

000 001 002 003 004 005 ***Memb*a: MEMBRANE-DRIVEN PARAMETER-EFFICIENT** 006 **FINE-TUNING FOR MAMBA**

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013 ABSTRACT

014 State Space Models (SSMs) have emerged as powerful alternatives to attention-
015 based Transformers, with Mamba demonstrating impressive efficiency and scal-
016 ability. As these models grow increasingly larger, the need for Parameter-Efficient
017 Fine-Tuning (PEFT) methods becomes critical to adapt pre-trained Mamba to
018 downstream tasks without prohibitive computational costs. However, previous
019 approaches simply apply traditional Transformer-tailored PEFT methods without
020 addressing the unique temporal processing dynamics of SSMs. To address this
021 limitation, we propose ***Memb*a**, a membrane-driven PEFT approach specifically de-
022 signed for Mamba. ***Memb*a** introduces Leaky Integrate Membrane (LIM) neurons
023 as bio-inspired gating mechanisms that naturally accumulate membrane potentials
024 over time, enhancing selective information retention. By strategically combining
025 LIM neurons with Low-Rank Adaptations (LoRA) and cross-layer membrane
026 transfer, our approach significantly improves Mamba’s temporal modeling capa-
027 bilities. Extensive experiments across language and vision tasks demonstrate that
028 ***Memb*a** achieves substantial improvements over existing PEFT methods.

029 1 INTRODUCTION

030 State Space Models (SSMs) (Gu et al., 2021b;a; Fu et al., 2022) have emerged as powerful alternatives
031 to Transformer (Vaswani et al., 2017) architectures, offering linear computational complexity with
032 respect to sequence length while maintaining competitive performance. SSMs share functional
033 similarities with recurrent architectures, including Long Short-Term Memory (LSTM) (Hochreiter
034 & Schmidhuber, 1997) and Gated Recurrent Unit (GRU) (Chung et al., 2014), through evolving
035 hidden states, though they employ different mathematical foundations based on state space theory (Gu
036 et al., 2020). Recent advancements, particularly Mamba (Gu & Dao, 2023; Dao & Gu, 2024), have
037 demonstrated remarkable success across language modeling (Pióro et al., 2024; Wang et al., 2024),
038 computer vision (Liu et al., 2024c; Zhu et al., 2024), and other domains (Wang et al., 2025; Quan & Li,
039 2024; Ota, 2024; Hu et al., 2024) by introducing selective SSMs with data-dependent parameters. As
040 these models scale, Parameter-Efficient Fine-Tuning (PEFT) methods become crucial for adaptation
041 with minimal trainable parameters. While PEFT techniques have shown success in Transformer-based
042 models (Hu et al., 2022; Houldsby et al., 2019), their application to SSMs remains limited. Recent
043 works (Yoshimura et al., 2024; Halloran et al., 2024; Ham et al., 2024) have begun exploring PEFT
044 for Mamba, but simply transfer Transformer-tailored methods without addressing the unique temporal
045 processing dynamics of SSMs.

046 Although Mamba is meticulously designed based on state-space theory (Gu et al., 2020), current
047 architecture lacks the sophisticated gating structures found in traditional recurrent networks such as
048 LSTM (Hochreiter & Schmidhuber, 1997) and GRU (Chung et al., 2014), relying instead on a single
049 linear transformation. Traditional recurrent networks incorporate multiple trainable gates to manage
050 memory retention and forgetting over time. In contrast, Mamba’s simplified gating mechanism
051 lacks temporal selectivity, structured memory, and nonlinear control capabilities. We believe that
052 this limitation, shared with earlier SSMs such as S4 (Gu et al., 2021a), can hinder the model’s
053 ability to adaptively capture task-specific temporal information during fine-tuning. Furthermore,
recent studies (Yoshimura et al., 2024; Ham et al., 2024) have revealed that directly fine-tuning the
state-space components often degrades Mamba’s performance, suggesting that modifying temporal
processing mechanisms during adaptation is particularly challenging. This phenomenon raises a

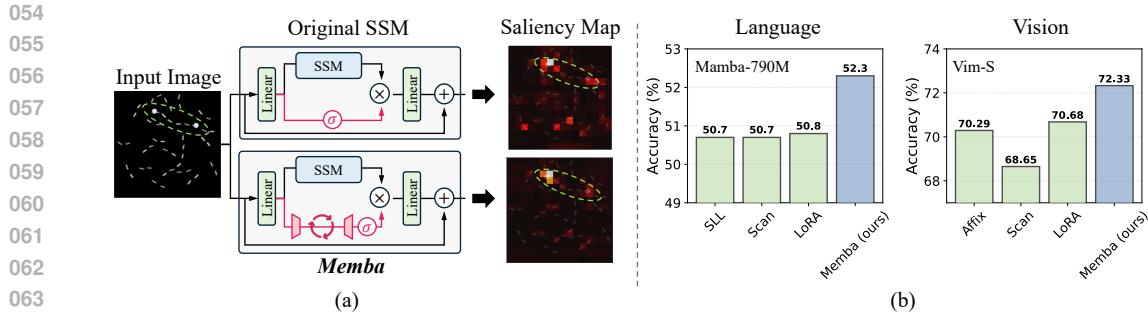


Figure 1: Overview of Memba architecture and performance comparison. (a) Architecture and saliency map comparison between original SSM and **Memba** on a Pathfinder dataset image. The pink lines in architectures represent gating branches, and the green dashed circle indicates the target path to be identified. (b) Performance comparison on language (commonsense reasoning) and vision (VTAB-1k) tasks using Mamba-790M and Vim-S architectures respectively. We compare **Memba** with SLL LoRA, Additional-scan, Affix-tuning, and LoRA in (Yoshimura et al., 2024).

fundamental question: *how can we effectively incorporate temporal adaptation during fine-tuning without disrupting the balanced dynamics of pre-trained SSMs?*

In this work, we propose **Memba**, which introduces a membrane-driven PEFT technique to enhance SSM gating. At the core of our approach is the Leaky Integrate Membrane (LIM) neuron, a novel mechanism that leverages the inherent hidden states of neuronal membrane potentials to strengthen the selective gating capabilities of SSM. Unlike traditional gating with feed-forward layers (Hochreiter & Schmidhuber, 1997; Chung et al., 2014), temporal chunked LIM neurons naturally accumulate the membrane potential over time, providing a sophisticated yet computationally efficient solution for temporal processing without requiring additional learnable parameters or complex recurrent structures. Furthermore, we design the LIM neuron with continuous flow of membrane information across layers, where each neuron transfers averaged membrane states to initialize neurons in subsequent layers, creating uninterrupted temporal processing throughout the network. Our approach combines LIM neurons with strategically placed LoRA on input and output projections, creating a comprehensive PEFT method specifically tailored for Mamba models. Consequently, **Memba** achieves superior performance to existing works (Yoshimura et al., 2024; Halloran et al., 2024) on Mamba fine-tuning while requiring only a fraction of the trainable parameters.

To demonstrate the effectiveness of membrane-driven gating in SSM architectures, Figure 1 presents an overall comparison between a standard SSM model and the proposed **Memba**. In Figure 1(a), **Memba** exhibits sharper saliency focused along the true path, while the original SSM's attention remains diffused, highlighting the inherent benefit of membrane dynamics in promoting selective information flow. Figure 1(b) shows the fine-tuning performance comparisons across language and vision tasks, where **Memba** consistently outperforms existing PEFT methods. The main contributions of our work are as follows:

- We propose **Memba**, a membrane-driven PEFT approach that enhances Mamba's gating mechanisms, effectively introducing temporal adaptation without modifying the core state-space components.
- We introduce a temporal chunked LIM neuron with cross-layer membrane propagation. This LIM neuron efficiently processes long sequences while preserving temporal information through evolving membrane potentials.
- Through extensive experiments across language (commonsense reasoning) and vision (vision adaptation) tasks, we demonstrate that **Memba** consistently achieves *state-of-the-art* performance compared to existing PEFT methods.

2 RELATED WORKS

2.1 STATE SPACE MODEL

State Space Models (SSMs) present a promising direction in sequence modeling that combines the parallel processing advantages of Transformer (Vaswani et al., 2017) with the recurrence properties of Recurrent Neural Networks (RNNs) (Hochreiter & Schmidhuber, 1997; Chung et al., 2014). (Gu

108 et al., 2021a) introduces structured state space sequence models (S4), which leverages efficient
 109 parameterization of continuous-time state space models for sequence modeling, enabling parallel
 110 computation. Subsequent architectures, including the Diagonal State Space (DSS) (Gupta et al.,
 111 2022), the Gated State Space (GSS) (Mehta et al., 2022), and Hungry Hungry Hippos (H3) (Fu
 112 et al., 2022), enhance the expressivity for better performance while simplifying the implementation
 113 complexity. Most recently, Mamba (Gu & Dao, 2023; Dao & Gu, 2024) introduces selective SSMs
 114 with data-dependent parameters, enabling dynamic adaptation to input context. This innovation has
 115 led to competitive performance across several domains (Pióro et al., 2024; Li et al., 2024; Wang
 116 et al., 2025; Quan & Li, 2024; Ota, 2024), establishing SSMs as viable alternatives to Transformer
 117 architectures for sequence processing. Given its promising performance and efficiency, Mamba shows
 118 significant potential to serve as a backbone for pre-trained foundation models (Gu & Dao, 2023;
 119 Ham et al., 2024; Hatamizadeh & Kautz, 2024), making efficient adaptation techniques increasingly
 120 important for future downstream applications.

121 2.2 PARAMETER-EFFICIENT FINE-TUNING

122 PEFT methods address the computational and storage challenges of adapting large pre-trained models
 123 to downstream tasks. These approaches modify only a small subset of model parameters while
 124 keeping the majority frozen. We differentiate the PEFT algorithms into four categories (Han et al.,
 125 2024): additive, selective, parameterized, and hybrid fine-tuning. Additive fine-tuning (Houlsby
 126 et al., 2019; He et al., 2021; Pfeiffer et al., 2020; Mahabadi et al., 2021) introduce new trainable
 127 components while preserving the original weights, including adapters that insert modules between
 128 layers, and prompt-based methods (Li & Liang, 2021; Liu et al., 2021; Lester et al., 2021; Liu
 129 et al., 2024b) that augment inputs with learnable tokens. Selective fine-tuning identifies and updates
 130 only a critical subset of the original parameters, either through unstructured approaches based on
 131 importance metrics (Guo et al., 2020; Sung et al., 2021; Xu et al., 2021) or structured methods
 132 targeting specific components (Zaken et al., 2021; He et al., 2023). Reparameterized fine-tuning
 133 transforms the optimization space, primarily through low-rank techniques like LoRA (Hu et al.,
 134 2022; Zhang et al., 2023) that decompose weight updates into smaller matrices, and its derivatives
 135 that incorporate quantization (Dettmers et al., 2023) or direction-magnitude decoupling (Liu et al.,
 136 2024a). Hybrid fine-tuning combines multiple paradigms to leverage their complementary strengths,
 137 integrating various strategies to achieve optimal performance-efficiency trade-offs (He et al., 2021;
 138 Zhang et al., 2024; Hu et al., 2023).

139 While PEFT methods have been extensively studied in Transformer architectures, their application to
 140 SSM models like Mamba remains relatively unexplored. (Halloran et al., 2024) investigates the appli-
 141 cation of LoRA to Mamba’s state space components, while (Yoshimura et al., 2024) comprehensively
 142 evaluates various PEFT techniques for SSMs, revealing unique challenges posed by their selective
 143 scanning mechanisms. (Ham et al., 2024) demonstrates that targeting projectors rather than state
 144 space components yields superior performance through their diagonal-based adaptation approach,
 145 focusing primarily on transfer learning. These initial studies indicate that although conventional
 146 PEFT methods can be applied to Mamba with reasonable success, there remains significant room
 147 for approaches that better leverage the architectural uniqueness of SSMs. The distinctive temporal
 148 processing characteristics of SSM architectures create opportunities for specialized adaptation tech-
 149 niques that enhance information flow control capabilities without modifying the core state-space
 150 components. Our work addresses this opportunity by introducing a bio-inspired mechanism designed
 to incorporate temporal dynamics into the gating mechanism.

