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# DISCRETE DIFFUSION FOR BUNDLE CONSTRUCTION

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## ABSTRACT

As a central task in product bundling, bundle construction aims to select a subset of items from huge item catalogs to complete a partial bundle. Existing methods often rely on the sequential construction paradigm that predicts items one at a time, nevertheless, this paradigm is fundamentally unsuitable for the essentially unordered bundles. In contrast, the non-sequential construction paradigm models bundle as a set, while it still faces two dimensionality curses: the combination complexity is exponential to the catalog size and bundle length. Accordingly, we identify two technical challenges: 1) how to effectively and efficiently model the higher-order intra-bundle relations with the growth of bundle length; and 2) how to learn item embeddings that are sufficiently discriminative while maintaining a relatively smaller search space other than the huge item set.

To address these challenges, we propose DDBC, a Discrete Diffusion model for Bundle Construction. DDBC leverages a masked denoising diffusion process to build bundles non-sequentially, capturing joint dependencies among items without relying on certain pre-defined order. To mitigate the curse of large catalog size, we integrate residual vector quantization (RVQ), which compresses item embeddings into discrete codes drawn from a globally shared codebook, enabling more efficient search while retaining semantic granularity. We evaluate our method on real-world bundle construction datasets of music playlist continuation and fashion outfit completion, and the experimental results show that DDBC can achieve more than 100% relative performance improvements compared with state-of-the-art baseline methods. Ablation and model analyses further confirm the effectiveness of both the diffusion backbone and RVQ tokenizer, where the performance gain is more significant for larger catalog size and longer bundle length. Our code is available at <https://anonymous.4open.science/r/DDBC-44EE>.

## 1 INTRODUCTION

Product bundling has been a pervasive business strategy, which originates from conventional retailing, evolves to e-commerce, and is further adopted by generic online services, such as music and video streaming (Chang et al., 2020). A product bundle is a set of relevant items assembled to satisfy users' needs (*e.g.*, games, outfits, playlists, meal kits) (Sun et al., 2024) and promote sales regarding sellers' pursuit. Bundle construction, *i.e.*, select a subset of items from the large item pools to build an entire bundle or complete a partial bundle, is the first and foremost problem among various bundle-centric studies, such as personalized bundle recommendation (Ma et al., 2022).

Existing studies, either specifically designed for bundle construction (Han et al., 2017; Bai et al., 2019; Gong et al., 2019; Deng et al., 2021) or general sequential recommendation (Kang & McAuley, 2018; Sun et al., 2019), have a fatal yet ever-overlooked flaw: most of them are based on a sequential construction paradigm, *i.e.*, predict the next item only rather than all the items in the entire bundle, however, such a sequential construction paradigm is essentially not suitable for bundle construction. Intuitively, a bundle is not a sequence of user's interacted items, and a user is not necessarily to follow a certain sequential order to consume the items within a bundle<sup>1</sup>. Thereby, sequential dependencies barely exist between consecutive items in a bundle, and sequential models bring marginal benefits to bundle construction. Delving deep into the technical foundations, consider  $N$  as the total number of items (item catalog size) and  $k$  as the bundle length (number of items

<sup>1</sup>Some bundles may have a sequential order by design, while here we focus on the general scenarios.

054 within a bundle), the theoretical space of modeling a bundle as a sequence is the permutation, *i.e.*,  
 055  $P(N, k)$ , while modeling it as a set by relaxing the sequential constraint will significantly downgrade  
 056 the space to the combination, *i.e.*,  $C(N, k)$ . Nonetheless, simply discarding the sequential construc-  
 057 tion paradigm, *e.g.*, existing non-sequential construction methods (Tomasi et al., 2024; Yang et al.,  
 058 2024), only partially addresses the problem, since the space of the combination is still exponential  
 059 to item catalog size  $N$  and bundle length  $k$ , which we call the two dimensionality curses.

060 These two dimensionality curses induce two technical challenges: First, bundle construction requires  
 061 to model the intra-bundle relations, such as similarity, compatibility, composability, *etc.*, among any  
 062 possible combinations of items, *e.g.*, pair-wise, tripartite, and quaternary (Chang et al., 2020). Thus,  
 063 how to effectively and efficiently preserve these higher-order relations, considering that the com-  
 064 plexity increases exponentially with the linear growth of  $k$ , remains the first key challenge. Second  
 065 and more seriously, the item catalogs, from which we draw items to build the bundle, are often  
 066 huge. For example,  $N$  could be tens of thousands or even millions on some online platforms such as  
 067 Spotify or Amazon. Conventional approaches typically leverage one embedding for each item (Ma  
 068 et al., 2024c), consequently, it is highly difficult to navigate through the huge candidate space and  
 069 precisely pick the desired item for a certain bundle. Therefore, how to learn item embeddings that  
 070 are sufficiently discriminative regarding different bundling functions while maintaining a relatively  
 071 small search space poses the second technical challenge.

072 To tackle the above two challenges, we propose a method that leverages *Discrete Diffusion for*  
 073 **Bundle Construction**, named as **DDBC**. Specifically, to model the higher-order intra-bundle item  
 074 relations, we introduce diffusion model as the backbone to replace the previous sequential or non-  
 075 sequential solutions. Basically, the diffusion model features with a non-sequential construction  
 076 paradigm, where it picks items according to the learned strategies regarding the entire bundle struc-  
 077 ture instead of following a certain pre-defined left-to-right sequential order. In terms of the second  
 078 challenge caused by huge item catalog size, we leverage the residual vector quantization tokenizer  
 079 (RVQ) to quantize the continuous item embedding into multiple discrete codes (Rajput et al., 2023).  
 080 The codes of each item are selected from a globally shared codebook that is significantly smaller  
 081 than the original item set, remarkably relaxing the dimensionality curse caused by  $N$ . By integrat-  
 082 ing the RVQ tokenizer into the diffusion backbone, we design our discrete diffusion model DDBC.  
 083 Concretely, we treat a complete bundle  $\bar{\mathbf{b}}$  as the clean state at  $t=0$ ; at each forward step we randomly  
 084 mask a subset of positions, eventually reaching an all- [MASK]. The training objective is to learn the  
 085 reverse denoising dynamics  $p_\theta(\mathbf{b}_{t-1} \mid \mathbf{b}_t, t)$ . During inference, given a partial bundle with unknown  
 086 slots marked [MASK], we iteratively denoise until the bundle is fully recovered. Importantly, ran-  
 087 dom masking exposes the model to rich contexts during training, thereby approximating the joint  
 088 distribution modeling over bundle items and providing the flexibility to accommodate different de-  
 089 coding priors. Also, the item codes learned by RVQ have different levels of semantic granularity,  
 090 therefore, the diffusion model can learn the bundling strategy more fine-grainedly.

091 Our contributions include: (1) We emphasize bundle construction should follow a non-sequential  
 092 construction paradigm and instantiate it with a masked denoising process. (2) We operate the diffu-  
 093 sion model in a vector-quantized discrete space using RVQ, which relaxing the dimensionality curse  
 094 caused by huge item catalog size. (3) We provide extensive and detailed empirical evidence that  
 095 our approach outperforms baselines in bundle construction, with benefits especially pronounced on  
 096 larger bundles and larger item catalogs. Comprehensive ablation and model studies further verify  
 097 the contribution of each component.

## 2 RELATED WORK

100 We review three lines of literature most relevant to our work: bundle construction, generative rec-  
 101 ommendation, and discrete diffusion models.

