Disparate effects of resonances in delay-based reservoir computers for chaotic attractor reconstruction

Keywords: time-multiplexed networks, reservoir computing, network topology, resonance effects, attractor reconstruction

Extended Abstract

Recurrent neural networks are well known for their suitability as high dimensional dynamical systems in the framework of reservoir computing (RC)[1]. The coupling-topology of the network has a strong impact on the nonlinear mixing of injected information and, consequently, on the performance of the network as a reservoir computer. Using the analogy between spatially extended systems and systems with time delay, time-multiplexed RC has become a promising concept that allows the use of dynamical systems as processing units for RC[2, 3]. In the delay-based time-multiplexed RC approach, analogous to the coupling-topology of networks, different internal coupling topologies can be realized by tuning the timescales of the delay τ or the data injection rate (inverse of the clock cycle T) [4]. In this paper we investigate the influence of the coupling topology of delay-based RC on chaotic time-series prediction tasks. Particularly, we demonstrate the differences in the performance arising when operating the trained reservoir open- and closed-loop (see Fig.1a). In the open-loop setup the reservoir is always fed the true timeseries in order to make predictions, whereas, in the closed-loop operation the reservoir output is fed in as the next input, thereby acting as a surrogate of the dynamical system which evolves autonomously after training.

In the limit where the dynamical system (reservoir) reacts much faster than the sampling time, the topology of delay-based time-multiplexed RCs is determined by the ratio between the delay (τ) and the input clock cycle (T). Equal values of τ and T (1:1 resonance) lead to a network topology of uncoupled nodes with delayed self-coupling, i.e. without inter-node coupling. Whereas, asynchronous settings for τ and T leads to delay-induced effective next-neighbor coupling, similar to a unidirectional ring (see Fig.1b). For open-loop chaotic timeseries prediction tasks, it is well known that τ-T resonances can have a detrimental effect on the performance [4, 5]. An example of the performance reduction at τ-T resonances can be seen in Fig.1c, where a 2D scan of a 2-step open-loop Mackey-Glass prediction task, performed with a semiconductor laser subjected to delayed self-feedback, is shown (prediction performance is plotted as color code with bright lines of bad performance appearing along resonances between T and τ).

For spatially multiplexed RC setups it was shown that an autonomous RC system operated in a closed loop configuration is more likely to produce a stable reconstruction of a chaotic timeseries if inter-node couplings are absent, i.e. only self-coupling [6]. Here we report that a similar effect can be observed in delay-based time-multiplexed RC. The impact of resonances, and therefore the impact of coupling topology, is disparate in delay-based RC as it depends on the operation mode after training. Figure 2 shows the performance of a reservoir build from a delayed-map both in open (a) and closed loop (b) operation as a function of the internal feedback delay time τ. The scans indicate that delay-based time-multiplexed RC shows bad performance at τ-T resonances if operated open-loop, however, improved performance is evident at the resonances in autonomous closed-loop operation. While the impact of resonances in the open-loop case can be traced back to the memory properties of the RC setup [4, 5], the open-loop case remains challenging and will be discussed in this contribution.

References

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Figures

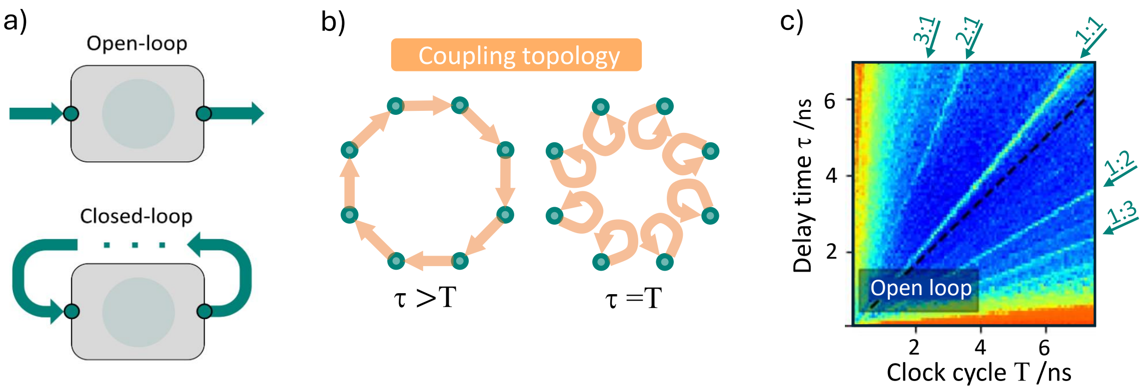

Figure 1: **Sketch** of a) open and closed loop RC configurations, b) Delay-induced coupling topology in time-multiplexed RC, c) Computing performance of a semiconductor laser with feedback for Mackey Glas open-loop 2-step prediction as a function of delay τ and clock cycle T (arrows point at resonances and yellow/red (blue) colors indicate bad(good) performance).

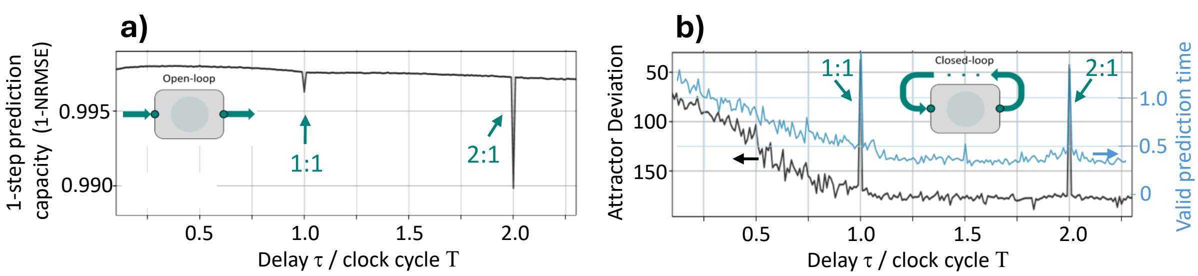

Figure 2: **Line scans** for increasing delay time τ. (a) Prediction capacity of a delayed map for open loop 1-step ahead prediction (Lorenz xyz prediction is bad at resonances), (b) closed-loop operation of the same RC showing valid prediction time (blue) and attractor deviation (black), which are best at resonances.