151 3 PRELIMINARIES

152 Mamba (Gu & Dao, 2023) architecture addresses the quadratic computational complexity of transform-
 153 ers by employing SSM with selective mechanisms. The standard continuous-time linear time-invariant
 154 SSM is defined by:

$$155 \mathbf{h}'(t) = \mathbf{A}\mathbf{h}(t) + \mathbf{B}x(t), \quad y(t) = \mathbf{C}\mathbf{h}(t) + \mathbf{D}x(t), \quad (1)$$

156 where $x(t) \in \mathbb{R}$ is the input sequence, $\mathbf{h}(t) \in \mathbb{R}^N$ is the hidden state with N being the state
 157 dimension that controls the model’s representational capacity, and $y(t) \in \mathbb{R}$ is the output. The
 158 parameters $\mathbf{A} \in \mathbb{R}^{(N \times N)}$, $\mathbf{B} \in \mathbb{R}^{(N \times 1)}$, $\mathbf{C} \in \mathbb{R}^{(1 \times N)}$, and $\mathbf{D} \in \mathbb{R}$ define the dynamics of the system.
 159 For practical implementation with discrete inputs like tokens, Mamba employs Zero-Order Hold
 160 (ZOH) discretization to derive equivalent discrete parameters with step size parameter Δ :
 161

$$\bar{\mathbf{A}} = \exp(\Delta \mathbf{A}), \quad \bar{\mathbf{B}} = (\Delta \mathbf{A})^{-1}(\exp(\Delta \mathbf{A}) - \mathbf{I})\Delta \mathbf{B}, \quad \bar{\mathbf{C}} = \mathbf{C}, \quad \bar{\mathbf{D}} = \mathbf{D}. \quad (2)$$

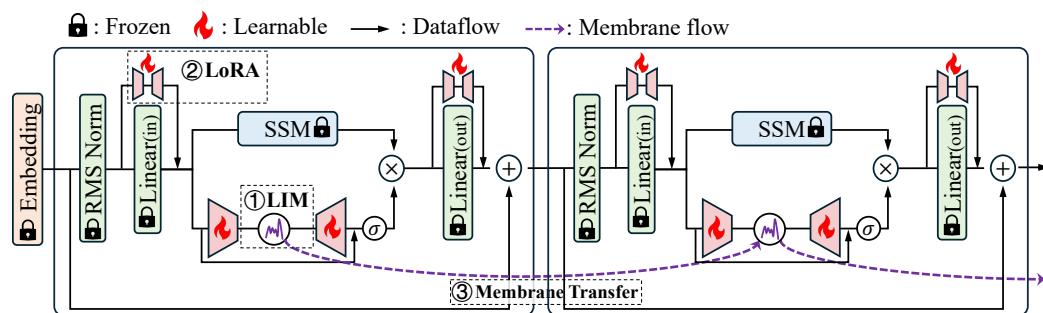


Figure 2: Overview of **Memba** architecture. On top of original Mamba architecture including embedding, normalization, linear layers, and SSM, our **Memba** is designed with ① Leaky Integrate Membrane (LIM), ② Low-Rank Adaptations (LoRAs) on input and output projection, and ③ membrane transfer across layers.

This discretization transforms the continuous system into its discrete counterpart called as selective scan (Scan):

$$\mathbf{h}_t = \bar{\mathbf{A}}\mathbf{h}_{t-1} + \bar{\mathbf{B}}x_t, \quad y_t = \bar{\mathbf{C}}\mathbf{h}_t + \bar{\mathbf{D}}. \quad (3)$$

The key innovation in Mamba lies in its selective parameterization mechanism, where Δ , \mathbf{B} , and \mathbf{C} become input-dependent, consequently making $\bar{\mathbf{A}}$, $\bar{\mathbf{B}}$, $\bar{\mathbf{C}}$, and $\bar{\mathbf{D}}$ input-dependent as well. This is achieved through a structured parameter matrix that computes these values dynamically based on input context, enabling adaptive behavior while maintaining linear computational complexity with respect to sequence length. Critically, Mamba employs a multiplicative gating mechanism that combines the output of selective scan with a transformed input:

$$\hat{y}_t = y_t \odot \sigma(x_t), \quad (4)$$

where $\sigma(\cdot)$ is a SiLU activation and \odot represents element-wise multiplication. This gating mechanism plays a crucial role in controlling information flow through the network. However, while the selective scan in the SSM branch effectively handles temporal processing with evolving hidden states, (Yoshimura et al., 2024; Ham et al., 2024) have shown that fine-tuning this component directly leads to suboptimal results. We address this limitation by introducing temporal adaptation capability to the gating branch during fine-tuning. Detailed notations for the original Mamba architecture are provided in Appendix D.

4 METHODOLOGY

In this section, we propose **Memba**, a membrane-driven PEFT approach for Mamba models that introduces temporal processing in the gating branch during fine-tuning. We first present the overall architecture of the **Memba** block, then introduce its three core components.

4.1 OVERALL ARCHITECTURE

We develop **Memba** by integrating three main components: ① *Leaky Integrate Membrane (LIM)*, which provides bio-inspired temporal processing; ② *optimal placement of Low-Rank Adaptations*, which strategically modifies key projection layers; and ③ *cross-layer membrane potential transfer*, which maintains temporal coherence across network depth. These modifications build upon the original Mamba architecture as illustrated in Figure 2.

When processing an input tensor $\mathbf{X}_{\text{input}} \in \mathbb{R}^{B \times L \times D}$, with B , L , and D representing batch size, length of input sequence, and feature dimension respectively, **Memba** first applies normalization and projection, then divides the resulting tensor into two parallel pathways:

$$\mathbf{X}_{\text{SSM}}, \mathbf{X}_{\text{gate}} = \text{Split}(\mathbf{W}_{\text{in}}(\text{RMS}(\mathbf{X}_{\text{input}}))), \quad (5)$$

where \mathbf{W}_{in} is the linear layer for input projection ("in_proj") doubling the channel dimension, RMS represents RMS normalization, and the Split represents channel-wise division by two. The $\mathbf{X}_{\text{SSM}} \in \mathbb{R}^{B \times L \times D}$ processes information through the selective scan, while the $\mathbf{X}_{\text{gate}} \in \mathbb{R}^{B \times L \times D}$ leverages our LIM mechanism. We detail the LIM mechanism in Section 4.2.

$$\mathbf{Y}_{\text{SSM}} = \text{Scan}(\mathbf{X}_{\text{SSM}}), \quad \mathbf{Y}_{\text{gate}} = \sigma(\mathbf{W}_{\text{out}}^{\text{gate}}(\text{LIM}(\mathbf{W}_{\text{in}}^{\text{gate}}(\mathbf{X}_{\text{gate}}))). \quad (6)$$

216 Here, Scan represents the selective scan computation as shown in equation 3, σ indicates SiLU
 217 activation, and $\mathbf{W}_{\text{in}}^{\text{gate}}$, $\mathbf{W}_{\text{out}}^{\text{gate}}$ are the linear projections before and after the LIM neuron, respectively.
 218 $\mathbf{W}_{\text{in}}^{\text{gate}}$ and $\mathbf{W}_{\text{out}}^{\text{gate}}$ reduce the computational overhead of LIM neuron operations through dimensionality
 219 reduction. \mathbf{Y}_{SSM} and \mathbf{Y}_{gate} are combined through multiplicative gating to produce the final output:
 220

$$\mathbf{Y}_{\text{out}} = \mathbf{W}_{\text{out}}(\mathbf{Y}_{\text{SSM}} \odot \mathbf{Y}_{\text{gate}}), \quad (7)$$

221 where \mathbf{W}_{out} is the linear layer for output projection ("out_proj"). This architecture modification
 222 enhances temporal adaptation through the gate path without directly altering the dynamics of the
 223 selective scan computation.
 224

226 4.2 KEY COMPONENTS OF MEMBA

227 ① LEAKY INTEGRATE MEMBRANE NEURON

228 **Implementation** To bring the temporal flow to
 229 Mamba’s gate branch, we introduce the Leaky
 230 Integrate Membrane (LIM) neuron shown in Figure 3. The LIM neuron is inspired by the Leaky
 231 Integrate-and-Fire (LIF) neuron, described in
 232 detail in Appendix A. Rather than processing each
 233 token as an individual step, which would be ex-
 234 pensive for long sequences, we adopt a chunking
 235 strategy for practical implementation. Given an
 236 input sequence of length L , we partition it into
 237 T equal-sized chunks $\{X[1], X[2], \dots, X[T]\}$,
 238 where each chunk $X[i] \in \mathbb{R}^{B \times L[i] \times D}$ for $i \in$
 239 $\{1, 2, \dots, T\}$ contains $L[i] = \lfloor L/T \rfloor$ tokens,
 240 with B and D representing batch size and fea-
 241 ture dimension respectively. Here, $\lfloor \cdot \rfloor$ denotes
 242 the floor function, ensuring each chunk contains
 243 the same number of tokens for consistent pro-
 244 cessing. For this uniformity in chunk size, we
 245 trim any remainder tokens when the sequence
 246 length is not evenly divisible by the number of
 247 chunks.

248 The key innovation in our LIM approach lies in
 249 maintaining membrane continuity across chunk
 250 boundaries while processing chunks sequentially.
 251 For each chunk (corresponding to one step), we
 252 process the input tokens using the leaky integrate
 253 membrane dynamics with reset:

$$\mathbf{u}[i+1]^l = r(\tau \mathbf{u}[i]^l + \mathbf{W}^l X[i]), \quad (8)$$

$$r(x) = \begin{cases} 0 & \text{if } x > V_{th}, \\ x & \text{otherwise} \end{cases}, \quad (9)$$

254 where $\mathbf{u}[i]^l \in \mathbb{R}^{B \times \lfloor L/T \rfloor \times D}$ is the membrane potential in l -th layer at i -th chunk, $\tau \in (0, 1]$ is the
 255 leaky factor for membrane potential leakage, \mathbf{W}^l is the weight of l -th layer, and $r(\cdot)$ is the reset
 256 function which sets values above threshold to 0 while preserving others. Note that τ and V_{th} are
 257 key parameters for deciding the membrane distribution, and we analyze the effects of τ and V_{th} in
 258 Section 5.3. After processing all chunks sequentially, we concatenate the outputs to reconstruct the
 259 full sequence representation of length L , zero padding if necessary to restore any trimmed tokens.
 260 The detailed algorithm of LIM neuron is shown in Appendix B.

261 **Membrane-Driven Temporal Processing** The LIM neuron is designed to retain the temporal infor-
 262 mation with selective memory through membrane dynamics. We analyze the membrane potential
 263 behavior of the LIM neuron as shown in Figure 4. The input is divided into four colored chunks,
 264 flattened into a sequence, and processed through the LIM neuron to generate membrane potentials. This
 265 visualization reveals two key characteristics of our approach. (1) Critical path features, highlighted
 266 in purple and pink boxes, generate pronounced peaks in the membrane potential, demon-
 267 strating the model’s selective attention to task-relevant information. (2) We observe a gradual decrease in
 268 the membrane potential over time, indicating the model’s ability to forget less relevant information.
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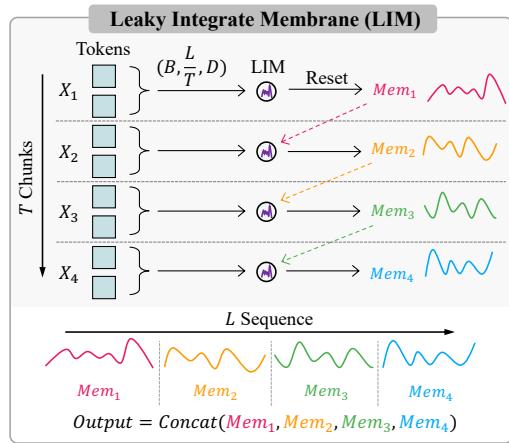


Figure 3: Overview of Leaky Integrate Membrane (LIM). Each token chunk is processed with LIM dynamics, and membrane outputs are concatenated to form the final sequence representation. In this figure, the input contains $L = 8$ tokens split into $T = 4$ chunks, with each chunk (X_1, X_2, X_3, X_4) containing 2 tokens.

For each chunk (corresponding to one step), we process the input tokens using the leaky integrate membrane dynamics with reset:

$$\mathbf{u}[i+1]^l = r(\tau \mathbf{u}[i]^l + \mathbf{W}^l X[i]), \quad (8)$$

$$r(x) = \begin{cases} 0 & \text{if } x > V_{th}, \\ x & \text{otherwise} \end{cases}, \quad (9)$$

where $\mathbf{u}[i]^l \in \mathbb{R}^{B \times \lfloor L/T \rfloor \times D}$ is the membrane potential in l -th layer at i -th chunk, $\tau \in (0, 1]$ is the leaky factor for membrane potential leakage, \mathbf{W}^l is the weight of l -th layer, and $r(\cdot)$ is the reset function which sets values above threshold to 0 while preserving others. Note that τ and V_{th} are key parameters for deciding the membrane distribution, and we analyze the effects of τ and V_{th} in Section 5.3. After processing all chunks sequentially, we concatenate the outputs to reconstruct the full sequence representation of length L , zero padding if necessary to restore any trimmed tokens. The detailed algorithm of LIM neuron is shown in Appendix B.