102 **Bundle construction** is the task of selecting a subset of items from the large item pools to build an  
 103 entire bundle or complete a partial bundle. It typically comprises two parts: (1) an encoder for users,  
 104 items, and bundles, and (2) a bundle generator. On the encoder side, early advances fuse semantics  
 105 feature (often via multimodal encoders *e.g.*, Elizalde et al. (2023)/Li et al. (2023)) with collaborative  
 106 signals (Sarwar et al., 2001; He et al., 2020) to learn stronger item embeddings (Ma et al., 2022;  
 107 2024a;c;b; Salganik et al., 2024). However, these encoders do not directly capture bundle-level  
 108 structure; semantically similar items may still not co-occur, whereas real user-constructed bundles

balance relevance, exhibit diversity, and maintain complementarity (Sun et al., 2024). Most bundle generators still follow a sequential construction paradigm (Chen et al., 2019; Bai et al., 2019; Chang et al., 2021; Deng et al., 2021; Liu et al., 2025; Han et al., 2017), however, the item order within a bundle does not necessarily reflect how users construct or consume bundles. Relying on a fixed order can introduce unnecessary order bias and harm generalization by overfitting to dataset-specific sequences (Yang et al., 2024). Several non-sequential approaches have been proposed. Wei et al. (2022) predicts all bundle items in parallel with a contrastive non-auto-regressive decoder, but it relies on predefined templates and fixed object types. Tomasi et al. (2024) uses a continuous-space diffusion model but only accepts text prompts as input. Yang et al. (2024) outputs an order-agnostic set in one shot; however, it lacks explicit intra-bundle relations interactions during generation. More importantly, all of these works generate bundles from scratch and do not address partial-bundle completion. Our approach targets this gap via discrete masked denoising, completing the missing items in an order-agnostic manner.

**Generative recommendation** is a paradigm that reframes recommendation as generating target item IDs rather than full-rank retrieving. (Wu et al., 2024; Li et al., 2024a) The line originates from generative retrieval (Rajput et al., 2023): using residual vector quantization (?Lee et al., 2022), items are quantized into multiple semantic IDs from coarse to fine, and an auto-regressive decoder generates these IDs conditioned on context. Subsequent work extends this paradigm to multimodal settings (Liu et al., 2024) and LLM backbones (Zheng et al., 2024; Zhai et al., 2025). As previously discussed, sequential construction paradigm is not suitable for bundling tasks; accordingly, we retain the multiple semantic-ID idea but replace AR backbone with a discrete diffusion model.

**Diffusion models** learn to generate data by inverting a forward noise process; in continuous spaces they have been widely used for images, audio, and trajectories (Ho et al., 2020; Janner et al., 2022; Kong et al., 2021; Liu et al., 2023; Yang et al., 2022). For discrete data, diffusion extends to categorical tokens by corrupting symbols (often to an absorbing [MASK]) and denoising to reconstruct them (Austin et al., 2021; Sahoo et al., 2024). Within recommendation, diffusion has largely been applied to sequential next-item prediction, operating on item latent embedding space (Wang et al., 2023; Yang et al., 2023; Li et al., 2024b), while discrete diffusion remains comparatively underexplored (Lin et al., 2024; Ju et al., 2025). In this work, we adopt an MDLM-style discrete diffusion backbone and leverage its order-agnostic nature to better model bundles. To the best of our knowledge, we are the first to study bundle construction with discrete diffusion.

### 3 METHOD

Our framework consists of two key components: (1) RVQ to discretize item embeddings, and (2) a DDM that operates over the code tokens for full bundle construction. The overall architecture is illustrated in Figure 1.

#### 3.1 PROBLEM FORMULATION

Let  $\mathcal{I} = \{i_1, i_2, \dots, i_N\}$  denote the item catalog and  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_M\}$  the collection of bundles, where  $N$  and  $M$  denote the number of items and bundles, respectively. Each bundle  $\mathbf{b} \in \mathcal{B}$  is set of items,  $\mathbf{b} = \{i_{j_1}, i_{j_2}, \dots, i_{j_{|\mathbf{b}|}}\}$ ,  $\{j_1, \dots, j_{|\mathbf{b}|}\} \subseteq [N]$ ,  $|\mathbf{b}| \geq 2$ . Each item  $i \in \mathcal{I}$  is mapped by a feature extractor to a latent vector  $E(i)$ , where the feature often encapsulates semantic signals and collaborative-filtering signals depending on the datasets. We formulate the bundle construction task as: for a bundle  $\bar{\mathbf{b}}$ , given a non-empty partial bundle (a subset of the entire bundle)  $\mathbf{b}_x \subseteq \bar{\mathbf{b}}$ , predict the rest part of the bundle (the complementary item set)  $\mathbf{b}_y = \bar{\mathbf{b}} \setminus \mathbf{b}_x$ .

#### 3.2 RESIDUAL QUANTIZATION OF ITEM EMBEDDINGS

To make masked discrete diffusion feasible on large catalogs, we first discretize continuous item embeddings into a compact, hierarchical code space via RVQ. We apply an  $L$ -level RVQ to obtain a tuple of discrete code indices  $\mathbf{z}(i) = (z^{(1)}(i), \dots, z^{(L)}(i))$ . The last level is a dedup code that carries no semantics and acts purely as an auto-increment field to ensure a one-to-one mapping from a code tuple back to a unique item ID. Formally,  $z^{(\ell)}(i) \in \{1, \dots, C_\ell\}$  indexes a codeword in

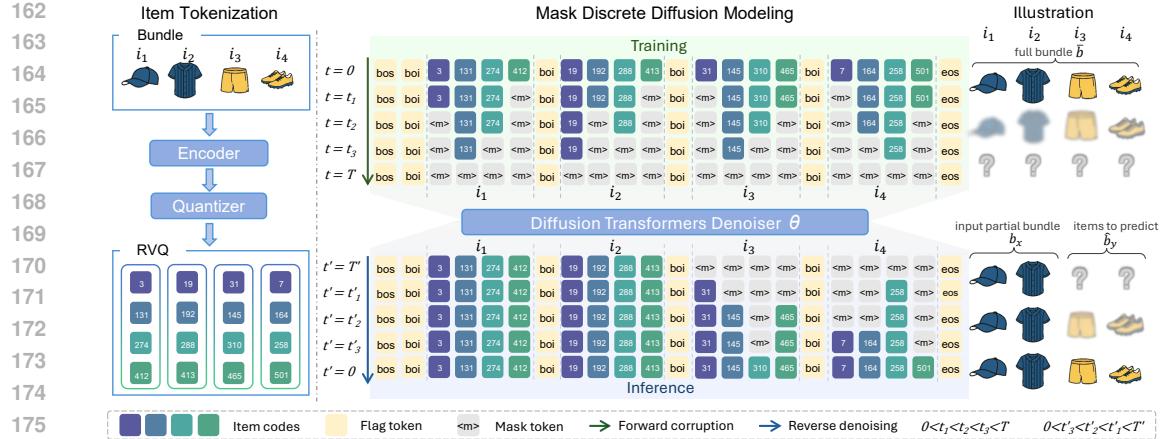


Figure 1: The overall framework of DDBC. The left side illustrates the item tokenization process via RVQ. The right side visualizes the training and inference stages of the masked discrete diffusion modeling. To be noted, we only show the forward process of the training stage, of which the backward process is just a reverse of the forward and omitted for simplicity.

codebook  $\mathcal{C}^{(\ell)} = \{\mathbf{e}_1^{(\ell)}, \dots, \mathbf{e}_{C_\ell}^{(\ell)}\} \subset \mathbb{R}^d$ . Let the residual be  $\mathbf{r}^{(0)} = E(i)$  and, for  $\ell = 1, \dots, L-1$ ,

$$z^{(\ell)}(i) = \arg \min_{c \in \{1, \dots, C_\ell\}} \|\mathbf{r}^{(\ell-1)} - \mathbf{e}_c^{(\ell)}\|_2^2, \quad \mathbf{r}^{(\ell)} = \mathbf{r}^{(\ell-1)} - \mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}. \quad (1)$$

The reconstruction uses only the semantic codebooks:  $\hat{E}(i) = \sum_{\ell=1}^{L-1} \mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}$ . Early codebooks capture coarse semantics; later residual codebooks refine details, inducing semantic smoothness among similar items. We train the codebooks with an RVQ loss that combines a reconstruction term and a codebook commitment term:

$$\mathcal{L}_{\text{RVQ}} = \|E(i) - \hat{E}(i)\|_2^2 + \beta \sum_{\ell=1}^{L-1} \left( \|\text{sg}[\mathbf{r}^{(\ell-1)}] - \mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}\|_2^2 + \|\mathbf{r}^{(\ell-1)} - \text{sg}[\mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}]\|_2^2 \right), \quad (2)$$

where  $\text{sg}[\cdot]$  denotes stop-gradient and  $\beta$  balances the commitment loss. We use a straight-through estimator for the discrete assignment; codebooks are updated via gradient descent. For a bundle  $\mathbf{b} = \{i_1, \dots, i_{|\mathbf{b}|}\}$ , RVQ yields a token matrix  $\mathbf{Z}^{(0)} \in \mathbb{N}^{|\mathbf{b}| \times L}$  with entries  $z_{j\ell} \in \{1, \dots, C_\ell\}$ ; the  $j$ -th row contains the  $L$  codes representing item  $i_j$ .

