Learning to Make Friends: Coaching LLM Agents toward Emergent Social Ties

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Abstract

Can large language model (LLM) agents reproduce the complex social dynamics that characterize human online behavior—shaped by homophily, reciprocity, and social validation—and what memory and learning mechanisms enable such dynamics to emerge? We present a multi-agent LLM simulation framework in which agents repeatedly interact, evaluate one another, and adapt their behavior through in-context learning accelerated by a coaching signal. To model human social behavior, we design behavioral reward functions that capture core drivers of online engagement, including social interaction, information seeking, self-presentation, coordination, and emotional support. These rewards align agent objectives with empirically observed user motivations, enabling the study of how network structures and group formations emerge from individual decision-making. Our experiments show that coached LLM agents develop stable interaction patterns and form emergent social ties, yielding network structures that mirror properties of real online communities. By combining behavioral rewards with in-context adaptation, our framework establishes a principled testbed for investigating collective dynamics in LLM populations and reveals how artificial agents may approximate or diverge from human-like social behavior.

18 1 Introduction

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- Have you ever been convinced by an AI? A recent study on Reddit [34]—where AI-generated content was used to test how effectively agents could persuade users—sparked public backlash.
- 21 Conducted without users' consent, the experiment was widely condemned for crossing ethical and
- legal boundaries. But why would anyone pursue such a study? And why does the scientific community
- 23 remain deeply invested in modeling human behavior online?
- 24 The short answer is that social media has permeated nearly every aspect of society. It shapes how
- 25 information flows, blurs the line between public and private spheres, and serves simultaneously as
- 26 a news source, marketplace, and social infrastructure. This raises a fundamental question: can we,
- 27 without compromising individual privacy, construct a digital twin that authentically captures the
- 28 behavior of real social media users?
- 29 While the linguistic, psychological, and cognitive capabilities of individual large language model
- 30 (LLM) agents have been demonstrated [6, 28, 36], their collective behavior in networked environments
- 31 remains far less understood. The implications of building such digital twins are far-reaching [4].
- 32 Policymakers could use them to test the effects of moderation strategies—such as content removal
- 33 [37]—before deploying them at scale. Researchers could simulate the spread of opinions, the
- 34 formation of online communities, or the contagion of consumer behavior. Security analysts could
- explore how to defend against manipulation or detect coordinated influence operations [2].

- In this paper, we introduce a framework for multi-agent LLM conversations designed to simulate
- both private and public user discussions. By equipping agents with reward-based tasks, we move
- beyond the recently demonstrated agent-based modeling (ABM) capabilities of LLMs and leverage
- 39 their cognitive abilities to study the emergence of social ties within dynamic simulations.

40 1.1 Unique Challenges

- 41 Although prior work has explored multi-agent LLM frameworks, simulating social media conversa-
- 42 tions raises three distinct challenges:
- 43 Agentic Personas. LLMs are trained to minimize prediction error in text generation, but this does not
- 44 guarantee realistic user personas. Poorly specified personas often exhibit similar linguistic patterns,
- 45 failing to reflect the diversity of real online users. Authentic simulations require agents that vary
- 46 in values, norms, and cognitive styles, supported by memory structures that track interactions and
- 47 evolve over time.
- 48 Learning Mechanisms. Games and other structured environments provide explicit rewards that
- 49 guide agents toward effective strategies. Social media, by contrast, involves open-ended exchanges
- 50 where memory must approximate the human ability to retain recent interactions while integrating
- 51 longer conversational histories. Because engagement spans a spectrum—from passive awareness
- 52 (viewing) to active participation (commenting, sharing)—agents need memory architectures that
- enable adaptive learning across contexts.
- 54 Social Topology. Users with shared interests tend to cluster, reinforcing in-group favoritism and
- out-group aversion that drive polarization and echo chambers. Structural features such as follower
- networks shape the spread of information. Yet a central open question remains: what network
- 57 structures emerge when LLM agents interact at scale?

58 1.2 Our Contributions

- To address these challenges, we introduce a multi-agent LLM platform that advances the modeling of
- online social dynamics through three core contributions:
- 61 Conversation Room. We build an interactive environment that integrates both public and private
- 62 communication channels. This design enables agents to deploy distinct strategies across contexts and
- allows systematic analysis of how channel choice shapes conversational dynamics.
- 64 Reward Structures. We formalize reward functions that translate empirically grounded human
- 65 motivations—including self-presentation, social interaction, coordination, and emotional support—
- into agent objectives. This framework establishes a principled bridge between behavioral theory and
- 67 multi-agent LLM learning.
- 68 **Tie Formation.** We design mechanisms that allow social ties to emerge endogenously from conversa-
- 69 tional interactions, without relying on pre-defined network structures. This enables systematic study
- of how support, alignment, and homophily drive the formation of group structures.

71 2 Preliminaries

- 72 Multi-agent LLMs. Early work established that LLM agents can display human-like routines
- 73 when equipped with memory and cognitive mechanisms. Park et al. [35] introduced the observa-
- 74 tion-planning-reflection cycle, showing that agents embedded in interactive environments develop
- 75 coherent daily behaviors. Subsequent studies demonstrated that LLM populations can approximate
- human-level attitudes and sociodemographic effects, reproducing patterns found in surveys and
- experiments [1, 3]. Beyond reproducing individual or population-level behavior, frameworks such
- 78 as CAMEL [25] and orchestration stacks like LangChain and AutoGen [41] enable coordination,
- 79 specialization, and standardized multi-agent dialogues. Recent surveys synthesize these design
- patterns and emphasize the need for systematic evaluation against human baselines [18, 14].
- Personas and diversity. A central challenge in simulation is ensuring credible heterogeneity across
- agents. Simple prompting often yields homogeneous responses, whereas personality induction
- methods, such as Big Five personality prompting, can generate stable stylistic variation [21]. The
- PERSONA benchmark extends this approach, offering pluralistic alignment across diverse profiles
- 85 [8]. At the same time, persona prompts may inadvertently surface reasoning biases [19], underscoring
- the importance of careful evaluation. More recent approaches move beyond prompting to train
- 87 agents in simulated societies, allowing them to internalize social norms and polarization dynamics

through interaction [26]. Together, these studies highlight both the promise and pitfalls of persona diversification for multi-agent LLM simulations.

From interactions to social structure. A key question is whether agent collectives reproduce network-level regularities. Emerging studies have examined LLM-generated networks, link formation, and paradoxes of visibility such as the friendship paradox [9, 33]. These experiments connect directly to canonical findings in human networks, including assortative mixing [30], modular community structure [13], and diffusion dynamics such as emotional contagion, misinformation spread, and echo chambers [24, 39, 10]. The ability of LLMs to produce persuasive or deceptive content further raises the stakes for safety-aware evaluation [43].

Our focus. Building on these streams of research, our framework centers on sequential, socially motivated behavior—spanning interaction, information seeking, self-presentation, coordination, and emotional support—operationalized through rewards, in-context coaching, and memory. We examine how such behavior gives rise to social ties, which form through micro-level signals (approval, reciprocity, latency) and aggregate into macro-level network structures (clustering, modularity, tie persistence). By allowing ties to emerge endogenously, without reliance on a fixed graph, we provide a testbed for studying phenomena such as homophily, community formation, and echo chambers.

3 Methodology

Figure 1 provides an overview of our multi-agent LLM platform for simulating social media conversations. The framework is organized into two main components: persona creation (A) and the social media simulation process (B). In the sections that follow, we describe how agents are initialized, outline the progression of the simulation, present the core of our learning paradigm—the task-specific reward structures of individual agents—and demonstrate how network structures emerge.

