(w.name)
    topo_nodes = [nodes[nm] for nm in topo]
    name_to_idx = {nd.name:i for i,nd in enumerate(topo_nodes)}
    idx_to_node = topo_nodes
    idx_to_name = [nd.name for nd in topo_nodes]
    N = len(topo_nodes)
    PI_idx = set(name_to_idx[nm] for nm in input_list if nm in name_to_idx)
    self_mask = [1 << i for i in range(N)]
    K = 6
    M = 20
    cuts = [[] for _ in range(N)]
    for i, nd in enumerate(topo_nodes):
        if i in PI_idx or nd.const_val is not None:
            cuts[i] = [self_mask[i]]
            continue
        fan_idxs = [name_to_idx[x] for x in nd.fanin_names if x in name_to_idx]
```

```
        c_list = None
        for f in fan_idxs:
            fcuts = cuts[f]
            if c_list is None:
                c_list = fcuts[:M]
            else:
                newset = set()
                for a in c_list[:M]:
                    for b in fcuts[:M]:
                        u = a | b
                        if u.bit_count() <= K:
                            newset.add(u)
                if newset:
                    lst = sorted(newset, key=lambda x: x.bit_count())
                    c_list = lst[:M]
                else:
                    c_list = []
            if not c_list:
                break
        if c_list is None:
            c_list = []
        s = set(c_list)
        s.add(self_mask[i])
        lst2 = sorted(s, key=lambda x: x.bit_count())
        cuts[i] = lst2[:M]
    INF = 10**18
    cost = [0]*N
    bestM = [None]*N
    for i, nd in enumerate(topo_nodes):
        if i in PI_idx or nd.const_val is not None:
            cost[i] = 0
            bestM[i] = None
            continue
        bestc = INF
        bm = None
        for cm in cuts[i]:
            if cm == self_mask[i]:
                continue
            s = 0
            m = cm
            while m:
                lsb = m & -m
                j = lsb.bit_length() - 1
                s += cost[j]
                m ^= lsb
            cval = s + 1
            if cval < bestc:
                bestc = cval
                bm = cm
        if bm is None:
            s = 0
            mask = 0
            for f in nd.fanin_names:
                if f in name_to_idx:
                    j = name_to_idx[f]
                    mask |= self_mask[j]
                    s += cost[j]
            bm = mask
            bestc = s + 1
        cost[i] = bestc
        bestM[i] = bm
    mapping = set()
    stack = [name_to_idx[nm] for nm in output_list if nm in name_to_idx]
    visited = set()
    while stack:
        u = stack.pop()
        if u in visited:
            continue
        visited.add(u)
        bm = bestM[u]
        if bm is None or bm == self_mask[u]:
            continue
        mapping.add(u)
        m = bm
        while m:
            lsb = m & -m
            j = lsb.bit_length() - 1
            m ^= lsb
            if j not in visited and j not in PI_idx and topo_nodes[j].const_val is None:
                stack.append(j)
    mapped = [i for i in range(N) if i in mapping]
    with open(solution_file, 'w') as f:
        f.write('.model ' + model_name + '\n')
```