Among many quantization strategies, we choose RVQ for three reasons. (1) Vocabulary compression. Its theoretical capacity is  $\prod_{\ell=1}^L C_\ell$ , enabling a small per-level vocabulary to index a very large item universe. (2) Denser supervision. Each item contributes  $L$  code tokens; at level  $\ell$ , a code typically aggregates roughly  $N/C_\ell$  items (for  $N$  total items), so code-level supervision is markedly denser than item-ID supervision. We quantify the increase in effective supervision in Section 4. (3) Coarse-to-fine granularity of semantics. Hierarchical residual codebooks perform implicit clustering at multiple granularities, which benefits downstream denoising. Additionally, when a candidate set is available in real applications, the finest semantic level and the disambiguation index may be unnecessary at inference time: one can decode with fewer RVQ levels and map prefixes to candidate items via an inverted index over code prefixes. We study this design choice in Section 4.

### 3.3 MASKED DISCRETE DIFFUSION OVER CODE TOKENS

We cast bundle construction as a masked discrete denoising process. In contrast to sequential construction paradigm, this formulation avoids imposing any arbitrary order on the items and instead allows the model to leverage the full set context in an order-agnostic manner. Importantly, our discrete diffusion does not directly impose a set-invariant objective: items are still flattened into a list and we do not explicitly enforce permutation invariance. Nevertheless, it serves as a good approximation to set modeling: at each training step, the already-revealed context and the positions to be revealed are randomized, providing rich, position-agnostic supervision so that the learned features

216 do not rely on absolute positions. To realize this idea, we adopt a discrete diffusion framework with  
 217 an absorbing-mask corruption mechanism. The key components are: an input tokenization design  
 218 for bundles, a forward corruption process that incrementally masks tokens, a bidirectional Trans-  
 219 former as the reverse denoiser, and an order-agnostic inference procedure with token-level validity  
 220 constraints.

221

222 **Input tokenization for bundles.** We serialize a bundle by inserting `<boi>` before each item and  
 223 wrapping with `<bos>` and `<eos>`. Let  $z_{j,\ell}$  denote the  $\ell$ -th code token of item  $j$  ( $j=1, \dots, |\mathbf{b}|$ ,  
 224  $\ell=1, \dots, L$ ). We use exactly two index sets:

$$225 \quad \Omega_{\text{flag}} = \{\text{<bos>}, \text{<boi>}, \text{<eos>}\}, \quad \Omega_{\text{code}} = \{(j, \ell) : j = 1, \dots, |\mathbf{b}|, \ell = 1, \dots, L\}. \quad (3)$$

226 Tokens in  $\Omega_{\text{flag}}$  are never masked; corruption and prediction operate only on  $z_{j,\ell}$  with  $(j, \ell) \in \Omega_{\text{code}}$ .

227 **Forward corruption.** We use an absorbing-mask Markov chain as in masked discrete diffusion. At  
 228 each step  $t \in \{1, \dots, T\}$ , each currently unmasked token  $z_{j,\ell}$  with  $(j, \ell) \in \Omega_{\text{code}}$  is independently  
 229 replaced by `[MASK]` with probability  $\beta_t \in (0, 1)$ :

$$231 \quad q\left(z_{j,\ell}^{(t)} = u \mid z_{j,\ell}^{(t-1)} = u\right) = 1 - \beta_t, \quad q\left(z_{j,\ell}^{(t)} = \text{[MASK]} \mid z_{j,\ell}^{(t-1)} \neq \text{[MASK]}\right) = \beta_t, \quad (4)$$

232 for any token value  $u \neq \text{[MASK]}$ , with the absorbing condition  $q(z_{j,\ell}^{(t)} = \text{[MASK]} \mid z_{j,\ell}^{(t-1)} =$   
 233  $\text{[MASK]}) = 1$  and independence across  $(j, \ell)$ . Let  $\alpha_t = \prod_{s=1}^t (1 - \beta_s)$  be the survival probability.  
 234 The closed-form transition from  $t=0$  to  $t$  is:

$$235 \quad q\left(z_{j,\ell}^{(t)} = v \mid z_{j,\ell}^{(0)} = u\right) = \alpha_t \mathbf{1}[v = u] + (1 - \alpha_t) \mathbf{1}[v = \text{[MASK]}], \quad (5)$$

236 *i.e.*, after  $t$  steps a token either survives with probability  $\alpha_t$  or is masked with probability  $1 - \alpha_t$ .

237 **Reverse denoising.** A bidirectional Transformer  $\theta$  is trained to predict the original token values  
 238 from a corrupted sequence. At inference, it produces a categorical distribution for each masked  
 239 position conditioned on the current noisy tokens, the timestep  $t$ :

$$240 \quad p_{\theta}(z_{j,\ell}^{(0)} \mid \mathbf{C}^{(t)}, t) \in \Delta^{C_{\ell}-1}, \quad \text{where } \Delta^{C_{\ell}-1} \triangleq \{p \in [0, 1]^{C_{\ell}} : \mathbf{1}^{\top} p = 1\}. \quad (6)$$

241 Following common practice in Sahoo et al. (2024), we use the “simple” reconstruction objective that  
 242 trains  $p_{\theta}$  to predict the original token  $z_{j,\ell}^{(0)}$  directly from a state  $\mathbf{Z}^{(t)}$  corrupted at a random timestep  $t$ .  
 243 Crucially, tokens that are unmasked, either because they belong to  $\mathbf{b}_x$  or because they have already  
 244 been generated in a previous step, are treated as clamped observations; they are never masked again  
 245 and remain fixed in all subsequent steps. This mechanism enables the model to unmask items in any  
 246 order during generation, without ever overwriting a code once it’s decided.<sup>2</sup>

247 **Training objective.** Let  $\mathcal{M}_t \subseteq \Omega_{\text{code}}$  be the set of positions masked by the forward process at step  
 248  $t$ . The discrete diffusion variational objective reduces to a weighted masked-token cross-entropy:

$$249 \quad \mathcal{L}_{\text{NELBO}} = \mathbb{E}_{t \sim \mathcal{U}\{1, \dots, T\}} \mathbb{E}_{\mathcal{M}_t} \sum_{(j, \ell) \in \mathcal{M}_t} -\log p_{\theta}(z_{j,\ell}^{(0)} \mid \mathbf{Z}^{(t)}, t). \quad (7)$$

250

251 **Inference.** We model bundle construction as iterative denoising of a partially masked token matrix.  
 252 Let the observed set be  $\mathbf{b}_x$  and the unknown complementary set be  $\mathbf{b}_y$  with  $|\mathbf{b}_y|$  items. Denote  
 253 by  $\Omega_x$  the positions (including all RVQ levels) that belong to items in  $\mathbf{b}_x$ , by  $\Omega_y$  the positions that  
 254 belong to items in  $\mathbf{b}_y$ , and by  $\Omega_{\text{flag}}$ . We construct an input sequence by flattening tokens row-wise  
 255 and inserting `<boi>` before each item’s  $L$  tokens to mark boundaries. Formally, the initial state  $\mathbf{Z}$   
 256 is:

$$257 \quad z_u = \mathbf{1}[u \in \Omega_x \cup \Omega_{\text{flag}}] x_u + \mathbf{1}[u \in \Omega_y] \text{[MASK]}. \quad (8)$$

258 Here,  $x_u$  represents the given token at position  $u$ , with  $\Omega_{\text{flag}}$  kept unmasked so the model knows item  
 259 segmentation. The tokens of  $\mathbf{b}_x$  remain clamped throughout. After decoding, a predicted item  $\hat{i}_j$  is  
 260 obtained by mapping its token tuple back to the catalog or to a reconstruction  $\hat{E}_j = \sum_{\ell=1}^{L-1}$ .

261

<sup>2</sup>While diffusion can be extended to allow re-masking to revise earlier decisions, we do not enable that option here and leave it to future work.

