

# 000 001 PHYSICS-GROUNDED MOTION FORECASTING VIA 002 EQUATION DISCOVERY FOR TRAJECTORY-GUIDED 003 IMAGE-TO-VIDEO GENERATION 004 005

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## ABSTRACT

013 Recent advances in video generation models have achieved remarkable visual  
014 realism. However, these models typically lack accurate physical alignment, failing  
015 to replicate real-world dynamics in object motion. This limitation arises primarily  
016 from their reliance on learned statistical correlations rather than capturing mech-  
017 anisms adhering to physical laws. To address this issue, we introduce a novel  
018 framework that integrates symbolic regression (SR) and trajectory-guided image-  
019 to-video (I2V) models for physics-grounded video forecasting. Our approach ex-  
020 tracts motion trajectories from input videos, uses a retrieval-based pre-training  
021 mechanism to enhance symbolic regression, and discovers equations of motion  
022 to forecast physically accurate future trajectories. These trajectories then guide  
023 video generation without requiring fine-tuning of existing models. We eval-  
024 uate our framework on scenarios from classical mechanics, including spring-mass,  
025 pendulums, and projectile motions. In these settings, our method successfully re-  
026 covers ground-truth analytical equations and improves the physical alignment of  
027 generated videos compared to baseline methods. This work provides a first step  
028 toward integrating equation discovery with video generation.<sup>1</sup>  
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## 1 INTRODUCTION

030 Recent advances in video generation models have significantly improved the realism of synthesized  
031 videos, driven primarily by diffusion-based and autoregressive models Blattmann et al. (2023); Yang  
032 et al. (2025); Team (2025); Kong et al. (2025). Incorporating motion trajectories enables precise con-  
033 trol over object movements, facilitating videos that more accurately capture intended dynamics Wu  
034 et al. (2025); Wang et al. (2024b); Namekata et al. (2025). However, existing trajectory-guided  
035 methods typically rely on text prompts, manually drawn or statistically derived trajectories Zhang  
036 et al. (2024); Team, none of which ensures adherence to the underlying laws of physics Kang et al.  
037 (2024); Motamed et al. (2025); Wang et al. (2025).  
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039 Physicists understand object dynamics by discovering physical laws from observational data and  
040 formulating these laws into symbolic equations. These equations reliably forecast object move-  
041 ments, unaffected by shifts in the underlying data distributions. Moreover, such equation discovery  
042 does not require extensive training data, unlike the scaling laws commonly adopted by current video  
043 generation models Kaplan et al. (2020). Therefore, for the *first* time, we investigate: i) whether  
044 AI methods can feasibly discover physics equations directly from video clips and subsequently use  
045 these equations to reliably forecast object motion trajectories, and ii) whether such equations can be  
046 identified from just one or a handful of video clip without extensive data-driven training.  
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048 To address the above research questions, we propose a novel *neuro-symbolic, inference-only* frame-  
049 work for forecasting object motion trajectories from a short video clip, followed by feeding the  
050 predicted trajectories into an image-to-video (I2V) model to produce physics-grounded videos. As  
051 illustrated in Figure 1, our approach first utilizes CoTracker Karaev et al. (2024) to extract initial  
052 object motion trajectories from a short video clip. We then employ a symbolic regression (SR)  
053 algorithm Crammer (2023a), an evolutionary search method that automatically discovers explicit

<sup>1</sup>The code and dataset are available at <https://anonymous.4open.science/r/ReSR-0083/>

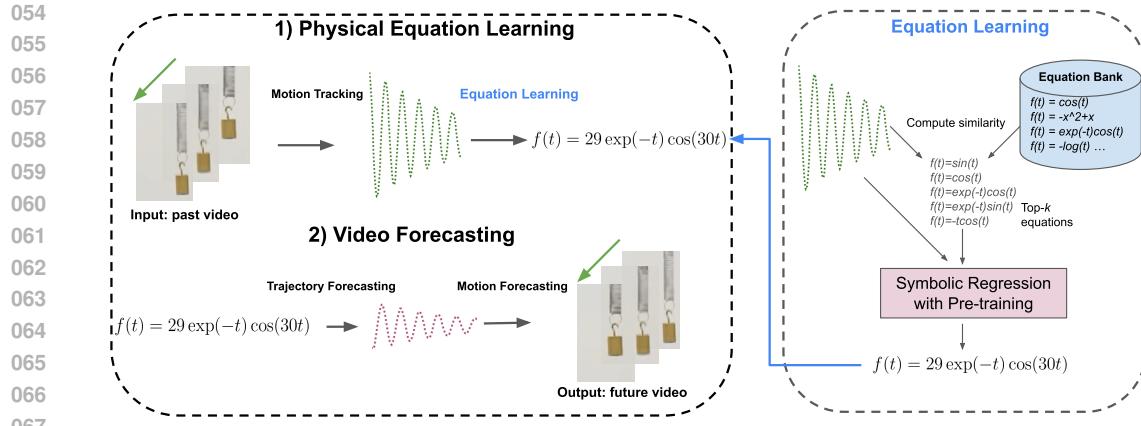


Figure 1: An overview of our framework. Given an input video, we first extract object (*i.e.*, spring or weight) motion trajectories, which are used to discover equations of motion via SR enhanced by our proposed retrieval-based pre-training mechanism (ReSR), where ReSR initializes the search with candidates retrieved from a curated equation bank of known physical laws. The learned symbolic equations forecast future object trajectories, serving as precise control signals to guide trajectory-guided video generation models, resulting in more physics-grounded video generation.

mathematical equations, to derive a *human-interpretable* symbolic equation characterizing the underlying physical law. Given the initial trajectories, this discovered equation can reliably produce future object movements of arbitrary length, consistently adhering to the underlying physics laws.