```
        f.write('.inputs ' + ' '.join(input_list) + '\n')
        f.write('.outputs ' + ' '.join(output_list) + '\n')
        for i in mapped:
            nd = topo_nodes[i]
            bm = bestM[i]
            leaves = [j for j in range(N) if (bm >> j) & 1]
            inputs = [idx_to_name[j] for j in leaves]
            k = len(inputs)
            Np = 1 << k
            full = (1 << Np) - 1
            masks = []
            for t in range(k):
                mm = 0
                for j in range(Np):
                    if (j >> t) & 1:
                        mm |= 1 << j
                masks.append(mm)
            val = {}
            for t,j in enumerate(leaves):
                val[j] = masks[t]
            vis = set(leaves)
            stack2 = [i]
            while stack2:
                u = stack2.pop()
                if u in vis:
                    continue
                vis.add(u)
                for p in topo_nodes[u].fanins:
                    pi = name_to_idx.get(p.name)
                    if pi is not None and pi not in vis:
                        stack2.append(pi)
            cone = [j for j in range(N) if j in vis and j not in leaves]
            for j in sorted(cone, key=lambda x: name_to_idx[topo_nodes[x].name]):
                v = topo_nodes[j]
                if v.const_val is not None:
                    val[j] = full if v.const_val == 1 else 0
                else:
                    if v.patterns_zero:
                        zm = 0
                        for p in v.patterns_zero:
                            mm = full
                            for t,ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0
                                    break
                                vm = val[pj]
                                if ch == '1':
                                    mm &= vm
                                elif ch == '0':
                                    mm &= (~vm) & full
                            zm |= mm
                        val[j] = (~zm) & full
                    else:
                        om = 0
                        for p in v.patterns_one:
                            mm = full
                            for t,ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0
                                    break
                                vm = val[pj]
                                if ch == '1':
                                    mm &= vm
                                elif ch == '0':
                                    mm &= (~vm) & full
                            om |= mm
                        val[j] = om
            root = val[i]
            f.write('.names ' + ' '.join(inputs) + ' ' + nd.name + '\n')
            for j in range(Np):
                if (root >> j) & 1:
                    bits = ''.join('1' if (j >> t) & 1 else '0' for t in range(k))
                    f.write(bits + ' 1\n')
        f.write('.end\n')
```

In the fourth iteration, the cut limit per node is increased from 20 to 30, enabling broader solution exploration and potentially improving quality. Additionally, the algorithm reduces redundant computations by caching precomputed scores for each cut.

```python
# iteration 4: explore larger solution space; reduce redundant computations

def solve(input_file: str, solution_file: str):
    from collections import deque
    raw = open(input_file).read().splitlines()
    lines = []
    i = 0
    while i < len(raw):
        l = raw[i].rstrip()
        if not l or l.lstrip().startswith('#'):
            i += 1
            continue
        if l.endswith('\\'):
            acc = l[:-1].rstrip()
            i += 1
            while i < len(raw):
                nl = raw[i].rstrip()
                if nl.endswith('\\'):
                    acc += ' ' + nl[:-1].rstrip()
                    i += 1
                else:
                    acc += ' ' + nl
                    i += 1
                    break
            lines.append(acc)
        else:
            lines.append(l)
            i += 1
    class Node:
        __slots__ = ('name','fanin_names','patterns_one','patterns_zero','const_val','fanins','fanouts')
        def __init__(self,n):
            self.name = n
            self.fanin_names = []
            self.patterns_one = []
            self.patterns_zero = []
            self.const_val = None
            self.fanins = []
            self.fanouts = []
    model = ''
    inputs = []
    outputs = []
    nodes = {}
    idx = 0
    while idx < len(lines):
        l = lines[idx].strip()
        if not l:
            idx += 1; continue
        if l.startswith('.model'):
            parts = l.split()
            if len(parts)>1: model = parts[1]
            idx += 1
        elif l.startswith('.inputs'):
            parts = l.split()
            for nm in parts[1:]:
                inputs.append(nm)
                if nm not in nodes: nodes[nm] = Node(nm)
            idx += 1
        elif l.startswith('.outputs'):
            parts = l.split()
            for nm in parts[1:]:
                outputs.append(nm)
            idx += 1
        elif l.startswith('.names'):
            parts = l.split()
            inps = parts[1:-1]; outp = parts[-1]
            if outp not in nodes: nodes[outp] = Node(outp)
            nd = nodes[outp]
            nd.fanin_names = inps[:]
            pats = []
            j = idx+1
            while j < len(lines) and not lines[j].startswith('.'):
                s = lines[j].strip()
                if s: pats.append(s)
                j += 1
            if not inps:
                if pats and pats[0]=='1': nd.const_val = 1
                else: nd.const_val = 0
            else:
```