270 **Token-validation constraints.** At generation time, we constrain the model’s predictions to ensure  
 271 that each set of  $L$  code tokens corresponds to a valid item from the catalog. Formally, for each  
 272 position  $(j, \ell)$  (code level  $\ell$  of item  $j$ ), we restrict the predicted token to the prefix-consistent subset  
 273  $\mathcal{V}_{\text{valid}}^{(\ell)}(j; \mathbf{z}_{j,<\ell}) \subseteq \{1, \dots, C_\ell\}$  and set the logits of any token not in  $\mathcal{V}_{\text{valid}}^{(\ell)}(j; \mathbf{z}_{j,<\ell})$  to  $-\infty$  before  
 274 the softmax, thereby preventing the model from selecting an invalid code combination. This validation  
 275 step ensures that the generated code tuples always decode to legitimate items, which is crucial  
 276 for maintaining recommendation feasibility.

## 278 4 EXPERIMENT

### 280 4.1 EXPERIMENTAL SETTINGS.

282 **Model settings.** We use CLHE (Ma et al., 2024c) as the item encoder, i.e.,  $E(i) = \text{CLHE}(i)$ .  
 283 Unless otherwise noted, our RVQ uses  $L = 4$  levels with fixed per-level codebook size  $C_\ell \equiv C$ ;  
 284 the diffusion horizon is  $T = \ell(L + 1)$ , where the per-item token length (including the boundary  
 285 marker) is denoted by  $\ell$ . We utilize a lightweight DDiT architecture for our Diffusion backbone,  
 286 with 6 transformer blocks, each with a hidden size of 64 and 8 self-attention heads. The model  
 287 operates with a linear noise scheduler  $\alpha(t) = 1 - t$ . All experiments are performed on four NVIDIA  
 288 A40 GPUs, and all models are trained in 20,000 steps.

289 **Datasets.** Following prior research on bundle construction Ma et al. (2024c); Liu et al. (2025), we  
 290 evaluate on two representative datasets, Spotify (Chen et al., 2018) and POG (Chen et al., 2019).  
 291 Unlike these works, our discrete diffusion model currently requires a fixed number of tokens per in-  
 292 stance, so we truncate bundles to a target length. For the Spotify playlist dataset, we create three sub-  
 293 sets by capping playlist length at 30/60/90 items (Spotify $_{k=30,60,90}$ ). For the POG fashion dataset,  
 294 whose average bundle length is small, we start from its denser variant and derive a fixed-length  
 295 version with four items (denoted POG $_{k=4}$ ). Unless noted, the input-predict ratio of the bundle,  
 296  $|\mathbf{b}_x| : |\mathbf{b}_y|$ , are set as 1 : 1, see Table 2 for other settings. Samples shorter than the target length  
 297 are dropped. Each dataset is split into train/validation/test with non-overlapping bundles. We also  
 298 perform data augmentation by swapping items within the bundle, and the details are describe in  
 299 Appendix B.

300 **Candidate size.** To standardize candidate pool, we set a candidate ratio  $\rho$  and construct a shortlist  
 301  $\mathcal{C}$  of size  $\rho |\mathbf{b}_y|$  by augmenting the ground-truth targets with randomly sampled non-targets:  $\mathcal{C} =$   
 302  $\mathbf{b}_y \cup \text{Random}_{(\rho-1)|\mathbf{b}_y|}(I \setminus \mathbf{b})$ . Unless otherwise stated, we fix  $\rho = 100$  in all experiments.

### 304 4.2 BASELINES.

306 We consider both non-sequential and sequential construction methods as baselines. To be fair, all  
 307 the baselines use the same item features, *i.e.*, pre-trained embeddings via CLHE (Ma et al., 2024c).

308 **Non-sequential construction methods.** They input the partial bundle  $\mathbf{b}_x$  and predict all the items  
 309 in the complementary set at once. *CLHE* (Ma et al., 2024c): A method that leverages contrastive  
 310 learning and hierarchical encoder to learn item and bundle representations. To be noted, CLHE was  
 311 not originally designed to predict all the items in the complementary set, while it follows the typical  
 312 top-k recommendation paradigm and evaluation protocol. We re-evaluate it against our metrics that  
 313 are pertinent to entire bundle construction. *BundleNAT* (Yang et al., 2024): A non-auto-regressive  
 314 generator that predicts a set of items in one shot using preference/compatibility signals. It was  
 315 originally used for the task of personalized bundle recommendation instead of bundle construction,  
 316 we adapt it for our task by removing the user inputs.

317 **Sequential construction methods.** They follow an auto-regressive construction strategy: initialize  
 318  $\mathbf{s}_0 = \mathbf{b}_x$ ; for  $j = 0, \dots, |\mathbf{b}_y| - 1$ , choose  $\hat{i}_j = \arg \max_{i \notin \mathbf{s}_j} \pi(i \mid \mathbf{s}_j)$  and update  $\mathbf{s}_{j+1} = \mathbf{s}_j \cup \{\hat{i}_j\}$   
 319 until  $|\mathbf{s}_{|\mathbf{b}_y|}| = |\bar{\mathbf{b}}|$ . *Bi-LSTM* (Han et al., 2017): It uses bi-directional LSTM to model the bundle  
 320 as a sequence. *SASRec* (Kang & McAuley, 2018): A Transformer-based sequential recommender  
 321 trained for next-item prediction. *TIGER* (Rajput et al., 2023): It generates items as discrete semantic  
 322 token sequences with an auto-regressive decoder. *BundleMLM* (Liu et al., 2025): It finetunes a  
 323 multimodal LLM for bundle construction. Its original evaluation is based on the multiple-choice  
 question protocol since it is impossible to input all the candidate items as input due to context

324  
325 Table 1: Overall performance comparison between our DDBC and baselines. "%Improv." denotes  
326 the relative improvement over the strongest baseline. Best in **bold**, second best underlined.

327 Model (A/beam)	328 Spotify <sub>k=30</sub>			329 Spotify <sub>k=60</sub>			330 Spotify <sub>k=90</sub>			331 POG <sub>k=4</sub>		
	332 F1 ↑	333 Jacc ↑	334 OAS ↑	332 F1 ↑	333 Jacc ↑	334 OAS ↑	332 F1 ↑	333 Jacc ↑	334 OAS ↑	332 F1 ↑	333 Jacc ↑	334 OAS ↑
328 CLHE	0.071	0.039	0.373	0.100	0.054	0.446	0.119	0.065	0.486	0.140	0.096	0.446
329 Bi-LSTM	0.124	0.071	<u>0.489</u>	0.062	0.034	0.430	0.047	0.025	0.426	0.035	0.024	0.390
330 SASRec	0.070	0.043	0.318	0.089	0.054	0.310	0.050	0.029	0.285	<u>0.169</u>	<u>0.114</u>	0.468
331 TIGER	0.093	0.053	0.329	<u>0.129</u>	<u>0.076</u>	0.413	<u>0.123</u>	<u>0.070</u>	0.480	<b>0.213</b>	<b>0.157</b>	<b>0.546</b>
332 BundleNAT	<u>0.153</u>	<u>0.090</u>	0.454	0.101	0.056	0.438	0.095	0.052	0.446	0.145	0.097	0.462
332 BundleMLLM	0.046	0.024	0.296	0.045	0.024	0.324	0.052	0.027	0.355	0.070	0.047	0.322
333 <b>DDBC</b>	<b>0.282</b>	<b>0.178</b>	<b>0.618</b>	<b>0.296</b>	<b>0.185</b>	<b>0.668</b>	<b>0.287</b>	<b>0.177</b>	<b>0.684</b>	0.139	0.098	<u>0.526</u>
334 %Improv. +	84.3%	97.8%	26.4%	129.5%	143.4%	49.8%	133.3%	152.9%	40.7%	—	—	—

335  
336 limitation of LLMs. Even though this setting is easier than our all-ranking setting, to be simple, we  
337 follow this paradigm and set the candidate set as 20.

338 To be noted, many other recommendation models can be adapted as baselines by following the  
339 paradigm of either the sequential or non-sequential construction. For example, the baselines im-  
340 plemented in Ma et al. (2024c): MultiDAE (Wu et al., 2016), MultiVAE (Liang et al., 2018), Hy-  
341 pergraph (Yu et al., 2022), and Transformer (Wei et al., 2023), *etc.* or the other advanced sequen-  
342 tial recommendation models. However, we do not include them because they either underperform  
343 CLHE or not highly relevant to the bundle scenario. We implement the most relevant and strongest  
344 baselines to the best of our knowledge, and more baselines could be implemented upon request.

