

000 HETEROGENEOUS GRAPH TEMPORAL FUSION 001 TRANSFORMER FOR TIME SERIES FORECASTING 002 IN MULTI-DOMAIN PHYSICAL SYSTEMS 003 004 005

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011 ABSTRACT

013 Existing Transformer-based models effectively capture multivariate dependencies,
 014 while pre-trained large models achieve strong generalization but are often confined
 015 to single-object or single-physics settings. Spatial-temporal approaches leverage
 016 graph structures but fall short in modeling heterogeneous entities with diverse
 017 inter-variable interactions, and they often lack mechanisms to enforce physical
 018 consistency. To address these challenges, we propose the Heterogeneous Graph
 019 Temporal Fusion Transformer (HGTFT), a pre-training and fine-tuning framework
 020 tailored for spatially and temporally structured physical environments. HGTFT to-
 021 kenizes observation points and generates embeddings that capture both temporal
 022 patterns and spatial correlations, enabling the integration of heterogeneous static
 023 and dynamic information. We further introduce optimized normalization and
 024 physics-informed loss functions that enhance predictive accuracy while improv-
 025 ing physical plausibility. Applied to temperature, flow, and energy-related datasets
 026 in building environments, our approach demonstrates strong zero-shot generaliza-
 027 tion and achieves substantial accuracy gains through few-shot fine-tuning with
 028 domain-specific data.

029 1 INTRODUCTION

031 Energy is a key factor in the development of AI, while energy systems are typical multiphysics
 032 systems involving coupled thermal, fluid, and electrical processes. More generally, multi-domain
 033 physical systems such as power grids and building energy networks consist of heterogeneous entities
 034 (e.g., sensors, actuators, and control devices) that interact across multiple physical fields. Accurate
 035 forecasting in these systems is critical for efficiency, safety, and sustainability, yet remains challeng-
 036 ing due to diverse data modalities, complex structural dependencies, and domain-specific physical
 037 dynamics. Existing approaches fall into four categories.

038 **Physics-prioritized/analytical methods:** These use known governing equations and boundary
 039 conditions directly. They provide strong interpretability but often struggle with complex domain
 040 coupling or computational cost in high-dimensional/irregular settings.

042 **Physics-guided neural solvers:** Physics-Informed Neural Networks (PINNs), e.g., DeepXDE Lu
 043 et al. (2021) and Neuromancer Drgona et al. (2023), embed PDE residuals, boundary conditions, or
 044 conservation laws into neural networks for strong physical consistency. Yet they incur high deriva-
 045 tive costs and are limited to continuous field simulations, not heterogeneous multi-entity systems.

046 **Purely data-driven methods:** Time-series models, e.g., LSTM and LTM (Dong et al. (2024), Liu
 047 et al. (2024c)), capture general temporal patterns but lack physical grounding, making extrapolation
 048 or safety-critical predictions unreliable. LLM-based time-series methods (Zhou et al. (2023), Liu
 049 et al. (2024b)) inherit this limitation, as tokenization treats inputs as numerical patches or sequences,
 050 ignoring spatial structure, static context, and multi-entity physical dependencies.

051 **Data-driven methods with physical constraints:** These approaches are primarily data-driven,
 052 leveraging network architectures to encode heterogeneous information while incorporating physics
 053 through appropriate loss functions and training pipelines. By doing so, physical consistency can
 be learned and enforced within the model. We propose the Heterogeneous Graph Temporal Fu-

sion Transformer (HGTFT), a framework that integrates heterogeneous spatial-temporal information while maintaining consistency with underlying physical constraints. In HGTFT, each token represents a node's state at a time step, embedding both dynamic and static attributes. The encoder captures temporal dynamics and heterogeneous spatial relations, while the decoder adapts these representations to task-specific forecasting. A tailored training pipeline incorporating domain-informed loss functions and improved normalization enhances predictive accuracy without compromising physical validity. Our contributions are threefold: (1) We define heterogeneous graph forecasting in multi-domain physical systems, covering a wide range of real-world infrastructures. (2) We introduce HGTFT with tailored tokenization and embedding strategies that fully leverage graph structures, static attributes, and dynamic features. (3) We develop a physics-informed training pipeline with improved normalization, enabling HGTFT to achieve higher accuracy and stronger consistency with domain physics across benchmarks.

2 RELATED WORK

Transformer for Time Series Forecasting. The Transformer model Vaswani et al. (2017) has revolutionized time series forecasting via attention. Extensions like Informer Zhou et al. (2021) use probabilistic sparse attention, while Frozen Pretrained Transformer Zhou et al. (2023) adapts pre-trained models from other domains, linking self-attention with principal component analysis. For multivariate forecasting, Crossformer Zhang & Yan (2023) implements a two-stage attention mechanism for temporal and cross-dimensional dependencies, and Temporal Fusion Transformer (TFT) Lim et al. (2021) provides interpretable mixed-input forecasting. Recent approaches like Time-Siam Dong et al. (2024) and Timer Liu et al. (2024c) use unlabeled data for representation learning. Unified training paradigms Woo et al. (2024) allow single models to handle multiple tasks, while decoder-only models Das et al. (2023) enhance prediction efficiency. These efforts demonstrate the potential of pre-training to improve generalization and accuracy in time series forecasting.

Spatial-Temporal Forecasting. Graph Neural Networks (GNNs) have made strides in graph-based learning through structural and positional encoding. Approaches like LSPE Ying et al. (2021) and NodeFormer Wu et al. (2022) address scalability, while LETR Xu et al. (2021) and Molecule Attention Transformer Maziarka et al. (2020) apply Transformers to specialized tasks. For heterogeneous graphs, HDGT Jia et al. (2023) and HAN Wang et al. (2019) use hierarchical attention to capture diverse node and edge types. In spatiotemporal forecasting, DCRNN Li et al. (2017) models spatial diffusion and temporal dependencies using diffusion convolution within a recurrent framework, while STEP Shao et al. (2022b) extends this approach with a pre-training enhanced GNN for long-range temporal patterns. Spacetimeformer Grigsby et al. (2021) and heuristic graphs Shao et al. (2022a) model complex temporal-spatial sequences, while Graph Neural ODEs Poli et al. (2019) incorporate differential equations for capturing dynamic temporal dependencies. Models like STS-GCN Song et al. (2020) and STSGT Banerjee et al. (2022) combine GCNs and Transformers to model synchronous spatial-temporal dependencies, applied to traffic and pandemic forecasting. Architectures such as HGT Hu et al. (2020) and PromptST Zhang et al. (2023) leverage pretraining and adaptive tuning for heterogeneous, multi-attribute graph predictions. Similarly, UniST Yuan et al. (2024) employs prompt-based learning and extensive pre-training to enhance generalization in urban spatio-temporal prediction.

Large Language Models for Time Series. Large Language Models (LLMs) have been adapted for time series tasks, particularly in few-shot and zero-shot settings. TimeGPT-1 Garza et al. (2023) reprograms LLMs for time series prediction by aligning embeddings with time-domain features, while Gruver Gruver et al. (2024) demonstrates zero-shot forecasting without fine-tuning. LLM4TS Chang et al. (2024) and TIME-LLM Jin et al. (2023) optimize LLMs for time series, improving adaptability to specialized datasets and temporal patterns. TimeCMA Liu et al. (2024a) introduces cross-modality alignment to enhance temporal understanding, and TimeChat Ren et al. (2024) expands this to multimodal contexts, integrating temporal information for applications like video understanding. These studies highlight LLMs' potential as general-purpose forecasters, though challenges in temporal representation, data efficiency, and interpretability remain.

Physics-informed methods. Another research line integrates physical constraints into neural networks to enhance interpretability and consistency with known dynamics. Representative frameworks such as PINNs Raissi et al. (2019), DeepXDE Lu et al. (2021), and Variational PINNs Kharazmi

108 et al. (2019) enforce the governing differential equations via loss regularization. Extensions in-
 109 cluding Neural Operators (Li et al. (2020b); Li et al. (2024)) and Fourier Neural Operators (Li
 110 et al. (2020a)) learn mappings between function spaces for efficient PDE simulation, while hybrid
 111 methods like Graph-based PINNs Gao et al. (2022) aims to connect continuous physics with graph
 112 structures.

113

114 3 PROBLEM DEFINITION

115

116 **General Spatial-Temporal Forecasting Problem.** Spatial-temporal forecasting problems, such
 117 as those involving traffic networks, the COVID-19 pandemic, or power grids Guo et al. (2019);
 118 Banerjee et al. (2022); Liu et al. (2023b), can typically be formulated using a spatial network $G =$
 119 (V, E, A) , where V is the set of node vertices, E represents the set of edges, and A is the adjacency
 120 matrix describing relationships between nodes. The goal is often to predict future observations for
 121 a single node type with a single relationship type. Each node entity v_i in the graph is associated
 122 with a graph signal matrix $X(t)_G \in \mathbb{R}^{N \times F}$, where F is the number of features per node, and t
 123 denotes the time step. $X(t)_G$ captures the spatial network observations at time t , with each entry
 124 $X_{i,t}$ representing the feature vector of node v_i at time t . The task is to predict future spatial-temporal
 125 data by learning a mapping function \mathcal{F} that maps historical series $\{X(t - T_{\text{past}} + 1)_G, \dots, X(t)_G\}$
 126 to future observations $\{X(t + 1)_G, \dots, X(t + T_{\text{future}})_G\}$, where T_{past} is the length of historical
 127 data and T_{future} is the forecast horizon.

128

129 **Extension to Heterogeneous Graph Forecasting in Multi-Domain Physical Systems.** In contrast,
 130 our problem involves a more complex heterogeneous graph comprising multiple node types and
 131 relationships. Each node v_i is characterized by static attributes s_i and time-variant features, which
 132 are further grouped into: 1) variables known for both past and future x_i , 2) variables known only for
 133 the past z_i , and 3) the prediction variable y_i . The problem can be formulated as:

134

$$\hat{y}_{i,t+1:t+T_{\text{future}}} = \mathcal{F}(s_i, x_{i,t-T_{\text{past}}+1:t+T_{\text{future}}}, z_{i,t-T_{\text{past}}+1:t}, y_{i,t-T_{\text{past}}+1:t}, N(v_i)), \quad (1)$$

135

136

$$N(v_i) = \bigcup_{r_l \in R} N_l(v_i), \quad (2)$$

137

138

139 where $\hat{y}_{i,t+1:t+T_{\text{future}}}$ denotes the predicted target sequence for node v_i over the future horizon
 140 $[t + 1, \dots, t + T_{\text{future}}]$, and \mathcal{F} is the learned forecasting function. $N(v_i)$ aggregates neighborhood
 141 information for node v_i across relation types r_l .

142

143 This extension is significant due to its ability to model complex multiphysics systems with diverse
 144 node types, features, and interrelationships, prevalent in real-world applications such as nuclear re-
 145 actors, aerospace vehicles, biomedical devices, combined heat and power systems, and smart build-
 146 ings. These systems necessitate advanced forecasting models capable of capturing intricate inter-
 147 dependencies and dynamic interactions across different physical domains. For illustrative examples
 148 and a discussion on the necessity of sophisticated modeling approaches, refer to Appendix A.

149

150

151 4 MODEL ARCHITECTURE

152 The proposed HGTFT model, outlined in Figure 1, is designed for the previously defined problem
 153 by aggregating multi-dimensional data across static and dynamic node features within a heteroge-
 154 neous graph structure. Features are aligned into unified embeddings per entity and time point, and
 155 these embeddings pass through neural layers that aggregate information across temporal and graph
 156 dimensions, resulting in fixed-dimension representations. The representations are then forwarded to
 157 task-specific model layer for dimension transformation tailored to each task.

158

159 **Fusion Layer.** Each node v_i is associated with static covariates s_i and time-varying features: future-
 160 known variables $x_{i,t}$, past-only variables $z_{i,t}$, and target variable $y_{i,t}$. We first map all available
 161 inputs into a shared d -dimensional latent space and fuse them using a Variable Selection Network
 162 (VSN) inspired by TFT Lim et al. (2021). This produces a time-dependent node representation:

163

$$h_{i,t}^{\text{node}} = \text{VSN}(\text{Proj}(s_i), \text{Proj}(x_{i,t}), \text{Proj}(z_{i,t}), \text{Proj}(y_{i,t})), \quad (3)$$

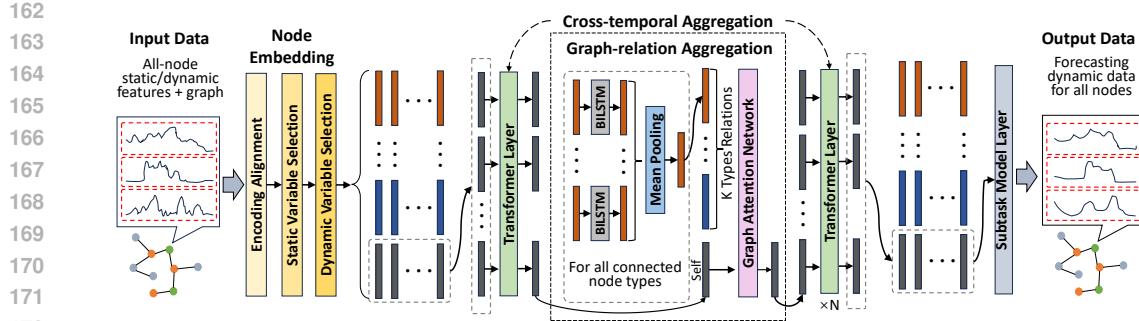


Figure 1: Architecture of the proposed HGTFT. The Fusion Layer converts heterogeneous data into unified-dimensional vectors, with colored bars indicating different object types. The Temporal Layer depicts processing for a single object type, shared across all objects. The Graph Layer shows processing at one time point, replicated across all time steps.

where $\text{Proj}(\cdot)$ denotes a linear transformation that maps each input variable to a fixed-dimensional vector, ensuring compatibility with subsequent layers.

Temporal Layer. To capture temporal dependencies, we employ a unified Transformer-based architecture for all temporal processing layers. Specifically, we apply Transformer encoder layer to the historical node representations across time:

$$h_{i,t}^{\text{temp}} = \text{Transformer}(\{h_{i,t-T_{\text{past}}+1}^{\text{node}}, \dots, h_{i,t}^{\text{node}}\})_t, \quad (4)$$

where the output $h_{i,t}^{\text{temp}}$ denotes the temporally encoded representation of node v_i at time step t . To effectively capture temporal dependencies, Transformer-based temporal layers are positioned both between the fusion and graph layers, and following the graph layer. This design enables the model to capture temporal dependencies in the node representations before and after relational aggregation, facilitating deeper modeling of time-evolving dynamics across heterogeneous entities.

Graph Layer. We adopt a two-stage relation-aware aggregation strategy tailored for heterogeneous physical graphs. In the first stage, neighbors of the same relation type are aggregated to capture the overall influence of each relation group, avoiding unnecessary complexity in modeling individual interactions. In the second stage, a multi-head graph attention mechanism (GAT) flexibly integrates these relation-specific embeddings, assigning adaptive importance to different relation types for each node. Instead of grouping neighbors merely by node type, we explicitly distinguish edge relations and assign separate parameters per relation. This design balances simplicity and expressiveness: it sufficiently models intra-relation effects while enabling fine-grained, context-aware weighting across relations, avoiding semantic entanglement and excessive parameterization typical of more complex HGNN methods, making it particularly suited for multiphysics systems. At each time step t , the system is modeled as a heterogeneous graph $\mathcal{G}_t = (V, E, R)$, where each node aggregates information from its multi-relational neighbors. For each relation type $r_\ell \in R$, we first compute:

$$h_{i,\ell}^{\text{agg}}(t) = \frac{1}{|N_\ell(v_i)|} \sum_{v_j \in N_\ell(v_i)} \text{BiLSTM}_\ell(h_{j,t}^{\text{temp}}), \quad (5)$$

$$\alpha_\ell^k = \text{softmax} \left(\text{LeakyReLU} \left(a^{k\top} [W^k h_{i,t}^{\text{temp}} \| W^k h_{i,\ell}^{\text{agg}}(t)] \right) \right), \quad (6)$$

$$h_{i,t}^{\text{graph}} = \frac{1}{K} \sum_{k=1}^K \sum_{\ell=1}^L \alpha_\ell^k W^k h_{i,\ell}^{\text{agg}}(t), \quad (7)$$

where K is the number of attention heads and $L = |R|$ is the number of relation types. $W^k \in \mathbb{R}^{d' \times d}$ is the learnable projection matrix for the k -th head, and $a^k \in \mathbb{R}^{2d'}$ is the shared attention vector for computing attention scores. This enables the model to selectively aggregate information from heterogeneous neighbor types in a multi-head attention manner.

216 **Subtask Model Layer.** To support diverse downstream forecasting objectives across heterogeneous
 217 entities, we adopt a modular subtask modeling framework. Each subtask shares a unified decoder
 218 architecture that transforms encoded representations into future predictions, as illustrated in Figure 9
 219 in Appendix F.1.