From another perspective, the equation discovery process can be viewed as training a symbolic model that characterizes motion trajectories. Current evolutionary search methods typically initialize their searches using randomly selected functions, often starting far from the global optimum and resulting in slow convergence. To mitigate this, we propose a novel **Retrieval-based** pre-training method for **Symbolic Regression**, called ReSR, which initializes the search from relevant equations retrieved from a physical equation bank. Unlike prior SR methods that rely on randomly initialized function sets, ReSR incorporates physics-inspired equations, reducing search space bias and improving both efficiency and interpretability by aligning candidates with established physics priors.

To investigate the fundamental challenges of learning equations from given videos, we conduct experiments on a set of videos in a controlled laboratory environment governed by the laws of classical mechanics. These videos depict systems, such as spring-mass oscillators, pendulums, and projectile motion. We choose this controlled setting because: i) it enables direct evaluation of discovered equations against ground-truth equations identified by physicists; ii) insights into object motion in classical mechanics can be easily extended into other types of motion; and iii) classical mechanics underpins a wide range of real-world applications, including physics simulation, scientific visualization, and physics education.

Our contributions are summarized as follows:

- We propose a novel neuro-symbolic framework for physics-grounded video forecasting. Specifically, our method first extracts motion trajectories from input videos, then discovers equations of motion. These equations are used to forecast future trajectories, which then guide I2V models to synthesize future videos that better align with physical laws. Importantly, our approach operates entirely at inference time and does not require fine-tuning of video generation models.
- We introduce a *retrieval-based pre-training mechanism* for SR, denoted as ReSR, which leverages a curated equation bank of known physical laws to provide strong initialization candidates. This substantially improves convergence speed and accuracy in discovering equations from observed trajectories.
- We empirically demonstrate that our framework not only recovers equations closely aligned with ground-truth analytical expressions and observed trajectories, but also generates videos with significantly improved physical consistency compared to existing baselines when conditioned on trajectories discovered by ReSR.

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## 2 PRELIMINARY

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Scientists have discovered empirical laws from observational data. For example, Johannes Kepler  
formulated the third law of planetary motion,  $(\text{period})^2 \propto (\text{radius})^3$ , after analyzing thirty years  
of astronomical data. Similarly, Planck's law was a function fitted to experimental data Planck  
(1900). However, modern scientific data is often high-dimensional and complex, making manual  
equation discovery a challenging task Virgolin & Pissis (2022). SR is a computational method for  
automatically deriving mathematical equations from data. Unlike traditional regression, which fits  
data to a predefined equation structure (e.g., linear or polynomial regression), SR searches for both  
the equation structure and parameters. This flexibility makes SR particularly valuable in scientific  
discovery Rudy et al. (2017); Meidani & Farimani (2023).119  
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SR approaches can be broadly categorized into two primary types: evolutionary algorithm (EA)-  
based methods and deep learning-based methods. EA-based approaches operate by evolving a pop-  
ulation of candidate equations over successive generations, using operations such as mutation and  
crossover to search for equations that best fit the data Brindle (1980); Goldberg & Deb (1991);  
Zhang & Shasha (1989); Stephens (2025); Cranmer (2023b). EA-based methods require minimal  
prior assumptions about equation structure, allowing them to explore a diverse space.125  
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Deep learning-based methods directly predict equations from data Biggio et al. (2021); Kamienny  
et al. (2022); Shojaee et al. (2023); Meidani et al. (2024). Devlin et al. (2019); Radford et al.  
(2019); Feng et al. (2023; 2025) typically train an end-to-end transformer-based model where the  
input is observational data and the output is a symbolic equation. However, deep learning models  
often struggle with out-of-distribution generalization Yang et al. (2024); Kim et al. (2024); Feng  
et al. (2024), and cannot guarantee the generated output forms a syntactically valid equation, lead-  
ing to non-executable equations. Inspired by the success of pre-training in deep learning Erhan  
et al. (2010); Devlin et al. (2019), we propose a pre-training mechanism for EA-based SR (see  
Section 3.3). We first construct an equation bank containing physics-related equations. During init-  
ialization, the SR algorithm retrieves equations that closely align with the observed data and uses  
them as initial candidates. This pre-training strategy significantly improves convergence speed and  
enhances the accuracy of the learned equations.136  
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## 3 METHODOLOGY

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### 3.1 TASK FORMULATION AND NOTATIONS

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The objective of this study is to achieve physics-grounded motion forecasting for trajectory-guided  
video generation. As illustrated in Figure 1, given an input video  $V_i$  depicting the initial motion of  
an object, our approach generates a video  $V_o$  representing the object's future motion. Our approach  
consists of three main steps. First, we extract the motion trajectories of moving objects in  $V_i$ . The  
extracted trajectories are represented as a set  $\mathbb{P} = \{p_1, p_2, \dots, p_n\}$ , where each trajectory  $p_i$  is a  
time series of object positions:  $p_i = [p_1, p_2, \dots, p_T]$ , where  $p_t = (x_t, y_t)$  denotes the image-space  
coordinate of the object at time step  $t$ . Next, we employ symbolic regression to learn equations that  
govern the motion of objects. Specifically, for each trajectory  $p_i$ , we aim to learn a pair of functions  
 $f_i^x(t)$  and  $f_i^y(t)$  such that:

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$$x_t = f_i^x(t), \quad y_t = f_i^y(t). \quad (1)$$

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that map time to object position. Using the learned equation  $f_i^x(t)$  and  $f_i^y(t)$ , we predict the future  
trajectory for time steps beyond the observed interval, i.e.,  $p_i = f_i(t), t \in \{T+1, T+2, \dots, T+K\}$ ,  
where  $K$  represents the forecast horizon. Finally, we utilize the predicted trajectories to guide  
trajectory-based video generation models, which then synthesize the future video  $V_o$ .155  
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### 3.2 EXTRACTION OF OBJECT MOTION TRAJECTORY

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To learn equations of object motion, we first extract object trajectories from the input video  $V_i$ . We  
employ CoTracker Karaev et al. (2024), a state-of-the-art point tracking model that performs joint  
point tracking and propagation across all frames. CoTracker requires a set of query points in the  
first frame to initiate tracking. While manual annotation is possible, it is not scalable across diverse  
video content. Instead, we adopt an automated approach by uniformly sampling query points on a  
2D  $M \times M$  grid across the first frame. Each query point is tracked throughout the entire video.

162 We perform all tracking in the original image coordinate system without additional preprocessing.  
 163 After collecting all trajectories, we compute the temporal variance of each trajectory. We then rank  
 164 the trajectories based on their positional variance across time and retain the top- $k$  trajectories with  
 165 the highest motion magnitude. This strategy is motivated by the observation that target objects in  
 166 physics-driven videos typically exhibit the most motion, while background regions tend to remain  
 167 static. As a result, selecting high-variance trajectories increases the likelihood of capturing the true  
 168 object dynamics and filtering out irrelevant background noise.

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 170 **3.3 SYMBOLIC REGRESSION WITH PRE-TRAINING**

171 In this step, we apply symbolic regression with retrieval-based pre-training (ReSR) to discover equa-  
 172 tions that fit extracted object trajectories. Instead of initializing the search process from scratch with  
 173 random equations, we retrieve a set of candidate equations from a curated equation bank composed  
 174 of physics-related equations. The retrieved equations then serve as priors to initialize the symbolic  
 175 regression. Given a trajectory  $\mathbf{p}_i = [p_1, p_2, \dots, p_T]$ , our goal is to learn Equation 1.

176 **Construction of Equation Bank.** We construct an equation bank containing a diverse set of equa-  
 177 tions derived from classical and empirical physics to serve as priors for symbolic regression. The  
 178 bank integrates equations from three sources: 1) The Feynman equation dataset Udrescu & Tegmark  
 179 (2020), which consists of equations extracted from the Feynman Lectures on Physics Feynman et al.  
 180 (1965). These equations typically take the form  $y = f(x_1, x_2, \dots)$ , with up to ten input vari-  
 181 ables. To adapt them for time-series motion, we substitute time-dependent variables (e.g., velocity,  
 182 acceleration, momentum) with the time variable  $t$ . Variables that are independent of time (e.g.,  
 183 mass, density) are replaced with constant values (e.g., 10), aiming to preserve equation structure.  
 184 We select a total of 106 equations after this adaptation. 2) The Nguyen dataset Uy et al. (2011),  
 185 which includes 10 commonly used empirical formulas in symbolic regression benchmarks. We ap-  
 186 ply the same time-variable substitution process. 3) We include 13 additional physics equations from  
 187 Thornton & Marion (2004), not present in the aforementioned datasets, to ensure the equation bank  
 188 represents a broad range of physical systems. All equations are stored as symbolic expressions in  
 189 Julia syntax Bezanson et al. (2017), enabling compatibility with our symbolic regression framework.

190 **Retrieval-based Pre-training Mechanism.**

191 Our proposed ReSR initializes symbolic regression with candidate equations retrieved from a cu-  
 192 rated equation bank. The retrieval is based on the similarity between the extracted object trajectory  
 193 and trajectories generated by each equation in the bank. Similarity is computed using Dynamic  
 194 Time Warping (DTW) Müller (2007), a sequence alignment algorithm that handles temporal mis-  
 195 alignments such as phase shifts and local time warping that are not captured by Euclidean distance.

196 However, standard DTW is unable to robustly handle spatial offsets and scale variations in trajectory  
 197 coordinates. To address this, we introduce *Normalized Dynamic Time Warping* (N-DTW), where the  
 198 extracted trajectory is rescaled to match the coordinate range of each equation-generated trajectory  
 199 before computing DTW. This helps the comparison to focus on shape similarity rather than absolute  
 200 position. Formally, given an extracted trajectory  $\mathbf{p}_i = [p_1, p_2, \dots, p_T]$ , where each  $p_t = (x_t, y_t)$ , we  
 201 normalize it as follows:

$$\bar{x}_t = (\hat{x}_{\max} - \hat{x}_{\min}) \cdot \frac{x_t - x_{\min}}{x_{\max} - x_{\min}} + \hat{x}_{\min} \quad (2)$$

$$\bar{y}_t = (\hat{y}_{\max} - \hat{y}_{\min}) \cdot \frac{y_t - y_{\min}}{y_{\max} - y_{\min}} + \hat{y}_{\min} \quad (3)$$

202 where  $x_{\min}, x_{\max}, y_{\min}, y_{\max}$  are the bounds of the extracted trajectory, and  $\hat{x}_{\min}, \hat{x}_{\max}, \hat{y}_{\min}, \hat{y}_{\max}$   
 203 are the bounds of the equation-generated trajectory.