### 345 4.3 EVALUATION METRICS

346  
347 We report retrieval-based metrics F1 and Jaccard (Jacc) (Manning et al., 2008; Ding et al., 2023), as  
348 well as a latent-space similarity-based metric OAS (Salton et al., 1975). Higher F1, Jacc, and OAS  
349 indicate better performance. Let  $\hat{\mathbf{b}}_y$  denote the set of predicted items, these metrics are calculated  
350 by:

$$354 \quad F1 := \frac{2PR}{P+R}, \quad Jacc := \frac{|\hat{\mathbf{b}}_y \cap \mathbf{b}_y|}{|\hat{\mathbf{b}}_y \cup \mathbf{b}_y|}, \quad OAS := \frac{1}{|\mathbf{b}_y|} \max_M \sum_{(\alpha, \beta) \in M} \cos(E(\alpha), E(\beta)), \quad (9)$$

355  
356 where  $P = \frac{|\hat{\mathbf{b}}_y \cap \mathbf{b}_y|}{|\hat{\mathbf{b}}_y|}$ ,  $R = \frac{|\hat{\mathbf{b}}_y \cap \mathbf{b}_y|}{|\mathbf{b}_y|}$ , and  $M$  is the optimal matching between items in  $\hat{\mathbf{b}}_y$  and  $\mathbf{b}_y$   
357 and  $\cos(\cdot, \cdot)$  denotes cosine similarity. Previous methods in bundle construction use popular next-  
358 item recommendation metrics, such as recall, ndcg, or hit rate (Ma et al., 2024c). However, these  
359 metrics are not suitable in the scenario of full bundle construction, which needs to assess the quality  
360 of the predicted entire item set instead of single item. Therefore, we propose these three metrics  
361 to collaboratively measure the performance of bundle construction, offering a comprehensive and  
362 consistent benchmark setting for future studies.

### 363 4.4 OVERALL PERFORMANCE COMPARISON

364  
365 Table 1 shows the overall performance of DDBC compared with baseline methods. First, among  
366 the baselines, BundleNAT and TIGER achieve the strongest performance. These results respec-  
367 tively highlight two key component of our model: the non-sequential construction paradigm and the  
368 advantages of discretizing items into multiple codes. Second, on the Spotify dataset series, DDBC  
369 clearly outperforms all baselines, achieving a 153% improvement in Jacc on Spotify<sub>k=90</sub>. Moreover,  
370 the performance gain of DDBC becomes more pronounced as the bundle sequence length increases.  
371 These results demonstrate that DDBC effectively captures the higher-order intra-bundle item  
372 relations, particularly for long-sequence bundles with rich structural dependencies. Third, on POG<sub>k=4</sub>,  
373 our model does not outperform TIGER. In fact, since we only predict two items, the task, to some  
374 extent reduces to a next-item prediction scenario, where auto-regressive methods such as TIGER  
375 have a clear advantage.

378

Table 2: Effect of input-predict ratio on Spotify<sub>k=60</sub>. Best in **bold**, second best underlined.

Model	5/55			10/50			30/30			45/15		
	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑
BundleNAT	0.106	0.059	0.443	0.128	0.072	0.463	0.101	0.056	0.438	0.084	0.046	0.359
SASRec	<u>0.119</u>	<u>0.070</u>	0.425	<u>0.131</u>	<u>0.078</u>	0.442	0.089	0.054	0.310	0.095	0.055	0.315
TIGER	0.087	0.050	0.365	0.100	0.059	0.381	0.129	0.076	0.413	0.154	<u>0.091</u>	0.426
<b>DDBC</b>	<b>0.237</b>	<b>0.144</b>	<b>0.637</b>	<b>0.268</b>	<b>0.164</b>	<b>0.664</b>	<b>0.296</b>	<b>0.185</b>	<b>0.668</b>	<b>0.260</b>	<b>0.161</b>	<b>0.614</b>
<i>Improv. +</i>	99.2%	105.7%	43.8%	104.6%	110.3%	43.4%	129.5%	143.4%	52.5%	68.8%	76.9%	44.1%

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387

Table 3: Effect of candidate ratio on Spotify<sub>k=60</sub>. Best in **bold**, second best underlined.

Model	$\rho=10$			$\rho=20$			$\rho=50$			$\rho=100$		
	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑	F1 ↑	Jacc ↑	OAS ↑
BundleNAT	0.266	0.163	0.508	<u>0.210</u>	0.124	0.485	0.153	0.088	0.464	0.101	0.056	0.438
SASRec	<u>0.292</u>	<u>0.194</u>	<u>0.519</u>	0.200	<u>0.126</u>	0.474	0.200	0.126	<u>0.475</u>	0.089	0.054	0.310
TIGER	0.191	0.151	0.326	0.107	0.080	0.295	0.108	0.081	0.296	0.129	<u>0.076</u>	0.413
<b>DDBC</b>	<b>0.599</b>	<b>0.447</b>	<b>0.763</b>	<b>0.503</b>	<b>0.355</b>	<b>0.727</b>	<b>0.380</b>	<b>0.250</b>	<b>0.689</b>	<b>0.296</b>	<b>0.185</b>	<b>0.668</b>
<i>Improv. +</i>	105.1%	130.4%	47.0%	139.5%	181.7%	49.9%	90.0%	98.4%	45.1%	129.5%	143.4%	52.5%

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## 397 4.5 MODEL STUDY

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**Effect of input-predict ratio.** We conduct experiments with different input-predict ratio on Spotify<sub>k=60</sub> and report results in Table 2. We observe that DDBC outperforms all baselines across different partial bundle sizes and exhibits a relatively consistent performance, demonstrating its robustness in scenarios with limited known items. Specifically, when the known partial bundles are small (e.g., 5/55, 10/50, 30/30), DDBC achieves substantial improvements over the best baseline by 106%, 110%, 143% on Jaccard, respectively. Although the performance gap narrows down as the number of input items grow, our method continues to maintain a leading position. These results highlight that DDBC is capable of generating coherent and distribution-aware bundles even when only a small subset of items is provided, validating the effectiveness of our masked denoising formulation and the discrete diffusion mechanism.

399

**Effect of candidate ratio.** In addition, we report the results for DDBC and the baselines under different candidate ratios in Table 3. The results indicate that while the absolute values of the evaluation metrics fluctuate as the candidate ratio ( $\rho$ ) increases, the relative improvements of DDBC over all baselines remain consistently substantial. Interestingly, among baselines, when  $\rho$  increases, TIGER start to bypass other baselines on F1 and Jacc ( $\rho=100$ ). This can be attributed to the fact that RVQ allows precise reconstruction of item IDs, implying the advantages of using RVQ. These findings highlight the adaptability of DDBC under varying candidate pool sizes, demonstrating its ability to maintain strong bundle representations even when the retrieval space becomes more challenging.

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**Efficiency analysis.** We record the inference time and parameter size of DDBC and the baseline methods, as reported in Figure 2, where the circle radius indicates each model’s overall performance. The inference time is measured on Spotify<sub>k=60</sub>. Specifically, although Bi-LSTM has fast inference and smallest parameters, its performance is not competitive (see Table 1). DDBC is highly parameter-efficient, containing only 0.79M parameters, and is significantly smaller than other baselines. Moreover, DDBC’s inference speed is comparable to the one-shot generation method BundleNAT and faster than all other baseline models; in particular, it is substantially faster than BundleMLLM, which relies on interactions with large language models.

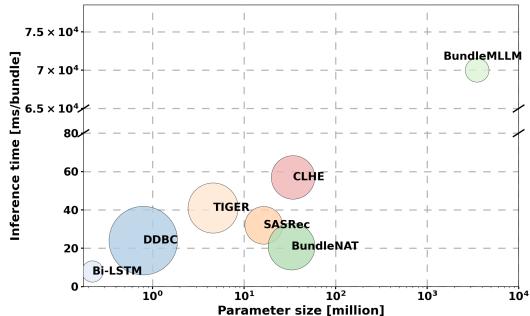


Figure 2: Illustration of the model efficiency comparison. The x-axis is parameter size (millions), y-axis is inference time (milliseconds per bundle), and the bubble radius corresponds to overall performance (larger is better).