Membrane-Driven Temporal Processing The LIM neuron is designed to retain the temporal information with selective memory through membrane dynamics. We analyze the membrane potential behavior of the LIM neuron as shown in Figure 4. The input is divided into four colored chunks, flattened into a sequence, and processed through the LIM neuron to generate membrane potentials. This visualization reveals two key characteristics of our approach. (1) Critical path features, highlighted in purple and pink boxes, generate pronounced peaks in the membrane potential, demonstrating the model’s selective attention to task-relevant information. (2) We observe a gradual decrease in the membrane potential over time, indicating the model’s ability to forget less relevant information.

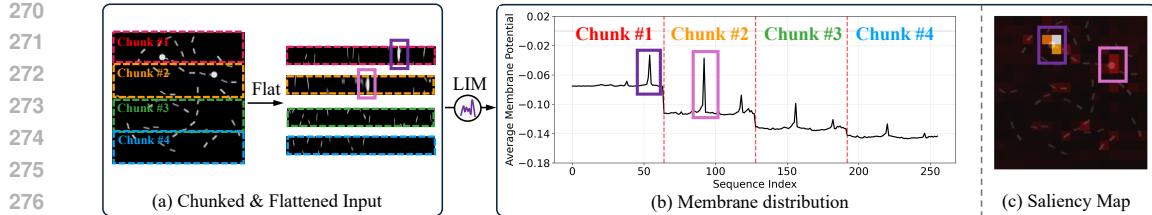


Figure 4: Membrane-driven temporal processing in **Memba**. (a) The input image is divided into spatial chunks and flattened into a sequential representation. (b) Membrane distribution and (c) saliency map through the LIM neuron show how the LIM neuron tracks main features while progressively decreasing baseline potentials across chunks, demonstrating adaptive temporal attention. We provide more visualizations in Appendix F.

baseline membrane potential across chunks, indicating progressive forgetting of memory as context accumulates. **This downward trend aligns with SSM’s natural behavior of retaining recent tokens while forgetting earlier ones.** Unlike the linear gate in the original Mamba, which delivers uniform sensitivity, our membrane-based approach in **Memba** naturally modulates temporal responsiveness.

The LIM neuron not only enables efficient processing of long sequences but also provides theoretical advantages for model optimization and generalization as follows:

Theorem 1. *Let $\mathcal{L}(\mathbf{y})$ be a twice-differentiable loss function, and let $\mathbf{y}_t = f_\theta(\mathbf{X}_t)$ be the output of a standard Mamba block. When augmented with our LIM mechanism, the effective output becomes $\hat{\mathbf{y}}_t = \mathbf{y}_t \odot g(\mathbf{u}_t)$, where \mathbf{u}_t is the membrane potential. The expected loss satisfies:*

$$\mathbb{E}[\mathcal{L}(\hat{\mathbf{y}}_t)] = \mathcal{L}(\mathbf{y}_t \odot g(\bar{\mathbf{u}}_t)) + \mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) + \mathcal{O}(\|\mathbf{\varepsilon}_t\|^3) \quad (10)$$

where $\bar{\mathbf{u}}_t = \mathbb{E}[\mathbf{u}_t]$, $\mathbf{\varepsilon}_t = \mathbf{u}_t - \bar{\mathbf{u}}_t$ with $\mathbb{E}[\mathbf{\varepsilon}_t] = 0$, and \mathcal{R} is a bounded regularization term satisfying $\mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) \leq \frac{\gamma}{2} \cdot \lambda_{\max} \cdot \mathbf{\varepsilon}_t^2$, where λ_{\max} is the maximum eigenvalue of the Hessian of \mathcal{L} and γ depends on the model outputs and gate sensitivity.

This theorem reveals LIM’s dual effect: the mean membrane component provides temporal context integration through leaky dynamics, while the fluctuation component introduces bounded regularization that adapts to model sensitivity and output magnitude. The smoother loss landscape geometry observed in Appendix C.6 represents empirical evidence of how this theoretical regularization manifests in practice. See Appendix C for complete derivation.

② PLACEMENT OF LOW-RANK ADAPTATIONS

To identify the optimal application points for our LIM neuron, we conduct an ablation study on the main components of **Memba**-130M. We evaluate applying LoRA to different projectors on common-sense reasoning tasks. “All” refers to simultaneously adapting four key projectors in original Mamba: input (“in_proj”), output (“out_proj”), time-scale (“dt_proj”), and selective state (“x_proj”) as shown in Appendix D. We then systematically exclude individual components, denoted by “-dt”, “-x”, “-out”, and “-in”. For example, in “-dt”, LoRAs are applied to in_proj, out_proj, and x_proj.

The results in Table 1 clearly show that input and output projectors are most critical for fine-tuning with the LIM neuron. Excluding either leads to performance drops of 1.2% and 0.8% respectively, suggesting these projectors act as crucial information bottlenecks in the **Memba** architecture. We further compare the accuracy of full fine-tuning against **Memba** with LoRA applied to both in_proj and out_proj on 790M and 1.4B architectures in Figure 5. Notably, our approach achieves higher performance than full fine-tuning while using fewer

Table 1: Ablation study on the impact of applying LoRA to different projection components in **Memba**-130M.

	All	-dt	-x	-out	-in
Avg. Acc. (%)	43.9	43.9	43.7	43.1	42.7
Acc. drop (%)	-	0.0	0.2 ↓	0.8 ↓	1.2 ↓

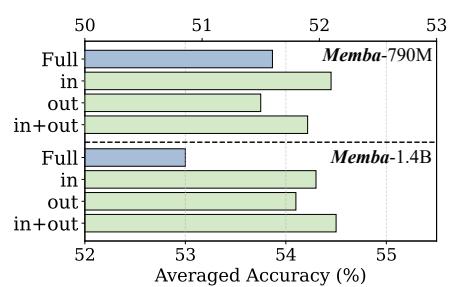


Figure 5: Performance comparison between full fine-tuning and **Memba** with LoRAs applied to in_proj and out_proj across 790M and 1.4B models.

324
 325 Table 2: Performance comparison between ***Memb*a** and prior methods on commonsense reasoning
 326 datasets. Reported values are accuracy percentages (%). **Bold** and underlined entries indicate the
 327 best and second-best performance, respectively. HS and WG refer to the HellaSwag and WinoGrande
 328 datasets. All results are from (Yoshimura et al., 2024), except for SLL LoRA (Halloran et al., 2024).

Model	Method	#Params(%)	BoolQ	PIQA	SIQA	HS	WG	ARC-e	ARC-c	OBQA	Avg.
Pythia 160M	Full	100	61.3	62.9	37.1	30.7	50.6	41.5	24.3	27.8	42.0
	LoRA	0.72	61.0	62.0	36.3	30.3	52.0	38.2	24.6	28.0	41.6
	Full	100	56.1	65.3	38.7	35.3	52.0	46.4	25.7	32.8	43.8
	SLL LoRA	1.45	56.3	63.3	38.2	34.6	51.6	43.5	23.6	30.6	42.7
	Additional-scan	0.51	57.8	64.1	37.5	34.5	53.0	41.3	23.5	30.0	42.7
	Affix-tuning	64.64	59.7	64.3	38.2	35.2	51.9	42.9	24.0	29.0	43.2
	LoRA (in_proj)	2.23	53.5	62.9	38.2	33.8	53.1	46.4	23.7	30.8	42.8
	LoRA _p (X)	2.67	61.7	64.0	39.5	34.3	52.2	43.5	25.3	29.4	43.7
	<i>Memb</i>a (in_proj)	3.95	56.3	64.4	37.7	34.3	52.4	48.9	23.8	30.0	43.5
	<i>Memb</i>a (out_proj)	3.10	58.4	64.9	38.8	34.4	51.8	50.0	24.2	30.0	44.0
	<i>Memb</i>a (in+out_proj)	5.20	58.8	65.8	40.1	34.7	51.6	47.7	24.7	31.2	44.3
Pythia 410M	Full	100	55.0	68.4	42.1	40.8	53.9	50.8	26.7	30.0	46.0
	LoRA	0.77	61.3	67.7	40.8	39.2	54.9	48.1	24.7	28.6	45.7
	Full	100	58.1	69.9	41.9	45.7	53.8	52.7	29.7	33.4	48.2
	SLL LoRA	2.30	59.5	69.6	42.2	44.1	54.9	50.6	26.3	30.8	47.3
	Additional-scan	0.47	61.9	69.3	41.2	45.3	54.9	49.5	28.4	31.4	47.7
	Affix-tuning	68.88	61.2	68.4	39.6	46.2	55.4	48.2	28.2	30.6	47.2
	LoRA (in_proj)	2.07	55.4	68.6	41.0	44.7	54.1	52.4	28.3	33.4	47.2
	LoRA _p (X)	2.67	60.8	68.8	42.1	44.7	56.2	50.4	27.4	32.2	47.8
	<i>Memb</i>a (in_proj)	3.67	59.1	69.2	42.7	45.1	54.0	55.2	28.0	33.0	48.3
	<i>Memb</i>a (out_proj)	2.88	58.0	70.0	42.5	45.4	54.9	55.4	26.7	31.2	48.0
	<i>Memb</i>a (in+out_proj)	4.83	58.7	69.8	42.5	45.4	53.7	55.6	28.0	34.0	48.5
Pythia 1B	Full	100	55.0	70.2	42.5	47.5	54.4	54.1	29.7	33.2	48.3
	LoRA	0.41	60.0	69.3	40.9	45.3	53.6	49.8	27.2	31.0	47.1
	Full	100	62.0	72.1	44.8	54.0	55.9	57.7	31.2	35.2	51.6
	SLL LoRA	3.1	60.7	72.0	42.4	54.7	56.9	55.3	29.4	34.2	50.7
	Additional-scan	0.33	63.0	71.9	41.9	54.2	57.1	54.9	30.0	32.6	50.7
	Affix-tuning	69.99	61.0	72.5	41.0	54.9	55.6	54.6	29.6	33.8	50.4
	LoRA (in_proj)	1.47	61.7	71.9	44.0	50.8	56.7	56.3	30.5	33.8	50.7
	LoRA _p (X)	1.75	59.9	72.2	44.2	52.8	58.0	53.7	30.8	34.8	50.8
	<i>Memb</i>a (in_proj)	2.61	62.2	72.6	43.8	54.4	57.5	61.0	31.7	34.0	52.1
	<i>Memb</i>a (out_proj)	2.04	57.9	72.0	44.3	55.2	56.7	60.4	31.2	34.0	51.5
	<i>Memb</i>a (in+out_proj)	3.45	62.4	72.8	44.1	54.8	57.3	61.3	31.6	34.3	52.3
Pythia 1.4B	Full	100	58.6	71.1	42.7	53.6	55.1	58.5	29.9	34.8	50.5
	LoRA	0.44	60.1	71.3	42.5	50.1	58.9	57.6	29.6	33.6	50.5
	Full	100	61.4	73.3	43.9	56.9	59.0	59.7	34.0	35.4	53.0
	SLL LoRA	4.64	59.7	73.5	43.1	56.9	60.7	59.7	31.7	36.0	52.7
	Additional-scan	0.26	63.0	73.5	42.8	57.5	60.5	60.9	32.4	37.4	53.5
	LoRA (in_proj)	1.13	62.6	73.6	43.7	55.6	59.7	58.3	31.7	35.6	52.6
	LoRA _p (X)	1.36	63.1	73.5	42.7	57.7	61.6	60.4	32.9	37.4	53.7
	<i>Memb</i>a (in_proj)	2.02	63.1	74.0	43.0	58.5	61.7	63.8	33.2	37.4	54.3
	<i>Memb</i>a (out_proj)	1.58	62.6	74.3	43.5	58.6	60.6	64.3	32.3	37.0	54.1
	<i>Memb</i>a (in+out_proj)	2.68	64.4	74.3	43.4	58.6	60.7	64.2	33.0	37.4	54.5

trainable parameters. Full fine-tuning often suffers from overfitting due to the relatively large number of trainable parameters compared to the size of downstream task datasets (Ham et al., 2024).