204 For each equation in the bank, we compute an N-DTW score with the normalized extracted trajec-  
 205 tory. Since the similarity between the extracted trajectory and each equation-generated trajectory  
 206 is computed independently, N-DTW retrieval can be easily parallelized across multiple CPU cores,  
 207 enabling scalability to large equation banks. We then select the top- $k$  equations with the lowest  
 208 distances as initial candidates for symbolic regression. This retrieval strategy emphasizes shape  
 209 similarity rather than proximity in raw values. For instance, consider a trajectory generated by  
 210  $y = 0.5 \cos(t + 3) + 100$ . Two candidate equations might be  $y = 100$  and  $y = \cos(t)$ . While  
 211 Euclidean distance may favor  $y = 100$  due to its proximity in magnitude, it fails to capture the

216 oscillatory structure. In contrast, N-DTW correctly identifies  $y = \cos(t)$  as the more structurally  
 217 similar trajectory.

218 **Initialization of ReSR.** We initialize a portion of population members with the top- $k$  retrieved  
 219 equations that closely match the target trajectory. We introduce an initialization weight hyperpa-  
 220 rameter  $\alpha \in [0, 1]$ , which determines the proportion of initial population members that are seeded  
 221 with retrieved equations, while the remaining are randomly generated. This hybrid initialization  
 222 strategy allows us to balance *exploration*—via randomly sampled equations that enable diversity in  
 223 the search space—and *exploitation*—via retrieved equations that act as informative priors. Higher  
 224 values of  $\alpha$  prioritize faster convergence, while lower values preserve the capacity for discovering  
 225 novel equation forms. If the available number of top- $k$  retrieved equations is insufficient to meet the  
 226 required number based on  $\alpha$ , we duplicate top- $k$  retrieved equations to fill the remaining positions.  
 227 This strategy ensures that the initial population predominantly contains equations closely matching  
 228 the observed dynamics, reducing the risk of including irrelevant or misleading equations that could  
 229 negatively impact the search efficiency. This initialization occurs only once at the beginning of the  
 230 symbolic regression run. All modifications, including retrieval-based pre-training and the integra-  
 231 tion of N-DTW, are implemented within a modified version of the `SymbolicRegression.jl`  
 232 framework Cranmer (2023b), ensuring compatibility with existing symbolic regression workflows  
 233 and reproducibility of our method.

### 234 3.4 TRAJECTORY-GUIDED VIDEO FORECASTING

235 To generate future video frames  $V_o$  that are physically consistent with learned motion dynamics,  
 236 we incorporate existing trajectory-guided I2V models, such as SG-I2V Namekata et al. (2025), Tora  
 237 Zhang et al. (2024), and MotionCtrl Wang et al. (2024b), into our framework. These models are  
 238 typically diffusion models Song et al. (2020) that synthesize temporally coherent video sequences  
 239 by denoising noise-perturbed images conditioned on a starting image and motion trajectories.

240 We use the final frame of the observed input video  $V_i$  as the starting image and condition on future  
 241 trajectories predicted by equations learned from ReSR. These trajectories are formatted as sequences  
 242 of  $(x, y)$  coordinates, sampled at temporal intervals that match the requirement of each I2V model.  
 243 This integration enables our framework to produce future video sequences that are not only visually  
 244 plausible but also governed by equations of motion inferred from past observations. Our approach is  
 245 *modular* and *model-agnostic*: it can be directly applied to any trajectory-guided I2V model without  
 246 retraining or fine-tuning.

## 247 4 EXPERIMENTS

248 We first assess whether the proposed ReSR enhances the performance of symbolic regression in  
 249 discovering equations. Then, we examine whether trajectories predicted by the learned equations  
 250 lead to videos that better align with real-world physical dynamics.

### 251 4.1 EVALUATION OF EQUATION DISCOVERY

252 **Datasets.** We evaluate equation discovery methods using trajectories extracted from videos of clas-  
 253 sical physics systems, divided into two categories: **1) systems with ground-truth trajectory equations**  
 254 (*i.e.*, systems with analytical solutions), including spring mass, damped spring mass, two body, and  
 255 projectile motion Huang et al. (2024); **2) systems without ground-truth trajectory equations**, includ-  
 256 ing single pendulum, double pendulum and fluid motion, where no closed-form analytical solution  
 257 exists Huang et al. (2024); Ohana et al. (2024). Each system includes ten videos with varying ini-  
 258 tial states. We use CoTracker Karaev et al. (2024) to extract uniformly sampling query points on  
 259 a  $10 \times 10$  grid from the first frame. From these, we select the top 5 trajectories with the high-  
 260 est temporal variance to serve as inputs for symbolic regression methods. Each trajectory is split  
 261 80%/10%/10% along the time dimension for training, validation, and evaluation, respectively. This  
 262 split aims to select equations that generalize from past states to unseen future states.

