Context-Aware Query Rewriting for Improving Users' Search Experience on E-commerce Websites

Anonymous ACL submission

Abstract

E-commerce queries are often short and ambiguous. E-commerce query understanding often uses query rewriting to disambiguate user-input queries. While using e-commerce search tools, users tend to enter multiple searches, which we call context, before purchasing. These history searches contain contextual insights about users' true shopping intents. Therefore, modeling such contextual information is critical to a better query rewriting model. However, existing query rewriting models ignore users' history behaviors and consider only the instant search query, which is often a short string offering limited information about the true shopping intent.

We propose an end-to-end context-aware query rewriting model to bridge this gap, which takes the search context into account. Specifically, our model builds a session graph using the history search queries, their contained words, and auxiliary category information. We then employ a weighted graph attention mechanism that models cross-query relations and computes contextual information of the session. The model subsequently calculates session representations by combining the contextual information with the instant search query using an aggregation network. The session representations are then decoded to generate rewritten queries. Empirically, we demonstrate the superiority of our method to state-of-the-art approaches under various evaluation metrics. Our code and data will be publicly available.

1 Introduction

Query rewriting is a task where a user inputs a potentially problematic query (e.g., typos or insufficient information), and we rewrite it to a new one that better matches the user's real shopping intent. This task plays an important role in e-commerce query understanding, where without proper rewriting, search engines often return undesired items, rendering the search experience unsatisfactory.

One major issue that impedes query rewriting is the ambiguity of queries. For example, Figure 1 (left) demonstrates searching for "bumblebee costumes" without considering search context. From the query alone, it is implausible to tell if the user's intent is for costumes of actual bumblebee, i.e., the animal, or the character from the movie franchise. This type of ambiguity is common in e-commerce search, where queries are usually short (only 2-3 terms) and insufficiently informative (He et al., 2016b). Therefore, it is not possible to disambiguate queries using only the instant search. A common solution is to use statistical rules to differentiate the possible choices. Specifically, in our example, suppose a total of 100 users entered the "bumblebee costumes" query, and 70 of them eventually purchased the movie character costume. When a new user searches for the same query, the recommended products will consist of 70% movie character costumes and 30% animal costumes. This procedure is problematic because each user has a specific intent, i.e., either the movie character costume or the animal costume, but rarely both, which the aforementioned method fails to address.

We propose to explore contextual information from users' history searches to resolve the query ambiguity issue. Taking the "bumblebee costumes" example again, in Figure 1 (right), suppose a rewriting model recognizes that the user searched for "Transformers movie" earlier, then it could infer that the user's purchase intent is the movie character costume, and hence can remove the input ambiguity. There have been existing works that utilize search logs for query rewriting. For example, Wang and Zhai (2007, 2008) use traditional TF-IDF-based similarity metrics to capture relational information among the user's history searches. These approaches are too restrictive to handle the increasingly complex corpus nowadays. As such, the rewritten queries significantly differ from the original one in intent. More recently, neu-

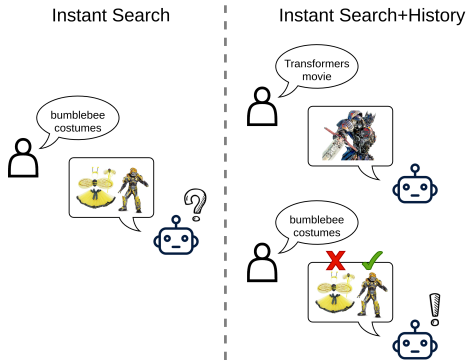


Figure 1: Searching for "bumblebee costumes" with (right) and without (left) history searches.

ral network-based query rewriting algorithms (He et al., 2016b; Xiao et al., 2019; Yang et al., 2019) are proposed. Most of such approaches employ a multi-stage training approach. Consequently, they involve complicated hand-crafted features or require excessive human annotations for the intermediate features (sometimes both).

To overcome the drawbacks of existing methods, we propose an end-to-end context-aware query rewriting algorithm. Our model's backbone is the Transformer (Vaswani et al., 2017). It is a sequence-to-sequence encoder-decoder model that exploits recent advances of the self-attention mechanism (Bahdanau et al., 2015). In our context-aware model, the Transformer encoder learns representations for individual history queries. The representations are further transformed to carry cross-query relational information using a weighted graph attention mechanism (wGAT (Velickovic et al., 2018)). Such a mechanism computes contextual information of a session based on a session graph, where its nodes contain the history queries, the tokens contained in the history queries, and the history queries' category information (see Section 3). After obtaining the contextual information from the wGAT, it is aggregated with the instant search using an aggregation network. The augmented information is subsequently fed into the Transformer decoder to generate rewritten queries. Previous works (Tu et al., 2019; Wang et al., 2020) that share the same spirit have shown to be effective in various natural language processing tasks.

We highlight that our proposed session graph formulation and the wGAT mechanism explicitly models cross-query relations, which is different from existing works. Previous approaches (e.g., (Dehghani et al., 2017)) capture such relations recursively, which is sub-optimal because such a structure suffers from the "forgetting" issue (Hochreiter and Schmidhuber, 1997), i.e., relation between queries far away will be lost. In contrast, wGAT associates any two queries by their contained words, enabling relation-modeling regardless of distance. Moreover, the proposed wGAT method takes category information into account, a component missing in prior works.