③ CROSS-LAYER MEMBRANE POTENTIAL TRANSFER

As large models scale in depth, maintaining temporal context across layers becomes challenging, yet crucial for effective sequence modeling. To address this, we implement a cross-layer membrane potential transfer technique that propagates temporal information throughout the network hierarchy without increasing computational cost. After processing all chunks within a layer and obtaining membrane potentials $\{\mathbf{u}^l[1], \mathbf{u}^l[2], \dots, \mathbf{u}^l[T]\}$, we compute the average membrane state, $\bar{\mathbf{u}}^l$ across all chunks. This averaged membrane potential $\bar{\mathbf{u}}^l$ serves as the initial state for the first chunk of the subsequent layer:

$$\bar{\mathbf{u}}^l = \frac{1}{T} \sum_{i=1}^T \mathbf{u}^l[i], \quad \mathbf{u}^{l+1}[1] = \bar{\mathbf{u}}^l. \quad (11)$$

By transferring this compressed temporal context, we enable each layer to begin processing with a summary of the temporal dynamics captured by the previous layer. This mechanism creates a hierarchical flow of temporal information through the network, allowing deeper layers to build upon

378
 379 Table 3: Performance comparison between ***Memb*a** and previous works on VTAB-1k datasets. Values
 380 shown are accuracy percentages (%). **Bold** and underlined values represent the best and second-best
 381 performance respectively. † represents our implementation, and other results are from (Yoshimura
 382 et al., 2024).

Model	Method	#Params(K)	Natural	Specialized	Structured	Avg.
ViT-S	Scratch	21,704	10.66	56.12	24.83	26.20
	Full	21,704	<u>51.79</u>	<u>72.29</u>	<u>45.27</u>	<u>53.47</u>
	LoRA	628	73.60	82.22	57.61	68.68
	Adaptformer	333	73.63	83.15	57.80	68.97
	Adapter+	122	74.68	83.57	58.82	69.87
Vim-S	Scratch	25,450	8.33	49.87	28.16	25.42
	Full	25,450	<u>59.35</u>	<u>68.74</u>	<u>34.39</u>	<u>50.08</u>
	LoRA (embed)	45	64.66	77.53	43.83	58.60
	LoRA (x.proj)	2,540	74.41	81.92	54.88	67.77
	LoRA (dt.proj)	2,442	75.35	83.05	57.12	69.30
	LoRA (out.proj)	2,663	76.42	83.96	60.08	71.12
	LoRA (in.proj)	1,483	76.58	84.08	60.16	71.25
	LoRA _p (Z)	1,778	76.15	84.26	59.72	70.94
	LoRA _p (X)	1,778	76.64	83.89	60.84	71.52
	LoRA (in+out.proj)	709	75.69	84.42	59.43	70.68
	Hybrid (w/ proj)	117,236	<u>77.00</u>	<u>84.41</u>	<u>61.55</u>	<u>72.05</u>
	Hybrid (w/o proj)	1,044	<u>76.85</u>	<u>84.42</u>	<u>61.06</u>	<u>71.80</u>
	<i>Memb</i>a (in.proj)	2,064	76.91	<u>85.10</u>	<u>60.70</u>	<u>71.81</u>
Vanilla-VMamba-S	<i>Memb</i>a (out.proj)	3,244	76.92	<u>85.32</u>	<u>61.18</u>	<u>72.06</u>
	<i>Memb</i>a (in+out.proj)	4,718	77.07	85.66	61.70	72.40
	LoRA (in.proj)†	3,993	77.76	86.05	63.44	73.48
	LoRA (out.proj)†	2,396	77.43	<u>86.06</u>	<u>64.33</u>	<u>73.73</u>
	LoRA (in+out.proj)†	6,389	<u>77.31</u>	<u>85.81</u>	<u>63.30</u>	<u>73.20</u>
<i>Memb</i>a	<i>Memb</i>a (in.proj)	5,591	<u>77.66</u>	<u>86.06</u>	<u>63.92</u>	<u>73.64</u>
	<i>Memb</i>a (out.proj)	3,993	77.77	85.98	64.87	74.07
	<i>Memb</i>a (in+out.proj)	7,987	77.14	86.14	63.93	73.48

405
 406 the representations learned by earlier layers. The use of averaged membrane potentials helps prevent
 407 information loss that might occur if only the final state were propagated, ensuring that the model
 408 maintains sensitivity to patterns that emerged at different points in the sequence.

409 5 EXPERIMENTS

410 In this section, we evaluate the proposed ***Memb*a** approach on both language and vision tasks. For
 411 language tasks, we fine-tune pre-trained Mamba on 8 commonsense reasoning benchmarks. For
 412 vision tasks, we fine-tune two different architectures, Vim (Zhu et al., 2024) and VMamba (Liu
 413 et al., 2024c), on the Visual Task Adaptation-Benchmark (VTAB)-1k dataset (Zhai et al., 2019). We
 414 also present ablation studies to analyze the ***Memb*a** architecture for a better understanding. Our
 415 experimental setup follows the framework established in MambaPeft (Yoshimura et al., 2024), and
 416 the experiments are implemented by A100 GPUs.

417 5.1 LANGUAGE TASK

418 **419 Experimental Details** To evaluate ***Memb*a**, we begin by loading pre-trained Mamba models (Gu &
 420 Dao, 2023) trained on the Pile dataset (Gao et al., 2020), apply the ***Memb*a** architecture modifications,
 421 and then fine-tune ***Memb*a** on a combined dataset of approximately 170k examples from commonsense
 422 reasoning tasks. We evaluate performance across eight individual benchmarks: BoolQ, PIQA, SIQA,
 423 HellaSwag, WinoGrande, ARC-Challenge, ARC-Easy, and OpenbookQA. For fine-tuning, we follow
 424 the setup of LLM-Adapter (Hu et al., 2023), and for evaluation, we use the `lm_eval` framework (Gao
 425 et al., 2024). Detailed hyperparameters are provided in Appendix E.

426 **427 Results** The overall results of ***Memb*a** on language tasks are shown in Table 2. Our comparison
 428 baselines are Transformer-based architecture Pythia (Biderman et al., 2023) and previous Mamba
 429 fine-tuning approaches, including SLL LoRA (Halloran et al., 2024) and MambaPEFT (Yoshimura
 430 et al., 2024). We present three variants of our approach for each model size: LoRA applied to
 431 input projections (`in.proj`), output projections (`out.proj`), and both. We observe that our
 432 ***Memb*a** consistently achieves *state-of-the-art* performance. Notably, with the Mamba 790M model,
 433 our `in.proj+out.proj` variant demonstrates a substantial 1.5% absolute improvement over the

Table 4: Ablation studies of (a) key components of LIM and (b) membrane parameters.

① LIM	② LoRA	③ Membrane Transfer	Mamba-130m	Vim-S	$\tau (V_{th} = 1)$	1/2	1/3	1/4	1/5
✓	✗	✗	43.1	83.8	Averaged Acc. (%)	85.7	85.2	85.2	85.0
✓	✗	✓	43.3	84.0	$V_{th} (\tau = 1/2)$	0.5	1	2	3
✗	✓	✗	43.8	85.3	Averaged Acc. (%)	85.4	85.7	85.3	85.2
✓	✓	✗	44.0	85.5					
✓	✓	✓	44.3	85.7					

(a) Accuracy comparison between combinations of key components of LIM neuron.

(b) Impact of membrane parameters on vision tasks. τ and V_{th} are leaky factor and threshold respectively.

best results reported in MambaPEFT (Yoshimura et al., 2024), highlighting the effectiveness of our membrane-based approach for larger models. The iso-parameter cases are shown in the Appendix H.2.

5.2 VISION TASK

Experimental Details We further investigate the effectiveness of **Memba** on vision tasks by fine-tuning two pre-trained Mamba architectures: Vim (Zhu et al., 2024) and VMamba (Hatamizadeh & Kautz, 2024), both initially trained on ImageNet-1k (Deng et al., 2009). Since the original VMamba architecture removes the gate branch, we utilize Vanilla-VMamba, which preserves the gating mechanism. We evaluate performance on the VTAB-1k image classification benchmark (Zhai et al., 2019), which comprises Natural, Specialized, and Structured domains. For fine-tuning, we adopt the DeiT (Touvron et al., 2021) training framework across all domains. Detailed hyperparameters are provided in Appendix E.

Results Table 3 presents our **Memba** results on Vim (Zhu et al., 2024) and VMamba (Hatamizadeh & Kautz, 2024) architectures, compared against ViT (Dosovitskiy et al., 2020) and previous PEFT approaches from MambaPEFT (Yoshimura et al., 2024). As in our language experiments, we evaluate variants with LoRA applied to input projections (`in_proj`), output projections (`out_proj`), or both. Our **Memba** outperforms previous PEFT methods on both Vim-S and Vanilla-VMamba-S architectures. Notably, with Vim-S, our `out_proj` variant achieves 72.40% average accuracy across all domains, surpassing the previous best result of Hybird method while using only 28% of the trainable parameters. Per-task performances of each categories are shown in Appendix G.

5.3 ANALYSIS

Contribution of Key Components To quantify the impact of each component in **Memba**, we conduct an ablation study examining combinations of our three key innovations: ① LIM neurons, ② LoRA, and ③ membrane transfer. Table 4(a) presents these results. For language tasks, we use the 130M parameter Mamba model on commonsense reasoning, while for vision tasks, we use the Vim-S architecture on the Specialized category of VTAB-1k, which includes Camelyon, EuroSAT, Resisc45, and Retinopathy datasets. Our results show that while LoRA enables effective fine-tuning of Mamba models, adding LIM neurons and membrane transfer further boosts performance.

Impact of Membrane Parameters In LIM neuron, two key hyperparameters control the membrane dynamics: the leaky factor (τ) and threshold (V_{th}) in equation 8 and equation 9. The leaky factor determines how much previous membrane potential is retained, while the threshold governs the reset mechanism. To understand their influence, we conduct experiments with various parameter combinations using the Vim-S architecture on the Specialized category of VTAB-1k, as shown in Table 4(b). For the leaky factor, higher values of τ yield better performance, confirming that stronger retention of previous states benefits temporal processing. For the threshold, $V_{th} = 1$ provides optimal performance, indicating lower values trigger excessive resets that disrupt information flow, while higher values reduce reset frequency, hindering the model’s ability to filter irrelevant information. These results demonstrate that balanced membrane dynamics are critical for effective temporal processing. Additional extensive ablation studies are presented in Appendix H.

6 CONCLUSION

We introduce **Memba**, a membrane-driven PEFT approach for Mamba models that integrates Leaky Integrate Membrane (LIM) neurons with strategic Low-Rank Adaptations (LoRAs). Without modifying state-space components, our method enhances temporal adaptation capabilities exclusively through the gating branch, preserving the tuned dynamics of pre-trained SSMs. Experiments across language and vision tasks demonstrate **Memba**’s consistent improvement over existing PEFT methods. Our approach represents an important step toward specialized adaptation techniques for SSMs, opening possibilities for effective fine-tuning of these architectures across diverse applications. Our separate limitation section is presented in the Appendix J.

486 REPRODUCIBILITY STATEMENT
487488 Our **Memb**a implementation builds upon the MambaPEFT codebase (Yoshimura et al., 2024).
489 Complete hyperparameters and experimental configurations are provided in Appendix E, with the
490 LIM algorithm detailed in Appendix B. All datasets follow standard evaluation protocols with exact
491 settings documented. Code will be made available upon acceptance.
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702 **A LEAKY INTEGRATE-AND-FIRE (LIF) NEURON**
703

704 The Leaky Integrate-and-Fire (LIF) neuron (Burkitt, 2006) has emerged as an important component
705 for energy-efficient computation in SNNs (Maass, 1997; Roy et al., 2019). The LIF neuron processes
706 temporal information through its membrane potential dynamics as follows:
707

708
$$\mathbf{u}[i+1]^l = \tau \mathbf{u}[i]^l + \mathbf{W}^l r(\mathbf{u}[i]^{l-1}), \quad (12)$$

709
$$r(\mathbf{u}[i]^l) = \begin{cases} 1 & \text{if } \mathbf{u}[i]^l > V_{th}, \\ 0 & \text{otherwise} \end{cases}, \quad (13)$$
710

712 where, $\mathbf{u}[i]^l$ is the membrane potential in l -th layer at timestep i , $\tau \in (0, 1]$ is the leaky factor
713 for membrane potential leakage, \mathbf{W}^l is the weight of l -th layer, and $f(\cdot)$ is the LIF function with
714 threshold V_{th} . When the membrane $\mathbf{u}[i]^l$ is higher than V_{th} , the LIF function generates a spike and
715 the membrane potential is reset to 0. The LIF neuron exhibits two core characteristics: (1) Leaky
716 Integration and (2) Reset Mechanism.
717

718 The term $\tau \mathbf{u}[i]^l$ models the leaky accumulation of membrane potential in biological neurons' mem-
719 branes. This leaky mechanism enables forgetting old, unnecessary information while focusing on
720 new inputs, mimicking how biological neurons naturally attenuate stale signals. The reset function
721 $r(\cdot)$ is analogous to the fire-and-reset mechanism in biological neurons, where the neuron discharges
722 its potential upon crossing a threshold V_{th} . In biological neurons, this generates a spike.
723