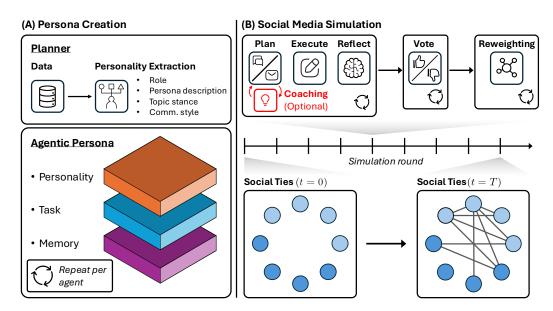


Figure 1: Multi-agent LLM social media conversation framework.

3.1 Persona Creation

Building agent-personas requires the design of psychologically grounded and behaviorally coherent profiles. To avoid randomly sampling attributes from a population of observed users, we begin by instantiating a *planner* agent. This agent orchestrates the creation of inherited persona data, drawing on corpora extracted from real-world online discussions on a specific topic. With the growing prevalence of platforms where users remain largely anonymous, many features commonly leveraged in frameworks such as [8]—including age, sex, race, or education—are no longer observable, even though people in real interactions might infer them from user profiles. Instead, we focus on features that can be reliably inferred from the content itself: *role*, *persona description*, *topic stance*, and

communication style. These attributes are directly grounded in the observed communications within the selected dataset.

Building on these content-based inferences, we construct agentic personas through a three-layer structure detailed below:

Personality. The first layer captures personality features derived from text data. We draw on the Big
Five traits and their cross-cultural stability [11, 27], complemented by more granular facets (e.g.,
sociability as an Extraversion facet; stubbornness as an Agreeableness facet) and constructs beyond
the Five (e.g., risk preference; interpersonal openness). High Extraversion is reflected in linguistic
markers such as frequent emoji use and elevated conversational activity, whereas low Agreeableness
is associated with increased negation and critical language. To assess these dimensions, we administer
the Mini-IPIP, a concise Big Five inventory [12, 15].

Task. Why individuals engage on social media platforms is shaped by a range of underlying motives.

Task. Why individuals engage on social media platforms is shaped by a range of underlying motives. Whiting and Williams [40] draw on gratification theory to explain such engagement, identifying 131 key drivers such as social interaction, information seeking, and entertainment. Assigning a persona a specific task based on these motivations results in markedly different behaviors that often align with the platform's social engagement hierarchy—ranging from passive users (often referred to as 134 lurkers) to highly active content creators. Related social media phenomena, such as agents exhibiting 135 stubbornness and resistance to opinion change, can likewise be integrated into the simulation. For 136 clarification of specific task formulations, we refer to Sec. 3.3, where we define the reward structures. 137 **Memory.** The role of memory in agents is essential for capturing the dynamics of evolving discussions. 138 We structure persona memory across three components: conversation memory, which records all 139 prior interactions; relationship memory, which tracks information about other agents the persona has interacted with; and opinion memory, which abstracts the content of a conversation into a singular 141 entry representing a stance or belief. Together, these form a lightweight analog to long-term human 142 memory (i.e. [42]) and enable agents to exhibit path-dependent behavior in extended dialogues. 143

3.2 Conversation Simulation

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Once the personas are initialized and their distinct personalities verified through a pre-survey assessment, we initiate the simulation. Each agent engages in a conversation on a predefined topic. At time t=0, we observe the social network $\mathcal{G}_0=(\mathcal{V},\mathcal{E}_0,A_0)$ with $\mathcal{E}_0=\emptyset$, indicating that there are no edges and no prior knowledge among users in \mathcal{V} ; the adjacency matrix satisfies $[A_0]_{uv}=0$ for all $u,v\in\mathcal{V}$. Our objective is to characterize the network structure \mathcal{G}_T that emerges after the agents' interactions. Messaging occurs through both direct (user-to-user) and public channels (visible to all agents), consistent with prior work showing that channel choice depends on privacy concerns, audience size, and message sensitivity. By incorporating both modes, we allow agents to adapt communication strategies to their emotional grounding and negotiation context.

In the opening round (t = 1), agents are unaware of one another and therefore required to make a public post (POST). In subsequent rounds, they may choose to post (POST), comment (COM), send a direct message (DM), or take no action (NOT). The per-round procedure unfolds in three phases.

Plan-Execute-Reflect. Each agent u plans a pre-specified number of actions N based on the batch of content from the previous round \mathcal{B}_{t-1}^{-1} , the reward $R_{t-1}(u)$, and additional side information (see Sec. A). Actions are selected from $\mathcal{A} = \{\text{POST}, \text{COM}, \text{DM}, \text{NOT}\}$. Planning is non-trivial: each action requires further specification (e.g., which post to comment on or which user to message) in addition to the strategic objective of maximizing future reward. As an optional step, we test whether a "coach" can simplify this process by providing a tip, enabling the agent to focus on constructing a well-structured response. Once a valid plan is constructed, the agent executes each action, generating content consistent with the action type (e.g., a DM reply is based on the specific message received in the prior round). Executed actions are stored in the agent's memory, and this procedure is repeated for all agents.

Vote. Social validation is central to how individuals perceive support or friendship online. Accordingly, for all publicly visible content (POST, COM), we enable agents to cast votes by liking, disliking, or remaining neutral toward the content.

Reweighting. After each round, agents update tie strengths by reweighting their relationships. This process leverages multiple behavioral signals to inform decisions about whether and how to adjust connections.

¹For each user u, this includes all public content plus direct messages addressed to u.

Rewards for In-Context Learning

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In the absence of explicit reward signals, agent behaviors tend to collapse toward purely greedy actions, failing to capture the diversity and goal-directed reactivity characteristic of real-world social 175 systems. To address this limitation, we define reward functions that align agent learning with task 176 objectives reflecting user motivations.

Social Interaction (SOC). Interacting with others and maintaining social ties are primary drivers of user engagement on social media platforms. To formalize this, we define the set of *direct-exchange* actions as $\mathcal{A}^{DIR} \subset \mathcal{A}$, $\mathcal{A}^{DIR} = \{DM, COM\}$, corresponding to direct messages and comments. Let \mathcal{B}_t be the set of all actions taken by users in the network \mathcal{V} in round t, and let $\mathcal{B}_t^{\mathtt{DIR}} = \{a \in \mathcal{B}_t : \mathtt{TYPE}(a) = \mathbf{B}_t : \mathtt{TYPE}(a) \in \mathcal{B}_t : \mathtt{TYPE}(a) \in \mathcal{B}_t : \mathtt{TYPE}(a) = \mathbf{B}_t : \mathtt{TYPE}(a) = \mathbf{B$ \mathcal{A}^{DIR} denote the subset of actions in round t that involve direct exchanges. We define the social interaction reward $R_t^{SOC}: \mathcal{V} \to [0, 1]$ as

$$R_t^{\rm SOC}(u) = (1-\beta^{\rm SOC}) \frac{I_t^{\rm sent}(u)}{N} \; + \; \beta^{\rm SOC} \frac{I_t^{\rm rec}(u)}{|\mathcal{B}_t^{\rm DIR}|}, \label{eq:Rsoc}$$

where $I_t^{\mathsf{sent}}(u) \coloneqq \sum_{a \in \mathcal{B}_{\bullet}^{\mathsf{DIR}}} \mathbb{1}\{a : u \to \cdot\}$, and $I_t^{\mathsf{rec}}(u) \coloneqq \sum_{a \in \mathcal{B}_{\bullet}^{\mathsf{DIR}}} \mathbb{1}\{a : \cdot \to u\}$, with $\beta^{\mathsf{SOC}} \in [0, 1]$. 184 Here, $I_t^{\rm sent}(u)$ counts the number of direct exchanges sent by user u in round t (i.e., actions with sender u and any recipient, denoted $u \to \cdot$), while $I_t^{\rm rec}(u)$ counts the number of direct exchanges received by user u. Both terms are normalized to ensure $R_t^{\rm SOC}(u) \in [0,1]$, where $N = |\mathcal{B}_t(u)|$ denotes the constant number of actions by any user $u \in \mathcal{V}$ per round. The hyperparameter $\beta^{\rm SOC}$ trades 185 186 187 188 off receiving and sending. 189