220 The decoder leverages a masked multi-head attention (MHA) mechanism to align the encoded inputs
 221 $\{h_{i,t}\}_{t=t-T_{\text{past}}+1}^{t+T_{\text{future}}}$ with their respective future time steps. The output is then passed through a task-
 222 specific dense projection to generate the predicted dynamics $\{\hat{y}_{i,t'}\}_{t'=t+1}^{t+T_{\text{future}}}$:

$$225 \quad \{\hat{y}_{i,t'}\}_{t'=t+1}^{t+T_{\text{future}}} = \text{Dense}(\text{MHA}(\{h_{i,t}\}_{t=t-T_{\text{past}}+1}^{t+T_{\text{future}}})) \quad (8)$$

227 To ensure stable information flow and consistent representation, the decoder incorporates Gated
 228 Residual Networks (GRNs), gating mechanisms, and Add & Norm layers.

229 The HGTFT framework has been validated for convergence on a simplified example in Ap-
 230 pendix A.1.

232 5 MODEL TRAINING METHODOLOGY

235 To forecast spatio-temporal dynamics in multi-domain physical systems, we design a progressive
 236 HGTFT training pipeline that integrates heterogeneous graph structure, temporal dynamics, and
 237 physical constraints. Key components such as multi-instance normalization and physics-informed
 238 loss terms ensure stable training and enforce physical consistency, while sequential stages of self-
 239 supervised learning, multi-task supervision, and subtask fine-tuning progressively improve general-
 240 ization and task-specific performance.

241 5.1 MULTI-INSTANCE NORMALIZATION

243 Normalization is critical for improving model stability, convergence, and generalization. However,
 244 standard methods often fall short in our setting due to large intra-type variability (e.g., cooling loads
 245 significantly differ by room size) resulting in suboptimal gradient updates and biased loss weighting.
 246 To address this, we propose Multi-Instance Normalization. For each variable type j and instance i ,
 247 we compute the time-series min and max values, then aggregate these across instances to derive the
 248 P_{\min} percentile of minima and P_{\max} percentile of maxima (e.g., 10th and 90th percentiles). These
 249 are used as normalization bounds:

$$251 \quad \tilde{v}_{i,j}(t) = \frac{v_{i,j}(t) - P_{\min}(\{\min_t v_{i,j}(t)\}_i)}{P_{\max}(\{\max_t v_{i,j}(t)\}_i) - P_{\min}(\{\min_t v_{i,j}(t)\}_i)} \quad (9)$$

253 This method ensures consistent scaling across instances of the same object type, improving learning
 254 dynamics and overall prediction accuracy. Further comparisons are provided in Appendix H.

256 5.2 SELF-SUPERVISED LEARNING

258 Training the HGTFT model requires strategies that effectively encode temporal and relational dy-
 259 namics. Self-supervised learning (SSL) offers a scalable approach by utilizing unlabeled spatio-
 260 temporal data via pretext tasks with pseudo-labels. Common tasks such as masked prediction and
 261 contrastive learning have demonstrated success in both graph and time-series domains Rani et al.
 262 (2023); Xie et al. (2022); Zhang et al. (2024). We formulate SSL as the joint optimization of the
 263 foundation model f_θ and auxiliary heads p_ϕ on an unlabeled dataset D_1 :

$$265 \quad (\theta^*, \phi^*) = \arg \min_{\theta, \phi} L_{\text{ssl}}(f_\theta, p_\phi, D_1), \quad (10)$$

267 where L_{ssl} combines two tasks to capture both temporal dependencies and structural relationships.

268 **Masked Time-Series Modeling.** Following Zerveas et al. (2021), portions of the input sequence
 269 are masked and reconstructed using Mean Squared Error (MSE) loss.

270 **Masked Edge Modeling.** A subset of graph edges is masked, and the model predicts them via
 271 Binary Cross-Entropy (BCE) loss, distinguishing true from randomly sampled negative edges.
 272

273 To balance the tasks, we use alternating training: the two SSL tasks switch during training, starting
 274 and ending with time-series modeling, emphasizing sequence learning while incorporating physical
 275 relational understanding. Loss formulations and the SSL training pipeline/results are provided in
 276 Appendix E.

277 5.3 MULTI-TASK SUPERVISED LEARNING

279 Building on the pre-trained model f_{θ^*} from SSL, we design a physics-informed multi-task super-
 280 vised learning (MTSL) framework to fine-tune parameters θ^{**} with task-specific heads q_ψ :
 281

$$282 \quad (\theta^{**}, \psi^*) = \arg \min_{\theta^*, \psi} L_{\text{MTSL}}(f_{\theta^*}, q_\psi, D_2, Y), \quad (11)$$

285 where D_2 is the labeled spatio-temporal dataset and Y denotes the task labels. Instead of sim-
 286 ultaneous MTS defense, which scales poorly with task count, we adopt a sequential training strategy that
 287 optimizes tasks one-by-one, reducing memory usage and promoting convergence in imbalanced
 288 multiphysics settings Vandenhende et al. (2021); Yu et al. (2024).

289 **Convergence Criterion.** Sequential training is considered converged when the average relative
 290 change in task losses falls below a threshold. Formally, for task i at iteration k :

$$292 \quad \Delta L_{\text{task},i}^{(k)} = \frac{|L_{\text{task},i}^{(k)} - L_{\text{task},i}^{(k-1)}|}{L_{\text{task},i}^{(k-1)}}, \quad \Delta L_{\text{avg}}^{(k)} = \frac{1}{N} \sum_{i=1}^N \Delta L_{\text{task},i}^{(k)}, \quad (12)$$

295 where N is the number of tasks. Convergence is reached when $\Delta L_{\text{avg}}^{(k)}$ falls below a predefined
 296 threshold (e.g., 2%).
 297

298 **Physics-informed Loss Design.** To embed physical consistency directly into model training, we
 299 augment the standard MSE loss with three domain-informed components: (1) Reasonableness
 300 Checks Score (RCS) discourages predictions that violate operational constraints or physical laws
 301 Appendix F.4; (2) Correlation-Based Score (CRS) promotes consistency with known correlations
 302 in multivariate time-series data Appendix F.5; (3) Frequency Domain Similarity (FDS) aligns pre-
 303 dicted and actual spectral characteristics Appendix F.6. The total loss for each task is the weighted
 304 sum of the four loss components, with learnable or pre-defined weights. We adopt a hard parameter
 305 sharing scheme with a shared encoder and task-specific decoders, enabling the model to generalize
 306 across tasks while retaining task-specific specialization. The weighting scheme and training settings
 307 for different stages are detailed in Appendix F.7.

308 5.4 SUBTASK FINE-TUNING

310 The subtask fine-tuning process consists of two stages: task fine-tuning and project-specific fine-
 311 tuning. **Task fine-tuning** adapts the pre-trained model to forecasting tasks by freezing shared en-
 312 coder layers and updating only task-specific parameters, enhancing performance and serving as
 313 pre-adaptation. **Project-specific fine-tuning** adapts the model to real-world scenarios with limited
 314 labels, updating only lightweight components (e.g., dense projection) to align with new data while
 315 retaining general representations from pretraining.

316 6 EXPERIMENTS

317 6.1 DATASETS

318 **Standard and Graph-Structured Datasets.** Common benchmarks for time-series forecasting
 319 fall into two categories. The first includes standard datasets such as ETT, Weather, and Electric-
 320 ity Haixu et al. (2022), which assess general temporal prediction under purely data-driven assump-
 321 tions. The second includes graph-structured datasets such as PeMSD4, PeMSD8 Chen et al. (2001),

and COVID-19 case data Dong et al. (2020); nyt, where each node has a time series and spatial dependencies are encoded in graphs. While valuable for studying spatiotemporal correlations, these datasets do not capture the complexity of multi-domain physical systems considered in this work.

Multi-domain physical System Datasets. Energy and building operations provide a more representative scenario for multi-domain physics forecasting. Building systems comprise diverse components governed by distinct physical mechanisms: rotational and flow devices (pumps, compressors, valves), heat exchange units (fan coils, radiators, exchangers), transport infrastructures (pipes, ducts, tanks), and sensing/control units (thermostats, flow meters, controllers). These interact through principles of heat transfer, fluid dynamics, thermodynamics, and mass/energy conservation. The diversity and interdependence of such components make building systems a meaningful and broadly applicable testbed for multi-domain physical forecasting.

We first include the Building Time-Series (BTS) dataset, recently released at NeurIPS 2024 Prabowo et al. (2024), which contains over ten thousand time-series variables collected from three real buildings over a three-year period, covering hundreds of unique ontologies. While valuable, its scale remains limited for comprehensive pre-training. We release the Multiphysics Building System (MBS) dataset, which combines real-world and simulated building data. Further details and access to the dataset via an anonymous link are provided in Appendix B.

6.2 BASELINES

We compare our approach against a diverse set of baselines, encompassing traditional machine learning models, graph-based methods, and recent advancements in large pre-trained time-series models. Traditional models such as LSTM Hochreiter (1997), as well as more recent architectures like Autoformer Wu et al. (2021), forecast each variable independently without incorporating structural information. TFT Lim et al. (2021) integrates static covariates with dynamic time-series inputs for multivariate forecasting. HTGNN Fan et al. (2022) and STD-MAE Gao et al. (2023) operate on graph-structured time-series data, with heterogeneous and homogeneous structures, respectively. Recent developments in large pre-trained models have shown significant promise. TimesFM Das et al. (2023) and MOIRAI Woo et al. (2024) represent general pre-trained time-series models. LLM-based approaches, including Time-LLM Jin et al. (2023) and LLMTimed Gruver et al. (2024), leverage large language models for time-series prediction.

6.3 MAIN RESULTS

We first assess HGTFT on graph-structured spatiotemporal datasets, where relational information is explicit but no physical constraints are provided. This setting evaluates general forecasting ability against data-driven baselines. As shown in Table 1, HGTFT consistently ranks top-2 on PeMSD4, PeMSD8, and COVID-19, confirming its strength in capturing structured relationships. In particular, the COVID-19 dataset exhibits complex, multi-scale dynamics driven by non-stationary interventions and heterogeneous regional attributes. Unlike more stable traffic datasets, it tests a model’s ability to capture diverse entities and their interactions, where HGTFT demonstrates clear advantages. For completeness, we also evaluated HGTFT on standard time-series benchmarks (e.g., ETT) in Appendix G, which are less aligned with the problem studied here.

Table 1: Performance on spatiotemporal datasets. COVID-19 (JHU): daily infection counts from 83 Michigan counties; COVID-19 (NYT): daily death counts from 50 U.S. states. All models are trained or fine-tuned on 10% of each dataset. Best results are in **bold**, second best are underlined.

Dataset	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTimed	Time-LLM	HGTFT (Ours)
PeMSD4	MAE	32.48	32.39	31.32	21.01	17.85	32.57	33.31	33.69	32.23	<u>19.94</u>
	RMSE	55.51	53.19	48.37	36.44	29.72	55.94	55.51	52.49	52.18	<u>32.16</u>
PeMSD8	MAE	24.98	25.56	24.63	18.22	13.67	23.93	24.03	26.68	27.74	<u>16.43</u>
	RMSE	41.71	41.65	39.74	27.04	22.62	42.41	42.49	43.94	40.01	<u>25.08</u>
COVID-19 (JHU)	MAE	122.42	115.77	121.81	46.24	47.75	99.75	105.74	115.37	95.01	41.54
	RMSE	232.11	198.67	261.77	102.73	92.62	216.63	234.24	216.74	201.14	<u>94.38</u>
COVID-19 (NYT)	MAE	70.59	62.37	71.36	31.16	<u>26.69</u>	57.07	81.05	72.24	83.17	25.69
	RMSE	139.18	133.35	158.93	75.98	<u>72.98</u>	113.45	134.57	157.31	146.45	<u>65.64</u>

We evaluate HGTFT on the open BTS dataset, which includes three anonymized buildings Prabowo et al. (2024), as a representative multi-domain physical system. Forecasting tasks use the previous 7 days (672 time steps) to predict the next day (96 time steps) at 15-minute intervals, with all metrics computed on normalized values to account for inter-variable scale differences. We follow three evaluation settings: (i) pretraining on 50 randomly selected MBS buildings, (ii) pretraining on the full MBS dataset, and (iii) direct training on 30 days of each BTS building’s data with the remaining days for evaluation. For the pretrained models, both zero-shot prediction (without BTS building-specific data) and few-shot adaptation (using 30 days of BTS data) are assessed. Settings (i) and (ii) leverage the training methodology and physics-informed losses introduced earlier. The experiments are repeated 10 times with different seeds for pretraining building selection and few-shot sampling, and results are averaged.

As shown in Table 2, HGTFT achieves strong zero-shot performance, further enhanced by few-shot adaptation. Even without pretraining and physics-informed losses, it consistently surpasses all baselines, reducing MSE by up to 38% and RCS by 25% relative to the second-best model. Physics-informed pretraining yields an order-of-magnitude improvement in RCS, demonstrating its effectiveness in enforcing physical consistency. Few-shot adaptation substantially lowers MSE, while keeping RCS only slightly higher yet still well-controlled, striking a balance between predictive accuracy and physical plausibility. Although the benefits of physics-informed pretraining are limited for purely temporal models, they extend to spatial-temporal approaches such as HTGNN and STD-MAE, highlighting the broader generalization potential of physics-aware training. The strongest gains are observed for HGTFT, reflecting its capacity to integrate heterogeneous dynamics with structured physical constraints.

Table 2: Time-series forecasting results on the BTS dataset under three settings: pretrained zero-shot, pretrained few-shot, and no pre-training. Best results are in **bold**, second-best are underlined.

Settings	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
Zero-shot (50 MBS)	MSE	0.0142	0.0136	0.0105	<u>0.0091</u>	0.0095	0.0163	0.0159	0.0232	0.0220	0.0056
	RCS	0.0903	0.0567	0.0321	<u>0.0179</u>	0.0197	0.0696	0.0507	0.0733	0.0786	0.0025
	CRS	0.9687	1.0019	1.1071	0.7302	<u>0.4494</u>	0.8852	1.0398	1.1942	1.1813	0.4437
	FDS	0.8347	0.9994	0.9747	1.0049	0.9038	0.8135	<u>0.6660</u>	1.2103	0.9644	0.5745
Few-shot (50 MBS)	MSE	0.0082	0.0064	<u>0.0062</u>	0.0063	0.0070	0.0083	0.0086	0.0120	0.0139	0.0036
	RCS	0.0750	0.0588	0.0281	<u>0.0297</u>	0.0277	0.0528	0.0458	0.0601	0.0556	0.0037
	CRS	0.7878	0.8703	0.8121	0.6518	0.4274	0.8937	0.8537	0.8695	0.8809	<u>0.5132</u>
	FDS	0.6062	0.5697	0.5936	0.5760	0.6780	<u>0.4943</u>	0.5085	0.7417	0.6437	0.4303
Zero-shot (Full MBS)	MSE	0.0133	0.0122	0.0107	0.0087	0.0084	0.0123	0.0179	0.0219	0.0181	0.0047
	RCS	0.0747	0.0514	0.0326	<u>0.0158</u>	0.0194	0.0645	0.0517	0.0704	0.0803	0.0018
	CRS	1.0924	1.1426	0.8729	0.7826	<u>0.6086</u>	1.0770	1.1654	1.2520	1.0313	0.4472
	FDS	0.7306	0.8617	0.8825	0.8798	1.1247	0.8398	<u>0.7114</u>	1.2927	1.0047	0.5045
Few-shot (Full MBS)	MSE	0.0073	0.0072	<u>0.0056</u>	0.0059	0.0065	0.0084	0.0087	0.0128	0.0112	0.0033
	RCS	0.0744	0.0553	0.0300	<u>0.0244</u>	0.0291	0.0554	0.0452	0.0600	0.0571	0.0032
	CRS	0.7383	0.8252	0.7387	0.6035	0.4107	0.7636	0.7942	0.8383	0.9016	0.4083
	FDS	0.5419	0.6185	0.5177	0.5945	0.6603	0.4495	0.4986	0.7663	0.6331	<u>0.4691</u>
No Pretrain	MSE	0.0082	0.0075	<u>0.0062</u>	0.0064	0.0072	0.0092	0.0086	0.0127	0.0113	0.0040
	RCS	0.0697	0.0537	<u>0.0278</u>	0.0315	0.0377	0.0560	0.0440	0.0596	0.0591	0.0209
	CRS	0.7378	0.8103	0.7110	0.6079	0.4271	0.8108	0.8096	0.8624	0.8215	<u>0.5814</u>
	FDS	0.6313	0.6399	0.5814	0.7077	0.6490	<u>0.5279</u>	0.4957	0.8392	0.6683	0.6255

6.4 ABLATION STUDY

To contextualize the ablation results, we additionally include baseline models on the MBS dataset, providing a horizontal reference for interpreting the performance levels before and after ablating each component. This allows us to more clearly quantify the contribution of static–dynamic fusion, structural graph modeling, temporal modules, and the subtask layer under a consistent evaluation setup.