432 4.6 ABLATION STUDY  
433

434 **Key components.** To further evaluate the effectiveness of key components of our model, we conduct  
435 ablation experiments (Table 4) to assess the contribution of each design choice in DDBC. Considering  
436 the resource and time overhead imposed by the extremely large vocabulary in Spotify (254,155),  
437 performing ablation studies without RVQ would be prohibitively expensive. Therefore, we adopt  
438 Spotify  $k = 30$ , the shortest sequence setting, as a more practical benchmark for these experiments.  
439 We study how each component contributes to  
440 performance: hierarchical (coarse-to-fine) de-  
441 coding, token validity filter, boi token, data aug-  
442 mentation, we also investigate how different  
443 RVQ depth affect performance. We derive the  
444 following insights. (1) Removing RVQ results  
445 in a dramatic performance drop. We encode  
446 items using their IDs and initialize their embed-  
447 dings with CLHE features, results in a dramatic  
448 drop in both F1 and Jaccard. This demonstrates  
449 RVQ mitigates the dimensionality curse due to  $N$ , which is crucial as it permits dense supervi-  
450 sion. (2) Discarding the boi token leads to a performance decline. Since Diffusion’s generation  
451 lacks inherent sequence, integrating the boi token is necessary to guide the model with positional  
452 information. (3) Data augmentation shows beneficial for modeling. The results of training on the  
453 original dataset show a slight performance reduction in this case, and simple data augmentation  
454 remains significant for diffusion modeling because it explicitly provides richer input context. (4)  
455 The token validity filter remains essential to guarantee the validity of the generated bundles, despite  
456 its removal leading to only a marginal decrease in performance. We evaluated the necessity of the  
457 token validity filter during inference. While removing the filter resulted in only a marginal decrease  
458 in overall performance, the invalid ratio concurrently rose to 2.5%. Therefore, the filter remains  
459 essential to guarantee the validity of the generated bundles.  
460

461 **Effect of RVQ depth.** To investigate the impact of the item  
462 embedding quantization level on model performance as dis-  
463 cussed in method section, with a fixed 4 levels RVQ, we train  
464 DDBC with utilizing different levels of RVQ. We report the  
465 results in Table 5. As the number of RVQ levels used increases,  
466 the model captures increasingly finer-grained item infor-  
467 mation, leading to substantial improvements in all the evalua-  
468 tion metrics. We state that the current setting represents a favorable  
469 trade-off between the representational capacity and compres-  
470 sion ratio of RVQ.  
471

472 5 CONCLUSION  
473

474 We recast bundle construction with a masked discrete diffusion model that progressively resolves  
475 unknown items in an order-agnostic manner. Conceptually, the formulation address the dual di-  
476 mensionality curses: (i) it removes spurious ordering, reducing the search space from permutations  
477 to combinations, preserves fine-grained, higher-order item relations, and (ii) shrinks the effective  
478 search space by mapping items to codes drawn from a globally shared codebook. Empirically, cou-  
479 pling DDM with RVQ yields consistent gains over prior sequential and non-sequential construction  
480 baselines, with especially strong improvements as bundle length grows.  
481

482 **Discussion.** Our current instantiation assumes fixed-length bundles, learning when to stop (*i.e.*,  
483 variable-length completion and principled halting criteria) remains open. Personalization is medi-  
484 ated by frozen encoders for user-item signals and item semantics; introducing explicit condition-  
485 ing into the diffusion process (*e.g.*, context features, or user instruction) could yield user-specific  
486 bundling. The RVQ design space (*e.g.*, number of levels, codebook sizes, and training regimes)  
487 deserves further study to balance identifiability, compression, and semantic smoothness. Finally,  
488 diffusion schedules and inference policies merit deeper optimization: adaptive timestep schedules,  
489 selective re-masking strategies, and entropy-guided decoding may improve sample efficiency and  
490 robustness.  
491

492 Table 4: Ablation study of key components.  
493

Variant	F1	Jacc	OAS
Our proposed DDBC	0.166	0.092	0.620
w/o RVQ	0.028	0.015	0.556
w/o boi token	0.116	0.063	0.536
w/o data augmentation	0.152	0.084	0.598
w/o token validity filter	0.163	0.090	–

494 Table 5: Effect of RVQ levels.  
495

$\ell \in$	F1	Jacc	OAS
{1}	0.096	0.051	0.555
{1, 2}	0.122	0.066	0.591
{1, 2, 3}	0.148	0.081	0.590
<i>Our proposed DDBC</i>			
{1, 2, 3, 4}	0.166	0.092	0.620

486 ETHICS STATEMENT  
487488 We affirm compliance with the ICLR Code of Ethics. Our study addresses bundle construction (e.g.,  
489 playlists and fashion outfits) and uses publicly available research datasets and splits released by prior  
490 work; no personally identifiable information (PII) or sensitive attributes are collected, inferred, or  
491 released. The inputs consist of item identifiers and non-sensitive metadata, and our models operate  
492 on discretized representations without accessing user profiles. Any code and models we release will  
493 be for research use only and will not include copyrighted media or proprietary assets.  
494495 REPRODUCIBILITY STATEMENT  
496497 We make our method reproducible by specifying the full training and evaluation pipeline, including  
498 the RVQ configuration, diffusion horizon, architecture, schedulers, and all hyperparameters. We pro-  
499 vide an anonymized repository <https://anonymous.4open.science/r/DDBC-44EE>,  
500 including the implementation of our model as well as the evaluation scripts for F1, Jaccard, and  
501 OAS. Upon publication, we plan to release checkpoints (where licenses permit) to reproduce all  
502 main and ablation results.  
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## 652 A THE USE OF LARGE LANGUAGE MODELS

653  
 654 We used large language models (*e.g.*, ChatGPT 5, Claude) as an assistive tool for writing polish  
 655 (grammar, phrasing, and LaTeX formatting), troubleshooting LaTeX errors, and scaffolding non-  
 656 critical scripts (plotting and small utilities). LLMs did not contribute novel scientific ideas, data  
 657 collection, or result selection, and any code snippets suggested by an LLM were reviewed, rewritten  
 658 where necessary, and validated by the authors. All technical claims, mathematical formulations,  
 659 and empirical results are the authors’ responsibility. LLMs are not listed as authors and have no  
 660 authorship rights.

## 662 B IMPLEMENTATION DETAILS

663  
 664 **Dataset statistics.** The statistics for the datasets used in our experiments are summarized in Table  
 665 6. All Spotify series dataset share a large item catalog size ( $N$ ) of 254,155, with a massive candidate  
 666 space. Density  $\text{Spotify}_{k=30} > \text{Spotify}_{k=60} > \text{Spotify}_{k=90}$ . Conversely, the  $\text{POG}_{k=4}$  dataset features  
 667 a substantially smaller item catalog size ( $N$ ) of 31,217 and contains a more consistent number of  
 668 bundles across the splits (*e.g.*,  $M_{\text{train}} = 29,704$ ). This variation in item catalog size and bundle  
 669 count allows for a comprehensive evaluation of our method’s scalability and performance under  
 670 different data density conditions.

671 **Data augmentation.** To improve the model’s robustness and prevent the bundle overfit to the default  
 672 sequential item order of the bundle, we employ a data augmentation strategy based on item swap-  
 673 ping. Specifically, for each original item sequence, we performed a series of adjacent item swaps  
 674 on a copy of the sequence. For the POG dense dataset, we set swap ratio 0.8, and for the Spotify  
 675 datasets, we used swap ratio 0.4. Subsequently, we randomly sampled a fixed-length subsequence  
 676 (sequence length) from the perturbed sequence, creating a new augmented training instance. This  
 677 data augmentation is an enhancement to the diffusion model, fulfilling the non-sequential modeling  
 678 objective while countering the potential issue of overfitting to the certain given sequential order in  
 679 the bundle.

680 **Over-retrieval.** To standardize the evaluation of generative models which can produce multiple  
 681 possible outputs, we employ an Over-Retrieval Strategy. This strategy aggregates the results from  
 682 multiple generation attempts, effectively forming the union  $\hat{B}_y$  used in the retrieval-based metrics.  
 683 For generative sampling models, we evaluate performance under a varying number of attempts  $b \in$   
 684  $\{1, 5, 10, 20, 50\}$  (denoted as Multiple Sampling, MS). For auto-regressive baselines that use beam  
 685 search, we report the results using beam width  $b \in \{1, 3, 5, 10, 20, 50\}$ , mapping the beam width to  
 686 the number of attempts ( $K = b$ ) for a fair comparison of computational cost.