Information-Seeking Reward (INF). Users on social platforms often seek to discover new topics while maintaining a diverse information consumption. To formalize this behavior, we assume that each content item c_t^u presented to user u at time t carries a single topic label $\tau(c) \in \{1, 2, \dots, K^{(\tau)}\}$, where $K^{(\tau)}$ denotes the total number of topics available on the platform. Let $\mathcal{C}_t(u)$ denote the set of items recommended to user u in round t, and define the set of topic labels encountered by u up to round t-1 as $\mathcal{H}_{t-1}^{\tau}(u) = \{\tau(c) : c \in \mathcal{H}_{t-1}(u)\}$. We define topic novelty as the number of previously unseen topics in the current recommendation $\mathcal{T}_t(u) = |\{\tau(c) : c \in \mathcal{C}_t(u)\} \setminus \mathcal{H}_{t-1}^{\tau}(u)|$, capturing the user's opportunity to encounter new information. In addition, we measure topic diversity using the empirical entropy over the topic distribution in $C_t(u)$. Let $p_{t,k}(u) := |\{c \in C_t(u) : \tau(c) = c\}|$ $k \} | / | \mathcal{C}_t(u) |$ for $k = 1, \dots, K^{(\tau)}$ denote the empirical frequency of topic k. The corresponding Shannon entropy is $H_t(u) \coloneqq -\sum_{k=1}^{K^{(\tau)}} p_{t,k}(u) \log p_{t,k}(u)$, which satisfies $0 \le H_t(u) \le \log K^{(\tau)}$. The per-round, per-user information-seeking reward is then defined as

$$R_t^{\text{INF}}(u) = (1 - \beta^{\text{INF}}) \frac{\mathcal{T}_t(u)}{K^{(\tau)}} + \beta^{\text{INF}} \frac{H_t(u)}{\log K^{(\tau)}}$$

where $\beta^{\text{INF}} \in [0,1]$ controls the trade-off between novelty and diversity. This ensures $R_t^{\text{INF}}(u) \in [0,1]$, where $\beta^{\text{INF}} = 0$ prioritizes the discovery of new topics, while $\beta^{\text{INF}} = 1$ encourages consuming a broad mix of content within the current topic space.

Self-Presentation Reward (PRE). Self-presentation is a fundamental motive for user participation on social media, where individuals share content to express identity, gain visibility, and seek validation through social feedback. To formalize this, we consider that in each round t, agent u executes a batch of actions $\mathcal{B}_t(u) \subset \mathcal{B}_t$, $\mathcal{B}_t(u) = \{a_{t,1}(u), \dots, a_{t,N}(u)\}$, where each action belongs to the action set \mathcal{A} . We define the subset of self-presentation actions as $\mathcal{B}_t^{\mathsf{POST}}(u) = \{a \in \mathcal{B}_t(u) : \mathsf{TYPE}(a) = \mathsf{POST}\}$, representing all posts created by user u in round t. The cardinality $|\mathcal{B}_t^{\mathsf{POST}}(u)|$ measures the quantity of self-generated content.

Beyond the act of posting, users also derive utility from feedback received on their content, reflecting 212 community approval or disapproval. For each post $a \in \mathcal{B}_t^{\mathsf{POST}}(u)$, let $\ell_t^+(a)$ and $\ell_t^-(a)$ denote the number of likes and dislikes received, respectively. The total positive and negative feedback accumulated by v on their posts during round t are then given by

$$L_t^+\big(\mathcal{B}_t^{\mathrm{POST}}(u)\big) = \sum_{a \in \mathcal{B}_t^{\mathrm{POST}}(u)} \ell_t^+(a), \quad L_t^-\big(\mathcal{B}_t^{\mathrm{POST}}(u)\big) = \sum_{a \in \mathcal{B}_t^{\mathrm{POST}}(u)} \ell_t^-(a).$$

The self-presentation reward combines the incentive to post content with the desire to receive positive social feedback. Formally, we define

$$R_t^{\text{PRE}}(u) = (1 - \beta^{\text{PRE}}) \frac{|\mathcal{B}_t^{\text{POST}}(u)|}{|\mathcal{B}_t(u)|} + \beta^{\text{PRE}} \frac{L_t^+ \left(\mathcal{B}_t^{\text{POST}}(u)\right) - L_t^- \left(\mathcal{B}_t^{\text{POST}}(u)\right)}{(|\mathcal{V}| - 1)N},$$

where $\beta^{\text{PRE}} \in [0,1]$ controls the trade-off between the intrinsic motivation to share content and the extrinsic motivation to receive social approval. The first term captures the relative frequency of self-presentation actions, while the second term measures net community feedback, normalized by the maximum possible number of other agents and actions per round to ensure scale consistency across simulations.

Coordination Reward (COORD). Coordination captures user behaviors that foster direct interaction and reciprocity within a social network, such as responding to others and explicitly mentioning peers to engage them in discussions. This reward models two forms of coordination: visibility through mentions and reciprocity through replies. We first define mentions as explicit references to other users (e.g., using "@user") in posts or comments. Let $\mathcal{B}_t^{\text{COORD}} = \left\{a \in \mathcal{B}_t : \text{TYPE}(a) \in \{\text{COM}, \text{POST}\}\right\}$ denote the set of all comments and posts created during round t. We define the number of times user v is mentioned in round t as $U_t^{@}(u) = |\{a \in \mathcal{B}_t^{\text{COORD}} : u \in \text{MENTIONS}(a)\}|$, where MENTIONS is a helper function that extracts the set of users explicitly mentioned in action a. The second form of coordination involves reciprocity through direct messaging. We identify the set of users who sent a direct message (DM) to user v in the previous round,

$$U_{t-1}(u) = \{v : \exists a \in \mathcal{B}_{t-1}, \text{ SNDR}(a) = v, \text{ RCPT}(a) = u, \text{ TYPE}(a) = DM\},$$

where SNDR and RCPT return the sender and recipient of action a, respectively. We then define the number of direct replies user v sends back to those who messaged them in the previous round as

$$U_t^{\mathrm{DM}}(u) = \Big| \big\{ a \in \mathcal{B}_t : \mathrm{TYPE}(a) = \mathrm{DM}, \; \mathrm{SNDR}(a) = u, \; \mathrm{RCPT}(a) \in U_{t-1}(u) \big\} \Big|.$$

This quantity captures the extent to which v reciprocates by responding to direct messages. The coordination reward is then defined as

$$R_t^{\text{COORD}}(u) = (1 - \beta^{\text{COORD}}) \frac{U_t^{@}(u)}{|\mathcal{B}_t^{\text{COORD}}|} + \beta^{\text{COORD}} \frac{U_t^{\text{DM}}(u)}{|U_{t-1}(u)|},$$

where $\beta^{\text{COORD}} \in [0, 1]$ controls the trade-off between prioritizing *visibility* through mentions and reciprocity through direct replies. This formulation incentivizes agents to balance being seen by others and actively maintaining responsive communication within the network.