We then ablate major design choices of HGTFT. For static–dynamic fusion, VSNs are replaced with dense layers. For structural modeling, we remove the graph encoder or substitute GAT-based aggregation. For temporal modeling, we vary Transformer depth, place a single layer before or after the graph layer (Pre-G/Post-G), or remove it. The subtask layer is simplified by removing GRU and Add & Norm units or retaining only a dense projection. Results on MSE, RCS, CRS, and FDS are

432 reported in Table 4. Model scaling (Appendix D) further shows performance improves with size up
 433 to 310M parameters, beyond which gains plateau, suggesting 310M as an efficient capacity balance.
 434

436 Table 3: Average forecasting performance over 10 runs on 50 randomly selected building cases from
 437 the MBS dataset. Best results in **bold**, second best underlined.

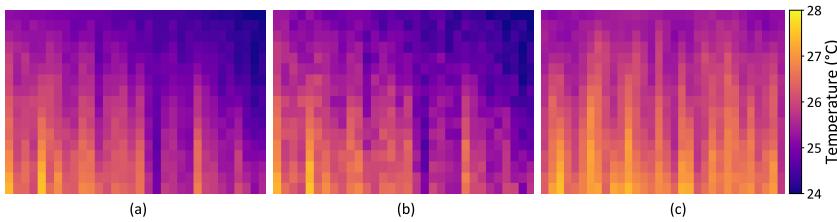
Metric	LSTM	Autoformer TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT zero-shot	HGTFT few-shot
MSE	0.0049	0.0053	0.0044	0.0048	0.0051	0.0064	0.0072	0.0093	0.0085	<u>0.0027</u> 0.0023
RCS	0.0376	0.0298	0.0152	0.0133	0.0206	0.0287	0.0243	0.0320	0.0307	0.0012 <u>0.0029</u>
CRS	0.4763	0.5207	0.5028	0.3855	<u>0.2804</u>	0.5311	0.5258	0.5527	0.5694	0.3123 0.2581
FDS	0.4382	0.4547	0.4288	0.4761	0.4930	<u>0.3684</u>	0.3899	0.5781	0.5139	0.4052 0.2919

444
 445 Table 4: Ablation results on architecture modifications and simplifications on the MBS dataset.

Metric	HGTFT	Fusion: dense	Graph: removed	Graph: GAT	Temporal: removed	Temporal: Pre-G	Temporal: Post-G	Temporal: Post-G	Subtask: w/o GRU	Subtask: dense
MSE	0.0027	0.0053	0.0065	0.0032	0.0072	0.0063	0.0064	0.0048	0.0067	
RCS	0.0012	0.0247	0.0343	0.0037	0.0157	0.0112	0.0103	0.0136	0.0297	
CRS	0.3123	0.5229	0.3551	0.3377	0.6324	0.5363	0.5289	0.4622	0.5435	
FDS	0.4052	0.4174	0.4139	0.4158	0.5890	0.5118	0.5152	0.4961	0.5803	

454 6.5 FURTHER ANALYSIS

455 To evaluate the model’s capacity to capture multiphysics interactions, Figure 2 visualizes predicted
 456 temperature fields on a sample floor at a selected time slice. The HGTFT model using full inputs
 457 (dynamic, static, and graph data) accurately reconstructs spatial temperature patterns. In contrast,
 458 the variant excluding static features (e.g., zone type, orientation) and spatial adjacency yields less
 459 coherent results, underscoring the importance of incorporating static and graph information.



470 Figure 2: Predicted temperature fields for a sample floor at a selected time slice. (a) ground truth,
 471 (b) prediction by HGTFT using full input, and (c) prediction without static or graph information.

472 To assess the impact of physics-aligned learning, Figure 3 presents model responses under low-
 473 frequency control changes, where the number of chillers increases from 2 to 3 and then to 4.
 474 The physics-aware HGTFT (solid lines) generates trends consistent with thermodynamic prin-
 475 ciples: adding chillers raises chilled water flow while reducing indoor temperature and humidity. In
 476 contrast, the MSE-only baseline (dashed lines) exhibits muted responses, with clustered curves that
 477 fail to capture the expected physical effects. These results demonstrate that incorporating physics-
 478 aligned supervision not only improves generalization to rare control actions but also enforces physi-
 479 cally consistent predictions. Metric ablation further supports this conclusion: removing RCS, CRS,
 480 or FDS degrades their corresponding scores from 0.0012 to 0.0134 (RCS), 0.312 to 0.384 (CRS),
 481 and 0.405 to 0.423 (FDS), while leaving other metrics largely unchanged. Among them, RCS proves
 482 to be the most influential.

483 To further validate our methodology, we conduct extensive analyses. First, we compare normaliza-
 484 tion strategies using CV-RMSE across variable types and observe that the proposed Multi-Instance
 485 Normalization consistently improves optimization stability and generalization over Min-Max and

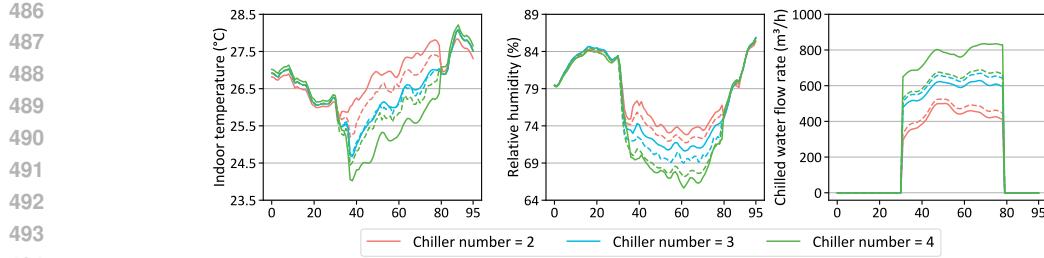


Figure 3: Predicted system responses to changes in control actions (solid: multi-metric training model; dashed: MSE-only training model).

Z-Score methods (Appendix H). For self-supervised learning, we evaluate three training strategies and find that while simultaneous task training impairs forecasting quality, our alternating task training method effectively balances time-series and relational representations with lower losses (Appendix J.1). In supervised learning, sequential multi-task training yields stable convergence, and reducing either task types or data coverage leads to moderate performance drops, highlighting the importance of task and case diversity (Appendix J.2). Input/output horizon analysis reveals that longer input patches enhance short- to mid-term forecasting accuracy, demonstrating the value of extended temporal context (Appendix J.3).

7 CONCLUSION

This paper addresses time series forecasting in heterogeneous multi-domain physical systems, where diverse entities, relations, and variables interact under physical constraints. We introduce the HGTFT, which integrates heterogeneous tokenization, graph-temporal fusion, and physics-aligned supervision within a pre-training and fine-tuning paradigm. Experiments show that HGTFT not only achieves performance comparable to state-of-the-art models on multiple spatiotemporal benchmarks, but also delivers clear advantages in realistic multiphysics scenarios, with strong zero-shot generalization and further gains through few-shot adaptation. These results highlight HGTFT as a robust and scalable framework for forecasting in complex physical environments. Limitations and future work are discussed in Appendix K.

540 **8 ETHICS STATEMENT**
541542 This work adheres to the ICLR Code of Ethics. The study does not involve human subjects, private
543 data, or personally identifiable information. All datasets used are publicly available, and additional
544 processed datasets are shared through an anonymized link in the supplementary materials to ensure
545 fair and ethical access. The methods and results do not pose foreseeable risks of discrimination,
546 unfair bias, or harmful applications. To the best of our knowledge, this work complies with standards
547 of research integrity, legal requirements, and ethical scientific conduct.
548549 **9 REPRODUCIBILITY STATEMENT**
550551 We have made significant efforts to ensure reproducibility. The model architecture and training
552 methodology are provided in Section 4 and Section 5 of the main text, with additional imple-
553 mentation and reproducibility details presented in Appendix E and Appendix F. The theoretical
554 foundations of the proposed framework are formally validated in Appendix A. The implementation
555 code is submitted as supplementary materials with clear instructions. All primary datasets em-
556 ployed in this study are publicly accessible. Furthermore, parts of our supplementary datasets are
557 released through an anonymized link (<https://drive.google.com/drive/folders/1fOG6SdFXXdJ0LtaELQA6o7obRxgTBfpq?usp=sharing>) to facilitate independent vali-
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810 A DESCRIPTION OF TYPICAL PROBLEMS

812 We present two examples to highlight the significance of extending the problem within the context
 813 of multiphysics systems (e.g., building systems). The first example involves a relatively simple
 814 dynamic system model, which begins with the fan coil unit (FCU) in relation to the space and
 815 cooling source. We then extend to a more complex system, which includes multiple types of objects
 816 and relationships, with each type of object potentially having a large number of instances.

818 A.1 EXAMPLE 1: HEAT EXCHANGE IN FAN COIL UNIT (FCU)

820 In the FCU, heat exchange occurs between air and water, and this process can be modeled using
 821 differential equations. Let's define the problem and derive the equations step by step.

822 Problem Definition

823 The heat exchange process involves the flow of air and water through the FCU, where air absorbs
 824 heat from the water and vice versa. The temperature dynamics for air and water are described as
 825 follows:

826 We first describe the air temperature dynamics. The rate of change of air temperature is governed
 827 by the following equation:

$$830 \dot{m}_{\text{air}} c_{\text{air}} \frac{dT_{\text{air}}(t)}{dt} = \dot{m}_{\text{air}} c_{\text{air}} (T_{\text{in,air}}(t) - T_{\text{out,air}}(t)) - Q_{\text{heat,air}}(t) \quad (13)$$

832 where: - \dot{m}_{air} is the mass flow rate of air (kg/s), - c_{air} is the specific heat capacity of air (J/kg·K), -
 833 $T_{\text{in,air}}(t)$ and $T_{\text{out,air}}(t)$ are the inlet and outlet air temperatures at time t (°C or K), - $Q_{\text{heat,air}}(t)$ is
 834 the heat exchanged between air and water at time t (W).

835 Next, we consider the water temperature dynamics. The change in water temperature over time can
 836 be described as:

$$839 \dot{m}_{\text{water}} c_{\text{water}} \frac{dT_{\text{water}}(t)}{dt} = Q_{\text{heat,water}}(t) - Q_{\text{water,out}}(t) \quad (14)$$

841 where: - \dot{m}_{water} is the mass flow rate of water (kg/s), - c_{water} is the specific heat capacity of water
 842 (J/kg·K), - $T_{\text{water}}(t)$ is the water temperature at time t (°C or K), - $Q_{\text{heat,water}}(t)$ is the heat ex-
 843 changed between air and water at time t (W), - $Q_{\text{water,out}}(t)$ is the heat lost by water to external
 844 factors at time t (W).

845 The heat exchange between air and water is modeled by the following equation:

$$848 Q_{\text{heat,air}}(t) = Q_{\text{heat,water}}(t) = h_{\text{air-water}} A_{\text{heat}} (T_{\text{air}}(t) - T_{\text{water}}(t)) \quad (15)$$

849 where: - $h_{\text{air-water}}$ is the heat transfer coefficient between air and water (W/m²·K), - A_{heat} is the
 850 heat exchange area (m²), - $T_{\text{air}}(t)$ and $T_{\text{water}}(t)$ are the air and water temperatures at time t (K).

852 Derivation of Differential Equations

853 Combining the heat exchange formulas with the temperature dynamics, we get a system of differen-
 854 tial equations:

$$857 \dot{m}_{\text{air}} c_{\text{air}} \frac{dT_{\text{air}}(t)}{dt} = \dot{m}_{\text{air}} c_{\text{air}} (T_{\text{in,air}}(t) - T_{\text{out,air}}(t)) - h_{\text{air-water}} A_{\text{heat}} (T_{\text{air}}(t) - T_{\text{water}}(t)) \quad (16)$$

$$860 \dot{m}_{\text{water}} c_{\text{water}} \frac{dT_{\text{water}}(t)}{dt} = h_{\text{air-water}} A_{\text{heat}} (T_{\text{air}}(t) - T_{\text{water}}(t)) - Q_{\text{water,out}}(t) \quad (17)$$

862 Introducing Temperature Difference

863 To simplify the equations, introduce the temperature difference:

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$$\Delta T(t) = T_{\text{air}}(t) - T_{\text{water}}(t) \quad (18)$$

867 Thus, the air temperature and water temperature can be expressed as:

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$$T_{\text{air}}(t) = T_{\text{water}}(t) + \Delta T(t) \quad (19)$$

871 Substituting this into the differential equations, we get:

872 For the air temperature equation:

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$$\dot{m}_{\text{air}}c_{\text{air}}\frac{dT_{\text{air}}(t)}{dt} = \dot{m}_{\text{air}}c_{\text{air}}(T_{\text{in,air}}(t) - T_{\text{out,air}}(t)) - h_{\text{air-water}}A_{\text{heat}}\Delta T(t) \quad (20)$$

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877 For the water temperature equation:878
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$$\dot{m}_{\text{water}}c_{\text{water}}\frac{dT_{\text{water}}(t)}{dt} = h_{\text{air-water}}A_{\text{heat}}\Delta T(t) - Q_{\text{water,out}}(t) \quad (21)$$

881 **Analytical Solution**882 For the temperature difference equation $\Delta T(t)$, we obtain:883
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$$\dot{m}_{\text{air}}c_{\text{air}}\frac{d\Delta T(t)}{dt} = \dot{m}_{\text{air}}c_{\text{air}}(T_{\text{in,air}}(t) - T_{\text{out,air}}(t)) - h_{\text{air-water}}A_{\text{heat}}\Delta T(t) \quad (22)$$

886 This is a first-order linear differential equation, which can be solved as:

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$$\Delta T(t) = \frac{\dot{m}_{\text{air}}c_{\text{air}}(T_{\text{in,air}}(t) - T_{\text{out,air}}(t))}{h_{\text{air-water}}A_{\text{heat}}} \left(1 - e^{-\frac{h_{\text{air-water}}A_{\text{heat}}}{\dot{m}_{\text{air}}c_{\text{air}}}t} \right) \quad (23)$$

890 Using the initial condition $\Delta T_0 = T_{\text{air},0} - T_{\text{water},0}$, we obtain:891
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$$T_{\text{air}}(t) = T_{\text{water}}(t) + \frac{\dot{m}_{\text{air}}c_{\text{air}}(T_{\text{in,air}}(t) - T_{\text{out,air}}(t))}{h_{\text{air-water}}A_{\text{heat}}} \left(1 - e^{-\frac{h_{\text{air-water}}A_{\text{heat}}}{\dot{m}_{\text{air}}c_{\text{air}}}t} \right) \quad (24)$$

894 The analytical solution for the water temperature is:

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$$T_{\text{water}}(t) = T_{\text{water},0} + \frac{h_{\text{air-water}}A_{\text{heat}}\Delta T_0}{\dot{m}_{\text{water}}c_{\text{water}}} \left(1 - e^{-\frac{h_{\text{air-water}}A_{\text{heat}}}{\dot{m}_{\text{water}}c_{\text{water}}}t} \right) - \frac{Q_{\text{water,out}}(t)}{\dot{m}_{\text{water}}c_{\text{water}}} \quad (25)$$

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900**Challenges and Complexities**901 The heat lost by water, $Q_{\text{water,out}}$, is influenced by heat/cooling source objects, while $T_{\text{in,air}}$ and $T_{\text{out,air}}$ are connected to spaces. Both heat/cooling source objects and spaces have their own distinct features and dynamics. The primary challenge in modeling such a system lies in the complex coupling of air and water dynamics, as well as the interactions between multiple spaces and Fan Coil Units (FCUs). As the number of spaces and FCUs increases, the complexity of the system grows exponentially, making it increasingly difficult to derive a closed-form solution. Therefore, the ability to integrate multiple object types and relationships through neural network algorithms is a critical requirement for addressing such problems.902
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904**Numerical Simulation and Prediction Using HGTFT**905
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913914 In this study, we assign different values to the static parameters in the previously defined mathematical model and apply time-varying functions to the external variables, $Q_{\text{water,out}}(t)$, $T_{\text{in,air}}(t)$, and $T_{\text{out,air}}(t)$. Through numerical simulations, a dataset is generated, which is then used to train the HGTFT-based model. The objective of this training is to predict the temperature profiles $T_{\text{water}}(t)$ and $T_{\text{air}}(t)$ based on the temporal variations of the external variables and the given static parameters.

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 973 Table 5: An example of an HVAC operation task, covering multiple object types—such as environment,
 974 general zone, chiller, chilled water pump (ACCCCP), cooling water pump (ACCCOP),
 975 cooling tower (ACCCOT), fan coil unit (ACATFC), and supply air fan (ACATFU)—along with
 976 their associated input and output variable types.