```
                for pt in pats:
                    sp = pt.split()
                    if len(sp)==1:
                        mask = sp[0]; bit = '1'
                    else:
                        mask,bit = sp[0],sp[1]
                    if bit=='1': nd.patterns_one.append(mask)
                    else: nd.patterns_zero.append(mask)
            idx = j
        elif l.startswith('.end'):
            break
        else:
            idx += 1
    for nd in nodes.values():
        nd.fanins = [nodes[nm] for nm in nd.fanin_names if nm in nodes]
    for nd in nodes.values():
        for f in nd.fanins:
            f.fanouts.append(nd)
    reachable = set()
    st = [nm for nm in outputs if nm in nodes]
    while st:
        u = st.pop()
        if u in reachable: continue
        reachable.add(u)
        for p in nodes[u].fanin_names:
            if p in nodes and p not in reachable:
                st.append(p)
    nodes = {nm:nd for nm,nd in nodes.items() if nm in reachable}
    for nd in nodes.values():
        nd.fanins = [f for f in nd.fanins if f.name in nodes]
        nd.fanouts = [f for f in nd.fanouts if f.name in nodes]
    indeg = {nm: len(nd.fanins) for nm,nd in nodes.items()}
    dq = deque([nm for nm,d in indeg.items() if d==0])
    topo = []
    while dq:
        u = dq.popleft(); topo.append(u)
        for w in nodes[u].fanouts:
            indeg[w.name] -= 1
            if indeg[w.name]==0: dq.append(w.name)
    topo_nodes = [nodes[nm] for nm in topo]
    N = len(topo_nodes)
    name_to_idx = {nd.name:i for i,nd in enumerate(topo_nodes)}
    idx_to_name = [nd.name for nd in topo_nodes]
    PI = set(name_to_idx[nm] for nm in inputs if nm in name_to_idx)
    cost = [0]*N
    bestM = [None]*N
    K = 6
    M = 30
    cuts = [[] for _ in range(N)]
    for i,nd in enumerate(topo_nodes):
        if i in PI or nd.const_val is not None:
            cost[i] = 0
            bestM[i] = None
            cuts[i] = [(1<<i, 0)]
            continue
        fans = [name_to_idx[nm] for nm in nd.fanin_names if nm in name_to_idx]
        c_list = None
        for f in fans:
            fcuts = cuts[f]
            if len(fcuts) > M: fcuts = fcuts[:M]
            if c_list is None:
                c_list = fcuts.copy()
            else:
                newm = {}
                for m1,sc1 in c_list:
                    for m2,sc2 in fcuts:
                        m = m1 | m2
                        if m.bit_count() <= K:
                            if m in newm: continue
                            t = m; sc = 0
                            while t:
                                lsb = t & -t; j = lsb.bit_length()-1
                                sc += cost[j]; t ^= lsb
                            newm[m] = sc
                if not newm:
                    c_list = []
                    break
                items = sorted(newm.items(), key=lambda x: x[1])
                c_list = items[:M]
        if c_list is None: c_list = []
        fb = 0
        scf = 0
        for f in fans:
```

```
            fb |= (1<<f)
            scf += cost[f]
        if fb.bit_count() <= K:
            if not any(m==fb for m,_ in c_list):
                c_list.append((fb, scf))
        bestc = 10**18; bm = None
        for m,sc in c_list:
            v = sc + 1
            if v < bestc:
                bestc = v; bm = m
        if bm is None:
            bm = fb; bestc = scf + 1
        cost[i] = bestc; bestM[i] = bm
        cuts[i] = sorted(c_list, key=lambda x: x[1])[:M]
    mapping = set()
    st = [name_to_idx[nm] for nm in outputs if nm in name_to_idx]
    vis = set()
    while st:
        u = st.pop()
        if u in vis: continue
        vis.add(u)
        bm = bestM[u]
        if bm is None: continue
        mapping.add(u)
        t = bm
        while t:
            lsb = t & -t; j = lsb.bit_length()-1; t ^= lsb
            if j in PI or topo_nodes[j].const_val is not None: continue
            if j not in vis: st.append(j)
    mapped = sorted(mapping)
    with open(solution_file, 'w') as f:
        f.write('.model ' + model + '\n')
        f.write('.inputs ' + ' '.join(inputs) + '\n')
        f.write('.outputs ' + ' '.join(outputs) + '\n')
        for i in mapped:
            nd = topo_nodes[i]
            bm = bestM[i]
            leaves = [j for j in range(N) if (bm>>j)&1]
            inputs_l = [idx_to_name[j] for j in leaves]
            k = len(inputs_l)
            Np = 1<<k
            full = (1<<Np) - 1
            masks = []
            for t in range(k):
                mm = 0
                for j in range(Np):
                    if (j>>t)&1: mm |= 1<<j
                masks.append(mm)
            val = {}
            for t,j in enumerate(leaves):
                val[j] = masks[t]
            vis2 = set(leaves)
            stk = [i]
            while stk:
                u = stk.pop()
                if u in vis2: continue
                vis2.add(u)
                for p in topo_nodes[u].fanins:
                    pj = name_to_idx.get(p.name)
                    if pj is not None and pj not in vis2:
                        stk.append(pj)
            cone = [j for j in topo if False]
            cone = [j for j in range(N) if j in vis2 and j not in leaves]
            for j in sorted(cone):
                v = topo_nodes[j]
                if v.const_val is not None:
                    val[j] = full if v.const_val==1 else 0
                else:
                    if v.patterns_zero:
                        zm = 0
                        for p in v.patterns_zero:
                            mm = full
                            for t,ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0; break
                                vm = val[pj]
                                if ch=='1': mm &= vm
                                elif ch=='0': mm &= (~vm)&full
                            zm |= mm
                        val[j] = (~zm)&full
                    else:
```

