Entanglement-free Motion Planning for Multiple Tethered Robots

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I. Motivation

Tethered robots maintain a connection to fixed or mobile anchors via tether cables [22], which can provide uninterrupted power, robust communication, or facilitate object transport. These capabilities make tethered robots well-suited for operations in environments with limited wireless coverage or where prolonged missions are required. The operation of collaborative multiple-tethered robots has promised increased efficiency over a single-tethered robot, with successful demonstrations in fruit collection [5] and sea trash removal tasks [17]. The potential applications for multiple tethered robots continue to expand, including search and exploration [18, 14], object gathering and removal [20, 1], and item transportation [9]. However, planning for multiple tethered robots presents unique challenges, particularly due to entanglement—the intertwining of tether cables, which restricts robot motion, compromises task success and incurs safety risks. Despite increasing interest, existing solutions typically rely on simplified models or problem constraints that limit their applicability.

Entanglement-free path planning is critical for multi-robot tethered systems, yet remains underexplored. While efficient algorithms have been developed for single tethered robots to navigate planar or 3D environments [21, 24, 8], planning for multiple robots introduces complex interactions and cable dynamics that are difficult to model and predict. Prior works have attempted various approaches, such as: (1) Restricting robots to planar workspaces with straight, non-crossing cables [19, 13]; (2) Allowing limited cable interactions through contact-based pushing [25]; (3) Developing hardware-based tension and angle measurement systems for entanglement detection [16]; (4) Assuming fully stretched cables to avoid bending or slack-induced complications [6, 12]. These assumptions often oversimplify real-world scenarios, where tether cables naturally possess slackness to reduce tension on the robots or allow sliding along surfaces. Consequently, there is no established solution for realistic slack cable models in collaborative tethered robot systems. A deeper understanding of entanglement, particularly in dynamic and obstacle-laden environments, is essential.

My research aims to address this challenge by *developing a framework that enables entanglement-free motion planning for multi-robot tethered systems*. This involves answering three key questions:

- How can we achieve a rigorous *mathematical characterization of entanglement* in multi-robot scenarios?
- What are the *conditions for both the occurrence and avoidance of entanglement*?
- How can we plan *computationally and dynamically feasible trajectories* that avoid entanglement while achieving mission objectives?

By leveraging concepts from topology and motion planning, my work focuses on:

- Topological representations of cable states to understand entanglement patterns;
- Integration of these representations into planning algorithms, including graph search and trajectory optimization for efficient motion planning.

II. Current Works

In my first attempt to address the problem [3], I explored the use of homotopy representations to characterize the topological relationship between robot paths and tether cables. Homotopy-based approaches have proven effective for a single tethered robot in static, 2D environments, where path similarity can be represented using tools such as the h-signature [8, 11]. However, extending these methods to multi-robot systems presents challenges due to the complex interactions between multiple cables and obstacles. To overcome these challenges, I introduced a *multi-robot tether-aware homotopy representation*, which encodes the crossings among cables and obstacles into a structured, string-based format. This representation allows each robot to estimate the risk of entanglement by tracking effective crossings with both static objects and other moving robots. Unlike prior approaches, which assume fully stretched or simplified cable models, our method captures the slackness and flexibility of real-world cables, enabling more accurate risk assessments. With this representation, I designed a *two-stage trajectory planning framework*. In the first stage, the planner searches for a feasible, non-entangling path in a homotopy-augmented graph. This path satisfies goal-reaching requirements while avoiding undesirable cable interactions. In the second stage, a convex trajectory optimization problem refines the path to minimize control effort, ensuring smooth and efficient robot motion while maintaining entanglement-free and collision-free constraints. The planning framework operates in a decentralized manner, allowing each robot to generate its trajectory independently and iteratively, which enhances scalability and real-time performance. Simulations involving two to eight robots demonstrate that the proposed planner significantly outperforms existing methods, achieving higher goal-reaching rates in both obstacle-free and obstacle-ridden environments. These results validate that combining topology-based representations with trajectory optimization is a promising approach to address the challenges of multi-tethered robot planning. However, this approach has limitations: the homotopy representation cannot capture all possible entanglements, particularly in highly dynamic scenarios with intricate multi-robot interactions. Consequently, complete prevention of entanglement remains an open problem.