977 Object type	978 Input			979 Output
	980 Static attribute	981 Dynamic variable for the 982 past	983 Dynamic attribute for the 984 future	
980 Environment		981 Outdoor temperature	982 Outdoor temperature	
980 GeneralZone	981 Area, volume, orientation	982 Indoor temperature, relative 983 humidity		984 Indoor temperature, relative 985 humidity
982 Chiller	983 Rated cooling capacity, 984 rated power	985 Chilled water supply tempera- 986 ture, chilled water return tempera- 987 ture, chilled water flow rate		988 Chilled water supply tempera- 989 ture, chilled water return tempera- 990 ture, chilled water flow rate
985 ACCCP	986 Rated power, rated flow 987 rate, rated head	988 Operating status, operat- 989 ing power, flow rate	990 Operating status	991 Operating power, flow 992 rate
986 ACCCOP	987 Rated power, rated flow 988 rate, rated head	989 Operating status, operat- 990 ing power, flow rate	991 Operating status	992 Operating power, flow 993 rate
987 ACCCOT	988 Rated power, rated air 989 flow, number of fans, 990 design outdoor wet-bulb 991 temperature	992 Number of operating 993 fans, air flow rate, leaving 994 water temperature, water 995 flow rate, leaving water 996 temperature setpoint	997 Leaving water tempera- 998 ture Setpoint	999 Number of Operating 999 fans, air flow rate, leaving 999 water temperature, water 999 flow rate
989 ACATFC	990 Rated power, rated air 991 flow, rated chilled water 992 flow rate	993 Supply air temperature, 994 return air temperature, 995 supply air temperature 996 setpoint	997 Supply air temperature 998 setpoint	999 Supply air temperature, 999 return air temperature
990 ACATFU	991 Rated power, rated air 992 flow	993 Fresh air flow rate, fan 994 speed	995 Fan speed	996 Fresh air flow rate

995 996 B MBS DATASET DETAILS

999 The Multi-physics Building System (MBS) dataset combines real-world and simulated building
 1000 data. A subset is publicly available at <https://drive.google.com/drive/folders/1fOG6SdFXXdJ0Lt aELQA6o7obRwgTBfp?usp=sharing>. Object and relationship definitions
 1001 in building operation systems are based on a standardized, publicly available data dictionary
 1002 commonly used in building automation. The training dataset primarily contains HVAC-related data,
 1003 including empirical data aggregated from diverse real-world deployments and synthetic data gen-
 1004 erated via a high-fidelity simulation environment. Figure 5 shows a partial 3D visualization from
 1005 the simulation setup, illustrating mappings between equipment and spatial zones, as well as detailed
 1006 interconnections such as piping and ductwork.

1008 1009 B.1 REAL PROJECT DATA

1010 We have accumulated a substantial dataset from a multitude of real-world projects, encompassing
 1011 various building subsystems such as energy management systems, security surveillance systems,
 1012 equipment and facility management systems, and building automation systems. The dataset com-
 1013 prises a total of 1045 projects, with 508 projects containing relatively comprehensive information.
 1014 The dataset contains about 5B tokens and 16B time points data.

1016 1017 B.2 SIMULATION DATA

1018 Compared to real-world project data, simulations can involve a much larger number of variables,
 1019 including many that are difficult or even impossible to measure in the real world project but can be
 1020 calculated in a simulation environment. Additionally, simulations allow for the alteration of many
 1021 operating conditions, covering a much broader range of scenarios than real projects can achieve.
 1022 Given the astronomical number of possible parameter combinations, it is necessary to reduce the
 1023 number of generated simulation cases. This can be achieved by carefully selecting variable param-
 1024 eters and applying orthogonal testing to optimize the case generation process. We constructed a
 1025 massive dataset of building energy simulations using EnergyPlus DOE (2015). By systematically
 varying key building parameters across 12 diverse base building models, we generated approxi-

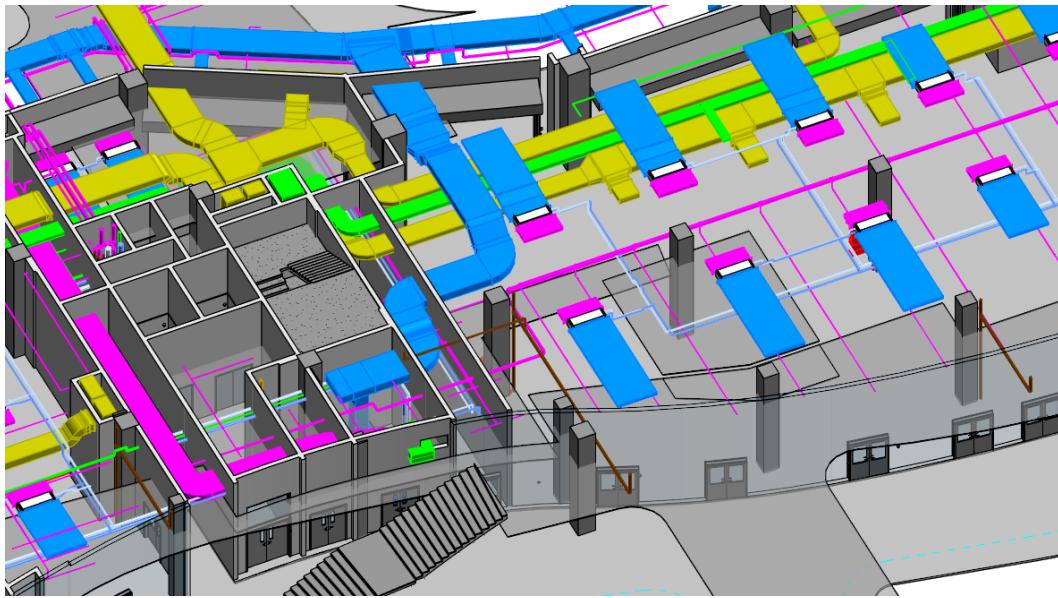


Figure 5: 3D illustration of a simulated building environment, showing spatial layout, service relationships between equipment and zones, and extensive duct and piping connections representing air and water flows in HVAC systems.

mately 5,000 simulation scenario cases. Each case provides high-resolution 15-minute data for a year, resulting in a dataset of over 80B tokens and 600B time points data.

B.3 COMPARISON BETWEEN REAL PROJECT AND SIMULATION DATA

We collected both simulated and real-world data for various variables, and Figure 6 illustrates a daily profile of chiller plant cooling power, for instance. Overall, the simulated data closely aligns with the real-world data, demonstrating a strong consistency. Due to the ability to simulate a wider range of operating conditions, the simulated data offers a broader coverage of scenarios. This increased diversity in the simulated conditions allows for a more comprehensive representation of potential system behaviors, enhancing the robustness of the model training and its ability to generalize to different operational contexts.

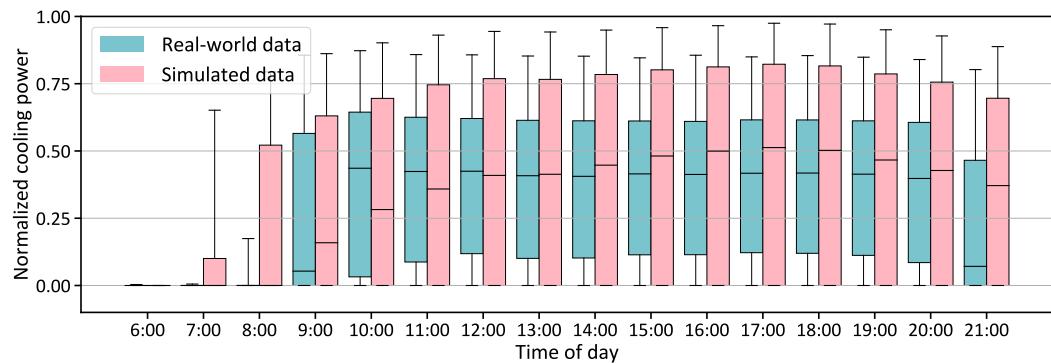


Figure 6: Comparison of a daily profile for chiller plant cooling power between real-world data and simulated data.

1080 C SUPPLEMENTARY EXPLANATION OF NETWORK UNITS AND FORMULAS
10811082 C.1 GATED RESIDUAL NETWORK (GRN)
1083

1084 The following description of the Gated Residual Network (GRN) is primarily based on the relevant
1085 sections from the Temporal Fusion Transformer (TFT) paper Lim et al. (2021). The GRN intro-
1086 duces a gating mechanism via the Gated Linear Unit (GLU) to regulate the flow of information
1087 and selectively pass only the most relevant inputs. This design is critical for handling diverse data
1088 inputs effectively. The GRN structure is described by Equations 26-28. The primary input a and
1089 the context input c are processed through the Exponential Linear Unit (ELU) activation function,
1090 linear transformation, GLU, and layer normalization. Weight matrices W_1, W_2, W_3 , and biases $b_1,$
1091 b_2 govern the transformation, providing flexibility through selective non-linear processing.

1092
$$\text{GRN}(a, c) = \text{LayerNorm}(a + \text{GLU}(\eta_1)), \quad (26)$$

1093

1094
$$\eta_1 = W_1 \eta_2 + b_1, \quad (27)$$

1095

1096
$$\eta_2 = \text{ELU}(W_2 a + W_3 c + b_2). \quad (28)$$

1097 The GLU is defined in Equation 29, where X is the input, W_4 and W_5 are learnable weights, b_3
1098 and b_4 are biases, and σ is the sigmoid function. The Hadamard product \odot modulates the GRN's
1099 influence on the input a , allowing it to potentially skip processing when the GLU output approaches
1100 zero. If no context vector is provided, c is set to zero.

1101
$$\text{GLU}(X) = \sigma(W_4 X + b_3) \odot (W_5 X + b_4). \quad (29)$$

1102 This modular structure enables the GRN to adapt flexibly to different input types and feature com-
1103 binations, enhancing the Variable Selection Networks' (VSNs) ability to identify and prioritize key
1104 variables efficiently.

1106 C.2 VARIABLE SELECTION NETWORK (VSN)
1107

1108 The variable selection weights α are computed to determine the contribution of each time-variant
1109 feature x_i to the aggregated embedding e^{agg} . This is achieved through a Gated Residual Network
1110 (GRN) and a softmax function as shown below:

1111
$$\alpha = [\alpha_1, \dots, \alpha_i, \dots, \alpha_m] = \text{Softmax}(\text{GRN}([e_1, \dots, e_i, \dots, e_m], c_s)), \quad (30)$$

1112

1113 where c_s is the static covariate encoder and e_i is the embedding vector of feature x_i . The aggregated
1114 entity embedding vector e^{agg} is a weighted sum of all the m time-variant variable embeddings:

1115
$$e^{\text{agg}} = \sum_{i=1}^m \alpha_i \text{GRN}(e_i). \quad (31)$$

1116
1117

1118 VSN can be also used for static feature selection, and Figure 7 presents the VSN architecture, with
1119 using GRN.

1121 C.3 TRANSFORMER
1122

1123 The self-attention mechanism in Transformer layers enhances the embeddings by considering the
1124 relationships between all elements in the input sequence, allowing the model to capture global con-
1125 text and complex dependencies. The mechanism works by calculating a similarity score between
1126 each query (Q) and key (K) pair, producing attention weights that reflect the importance of each
1127 element in relation to others. These attention weights enable each element to be influenced by other
1128 relevant elements in the sequence, leading to a dynamic and context-aware representation.

1129 The self-attention mechanism computes the attention weights for a given set of query, key, and value
1130 matrices Q, K , and V as follows:

1132
$$\text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V, \quad (32)$$

1133

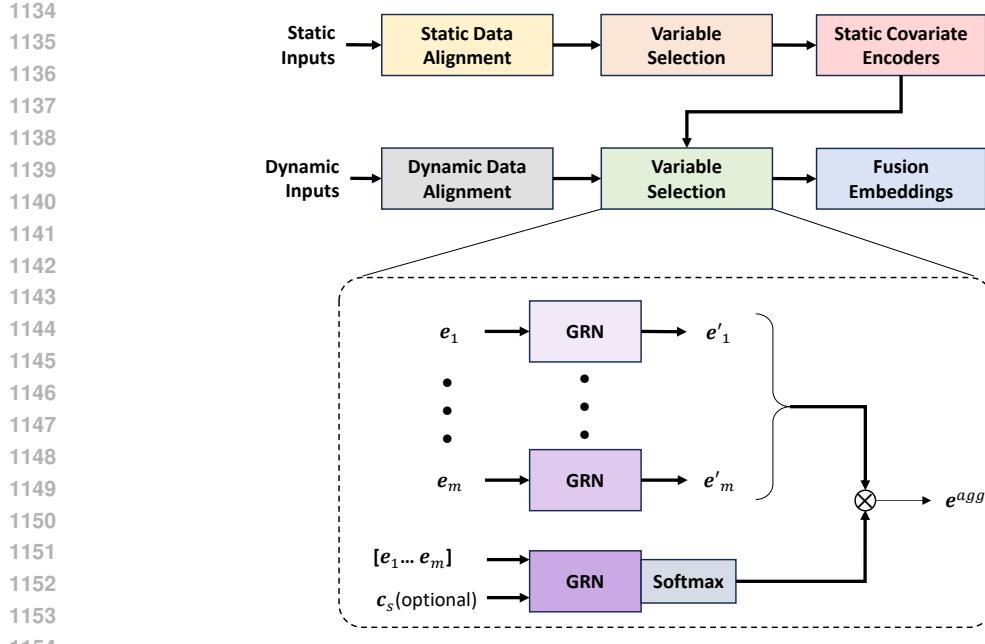


Figure 7: Overview of the entire workflow of the Fusion Layer, where both static and dynamic data pass through two Variable Selection Networks (VSN) with distinct parameters. The static features are selected by themselves, while dynamic data is filtered based on selected static covariates. The calculation mechanism of the VSN is also depicted in the diagram.

where Q , K , and V are the query, key, and value matrices, respectively. d_k is the dimensionality of the key vectors. The term $\frac{QK^T}{\sqrt{d_k}}$ ensures that the dot-product similarity is normalized by the square root of the dimensionality, preventing large values that could make the softmax function too sharp. The softmax function is applied to the similarity scores to generate a probability distribution, which is then used to weight the values in V .

The multi-head attention mechanism allows the model to capture information from multiple representation subspaces. Instead of computing a single attention output, multiple attention heads are computed in parallel, and their results are concatenated and projected back to the original space. The multi-head attention mechanism is defined as:

$$\text{MultiHead}(Q, K, V) = [H_1 \oplus \dots \oplus H_h \oplus \dots \oplus H_H]W_H, \quad (33)$$

where H_h represents the output of the h -th attention head, computed as:

$$H_h = \text{Attention}(QW_h^Q, KW_h^K, VW_h^V), \quad (34)$$

and W_h^Q , W_h^K , and W_h^V are learned weight matrices for the query, key, and value matrices, respectively, for the h -th head. The symbol \oplus denotes concatenation, meaning the outputs from all attention heads are concatenated into a single vector. W_H is a learned weight matrix that projects the concatenated output back into the model's desired output dimension.

After the multi-head attention step, a feed-forward network (FFN) is applied to introduce non-linearity. The FFN consists of two fully connected layers with a ReLU activation function applied between them. This non-linearity enables the model to capture more complex relationships and dependencies within the data.

Thus, the combination of self-attention and multi-head attention allows the Transformer model to focus on different parts of the input sequence simultaneously, creating a more dynamic and con-

1188 textually aware representation, especially useful for tasks involving long-range dependencies and
 1189 complex sequence data.
 1190

1191 C.4 INTRA-RELATION AGGREGATION 1192

1193 To preserve graph heterogeneity and enable fine-grained relation modeling, we perform relation-
 1194 specific neighborhood aggregation using distinct BiLSTM encoders for each relation type.
 1195

1196 At time step t , the system is represented as a heterogeneous graph $\mathcal{G}_t = (V, E, R)$, where R denotes
 1197 the set of edge relation types. For each node $v_i \in V$ and relation $r_\ell \in R$, we aggregate temporal
 1198 embeddings $h_{j,t}^{\text{temp}}$ from neighbors $v_j \in N_\ell(v_i)$ using:
 1199

$$1200 h_{i,\ell}^{\text{agg}}(t) = \frac{1}{|N_\ell(v_i)|} \sum_{v_j \in N_\ell(v_i)} \text{BiLSTM}_\ell(h_{j,t}^{\text{temp}}), \quad (35)$$

1201 Unlike HetGNN, which shares encoders across neighbor types, we assign a distinct BiLSTM per
 1202 relation type r_ℓ , allowing the model to disentangle heterogeneous physical or logical interactions.
 1203 For example, a room might be connected to others via either airflow or control signals—relations
 1204 that are semantically different and thus require different encoding strategies.
 1205

1206 C.5 INTER-RELATION AGGREGATION 1207

1208 To integrate information from multiple relation types, we adopt a multi-head attention mechanism
 1209 over the aggregated embeddings $h_{i,\ell}^{\text{agg}}(t)$. For each attention head $k = 1, \dots, K$, attention coeffi-
 1210 cients α_ℓ^k are computed as:
 1211

$$1212 \alpha_\ell^k = \text{softmax} \left(\text{LeakyReLU} \left(a^{k\top} [W^k h_{i,t}^{\text{temp}} \| W^k h_{i,\ell}^{\text{agg}}(t)] \right) \right), \quad (36)$$

1213 where $W^k \in \mathbb{R}^{d' \times d}$ is a learnable projection matrix and $a^k \in \mathbb{R}^{2d'}$ is a shared attention vector for
 1214 the k -th head. The final graph-based embedding for node v_i is:
 1215

$$1216 h_{i,t}^{\text{graph}} = \frac{1}{K} \sum_{k=1}^K \sum_{\ell=1}^L \alpha_\ell^k W^k h_{i,\ell}^{\text{agg}}(t), \quad (37)$$

1217 This fusion mechanism allows the model to assign adaptive weights to different relation types per
 1218 attention head, enabling robust modeling of heterogeneous dependencies. Compared to early fusion
 1219 approaches, this method provides enhanced flexibility and improved representation quality for nodes
 1220 participating in multi-relational contexts.
 1221

1222 D MODEL VERSION COMPARISON 1223

1224 This section presents a systematic comparison of different model variants for time-series forecasting
 1225 in complex building operation systems. All models are trained on the MBS dataset, using the pro-
 1226 posed HGTFT architecture. The resulting pretrained model, specialized for the building domain, is
 1227 termed BOSG (Building Operation System Generator). We explore a range of model configurations
 1228 by varying embedding dimensions, network depth, and overall parameter count to analyze trade-offs
 1229 between predictive performance, model size, and training efficiency.
 1230

1231 D.1 EMBEDDING DIMENSION ADJUSTMENT 1232

1233 We tested four embedding dimensions (64, 128, 256, and 512), while keeping the architecture con-
 1234 stant: one temporal layer, one graph layer, and two additional temporal layers. Results in Table 6
 1235 show that 256-d offers a strong trade-off between accuracy and efficiency. Although 512-d pro-
 1236 vides marginal MSE improvements, the parameter increase is substantial, with limited performance
 1237 benefit.
 1238

1242
 1243 Table 6: Performance comparison of various model configurations with different embedding dimen-
 1244 sions.