Our proposed method improves upon existing works from three aspects. First, our model does not involve recursion, unlike conventional recurrent neural network-based approaches (He et al., 2016b; Yang et al., 2019; Xiao et al., 2019). Our proposed attention-based method can be trained in full parallel and avoids gradient explosion and gradient vanishing problems (Pascanu et al., 2013), from which existing models suffer. These advantages facilitate training deep models containing dozens of layers capable of capturing high-order information. Second, our end-to-end sequence-to-sequence learning formulation eliminates the necessity of excessive labeled data. Previous approaches (Yang et al., 2019; Xiao et al., 2019) require the judgment of "semantic similarity", and thus crave for human annotations, which are expensive to obtain. In contrast, our method uses search logs as supervision, which does not involve human effort, and are cheap to acquire. Third, our method can leverage powerful pre-trained language models, such as BART (Lewis et al., 2020). Such models contain rich semantic information and are successful in numerous natural language processing tasks (Devlin et al., 2019; Liu et al., 2019; Radford et al., 2019).

We demonstrate the efficacy of our method on in-house data from an online shopping platform. Our context-aware query rewriting model outperforms various baselines by large margins. Notably, comparing with the best baseline method (Transformer-based model), our model achieves 22.5% relative improvement under the MRR (Mean Reciprocal Rank) metric and 11.7% relative improvement under the HIT@16 metric (a hit rate metric).

2 Related Works

◇ **Context-based query rewriting** One line of work uses statistical methods. For example, Cui et al. (2002, 2003) extract probabilistic correlations between the search queries and the product descriptions. Other works extract features that are related to the user's current search (Huang et al., 2003; Huang and Efthimiadis, 2009), or

from relational information among the user's history searches (Billerbeck et al., 2003; Baeza-Yates and Tiberi, 2007; Wang and Zhai, 2007; Cao et al., 2008; Wang and Zhai, 2008). There are also statistical machine translation-based models (Riezler et al., 2007; Riezler and Liu, 2010) that employ sequence-to-sequence approaches. The aforementioned statistical methods suffer from unreliable extracted features, such that the rewritten queries differ from the original one in intent.

Another line of work focuses on neural query rewriting models (He et al., 2016b; Xiao et al., 2019; Yang et al., 2019). These models adopt recurrent neural networks (RNNs, Hochreiter and Schmidhuber 1997; Sutskever et al. 2014) to learn a vectorized representation for the user's search query, after which KNN-based methods are used to find queries that yield similar representations. One major limitation is that the rewritten queries are limited to the previously presented ones. Also, these methods often involve complicated and ungrounded feature function designs, e.g., He et al. (2016b) and Xiao et al. (2019) hand-crafted 18 feature functions, or require excessive labeled data (Yang et al., 2019). There are other works (Sordoni et al., 2015; Dehghani et al., 2017; Jiang and Wang, 2018) that use RNNs for generative query suggestion, but they inherit the weaknesses of RNNs and yield unsatisfactory performance in practice.

Note that Grbovic et al. (2015) construct context-aware query embeddings using word2vec (Mikolov et al., 2013). In their approach, an embedding is learned for each distinct query in the dataset. As such, the quality of the learned embeddings rely heavily on the number of occurrences of each query. This method is not applicable to our case because in our dataset, almost all the queries are distinct.

3 Problem Setup

◇ **Category information of queries** Each search query results in multiple recommended products, and each of these products belongs to multiple categories, e.g., the movie character costume in Figure 1 belongs to both the "entertainment" category and the "fashion" category. For each search query, the user may react to multiple returned products, e.g., click, add to cart, and purchase. If a user reacts to a specific product, we say the user takes an action on each category corresponding to that product, e.g., if a user clicks on the movie character costume, we say the user takes one action on category "entertainment" and one action on category "fashion". For a specific query, we collect user actions on all the recommended products, and we obtain

$$\{\text{Category}_1 : \#\ \text{actions}_1, \cdots\},$$

where Categories $1-N$ are pre-specified. Then the category information of the query is defined as

$$\left\{\mathbb{P}[C_1] = \frac{\#\ \text{actions}_1}{\sum_{i=1}^{N} \#\ \text{actions}_i}, \cdots\right\},$$

where C_i stands for "Category$_i$".

◇ **Session data** The session data are collected from search logs. First, we collect all the searches from a specific user within a time window, and we call the searches a "session". After the user purchases a product, the session ends, i.e., we do not consider subsequent queries and behaviors after a purchase happens. This is because, after a purchase, the user's intent often change. Note that different sessions may be collected from different users.

Each session contains multiple searches, where each consists of a search query and its category information. We call the last query in the session the "target" query, the second to the last query the "source" query, and the others the "history" queries. The intuition behind this is that because sessions always end with a purchase, the last search (i.e., the target) reflects the user's real intent. When the user enters the second to the last search (i.e., the source), if we can rewrite it to the target query, the user's intent will be fulfilled.

We collect about 3 million (M) sessions, where each session consists of at least 3 history queries, a source query (i.e., the one we need to rewrite), and a target query (i.e., the ground-truth query that is associated with the purchase). We have roughly 18.7M queries, and on average, each session contains 4 history queries. Query rewriting is consequently formulated as a sequence-to-sequence learning problem. We highlight that per our formulation, we do not need human annotations, unlike existing approaches. To open new research opportunities, our data is currently undergoing internal procedures for release.