724 **B THE DETAILS OF LIM ALGORITHM**
725

726 We present the details of the LIM process, which involves efficient processing of sequential data
727 through temporal chunking and cross-layer membrane potential transfer.
728

729 **Algorithm 1** Leaky Integrate Membrane
730

731 **Input:** Input sequence tensor $\mathbf{X}_{\text{input}} \in \mathbb{R}^{B \times L \times D}$, number of chunks T , leak factor τ , threshold
732 V_{th} , linear projection $f(\cdot)$, previous membrane potential $\mathbf{u}_{\text{prev}} \in \mathbb{R}^{B \times \lfloor L/T \rfloor \times D}$
733 **Output:** Output membrane $\mathbf{u}_{\text{output}} \in \mathbb{R}^{B \times L \times D}$, averaged membrane $\mathbf{u}_{\text{prev}} \in \mathbb{R}^{B \times \lfloor L/T \rfloor \times D}$
734 $l \leftarrow \lfloor L/T \rfloor \quad \# \text{Chunk size using floor function}$
735 $r \leftarrow L - l \times T \quad \# \text{Calculate remainder}$
736 **if** $r \neq 0$ **then**
737 $\mathbf{X} \leftarrow \mathbf{X}_{\text{input}}[:, :, -r, :]$ *# Trim sequence to be divisible by T*
738 $L \leftarrow L - r$
739 **end if**
740 **if** \mathbf{u}_{prev} is None **then**
741 $\mathbf{u}_{\text{current}} \leftarrow 0$
742 **else**
743 $\mathbf{u}_{\text{current}} \leftarrow \mathbf{u}_{\text{prev}} \quad \# \text{Transfer membrane state from previous layer}$
744 **end if**
745 **for** $i = 0$ **to** $T - 1$ **do**
746 $\text{start_idx} \leftarrow i \times l$
747 $\text{end_idx} \leftarrow (i + 1) \times l$
748 $\mathbf{X}_i \leftarrow \mathbf{X}[:, \text{start_idx} : \text{end_idx}, :]$ *# Extract current chunk*
749 $\mathbf{u}_{\text{current}} \leftarrow \tau \cdot \mathbf{u}_{\text{current}} + f(\mathbf{X}_i) \quad \# \text{Leaky integrate dynamics}$
750 $\mathbf{u}_{\text{current}} \leftarrow \text{Reset}(\mathbf{u}_{\text{current}}) \quad \# \text{Apply Reset with threshold } V_{th}$
751 $\mathbf{u}_{\text{list}}[i] = \mathbf{u}_{\text{current}}$
752 **end for**
753 $\mathbf{u}_{\text{output}} \leftarrow \text{concatenate}(\mathbf{u}_{\text{list}}, \text{dim} = 0) \quad \# \text{Combine all chunks}$
754 $\mathbf{u}_{\text{prev}} \leftarrow \text{mean}(\mathbf{u}_{\text{list}}, \text{dim} = 0) \quad \# \text{Average the membrane for transfer to next layer}$
755 **if** $r \neq 0$ **then**
756 $\mathbf{u}_{\text{output}} \leftarrow \text{pad}(\mathbf{u}_{\text{output}}) \quad \# \text{Pad to original length}$
757 **end if**
758 **return** $\mathbf{u}_{\text{output}}, \mathbf{u}_{\text{prev}}$

756 C THEORETICAL DERIVATION
757758 C.1 DECOMPOSITION OF LIM SIGNAL
759760 In our LIM mechanism, the membrane potential evolves according to the dynamics defined in equa-
761 tion 8:
762

$$\mathbf{u}_t = r(\tau \mathbf{u}_{t-1} + \mathbf{W} \mathbf{X}_t) \quad (14)$$

763 where $r(\cdot)$ is the reset function from equation 9.
764765 We decompose the membrane potential \mathbf{u}_t into a mean component and a fluctuation component:
766

$$\mathbf{u}_t = \bar{\mathbf{u}}_t + \varepsilon_t, \quad \text{where } \mathbb{E}[\varepsilon_t] = \mathbf{0} \quad (15)$$

767 The mean component $\bar{\mathbf{u}}_t = \mathbb{E}[\mathbf{u}_t]$ captures the expected membrane potential, which represents the
768 temporal integration of input history. The fluctuation component $\varepsilon_t = \mathbf{u}_t - \bar{\mathbf{u}}_t$ represents deviations
769 from this expected behavior.
770771 **Boundedness of fluctuations:** Due to the reset mechanism in $r(\cdot)$, the membrane potential is
772 bounded:
773

$$0 \leq \mathbf{u}_t \leq V_{th} \quad \text{for all } t \quad (16)$$

774 Since the mean $\bar{\mathbf{u}}_t = \mathbb{E}[\mathbf{u}_t]$ is also bounded within the same range, the fluctuations are bounded:
775

$$\|\varepsilon_t\| \leq V_{th} \quad (17)$$

776 The decomposition $\mathbf{u}_t = \bar{\mathbf{u}}_t + \varepsilon_t$ enables us to analyze how membrane dynamics affect the loss
777 function: the mean component $\bar{\mathbf{u}}_t$ provides stable temporal context integration, while the bounded
778 fluctuation component ε_t introduces controlled variability that, as we show in the following sections,
779 acts as an adaptive regularization mechanism.
780782 C.2 MULTIPLICATIVE GATING ANALYSIS
783784 In our architecture, the membrane potential influences the output through multiplicative gating:
785

$$\hat{\mathbf{y}}_t = \mathbf{y}_t \odot g(\mathbf{u}_t) \quad (18)$$

786 where \mathbf{y}_t is the output from the selective scan, $g(\cdot)$ is the gating function applied element-wise
787 (typically SiLU activation), and \odot denotes element-wise multiplication.
788789 Using the decomposition of \mathbf{u}_t from the previous section:
790

$$g(\mathbf{u}_t) = g(\bar{\mathbf{u}}_t + \varepsilon_t) \quad (19)$$

791 We apply an element-wise Taylor expansion of g around $\bar{\mathbf{u}}_t$:
792

$$g(\mathbf{u}_t) = g(\bar{\mathbf{u}}_t) + g'(\bar{\mathbf{u}}_t) \odot \varepsilon_t + \frac{1}{2} g''(\bar{\mathbf{u}}_t) \odot \varepsilon_t^2 + O(\|\varepsilon_t\|^3) \quad (20)$$

793 where $g'(\bar{\mathbf{u}}_t)$ and $g''(\bar{\mathbf{u}}_t)$ denote element-wise derivatives, and ε_t^2 represents element-wise squaring.
794795 Therefore, the gated output becomes:
796

$$\hat{\mathbf{y}}_t = \mathbf{y}_t \odot g(\bar{\mathbf{u}}_t) + \mathbf{y}_t \odot g'(\bar{\mathbf{u}}_t) \odot \varepsilon_t + \frac{1}{2} \mathbf{y}_t \odot g''(\bar{\mathbf{u}}_t) \odot \varepsilon_t^2 + O(\|\varepsilon_t\|^3) \quad (21)$$

803 C.3 ANALYSIS OF EXPECTED LOSS
804805 We analyze the expected loss where the expectation is taken over the randomness in the membrane
806 reset process, treating the input sequence and resulting selective scan output \mathbf{y}_t as deterministic for a
807 given input.
808809 We expand around $\mathbf{y}_t^* = \mathbf{y}_t \odot g(\bar{\mathbf{u}}_t)$, which represents the output when the membrane potential is
810 at its expected value. Note that this is not exactly $\mathbb{E}[\hat{\mathbf{y}}_t]$ due to the nonlinearity of g , but serves as a
811 natural reference point that isolates the effects of membrane fluctuations.
812

810 Let $\delta \mathbf{y}_t = \hat{\mathbf{y}}_t - \mathbf{y}_t^*$. From the multiplicative gating analysis:

$$811 \quad 812 \quad 813 \quad \delta \mathbf{y}_t = \mathbf{y}_t \odot g'(\bar{\mathbf{u}}_t) \odot \varepsilon_t + \frac{1}{2} \mathbf{y}_t \odot g''(\bar{\mathbf{u}}_t) \odot \varepsilon_t^2 + O(\|\varepsilon_t\|^3) \quad (22)$$

814 The Taylor expansion of the loss function around \mathbf{y}_t^* gives:

$$815 \quad \mathcal{L}(\hat{\mathbf{y}}_t) = \mathcal{L}(\mathbf{y}_t^* + \delta \mathbf{y}_t) \quad (23)$$

$$816 \quad 817 \quad = \mathcal{L}(\mathbf{y}_t^*) + \nabla \mathcal{L}(\mathbf{y}_t^*)^\top \delta \mathbf{y}_t + \frac{1}{2} \delta \mathbf{y}_t^\top \nabla^2 \mathcal{L}(\mathbf{y}_t^*) \delta \mathbf{y}_t + O(\|\delta \mathbf{y}_t\|^3) \quad (24)$$

818 Taking expectation over the membrane fluctuations:

$$819 \quad 820 \quad \mathbb{E}[\mathcal{L}(\hat{\mathbf{y}}_t)] = \mathcal{L}(\mathbf{y}_t^*) + \nabla \mathcal{L}(\mathbf{y}_t^*)^\top \mathbb{E}[\delta \mathbf{y}_t] + \frac{1}{2} \mathbb{E}[\delta \mathbf{y}_t^\top \nabla^2 \mathcal{L}(\mathbf{y}_t^*) \delta \mathbf{y}_t] + O(\mathbb{E}[\|\delta \mathbf{y}_t\|^3]) \quad (25)$$

822 Since \mathbf{y}_t and $\bar{\mathbf{u}}_t$ are deterministic given the input, and $\mathbb{E}[\varepsilon_t] = \mathbf{0}$:

$$823 \quad 824 \quad \mathbb{E}[\delta \mathbf{y}_t] = \mathbf{y}_t \odot g'(\bar{\mathbf{u}}_t) \odot \mathbb{E}[\varepsilon_t] + \frac{1}{2} \mathbf{y}_t \odot g''(\bar{\mathbf{u}}_t) \odot \mathbb{E}[\varepsilon_t^2] \quad (26)$$

$$825 \quad 826 \quad = \frac{1}{2} \mathbf{y}_t \odot g''(\bar{\mathbf{u}}_t) \odot \mathbb{E}[\varepsilon_t^2] \quad (27)$$

827 For the second-order term, we assume independent fluctuations across dimensions with $\mathbb{E}[\varepsilon_{t,i} \varepsilon_{t,j}] = \delta_{ij} \sigma_i^2$, which is reasonable given the independent reset behavior in each dimension. The dominant contribution comes from the first-order term in $\delta \mathbf{y}_t$:

$$828 \quad 829 \quad \mathbb{E}[\delta \mathbf{y}_t^\top \nabla^2 \mathcal{L}(\mathbf{y}_t^*) \delta \mathbf{y}_t] \approx \mathbb{E}[(\mathbf{y}_t \odot g'(\bar{\mathbf{u}}_t) \odot \varepsilon_t)^\top \nabla^2 \mathcal{L}(\mathbf{y}_t^*) (\mathbf{y}_t \odot g'(\bar{\mathbf{u}}_t) \odot \varepsilon_t)] \quad (28)$$

$$830 \quad 831 \quad = \sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 [\nabla^2 \mathcal{L}(\mathbf{y}_t^*)]_{i,i} \sigma_i^2 \quad (29)$$

834 We define the regularization term to include both the bias correction and the quadratic regularization:

$$835 \quad 836 \quad \mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) = \nabla \mathcal{L}(\mathbf{y}_t^*)^\top \left(\frac{1}{2} \mathbf{y}_t \odot g''(\bar{\mathbf{u}}_t) \odot (\sigma_1^2, \dots, \sigma_d^2)^\top \right) + \frac{1}{2} \sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 [\nabla^2 \mathcal{L}(\mathbf{y}_t^*)]_{i,i} \sigma_i^2 \quad (30)$$

839 Therefore:

$$840 \quad \mathbb{E}[\mathcal{L}(\hat{\mathbf{y}}_t)] = \mathcal{L}(\mathbf{y}_t \odot g(\bar{\mathbf{u}}_t)) + \mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) + \mathcal{O}(\|\varepsilon_t\|^3) \quad (31)$$

842 C.4 BOUNDEDNESS OF THE LIM LOSS

844 To establish the theoretical foundation for our regularization analysis, we first demonstrate the boundedness of $\mathbb{E}[\mathcal{L}(\hat{\mathbf{y}}_t)]$ by showing that both components of the gated output $\mathbf{y}_t \odot g(\bar{\mathbf{u}}_t)$ remain bounded.