Emotional Support Reward (EMO). Emotional support is a core element of social interactions, reflecting the encouragement, affirmation, or criticism that users receive during their engagements. We define $\mathcal{B}_t^{\text{DIR}}$ as the set of all direct messages (DM) and comments (COM) exchanged in round t. For each action $a \in \mathcal{B}_t^{\text{DIR}}$, let RCPT(a) denote the recipient and let $s(a) \in [-1, 1]$ be the sentiment

score, where positive values indicate supportive content and negative values indicate hostility. We compute the total positive and negative sentiment directed toward user v in round t as

$$U_t^+(u) = \sum_{\substack{a \in \mathcal{B}_t^{\text{DIR}} \\ \text{RCPT}(a) = u}} \max\{0, s(a)\}, \quad U_t^-(u) = \sum_{\substack{a \in \mathcal{B}_t^{\text{DIR}} \\ \text{RCPT}(a) = u}} \max\{0, -s(a)\}.$$

To capture the role of sentiment in shaping user experience, we define the emotional support reward using a trade-off parameter $\beta^{\text{EMO}} \in \mathbb{R}$ and a small constant $\varepsilon > 0$ for numerical stability:

$$R_t^{\mathrm{EMO}}(u) = \frac{1}{2} \left(1 + \frac{U_t^+(u) + \beta^{\mathrm{EMO}} U_t^-(u)}{U_t^+(u) + |\beta^{\mathrm{EMO}}| U_t^-(u) + \varepsilon} \right).$$

This formulation ensures that $R_t^{\text{EMO}}(u)$ remains within the unit interval. The parameter β^{EMO} allows the model to reflect different platform dynamics: positive values imply that negative sentiment contributes to engagement, negative values penalize negative sentiment more strongly, and setting $\beta^{\text{EMO}}=1$ treats positive and negative sentiment symmetrically. Under this formulation, interactions dominated by a single sentiment polarity yield rewards close to one, while balanced sentiment or the absence of interactions result in a value near one half, indicating a neutral emotional environment. Compositional Reward. User motivations on social platforms rarely stem from a single objective but instead reflect a blend of social interaction (SOC), information seeking (INF), self-presentation (PRE), coordination (COORD), and emotional support (EMO). To capture this complexity, we define the overall reward as a weighted combination of these components, allowing the simulation to reflect diverse user goals and the interplay among engagement drivers. Formally, we write the compositional reward as

$$R_t(u) = \sum_{r \in \mathcal{R}} \lambda_r R_t^r(u), \quad \sum_{r \in \mathcal{R}} \lambda_r = 1, \quad \lambda_r \ge 0,$$

where the set $\mathcal{R} = \{\text{SOC, INF, PRE, COORD, EMO}\}$ denotes the included reward components. The coefficients λ_r specify the relative importance of each component, enabling the model to represent users with distinct goals or to reflect platform designs that emphasize particular forms of engagement. This compositional framework ensures that while each reward remains interpretable and bounded, their aggregation can flexibly capture the multidimensional objectives underlying behavior in online environments.

3.4 Mechanisms of Social Tie Formation

We model the evolution of directed social ties over time by representing the interaction structure at each round t as a weighted, directed network $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t, A_t)$, where $\operatorname{diag}(A_t) = 0$. The entry $[A_t]_{uv} \in [0,1]$ stores the strength of the directed tie from user v to user v. Ties are initialized at zero and strengthened or weakened over time in response to interaction, allowing us to separate the formation of a tie from its subsequent development. This approach builds on actor-oriented models for dynamic social networks [38] and is consistent with empirical evidence that the persistence of communication ties depends strongly on the frequency and recency of interaction [32, 23].

To determine whether a directed tie should be updated in a given round, we introduce a binary activation denoted as $\zeta_t(v \to u) \in \{0,1\}$ that encodes whether user v actively interacted with user u in round t. We operationalize this by defining two directed interaction channels. The ADDRESS channel activates when v directly addresses u—either by sending a private message or by publicly mentioning u. The ENGAGE channel activates when v interacts with content created by u—either through commenting or voting. Formally, let \mathcal{B}_t be the batch of all actions in round t, then

$$\begin{aligned} \mathsf{ADDRESS}_{v \to u}(t) &\coloneqq \mathbb{1} \left\{ \exists \, a \in \mathcal{B}_t : \, \mathsf{isDmTo}(a, u, v) \lor \mathsf{isMention}(a, u, v) \right\}, \\ &\mathsf{ENGAGE}_{v \to u}(t) \coloneqq \mathbb{1} \left\{ \exists \, a \in \mathcal{B}_t : \, \mathsf{isComTo}(a, u, v) \lor \mathsf{isVoteFor}(a, u, v) \right\}, \end{aligned}$$

and we define the overall activation as $\zeta_t(v \to u) \coloneqq \mathsf{ADDRESS}_{v \to u}(t) \lor \mathsf{ENGAGE}_{v \to u}(t)$. Once activation is determined, the tie strength is updated using a gated update rule that differentiates between active and passive rounds. On active rounds, the tie is strengthened based on a scalar evidence score $e_t(v \to u) \in [0,1]$, which aggregates the quality of the interaction. On passive rounds—when no directed interaction from v to v occurs—the tie strength decays. The update is defined as

$$[A_{t+1}]_{uv} := \begin{cases} [A_t]_{uv} + \min\left\{\Delta_{\max}, \ (1 - [A_t]_{uv})[e_t(v \to u) - \xi]_+\right\}, & \text{if } \zeta_t(v \to u) = 1, \\ (1 - \delta) \cdot [A_t]_{uv}, & \text{if } \zeta_t(v \to u) = 0, \end{cases}$$
(1)

where $\xi \in [0,1)$ is a minimum-evidence threshold, $\Delta_{\max} \geq 0$ caps the per-round increase, $\delta \in [0,1)$ controls decay, and $[x]_+ = \max\{0,x\}$. The scaling term $1 - [A_t]_{uv}$ ensures that increases become smaller as the tie approaches its maximum value, while decay acts multiplicatively to gradually fade inactive ties. To interpret δ , one can parameterize it via a half-life h such that $\delta = 1 - 2^{-1/h}$.

The evidence score $e_t(v \to u)$ is computed by aggregating several dyadic signals that capture distinct social mechanisms. These include: the *novelty* of the interaction—capturing whether v introduces new information to u, drawing on theories of brokerage and weak ties [17, 7]; the *reciprocity* of interactions [16, 31]—captured by the symmetry of feedback between users—and the *approval* expressed through likes and other positive evaluations [29]; and the *affective tone* of communication—measured by the sentiment of direct messages, reflecting relational support [20]. Each signal is bounded and passed through a monotone mapping G_{θ} , which ensures that stronger combinations of signals yield higher evidence scores. Further details on the construction of individual signals and the specification of G_{θ} are provided in the Supplementary Materials.

An alternative approach to constructing signals heuristically is to score the text from interactions in each round, as demonstrated in the prompt in Appendix A. In this approach, we replace the evidence score $e_t(v \to u)$ with a normalized score obtained from the prompt to update the ties.