1245 Embedding 1246 dimension	1247 Model size	1248 MSE	1249 RCS	1250 CRS	1251 FDS
64-d	22,241,773	0.0098	0.0168	0.434	0.491
128-d	81,437,154	0.0059	0.0045	0.396	0.448
256-d	310,800,689	0.0027	0.0012	0.312	0.405
512-d	1,173,418,849	0.0026	0.0012	0.320	0.413

1252
 1253 D.2 MODEL LAYER ADJUSTMENT

1254
 1255 We compared multiple network layer configurations, modifying the order and count of temporal and
 1256 graph layers (see Table 7). Results indicate that placing a temporal layer before the graph layer is
 1257 essential for capturing temporal context prior to modeling inter-object relations. Additional temporal
 1258 layers after the graph layer further improve performance, but benefits plateau beyond two layers.

1260
 1261 Table 7: Performance comparison of various model configurations with different network layer
 1262 architectures.

1263 Layer configuration	1264 Model size	1265 MSE	1266 RCS	1267 CRS	1268 FDS
Graph+Temporal	197,075,249	0.0068	0.0025	0.487	0.536
Temporal+Graph	197,075,249	0.0056	0.0021	0.469	0.480
Temporal+Graph $\times 2$	213,697,841	0.0053	0.0020	0.454	0.477
(Temporal+Graph) $\times 2$	270,560,561	0.0039	0.0018	0.413	0.439
(Temporal+Graph) $\times 3$	344,045,873	0.0033	0.0015	0.375	0.414
Temporal+Graph+Temporal	253,937,969	0.0037	0.0016	0.395	0.468
Temporal $\times 2$ +Graph+Temporal	310,800,689	0.0036	0.0016	0.386	0.442
Temporal $\times 2$ +Graph+Temporal $\times 2$	367,663,409	0.0028	0.0013	0.306	0.399
Temporal+Graph+Temporal $\times 2$	310,800,689	0.0027	0.0012	0.312	0.405
Temporal+Graph+Temporal $\times 3$	367,663,409	0.0026	0.0012	0.304	0.397

1275
 1276 D.3 SCALING STUDY AND MODEL VARIANTS

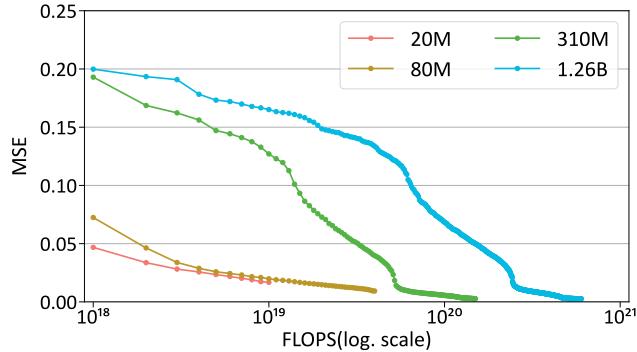
1277
 1278 We conducted a scaling study on the BOSG model to investigate the relationship between model size,
 1279 computation, and forecasting performance. Four BOSG configurations were trained with parameter
 1280 sizes of 20M, 80M, 310M, and 1.26B, each using 30K iterations and a fixed global batch size of 64.
 1281 All model variants adopted 8 attention heads and incorporated up/down projection layers to enhance
 1282 feature representation. Their architectural details and evaluation results are summarized in Table 8.
 1283 As model size increased, the primary forecasting metric (MSE) consistently decreased from 0.0107
 1284 to 0.0025, with notable gains up to 310M parameters. However, performance improvement between
 1285 the 310M and 1.26B models was marginal, indicating diminishing returns at larger scales.

1286
 1287 To better understand compute-performance efficiency, we saved model checkpoints at specific
 1288 FLOPS intervals during training and plotted the resulting MSE values on a log scale. As shown
 1289 in Figure 8, training performance improved with increasing computational budget, although the
 1290 rate of improvement flattened beyond the 310M model. All experiments were conducted on a high-
 1291 performance system consisting of eight NVIDIA A800 GPUs (80GB memory each), providing 3456
 1292 tensor cores in total. This setup enabled efficient parallel training, with the largest model (1.26B)
 1293 completing 30K iterations in approximately three days. These findings provide practical guidance
 1294 for compute-optimal scaling in time-series modeling.

1296

1297 Table 8: Performance comparison of BOSG model variants with varying parameter sizes and con-
1298 figurations.

1299 Version	1300 Params	1301 Embedding	1302 Layer configuration	1303	MSE	RCS	CRS	FDS
20M	19,164,316	64-d	Graph+Temporal		0.0107	0.0184	0.496	0.519
80M	77,202,922	128-d	Temporal+Graph+Temporal		0.0073	0.0055	0.435	0.481
310M	310,800,689	256-d	Temporal+Graph+Temporal $\times 2$		0.0027	0.0012	0.312	0.405
1.26B	1,258,271,153	512-d	Temporal+Graph+Temporal $\times 3$		0.0025	0.0012	0.307	0.416

1304
1305 Figure 8: Training MSE vs. FLOPS (log scale) for different BOSG model sizes (20M, 80M, 310M,
1306 1.26B) on the SSL masked time-series modeling task.
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E SELF-SUPERVISED LEARNING

1318

E.1 LOSS FUNCTION DETAILS FOR SELF-SUPERVISED LEARNING

1319
1320 The following describes the loss functions employed in our self-supervised learning (SSL) tasks,
1321 aimed at ensuring clarity and reproducibility. For masked time-series modeling, the reconstruction
1322 error is quantified using the Mean Squared Error (MSE) as follows:
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where N represents the number of nodes, F_i denotes the number of features for the i -th node, and $M_{i,f}$ is the number of masked time points for each feature.

For the graph-based task, the Binary Cross-Entropy (BCE) loss function is utilized to evaluate the classification accuracy of edge predictions, defined as:

$$\text{BCE} = -\frac{1}{N_r} \sum_{i=1}^{N_r} (r_i \log(\hat{r}_i) + (1 - r_i) \log(1 - \hat{r}_i)), \quad (39)$$

where N_r represents the number of samples, \hat{r}_i is the predicted relation, and r_i denotes the true relation value.

E.2 MODEL TRAINING EXPERIMENTS FOR SELF-SUPERVISED RELATIONSHIP LEARNING TASK

We conducted experiments for self-supervised relationship learning task with the HGTFT model to identify which network layers are essential to update and which can remain fixed. Additionally, we evaluated the prediction results when the parameters of the task-specific linear transformation layer were either initialized randomly without updates or jointly updated alongside HGTFT. Further, we examined the effect of initializing HGTFT parameters either randomly or using pre-trained

weights from a masked time-series modeling task. The results of these validation experiments are summarized in Table 9.

Table 9: Experimental results of masked edge modeling for various model update approaches.

Case No.	HGTFT update layer	Task NW	Initialization	loss (BCE)
Case 1	node, temporal, graph	Update	Random	0.34
Case 2	temporal, graph	Update	Random	0.35
Case 3	graph	Update	Random	0.42
Case 4	temporal, graph	w/o update	Random	0.35
Case 5	temporal, graph	w/o update	Masked time-series modeling	0.28

The experimental results revealed that updating the network layers responsible for the temporal and graph embeddings is crucial for task performance. Additionally, reusing the pre-trained parameters from the masked time-series modeling task provided a significant improvement over random initialization. Interestingly, the task-specific linear output layer primarily acted as a dimensionality transformation and had minimal impact on the prediction results. Based on these observations, we determined that the optimal approach involves initializing the base HGTFT model parameters from the trained masked time-series modeling task, updating only the temporal and graph embeddings, and leaving the task-specific linear output layer randomly initialized and fixed during training.

E.3 TRAINING PIPELINE FOR SELF-SUPERVISED LEARNING

In our self-supervised learning approach, we prioritized the masked time-series modeling task as the primary objective, with the masked edge modeling task as a secondary target. The goal was to minimize the loss of the masked edge modeling task while ensuring that the loss of the masked time-series modeling task increased by no more than 10% from its optimal value. A series of sequential training experiments were conducted to achieve this balance, and the results are summarized in Table 10.

Table 10: Experiment results for the self-supervised learning pipeline.

Step No.	Masked time-series modeling			Masked edge modeling		
	Task on/off	Starting loss	Ending loss	Task on/off	Starting loss	Ending loss
Step 1	On	1.8421	0.0027	Off	N/A	0.6942
Step 2	Off	0.0027	0.6439	On	0.6942	0.2885
Step 3	On	0.6439	0.0028	Off	0.2885	0.4526
Step 4	Off	0.0028	0.2781	On	0.4526	0.2640
Step 5	On	0.2781	0.0026	Off	0.2640	0.3304
Step 6	Off	0.0026	0.2673	On	0.3304	0.2595
Step 7	On	0.2673	0.0026	Off	0.2595	0.3184

Through a total of seven rounds of alternating training between the two tasks, we observe a consistent decrease in the loss for the masked time-series modeling task before each training session, with little change in the loss after training. In contrast, for the masked edge modeling task, the loss values showed noticeable reductions both before and after training in each round. Notably, the final round of training for the masked time-series modeling task had minimal impact on the graph relationship prediction, suggesting that the model had converged and further training on this task no longer significantly affected the performance of the masked edge modeling task.

F SUPERVISED LEARNING

F.1 SUPERVISED LEARNING SUBTASK MODEL

Each subtask shares a unified decoder structure (see Figure 9), where masked attention connects historical embeddings to future targets. GRN blocks and lightweight dense projections are included

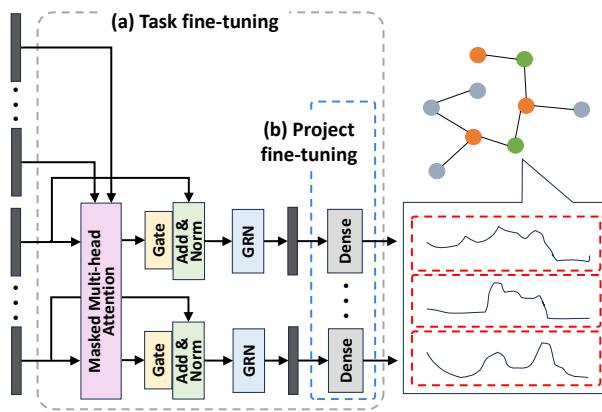


Figure 9: Model structure for a typical prediction subtask with two fine-tuning phases.

for stable adaptation. Fine-tuning is performed in two stages: task-level tuning updates only task-specific parameters, while project-level tuning adjusts the dense head to align with limited real-world data, preserving general representations learned during pretraining.

F.2 SUPERVISED LEARNING TRAINING TASK

Forecasting tasks in multiphysics systems exhibit substantial diversity due to the heterogeneity of entities, variable types, and interaction structures. To capture this complexity, we construct a suite of supervised learning tasks based on scenario-specific interaction topologies. Each scenario is represented as a heterogeneous graph comprising distinct physical entities (e.g., thermal zones, fluid circulation units, environmental sensors) and their relationships, as illustrated in Figure 10 for scenario 3.3.

Beyond structural diversity, variations in variable availability across entities further contribute to task differentiation. We first define original tasks by selecting strongly correlated entities and predicting all of their dynamic variables for future time points. Derived tasks are then generated by selectively masking or revealing subsets of variables in the future, simulating diverse observability conditions. An example of such task construction is provided in Table 5.

F.3 MEAN SQUARE ERROR FOR SUPERVISED LEARNING

The accuracy loss, denoted as L_{MSE} , is quantified using the Mean Squared Error (MSE) across all entities for each task, as formally defined below:

$$L_{\text{MSE}} = \frac{1}{N} \sum_{i=1}^N (y_i(t, T_{\text{future}}) - \hat{y}_i(t, T_{\text{future}}))^2, \quad (40)$$

where N represents the number of entities, which may vary across different tasks. The terms $y_i(t, T_{\text{future}})$ and $\hat{y}_i(t, T_{\text{future}})$ refer to the true and predicted values, respectively, for the time period from $t + 1$ to $t + T_{\text{future}}$, corresponding to all dynamic prediction features of the i -th entity. For the sake of brevity and clarity, the feature dimension is omitted from the formula.

F.4 REASONABILITY CHECKS SCORE

In complex physical systems, time series predictions must not only achieve numerical accuracy but also respect fundamental physical laws and operational constraints. We propose the *Reasonability Checks Score* (RCS) as an auxiliary evaluation metric to quantify the degree to which predicted values conform to domain-specific physical expectations. Rather than being limited to any particular field, the RCS framework is designed to be modular and extensible, supporting multi-domain constraints across various physical and engineered systems.

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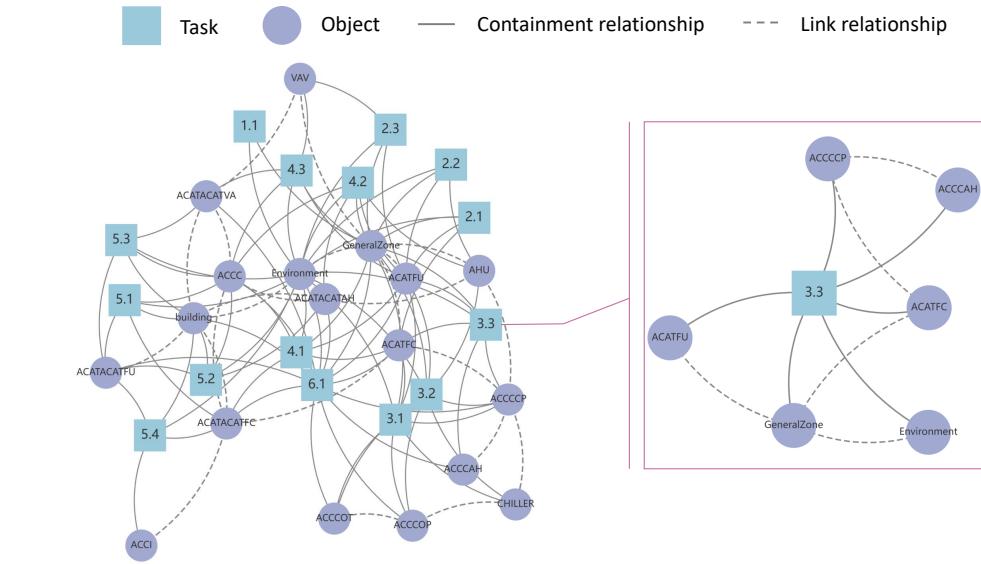


Figure 10: Topology of tasks and entity types in a building multiphysics system. Each scenario defines a specific combination of interconnected entities, identified by a unique scenario ID. Highlighted example 3.3 includes Environment, General Zone, Chiller, Chilled/ Cooling Water Pumps (ACCCCP/ACCCOP), Cooling Tower (ACCCOT), Fan Coil Unit (ACATFC), and Supply Air Fan (ACATFU).

To structure this assessment, we categorize reasonableness checks into four generalized dimensions:

1. **Physical State Bounds:** Core physical quantities (e.g., temperature, pressure, flow rate, power) should remain within known feasible or safe ranges, derived from empirical knowledge or physical laws.
2. **Energy and Resource Balance:** Energy usage, mass flow, or material consumption should be consistent with input-output relationships and operational schedules. Sudden discontinuities or unrealistic surges may indicate violations of conservation principles or faulty control.
3. **System Operating Constraints:** Devices or subsystems should operate in valid configurations, respecting timing constraints, activation conditions, and logical dependencies (e.g., cooling should not activate when the system is already below the lower threshold).
4. **Inter-Component Consistency:** Multiple subsystems interacting within the same environment should exhibit consistent behavior. For example, responses to a shared external stimulus should not contradict each other or physical causality.