687 Table 6: Dataset statistics.  $N$  is the catalog size (total number of items in the dataset);  $M_{\text{train/val/test}}$   
 688 are the number of bundles in train/val/test sets.

Dataset	$N$	$M_{\text{train}}$	$M_{\text{val}}$	$M_{\text{test}}$
$\text{Spotify}_{k=30}$	254,155	321,929	1,374	2,744
$\text{Spotify}_{k=60}$	254,155	253,358	798	1,582
$\text{Spotify}_{k=90}$	254,155	188,618	463	969
$\text{POG}_{k=4}$	31,217	29,704	1,303	2,521

696 **Input tokenization for bundles (details).** Given  $b = \{i_1, \dots, i_{|b|}\}$  with item codes  $\mathbf{z}(i_j) =$   
 697  $(z_{j,1}, \dots, z_{j,L})$ , the serialized sequence is  
 698  $\mathbf{x} = (\text{<bos>}, \text{<boi>}, z_{1,1}, \dots, z_{1,L}, \text{<boi>}, z_{2,1}, \dots, z_{2,L}, \dots, \text{<boi>}, z_{|b|,1}, \dots, z_{|b|,L}, \text{<eos>}).$   
 699 (10)

700 Its length is  $U = 2 + |b|(L+1)$ . Define the index map for item  $j$  and level  $\ell$ :

$$701 u(j, 0) = 1 + (j-1)(L+1) + 1, \quad u(j, \ell) = u(j, 0) + \ell, \quad \ell \in \{1, \dots, L\}. \quad (11)$$

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702 **Algorithm 1** RVQ-ENCODE for an item embedding  
703

704 **Require:** Item embedding  $E(i) \in \mathbb{R}^d$ ; semantic codebooks  $\{\mathcal{C}^{(1)}, \dots, \mathcal{C}^{(L-1)}\}$  with  $\mathcal{C}^{(\ell)} = \{\mathbf{e}_c^{(\ell)}\}_{c=1}^{C_\ell} \subset \mathbb{R}^d$ ;  
705 dedup indexer  $\text{DEDUP}(i) \in \{1, \dots, C_L\}$   
706 **Ensure:** Code indices  $\mathbf{z}(i) = (z^{(1)}(i), \dots, z^{(L)}(i))$  and reconstruction  $\hat{E}(i)$

707 1:  $\mathbf{r} \leftarrow E(i); \hat{E} \leftarrow \mathbf{0}_d$   
708 2: **for**  $\ell = 1$  to  $L - 1$  **do** ▷ semantic levels  
709 3:  $z^{(\ell)}(i) \leftarrow \underset{c \in \{1, \dots, C_\ell\}}{\operatorname{argmin}} \|\mathbf{r} - \mathbf{e}_c^{(\ell)}\|_2^2$  (tie-break: smallest index)  
710 4:  $\hat{E} \leftarrow \hat{E} + \mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}; \mathbf{r} \leftarrow \mathbf{r} - \mathbf{e}_{z^{(\ell)}(i)}^{(\ell)}$   
711 5: **end for**  
712 6:  $z^{(L)}(i) \leftarrow \text{DEDUP}(i)$  ▷ non-semantic dedup level  
713 7: **return**  $\mathbf{z}(i), \hat{E}(i) = \hat{E}$

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715 **Algorithm 2** Constraint-aware order-agnostic decoding (inference)  
716

717 **Require:** Observed item set  $\mathbf{b}_x$ ; index maps  $u(j, \ell)$  and  $\text{INVIDX}(u) \rightarrow (j, \ell)$ ; code-domain valid sets  
718  $\{\mathcal{V}_{\text{valid}}^{(\ell)}(j; \mathbf{z}_{j, < \ell})\}$ ; diffusion model  $p_\theta$ ; horizon  $T$   
719 **Ensure:** Clean token matrix  $\mathbf{Z}^{(0)}$  and completed bundle  $\hat{\mathbf{b}}$

720 1: Initialize  $\mathbf{Z}$  with tokens for  $\Omega_x$  and for  $\Omega_{\text{flag}}$ ; set  $z_u \leftarrow [\text{MASK}]$  for all  $u \in \Omega_y$   
721 2: **while** there exists  $u \in \Omega_y$  with  $z_u = [\text{MASK}]$  **do**  
722 3: Choose a timestep  $t \in \{1, \dots, T\}$  (e.g.,  $t = T - s + 1$  at step  $s$ , or by a schedule)  
723 4: **for all**  $u \in \Omega_y$  with  $z_u = [\text{MASK}]$  **do**  
724 5:  $(j, \ell) \leftarrow \text{INVIDX}(u)$   
725 6:  $\pi \leftarrow p_\theta(\cdot \mid \mathbf{Z}^{(t)} = \mathbf{Z}, t)$  categorical over  $\{1, \dots, C_\ell\}$  (no [MASK])  
726 7: **mask out invalids:**  $\pi[c] \leftarrow 0$  for  $c \notin \mathcal{V}_{\text{valid}}^{(\ell)}(j; \mathbf{z}_{j, < \ell})$ ;  $\pi \leftarrow \pi / \sum_c \pi[c]$   
8:  $P(u) \leftarrow \pi$   
9: **end for**  
10: Select a reveal set  $S \subseteq \{u \in \Omega_y : z_u = [\text{MASK}]\}$  (e.g., top- $k$  by  $\max P(u)$ , lowest-entropy, or  
11: reveal ratio  $\eta$ )  
12: **for all**  $u \in S$  **do**  
13:   **decode:**  $z_u \leftarrow \arg \max P(u)$  (or sample with temperature/top- $p$ )  
14:   **clamp:**  $z_u$  stays unmasked thereafter  
15: **end while**  
16:  $\mathbf{Z}^{(0)} \leftarrow \mathbf{Z}$   
17: **return**  $\mathbf{Z}^{(0)}, \hat{\mathbf{b}}$  via  $\hat{i}_j = \text{CODE2ITEM}(z_{j, 1:L})$  for all  $j$

---

736  
737 We then specify exactly two sets:

738 
$$\Omega_{\text{flag}} = \{1, U\} \cup \{u(j, 0)\}_{j=1}^{|b|}, \quad \Omega_{\text{code}} = \{u(j, \ell) : j = 1, \dots, |b|, \ell = 1, \dots, L\}. \quad (12)$$

739 By construction,  $\Omega_{\text{flag}} \cap \Omega_{\text{code}} = \emptyset$  and  $\Omega_{\text{flag}} \cup \Omega_{\text{code}} = [U]$ . Positions in  $\Omega_{\text{flag}}$  are never masked;  
740 corruption and prediction operate only on  $\Omega_{\text{code}}$  (including the dedup level  $\ell = L$ ).  
741

742 **RVQ encoding pseudocode.** We elucidate the pseudocode in Algorithm 1, specifying the encoding  
743 for an item embedding.

744 **Evaluation metric setting.** To quantify the similarity between  $p_{\text{Predict}}$  and  $p_{\text{Target}}$ , we compute the  
745 pairwise similarity  $S(t_i, \hat{t}_j)$  for all tracks  $t_i \in p_{\text{Target}}$  and  $t_j \in p_{\text{Predict}}$ . This setup forms a bipartite  
746 graph, where the nodes correspond to tracks in the two playlists, and the edge weights represent  
747 their pairwise similarity scores. The total similarity is defined as the *Optimal Weighted Bipartite*  
748 *Matching*:

749  
750 
$$M^* = \arg \max_M \sum_{(t_i, t_j) \in M} S(t_i, t_j), \quad (13)$$

751 where  $M$  is a bijective mapping between  $p_{\text{Target}}$  and  $p_{\text{Predict}}$ .

752 **Hungarian algorithm.** We employ the Hungarian algorithm to solve the optimal matching problem.  
753 The steps are detailed with pseudocode format in Algorithm 3.