For reporting purposes, we optionally export the final adjacency matrix in an undirected form, defined as $[A_t^{\mathrm{ud}}]_{uv} = \frac{1}{2}([A_t]_{uv} + [A_t]_{vu})$, which symmetrizes tie strength for visualization and summary statistics. After the final simulation round T, we apply a threshold θ to A_T^{ud} to obtain a binarized undirected graph, which is used in the network analysis. This transformation is used only for reporting and does not affect the directed tie dynamics or update rules described above.

4 Results

We propose an overall framework for studying the formation of social ties. Our experiments were conducted with $|\mathcal{V}|=30$ agents over T=15 simulation rounds, with discussions centered on climate change, where the persona planner utilized data from [22].² We present preliminary findings, beginning with an evaluation of whether specific tasks, referred to here as policies, can be learned. In Fig. 2, we observe that performance generally increases over simulation rounds, both with and

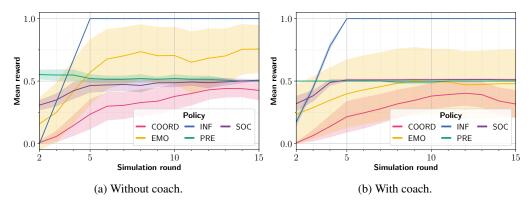


Figure 2: Comparison between augmenting in-context learning with coach.

without the coach. The information-seeking (INF) policy is typically the easiest to learn, as it is primarily driven by exposure to new content. Emotional support (EMO) also achieves relatively high values, reflecting the generally positive behavior of most agents. Self-presentation (PRE) can be partially controlled through personal posts, but policies that depend on coordination are more difficult to learn. Comparing Fig. 2a to Fig. 2b, we find that the coach accelerates learning in the early stages for some policies. However, in the long run, the simulation without the coach performs better. Binding agents to tips provided by the coach appears to reduce exploration, consistent with our earlier observation that optimality depends not only on the agent's own actions but also on targeting the right users. Further testing is therefore required to identify the root causes of these results. While having the coach generate strategies and guide the plan stage provided some structure, it yielded only limited improvements.

Next, we examine the reweighting step when it is performed via (i) a heuristic based on predefined signals versus (ii) a LLM text-based approach. After constructing the directed weighted graph at the final simulation step T=15, we convert it into an undirected unweighted graph. The conversion threshold θ plays a central role in this process. We also allow ties to decay, and bound tie strength updates at each step by $\Delta_{\rm max}$. The main evaluation metrics are outlined in Appendix C. These metrics, together with baseline statistics from real networks reported in [9], serve as benchmarks. Comparing the two reweighting methods, we find that the heuristic shows divergence, for example in average clustering and average shortest path length. By contrast, the LLM-based text approach yields results that are more stable and more closely resemble real networks, largely independent of the threshold θ .

We acknowledge that the analysis presented here is an initial step. Further work is required, particularly since the generated graphs are dynamic and the interplay between agent relationships, tie strength updates, and topical alignment remains to be fully understood.

5 Conclusion

We present a platform that endogenously learns social ties among interacting LLM agents from their private and public exchanges. Each agent optimizes a task-specific behavioral reward grounded in gratifications theory [40] and adapts via a plan-execute-reflect loop with bandit-style updates. We show that these rewards are learnable in practice, with varying difficulty under bounded rationality and strategic influence, and that some objectives are unattainable in principle because achieving

²Due to API rate limits, the experiments presented in this work were conducted using OpenAI's gpt-4.1-mini.

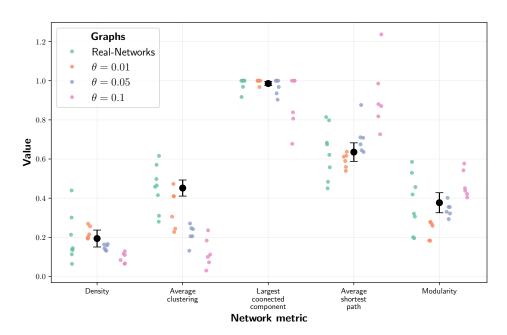


Figure 3: Network metrics for heuristic tie formation.

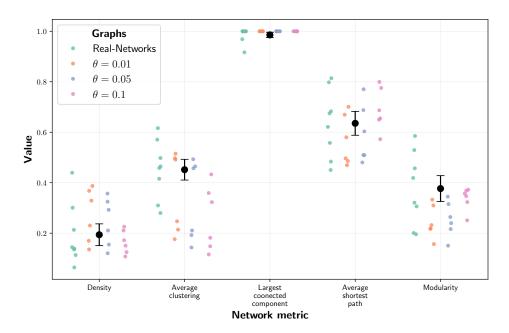


Figure 4: Network metrics for text-based tie formation.

them would require control over other agents. Using the evaluation protocol of [9], the emergent networks match key statistics of real social graphs, establishing the platform as a controlled testbed for studying echo-chamber formation, the dynamics of niche communities, and the design of mitigating interventions. The study is conservative in scale ($|\mathcal{V}|=30,\ T=15$), uses limited replications, and starts from empty networks; scaling to larger cohorts and horizons, seeding pre-existing ties, and running intervention stress tests are clear next steps to strengthen external validity and policy relevance. Finally, with a small number of actions per round, the coaching component did not yield consistent gains; its efficacy should be re-evaluated in larger settings with more agents and richer action budgets.

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455 A Prompt Specifications

In this section, we present the prompts underlying our multi-agent LLM simulation: *Coach, Plan, Vote*, and *Tie-Update*. As detailed in the methodology, the *Coach* prompt provides action recommendations that streamline the planning of *N* actions in each simulation round *t*. Although the *Plan* prompt contains nearly all required information on its own, planning remains nontrivial—particularly when integrating private (direct messages) and public (posts and comments) channels—and can overload the agent. Producing a valid, executable action list is therefore essential.

The *Plan* prompt can run with or without the coach's *tip*. If a candidate plan fails our validation checks, we re-prompt (up to three times) until a valid plan is produced. Finally, we present the *Vote* prompt, a key component of the social-media simulation that captures how each agent, given its personality parameters, evaluates other users' content.

Coach Prompt

System: You are a strategy coach in a multi-round social media game. Your job is to help the agent maximize long-term rewards by closing the gaps between their current reward scores and their target weights. Priorities are ordered by importance, so the first items need the most improvement. Output 3-5 one-sentence bullet tips that are concrete, executable, and encourage variety in partners and topics across rounds to promote diversity and exploration. No preamble, no explanations. Remember: the total reward per round is capped at 1, and many reward components depend on how others engage with the agent, so your tips should encourage responses and collaboration.

```
User: Priority this round (high\rightarrowlow): {focus} Guidelines: {guidelines}
```

Context:

- Users: {users}
- DM senders last round: {dm_last_senders}
- Recent partners: {recent_partners}
- Known topics so far: {seen_topics}
- Recent topics (last 3): {last_recent}

Each tip must include:

- action type (POST/COM/DM),
- \cdot target user (from lists) or 'top-liked post',
- topic (choose a relevant or new topic; exploring new topics helps INF),
- whether you invite them to @mention you back (only if aiming at COORD),
- tone (supportive/neutral/critical).

Focus items are ordered by how far below target the corresponding reward is; design actions to improve the highest-priority components. Vary topics and partners to support learning and social diversity.

Return ONLY the bullets.