Each reasonableness check can be modeled as a differentiable function that penalizes violations of soft physical constraints. The total RCS loss is defined as:

$$L_{\text{RCS}} = \sum_{k=1}^K g_k(\hat{y}_k(t, T_{\text{future}})), \quad (41)$$

where $g_k(\cdot)$ denotes the k -th check function applied to the predicted output $\hat{y}_k(t, T_{\text{future}})$, and K is the total number of checks relevant to the task.

Example 1: Bounded Range Check. For a physical variable $\hat{y}(t)$ constrained within a range $[y_{\min}, y_{\max}]$, the penalty term can be formulated as:

$$g_{\text{range}}(\hat{y}(t)) = \lambda \cdot \left[\max(0, \hat{y}(t) - y_{\max})^2 + \max(0, y_{\min} - \hat{y}(t))^2 \right], \quad (42)$$

1512 where λ is a weighting coefficient controlling the penalty strength at each time point t .
 1513

1514 **Example 2: Energy Conservation Check.** In multi-physical systems, the principle of energy con-
 1515 servation often serves as a key constraint. For instance, in a thermal process involving heat exchange,
 1516 the heat entering a system at time t should approximately equal the sum of the heat leaving the sys-
 1517 tem and the internal losses, i.e., $Q_{\text{in}}(t) \approx Q_{\text{out}}(t) + Q_{\text{loss}}(t)$. To enforce this physical constraint on
 1518 predicted outputs, we define the energy conservation check function as:
 1519

$$1520 \quad g_{\text{energy}}(t) = \gamma \cdot \left(\hat{Q}_{\text{in}}(t) - \hat{Q}_{\text{out}}(t) - Q_{\text{loss}}(t) \right)^2, \quad (43)$$

1522 where $\hat{Q}_{\text{in}}(t)$ and $\hat{Q}_{\text{out}}(t)$ denote the predicted input and output energy at time t , and $Q_{\text{loss}}(t)$
 1523 is a predefined (or estimated) time-dependent loss term. The scalar γ controls the importance of
 1524 this check. This function penalizes deviations from the expected energy balance at each timestep,
 1525 thereby promoting physically consistent predictions.
 1526

1527 F.5 CORRELATION-BASED SCORE

1529 The Correlation-Based Score (CBS) evaluates the statistical correlation between predicted and true
 1530 values in time-series forecasting by computing the Pearson correlation coefficients for both predicted
 1531 and true values, determining the deviation between these correlations for each variable pair, and then
 1532 calculating the loss as the Mean Squared Error (MSE) of these deviations.

1533 The formula for the CBS loss L_{corr} is given by:
 1534

$$1535 \quad L_{\text{CBS}} = \frac{1}{L} \sum_{l=1}^L \left(|\rho(\hat{y}_i, \hat{y}_j) - \rho(y_i, y_j)|^2 \right), \quad (44)$$

1538 Where L is the number of variable pairs in the prediction task. $\rho(\hat{y}_i, \hat{y}_j)$ is the Pearson correlation
 1539 coefficient between the predicted values \hat{y}_i and \hat{y}_j , and $\rho(y_i, y_j)$ is the Pearson correlation coefficient
 1540 between the true values y_i and y_j .
 1541

1542 F.6 FREQUENCY DOMAIN SIMILARITY

1544 To calculate the similarity between two time-series datasets in the frequency domain, we can use the
 1545 Fourier Transform to convert both datasets from the time domain to the frequency domain and then
 1546 compare their frequency components. The steps are stated as following:
 1547

- 1548 **Fourier Transform:** Apply the Fourier Transform to each time series to obtain the am-
 1549 plitude and phase spectra. Let $A_X(f)$ and $\theta_X(f)$ be the amplitude and phase of the first
 1550 time-series data across frequencies f . Similarly, $A_Y(f)$ and $\theta_Y(f)$ represent the amplitude
 1551 and phase of the second time series.
- 1552 **Amplitude Cosine Similarity:** Define the cosine similarity for the amplitude spectra of
 1553 the two time-series datasets as follows:

$$1554 \quad S_{\text{amp}} = \frac{\sum_{f=1}^N A_X(f) \cdot A_Y(f)}{\sqrt{\sum_{f=1}^N A_X(f)^2} \cdot \sqrt{\sum_{f=1}^N A_Y(f)^2}} \quad (45)$$

1557 where N is the number of frequency components. This metric evaluates the similarity in
 1558 amplitude between the two datasets.
 1559

- 1560 **Phase Cosine Similarity:** Define the cosine similarity for the phase spectra by converting
 1561 the phase angles into their respective sine and cosine components:
 1562

$$1563 \quad S_{\text{phase}} = \frac{\sum_{f=1}^N (\cos(\theta_X(f)) \cdot \cos(\theta_Y(f)) + \sin(\theta_X(f)) \cdot \sin(\theta_Y(f)))}{\sqrt{\sum_{f=1}^N (\cos(\theta_X(f))^2 + \sin(\theta_X(f))^2)} \cdot \sqrt{\sum_{f=1}^N (\cos(\theta_Y(f))^2 + \sin(\theta_Y(f))^2)}} \quad (46)$$

1564 This metric evaluates the alignment of phase angles between the two time series.
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Table 11: Supervised learning pipeline with loss weights.

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4. **Combined Frequency Domain Similarity:** Finally, define the combined frequency domain similarity S_{freq} using a weighted sum of the amplitude and phase similarities:

$$S_{\text{freq}} = \alpha S_{\text{amp}} + \beta S_{\text{phase}} \quad (47)$$

where α and β are weights that can be adjusted based on the relative importance of amplitude and phase similarity. This combined metric S_{freq} captures both amplitude and phase alignment, offering a comprehensive measure of similarity between the two time-series datasets in the frequency domain. The loss for Frequency Domain Similarity (FDS), L_{FDS} , is $1 - S_{\text{freq}}$.

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F.7 SUPERVISED LEARNING PIPELINE

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$$L_{\text{task},i} = a_1 L_{\text{MSE}} + a_2 L_{\text{RCS}} + a_3 L_{\text{CRS}} + a_4 L_{\text{FDS}}, \quad (48)$$

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where a_1, a_2, a_3, a_4 denote the respective weights of each loss component.

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In Stage 1, the parameters of the foundation model, initialized from the self-supervised phase, are frozen, with only the task-specific parameters being updated. This allows the model to quickly adapt to a reasonable accuracy range, using a learning rate of 0.1, while prioritizing the MSE loss function.

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In Stage 2, both the foundation model and task-specific models are jointly trained, with the learning rate gradually decaying from 0.1 to 0.01. This stage aims to improve prediction accuracy and gradually bring it closer to optimal performance. The loss weights are adjusted to strike a balanced consideration of the different loss functions.

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In Stages 3, 4, and 5, the task-specific parameters are frozen, and the foundation model is further refined to enhance generalization capability. The learning rate is progressively reduced to 0.005, 0.003, and 0.001, respectively. During these stages, the loss weights are adjusted to refine model performance. In Stage 3, the focus is on improving accuracy with minimal adjustments to the consistency and rationality losses. In Stages 4 and 5, the loss weights are updated to place greater emphasis on L_{RCS} , promoting improved rationality and domain-specific reasoning.

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G BASELINE METHODS SELECTION

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To evaluate the performance of HGTFT across zero-shot and few-shot forecasting tasks, we compare it against diverse baselines, including classic models (No LMs), time-series large models (Time LMs), and large language model-based methods (LLMs). These methods differ in how they handle input modalities such as time-series (TS), static metadata (Static), and graph structure (Graph), as

1620 detailed in Table 12. For each method, we selected the most capable open-source version available
 1621 to ensure a fair comparison.
 1622

1623 Below, we provide an overview of the selected baseline methods and their respective adaptations to
 1624 our setting:

- 1625 • **LSTM** and **Autoformer**: Forecast each variable independently, without incorporating
 1626 static or relational information. Their outputs are aggregated through post-processing to
 1627 construct full multivariate predictions.
- 1628 • **TFT**: Combines multivariate time-series data with static features to perform object-level
 1629 forecasting. It supports variable selection and interpretable attention mechanisms but does
 1630 not model inter-object dependencies.
- 1631 • **HTGNN**: Utilizes graph-structured time-series inputs, leveraging the relationships between
 1632 objects to perform dynamic variable forecasting in a heterogeneous setting.
- 1633 • **STD-MAE**: Utilizes graph-structured time-series inputs in a homogeneous setting, where
 1634 the system is decomposed into multiple homogeneous subgraphs. Each subgraph is mod-
 1635 eled independently to capture localized spatial-temporal patterns, and the predictions are
 1636 subsequently aggregated to form the overall system-level forecast.
- 1637 • **TimesFM** and **MOIRAI**: Encode each object type’s time-series data sequentially, forecast-
 1638 ing each variable independently. These models do not utilize static or graph information;
 1639 instead, multivariate predictions are obtained by batching univariate forecasts.
- 1640 • **LLMTime** and **Time-LLM**: Process multiple object instances simultaneously using only
 1641 time-series data. These LLM-based models do not account for static metadata or inter-
 1642 instance relationships, but benefit from large-scale pretraining and context-aware genera-
 1643 tion.

1645 Table 12: Baseline methods summary.

1647 Method	1648 Input Type	1649 Category	1650 Model Version
1648 LSTM	1649 TS	1650 No LM	1651 —
1649 Autoformer	1650 TS	1651 No LM	1652 —
1650 TFT	1651 TS, Static	1652 No LM	1653 —
1651 HTGNN	1652 TS, Graph	1653 No LM	1654 —
1652 STD-MAE	1653 TS, Graph	1654 No LM	1655 —
1653 TimesFM	1654 TS	1655 Time LM	1656 200M
1654 MOIRAI	1655 TS	1656 Time LM	1657 1.1-R-large
1655 LLMTime	1656 TS	1657 LLM	1658 LLaMA-2 70B
1656 Time-LLM	1657 TS	1658 LLM	1659 LLaMA 7B

1656 In recent years, there has been a large number of work focusing on spatial-temporal forecasting
 1657 in relatively simple settings involving homogeneous object types and graph structures. Although
 1658 these works differ from the problem definition and setting in our study, we include several widely
 1659 recognized spatial-temporal forecasting algorithms from the past 4 years for a more comprehensive
 1660 comparison. We evaluate their performance on four standard datasets: PEMS04, PEMS08, COVID-
 1661 19 (JHU), and COVID-19 (NYT). As shown in Table 13, while recent methods continue to make
 1662 marginal improvements in these benchmarks, the performance gap is narrowing. This highlights a
 1663 critical limitation: the lack of methods and datasets capable of handling more complex scenarios.
 1664 Addressing this gap is the primary motivation of our work.

1665 We also evaluate our method and selected baselines on commonly used standard time-series datasets,
 1666 including ETT, Weather, Electricity, Traffic and ILI. Although these datasets are primarily bench-
 1667 marks for purely data-driven forecasting and are not the main focus of our study, our method
 1668 achieves performance comparable to state-of-the-art models (Table 14).

1670 H NORMALIZATION METHODS

1671 To evaluate the effectiveness of different normalization methods, using MSE directly on normalized
 1672 data is not appropriate, as each method applies a unique scaling to the variables, which would

1674

1675 Table 13: Performance comparison on PEMS04, PEMS08, COVID-19 (JHU), COVID-19 (NYT)
1676 datasets. Best results are in **bold**, second best are underlined.

Model	PEMS04				PEMS08				COVID-19 (JHU)		COVID-19 (NYT)	
	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
LSTM Hochreiter (1997)	32.48 \pm 0.38	55.51 \pm 0.74	24.98 \pm 0.38	41.71 \pm 0.43	122.42 \pm 1.41	232.11 \pm 3.51	70.59 \pm 0.85	139.18 \pm 1.71				
Autoformer Wu et al. (2021)	32.39 \pm 0.43	53.19 \pm 0.75	25.56 \pm 0.34	41.65 \pm 0.44	115.77 \pm 1.08	198.67 \pm 2.48	62.37 \pm 0.76	133.35 \pm 1.68				
TFT Lim et al. (2021)	31.32 \pm 0.35	48.37 \pm 0.67	24.63 \pm 0.36	39.74 \pm 0.42	121.81 \pm 1.43	261.77 \pm 3.20	71.36 \pm 0.84	158.93 \pm 2.30				
STformer Grigsby et al. (2021)	31.69 \pm 0.48	55.70 \pm 0.71	24.91 \pm 0.44	43.23 \pm 0.59	72.42 \pm 0.90	166.86 \pm 2.13	49.63 \pm 0.64	123.01 \pm 1.51				
TimesFM Das et al. (2023)	32.57 \pm 0.43	55.94 \pm 0.68	23.93 \pm 0.38	42.41 \pm 0.56	99.75 \pm 1.10	216.63 \pm 2.99	57.07 \pm 0.72	113.45 \pm 1.62				
MOIRAI Woo et al. (2024)	33.31 \pm 0.45	55.51 \pm 0.72	24.03 \pm 0.32	42.49 \pm 0.56	105.74 \pm 1.26	234.24 \pm 2.70	81.05 \pm 0.96	134.57 \pm 1.94				
LLMTime Grudev et al. (2024)	33.69 \pm 0.52	52.49 \pm 0.76	26.68 \pm 0.40	43.94 \pm 0.44	115.37 \pm 1.22	216.74 \pm 2.71	72.24 \pm 0.90	157.31 \pm 1.87				
Time-LLM Jin et al. (2023)	32.23 \pm 0.40	52.18 \pm 0.67	27.74 \pm 0.48	40.01 \pm 0.44	95.01 \pm 1.13	201.14 \pm 2.68	83.17 \pm 1.01	146.45 \pm 2.02				
ASTGCN Guo et al. (2019)	23.46 \pm 0.27	34.88 \pm 0.59	17.91 \pm 0.31	28.80 \pm 0.47	58.10 \pm 0.61	109.14 \pm 1.91	33.71 \pm 0.48	93.55 \pm 1.37				
STGCN Han et al. (2020)	21.72 \pm 0.34	34.61 \pm 0.57	18.73 \pm 0.35	28.05 \pm 0.50	52.94 \pm 0.63	110.63 \pm 1.68	37.25 \pm 0.44	88.62 \pm 1.27				
STGCN Song et al. (2020)	21.26 \pm 0.28	34.42 \pm 0.50	17.88 \pm 0.32	27.45 \pm 0.45	53.19 \pm 0.55	111.51 \pm 1.65	34.58 \pm 0.46	89.42 \pm 1.31				
STFGNN Li & Zhu (2021)	19.24 \pm 0.25	31.18 \pm 0.55	16.76 \pm 0.29	25.74 \pm 0.44	51.45 \pm 0.56	101.48 \pm 1.72	33.00 \pm 0.48	82.17 \pm 1.38				
STGODE Fang et al. (2021)	21.63 \pm 0.31	33.30 \pm 0.49	16.14 \pm 0.31	25.46 \pm 0.45	56.84 \pm 0.66	106.01 \pm 1.66	32.09 \pm 0.44	81.72 \pm 1.33				
STNorm Deng et al. (2021)	19.07 \pm 0.28	31.91 \pm 0.52	15.05 \pm 0.27	25.64 \pm 0.41	46.68 \pm 0.53	99.34 \pm 1.58	30.59 \pm 0.40	80.32 \pm 1.21				
DSTAGNN Lan et al. (2022)	19.87 \pm 0.33	30.80 \pm 0.53	15.93 \pm 0.32	24.53 \pm 0.38	50.45 \pm 0.57	100.51 \pm 1.57	31.49 \pm 0.46	76.50 \pm 1.31				
HTGNN Fan et al. (2022)	21.01 \pm 0.37	36.44 \pm 0.56	18.22 \pm 0.38	27.04 \pm 0.48	46.24 \pm 0.51	102.73 \pm 1.56	31.16 \pm 0.49	75.98 \pm 1.29				
PDFformer Jiang et al. (2023)	18.60 \pm 0.29	29.94 \pm 0.51	12.82 \pm 0.26	<u>22.62 \pm 0.35</u>	46.57 \pm 0.58	93.48 \pm 1.21	28.69 \pm 0.44	71.70 \pm 1.12				
STAformer Liu et al. (2023a)	18.62 \pm 0.30	29.65 \pm 0.44	12.97 \pm 0.26	24.21 \pm 0.35	47.58 \pm 0.59	96.87 \pm 1.34	24.62 \pm 0.41	<u>77.43 \pm 1.30</u>				
STD-MAE Gao et al. (2023)	17.85 \pm 0.27	<u>29.72 \pm 0.44</u>	13.67 \pm 0.28	22.62 \pm 0.36	47.75 \pm 0.60	92.62 \pm 1.27	26.69 \pm 0.45	72.98 \pm 1.21				
HGTFT (Ours)	19.94 \pm 0.34	32.16 \pm 0.54	16.43 \pm 0.34	25.08 \pm 0.41	41.54 \pm 0.44	94.38 \pm 1.20	<u>25.69 \pm 0.42</u>	65.64 \pm 1.04				

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1692 Table 14: Performance on standard time-series forecasting. ETT results are averaged over four
1693 subsets: ETT1, ETT2, ETTm1, and ETTm2. All models are trained or fine-tuned on 10% of each
1694 dataset. Best results are in **bold**, second best are underlined.