756	<b>Algorithm 3</b> OAS via Hungarian Algorithm (maximize sum of cosine similarities)
757	
758	<b>Require:</b> Predicted set $\hat{\mathbf{b}}_y = \{\hat{i}_1, \dots, \hat{i}_{\hat{n}}\}$ (duplicates removed); ground-truth set $\mathbf{b}_y = \{i_1, \dots, i_n\}$ ; embeddings $E(\cdot)$
759	<b>Ensure:</b> Optimal matching $M \subseteq \{1, \dots, \hat{n}\} \times \{1, \dots, n\}$ and OAS
760	1: <b>build similarity:</b> $S \in \mathbb{R}^{\hat{n} \times n}$ with $S[a, b] \leftarrow \cos(E(\hat{i}_a), E(i_b))$
761	2: $m \leftarrow \max(\hat{n}, n)$
762	3: <b>build square cost:</b> $\tilde{C} \in \mathbb{R}^{m \times m}$ <span style="float: right;">▷ convert max-sim to min-cost</span>
763	4: <b>for</b> $a = 1$ to $m$ <b>do</b>
764	5: <b>for</b> $b = 1$ to $m$ <b>do</b>
765	6: <b>if</b> $a \leq \hat{n}$ <b>and</b> $b \leq n$ <b>then</b>
766	7: $\tilde{C}[a, b] \leftarrow 1 - S[a, b]$ <span style="float: right;">▷ cost <math>\in [0, 2]</math> since <math>\cos \in [-1, 1]</math></span>
767	8: <b>else</b>
768	9: $\tilde{C}[a, b] \leftarrow 1$ <span style="float: right;">▷ dummy pairs have similarity 0</span>
769	10: <b>end if</b>
770	11: <b>end for</b>
771	12: <b>end for</b>
772	13: <b>row reduction:</b> $\tilde{C}[a, \cdot] \leftarrow \tilde{C}[a, \cdot] - \min_b \tilde{C}[a, b]$ for all $a$
773	14: <b>column reduction:</b> $\tilde{C}[\cdot, b] \leftarrow \tilde{C}[\cdot, b] - \min_a \tilde{C}[a, b]$ for all $b$
774	15: <b>repeat</b>
775	16:     Cover all zeros in $\tilde{C}$ by the minimum number of horizontal/vertical lines
776	17: <b>if</b> (#lines < $m$ ) <b>then</b>
777	18: $\Delta \leftarrow \min\{\tilde{C}[a, b] : \tilde{C}[a, b] \text{ is uncovered}\}$
778	19:         Subtract $\Delta$ from every <i>uncovered</i> entry
779	20:         Add $\Delta$ to every <i>doubly-covered</i> entry
780	21:         (singly-covered entries unchanged)
781	22: <b>end if</b>
782	23: <b>until</b> #lines = $m$
783	24: <b>extract assignment:</b> find $m$ independent zeros (no two share a row/column) to form an optimal assignment
784	$\tilde{M} \subseteq \{1, \dots, m\}^2$
785	25: <b>restrict to real items:</b> $M \leftarrow \{(a, b) \in \tilde{M} : a \leq \hat{n}, b \leq n\}$
786	26: $S_{\text{sum}} \leftarrow \sum_{(a, b) \in M} S[a, b]$
787	27: OAS $\leftarrow \frac{S_{\text{sum}}}{n}$ <span style="float: right;">▷ denominator is <math> \mathbf{b}_y  = n</math></span>
788	28: <b>return</b> $M, \text{OAS}$

## C ADDITIONAL RESULTS

**Comparison across datasets using Jaccard.** We compare the results across datasets using Jaccard with multiple attempts. As Table 7 shows, among the established baselines, BundleNAT generally achieves the best Jaccard performance across the Spotify datasets. This suggests that BundleNAT’s non-auto-regressive architecture is particularly effective at generating relevant set-based results compared to the sequential models. What’s more, our proposed method, DDBC, consistently and significantly outperforms all baselines across every dataset and attempt level. On Spotify <sub>$k=30$</sub> , the DDBC model achieves a Jaccard@1 of 0.164, nearly doubling the performance of the best baseline (BundleNAT at 0.090). This performance gap confirms the efficacy and advanced capability of our model, especially when generating predictions with multiple attempts.

Table 7: Comparison across datasets using Jaccard with  $A \in \{1, 5, 20\}$ . Best in **bold**, second best underlined.

Model (A/beam)	Spotify <sub><math>k=30</math></sub>			Spotify <sub><math>k=60</math></sub>			Spotify <sub><math>k=90</math></sub>			POG <sub><math>k=4</math></sub>		
	Jacc@1	Jacc@5	Jacc@20	Jacc@1	Jacc@5	Jacc@20	Jacc@1	Jacc@5	Jacc@20	Jacc@1	Jacc@5	Jacc@20
CLHE	.009	.005	.002	.008	.004	.002	.007	.003	.001	.059	.048	.020
SASRec	.043	.023	.009	.054	.030	.013	.029	.013	.005	.114	.066	.024
TIGER	.053	.028	.011	<u>.076</u>	.036	.013	<u>.070</u>	.034	.013	<u>.157</u>	<u>.094</u>	<u>.038</u>
BundleNAT	<u>.090</u>	<u>.076</u>	<u>.033</u>	.056	<u>.055</u>	<u>.027</u>	.052	<u>.052</u>	<u>.026</u>	.097	.055	.023
<b>DDBC</b>	<b>.164</b>	<b>.130</b>	<b>.053</b>	<b>.185</b>	<b>.137</b>	<b>.055</b>	<b>.177</b>	<b>.132</b>	<b>.054</b>	.098	<u>.073</u>	<u>.032</u>

**Latent-space quality.** To better explore the latent-space quality of the items generated by our method, we report the OAS metric at  $A = 50$ , as shown in Table 8. For the Spotify dataset series,

when the bundle length ( $k$ ) increases from 30 to 90, the latent-space quality improves substantially and stable. Specifically, the average OAS decreases consistently from 0.397 ( $\text{Spotify}_{k=30}$ ) to 0.338 ( $\text{Spotify}_{k=60}$ ) and finally to 0.315 ( $\text{Spotify}_{k=90}$ ). Since a lower OAS score indicates lower distance and higher similarity, this trend suggests that our method has an improved capacity to model longer bundles.

Table 8: Latent-space quality at  $A=50$ . We report the  $\{\text{min}, \text{avg}, \text{max}, \text{var}\}$  OAS over test bundles, corresponding to minimal, average, maximal, and variance, respectively.

Dataset	min	avg	max	var
$\text{Spotify}_{k=30}$	0.477	0.603	0.717	0.003
$\text{Spotify}_{k=60}$	0.582	0.662	0.733	0.001
$\text{Spotify}_{k=90}$	0.625	0.685	0.738	0.001
$\text{POG}_{k=4}$	0.246	0.525	0.791	0.018

**Additional ablation study results.** We report ablation study results on  $\text{Spotify}_{k=30}$  ( $A=1$  or  $A=10$ ) in Table 9. The results obtained at  $A=10$  are consistent with those observed at  $A=1$ . (1) The Residual Vector Quantization (RVQ) component exhibits to be absolutely indispensable. This result confirms the substantial mechanism of RVQ to mitigate the dimensionality curse caused by the huge item catalog size ( $N$ ). (2) The boi token could provide positional guidance and improve latent space quality largely. Removing the boi token results in a significant performance degradation. (3) Simple data augmentation (item swapping) proves to be a beneficial technique for enhancing order-agnostic modeling and improving model robustness by mitigating overfitting to specific bundle arrangements. (4) Although the Token Validity Filter yields only a marginal performance improvement, its inclusion remains necessary to guarantee the validity of the generated bundles during inference.

Table 9: Ablation study on  $\text{Spotify}_{k=30}$  ( $A=1$  vs.  $A=10$ ). “Proposed” is our model DDBC; each of the other variants changes exactly one component that is removed from the proposed method.

Variant	A=1			A=10		
	F1 $\uparrow$	Jacc $\uparrow$	OAS $\uparrow$	F1 $\uparrow$	Jacc $\uparrow$	OAS $\uparrow$
Our proposed DDBC	0.282	0.178	0.618	0.166	0.092	0.620
w/o RVQ	0.021	0.011	0.557	0.028	0.015	0.556
w/o token validity filter	0.276	0.173	–	0.163	0.090	–
w/o boi token	0.176	0.104	0.538	0.116	0.063	0.536
w/o data augmentation	0.254	0.158	0.599	0.152	0.084	0.598