Plan Prompt

 $\label{thm: System: You are {name}, a user participating in a multi-round social media game. In each round you will execute exactly {n_actions} actions and your choices accumulate across rounds, affecting future relationships, topics seen and rewards. Recall past interactions and relationships from your memory when planning your actions.$

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```
Your best was round {best_round} with score {best_reward} when you did
{best_summary}.
Your only goal is to grow what you value most-{active_comps}-in these proportions:
{active_weights}.
[You got this coaching tip: {tip}]
Here are the reward definitions and how they translate into behaviours:
{selected_reward_docs}
Here's how they combine:
Your total score is a mix of these parts (SOC, INF, PRE, COORD, EMO), weighted
by \lambda's that sum to 1. Each \lambda decides a component's importance. Current weights:
{active_weights}
Reward-aware rules (apply when the corresponding \lambda is large):
· COORD: For POST/COM, set "mention_flag": true when you plan to include a literal
  @mention (e.g., @post_author or a user from Users).
• SOC: Balance COM/DM between initiating and replying; if someone messaged you last
  round, reply.
• INF: Prefer topics not seen recently; otherwise keep a diverse mix from Topics.
• PRE: Include at least one POST (keep it concise and end with a clear question).
• EMO: Set "tone": "supportive" for COM/DM to elicit supportive replies.
Plan exactly {n_actions} actions and respond with a raw JSON array only (no code
fences, no comments, no extra keys, no trailing commas).
Each action must be an object with EXACTLY these keys:
"type": one of "POST", "COM", "DM", "NOT"
"recipient": user ID for DM, otherwise null
"topic": string for POST/COM/DM, null for NOT
- "target_id": COM \rightarrow a post ID from the list; DM \rightarrow a DM reply ID (reply mode) or
 null (cold DM); null for POST and NOT
· "mention_flag": boolean; true only if you will include an @mention (POST/COM only;
  must be false for DM and NOT)
"tone": one of "supportive", "neutral", "critical"
CONSTRAINTS:

    POST: recipient=null, target_id=null

• COM: recipient=null, target_id MUST be a valid post ID from the list
   - Reply DM: recipient MUST be the author of a listed DM reply AND target_id MUST
     be that DM id
   - Cold DM: recipient MUST be a valid user ID from the list AND target_id MUST
     be null

    NOT: recipient=topic=target_id=null

Topic guidance:
• For COM: use the target post's topic if provided; otherwise pick a relevant
  engaging topic.
• For POST/DM: choose any topic likely to engage the audience or recipient; exploring
  new topics can increase INF reward.
• There is no fixed allowed topic list; you may introduce new topics.
User: Last actions (most recent round): {last_actions},
Last observed reward scores: {observed_rewards},
Users you can interact with: {users},
Known topics so far: {topics} (you may introduce new topics),
DM replies (users who DM'd you last round):
- author: {dm_author} (DM id: {dm_id}, topic: {dm_topic}, content: {dm_content})
Commentable posts (choose a target_id from this list ONLY):
- target_id: {p_id} (POST author: {p_author}, topic: {p_topic}, content:
{p_content})
Based on the above context, decide on \{n\_actions\} actions that best improve your
priority rewards. For Reply DM, use the listed DM IDs; cold DM can be sent to any
user in the list. For COM, you must supply a valid post ID from the list above.
Remember: replies help SOC, mentions help COORD, and supportive tone helps EMO.
```

469 The Vote prompt allows a user to express either liking or disliking of exposed content.

```
Vote Prompt
System: You are {name}, a user in a multi-round social media game. You are a
{persona_type} with {personality} personality - use these traits to guide your
Your choices to "like" (1), "dislike" (-1), or remain neutral (0) on each piece
of content affect how others perceive you (PRE) and the emotional tone of your
interactions (EMO), and therefore influence your future rewards. Use your persona
traits[ and your relationship history] to decide which content to support, oppose,
or ignore. A neutral vote (0) means you have no strong opinion or the relationship
context is neutral.
Respond ONLY with a JSON array of objects. Each object must have:
• id: the content ID (integer)
\cdot vote: 1 for "like", -1 for "dislike", or 0 for "no vote"
Example output:
"id": 42, "vote": 1,
"id": 73, "vote": -1,
"id": 99, "vote": 0
No extra keys, no free-form text, no explanation.
User: Here are the latest items to vote on (id, sender, content[, relationship]):
      • {id} {sender} {content} {relationship}
      • {id} {sender} {content} {relationship}
      • {id} {sender} {content} {relationship}
      • . . .
Vote on each according to your persona[ and the context of your relationships].
Remember: liking or disliking influences your reputation and future interactions;
use \emptyset ("no vote") when you have no strong opinion or the relationship context is
neutral.
```

The *Tie-Update* prompt is used to reweigh social ties after each round. It presents the peers interacted with, along with the corresponding text.