263 **Evaluation.** For systems with ground-truth equations, we evaluate symbolic similarity between pre-  
 264 dicted equations and ground-truth equations using normalized Tree Edit Distance (TED) Zhang &  
 265 Shasha (1989), which measures how many edit operations (*i.e.*, insertions, deletions, substitutions)

Methods	Baseline Comparison			
	with AS		w/o AS	
	TED (↑)	MSE (↓)	MSE (↓)	ITB (↓)
APO	0.33 <sub>0.11</sub>	7.92 <sub>0.21</sub>	76.52 <sub>7.56</sub>	68.21 <sub>16.51</sub>
gplearn	0.40 <sub>0.11</sub>	3.87 <sub>0.21</sub>	61.95 <sub>7.44</sub>	83.14 <sub>12.97</sub>
uDSR	0.41 <sub>0.12</sub>	3.73 <sub>0.11</sub>	50.35 <sub>6.10</sub>	74.83 <sub>13.30</sub>
KAN	0.22 <sub>0.14</sub>	11.14 <sub>0.49</sub>	91.43 <sub>9.57</sub>	N/A
PySR	0.47 <sub>0.16</sub>	2.95 <sub>0.05</sub>	45.25 <sub>4.39</sub>	61.43 <sub>10.37</sub>
LaSR	0.54 <sub>0.15</sub>	1.91 <sub>0.05</sub>	32.56 <sub>4.05</sub>	59.93 <sub>11.93</sub>
<b>ReSR (Our)</b>	<b>0.80<sub>0.08</sub></b>	<b>1.52<sub>0.04</sub></b>	<b>27.58<sub>3.61</sub></b>	<b>44.31<sub>7.61</sub></b>

Ablation Study				
Varying Initialization Weight $\alpha$				
ReSR-0	0.47 <sub>0.16</sub>	2.95 <sub>0.05</sub>	45.25 <sub>4.39</sub>	61.43 <sub>10.37</sub>
ReSR-0.25	0.60 <sub>0.13</sub>	1.84 <sub>0.06</sub>	30.90 <sub>3.16</sub>	57.24 <sub>8.29</sub>
ReSR-0.5	0.70 <sub>0.12</sub>	1.74 <sub>0.05</sub>	28.19 <sub>3.12</sub>	49.58 <sub>8.82</sub>
ReSR-0.75	0.80 <sub>0.08</sub>	1.52 <sub>0.04</sub>	27.58 <sub>3.61</sub>	44.31 <sub>7.61</sub>
ReSR-1.0	0.77 <sub>0.10</sub>	1.55 <sub>0.04</sub>	28.32 <sub>2.90</sub>	46.63 <sub>8.94</sub>

Ablation Study				
Varying Training/Test Split Proportion ( $\alpha = 0.75$ )				
ReSR-2.7	0.54 <sub>0.17</sub>	31.96 <sub>5.93</sub>	137.04 <sub>9.17</sub>	33.76 <sub>5.14</sub>
ReSR-4.5	0.61 <sub>0.14</sub>	18.41 <sub>2.32</sub>	89.28 <sub>5.20</sub>	37.66 <sub>6.05</sub>
ReSR-6.3	0.68 <sub>0.13</sub>	7.19 <sub>0.21</sub>	57.51 <sub>6.47</sub>	42.83 <sub>6.39</sub>
ReSR-8.1	0.80 <sub>0.08</sub>	1.52 <sub>0.04</sub>	27.58 <sub>3.61</sub>	44.31 <sub>7.61</sub>

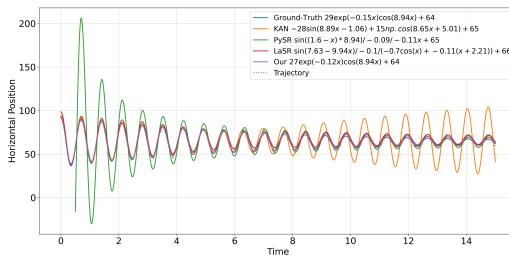
Table 1: Quantitative comparison with baselines and ablation study of ReSR. AS indicates analytical solutions; Conv. indicates convergence. Best results in **bold**.

are required to transform one equation tree into another, normalized by the maximum node count of two equation trees. For systems without ground-truth equations, we measure the Mean Squared Error (MSE) between the trajectory generated by predicted equations and the actual observed trajectory. To compare the convergence speed of different methods, we report the *iteration-to-best* (ITB) metric, which measures the number of iterations required to reach the method's lowest MSE on the validation set Xing et al. (2018); Smith & Topin (2019).

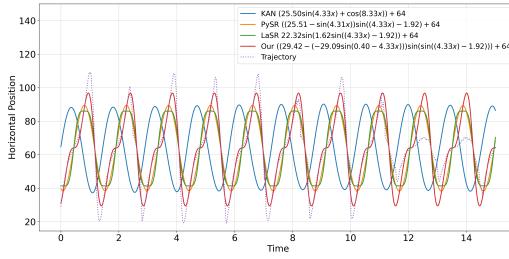
**Baselines.** We compare against the following methods: **APO** Schmidt & Lipson (2010): A symbolic regression method using Age-fitness Pareto Optimization. **gplearn** Stephens (2025): An EA-based symbolic regression with a scikit-learn-style API. **uDSR** Landajuela et al. (2022): A hybrid approach that combines deep learning models with evolutionary algorithms to discover equations. **KAN** Liu et al. (2025): Kolmogorov-Arnold Networks (KANs) replace each weight in Multi-Layer Perceptrons (MLPs) with a univariate function parameterized as a spline, enabling symbolic equation extraction after training. **PySR** Cranmer (2023b): A symbolic regression framework based on evolutionary search, which can be viewed as an ablation model without retrieval-based pre-training. **LaSR** Grayeli et al. (2024): A symbolic regression approach that leverages large language models to propose initial equations.