4 Method

Figure 2 illustrates our context-aware query rewriting model. The model contains four parts: a

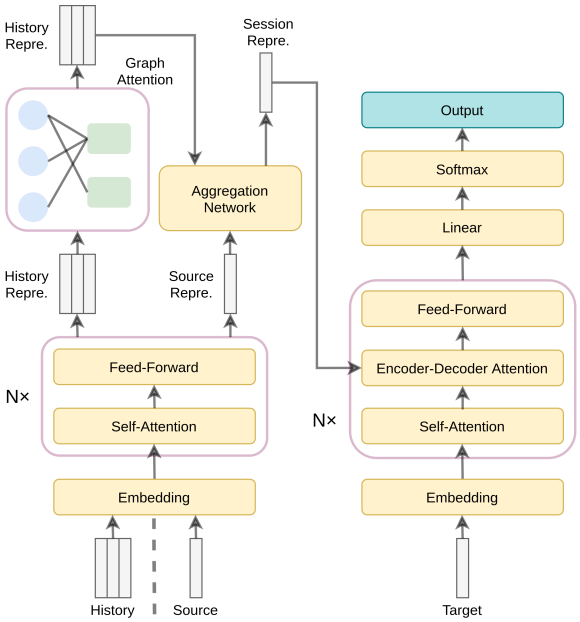


Figure 2: Overview of model.

conventional Transformer (Vaswani et al., 2017) encoder, a weighted graph attention mechanism (Velickovic et al., 2018) that captures the user's purchase intent, an aggregation network that encodes the history searches, and a conventional Transformer decoder that generates the rewritten query candidates.

4.1 Transformer Encoder

For a given source query, we first pad it with a ⟨boq⟩ (begin-of-query) token. Then, we pass the padded query through a token embedding layer and a position embedding layer, and we obtain $Y_s \in \mathbb{R}^{L_s \times d}$. Here L_s is the length of the padded source query, and d is the embedding dimension. We then pass Y_s through N layers of encoder blocks, where each of these blocks contains a self-attention mechanism and a position-wise feed-forward neural network, and then we obtain an encoded representation $H_s \in \mathbb{R}^{L_s \times d}$.

For the history queries in this session, we also pad them with ⟨boq⟩ tokens. Suppose that we have N_h padded history queries (recall a session contains multiple history queries), and their respective length is denoted by $L_h^1, \cdots, L_h^{N_h}$. We pad the history queries to the same length, and we obtain the history query matrix $X_h \in \mathbb{R}^{N_h \times L_h}$, where $L_h = \max\{L_h^1, \cdots, L_h^{N_h}\}$. Then, following the same procedures as encoding the source query, we pass X_h through the embedding layers and the encoder blocks, after which we obtain the history query representations $U_h \in \mathbb{R}^{N_h \times L_h \times d}$.

4.2 Contextual Information from Session Graphs

After we obtain the history query representations U_h, the next step is to refine them. Such refinement is necessary because the encoder considers the history queries separately, such that their interactions are not taken into account. However, since each search depends on its previous searches in the same session, modeling cross-query relations are imperative for determining the user's purchase intent. To this end, we use a weighted graph attention mechanism (Velickovic et al., 2018; Wang et al., 2020) to capture contextual information from U_h.

4.2.1 Session Graph Construction

First we specify how to build a graph for each session, which we call the session graph. Suppose we have a session that contains three history queries:

$$\begin{aligned} Q_1 : \big\{&\textbf{Search query} : T_1, T_3; \\ &\textbf{Category} : \mathbb{P}[C_1] = 1.0\big\}, \\ Q_2 : \big\{&\textbf{Search query} : T_1, T_2, T_3; \\ &\textbf{Category} : \mathbb{P}[C_1] = 0.6, \mathbb{P}[C_2] = 0.4\big\}, \\ Q_3 : \big\{&\textbf{Search query} : T_1, T_2, T_3, T_4, T_5; \\ &\textbf{Category} : \mathbb{P}[C_2] = 0.7, \mathbb{P}[C_3] = 0.3\big\}, \end{aligned} \tag{1}$$

where Q_1, Q_2, Q_3 are the three queries, $T_1, \cdots, T_5$ are the five tokens that appear in the three queries, and C_1, C_2, C_3 are the three categories to which the queries belong. Recall Section 3 for the problem setup and the definition of category information. Figure 3 illustrates the session graph. In this 3-partite graph, the blue circles are token nodes $(T_1, \cdots, T_5)$; the green rectangles are query nodes (Q_1, Q_2, Q_3); and the red diamonds are category nodes (C_1, C_2, C_3). In our example, the history query representations have size $U_h \in \mathbb{R}^{3 \times 6 \times d}$, that is, we have 3 queries, and the maximum query lengths is 6 (recall we prepend a ⟨boq⟩ token to each query).

4.2.2 Node Representations

The next step is to refine the node representations. Each of the nodes in the session graph has its own representation.

- The token representations are simply the corresponding representations of the tokens, extracted from the token embedding matrix.
- The query representations are the representations of the ⟨boq⟩ token in each padded history query, i.e., the representation of the Q_1 query node in Figure 3 is found by $U_h[0, 0, :] \in \mathbb{R}^d$. Note that this is akin

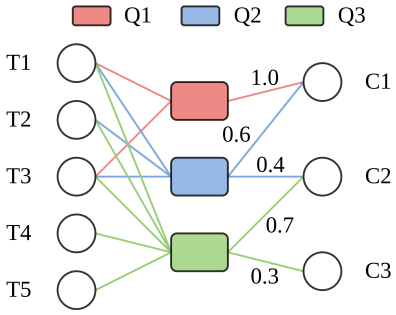


Figure 3: Session graph. Here "*T*" stands for tokens, "*Q*" stands for queries, and "*C*" stands for categories.

to BERT, where a <cls> token is inserted and its representation is used for classification tasks.