```
                        om = 0
                        for p in v.patterns_one:
                            mm = full
                            for t,ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0; break
                                vm = val[pj]
                                if ch=='1': mm &= vm
                                elif ch=='0': mm &= (~vm)&full
                            om |= mm
                        val[j] = om
            root = val[i]
            f.write('.names ' + ' '.join(inputs_l) + ' ' + nd.name + '\n')
            for j in range(Np):
                if (root>>j)&1:
                    pat = ''.join('1' if (j>>t)&1 else '0' for t in range(k))
                    f.write(pat + ' 1\n')
        f.write('.end\n')

```

In the fifth iteration, it introduces a dynamic pruning method by adapting the cut limit based on circuit size—more cuts for smaller circuits (higher quality), and fewer for large circuits (better yield).

```
# iteration 5: dynamic pruning of cut enumeration space

def solve(input_file: str, solution_file: str):
    from collections import deque
    raw = open(input_file).read().splitlines()
    lines = []
    i = 0
    while i < len(raw):
        l = raw[i].rstrip()
        if not l or l.lstrip().startswith('#'):
            i += 1
            continue
        if l.endswith('\\'):
            acc = l[:-1].rstrip()
            i += 1
            while i < len(raw):
                nl = raw[i].rstrip()
                if nl.endswith('\\'):
                    acc += ' ' + nl[:-1].rstrip()
                    i += 1
                else:
                    acc += ' ' + nl
                    i += 1
                    break
            lines.append(acc)
        else:
            lines.append(l)
            i += 1
    class Node:
        __slots__ = ('name','fanin_names','patterns_one','patterns_zero','const_val','fanins','fanouts')
        def __init__(self,n):
            self.name = n
            self.fanin_names = []
            self.patterns_one = []
            self.patterns_zero = []
            self.const_val = None
            self.fanins = []
            self.fanouts = []
    model = ''
    inputs = []
    outputs = []
    nodes = {}
    idx = 0
    while idx < len(lines):
        l = lines[idx].strip()
        if not l:
            idx += 1; continue
        if l.startswith('.model'):
            parts = l.split()
            if len(parts) > 1: model = parts[1]
            idx += 1
        elif l.startswith('.inputs'):
            parts = l.split()
            for nm in parts[1:]:
                inputs.append(nm)
                if nm not in nodes: nodes[nm] = Node(nm)
```