**Implementation Details.** For EA-based symbolic regression methods, including both our method and baselines, we run 100 iterations with a population size of 30 across 30 populations. The search space operators include basic arithmetic ( $+$ ,  $-$ ,  $*$ ,  $/$ ), power functions, and common mathematical functions:  $\cos$ ,  $\sin$ ,  $\exp$ ,  $\log$ ,  $\tan$ , and  $\sqrt{}$ . For KAN, we perform grid-based hyperparameter tuning and report results using the best-performing configuration on the validation set. All experiments are conducted on a machine with 32-core CPUs and a single 80GB A100 GPU.

**Results and Analysis.** Table 1 presents the comparison between ReSR, baseline methods, and ablation variants. ReSR consistently outperforms all baselines in both symbolic similarity (TED) and trajectory error (MSE), demonstrating improved accuracy in discovering physical equations. Additionally, it achieves the fastest convergence (lowest ITB), highlighting the effectiveness of retrieval-based pre-training. For the ablation study, we first analyze the effect of the initialization weight hyperparameter  $\alpha$ . Performance improves steadily as  $\alpha$  increases, peaking at  $\alpha = 0.75$ , which supports both exploitation (using physics-aligned priors) and exploration (diversity through random sampling). Another ablation study investigates the impact of varying the training/test split while keeping the validation set fixed at 10% of the data. Results show that increasing the training set size improves equation discovery accuracy, indicating that ReSR benefits from larger datasets to better fit observational data.



(a) Damped spring–mass system.



(b) Single pendulum system.

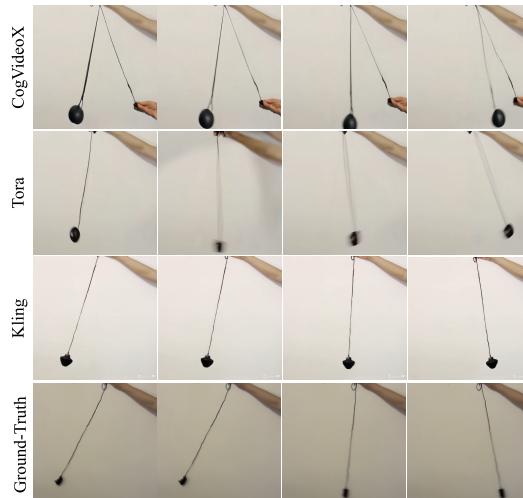
Figure 2: Qualitative comparison of equations discovered by different methods.

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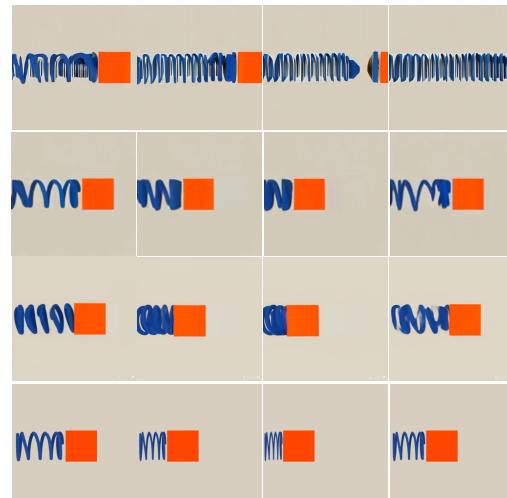
378 thing Wu et al. (2025), MotionCtrl Wang et al. (2024b), SG Namekata et al. (2025), Tora Zhang  
 379 et al. (2024) and Kling (a commercial model) Team. These models are conditioned on the initial  
 380 image and the given trajectory. For models that accept a single trajectory, we use the one with the  
 381 highest motion magnitude, while for models that support multiple trajectories, we use the top-5 with  
 382 the highest motion magnitudes. For synthetic data, we also use a physics simulator Huang et al.  
 383 (2024) to generate future videos serving as a reference for upper-bound performance.

384 **Implementation Details.** We resize initial im-  
 385 ages to match each model’s input resolution.  
 386 The video length is fixed at 5 seconds, with  
 387 frames per second (FPS) set per model require-  
 388 ments. For models requiring text prompts, we  
 389 use either official prompt guidelines or gen-  
 390 erate prompts using GPT-4o Yang et al. (2025).  
 391 Trajectories are normalized and scaled to match  
 392 each model’s spatial resolution and sampled  
 393 uniformly at 2 points per second. All models  
 394 are run on a machine with an NVIDIA 80G  
 395 A100 GPU or API without fine-tuning.  
 396

396 **Results and Analysis.** Table 2 presents auto-  
 397 matic evaluation results on all models. Models  
 398 guided by trajectory consistently outperform  
 399 trajectory-free baselines in both visual quality  
 400 and physics alignment. Among trajectory-guided I2V  
 401 models, Kling guided by trajectories pre-  
 402 dicted by our method achieves the best performance,  
 403 closely approaching Kling with ground-truth  
 404 trajectories. Real initial frame settings have lower  
 405 performance than synthetic settings, likely due  
 406 to background noises and systems complexity.  
 407 In the synthetic setting, all models perform signif-  
 408 icantly worse than the simulator, indicating that  
 409 current data-driven video generation models struggle  
 410 to capture physical dynamics, even when guided by  
 411 ground-truth trajectory. This highlights the need  
 412 for future work to improve the physical alignment of  
 413 data-driven approaches.



