Thermal Conductance Graph (TCG): A Novel Physics-Inspired Method for Time Series to Complex Network Mapping

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We propose a new method for mapping a univariate time series into a weighted graph, called the Thermal Conductance Graph (TCG). We adopt a physics-inspired abstraction by treating each time-indexed observation as a ”thermal” node with a temperature proportional to its value. Formally, given a time series $X = \{x_1, x_2, \ldots, x_n\}$, we construct a graph $G_{\text{TCG}} = (V, E)$ where each vertex $v_t \in V$ corresponds to the point (t, x_t). Initially, every pair of distinct nodes i, j is connected by an undirected edge (a complete graph). We assign each edge a conductance weight W_{ij} following a Fourier heat conduction analog: the weight is proportional to the difference in their values and inversely proportional to the time separation. In the simplest (unit-exponent) case, we take: $W_{ij} = \kappa \frac{|x_i - x_j|}{|j-i|}$, for $i \neq j$, with $W_{ii} = 0$ and $\kappa = 1$. This construction means that a large temperature (value) contrast between two points yields a strong connection. In contrast, points with nearly equal values have only a weak link- especially if they are far apart in time (since a given difference ”spread” over a long interval represents a small gradient). We include tunable exponents (α, β) in a generalized form of edge conductance weight $W_{ij} = \kappa |x_i - x_j|^{\alpha} / |j-i|^{\beta}$ (With exponents $\alpha, \beta \geq 0$ and a global scale $\kappa > 0$) to adjust nonlinearity in amplitude difference and distance penalty. In our default setting we use $\alpha = 1, \beta = 1$ unless there is reason to assume superdiffusive or subdiffusive coupling. Here, W_{ij} is defined as a non-negative ”conductance” magnitude; unlike the Electrostatic Graph (ESG)[1] approach, TCG weights have no sign because they represent coupling strength rather than a directed force. By the proposed TCG mapping, the time series is converted to a dense weighted network encoding its thermal (value) landscape. In principle, any two time points(nodes) can exchange heat, which denotes that the proposed TCG graph is complete and the dense connectivity thus ”privileges” recent, high-contrast interactions (strong edges for close-by points with big value differences) while still allowing long-range influence through multi-hop diffusion paths for distant points. This means even far-apart parts of the series can affect each other’s network proximity indirectly, providing a global coherence, but without overweighting those distant low-similarity connections. A prominent issue with classic Visibility Graphs[2] is that they are inherently unweighted. The NVG/HVG[3] algorithms connect points based on geometric visibility criteria (line-of-sight in the time-value plane), preserving the ordering and some structural patterns of the series, but every visible connection is treated equally, which results in a loss of amplitude information. Our TCG method preserves this missing information by assigning a weight proportional to the actual value difference between points. On the other hand, the Electrostatic Graph (ESG)[1] mapping by Tsiotas et al. was a recent physics-inspired attempt to overcome the VG’s lack of weights. In ESG, each time point i is treated as a charged particle with charge $q_i = x_i$, and the interaction between any two points is defined by Coulomb’s law: $F_{ij} \propto \frac{x_i x_j}{(j-i)^2}$. This yields a complete weighted graph where the edge weight (force) is proportional to the product of the values and decays with the square of their time separation. ESG embedded the series into a topology that was more ”natural” and structurally informative for different real-world signals. Though ESG has its limitations. In practice, one needs to impose a threshold F_c to cancel out the infinitely small forces and obtain a sparser, more interpretable complex network. Tsiotas et al. suggest a data-dependent threshold in their work, but the need for this extra parameter adds an ad-hoc element to the method. TCG, on the other hand, while also producing a complete graph initially, naturally down-weights distant or low-contrast connections so heavily that one can sparsify the network gradually without a sharp loss of meaningful structure. In short, TCG does not require an arbitrary force cutoff to create a sensible network; the weighting itself imparts a continuum from strong to negligible connections. Secondly, the nature of the weighting in TCG versus ESG leads to different emphases. The weight formula for ESG is multiplicative. It multiplies the links between two large-magnitude values (e.g., two large peaks increase their strength, independent of whatever is between them). This is helpful for identifying co-occurrences of large values or scale differences in general, but it also indicates that ESG overemphasises outliers or is sensitive to nonstationary variance (as any $x_i \times x_j$ product is conspicuous). TCG's weighting, by contrast, is differential: it directly emphasises the value difference or gradient.This means TCG naturally spotlights transitions, jumps, and changes in the series.In summary, TCG provides a diffusive network model of the time series: values correspond to temperatures, and edges represent potential heat flow, naturally incorporating both magnitude differences and temporal distance in a single weighted graph structure and it preserves the strengths of prior mappings - it keeps the time ordering and some geometric intuition of visibility graphs, and it preserves quantitative information like the ESG - while avoiding their weaknesses. Further, we assess how closely the original time series X co-varies with the node degree series derived from the weighted network (TCG) by computing bivariate correlations shown in Figure 1.

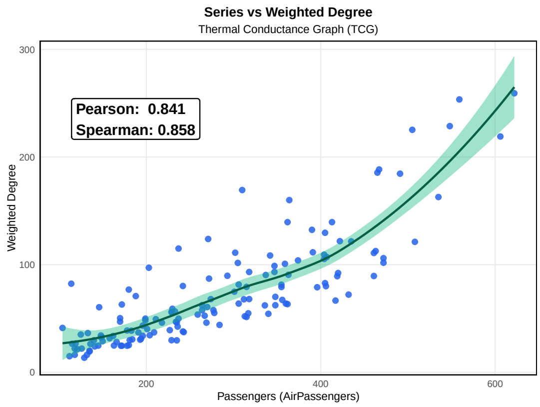

Figure 1: **Correlation analysis:** We compare X with the weighted-degree node series $\{D_t\}$. Using standard measures, we obtain a strong positive association: Pearson $r = 0.841$ and Spearman $\rho = 0.858$. Both coefficients lie in $[-1,1]$; values close to $+1$ indicate a robust relationship between the time-series dynamics and the network-based representation.

References

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