```
            idx += 1
        elif l.startswith('.outputs'):
            parts = l.split()
            for nm in parts[1:]:
                outputs.append(nm)
            idx += 1
        elif l.startswith('.names'):
            parts = l.split()
            inps = parts[1:-1]; outp = parts[-1]
            if outp not in nodes: nodes[outp] = Node(outp)
            nd = nodes[outp]
            nd.fanin_names = inps[:]
            pats = []
            j = idx + 1
            while j < len(lines) and not lines[j].startswith('.'):
                s = lines[j].strip()
                if s: pats.append(s)
                j += 1
            if not inps:
                nd.const_val = 1 if pats and pats[0] == '1' else 0
            else:
                for pt in pats:
                    sp = pt.split()
                    if len(sp) == 1:
                        mask, bit = sp[0], '1'
                    else:
                        mask, bit = sp[0], sp[1]
                    if bit == '1': nd.patterns_one.append(mask)
                    else: nd.patterns_zero.append(mask)
            idx = j
        elif l.startswith('.end'):
            break
        else:
            idx += 1
    for nd in nodes.values():
        nd.fanins = [nodes[nm] for nm in nd.fanin_names if nm in nodes]
    for nd in nodes.values():
        for f in nd.fanins:
            f.fanouts.append(nd)
    reachable = set()
    st = [nm for nm in outputs if nm in nodes]
    while st:
        u = st.pop()
        if u in reachable: continue
        reachable.add(u)
        for p in nodes[u].fanin_names:
            if p in nodes and p not in reachable:
                st.append(p)
    nodes = {nm:nd for nm,nd in nodes.items() if nm in reachable}
    for nd in nodes.values():
        nd.fanins = [f for f in nd.fanins if f.name in nodes]
        nd.fanouts = [f for f in nd.fanouts if f.name in nodes]
    indeg = {nm: len(nd.fanins) for nm,nd in nodes.items()}
    dq = deque([nm for nm,d in indeg.items() if d == 0])
    topo = []
    while dq:
        u = dq.popleft(); topo.append(u)
        for w in nodes[u].fanouts:
            indeg[w.name] -= 1
            if indeg[w.name] == 0: dq.append(w.name)
    topo_nodes = [nodes[nm] for nm in topo]
    N = len(topo_nodes)
    name_to_idx = {nd.name: i for i, nd in enumerate(topo_nodes)}
    idx_to_name = [nd.name for nd in topo_nodes]
    PI = set(name_to_idx[nm] for nm in inputs if nm in name_to_idx)
    cost = [0] * N
    bestM = [None] * N
    K = 6
    if N <= 1500:
        M = 64
    elif N <= 3000:
        M = 48
    elif N <= 5000:
        M = 32
    else:
        M = 20
    cuts = [[] for _ in range(N)]
    def prune_cuts(items, limit):
        items_sorted = sorted(items, key=lambda x: (x[1], x[0].bit_count()))
        pr = []
        for m, sc in items_sorted:
            dom = False
            for pm, psc in pr:
```

```
                if psc <= sc and (pm & m) == pm:
                    dom = True
                    break
            if not dom:
                pr.append((m, sc))
                if len(pr) >= limit:
                    break
        return pr
    for i, nd in enumerate(topo_nodes):
        if i in PI or nd.const_val is not None:
            cost[i] = 0
            bestM[i] = None
            cuts[i] = [(1 << i, 0)]
            continue
        fans = [name_to_idx[nm] for nm in nd.fanin_names if nm in name_to_idx]
        fans.sort(key=lambda x: len(cuts[x]))
        c_list = None
        for f in fans:
            fcuts = cuts[f]
            if not fcuts:
                c_list = []
                break
            fcuts = fcuts[:M]
            if c_list is None:
                c_list = fcuts.copy()
            else:
                newm = {}
                for m1, sc1 in c_list:
                    for m2, sc2 in fcuts:
                        m = m1 | m2
                        if m.bit_count() <= K:
                            s2 = sc1 + sc2
                            prev = newm.get(m)
                            if prev is None or s2 < prev:
                                newm[m] = s2
                if not newm:
                    c_list = []
                    break
                c_list = prune_cuts(list(newm.items()), M)
        if not c_list:
            um = 0; usc = 0
            for f in fans:
                um |= (1 << f)
                usc += cost[f]
            c_list = [(um, usc)]
        um = 0; usc = 0
        for f in fans:
            um |= (1 << f)
            usc += cost[f]
        if um.bit_count() <= K and all(m != um for m, _ in c_list):
            c_list.append((um, usc))
        c_list = prune_cuts(c_list, M)
        bestc = 10**18; bm = None
        selfm = (1 << i)
        for m, sc in c_list:
            if m == selfm:
                continue
            v = sc + 1
            if v < bestc:
                bestc = v; bm = m
        if bm is None:
            bm = um
            bestc = usc + 1
        cost[i] = bestc
        bestM[i] = bm
        cuts[i] = c_list
    mapping = set()
    st = [name_to_idx[nm] for nm in outputs if nm in name_to_idx]
    vis = set()
    while st:
        u = st.pop()
        if u in vis: continue
        vis.add(u)
        bm = bestM[u]
        if bm is None: continue
        mapping.add(u)
        t = bm
        while t:
            lsb = t & -t
            j = lsb.bit_length() - 1
            t ^= lsb
            if j in PI or topo_nodes[j].const_val is not None: continue
            if j not in vis:
```