• The category representations are extracted from a category embedding matrix $E_C \in \mathbb{R}^{d\times|C|}$, where $|C|$ is the number of categories. Then the representation for category k is the k-th column of E_C.

Denote $\mathcal{G}_q = \{q_i\}_{i=1}^{N_q}$, $\mathcal{G}_t = \{t_i\}_{i=1}^{N_t}$, and $\mathcal{G}_c = \{c_i\}_{i=1}^{N_c}$ the sets of representations for the query, token, and category nodes, respectively. Here N_q is the number of query nodes, N_t is the number of token nodes, and N_c is the number of category nodes. Note that all the node representations have the same size, i.e., $q_i, t_i, c_i \in \mathbb{R}^d$.

4.2.3 Update Node Representations

We use a weighted multi-head graph attention mechanism to update the node representations. For simplicity, denote $N_g = N_q + N_t + N_c$ the number of distinct nodes in the session graph, and $\mathcal{G} = \mathcal{G}_q \cup \mathcal{G}_t \cup \mathcal{G}_c = \{g_i\}_{i=1}^{N_g}$ the set of all the node representations. We define e_{ij} as the edge weight between g_i and g_j, and it equals to the probability term in the category information (see (1)). We set $e_{ij} = 1$ if such weights are not defined, e.g., when updating the query representations $\mathcal{G}_q$ using the token representations $\mathcal{G}_t$.

With the above notations, a weighted single-head graph attention mechanism is defined as

$$\begin{aligned} z_{ij} &= \text{LeakyReLU}\left(W_a[W_q g_i; W_k g_j]\right) \cdot e_{ij}, \\ \alpha_{ij} &= \frac{\exp(z_{ij})}{\sum_{\ell\in\mathcal{N}_i}\exp(z_{i\ell})}, \\ h_i &= g_i + \text{ELU}\left(\textstyle\sum_{j\in\mathcal{N}_i}\alpha_{ij}W_v g_j\right). \end{aligned} \tag{2}$$

Here $\text{ELU}(x) = x\cdot 1\{x>0\} + (\exp(x)-1)\cdot 1\{x\le 0\}$ is the exponential linear unit, $\mathcal{N}_i$ denotes the neighbor of the i-th node, and W_a, W_q, W_k, W_v are trainable weights.

The edge weight e_{ij} essentially controls the "importance" of the category information to the query nodes. That is, if e_{ij} is small, i.e., it is unlikely, although not impossible, that a query belongs to a certain category, then our model will pay less attention to the corresponding category information. The session graph only induces attention between nodes that are connected. Note that a residual connection (He et al., 2016a) is added to the last equation in Eq. 2. This has proven to be an effective technique to prevent gradient vanishing, and hence, to stabilize training.

A weighted multi-head graph attention mechanism is then defined as the concatenation of $[h_i^1, h_i^2, \cdots, h_i^K]$, where K is the number of heads, and each of the h_i is calculated via Eq. 2.

The token node representations, the category node representations, and the query node representations are updated iteratively. Specifically, we adopt the following update procedure:

$$\text{Token} \underset{3}{\overset{2}{\rightleftarrows}} \text{Query} \underset{4}{\overset{1}{\rightleftarrows}} \text{Category}.$$

In more details,

• Step 1 updates the query representations ($\mathcal{G}_q$) using the categories ($\mathcal{G}_c$), such that $\mathcal{G}_q$ is aware of the category knowledge.

• Steps 2 and 3 model cross-query relations. First, we update the token representations ($\mathcal{G}_t$) using $\mathcal{G}_q$, in order that the tokens acknowledge to which queries they belong. Then, $\mathcal{G}_q$ is re-computed using the updated version of $\mathcal{G}_t$, which essentially evaluates cross-query relations, using the token nodes as intermediaries.

• Finally, step 4 updates $\mathcal{G}_c$ using $\mathcal{G}_q$. This step enriches each category node's representation by incorporating information of all the queries that belong to this category.

The weighted graph attention mechanism (wGAT) used in each of the four steps are distinct, i.e., there are four different sets of weights $[W_a, W_q, W_k, W_v]$. We highlight that the weighted graph attention enables modeling of cross-query relations, which is implausible for conventional attention methods.

Eventually, we obtain the updated vectorized representations $\{h_i\}_{i=1}^{N_g}$ for all the nodes. We collect $\{h_i\}_{i=1}^{N_h+N_t}$, the updated representations that correspond to the query nodes and the token nodes, and we treat them as the contextual information of the session. Note that here, we exclude the category node representations. This is because such representations contribute to all the sessions, and do not constitute session-specific knowledge.

We remark that the wGAT mechanism explicitly models cross-query relations by associating query representations with word representations. This is fundamentally different from existing methods, where the relations are captured via recursion.

4.3 Session Representation from Aggregation Network

Recall that we pass the source query through an encoder and obtain $H_s \in \mathbb{R}^{L_s \times d}$, which contains representations for all the tokens in the source query. We use that of the prepended $\langle \text{boq} \rangle$ token as the representation of the source query, which is denoted $h_s \in \mathbb{R}^d$. We adopt an aggregation network to extract useful information with respect to h_s from the contextual information $\{h_i\}_{i=1}^{N_h+N_t}$. The network employs an attention mechanism that determines to what extent each vector h_i contributes to the source query h_s. Concretely,

$$\begin{aligned} z_i &= (W_k h_i)^\top h_s, \quad \alpha_i = \frac{\exp(z_i)}{\sum_{j=1}^{N_g} \exp(z_j)}, \\ v &= \textstyle\sum_{i=1}^{N_g} \alpha_i W_v h_i, \quad H_{\text{sess}} = H_s + v, \end{aligned} \tag{3}$$

where W_k and W_v are trainable weights. The last equation in Eq. 3 is summed row-wise, wherein $H_{\text{sess}}, H_s \in \mathbb{R}^{L_s \times d}$, and $v \in \mathbb{R}^d$.