Dataset	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
ETT	MSE	0.589	0.465	<u>0.400</u>	0.455	0.480	0.421	0.391	0.575	0.408	0.425
	MAE	0.597	0.459	<u>0.412</u>	0.484	0.534	0.437	0.404	0.577	0.428	0.441
Weather	MSE	0.332	0.338	0.292	0.335	0.393	0.299	<u>0.259</u>	0.345	0.237	0.299
	MAE	0.363	0.382	0.311	0.366	0.383	0.321	<u>0.287</u>	0.412	0.264	0.334
Electricity	MSE	0.268	0.227	0.239	0.263	0.257	0.245	<u>0.192</u>	0.276	0.163	0.219
	MAE	0.365	0.338	0.318	0.358	0.394	0.330	<u>0.295</u>	0.390	0.264	0.317
Traffic	MSE	0.804	0.628	0.646	0.552	0.596	0.521	0.620	0.813	0.383	0.481
	MAE	0.509	0.379	0.398	0.389	0.433	0.344	<u>0.336</u>	0.498	0.264	0.350
ILI	MSE	4.753	3.125	3.343	4.365	3.894	2.435	<u>1.573</u>	2.868	1.437	2.432
	MAE	1.580	1.168	1.281	1.550	1.489	1.021	<u>0.935</u>	1.047	0.805	1.077

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1707 make MSE comparisons unfair. Instead, we reverse-normalize the variables before calculating the
1708 evaluation metrics to ensure a fair comparison of methods. However, the diverse ranges of the
1709 original variables after reverse normalization pose challenges in balancing weights across variables.
1710 To address this, we focus on key variables from the training tasks and compute statistical metrics
1711 for each individually. Their CV-RMSE values are listed in Table 15. As shown, the "Multi-Instance
1712 Normalization" method achieves more balanced prediction performance across various variables
1713 compared to the other methods.

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I TIME SERIES FORECASTING VISUALIZATION

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1717 To facilitate a qualitative analysis of the zero-shot and few-shot prediction results based on the
1718 BOSG-310M, we present time-series prediction plots for several key variables. The plots illustrate
1719 the forecasting performance of the proposed model on three critical objects: room, fan coil unit
1720 (FCU), and chiller system, under both zero-shot and fine-tuned conditions.

1721

1722 As shown in Figures 11 to 13, the zero-shot predictions capture the overall trends and patterns for
1723 each variable, although the accuracy of the predictions varies across different variables. While the
1724 model is able to predict the general shape of the curves, the degree of precision differs, reflecting
1725 the inherent challenges of making predictions without prior task-specific fine-tuning. In addition,
1726 we present the results of predictions following fine-tuning with one month of data. The improve-
1727 ments are evident, with significantly enhanced accuracy across all variables, particularly in capturing
1728 short-term dynamics. However, it is important to note that fine-tuning with a relatively short period
1729 of data, although it improves predictions for recent time period and conditions similar to those seen

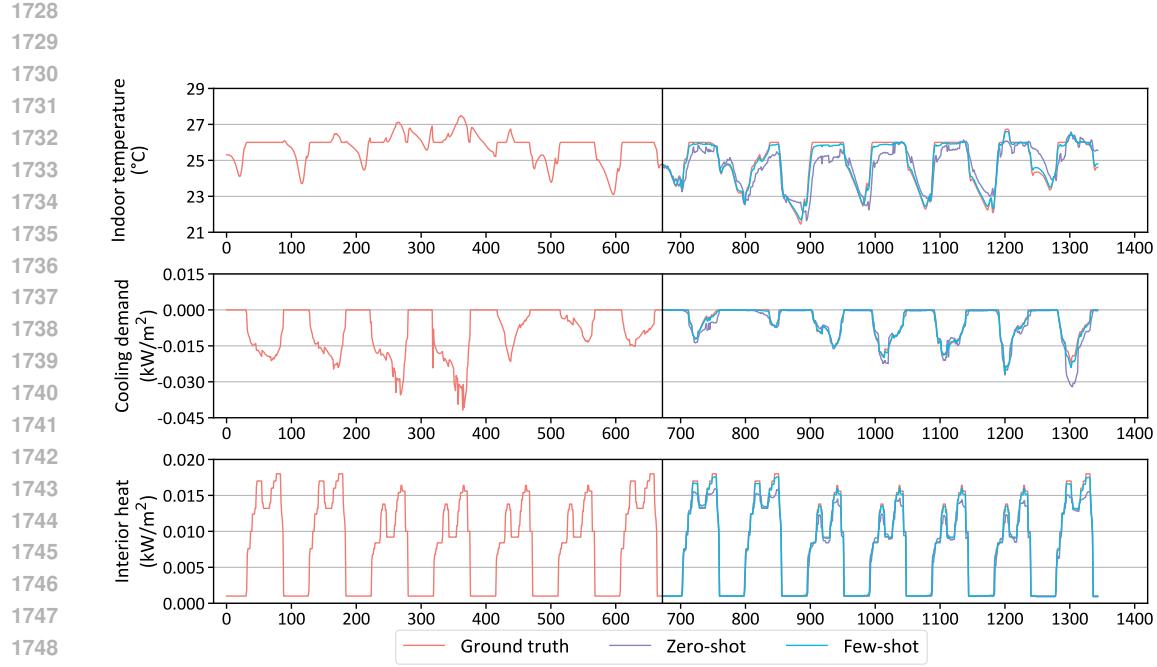


Figure 11: Visualization of time-series forecasting for key variables of a room object, predicting the next 7 days based on the past 7 days. Predictions include zero-shot and few-shot (with one month of fine-tuning data).

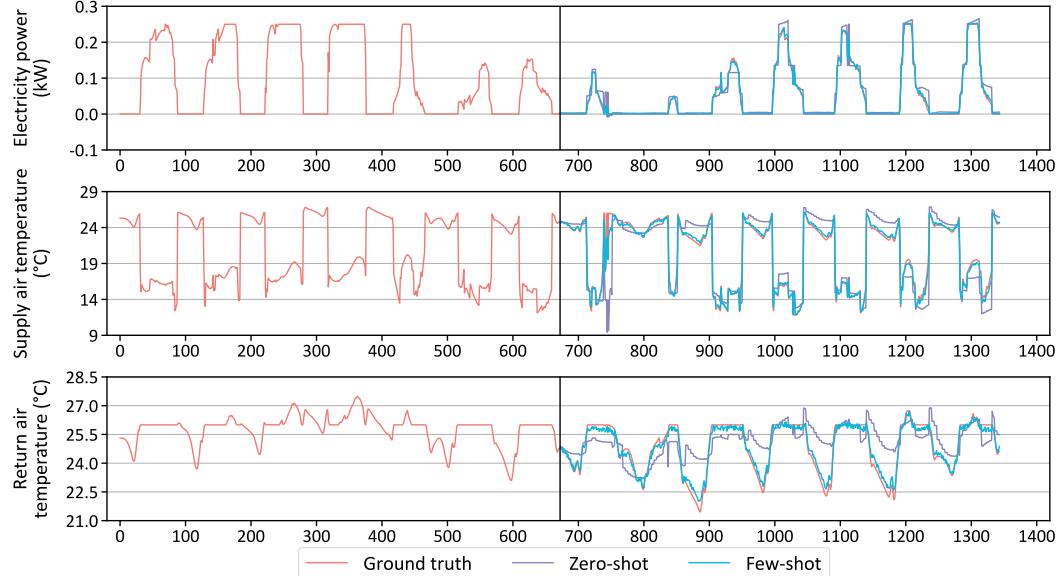
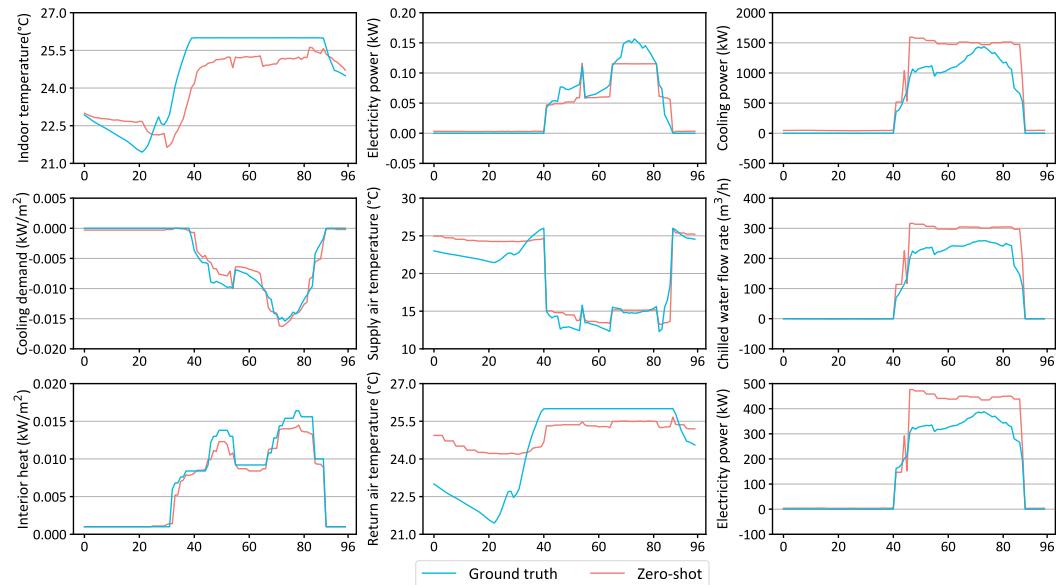
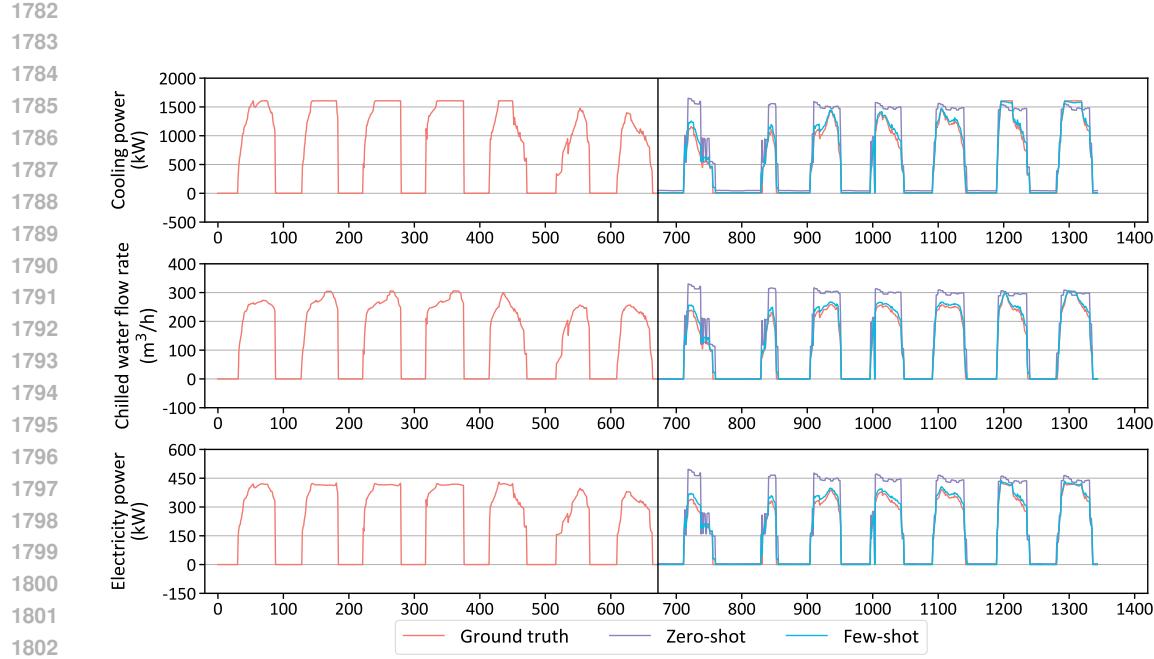


Figure 12: Visualization of time-series forecasting for key variables of a fan coil unit object, predicting the next 7 days based on the past 7 days. Predictions include zero-shot and few-shot (with one month of fine-tuning data).



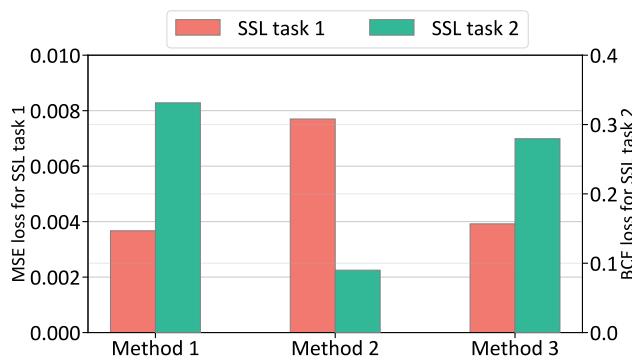


Figure 15: Comparison of three SSL training strategies: separate training, simultaneous training, and alternating training. Task 1 (masked time-series modeling) uses MSE loss, while Task 2 (masked edge modeling) uses BCE loss.

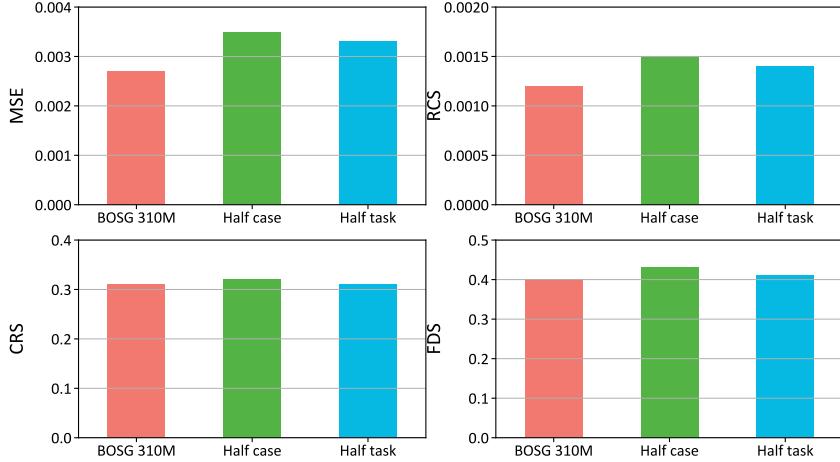


Figure 16: Performance comparison of the BOSG-310M model under three settings: full training, half the number of training cases, and half the number of tasks. Evaluation is based on multiple metrics.

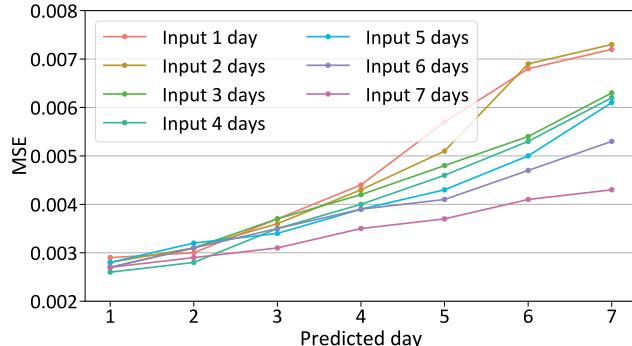


Figure 17: Prediction accuracy for daily rolling forecasts over a 7-day horizon. Each line represents a different input patch length, ranging from 1 to 7 days.

1890
 1891 Table 15: Comparison of average relative errors among various normalization methods. The average
 1892 error is calculated using the difference between the maximum and minimum values of the actual data
 1893 for each instance variable as the base for CV-RMSE computation, excluding outlier instances such as
 1894 devices that have been continuously inactive. The average CV-RMSE is computed for all instances
 1895 of the same variable type.

Object Type	Typical Variable	Normalization Method		
		Min-Max	Z-score	Multi-Instance
Room	Indoor Temperature	2.1%	2.0%	2.9%
	CO ₂	2.4%	2.5%	2.6%
Chiller	Chilled Water Supply Temperature	7.9%	8.1%	6.8%
	Chilled Water Return Temperature	16.3%	14.6%	11.2%
	Chilled Water Flow Rate	45.6%	33.9%	16.8%
Chilled Water Pump	Operating Power	34.6%	33.7%	13.6%
	Flow Rate	39.9%	38.2%	12.8%
Cooling Water Pump	Operating Power	42.5%	42.6%	15.7%
	Flow Rate	45.5%	48.4%	16.1%
Cooling Tower	Leaving Tower Water Temperature	23.0%	24.3%	7.6%
	Water Flow Rate	35.3%	33.8%	15.5%
Fan Coil Unit	Supply Air Temperature	9.8%	9.2%	8.3%
	Return Air Temperature	5.7%	6.1%	3.6%
Supply Air Fan	Fresh Air Flow Rate	41.1%	40.7%	21.3%

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 1914
 1915 in the fine-tuning phase, may deteriorate predictions for longer time horizons or when faced with
 1916 highly divergent operational scenarios. To investigate this, we also tested fine-tuning with longer
 1917 data windows (three months and six months), and found that, overall, predictions for extended time-
 1918 frames benefited from the use of larger fine-tuning datasets. This suggests that a more extended
 1919 fine-tuning period helps to mitigate overfitting and ensures better generalization for long-term pre-
 1920 dictions. However, a key challenge remains: how to fine-tune effectively with limited data while
 1921 avoiding overfitting and preserving our foundation model’s ability to learn the underlying physical
 1922 dynamics. This continues to be an area of significant research interest.