To address this, my second line of research [2] focuses on developing a more general *topological representation of entanglement using braid theory*, which is a branch of low-dimensional topology. Braids provide a dual algebraic and geometric framework for analyzing spatiotemporal patterns in multi-robot systems [15, 10]. By abstracting robot trajectories as strands in a topological braid, I identified the key interaction patterns that lead to entanglement. A critical finding of this study is that all complex entanglements can be decomposed into a finite set of basic interactions between pairs or triplets of robots. This insight forms the basis for my design of a topology-based planning algorithm that searches for paths in an abstract space defined by these braid patterns. The algorithm avoids paths that correspond to known entangling patterns, ensuring that the output is a guaranteed entanglement-free topological plan for all robots.

This second work provides a rigorous topological framework for understanding and preventing entanglement. However, the high-level plans generated by this approach are purely topological and do not account for robot dynamics, collision avoidance, or control constraints. As such, they are not directly executable by real robots. In my third study [4], I bridge the gap between topological planning and practical execution by developing a *topology-guided planning framework*. This framework builds on the success of decentralized planning from my first work [3] while incorporating braid-based constraints to enforce adherence to the topological plan. First, the topological graph search generates a plan that avoids entangling braid patterns; then, trajectory optimization produces dynamically feasible, collision-free motion plans that conform to the chosen topological structure. This approach allows each robot to independently generate its trajectory while collectively ensuring that the entire team avoids entanglement. Simulations with four to ten robots show that this method achieves complete task success in scenarios where existing approaches fail, highlighting the scalability and robustness of the framework. Furthermore, I validated the approach through flight experiments with three UAVs, demonstrating that the planner is practical for real-world tethered systems. To our knowledge, this is the first comprehensive solution that achieves entanglement-free planning for multiple tethered robots with realistic slack cable models.

III. Future Work

My research has demonstrated the effectiveness of a multidisciplinary approach that integrates topological representations with motion planning to address the problem of multi-tethered robot navigation. I plan to move forward in two key directions: (1) Enabling multi-tethered robots to perform autonomous operations in complex real-world environments, and (2) Expanding the use of topology-inspired algorithms to broader contexts and applications beyond tethered robots.

Real-world multi-tethered robot autonomy in complex environments. Our work has enabled theoretically guaranteed entanglement-free motion for multiple tethered robots in structured 3D environments with relatively simple obstacle geometries. These environments, which resemble warehouses or other indoor industrial spaces, enable tethered robots to function as inventory monitoring agents or mobile transportation units. However, real-world unstructured environments, such as forests or underwater regions, present irregular obstacles that challenge our existing assumptions. Developing robust planning methods capable of handling such environments is a critical next step. One key question is how to dynamically identify and leverage critical topological features of such complex surroundings within a planning framework that is both topology-aware and geometry-sensitive. Unlike in structured environments with predictable obstacle configurations, real-world scenarios require flexible and adaptive representations to handle unknown and evolving conditions. Given the recent advancements in learning-based techniques, I plan to investigate the integration of data-driven methods for enhanced topological and geometric classification of obstacles.

Topology-inspired algorithms for better task assignment and motion planning One of my research's key insights is that *topology is an effective abstraction* for solving complex multi-robot motion planning problems. The ability to represent entanglement through topological features has not only guided motion planning but also revealed broader potential applications for topology-inspired algorithms. Beyond tethered robots, many robotic planning tasks can benefit from topological representations that abstract unnecessary geometric details while preserving essential spatial relationships. For instance, humans often rely on high-level spatial information—such as the relative locations of landmarks—rather than precise geometric coordinates when navigating through unfamiliar environments [7]. Robots, similarly, can use topological task representations to capture the order and dependencies of sub-tasks or the spatial relationships between agents and obstacles [23]. This level of abstraction simplifies planning in high-dimensional spaces by shrinking the solution space, enabling fast computation of high-level plans while providing low-level motion planners with flexible constraints to optimize trajectories. Selecting an appropriate topological representation could potentially offer new perspectives to existing complex high-dimensional planning problems, such as multi-robot search and rescue and multi-joint robot manipulator planning, and enhance computational efficiency and solution quality.

Ultimately, these future directions aim to generalize the benefits of topological thinking to a wide range of robotic applications, pushing the boundaries of autonomous multi-robot systems in complex, real-world settings.

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