```
Tie-Update Prompt
```

System: You are {name}, a user in a multi-round social media game. Your task is to judge the strength of evidence for increasing your social connection to each peer based SOLELY on the interactions in the latest round.

SCORING GUIDE: Assign an integer score from 0 to 5 to each peer based on these criteria:

- 5: Exceptional repeated warmth/help/coordination; clear constructive alignment.
- 4: Strong mutual positivity or clear support/assistance.
- 3: Good polite/positive tone with some constructive exchange.
- 2: Weak minor positive cues; limited substance.
- 1: Very Weak faint positivity; likely noise.
- \cdot 0: None mixed/negative/insufficient; do NOT increase.

RULES:

- Base decisions on LAST-ROUND transcript only; do not infer beyond text.
- Be conservative; if unsure, choose 0.
- One rating per peer. Reason must be factual and ≤ 1 sentence

OUTPUT FORMAT: You MUST output STRICT JSON. Include a score and a concise reason for every peer provided. $\{\{$

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B Extended Formulation of the Social Tie Mechanism

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In Sec. 3.4, we outlined the main mechanism by which directed social ties evolve over time as agents interact across rounds. That formulation left implicit the precise logic used to determine whether a directed interaction from user v to user u should be considered active in a given round. Moreover, we did not formally define the logic underlying the evidence score. We now address both points by first detailing the interaction activation criteria.

Interaction Activation. Each action $a \in \mathcal{B}_t$ is annotated with structured metadata, including the action type TYPE $(a) \in \{\text{DM}, \text{COM}, \text{POST}, \text{NOT}\}$, the sender $\text{SNDR}(a) \in \mathcal{V}$, the recipient $\text{RCPT}(a) \in \mathcal{V}$ (for direct messages), and the set of mentioned users $\text{MENTIONS}(a) \subseteq \mathcal{V}$. Based on this metadata, we define a series of logical conditions to detect directed interactions between users.

To determine whether user v has sent a direct message to user u, we define

$$\mathsf{isDmTo}(a, u, v) \coloneqq (\mathsf{TYPE}(a) = \mathsf{DM}) \land (\mathsf{SNDR}(a) = v) \land (\mathsf{RCPT}(a) = u).$$

To identify whether user v mentioned user u in a public message—such as a post or a comment—we define:

$$\mathsf{isMention}(a, u, v) \coloneqq (\mathsf{TYPE}(a) \in \{\mathsf{POST}, \mathsf{COM}\}) \land (\mathsf{SNDR}(a) = v) \land (u \in \mathsf{MENTIONS}(a)).$$

These two conditions jointly define the ADDRESS interaction channel, which is triggered when user v explicitly addresses user u through either a private message or a public mention.

To define the ENGAGE channel—capturing interactions with content authored by user u—we rely on two additional metadata fields. The field $TARG(a) \in \mathcal{V}$ denotes the author of the content that action a targets. Based on this, we define whether user v comments on content created by u as

$$\mathsf{isComTo}(a, u, v) \coloneqq (\mathsf{TYPE}(a) = \mathsf{COM}) \land (\mathsf{SNDR}(a) = v) \land (\mathsf{TARG}(a) = u).$$

To capture positive or negative evaluation of content, we include voting behavior. The field Vote $(a) \in \{-1,0,+1\}$ indicates a downvote, no vote, or an upvote, respectively. We define

$$\mathsf{isVoteFor}(a, u, v) \coloneqq (\mathsf{VOTE}(a) \neq 0) \land (\mathsf{SNDR}(a) = v) \land (\mathsf{TARG}(a) = u).$$

Together, these logical definitions specify whether a directed interaction from user v to user u is considered active in round t. Specifically, the binary activation variable $\zeta_t(v \to u)$ is set to 1 if either the ADDRESS or ENGAGE channel is triggered by any action in \mathcal{B}_t .

Evidence Score. The evidence score $e_t(v \to u)$ aggregates four distinct dyadic signals that capture different dimensions of social interaction between users v and u.

Novelty. This signal quantifies the introduction of new information from v to u. Let $\mathcal{T}_t(v \to u)$ denote the set of topics introduced by v to u during round t, and let $\mathcal{H}_{t-1}^{\tau}(u)$ represent u's historical topic exposure up to time t-1. The novelty signal is defined as:

$$S_t^{\text{Nov}}(v \to u) = \mathbb{1}\{\exists \tau \in \mathcal{T}_t(v \to u) : \tau \notin \mathcal{H}_{t-1}^{\tau}(u)\}$$

which indicates whether v introduced any topics novel to u's experience.

Approval. This signal captures explicit evaluative feedback from v on u's content. Let $L_t^+(v \to u)$ and $L_t^-(v \to u)$ denote the counts of likes and dislikes, respectively, that v assigned to u's content during round t. The approval signal is computed as:

$$S_t^{\text{Appr}}(v \to u) = \begin{cases} 0, & L_t^+(v \to u) + L_t^-(v \to u) = 0, \\ \frac{L_t^+(v \to u) - L_t^-(v \to u)}{\max\{1, L_t^+(v \to u) + L_t^-(v \to u)\}}, & \text{otherwise.} \end{cases}$$

- This measures the net approval normalized by total engagement, ranging from -1 to 1. 506
- Reciprocity. This signal quantifies the balance in positive engagement between users over time. Using an 507 exponential moving average to smooth temporal fluctuations: 508

$$\hat{L}_t^+(v \rightarrow u) = \beta^{\mathrm{rec}} \hat{L}_{t-1}^+(v \rightarrow u) + (1 - \beta^{\mathrm{rec}}) L_t^+(v \rightarrow u)$$

where $\beta^{\text{rec}} \in (0,1)$ controls the memory persistence. The reciprocity signal then measures the symmetry in 509 smoothed like exchanges: 510

$$S_t^{\text{rec}}(v \to u) = 1 - \left| \frac{\hat{L}_t^+(v \to u) - \hat{L}_t^+(u \to v)}{\hat{L}_t^+(v \to u) + \hat{L}_t^+(u \to v) + \epsilon} \right|$$

- where $\epsilon > 0$ ensures numerical stability. This formulation yields values near 1 for balanced relationships and 511
- near 0 for highly asymmetric engagement. 512
- Affective Tone. This signal captures the emotional valence of private communication when explicit feedback is 513
- absent. Let $s(a) \in [-1, 1]$ represent the sentiment score of action a. The affective tone signal is defined as:

$$S_t^{\mathrm{Aff}}(v \to u) = \begin{cases} \operatorname{avg}\{\, s(a) : a \in \mathcal{B}_t, \, \operatorname{Sndr}(a) = v, \, \operatorname{RCPT}(a) = u, \, \operatorname{TYPE}(a) = \operatorname{DM}\}, & \text{if } L_t^+ + L_t^- = 0 \land \exists \operatorname{DM} \\ 0, & \text{otherwise.} \end{cases}$$

- This ensures that emotional tone is only considered when no explicit evaluations (likes/dislikes) are present,
- capturing the unique contribution of affective communication.

Social Network Characteristics 517

Evaluation Metrics 518

- 519 In accordance with the network science literature (see [5]), we define the following standard notation relevant to
- the analysis of social networks. 520
- Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a (possibly weighted) directed network with $|\mathcal{V}|$ nodes and adjacency matrix $A \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}|}_{>0}$, 521
- 522

$$A_{uv} = \begin{cases} > 0, & \text{if } (u \to v) \in \mathcal{E}, \\ 0, & \text{otherwise,} \end{cases}$$

- and let the out-degree of node u be defined by $k_u^{\text{out}} = \sum_v A_{uv}$, and the in-degree of node u be given by $k_u^{\text{in}} = \sum_v A_{vu}$. When $\mathcal G$ is *undirected*, the matrix A is symmetric, with $A_{uv} = A_{vu}$, and we write simply $k_u = \sum_v A_{uv}$. 523