```
                st.append(j)
    mapped = sorted(mapping)
    with open(solution_file, 'w') as f:
        f.write('.model ' + model + '\n')
        f.write('.inputs ' + ' '.join(inputs) + '\n')
        f.write('.outputs ' + ' '.join(outputs) + '\n')
        for i in mapped:
            nd = topo_nodes[i]
            bm = bestM[i]
            leaves = [j for j in range(N) if (bm >> j) & 1]
            inputs_l = [idx_to_name[j] for j in leaves]
            k = len(inputs_l)
            Np = 1 << k
            full = (1 << Np) - 1
            masks = []
            for t in range(k):
                mm = 0
                for j in range(Np):
                    if (j >> t) & 1:
                        mm |= 1 << j
                masks.append(mm)
            val = {}
            for t, j in enumerate(leaves):
                val[j] = masks[t]
            seen = set(leaves)
            stk = [i]
            cone = []
            while stk:
                u = stk.pop()
                if u in seen: continue
                seen.add(u)
                cone.append(u)
                for p in topo_nodes[u].fanins:
                    pj = name_to_idx.get(p.name)
                    if pj is not None and pj not in seen:
                        stk.append(pj)
            cone.sort()
            for j in cone:
                v = topo_nodes[j]
                if v.const_val is not None:
                    val[j] = full if v.const_val == 1 else 0
                else:
                    if v.patterns_zero:
                        zm = 0
                        for p in v.patterns_zero:
                            mm = full
                            for t, ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0
                                    break
                                vm = val[pj]
                                if ch == '1':
                                    mm &= vm
                                elif ch == '0':
                                    mm &= (~vm) & full
                            zm |= mm
                        val[j] = (~zm) & full
                    else:
                        om = 0
                        for p in v.patterns_one:
                            mm = full
                            for t, ch in enumerate(p):
                                fn = v.fanin_names[t]
                                pj = name_to_idx.get(fn)
                                if pj is None:
                                    mm = 0
                                    break
                                vm = val[pj]
                                if ch == '1':
                                    mm &= vm
                                elif ch == '0':
                                    mm &= (~vm) & full
                            om |= mm
                        val[j] = om
            root = val[i]
            f.write('.names ' + ' '.join(inputs_l) + ' ' + nd.name + '\n')
            for j in range(Np):
                if (root >> j) & 1:
                    pat = ''.join('1' if (j >> t) & 1 else '0' for t in range(k))
                    f.write(pat + ' 1\n')
        f.write('.end\n')
```

H Datasets

We summarize the original data sources for each problem in Table 26 and the number of data instances in Table 27. All datasets are derived from real-world applications. We further partition or transform them into standardized input formats, ensuring the inclusion of both small-scale instances for demonstration purposes and large-scale instances for evaluation. For detailed data organization, please refer to our repository.

Table 26: Datasets used in our benchmark.

Problem	**Original Data Source**
Operator scheduling	EXPRESS [144]
Technology mapping	EPFL [7] and ISCAS85 [53]
Global routing	ISPD'24 Contest [79]
E-graph extraction	SmoothE [15]
Intra-op parallelism	ASPLOS'24 Contest [94]
Protein sequence design	Protein Data Bank (PDB) [36]
Mendelian error detection	Cost Function Library [119, 123]
Airline crew pairing	China Graduate Mathematical Modeling Competition'21 F [28]
Pickup and delivery w/ time windows	MetaPDPTW [76]

Table 27: Number of instances of each problem in HeuriGym.

Problem	**# of Instances**
Operator scheduling	24
Technology mapping	31
Global routing	24
E-graph extraction	23
Intra-op parallelism	28
Protein sequence design	24
Mendelian error detection	20
Airline crew pairing	14
Pickup and delivery w/ time windows	30
Total	218