(a) Single pendulum system.



(b) Damped spring-mass system.

Figure 3: Qualitative comparisons across models.

425 For human evaluation in Table 3, annotators consistently preferred trajectory-guided models over  
 426 trajectory-free baseline on both physical alignment and visual quality. Inter-annotator agreement  
 427 was measured using Fleiss’ Kappa Fleiss (1971), yielding a score of 0.73, which indicates substan-  
 428 tial agreement among annotators. Notably, Kling with ReSR-guided trajectories was preferred over  
 429 its manually guided counterpart, confirming that learned equations offer more accurate and reliable  
 430 motion control. In the synthetic setting, the physics simulator was consistently rated as the most  
 431

432 physically accurate, highlighting the limitations of data-driven approaches. We attribute this to a  
 433 fundamental difference: current video generation models are trained to capture statistical correlations  
 434 in large-scale datasets, but lack explicit modeling of physical causality. In contrast, physics  
 435 simulators generate motion directly from governing equations, ensuring high physical fidelity. How-  
 436 ever, simulators have their own limitations. They are not scalable across diverse scenarios and tend  
 437 to lack realism when applied to real-world scenarios. This highlights the value of our method,  
 438 which seeks to combine the interpretability and physical grounding of governing equations with the  
 439 flexibility and realism of data-driven generative models.

440 Figure 3 illustrates qualitative comparisons. Trajectory-guided models exhibit improved global motion  
 441 consistency, while trajectory-free models (e.g., CogVideoX) often produce erratic or implausible  
 442 dynamics. Even the strongest model, Kling, fails to capture fine-grained physical details such  
 443 as spring deformation, suggesting that while trajectory conditioning improves high-level motion,  
 444 current models still lack the physical inductive biases needed for fine-grained dynamic synthesis.

## 446 5 RELATED WORK

447  
 448  
 449 **Equation Discovery from Video.** Chari et al. (2019); Luan et al. (2021); Tetriyani et al. (2024);  
 450 Garcia et al. (2024) aim to extract physical laws of dynamic systems directly from video, using  
 451 symbolic regression or ODE-based methods. However, many of these approaches impose strong  
 452 constraints on the equation structure, such as assuming linearity, or focus solely on estimating pa-  
 453 rameters of pre-defined models. Huang et al. (2024) uses autoencoders to encode video sequences  
 454 into low-dimensional latent vectors and attempt to learn system dynamics in that space. These la-  
 455 tent variables often lack physical interpretability, and the resulting dynamics are not expressed as  
 456 symbolic equations. In contrast, our approach employs symbolic regression to directly learn explicit  
 457 symbolic equations, capturing physically meaningful variables that map time to object positions,  
 458 thus ensuring interpretability and physical alignment.

459 **Physics-aligned Video Generation.** Millington (2007); Todorov et al. (2012); Bonnet et al. (2022);  
 460 Kohl et al. (2024); Ohana et al. (2024); Lv et al. (2024) use physics simulators to ensure physical  
 461 realism in video generation, where dynamics are modeled via hard-coded rules and equations. While  
 462 highly accurate, these simulators are typically limited to specific domains, require hand-crafted  
 463 scenario design. On the other hand, Blattmann et al. (2023); Wang et al. (2024a); Yang et al. (2025);  
 464 Team (2025); Kong et al. (2025) use diffusion or autoregressive architectures to synthesize diverse  
 465 scenes from image or text prompts but often lack physical consistency, leading to unrealistic object  
 466 motion Motamed et al. (2025).

467 Trajectory-guided video generation is a motion-aware video synthesis framework where object  
 468 movement is explicitly controlled by numerical trajectories, which are typically represented as se-  
 469 quences of  $(x, y)$  coordinates over time Xing et al. (2025); Ho et al. (2020); Song et al. (2020); Wang  
 470 et al. (2024b); Wu et al. (2025); Namekata et al. (2025); FU et al. (2025); Zhang et al. (2024). In  
 471 prior work, trajectories are manually drawn, which does not ensure physical alignment. In contrast,  
 472 we use learned equations from observational data to generate future trajectories, ensuring that the  
 473 future object motion follows discovered physical dynamics.

## 474 6 CONCLUSION

475  
 476 We introduce a novel physics-grounded, inference-only framework for motion forecasting in  
 477 trajectory-guided video forecasting, which employs ReSR for equation discovery. Experimental  
 478 results demonstrate that our approach can reliably generate future motion trajectories closely match-  
 479 ing equations derived from classical mechanics. Experimental results also highlight the limitations  
 480 of current SOTA I2V models. Even with accurate trajectories, generated videos may deviate in  
 481 fine-grained details such as velocity or deformation. Addressing these limitations requires advances  
 482 in controllable video generation models. Overall, our work illustrates the potential of integrating  
 483 interpretable equation discovery with I2V models and paves the way for applications in scientific  
 484 discovery and simulations for robotics. An exciting next step is to apply our approach to multi-body  
 485 systems involving collisions and contact dynamics. We expect this will require integrating symbolic  
 486 regression with hybrid modeling frameworks to capture discontinuous transitions.