847 **Boundedness of \mathbf{y}_t (Mamba output):** The HiPPO theory (Gu et al., 2020) ensures that Mamba’s hidden state remains bounded through its initialization strategy. Specifically, HiPPO initializes the state matrix \mathbf{A} with all negative real parts of eigenvalues, making the discretized system matrix $\bar{\mathbf{A}} = \exp(\Delta \mathbf{A})$ contractive with spectral radius less than 1. This contractivity property ensures that the recurrent dynamics (equation 3) cannot grow unboundedly. Even with Mamba’s input-dependent discretization parameter Δ_t , the constraint $\Delta_t > 0$ preserves the contractivity property since $\exp(\Delta_t \mathbf{A})$ maintains the same spectral properties as $\exp(\mathbf{A})$ when $\Delta_t > 0$. Therefore, for bounded inputs $\|\mathbf{x}_t\| \leq X_{\max}$, we have $\|\mathbf{h}_t\| \leq H$ for some constant H , which ensures $\|\mathbf{y}_t\| \leq M$ for some bound M .

856 **Boundedness of $g(\bar{\mathbf{u}}_t)$:** The gating function is bounded by the design of our LIM mechanism. Since the reset function constrains $0 \leq \bar{\mathbf{u}}_t \leq V_{th}$ and the activation function $g(\cdot)$ (typically SiLU) is bounded on this interval, we have $|g(\bar{\mathbf{u}}_t)| \leq G_{\max}$ for some constant G_{\max} .

859 **Final boundedness:** Since both components are bounded, the gated output satisfies $\|\mathbf{y}_t \odot g(\bar{\mathbf{u}}_t)\| \leq 860 M \cdot G_{\max}$. For a Lipschitz continuous loss function \mathcal{L} with Lipschitz constant L_{lip} , we obtain:

$$861 \quad 862 \quad \mathbb{E}[\mathcal{L}(\hat{\mathbf{y}}_t)] \leq \mathcal{L}(\mathbf{0}) + L_{\text{lip}} \cdot M \cdot G_{\max} \quad (32)$$

863 Therefore, the expected loss under our LIM mechanism is bounded, providing the theoretical foundation for our subsequent regularization analysis.

864 C.5 BOUNDING THE REGULARIZATION TERM
865866 We now establish the bound on the regularization term $\mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t)$ as stated in Theorem 1.
867868 From the analysis in the previous section, the dominant contribution to $\mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t)$ comes from the
869 quadratic term:
870

871
$$\mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) \approx \frac{1}{2} \sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 [\nabla^2 \mathcal{L}(\mathbf{y}_t^*)]_{i,i} \sigma_i^2 \quad (33)$$

872

873 Bounding the diagonal Hessian elements by the maximum eigenvalue λ_{\max} :
874

875
$$[\nabla^2 \mathcal{L}(\mathbf{y}_t^*)]_{i,i} \leq \lambda_{\max} (\nabla^2 \mathcal{L}(\mathbf{y}_t^*)) \quad (34)$$

876

877 Therefore:
878

879
$$\sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 [\nabla^2 \mathcal{L}(\mathbf{y}_t^*)]_{i,i} \sigma_i^2 \leq \lambda_{\max} (\nabla^2 \mathcal{L}(\mathbf{y}_t^*)) \sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 \sigma_i^2 \quad (35)$$

880

881 The term γ in Theorem 1 captures the combined effect of model outputs, gate sensitivity, and
882 fluctuation variance. We can bound:
883

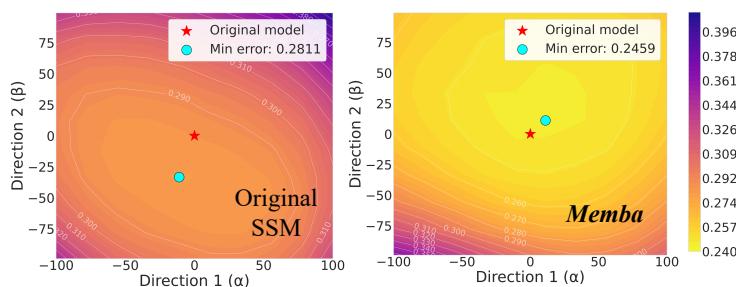
884
$$\sum_i (y_{t,i} g'(\bar{u}_{t,i}))^2 \sigma_i^2 \leq \gamma \epsilon^2 \quad (36)$$

885

886 where γ is a dimensionless constant and $\epsilon^2 = \mathbb{E}[\|\varepsilon_t\|^2] = \sum_i \sigma_i^2$.
887888 Let $\lambda_{\max} = \max_t \lambda_{\max} (\nabla^2 \mathcal{L}(\mathbf{y}_t^*))$.
889890 Therefore:
891

892
$$\mathcal{R}(\mathbf{y}_t, \bar{\mathbf{u}}_t) \leq \frac{1}{2} \lambda_{\max} \cdot \gamma \epsilon^2 = \frac{\gamma}{2} \cdot \lambda_{\max} \cdot \epsilon^2 \quad (37)$$

893

894 This establishes the bound stated in Theorem 1.
895896 C.6 LOSS LANDSCAPE VISUALIZATION
897898 To qualitatively verify our theorized regularization effect, Figure 6 presents a comparison of loss
899 landscapes between Mamba fine-tuned with standard LORA and our **Memb** approach. The land-
900 scapes are visualized for models fine-tuned on the Diabetic Retinopathy dataset from the VTAB-1k
901 benchmark. The visualization reveals two key findings that align with our theoretical analysis: (1)
902 **Memb** achieves a lower overall loss (minimum error 0.2459 vs. 0.2811), demonstrating its better
903 optimization capability; and (2) **Memb** produces a smoother, more convex loss landscape with
904 more gradual contour transitions. The smoother geometry suggests LIM helps avoid sharp minima,
905 potentially contributing to its improved generalization performance and stability during fine-tuning.
906916 Figure 6: Loss landscapes of original SSM and Memb.
917

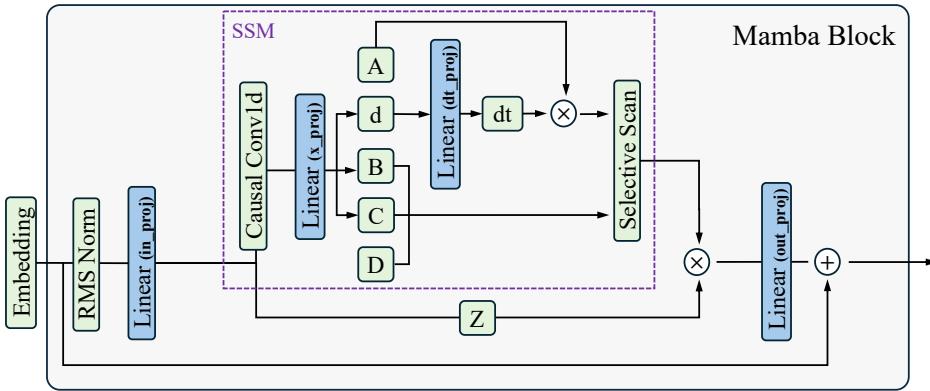


Figure 7: Overall architecture of the original Mamba block.

D ORIGINAL MAMBA ARCHITECTURE

To facilitate understanding of our LoRA placement strategy in conjunction with the LIM neuron, we present the overall architecture of the original Mamba block in Figure 7. The SSM core, enclosed in the purple dotted box, follows the computations described in Equations equation 1, equation 2, equation 3, where Z represents the gating values. We highlight the four linear layers used for LoRA insertion in blue: `in_proj`, `x_proj`, `dt_proj`, and `out_proj`. The ablation results reported in Table 1 correspond to the impact of selectively applying LoRA to each of these layers in combination with the LIM neuron.

E HYPERPARAMETER DETAILS

E.1 LANGUAGE TASK

For commonsense reasoning evaluation, we adopt the fine-tuning setup from (Hu et al., 2023; Yoshimura et al., 2024). We train on a combined commonsense reasoning dataset containing approximately 170K examples for 3 epochs with a batch size of 16. Our LoRA configuration applies rank-64 adapters to the projection components (`in_proj` and `out_proj`, denoted as \mathbf{W}_{in} and \mathbf{W}_{out}), and rank-32 adapters to the gate components (\mathbf{W}_{in}^{gate} and \mathbf{W}_{out}^{gate}). For the LIM neuron, we configure 4 chunks with a threshold of 1.0 and a leaky factor of 2.0. We employ model-specific learning rates: 1e-3 for **Memb**a-130M, 5e-4 for **Memb**a-370M and **Memb**a-790M, and 1e-4 for **Memb**a-1.4B. All models are trained using a linear learning rate scheduler with 100 warmup steps.

E.2 VISION TASK

For vision evaluation, we use the VTAB-1K benchmark, which comprises 19 diverse datasets with 1,000 training examples each. Our training protocol consists of 100 epochs with a cosine learning rate scheduler, 10 epochs of warmup, a learning rate of 1e-3, weight decay of 1e-4, and batch size of 32. For parameter-efficient fine-tuning, we employ architecture-specific LoRA configurations. The Vim (Zhu et al., 2024) architecture uses ranks of 32 for \mathbf{W}_{in} , 96 for \mathbf{W}_{out} , and 16 for gate-related linear layers (\mathbf{W}_{in}^{gate} and \mathbf{W}_{out}^{gate}). The VMamba (Liu et al., 2024c) architecture employs ranks of 64 for both \mathbf{W}_{in} and \mathbf{W}_{out} , with rank 16 for gate components. For both architectures, we configure the LIM neuron with 4 temporal chunks, a threshold of 1.0, and a leaky factor of 2.0.

E.3 S4 IMPLEMENTATION

We train the **Memb**a-S4 architecture from scratch on the Long Range Arena (LRA) benchmarks (Tay et al., 2020), which include ListOps, Text, Retrieval, Image, and Pathfinder tasks, in Section H.4. These benchmarks are specifically designed to evaluate model performance on long-range sequence processing. Table 5 provides the experimental configuration for our S4 implementation. For the

gate modification component, we employ a 32-rank for gate components ($\mathbf{W}_{\text{in}}^{\text{gate}}$ and $\mathbf{W}_{\text{out}}^{\text{gate}}$), with 4 chunks. Additionally, we set the LIM threshold to 1.0 and use a leaky factor of 2.0.

Table 5: Experimental settings on training S4 architecture. #layers: the number of layers. d_features: dimension of feature maps. d_state: dimension of state. lr: learning rate. we adopt the S5 (Smith et al., 2022) training setup for other hyperparameters.

F VISUALIZATION OF TEMPORAL PROCESSING

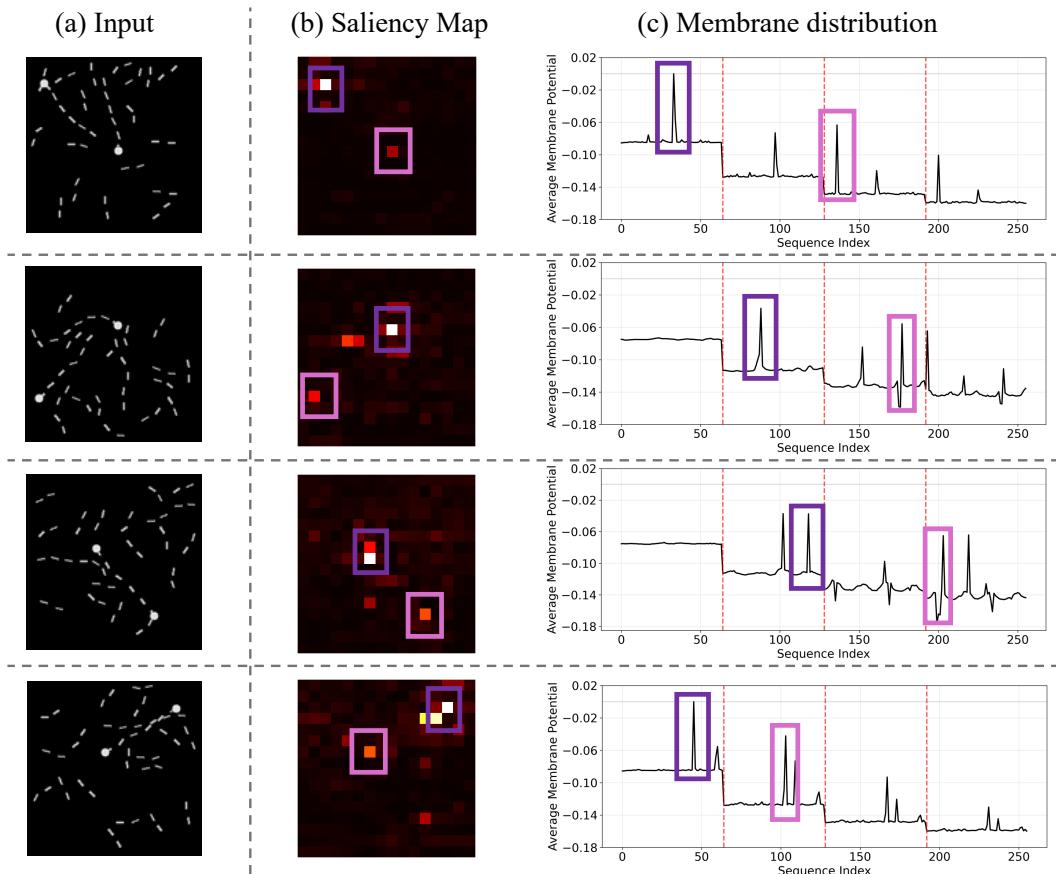


Figure 8: Membrane-driven temporal processing in **Memba**. Given input image (a), we present saliency maps (b) and associated membrane distributions (c) throughout the sequence processing.