The matrix H_{sess} serves as the representation of the session. Intuitively, by incorporating the aggregation network, we can filter out redundant information from the session history and only keep the ones pertinent to the source query.

After the Transformer encoder, the weighted graph attention mechanism, and the aggregation network, we obtain H_{sess}, the session representation that contains information on both the source query and its history searches. Subsequently, H_{sess} is fed into the Transformer decoder to generate rewritten query candidates.

The algorithm is detailed in Algorithm 1 in Appendix D.

5 Experiments

We conduct experiments on in-house data from an online shopping platform[1]. Notice that we focus on session-based query reformulation, a scenario that is rare in existing datasets (see Section 3 for details). We implement two methods with different backbone: *Transformer+Aggregation+Graph* and *BART+Aggregation+Graph*. The first one is constructed in the previous section, and the second one employs a fine-tuning approach instead of training-from-scratch. Training details are deferred to Appendix C.

[1]The dataset is undergoing internal processes for release.

5.1 Baselines

For baselines with pre-training, we use MeshBART (Chen and Lee, 2020). For baselines without pre-training, we use MeshTransformer (Chen and Lee, 2020) (a variant of MeshBART where we train the model from scratch), LQRW (He et al., 2016b) and HRED (Sordoni et al., 2015). We also compare our algorithm with two model variants: *Transformer+Aggregation* and *BART+Aggregation*, where we use the aggregation network but not the wGAT mechanism. Please refer to Appendix B for details.

5.2 Evaluation Metrics

We use both offline metrics, e.g., BLEU, and online metrics, e.g., MRR (Mean Reciprocal Rank), HIT@1, and HIT@16, to evaluate the query rewriting models. For online metrics, we report the gains over the the results calculated by using only source queries. We remark that the online metrics (i.e., MRR, HIT@1, and HIT@16) are more important than the offline metric (i.e., BLUE), because MRR and HIT are directly linked to user experience.

We use the BLEU score (Post, 2018) as an offline evaluation metric. This metric is constantly used to evaluate the quality of translation. We adopt it here because similar to machine translation, we formulate query rewriting as a seq2seq learning task. The correlation between the rewritten query and the target query reflects the model's ability to capture the user's purchase intent.

The online MRR metric describes the accuracy of the rewritten queries. As an online test, for each source query in the test set, we generate 10 candidate queries $r_1, \cdots, r_{10}$. Then we search each of these candidates using our production search engine, and we obtain the returned products, of which we only keep the top 32. Recap that we know the actual product that the customer purchased. The next step is to calculate the reciprocal of the actual product's rank for each of $r_1, \cdots, r_{10}$. For example, suppose for r_1, the actual purchased product is the second within the 32 returned products, then the score for r_1 is $\text{score}_1 = 1/2 = 0.5$. The score of the rewritten queries $r_1, \cdots, r_{10}$ is then defined as $\max\{\text{score}_i\}_{i=1}^{10}$. Finally, the score for the query

Number of candidates	#Candidates=5			#Candidates=10			BLEU
Metric	MRR	HIT@1	HIT@16	MRR	HIT@1	HIT@16	
Target Query	+0.161	+0.106	+0.290	+0.161	+0.106	+0.290	—
Baseline methods							
LQRW	+0.035	+0.025	+0.064	+0.068	+0.049	+0.126	29.38
HRED	+0.047	+0.032	+0.084	+0.081	+0.057	+0.142	25.67
MeshBART	+0.046	+0.031	+0.082	+0.082	+0.055	+0.148	30.87
MeshTransformer	+0.043	+0.026	+0.092	+0.085	+0.056	+0.159	25.33
Our methods							
BART+Aggregation	+0.063	+0.039	+0.109	+0.097	+0.064	+0.171	31.89
Transformer+Aggregation	+0.052	+0.029	+0.108	+0.102	+0.070	+0.173	27.22
BART+Aggregation+Graph	**+0.069**	**+0.046**	+0.118	+0.105	+0.075	+0.176	32.85
Transformer+Aggregation+Graph	+0.066	**+0.046**	**+0.120**	**+0.116**	**+0.083**	**+0.201**	28.15

Table 1: Experimental results. The results of MRR, HIT@1, and HIT@16 are shown as gain over the source query. The best result(s) under the MRR, HIT@1, and HIT@16 metrics are shown in **bold**.

rewriting model is the average over all the source query scores.

We also use HIT@1 and HIT@16 as evaluation metrics. HIT@16 is the percentage that the actual product is ranked within the first 16 products (the first page) when we search the rewritten query. And HIT@1 is similarly defined.

5.3 Experimental Results

Table 1 summarizes experimental results. The power of of our proposed query rewriting approach is well-demonstrated from the results. Recall that in our formulation, we rewrite a source query to a target query. The "target query" entry in Table 1 is the performance gain of the ground truth target query, i.e., this entry signifies upper bounds of performance gain that any model can achieve.