486 ETHICS STATEMENT  
487488 As part of our study, we hired human evaluators to assess the visual and physical quality of generated  
489 videos. All participants were recruited voluntarily through an online platform and provided informed  
490 consent before participating. We ensured anonymity of all responses and did not collect personally  
491 identifiable information. Participants were compensated above the local minimum wage relative to  
492 task duration.494 REPRODUCIBILITY STATEMENT  
495496 We release code and data at <https://anonymous.4open.science/r/ReSR-0083/>. Ex-  
497 perimental details can be found in section 4.1 and 4.2. All video generation models used in our  
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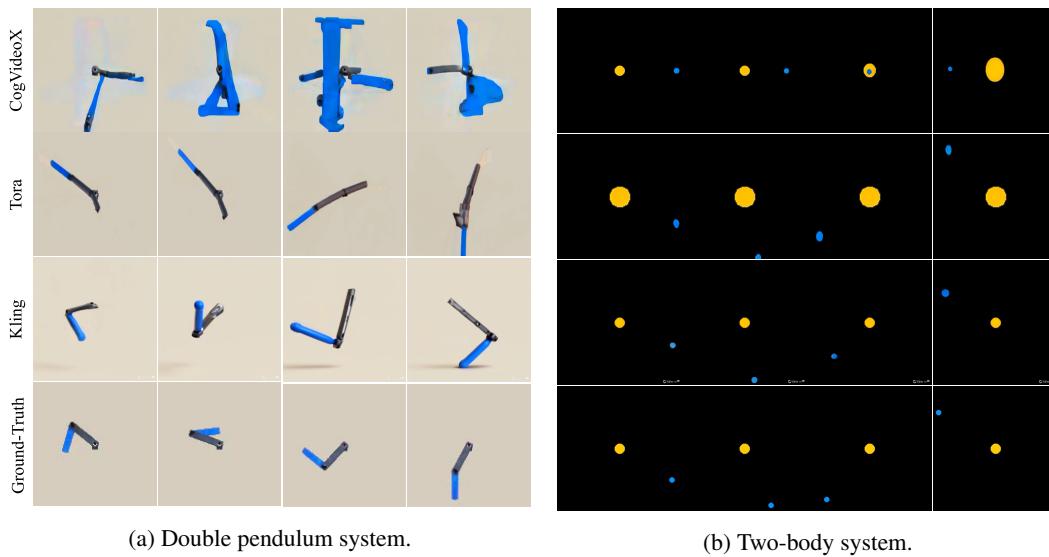
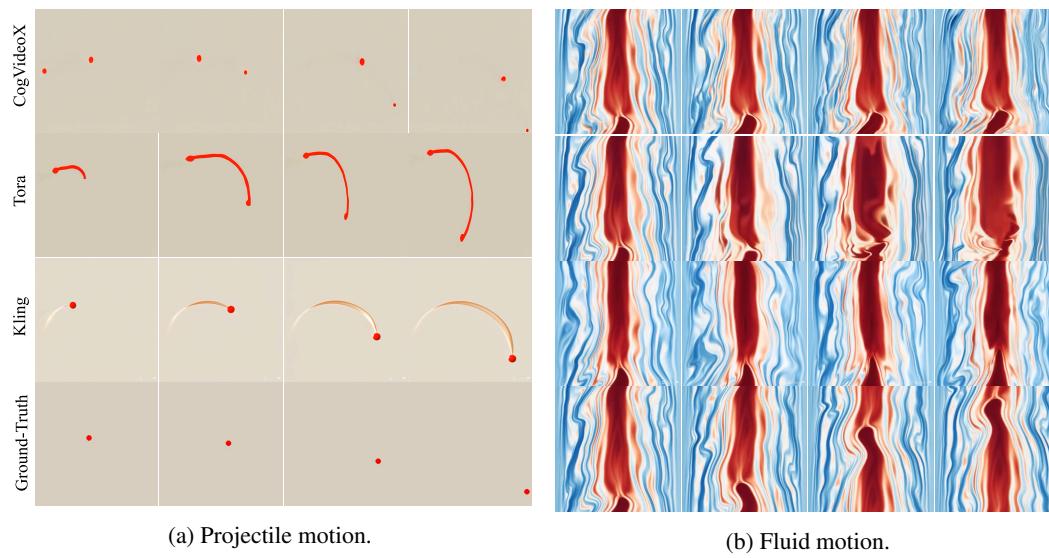
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756 A THE USE OF LARGE LANGUAGE MODELS  
757

758 In this research, large language models are employed as tools to support writing. Specifically, I used  
759 LLMs to check and refine the grammar of drafts. In addition, LLMs were applied as debugging aids  
760 during code development. Importantly, all core research ideas, experimental designs, analyses, and  
761 conclusions presented in this thesis remain our own.

762  
763 B QUALITATIVE EVALUATION  
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765 Figure 4 and 4 illustrate qualitative comparison on different physical systems across models.  
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785 Figure 4: Qualitative comparisons across models.  
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787804  
805 Figure 5: Qualitative comparisons across models.  
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