1923 We present daily profile curves for multiple dynamic variables of three different types of objects,
 1924 along with their zero-shot prediction results. In Figure 14, we observe that the temporal relation-
 1925 ships between the associated objects are effectively captured. In particular, the predictions for the
 1926 chiller system and FCU demonstrate that the forecasted surge in cooling power for the Chiller Sys-
 1927 tem at time point 42 closely aligns with the predicted supply air temperature of the FCU at the same
 1928 time. Similarly, the slight decrease in the room’s indoor temperature at time point 66 is well-aligned
 1929 with the small increase in the electricity power consumption of the FCU. In some instances, these
 1930 temporal relationships are even more pronounced in the predicted data than in the actual observa-
 1931 tions, highlighting the model’s capability to effectively capture interdependencies across various
 1932 components in the system.

J ADDITIONAL RESULTS

J.1 SELF-SUPERVISED LEARNING COMPARISON.

1939 We investigate the interaction between two self-supervised learning (SSL) tasks: masked time-series
 1940 modeling and masked edge prediction. Figure 15 compares three SSL training strategies: (1) training
 1941 each task independently, (2) simultaneous multi-task training, and (3) our proposed alternating task
 1942 training. Results show that while simultaneous training impairs the performance of the time-series
 1943 task, the alternating training method maintains low loss for both tasks, offering a better trade-off
 between sequence forecasting and structural relation modeling.

1944

1945 Table 16: Time-series forecasting results on the Hydronic Domain dataset under three settings:
1946 pretrained zero-shot, pretrained few-shot, and no pre-training. Best results are in **bold**, second-best
1947 are underlined.

Settings	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
Zero-shot	MSE	0.0066	0.0063	0.0046	0.0039	0.0038	0.0078	0.0073	0.0109	0.0093	0.0024
	RCS	0.0458	0.0260	0.0148	<u>0.0084</u>	0.0096	0.0315	0.0241	0.0346	0.0381	0.0013
	CRS	0.4812	0.5126	0.5457	<u>0.3328</u>	0.1949	0.3789	0.4730	0.6334	0.5462	<u>0.2239</u>
	FDS	0.4089	0.4989	0.5130	0.4619	0.4296	<u>0.3334</u>	0.3428	0.5223	0.4384	0.2922
Few-shot	MSE	0.0041	0.0035	0.0039	0.0032	<u>0.0030</u>	0.0036	0.0042	0.0062	0.0065	0.0017
	RCS	0.0344	0.0285	0.0136	0.0157	<u>0.0120</u>	0.0252	0.0195	0.0282	0.0285	0.0018
	CRS	0.3453	0.3587	0.3428	0.3079	<u>0.2340</u>	0.4287	0.3710	0.4202	0.3988	0.2280
	FDS	0.2643	0.2736	0.2916	0.2966	0.2960	<u>0.2176</u>	0.2499	0.3465	0.3052	0.1967
No Pretrain	MSE	0.0035	0.0039	<u>0.0031</u>	0.0036	0.0035	0.0043	0.0039	0.0062	0.0050	0.0019
	RCS	0.0326	0.0275	<u>0.0130</u>	0.0157	0.0190	0.0262	0.0197	0.0257	0.0264	0.0090
	CRS	0.3429	0.3486	0.3489	0.3069	<u>0.2361</u>	0.3648	0.3993	0.3957	0.3641	0.2322
	FDS	0.2928	0.3307	0.2556	0.3377	0.3092	<u>0.2493</u>	0.2310	0.4098	0.3020	0.2716

1959

1960 Table 17: Time-series forecasting results on the Airflow / HVAC Domain dataset under
1961 three settings: pretrained zero-shot, pretrained few-shot, and no pre-training. Best results are in
1962 **bold**, second-best are underlined.

Settings	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
Zero-shot	MSE	0.0054	0.0051	0.0038	<u>0.0033</u>	0.0035	0.0069	0.0060	0.0092	0.0079	0.0022
	RCS	0.0386	0.0227	0.0120	<u>0.0074</u>	0.0084	0.0278	0.0216	0.0306	0.0327	0.0010
	CRS	0.4028	0.4567	0.4422	0.2953	<u>0.1870</u>	0.3401	0.4071	0.5263	0.4750	0.1825
	FDS	0.3542	0.4452	0.4207	0.4023	0.3598	0.3084	<u>0.2545</u>	0.4547	0.3935	0.2499
Few-shot	MSE	0.0036	0.0026	0.0025	0.0028	<u>0.0024</u>	0.0029	0.0035	0.0051	0.0058	0.0015
	RCS	0.0283	0.0242	0.0112	0.0132	<u>0.0098</u>	0.0208	0.0168	0.0250	0.0235	0.0016
	CRS	0.2845	0.2987	0.2792	0.2579	<u>0.2035</u>	0.3656	0.3076	0.3395	0.3366	0.1992
	FDS	0.2170	0.2288	0.2376	0.2632	0.2480	<u>0.1941</u>	0.2055	0.3048	0.2593	0.1599
No Pretrain	MSE	0.0030	0.0033	<u>0.0025</u>	0.0029	0.0029	0.0036	0.0034	0.0051	0.0044	0.0018
	RCS	0.0263	0.0235	<u>0.0116</u>	0.0136	0.0162	0.0231	0.0158	0.0231	0.0213	0.0081
	CRS	0.2900	0.2797	0.3093	0.2711	<u>0.2180</u>	0.3053	0.3300	0.3403	0.3143	0.2029
	FDS	0.2479	0.2646	<u>0.2288</u>	0.2772	0.2534	0.2335	0.2446	0.3467	0.2692	0.2278

1975

1976 J.2 IMPACT OF TASK DIVERSITY AND DATA QUANTITY.

1977

1978 Leveraging SSL-pretrained weights, we adopt a sequential multi-task learning framework where
 1979 downstream tasks are optimized one after another. During training, the average task loss consis-
 1980 tently decreases across rounds, and the rate of change stabilizes, indicating convergence under the
 1981 serialized learning schedule. We further conduct ablation studies by halving the number of training
 1982 tasks and the number of training cases, respectively. As shown in Figure 16, both reductions lead
 1983 to moderate performance degradation, highlighting the importance of maintaining sufficient task
 1984 diversity and data coverage for robust generalization.

1985

1986 J.3 EFFECT OF INPUT PATCH LENGTH AND FORECASTING HORIZON.

1987

1988 We evaluate the model’s performance across different input and output durations, ranging from 1
 1989 to 7 days. Figure 17 presents the results of daily rolling forecasts, where the x-axis denotes the
 1990 target prediction day and the y-axis represents the corresponding MSE. Each curve corresponds to
 1991 a different input patch length. The results demonstrate that longer input sequences generally yield
 1992 improved accuracy, particularly for longer forecasting horizons.

1993

1994 J.4 MULTIPHYSICS DECOMPOSITION OF BUILDING SYSTEMS AND CROSS-DOMAIN
1995 FORECASTING RESULTS

1996

1997 Although our experiments are conducted in the building domain, this environment is inherently
 1998 multiphysics, consisting of several interacting sub-domains. A modern building comprises numer-
 1999 ous subsystems, each governed by distinct physical principles and involving heterogeneous object

1998

1999 Table 18: Time-series forecasting results on the Thermal Envelope Domain dataset under three
2000 settings: pretrained zero-shot, pretrained few-shot, and no pre-training. Best results are in **bold**,
2001 second-best are underlined.

Settings	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
Zero-shot	MSE	0.0125	0.0115	0.0085	<u>0.0076</u>	0.0078	0.0133	0.0130	0.0197	0.0186	0.0047
	RCS	0.0766	0.0495	0.0260	<u>0.0147</u>	0.0165	0.0616	0.0441	0.0603	0.0641	0.0022
	CRS	0.8594	0.8815	0.9421	<u>0.6541</u>	0.3721	0.7361	0.9285	1.0628	1.0600	<u>0.3925</u>
	FDS	0.7280	0.8681	0.8573	0.8716	0.7755	0.7171	<u>0.5979</u>	1.0270	0.7797	0.4871
Few-shot	MSE	0.0071	<u>0.0053</u>	0.0055	0.0056	0.0057	0.0070	0.0072	0.0104	0.0112	0.0029
	RCS	0.0656	0.0520	0.0251	0.0264	<u>0.0237</u>	0.0465	0.0367	0.0535	0.0482	0.0030
	CRS	0.6572	0.7032	0.6572	0.5697	0.3595	0.7748	0.7198	0.7372	0.7755	<u>0.4398</u>
	FDS	0.5228	0.5040	0.4999	0.5026	0.5515	0.4346	0.4334	0.6498	0.5284	0.3595
No Pretrain	MSE	0.0069	0.0066	<u>0.0054</u>	0.0055	0.0059	0.0079	0.0074	0.0112	0.0091	0.0033
	RCS	0.0600	0.0476	<u>0.0246</u>	0.0282	0.0320	0.0501	0.0369	0.0491	0.0491	0.0181
	CRS	0.6514	0.6875	0.5858	0.5388	<u>0.4794</u>	0.6665	0.6798	0.7483	0.6874	0.4525
	FDS	0.5668	0.5731	0.5054	0.6219	0.5356	<u>0.4585</u>	0.4299	0.7306	0.5622	0.5195

2013

2014 Table 19: Time-series forecasting results on the Refrigeration / Plant Domain dataset under three
2015 settings: pretrained zero-shot, pretrained few-shot, and no pre-training. Best results are in **bold**,
2016 second-best are underlined.

Settings	Metric	LSTM	Autoformer	TFT	HTGNN	STD-MAE	TimesFM	MOIRAI	LLMTime	Time-LLM	HGTFT (Ours)
Zero-shot	MSE	0.0102	0.0103	0.0074	<u>0.0067</u>	0.0075	0.0129	0.0126	0.0166	0.0163	0.0041
	RCS	0.0638	0.0448	0.0240	<u>0.0133</u>	0.0142	0.0551	0.0392	0.0537	0.0627	0.0020
	CRS	0.7575	0.7447	0.8515	0.5506	<u>0.3466</u>	0.6324	0.7807	0.9340	0.9009	<u>0.3393</u>
	FDS	0.6069	0.7913	0.7669	0.7728	0.6959	0.6359	<u>0.5280</u>	0.9461	0.7022	0.4505
Few-shot	MSE	0.0058	0.0047	<u>0.0044</u>	0.0047	0.0054	0.0062	0.0068	0.0084	0.0104	0.0027
	RCS	0.0560	0.0470	<u>0.0212</u>	0.0224	0.0199	0.0407	0.0337	0.0437	0.0407	0.0026
	CRS	0.6011	0.6550	0.6379	0.4607	0.3331	0.7084	0.6494	0.6822	0.6688	<u>0.3730</u>
	FDS	0.4422	0.4004	0.4726	0.4111	0.4893	0.3848	<u>0.3806</u>	0.5345	0.4996	0.3027
No Pretrain	MSE	0.0065	0.0057	<u>0.0045</u>	0.0050	0.0056	0.0069	0.0061	0.0094	0.0082	0.0031
	RCS	0.0556	0.0402	<u>0.0219</u>	0.0239	0.0273	0.0417	0.0313	0.0465	0.0433	0.0148
	CRS	0.5184	0.5905	0.5669	<u>0.4500</u>	0.3206	0.6289	0.6133	0.6671	0.6154	0.4650
	FDS	0.4610	0.4556	0.4504	0.5027	0.4599	<u>0.4176</u>	0.4249	0.6336	0.4770	0.4002

2028

2029 types. For clarity, we decompose the building system into four canonical sub-domains, as commonly
2030 recognized in building science and HVAC engineering.

2032

1. **Hydronic Domain Representative objects:** pumps, valves, tanks, hydronic loops, and water distribution networks
Governing physics: fluid dynamics, mass conservation, hydraulic balance
2. **Airflow / HVAC Domain Representative objects:** fans, ducts, dampers, variable air volume (VAV) components, heat exchangers
Governing physics: airflow mechanics, convective heat transfer, pressure–flow coupling
3. **Thermal Envelope Domain Representative objects:** walls, windows, shading devices, indoor zones, outdoor environment
Governing physics: conduction, radiation, heat storage, thermodynamic balance
4. **Refrigeration / Plant Domain Representative objects:** chillers, compressors, cooling towers, condensers, evaporators
Governing physics: vapor compression cycles, phase-change thermodynamics, energy balance

2047 These sub-domains collectively span multiple physical fields—including heat transfer, fluid flow,
2048 thermodynamics, mechanical work, and cyber–physical control—and exhibit cross-physics and
2049 cross-entity couplings. Demonstrating consistent forecasting performance across these heterogeneous
2050 components provides evidence that our framework is not restricted to a single physical mechanism
2051 but instead supports general **multi-entity, multi-variable forecasting with heterogeneous interactions**.

To further illustrate the generality of our approach, Tables 16 to 19 present the forecasting results across the four canonical sub-domains. The tables report evaluation metrics under three settings: **no pre-training**, **zero-shot performance**, and **few-shot fine-tuning**. This experiment follows a setup similar to that of Table 2 in the main text; however, it is conducted on the MBS dataset—using 50 randomly sampled buildings for testing—because the BTS dataset does not provide full or consistent coverage of all four sub-domains.

J.5 PERFORMANCE GAINS UNDER FIXED CAPACITY: JUSTIFYING MODEL COMPLEXITY

To assess when a complex architecture like HGTFT is justified compared to simpler models such as LSTM, we conduct two sets of comparative experiments:

1. **Scenario 1: Single-variable prediction** We use a univariate time-series forecasting dataset (ETT) to evaluate performance when only a single object type is involved.
2. **Scenario 2: Multi-object prediction** We construct a multi-object scenario containing two object types, each with three information channels (two dynamic variables and one static feature), to evaluate the benefits of HGTFT in capturing cross-entity interactions.

Table 20 and Table 21 present the corresponding forecasting results for the two scenarios. This experimental setup allows us to characterize practical trade-offs: while HGTFT and LSTM perform comparably in the single-object scenario with limited interactions, HGTFT demonstrates clear advantages in the multi-object setting, benefiting from its graph–temporal fusion and ability to model heterogeneous interactions. These results highlight the regimes where a more complex architecture is warranted, and when simpler models suffice.

Table 20: Comparison of HGTFT and LSTM under different model sizes (Scenario 1). FLOPS are reported in GFLOPS (10^9 FLOPS).

Parameters	Embedding Dim	HGTFT GFLOPS	HGTFT MSE	LSTM GFLOPS	LSTM MSE
0.8M	64	2,108	0.683	3,494	0.694
3M	128	16,369	0.521	13,753	0.607
10M	256	128,965	0.413	54,561	0.593

Table 21: Comparison of HGTFT and LSTM under different model sizes (Scenario 2). FLOPS are reported in GFLOPS (10^9 FLOPS).

Parameters	Embedding Dim	HGTFT GFLOPS	HGTFT MSE	LSTM GFLOPS	LSTM MSE
1.5M	64	4,217	0.0097	6,989	0.0162
6M	128	32,738	0.0056	27,505	0.0149
20M	256	257,930	0.0025	109,121	0.0144

K LIMITATIONS AND FUTURE WORK

Despite the promising results, this work still faces several limitations and open challenges:

1. **Generality across physical domains:** Our experiments focus on building operation systems as representative multiphysics environments, capturing rich interactions among thermal, hydraulic, and control processes. Future work can extend validation to other complex physical systems (e.g., energy grids or manufacturing processes) to further establish and demonstrate the generality of the proposed framework.
2. **Dataset coverage:** Public datasets for multiphysics forecasting remain limited. While our MBS dataset is larger and more comprehensive than prior resources, expanding it to include additional object types, physical processes, and control scenarios would further enhance its representativeness and support broader evaluation, fostering community progress.

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3. **Few-shot adaptation:** While few-shot finetuning yields clear benefits under short horizons
2107 or near-distribution conditions, its performance can degrade when forecasting over longer
2108 horizons or under substantial distributional shifts. Developing selective adaptation strate-
2109 gies that automatically identify which parameters or modules to adapt will be crucial for
2110 improving robustness in such settings.

2111
2112 Addressing these challenges will strengthen the robustness, flexibility, and scalability of multi-
2113 physics forecasting, paving the way for broader deployment in real-world complex physical systems.

2114 L LLM USAGE

2115
2116 Large language models (LLMs) were used solely for grammar correction and stylistic refinement
2117 of the manuscript. They did not contribute to research ideation, model design, data analysis, or
2118 experimental results. The authors take full responsibility for the scientific content of the paper.