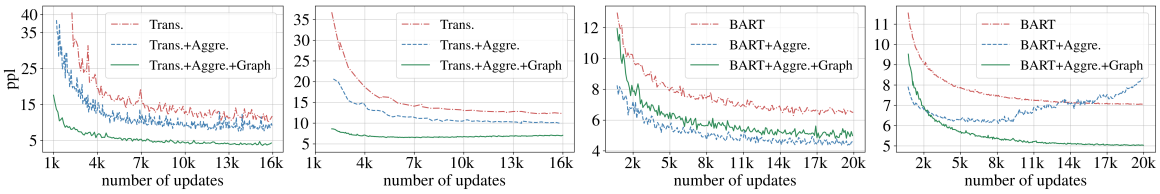
We can see that the attention-based models (i.e., MeshBART and MeshTransformer) outperforms the recurrent neural network-based approach (i.e., LQRW and HRED). This is because RNNs suffer from both the forgetting and the training issues. In contrast, Transformer-based models use the attention mechanism instead of recursion to capture dependencies, which has proven to be more effective. Moreover, by aggregating history searches into the models, BART+Aggregation and Transformer+Aggregation consistently outperform their vanilla alternatives. Essentially performance of these two methods indicate that integrating history queries into training is critical. The performance is further enhanced by incorporating the session graphs. Specifically, Transformer+Aggregation+Graph achieves the best performance under almost all the metrics. Notice that the HIT@16 metric gain improves from +0.159 to +0.201 when employing both the aggregation network and the session graph formulation for the Transformer-based models. We highlight that the weighted graph attention mechanism can directly captures cross-query relations, which is implausible for all the baselines. We can see that this property indeed contributes to model performance, i.e., HIT@16 increases from +0.173 to +0.201 when further equip Transformer+Aggregation with the wGAT mechanism.

Notice that BLEU is not a definitive metric. For example, the online metrics of HRED are consistently higher than those of LQRW, even though the BLEU score of the former is significantly lower than the latter. Also, compared with Transformer-based models, the BLEU score is consistently higher when using the BART model as the backbone. This is because a pre-trained language model contains more semantic information. However, the online metrics of the BART-based models are worse than those of the Transformer-based models.

However, the BLEU score is comparable for models with the same backbone. For example, for Transformer+Aggregation vs. Transformer+Aggregation+Graph, the BLEU scores are 27.22 vs. 28.15. Such a tendency coincides with the online metrics. We observe the same results from BART-based models.

5.4 Analysis

◇ **BART vs. Transformer** Even though BART contains twice the number of parameters than Transformer (140M vs. 70M), models fine-tuned on BART yield lower MRR and HIT metrics (with 10 generated candidate queries). One reason is that publicly available models are pre-trained on natural language corpus, but queries are usually short and have distinct structures. This raises doubts on whether current pre-trained models are suitable for the query domain. Indeed, the rich semantic



(a) Transformer (train). (b) Transformer (valid). (c) BART (train). (d) BART (valid).

Figure 4: Training and validation perplexity using Transformer and BART as backbone.

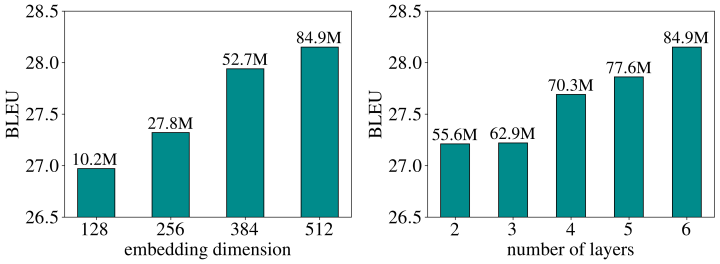


Figure 5: Model performance (in BLEU scores) vs. model size. The model size (in millions of parameters) are shown above the bars.

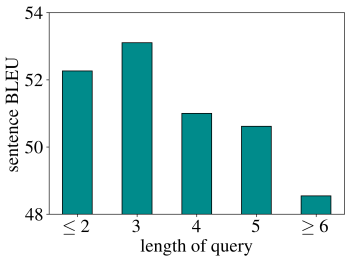


Figure 6: Query length vs. rewriting quality.

information enables a much better BLEU score (32.85 vs. 28.15), but the online tests suggest the fine-tuned models' inferior performance.

◇ **Training from scratch vs. fine-tuning** Figure 4 plots the training and validation perplexity (ppl) of the training-from-scratch approach and the fine-tuning approach. From Figure 4a and Figure 4b, we can see that by employing the aggregation network, Transformer+Aggregation fits the data better and exhibits enhanced generalization. The training and validation ppls are further significantly improved by incorporating the weighted graph attention mechanism, i.e., Transformer+Aggregation+Graph.

Notice that in Figure 4c, BART+Aggregation outperforms BART+Aggregation+Graph in terms of training ppl, which is different from the training-from-scratch approach. As indicated by Figure 4d, BART+Aggregation shows clear sign of overfitting. This is because even though pre-trained language models contain rich semantic information, much of it is considered "noisy" for query rewriting. Thus feature enhancement initiated by the weighted graph attention mechanism is needed.

◇ **Model size vs. performance** Figure 5 illustrates the relation between model size and performance, where we decrease the embedding dimension (correspondingly hidden dimensions of the feed-forward neural network) and the number of layers. We can see that even with $1/8$ of the parameters, model performance does not decrease much. Moreover, our model is more than 20% smaller than a BERT-base model (85M vs. 110M), rendering online deployment more than possible.

