

# Characterizing the dynamics of unlabelled temporal network trajectories

*Keywords: temporal networks, unlabelled graphs, graph invariants, chaotic networks, periodic networks*

## Extended Abstract

Temporal networks [1] describe systems where interactions evolve over time. In many applications, however, node identities cannot be tracked reliably. This may be due to technical limitations, such as in biological swarms or molecular structures, or to privacy constraints in human-centered data like mobility traces or face-to-face interactions. In such settings, one obtains unlabelled temporal networks, where each snapshot is defined only up to node permutations. This makes it nontrivial to characterize temporal dynamics, since adjacency matrices are not uniquely defined.

To address this challenge, we develop methods based on permutation-invariant graph metrics. Using graph invariants—properties that remain unchanged under node relabelling—we define pseudo-distances between snapshots. In particular, we construct measures from degree sequences, eigenvector centrality, and adjacency spectra. These pseudo-distances are then used to build dynamical quantifiers such as an unlabelled autocorrelation function and indicators of sensitivity to initial conditions, extending standard tools from dynamical systems theory to the unlabelled setting.

Our framework is validated on synthetic and empirical data. For synthetic chaotic temporal networks, generated from logistic maps and globally coupled maps [2], pseudo-distances capture the divergence of close trajectories, reflecting chaotic instability even without labels. The unlabelled autocorrelation function successfully detects periodicity in noisy networks and reveals finite memory in temporal networks built with an autoregressive model. On real data, including U.S. air traffic [3] and face-to-face contact networks [4], the method recovers meaningful dynamical signatures such as daily periodicity and long-range correlation patterns (Fig. 1).

These results show that unlabelled temporal networks retain rich dynamical information. By exploiting graph invariants, one can recover fingerprints of chaos, periodicity, and memory without requiring node-specific tracking. This approach broadens the scope of temporal network analysis to settings where labels are inaccessible or sensitive, with potential applications from collective animal behaviour to privacy-preserving studies of human interactions.

**Ethical Considerations:** The datasets analyzed in this work are publicly available and already anonymized, which ensures compliance with ethical standards and minimizes risks of re-identification. Our methodological focus on unlabelled network representations further reduces the use of sensitive node-level information. Nonetheless, as structural patterns can sometimes reveal group-level behaviors, responsible application of this framework requires awareness of potential societal impacts and adherence to appropriate data governance practices.

## References

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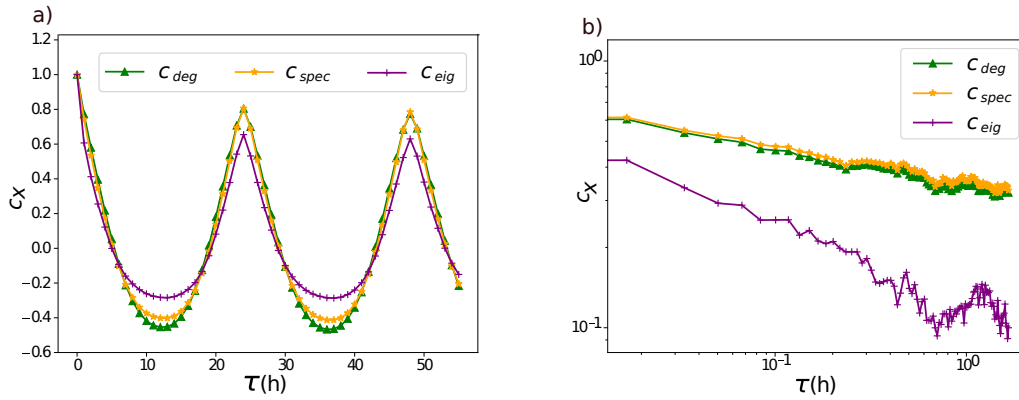


Figure 1: **Unlabelled network autocorrelation function** for the US air traffic and Malawi Village contact network. We validate the autocorrelation function for unlabelled networks based on three different graph invariants: degree sequences (triangle), adjacency spectra (star), and eigenvector centrality (cross), for two different types of dynamics. Panel (a): US air traffic, where nodes are airports and edges represent flights. The autocorrelation function detects the typical 1-day period even after label removal. Panel (b): Malawi Village contact network, where temporal correlations remain detectable despite the absence of labels.