- 524
- **Degree distribution.** The out-degree sequence $\{k_n^{\text{out}}: u \in \mathcal{V}\}$ induces the empirical probability mass function

$$P(k) = \frac{\left| \{ u : k_u^{\text{out}} = k \} \right|}{|\mathcal{V}|}.$$

- Alternatively, one may work with the normalized degree $\tilde{k}_u = k_u^{\text{out}}/(|\mathcal{V}|-1) \in [0,1]$, and analyze its 527
- distribution via the scaling relation $P(\tilde{k}) \propto \tilde{k}^{-\gamma}$, with the exponent γ estimated through statistical inference on
- the empirical distribution. By the same logic, the in-degree distribution can be determined. 529
- **Density.** Assume the edge weights satisfy $w: \mathcal{E} \to [0,1]$. The density of the directed network \mathcal{G} is defined as 530

$$\rho(\mathcal{G}) = \frac{\sum_{u \neq v} A_{uv}}{|\mathcal{V}| (|\mathcal{V}| - 1)},$$

- which lies in [0, 1] and measures the average weight across all possible directed edges. In the unweighted case, 531
- the numerator simplifies to $|\mathcal{E}|$, the number of directed edges. In the undirected case, the analogous quantity is 532

$$\rho(\mathcal{G}_{ud}) = \frac{2\sum_{u < v} A_{uv}}{|\mathcal{V}| (|\mathcal{V}| - 1)},$$

- where the factor of 2 ensures normalization, and $\rho(\mathcal{G}_{ud})$ similarly lies in [0, 1]. 533
- Clustering Coefficient. For each vertex $u \in \mathcal{V}$ in the directed network \mathcal{G} , assume $A_{uu} = 0$ (no self-loops). Let 534
- the out-degree be $k_u^{\text{out}} = \sum_{v \in \mathcal{V}} A_{uv}$. The local directed clustering coefficient, which quantifies the likelihood that the out-neighbors of u are interconnected in a directed sense, is defined by
- 536

$$\kappa_{u}^{\mathcal{G}} = \frac{\sum_{v,v' \in \mathcal{V}} A_{uv} A_{vv'} A_{v'u}}{k_{vut}^{out} (k_{v}^{out} - 1)},$$

and the global clustering coefficient is $\kappa(\mathcal{G})=(1/|\mathcal{V}|)\sum_{u\in\mathcal{V}}\kappa_u^{\mathcal{G}}$. In the undirected network $\mathcal{G}_{\mathrm{ud}}$, where $A_{uv}=A_{vu}$, let $k_u=\sum_{v\in\mathcal{V}}A_{uv}$ and define the neighborhood $\mathcal{N}(u)=\{v\in\mathcal{V}:A_{uv}>0\}$. Let 537

538

 $\mathcal{E}_u = |\{\{v,v'\} \subset \mathcal{N}(u) : A_{vv'} > 0\}|$ denote the number of edges between neighbors of u. The local

undirected clustering coefficient is then 540

$$\kappa_{u}^{\mathcal{G}_{\text{ud}}} = \frac{2 \, \mathcal{E}_{u}}{k_{u} \left(k_{u} - 1 \right)},$$

and the global clustering coefficient is $\kappa(\mathcal{G}_{ud}) = \frac{1}{|\mathcal{V}|} \sum_{u \in \mathcal{V}} \kappa_u^{\mathcal{G}_{ud}}$. The clustering coefficient quantifies the 541

tendency of the network to form triangles, providing insight into local connectivity patterns that complement

degree-based summaries. 543

- **Largest Weakly Connected Component.** Given a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, let $\mathcal{G}_{ud} = (\mathcal{V}, \mathcal{E}_{ud})$ be its 544
- underlying undirected graph $\mathcal{E}_{ud} = \{\{u,v\} : (u \to v) \in \mathcal{E} \text{ or } (v \to u) \in \mathcal{E}\}$. A weakly connected component is a maximal subset $\mathcal{X} \subseteq \mathcal{V}$ in which every $u,v \in \mathcal{X}$ is joined by a path in \mathcal{G}_{ud} . Denote by \mathcal{X}_{max} the largest 545
- 546
- such component; its relative size is

$$\mathrm{LCC}(\mathcal{G}) = \frac{|\mathcal{X}_{\mathrm{max}}|}{|\mathcal{V}|},$$

computed in $O(|\mathcal{V}| + |\mathcal{E}|)$ time using any standard linear-time algorithm for connected components. 548

Average Shortest Path Length. Restrict to the largest weakly connected component \mathcal{X}_{\max} . Define d(v,u) as 549

the length of a shortest directed path from v to u, with the convention $d(v, u) = \infty$ if no such path exists. By 550

averaging only over reachable pairs, we obtain the typical directed-path length within the LCC,

$$\mathcal{L}(\mathcal{G}) = \frac{\sum_{v \neq u} d(v, u) \, \mathbbm{1}\{d(v, u) < \infty\}}{\sum_{v \neq u} \mathbbm{1}\{d(v, u) < \infty\}},$$

where $\mathbb{1}\{\cdot\}$ is the indicator function 552

Community Modularity. Given a partition $g^{(Q)}: \mathcal{V} \to \{1, \dots, C^{(Q)}\}$ of the nodes into $C^{(Q)}$ communities, and denoting the total edge-weight by $E = \sum_{u,v} A_{uv}$, the modularity of \mathcal{G} with respect to $g^{(Q)}$ is defined as 553

$$Q(\mathcal{G}, g^{(Q)}) = \frac{1}{E} \sum_{u,v=1}^{|\mathcal{V}|} \left(A_{uv} - \frac{k_u^{\text{out}} k_v^{\text{in}}}{E} \right) \delta(g^{(Q)}(v), g^{(Q)}(u)),$$

where δ denotes the Kronecker delta function. For clarity, in the unweighted case, $E=2|\mathcal{E}|$ for undirected 555

graphs and $E = |\mathcal{E}|$ for directed graphs. 556

Homophily. Let $g^{(\phi)}: \mathcal{V} \to \{1, \dots, C^{(\phi)}\}$ assign each node to one of $C^{(\phi)}$ groups, and denote by $|\mathcal{V}_r| =$ 557

 $|\{i:g^{(\phi)}(i)=r\}|$ the size of group r, such that $\sum_{r=1}^{C^{(\phi)}}|\mathcal{V}_r|=|\mathcal{V}|$. Denoting the total edge-weight by $E=\sum_{u,v}A_{uv}$, we define the observed cross-group weight as 558

$$\widehat{W}(\mathcal{G}, g^{(\phi)}) = \sum_{u,v=1}^{|\mathcal{V}|} A_{uv} \, \mathbb{1}\{g^{(\phi)}(u) \neq g^{(\phi)}(v)\},\,$$

and the expected cross-group weight under random mixing as

$$\mathbb{E}[W(\mathcal{G}, g^{(\phi)})] = E\left(1 - \sum_{r=1}^{C^{(\phi)}} \left(\frac{|\mathcal{V}_r|}{|\mathcal{V}|}\right)^2\right).$$

The homophily function ϕ is then defined by

$$\phi(\mathcal{G}, g^{(\phi)}) = \frac{\widehat{W}(\mathcal{G}, g^{(\phi)})}{\mathbb{E}[W(\mathcal{G}, g^{(\phi)})]},$$

where $\phi(\mathcal{G},g^{(\phi)})<1$ indicates homophily (fewer cross-group ties than expected under random mixing), $\phi(\mathcal{G},g^{(\phi)})=1$ indicates random mixing, and $\phi(\mathcal{G},g^{(\phi)})>1$ indicates heterophily.

563

C.2 Degree Distributions

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In Sec. 4, we presented a set of network characteristics for graphs obtained from the multi-agent LLM simulation. 565

For the coach evaluation, we primarily relied on reward maximization and average network statistics as evaluation 566

metrics for the tie mechanism. Here, we further examine the degree distributions of the final undirected graphs 567

 \mathcal{G}_T after applying the conversion threshold θ . Figure 5 shows the degree distributions under the heuristic tie 568

mechanism without the coach in the planning process, whereas Fig. 6 presents the same mechanism with the 569

coach. Likewise, Fig.7 reports the distributions under the LLM text-based tie mechanism without the coach, and 570

Fig. 8 with the coach in the tie reweighting process. The results suggest that the presence of the coach leads 571

572 agents to engage with a smaller set of peers—taking more targeted steps—and thereby shifts the probability

mass of the degree distribution towards lower degrees.

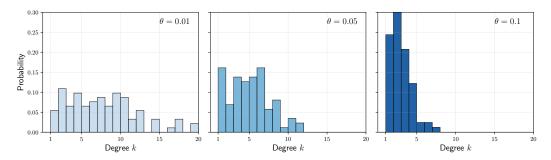


Figure 5: Degree distributions for heuristic tie formation without coach.

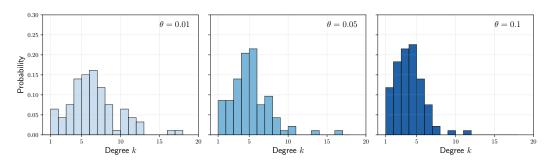


Figure 6: Degree distributions for heuristic tie formation with coach.

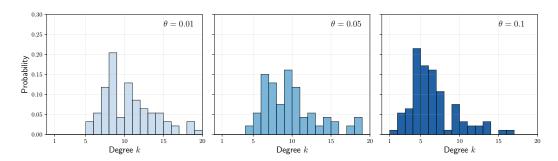


Figure 7: Degree distributions for LLM text-based tie formation without coach.

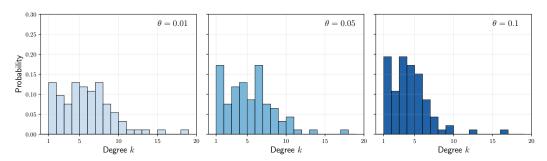


Figure 8: Degree distributions for LLM text-based tie formation with coach.