To understand **Memba**’s temporal processing, we analyze membrane dynamics on the Pathfinder task. Figure 8 shows the relationship between visual attention and membrane activity during sequence processing. Task-critical path segments, highlighted in purple and pink, trigger pronounced membrane responses, demonstrating how the LIM neuron selectively amplifies relevant visual elements. Additionally, the membrane baseline declines across temporal chunks, indicating adaptive forgetting as new information accumulates.

1026 G INDIVIDUAL TASK ACCURACY FOR VTAB-1K BENCHMARK

1028 We provide the details of individual task performance on VTAB-1K benchmark as shown in Table 6.
1029 The Table 3 shows the averaged values of each category (Natural, Specialized, and Structured).

1031 Table 6: Per-task performances of VTAB-1k benchmark in Table 3. We separate three tables according
1032 to baseline architecture: ViT-S, Vim-S, and Vanilla-VMamba-S. **Avg.** and **Overall Avg.** represent
1033 average accuracy of each category and the average accuracy of overall tasks. **Bold** and underlined
1034 values represent the best and second-best performance respectively.

Method	Natural						Specialized						Structured						Overall Avg.				
	CIFAR-100	Caltech101	DTD	Flowers102	Pets	SVHN	Sun397	Avg.	Camelyon	EuroSAT	Resisc45	Retinopathy	Avg.	Clev-Count	Clev-Dist	DMLab	KITTI-Dist	dSpr-Loc	sNORB-Azim	sNORB-Elev			
Scracth	4.9	11.1	9.7	24.4	3.4	19.4	1.6	10.7	65.0	57.3	28.6	73.6	56.1	20.9	48.1	26.2	45.0	7.3	27.1	6.2	17.8	24.8	26.2
Full	30.4	69.3	37.6	65.8	50.2	87.8	21.5	51.8	76.2	85.2	66.8	63.0	72.8	33.5	56.8	73.5	40.4	28.0	22.8	45.3	53.5	52.5	
LoRA	<u>55.3</u>	<u>87.5</u>	<u>67.3</u>	<u>90.2</u>	<u>85.1</u>	<u>39.6</u>	<u>73.6</u>	<u>81.7</u>	<u>94.1</u>	<u>80.5</u>	<u>82.7</u>	<u>82.7</u>	<u>82.7</u>	<u>72.6</u>	<u>59.1</u>	<u>47.0</u>	<u>81.6</u>	<u>81.5</u>	<u>49.6</u>	<u>29.4</u>	<u>35.1</u>	<u>57.6</u>	<u>68.7</u>
Adaptformer	56.0	88.0	65.6	90.5	90.5	85.3	39.5	73.6	84.0	93.8	82.2	72.7	83.2	77.5	59.0	46.7	79.3	83.7	50.4	30.5	35.4	57.8	69.0
Adapter+	56.6	89.2	66.8	91.1	90.2	88.2	40.7	74.7	84.8	94.2	82.9	72.4	83.6	76.8	59.7	48.4	80.5	87.8	51.9	32.4	33.1	58.8	69.9

(a) ViT-S architecture pre-trained with ImageNet-1K.

Method	Natural						Specialized						Structured						Overall Mean				
	CIFAR-100	Caltech101	DTD	Flowers102	Pets	SVHN	Sun397	Avg.	Camelyon	EuroSAT	Resisc45	Retinopathy	Avg.	Clev-Count	Clev-Dist	DMLab	KITTI-Dist	dSpr-Loc	sNORB-Azim	sNORB-Elev			
Scracth	5.6	11.9	5.9	12.1	5.0	16.3	1.6	8.3	61.6	62.3	13.8	61.6	49.8	28.9	53.2	22.5	40.9	38.6	11.8	11.3	18.3	28.2	25.4
Full	51.6	83.0	61.5	71.1	45.0	88.4	14.9	59.4	66.5	88.1	63.3	57.1	68.7	48.8	60.4	38.8	55.0	6.3	11.8	30.0	24.1	34.4	
LoRA(<u>embed</u>)	51.7	84.9	59.2	<u>75.5</u>	<u>88.0</u>	<u>53.4</u>	<u>39.8</u>	<u>64.7</u>	<u>77.5</u>	<u>91.2</u>	<u>71.5</u>	<u>70.0</u>	<u>77.5</u>	<u>48.8</u>	<u>60.4</u>	<u>38.8</u>	<u>55.0</u>	<u>7.3</u>	<u>19.7</u>	<u>26.2</u>	<u>43.8</u>	<u>58.6</u>	
LoRA(<u>Z-proj</u>)	59.2	88.0	67.8	88.9	90.4	83.9	43.0	74.4	82.7	93.9	80.3	78.1	81.1	42.9	52.0	34.9	73.0	54.7	47.0	27.8	33.1	54.9	67.8
LoRA(<u>dt.proj</u>)	61.6	90.1	66.7	89.0	90.7	86.1	43.4	75.4	83.2	94.9	81.2	72.9	83.1	76.9	59.8	60.7	43.6	80.6	75.0	45.8	27.8	37.0	57.1
LoRA(<u>out.proj</u>)	64.2	89.8	68.6	91.4	91.4	91.2	86.0	43.8	76.4	84.9	94.9	83.4	72.7	84.0	48.5	62.7	49.5	81.3	77.1	52.0	32.8	40.9	60.1
LoRA(<u>in.proj</u>)	63.4	90.3	68.2	91.0	91.0	88.4	43.5	76.6	84.7	95.0	83.8	72.9	84.1	84.0	61.3	48.6	80.0	83.5	52.6	31.8	39.5	60.2	71.3
LoRA _p (<u>X</u>)	61.7	90.6	68.1	90.3	90.7	88.4	43.2	76.2	85.6	95.2	83.5	72.8	84.3	82.4	60.6	48.2	81.9	81.6	52.4	31.4	39.5	59.7	70.9
LoRA _p (<u>Hybrid</u>)	63.7	90.2	69.4	90.6	91.5	88.9	43.6	76.6	84.5	95.3	83.4	72.4	83.3	84.9	62.9	48.6	81.4	82.8	52.7	33.1	40.4	60.8	71.5
<u>Memb</u> (<u>in.proj</u>)	63.4	91.0	<u>69.2</u>	<u>90.6</u>	<u>91.4</u>	<u>89.2</u>	<u>43.6</u>	<u>76.9</u>	<u>86.2</u>	<u>95.2</u>	<u>83.6</u>	<u>75.1</u>	<u>85.7</u>	<u>83.6</u>	<u>62.3</u>	<u>49.5</u>	<u>82.3</u>	<u>83.8</u>	<u>52.7</u>	<u>32.8</u>	<u>38.8</u>	<u>60.7</u>	<u>71.8</u>
<u>Memb</u> (<u>out.proj</u>)	63.1	91.4	68.8	91.4	91.5	88.4	43.9	76.9	86.6	95.3	84.1	75.3	85.3	84.6	63.2	50.3	82.7	82.8	52.4	33.4	40.1	61.2	72.1
<u>Memb</u> (<u>in+out.proj</u>)	63.2	<u>91.1</u>	68.6	<u>91.3</u>	<u>91.4</u>	<u>89.9</u>	<u>44.0</u>	<u>77.1</u>	<u>87.2</u>	<u>95.4</u>	<u>84.5</u>	<u>75.5</u>	<u>85.7</u>	<u>84.8</u>	<u>62.4</u>	<u>50.4</u>	<u>83.3</u>	<u>85.1</u>	<u>52.5</u>	<u>34.7</u>	<u>40.3</u>	<u>61.7</u>	<u>72.4</u>

(b) Vim-S architecture pre-trained with ImageNet-1K.
(c) Vanilla-VMamba-S architecture pre-trained with ImageNet-1K.

H ADDITIONAL ABLATION STUDY

In this section, we conduct comprehensive ablation studies to provide deeper insights into Memb's design choices and performance characteristics. We systematically examine five key aspects: (1) the impact of chunk count on LIM neuron performance, (2) accuracy comparisons under iso-parameter conditions, (3) parameter allocation within the LIM neuron, (4) performance comparison with traditional recurrent gating mechanisms, and (5) computational overhead analysis through inference time measurements.

H.1 NUMBER OF CHUNKS

The number of chunks in the LIM neuron determines the computational iteration of membrane dynamics. Table 7 presents accuracy comparisons across different numbers of chunks for both language and vision tasks. For language tasks, we employ the **Memb**-130M architecture on commonsense reasoning benchmarks. For vision tasks, we utilize the Vim-S architecture evaluated on Camelyon, EuroSAT, Resisc45, and Retinopathy datasets from the VTAB-1K benchmark. Across both task domains, LIM with 4 chunks achieves optimal performance while maintaining reasonable

1080 inference time. The reported inference time corresponds to LIM neuron processing only. Notably,
 1081 inference time does not scale proportionally with the number of chunks, as increasing the chunk
 1082 count results in smaller sequence lengths per chunk.
 1083

1084 Table 7: Accuracy comparison across different numbers of chunks. #chunk represents the number
 1085 of chunks in the LIM neuron. For language and vision tasks, we use ***Memb*a**-130M and Vim-S
 1086 architectures, respectively. Inference times are measured during vision task evaluation.
 1087

	#chunk	Language (%)	Vision (%)	Inference time (ms)
	2	43.9	85.60	1.5
	4	44.3	85.66	1.7
	6	44.1	85.43	1.8
	8	43.7	85.36	2.0

1094 H.2 ISO-PARAMETER ACCURACY COMPARISON

1095 To verify the effectiveness of ***Memb*a** under fair comparison conditions, we evaluate accuracy with
 1096 the same number of learnable parameters as previous works, particularly MambaPeft (Yoshimura
 1097 et al., 2024), as shown in Table 8. To match the parameter count, we apply LoRA only to the output
 1098 projection (\mathbf{W}_{out}) and reduce the rank of gate components ($\mathbf{W}_{\text{in}}^{\text{gate}}$ and $\mathbf{W}_{\text{out}}^{\text{gate}}$) to 16. This configuration
 1099 matches the parameter count of MambaPeft’s ”LoRA (in_proj)” setting, and our ***Memb*a** consistently
 1100 achieves better performance across all architecture sizes.
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1102 Table 8: Performance comparison between ***Memb*a** and MambaPeft (Yoshimura et al., 2024) on com-
 1103 monsense reasoning datasets. Reported values are accuracy percentages (%). **Bold** and underlined
 1104 entries indicate the best and second-best performance, respectively. HS and WG refer to the Hel-
 1105 laSwag and WinoGrande datasets.
 1106