◇ **Query length vs. performance** Figure 6 demonstrates model performance with respect to length of the instant query. We can see that the BLEU score gradually decreases when the length increases. This is because long queries are often very specific (e.g., down to specific models or makes), making the rewriting task harder.

◇ **Case study** We examine advantages of leveraging history information and diversity of query generation. Results are deferred to Appendix A.

6 Conclusion and Discussion

We propose an end-to-end context-aware query rewriting model that can efficiently leverage user's history behavior. Our model infers a user's purchase intent by modeling her history searches as a graph, on which a weighted graph attention mechanism is applied to generate informative session representations. The representations are subsequently decoded into rewritten queries. We conduct experiments using in-house data from an online shopping platform, where our model achieves 11.7% and 22.5% relative improvement under the online MRR and HIT@16 metrics, respectively.

Our proposed session graph is flexible, and can be extended to incorporate more information. In this paper, we present a 3-partite graph, which contains words, queries, and categories. Additional components can be added as extra layers to the session graph.

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A Case Study

◇ **Advantages of leveraging history information** Two examples are shown in Table 3. The first example is error correction. In the example, the customer wishes to purchase dodge (a car brand) posters, but she mistakenly searches for dodger (a baseball team) posters. Without history information, it is impossible to determine the customer's true intent. However, by looking at session histories, we find that all the previous searches are related to automobiles (e.g., dodge and mopar), and therefore the query should be rewritten to "dodge posters". Our model successfully captures this pattern. Notice that the rewritten query without leveraging context does not match the user's intent.

The second example is keyword refinement. In the example, by looking at the history searches, it is obvious that the customer wishes to find phone cases, instead of phones. However, this intent is impossible to capture by using only the source query. Our model automatically adds the keyword "case" to the source query and matches the target query. On the other hand, without the context information, the rewritten result is not satisfactory.

◇ **Diversity of query generation** Table 2 demonstrates two examples. In the first example (the left three columns), notice that our model can grep information from history queries, e.g., "iphone 11 case sailor moon", and can delete keywords that are deemed insignificant or too restrictive, e.g., "iphone 11 case leopard" instead of "snow leopard". Also, our model can effectively capture domain information. For example, some of the history query keywords (e.g., pokemon, eevee) are often described as "cute", and our model recommends this keyword. All the history keywords are from Japanese anime series, therefore our model suggests another popular character, "totoro". Additionally, the "disney" and "disney princess" keywords are generated based on the interest to virtual characters. Finally, notice that the likelihood of all the suggested queries is similar, which means our model cannot single out a significantly better query than the others. Therefore our model generated a diverse group of queries.

In the second example (the right two columns), the generated query successfully matches the target query. Note that the top two generated queries have high likelihood, and the likelihood decreases drastically as the suggested queries become more and more implausible. In this example, the first query is 172% more likely than the tenth query, whereas this number is only 41% in the previous example. This suggests that our model can differentiate between good quality suggestions and poor quality alternatives.

B Baselines

The baselines are split into two groups: without pre-training and with pre-training. For the w/o pre-training group, we build the following models:

◇ ***Learning to Rewrite Queries (LQRW)*** (He et al., 2016b) is one of the first methods that applies deep learning techniques to query rewriting. Specifically, the LQRW model combines a sequence-to-sequence LSTM (Hochreiter and Schmidhuber, 1997; Sutskever et al., 2014) model with statistical machine translation (Riezler and Liu, 2010) techniques to generate queries. The candidates are subsequently ranked using hand-crafted feature functions.

◇ ***Hierarchical Recurrent Encoder-Decoder (HRED)*** (Sordoni et al., 2015) employs a hierarchical recurrent neural network for generative query suggestion. The model is a step forward from its predecessors in that it is sensitive to the order of queries and it is able to suggest rare and long-tail queries.

◇ ***Transformer+Aggregation*** is the model where we use the aggregation network to encode history search queries, i.e., without the weighted graph attention mechanism. Specifically, we first obtain the representations of the source query and the history queries from the Transformer encoder. Then, we extract information related to the source query from the history representations using an aggregation network. Such information is added to the source representation, and we follow a standard decoding procedure using these two factors. See Section 4.3 for details.

The second group of methods adopt pre-trained language models for query rewriting. BART (Lewis et al., 2020) is a pre-trained seq2seq model. We adopt this particular model instead of, for example, BERT (Devlin et al., 2019) or GPT-2 (Radford et al., 2019), because we treat query rewriting as a seq2seq task. And the aforementioned architectures have either the Transformer encoder (e.g., BERT) or the Transformer decoder (e.g., GPT-2), but not both. In our experiments,

Type	Query	Likelihood	Query	Likelihood
History	iphone 11 pro case pokemon; iphone 11 pro case eevee; iphone 11 pro case hetalia; iphone 11 pro case sailor moon	—	colorado 2005 tail lights; colorado 2005 door colorado 2005 accessories	—
Source	iphone 11 pro case snow leopard	—	colorado headlights	—
Target	iphone 11 pro case tiger	—	colorado 2005 headlights	—
Rewritten	iphone 11 pro case disney	0.497	2005 colorado headlights	0.566
	iphone 11 pro case sailor moon	0.492	colorado headlights 2005	0.458
	iphone 11 pro case harry potter	0.445	colorado headlights led	0.357
	iphone 11 pro case	0.440	colorado headlights assembly	0.301
	iphone 11 pro case cute	0.419	colorado tail lights	0.289
	iphone 11 pro case leopard	0.391	colorado headlights housing	0.237
	iphone 11 pro case clear	0.379	colorado led headlights	0.234
	iphone 11 pro case disney princess	0.372	2004 colorado headlights	0.230
	iphone 11 pro case pink	0.364	colorado 2004 headlights	0.214
	iphone 11 pro case totoro	0.353	colorado headlights 2004	0.208

Table 2: Two examples of generated queries and their associated likelihood.