Model	Method	#Params(%)	BoolQ	PIQA	SIQA	HS	WG	ARC-e	ARC-c	OBQA	Avg.
Mamba 130M	Full	100	56.1	65.3	38.7	35.3	52.0	46.4	25.7	32.8	43.8
	Additional-scan	0.51	57.8	64.1	37.5	<u>34.5</u>	<u>53.0</u>	41.3	23.5	30.0	42.7
	Affix-tuning	64.64	<u>59.7</u>	<u>64.3</u>	38.2	35.2	51.9	42.9	24.0	29.0	43.2
	LoRA (in.proj)	2.23	53.5	62.9	38.2	33.8	53.1	<u>46.4</u>	23.7	30.8	42.8
	LoRA _p (X)	2.67	61.7	64.0	39.5	34.3	52.2	43.5	25.3	29.4	43.7
	<i>Memb</i>a	2.23	56.3	64.5	<u>39.1</u>	34.4	51.6	48.0	24.0	29.4	43.4
Mamba 370M	Full	100	58.1	69.9	41.9	45.7	53.8	52.7	29.7	33.4	48.2
	Additional-scan	0.47	61.9	69.3	<u>41.2</u>	<u>45.3</u>	54.9	49.5	28.4	31.4	47.7
	Affix-tuning	68.88	<u>61.2</u>	68.4	39.6	46.2	55.4	48.2	28.2	30.6	47.2
	LoRA (in.proj)	2.07	55.4	68.6	41.0	44.7	54.1	<u>52.4</u>	<u>28.3</u>	33.4	47.2
	LoRA _p (X)	2.67	60.8	68.8	42.1	44.7	56.2	50.4	27.4	<u>32.2</u>	47.8
	<i>Memb</i>a	2.07	56.2	69.2	42.1	44.8	<u>55.7</u>	56.0	28.3	33.4	48.2
Mamba 790M	Full	100	62.0	72.1	44.8	54.0	55.9	57.7	31.2	35.2	51.6
	Additional-scan	0.33	63.0	71.9	41.9	54.2	57.1	54.9	30.0	32.6	50.7
	Affix-tuning	69.99	61.0	72.5	41.0	54.9	55.6	54.6	29.6	33.8	50.4
	LoRA (in.proj)	1.47	<u>61.7</u>	71.9	44.0	50.8	56.7	<u>56.3</u>	30.5	33.8	50.7
	LoRA _p (X)	1.75	59.9	<u>72.2</u>	44.2	52.8	<u>58.0</u>	53.7	<u>30.8</u>	34.8	<u>50.8</u>
	<i>Memb</i>a	1.47	58.8	72.2	44.1	54.7	58.3	61.5	31.1	34.0	51.8
Mamba 1.4B	Full	100	61.4	73.3	43.9	56.9	59.0	59.7	34.0	35.4	53.0
	Additional-scan	0.26	63.0	73.5	42.8	57.5	60.5	<u>60.9</u>	<u>32.4</u>	<u>37.4</u>	53.5
	LoRA (in.proj)	1.13	62.6	<u>73.6</u>	43.7	55.6	59.7	58.3	31.7	35.6	52.6
	LoRA _p (X)	1.36	<u>63.1</u>	73.5	42.7	<u>57.7</u>	61.6	60.4	32.9	<u>37.4</u>	<u>53.7</u>
	<i>Memb</i>a	1.13	64.1	73.9	43.4	58.3	<u>60.8</u>	65.1	32.3	37.6	54.5

1128 H.3 PARAMETER ALLOCATION ANALYSIS

1129 To provide deeper insights into *Memb*a’s parameter efficiency, Table 9 presents a detailed breakdown
 1130 of parameter allocation across different model sizes. Unlike traditional PEFT methods that distribute
 1131 parameters across various projection layers, *Memb*a strategically allocates all trainable parameters to
 1132 the gate projections ($\mathbf{W}_{\text{in}}^{\text{gate}}$ and $\mathbf{W}_{\text{out}}^{\text{gate}}$) within the LIM neuron. For instance, in the Mamba-790M
 1133 configuration, *Memb*a allocates 4.72M parameters to input gate projections and 7.08M parameters

to output gate projections, totaling 11.80M parameters, identical to the baseline LoRA (in_proj) method. This targeted allocation enables Memba to enhance temporal processing capabilities while maintaining parameter efficiency. Notably, Memba consistently outperforms baseline methods across all model sizes: achieving 1.1% improvement over LoRA.p(X) for Mamba-790M (51.8% vs 50.8%) and 0.8% improvement for Mamba-1.4B (54.5% vs 53.7%), demonstrating that strategic parameter placement in the gating mechanism is more effective than conventional projection-based adaptations.

Table 9: Parameter allocation and performance comparison for Memba. The ”#Params” column represents the total number of trainable parameters, ” $\mathbf{W}_{\text{in}}^{\text{gate}} + \mathbf{W}_{\text{out}}^{\text{gate}}$ ” shows the parameters dedicated to the gate modules in the LIM neuron, and ”Others” represents all remaining parameters excluding the gate modules.

Model	Method	# Params	$\mathbf{W}_{\text{in}}^{\text{gate}} + \mathbf{W}_{\text{out}}^{\text{gate}}$	Others	Avg. Acc. (%)
Mamba-130M	LoRA (in_proj)	2.95M	-	-	42.8
	LoRA.p(X)	3.54M	-	-	43.7
	Memba	2.95M	1.18M	1.77M	43.4
Mamba-370M	LoRA (in_proj)	7.87M	-	-	47.2
	LoRA.p(X)	9.44M	-	-	47.8
	Memba	7.87M	3.15M	4.72M	48.2
Mamba-790M	LoRA (in_proj)	11.80M	-	-	50.7
	LoRA.p(X)	14.16M	-	-	50.8
	Memba	11.80M	4.72M	7.08M	51.8
Mamba-1.4B	LoRA (in_proj)	15.73M	-	-	52.6
	LoRA.p(X)	18.87M	-	-	53.7
	Memba	15.73M	6.29M	9.44M	54.5

H.4 COMPARISON WITH TRADITIONAL RECURRENT GATING MECHANISMS

Our LIM neuron is designed to enhance Mamba’s temporal adaptation capabilities through recurrently evolving membrane potentials. To evaluate this approach against established alternatives, we compare LIM with traditional recurrent architectures by replacing LIM with LSTM (Hochreiter & Schmidhuber, 1997) and GRU (Chung et al., 2014) in the gate path.

Table 10 presents a comprehensive comparison across two distinct scenarios: vision adaptation tasks using the Vim-S architecture on VTAB-1k, and sequence modeling tasks on the Long Range Arena (LRA) benchmark (Tay et al., 2020) using S4 (Gu et al., 2021a) trained from scratch. Our LIM neuron consistently outperforms both LSTM and GRU across domains while offering two significant advantages: (1) zero learnable parameters and (2) lower inference latency. The superior performance of LIM stems from its temporal processing driven by membrane dynamics, whereas LSTM and GRU rely on parameter-heavy gating units that increase computational cost with trivial improvement to temporal modeling capability.

Table 10: Comparison of LIM with other recurrent units in terms of performance, parameter count, and latency. We report average accuracy (%) on two benchmarks: VTAB (using the Vim-S architecture) and Long Range Arena (using the S4 architecture). # Params denotes the number of learnable parameters per unit given a hidden dimension H , excluding the input and output projection layers ($\mathbf{W}_{\text{in}}^{\text{gate}}$ and $\mathbf{W}_{\text{out}}^{\text{gate}}$) which are applied consistently across all methods. Latency refers to inference time per batch measured on the S4 architecture using an A100 GPU.

Gate	# Params	Vim-S	S4						Latency (s)
			VTAB	ListOps	Text	Retrieval	Image	Pathfinder	
LSTM	$8H^2 + 4H$	71.59	62.10	88.00	91.37	89.60	96.75	85.57	0.189
GRU	$6H^2 + 3H$	71.68	61.80	87.97	91.70	89.02	96.81	85.46	0.188
LIM (ours)	0	72.33	62.05	89.60	91.52	89.88	97.12	86.04	0.150

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H.5 INFERENCE TIME ANALYSIS

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The proposed **Memba** incorporates the LIM neuron, which introduces inevitable recurrent computation for membrane accumulation and reset mechanisms. To quantify this computational overhead compared to previous PEFT methods on Mamba, we provide an inference time comparison based on Table 4(a). Table 11 demonstrates how the LIM module affects both accuracy and inference time. Since membrane transfer has a negligible impact on inference time, we exclude its influence from our analysis. We conduct experiments on language and vision tasks using the **Memba**-130M and Vim-S architecture, with inference times measured on single-batch evaluation.

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Memba, which integrates LIM, LoRA, and membrane transfer, achieves higher performance on both language and vision tasks. However, this comes with an 8.8% inference time overhead compared to Mamba with LoRA. This overhead stems from the iterative operations required for membrane accumulation in the LIM neuron. Nevertheless, the 8.8% increase represents a modest computational cost, and our chunked sequence processing approach in the LIM neuron effectively balances performance gains with inference efficiency.

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Table 11: Inference time analysis with different **Memba** components. The table shows the impact of each component on accuracy and computational overhead. For language and vision tasks, we use **Memba**-130M and Vim-S architectures, respectively. Inference times are measured during vision task evaluation on a single batch.

① LIM	② LoRA	③ Membrane Transfer	Language (%)	Vision (%)	Inference Time (s)
✗	✓	✗	43.8	85.3	0.517
✓	✓	✓	44.3	85.7	0.563

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I COMPREHENSIVE MEMORY AND LATENCY ANALYSIS

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To provide a complete understanding of the computational requirements of Memba, we present comprehensive GPU memory and inference latency comparisons across baseline methods, model sizes, and task domains. For language tasks, we measure GPU peak memory during fine-tuning and per-sample inference latency on the ARC-Easy benchmark, shown in Table 12.

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Table 12: GPU memory and inference latency comparison on language tasks (ARC-Easy benchmark). Latency represents per-sample inference time.

Model	Method	Learnable Param (%)	GPU Peak Memory (GB)	Latency (s)
Mamba-130M	SLL LoRA	1.45	2.62	0.015
	Additional-scan	0.51	2.17	0.014
	Affix-tuning	0.17	2.49	0.013
	LoRA (in+out)	3.53	2.51	0.014
	Memba	3.95	2.82	0.020
Mamba-370M	SLL LoRA	2.30	5.00	0.027
	Additional-scan	0.47	4.15	0.023
	Affix-tuning	0.16	4.50	0.021
	LoRA (in+out)	2.28	5.00	0.024
	Memba	3.67	5.80	0.044
Mamba-790M	SLL LoRA	3.10	8.20	0.028
	Additional-scan	0.33	6.56	0.023
	Affix-tuning	0.11	6.97	0.021
	LoRA (in+out)	2.32	8.03	0.024
	Memba	2.61	9.21	0.045
Mamba-1.4B	SLL LoRA	4.64	12.49	0.028
	Additional-scan	0.26	9.65	0.024
	Affix-tuning	0.09	9.71	0.021
	LoRA (in+out)	1.80	11.90	0.025
	Memba	2.02	13.60	0.045

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Memba incurs approximately 12-14% additional GPU memory overhead compared to LoRA (in+out), which is required for storing membrane potentials across layers to enable cross-layer membrane

transfer. We observe that inference latency appears to be largely independent of hidden dimension size. Mamba-370M, 790M, and 1.4B exhibit similar latency despite having different hidden dimensions, as they share the same number of layers. In contrast, Mamba-130M shows noticeably lower latency due to its shallower architecture with fewer layers. Regarding the latency overhead introduced by recurrent operations in the LIM neuron, as discussed in the Limitation section (Section J), this latency can be substantially reduced through CUDA kernel optimization.

Table 13: GPU memory and inference latency comparison on vision tasks (Caltech101 dataset in VTAB-1k benchmark).

Model	Method	Learnable Param (%)	GPU Peak Memory (GB)	Latency (s)
Vim-S	LoRA (in+out)	13.98	7.68	0.517
	Memb	15.60	9.85	0.563

For vision tasks in Table 13, we observe similar patterns on the VTAB-1k benchmark using the Vim-S architecture. The memory overhead is approximately doubled compared to language tasks because Vim-S uses bidirectional SSM with two gate paths, requiring storage of membrane potentials for two LIM neurons. The inference latency increase remains modest. These resource requirements represent a reasonable trade-off given the substantial performance improvements demonstrated in Tables 2 and 3.

J LIMITATIONS

We propose **Memb** to enable temporal adaptation during fine-tuning without disrupting the balanced dynamics of pre-trained SSMs. To achieve this, we introduce the LIM neuron in the gate pathway while preserving the original SSM components. The LIM neuron accumulates membrane dynamics with a reset function, which inherently introduces recurrent computation and consequently incurs additional computational overhead. Although the selective scan operation in SSMs also involves recurrent computation, Gu et al. (Gu & Dao, 2023) address this efficiency challenge through specialized hardware kernel modifications. Similarly, our LIM algorithm can be integrated into optimized CUDA kernels in future implementations. The widely-used SpikingJelly framework (Fang et al., 2023) demonstrates that custom CUDA kernels for LIF neurons achieve up to 30 \times speedups through operator fusion, where element-wise operations (leaky decay, addition, reset) are fused into a single kernel launch. Our chunk-based recurrent operations can directly leverage this optimization approach, as the sequential dependency between chunks does not prevent intra-chunk parallelization. We expect this would achieve negligible computational overhead comparable to the selective scan kernel.