Example 1 History	dodge led sign; dodge banners; mopar banner; mopar poster
Source	dodger posters
Target	dodge posters
Rewritten w/o context	dodger flag
Rewritten w/ context	dodge poster
Example 2 History	samsung galaxy case; samsung galaxy a11 case; samsung a11 case
Source	samsung galaxy a7
Target	samsung galaxy a7 case
Rewritten w/o context	samsung galaxy a7 charger
Rewritten w/ context	samsung galaxy a7 case

Table 3: Two examples of context-aware query rewriting with and without context.

BART is fine-tuned in a setting similar to training the Transformer model. We adopt the BART-base architecture in all the experiments, which contains about 140M parameters.

◇ ***MeshBART*** (Chen and Lee, 2020) is a BART-based model that first concatenates the history query, and then feeds it to a pre-trained BART model for query generation. Note that the original method requires click information. We remove this component as the proposed method do not need such data.

◇ ***BART+Aggregation*** is similar to Transformer+Aggregation, except we replace the Transformer backbone with the pre-trained seq2seq BART model.

C Training details

We use the *Fairseq* (Ott et al., 2019) code-base with *PyTorch* (Paszke et al., 2019) as the back-end to implement all the methods. All the experiments are conducted using 8 NVIDIA V100 (32GB) GPUs.

For training a Transformer model from scratch, we adopt the Transformer-base (Vaswani et al., 2017) architecture. We use Adam (Kingma and Ba, 2015) as the optimizer, and the learning rate is chosen from $\{3 \times 10^{-4}, 5 \times 10^{-4}, 1 \times 10^{-3}\}$. We use 4 heads for the weighted multi-head graph attention mechanism, where the head dimension is set to be 128 (note that the Transformer-base architecture has embedding dimension 512).

For fine-tuning a BART model, we adopt the BART-base (Lewis et al., 2020) architecture. We use AdamW (Loshchilov and Hutter, 2019) as the optimizer, and the learning rate is chosen from $\{3 \times 10^{-5}, 5 \times 10^{-5}, 1 \times 10^{-4}\}$. Similar to the training from scratch scheme, we adopt 4 heads, each with dimension 192, for the weighted graph attention mechanism.

For both training-from-scratch and fine-tuning, please refer to[2] Ott et al. (2019) for more details such as pre-processing steps and other hyper-parameters.

[2] https://github.com/pytorch/fairseq/blob/master/examples/translation/README.md

D Detailed Algorithm

Algorithm 1: Context-aware query rewriting.

Input: $\mathcal{D}$: dataset containing sessions; Initial parameters for the Transformer encoder and the Transformer decoder; Initial parameters for four weighted graph attention mechanism (Eq. 2): $\mathrm{wGAT}_{\mathrm{t}\to\mathrm{q}}$, $\mathrm{wGAT}_{\mathrm{q}\to\mathrm{c}}$, $\mathrm{wGAT}_{\mathrm{c}\to\mathrm{q}}$, $\mathrm{wGAT}_{\mathrm{q}\to\mathrm{t}}$; Initial parameters for the aggregation network (Eq. 3); K: the number of updates on the session graph; N: the number of rewritten queries for each session.

Output: A list that contains N generated queries for each session in the dataset.

Set result list: `rewritten` $= []$;

for *each session in* $\mathcal{D}$ **do**

  /* **Encode input data.** */

  Compute source representation H_s and history representation U_h using the Transformer encoder;

  /* **Apply weighted graph attention.** */

  Obtain initial representations $\mathcal{G}_t^0$, $\mathcal{G}_q^0$, $\mathcal{G}_c^0$;

  for $k = 1 \cdots K$ **do**

    $\mathcal{G}_q^{k-1/2} = \mathrm{GAT}_{\mathrm{c}\to\mathrm{q}}(\mathcal{G}_t^{k-1}, \mathcal{G}_q^{k-1})$;

    $\mathcal{G}_t^{k} = \mathrm{GAT}_{\mathrm{q}\to\mathrm{t}}(\mathcal{G}_q^{k-1/2}, \mathcal{G}_t^{k-1})$;

    $\mathcal{G}_q^{k} = \mathrm{GAT}_{\mathrm{t}\to\mathrm{q}}(\mathcal{G}_t^{k}, \mathcal{G}_q^{k-1/2})$;

    $\mathcal{G}_c^{k} = \mathrm{GAT}_{\mathrm{q}\to\mathrm{c}}(\mathcal{G}_q^{k-1/2}, \mathcal{G}_c^{k-1})$;

  end

  Set history representation $\{h_i\}_{i=1}^{N_t+N_h} = \mathcal{G}_t^K \cup \mathcal{G}_h^K$;

  /* **Apply aggregation network.** */

  Compute session representation H_{sess} from H_s and $\{h_i\}_{i=1}^{N_t+N_h}$ using Eq. 3;

  /* **Generate rewritten queries.** */

  Generate N rewritten queries $\{q_i\}_{i=1}^{N}$ using the Transformer decoder and a beam search procedure;

  `rewritten` $=$ `rewritten` $+ \{q_i\}_{i=1}^{N}$;

end

Output: Rewritten queries `rewritten`.